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A Generalized Proportional Integral Controller for an Ankle Rehabilitation Machine based on an XY table

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Abstract

This paper addresses the design and implementation of a Generalized Proportional Integral (GPI) controller for ankle rehabilitation based on an XY table. The horizontal linear guide provide the abduction/adduction motion and the vertical linear guide provide the dorsiflexión/plantarflexión motion. The ankle rehabilitation machine is of Continuous Passive Motion kind and it be able to perform single and combined movements.

Some numerical and experimental results are included to illustrate the dynamic performance of the GPI controller.

1. Introduction

In recent years there has been considerable interest in developing rehabilitation machines by technology development companies, institutions and universities from around the world to fully rehabilitate the affected part (e.g., knee, ankle, hands, etc.).

These machines are used to begin motion as soon as possible following surgery, and hopefully alleviate the problem of post-operative stiffness. One advantage of these machines is their portability. The machine can be used in a hospital, but it can also be used in the home of the patient.

Rehabilitation is the process of restoration of skills by a person who has had an illness or injury so as to regain maximum self-sufficiency and function in a normal or as near normal manner as possible. The rehabilitation is beneficial to reduce spasticity, to increase the muscle mass and control the muscle movement. There are three types of exercises used in rehabilitation processes: passive, active and assisted. In passive exercises the nurse or therapist moves the joint without the patient's muscles being used. An medium mode is the assisted, which combines the efforts of patient and therapist. Active rehabilitation are the purposeful voluntary motion that are performed by the person himself, without resistance.

Devices called Continuous Passive Motion (CPM) shown in Fig. 1 are widely used in medical centers for rehabilitation purposes and others are in research process [1-12]. The CPM of ankle is used to perform smooth and control motions in rehabilitation therapies to help patients to perform repetitive movements in a well-defined interval and a given speed. Some of the ankle rehabilitation machine have been proposed based on the configuration of parallel robots whose mechanical structure is formed by a closed chain mechanism where the end effector is attached to the base by at least two independent kinematic chains [1-10].

Figure 1. CPM Machines for ankle rehabilitation.

GPI control, or control based on Integral Reconstructors, is a recent development in the literature about automatic control. Its main line of development rests within the finite dimensional linear systems case with some extensions to linear delay differential systems and nonlinear systems.

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GPI controller is proposed for the efficient rejection of a completely unknown perturbation input in a controlled mass system attached to an uncertain mass-spring-damper mechanical system, [13]. For other developments concerning GPI control in active vibration control of rotting machinery is presented in [14].

Fig. 2 shows the three movements in the ankle can be made: 1) dorsiflexion-plantarflexion, 2) inversion-eversion and 3) abduction-adduction. Table 1 shows the maximum intervals [5] for each movement.

TABLE 1: Ankle range of motion

Type of motion	Max. allowable motion
Dorsiflexion	20.3º to 29.8º
Plantarflexion	37.6º to 45.8º
Inversion	14.5º to 22.0º
Eversion	10.0º to 17.0º
Abduction	15.4º to 25.9º
Adduction	22.0º to 36.0º

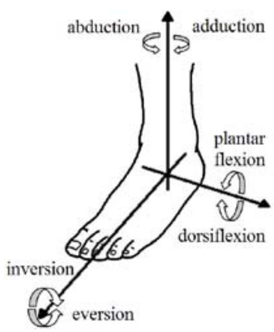


Figure 2. Foot-ankle movements.

Generalized Proportional Integral (GPI) control was introduced, in the context of Predictive Control of Differentially Flat systems in an article by Fliess and his coworkers [16].

Our GPI control design is an output feedback trajectory tracking controller for the controlled movable platform, of an ankle rehabilitation machine based on an XY table, which is made robust with respect to an constant perturbation input.

