# linear classifier

2024年9月12日 11:45

streches pixels → col. × respect spatial struc of img

1 General case:

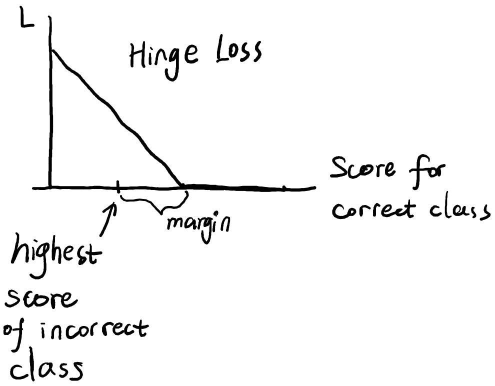
ds $\{(x_i, y_i)\}_{i=1}^{N}$

$x_i$: img; $y_i$: label index

Loss L: how good our classifier is. $f(x, W) = Wx$

$$L = \frac{1}{N} \sum_i L_i(\underbrace{f(x_i, W)}_{\text{classifier score}}, \underbrace{y_i}_{\text{lb index}})$$

2 Multiclass SVM Loss




The score of the correct class should be higher than all the other scores.

Scores $s = f(x_i, W)$

Loss
$$L = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{o.w.} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$j \neq y_i$: not inc corr label

e.g.

| | | | |
|---|---|---|---|
| cat | 3.2 | 1.3 | 2.2 |
| car | 5.1 | 4.9 | 2.5 |
| frog | -1.7 | 2.0 | -3.1 |

1° Compute the loss of a cat:

loop over all the incorrect classes

$L_{cat} = \max(0, 5.1-3.2+1)$
$\quad + \max(0, -1.7-3.2+1)$
$= 2.9 + 0$
$= 2.9$

$L_{car} = \max(0, 1.3-4.9+1)$
$\quad + \max(0, 2.0-4.9+1)$
$= 0$

$L_{frog} = \max(0, 2.2-(-3.1)+1)$
$\quad + \max(0, 2.5-(-3.1)+1)$
$= 6.3 + 6.6$
$= 12.9$

2° What happens to $L_{car}$ if the scores for car img change a little bit?

Still 0. ∵ $S_{car}$ a lot > any of other scores of incorrect classes

3° Min, max of loss?

min loss : 0 Correct cate has a s much higher than all incorrect cate.

max $\infty$ very ~ lower

4° If all the scores are random, what loss would we exp?

Supp: draw on scores from Gaussian dist. with very small $\sigma$,

$\Rightarrow$ s are small rand values

$\Rightarrow$ $E(s_j - s_{y_i}) \approx 0$

$\Rightarrow$ $\underbrace{\max(0, 0+1)}_{1 \text{ per incorrect cate}}$

$\Rightarrow$ $L_i = C - 1$

cate correct

5° If sum over all the classes inc $i = y_i$ ?

All L to be inflated by 1.

多了1个 term $\max(0, s_{y_i} - s_{y_i} + 1)$

6° What if the loss used a mean instead of a $\Sigma$

Preference of the weight matrix remains the same.

7° What if we use $L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$

Change scores in non-linear way

Cannot call it multi-class SVM loss, it shows dif pref on weight mat.

8° Sup some W with $L=0$. Unique?

No.

**Original W:**

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

**Using 2W instead:**

= max(0, 2.6 - 9.8 + 1)
+max(0, 4.0 - 9.8 + 1)
= max(0, -6.2) + max(0, -4.8)
= 0 + 0
= 0

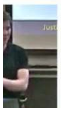
9° How should we choose between W and 2W if they both perform the same on the training data?

Other terms to eval pref on W.

