# Convolutional neural network

2024年9月12日 22:23

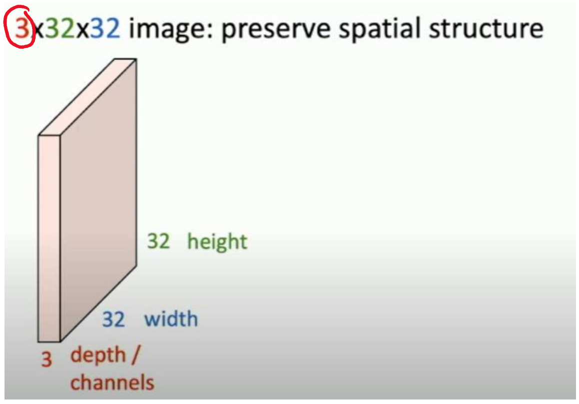
$\underbrace{32\times32}_{1024}\times3 \to 3072\times1$ fully connected

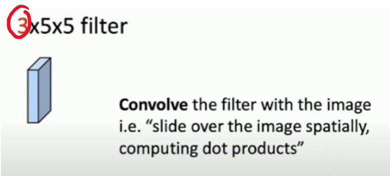
3 Components: Conv layers, Pooling, Normalization

1 Convolution Layers

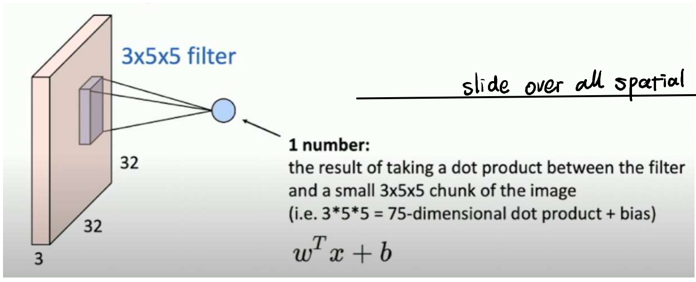
1)

1°



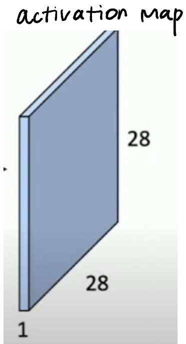




slide over all spatial loc



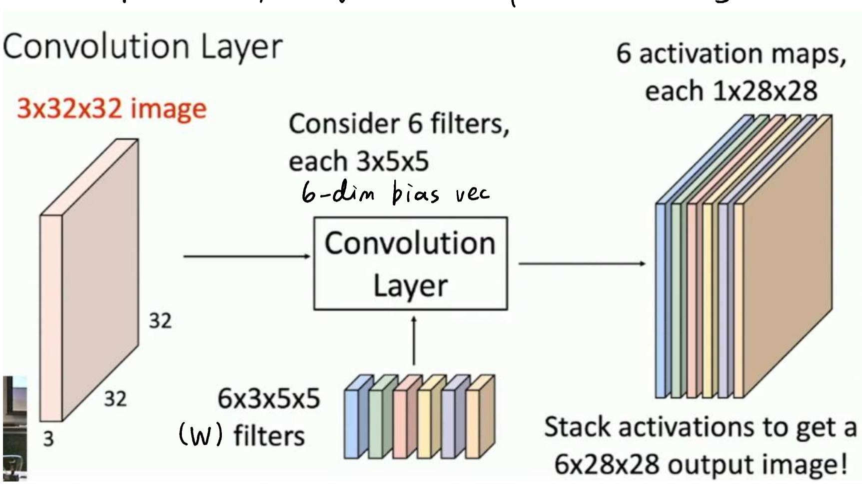

3×32×32

activation map




1×28×28

2° 可以有更多 filters，提取不同特征 → act- maps, stack them together.




6-dim bias vec

(W)

3° Batch of img $2\times3\times32\times32 \xrightarrow{6\times3\times5\times5 \text{ filters}} 2\times6\times28\times28$ Batch of outp

$N\times C_{in}\times H\times W$ $C_{out}\times C_{in}\times K_w\times K_h$ $N\times C_{out}\times H'\times W'$

3° Batch of img $2\times3\times32\times32$ $\xrightarrow{6\times3\times5\times5 \text{ filters}}$ $2\times6\times28\times28$ Batch of outp

$N\times C_{in}\times H\times W$ $\quad C_{out}\times C_{in}\times K_w\times K_h$ $\quad N\times C_{out}\times H'\times W'$

inp channels

mayb dif

2) Stacking Convolutions

1° inp → Conv → 1st hid → Conv → 2nd hid → Conv ...

$N\times3\times32\times32$ $\quad W_1: 6\times3\times5\times5$ $\quad N\times6\times28\times28$ $\quad W_2: 10\times6\times3\times3$ $\quad N\times10\times26\times26$

$b_1: 6$ $\quad 32-5+1$ $\quad b_2: 10$ $\quad 28-3+1$

oriented edges,
opposing colors

Stacking 2 conv → another conv $\quad y=W_2W_1x$ linear classifier

2° Size

① In: W $\quad 7\times7$

Fil: K $\quad 3\times3$

Out $W-K+1$ $\quad 5\times5$

Feature map shrink, lim #layers, sol:

② Padding, +0s around the input

In: W

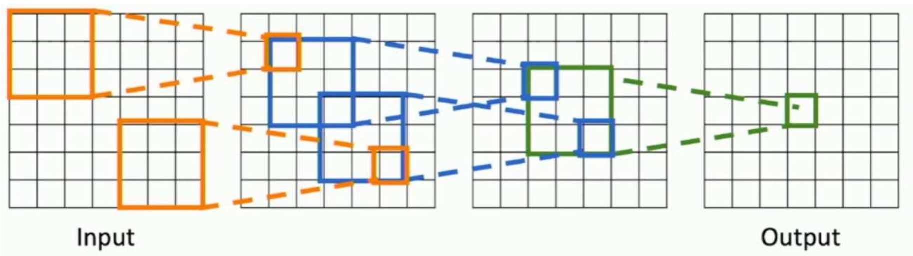
Fil: K

Pad: P

Out: $W-K+1+2P$

(common: $P=(K-1)/2$ to make size in = out)

3° Receptive Fields




| Layer | 3 | 2 | 1 | 0 |
|---|---|---|---|---|
| Field size | $1+3\times2=7$ | $1+2\times2=5$ | $1+1\times2=3$ | 1 |

L Layer receptive field size $= 1 + L\times(K-1)$

fil size

e.g. input $1000\times1000$, $K=3$, $L=?$

$1000 = 1 + L\times(3-1)$

$L = 999\div2 = 499.5$

Large imgs need many layers for each outps to "see" the whole img, sol:
(gloabal context)

4° Downsample inside the network

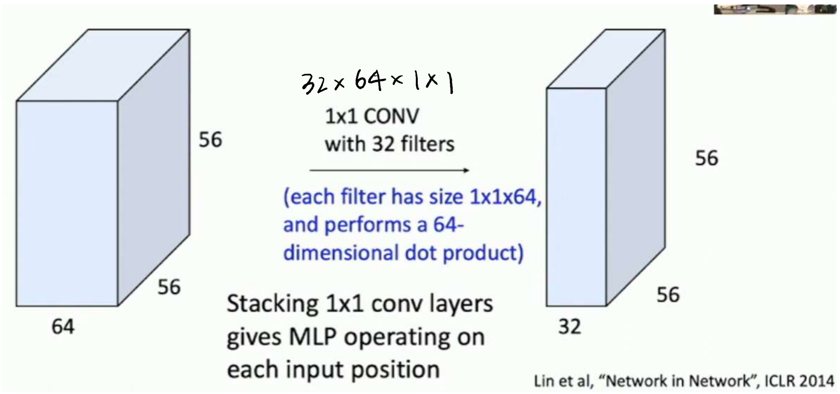
Controlling the stride: indirectly downsampling ∵ fewer data pt are being processed.

① Strided conv:

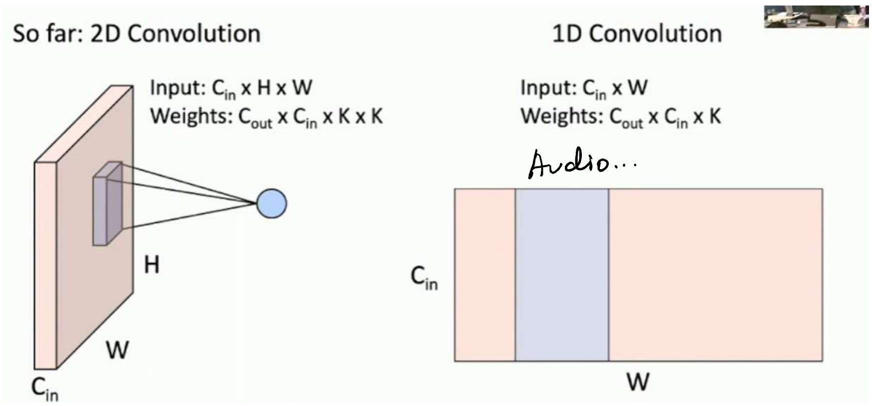
| | | |
|---|---|---|
| In | W | $3 \times 32 \times 32$ |
| Fil | K | $10 \times 5 \times 5$ |
| Pad | P | 2 |
| Str | S | 1 |
| Out | $(W - K + 2P)/S + 1$ | $(32-5+2\times2)/1+1 = 32$   $10 \times 32 \times 32$ |