## Regularization: Beyond Training Error

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)} + \underbrace{\lambda R(W)}$$

$\lambda$ = regularization strength (hyperparameter)



**Data loss**: Model predictions should match training data

**Regularization**: Prevent the model from doing *too well* on training data

**Simple examples**

pref using more fea

L2 regularization: $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization: $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2): $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

pref weight on single fea

**More complex**:

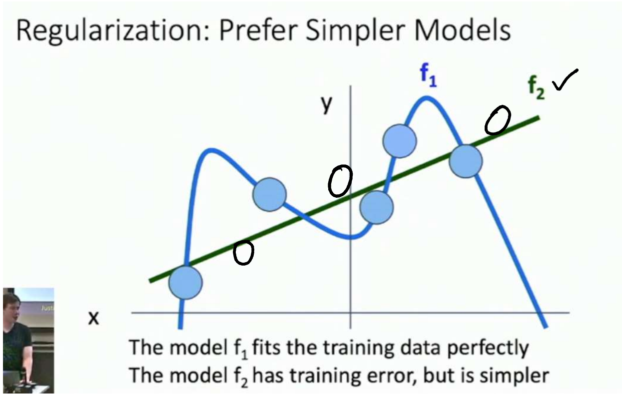
Dropout

Batch normalization

Cutout, Mixup, Stochastic depth, etc…

**Purpose of Regularization:**

- Express preferences in among models beyond "minimize training error"
- Avoid **overfitting**: Prefer simple models that generalize better
- Improve optimization by adding curvature

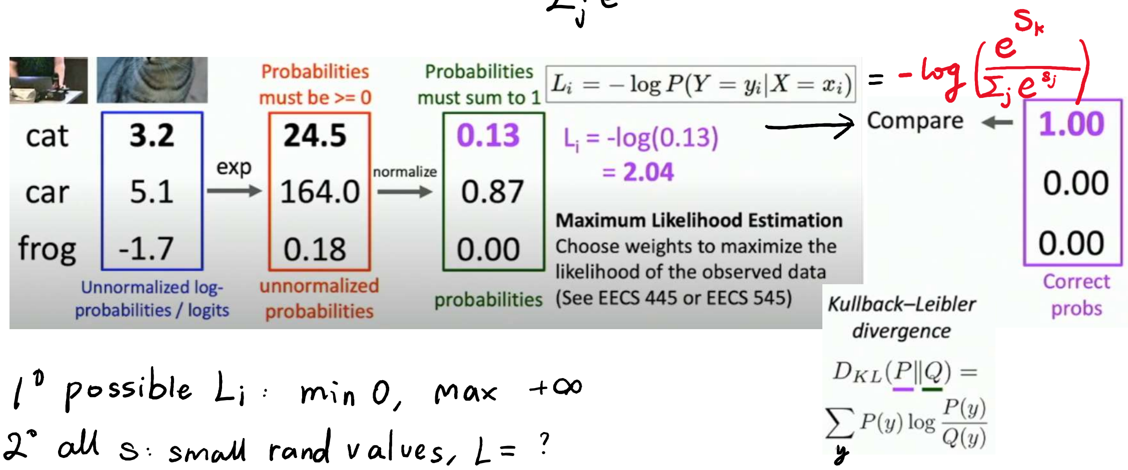



Regularization, u should usually use it.

3 Cross - Entropy Loss (Multinomial Logistic Regression)

raw scores → prob

$s = f(x_i; W)$ $\qquad P(Y=k \mid X=x_i) = \dfrac{e^{s_k}}{\sum_j e^{s_j}}$




1° possible $L_i$: min 0, max $+\infty$

2° all $s$: small rand values, $L = ?$

⇒ uniform in softmax

⇒ uniform prob $L = -\log(\frac{1}{C})$

① 训练开始时的 L 应该接近 $-\log(\frac{1}{C})$，
否则有大 bug

② 后续看到越来越↑于 $-\log(\frac{1}{C})$，
有大 bug. 你的 classifier 比 random 还差

4 scores [10, -2, 3]

[10, 9, 9]

[10, -100, -100],

& $y_i = 0$

1° cross entropy loss ? $>0$

SVM $=0$

2° slightly change s of the last data pt ?

CE 会把 correct class 推向 $+\infty$, 反之 $-\infty$

SVM stay the same

3°

**Q**: What happens to each loss if I double the score of the correct class from 10 to 20?

**A**: Cross-entropy loss will decrease, SVM loss still 0