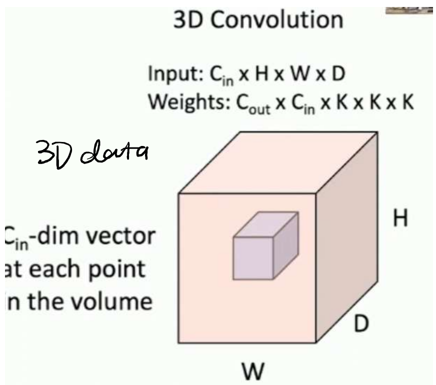
\# learnable para
- ① para/fl: $3 \times 5 \times 5 + 1 = 76$ (bias)
- ② × #fl: $76 \times 10 = 760$

\# multiply-add oper
- \# outs: $10 \times 32 \times 32 = 10240$
- inner product/out: $3 \times 5 \times 5 = 75$
- total: $75 \times 10240 = 768$ K

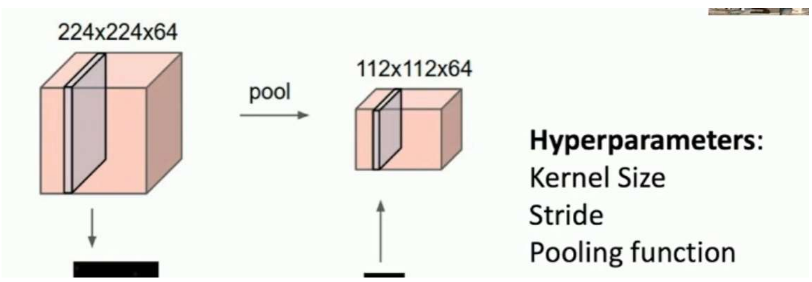


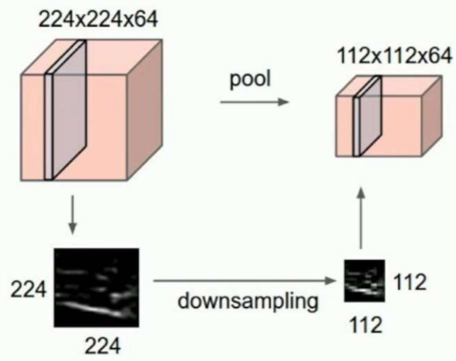

5° Other conv



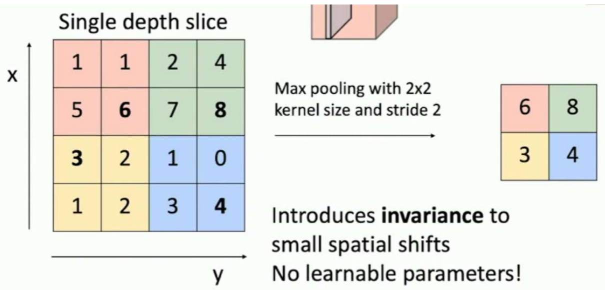

2 Pooling Layer: downsample 另一法






**Hyperparameters**:
Kernel Size
Stride
Pooling function

1° Max Pooling

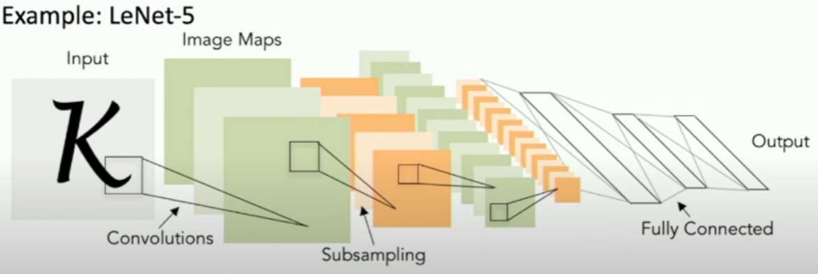



Introduces **invariance** to small spatial shifts
No learnable parameters!

## 3 Convolutional Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

Example: LeNet-5




| Layer | Output Size | Weight Size |
|---|---|---|
| Input | 1 x 28 x 28 | |
| Conv ($C_{out}$=20, K=5, P=2, S=1) | 20 x 28 x 28 | 20 x 1 x 5 x 5 |
| ReLU | 20 x 28 x 28 | |
| MaxPool(K=2, S=2) | 20 x 14 x 14 | |
| Conv ($C_{out}$=50, K=5, P=2, S=1) | 50 x 14 x 14 | 50 x 20 x 5 x 5 |
| ReLU | 50 x 14 x 14 | |
| MaxPool(K=2, S=2) | 50 x 7 x 7 | |
| Flatten | 2450 | |
| Linear (2450 -> 500) | 500 | 2450 x 500 |
| ReLU | 500 | |

Spatial size↓ (pl & strided)

# channels ↑ (total volume is preserved)

ReLU 不一定需要.

Deep NN: hard to train (converge), sol:

## 4 Normalization

1) Batch Nor in fully-connected

1° Idea: "Normalize" the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce "internal covariate shift", improves optimization

We can normalize a batch of activations like this:

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We can normalize a batch of activations like this:

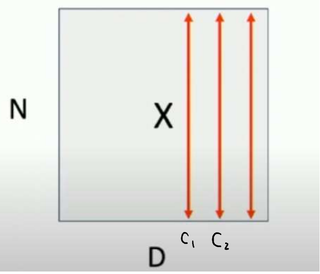
$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

This is a **differentiable function**, so we can use it as an operator in our networks and backprop through it!

convert inp → more standardized dist

**Input:** $x : N \times D$

batch dim $i$




# dim each vec $j$

per channel mean, shape D $\qquad \mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$ ①

~ std, shape D $\qquad \sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$ ②

Normalized X, $N \times D$ $\qquad \hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$ ③

What if $\mu = 0$, unit vec: too hard of a constrained, sol:

2° + Learnable scale & shift para: $\gamma, \beta$: D

Learning $\gamma = \sigma$, $\beta = \mu$, recover identity func.

① ② ③

$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$ ④ $\quad N \times D$

① ~ ③ Estimated depend on minibatch; × do this at test-time! sol:

3° Test-Time use $\mu_j$, $\sigma_j^2$ got from training

## Batch Normalization: Test-Time

**Input:** $x : N \times D$

**Learnable scale and shift parameters:**

$\gamma, \beta : D$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$ — Normalized x, Shape is N x D

$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$ — Output, Shape is N x D

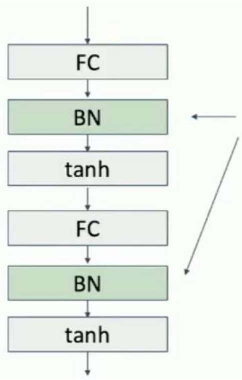
During testing batchnorm becomes a linear operator! Can be fused with the previous fully-connected or conv layer

v. 融合.

2) Batch N~ in Conv

| Batch Normalization for **fully-connected** networks | Batch Normalization for **convolutional** networks (Spatial Batchnorm, BatchNorm2D) |
|---|---|
| $\mathbf{x}: N \times D$ | $\mathbf{x}: N \times C \times H \times W$ |
| Normalize ↓ (N) | Normalize ↓ (N, H, W) |
| $\mu, \sigma: 1 \times D$ | $\mu, \sigma: 1 \times C \times 1 \times 1$ |
| $\gamma, \beta: 1 \times D$ | $\gamma, \beta: 1 \times C \times 1 \times 1$ |
| $y = \gamma(x-\mu)/\sigma+\beta$ | $y = \gamma(x-\mu)/\sigma+\beta$ |
| only on batch | on batch, spatial dims |

3)

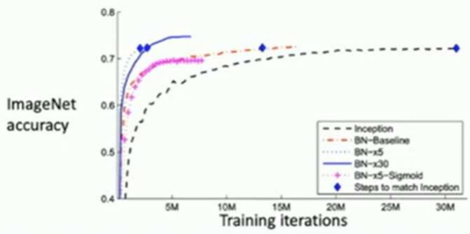



Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

⊕

- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!




⊖

- Not well-understood theoretically (yet)
- Behaves differently during training and testing: this is a very common source of bugs!

4) Others Norm