2. Ankle rehabilitation machine based on an XY table

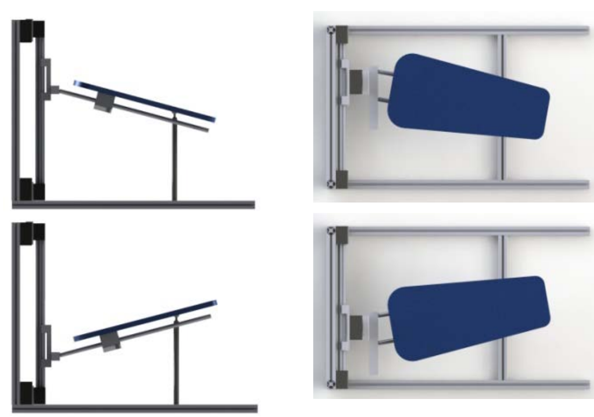
2.1 System Description

The virtual prototype of ankle rehabilitation machine (ARM) [9] capable to provide dorsi/plantarflexion and abduction/adduction movements is shown in Fig. 3. The ankle rehabilitation machine is based on parallel mechanism because they have the desirable characteristics of low inertia, high rigidity, compactness, greater portability, and precise resolution compared with serial robots. The strut and the linear actuator support the top movable platform. This strut is positioned in order to compensate the force of the foot weight.

The mechatronic design of the ankle rehabilitation machine is shown in Fig. 3. The ankle rehabilitation machine based on an XY table has 2 degrees of freedom (dof), and consists of: two actuators (gear CD motor) for the horizontal and vertical movements, a movable platforms to support the foot-ankle to be rehabilitated. Spherical joints are used to connect the movable platform with the XY table, see Figs. 3 and 4.

The machine has workspace enough to cover the required range of motion of the ankle movements of dorsi/plantarflexion and abduction/adduction, see Table 1.

Figure 3. Isometric view of the ankle rehabilitation machine.



a) b)

Figure 4. Ankle rehabilitation machine: a) dorsi/plantarflexion (front view) and b) abduction/adduction positions (up).

The ankle rehabilitation machines was designed with few custom components (6 elements), two standard linear guides and modular T-slot aluminum profile is used for the fixed structure.

2.2. Mathematical modelling

In this analysis only independent single movements are considered of abduction/adduction and dorsi/plantarflexion, see Fig. 4, which represent the movements of an XY table in the X and Y directions, respectively. The XY table consists of two linear guides, which provide the movements in the directions of the axes X (horizontal) and Y (vertical), Figs. 5 and 6. For X axis motion, the mass m_1 is considered which corresponds to the sum of the movable platform mass and the mass of the whole linear guide system for the Y axis. For the Y axis the mass m_2 is considered due only to the carriage. The Viscous damping is considered, b, in both linear guides between the carriage and the guide, see Fig. 5. F_x and F_y are the control forces for the motion of X and Y axes, respectively. The forces P_1 y P_2 are considered as constant disturbances due to interaction with the movable platform, W represents the foot weight.

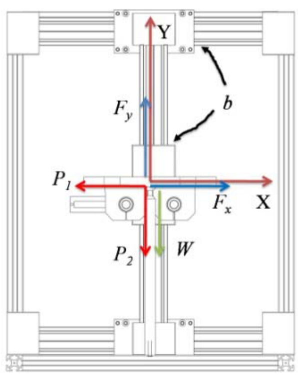


Figure 5. Free body diagram of the XY table.

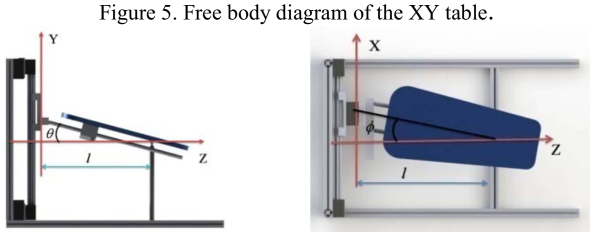


Figure 6. Angle θ and φ to denote the angular position of dorsi/plantarflexion and abduction/adduction, respectively.

The mathematical model governing the dynamics of the XY table can be obtained by applying Newton's second law, which is given by

$$\ddot{x}+\frac{b}{\left(m_1+m_2\right)}\dot{x}-\frac{F_x-P_1}{\left(m_1+m_2\right)}=0 \tag{1}$$

$$\ddot{y}+\frac{b}{m_2}\dot{y}-\frac{F_y+W-P_2}{m_2}=0 \tag{2}$$

From figure 6 we can get the relationships between the linear displacements (x, y) and the angular positions φ and θ, given by:

$$\tan\varphi=\frac{x}{l},\quad \tan\theta=\frac{y}{l} \tag{3}$$

2.3 GPI controller

To design a controller for position reference tracking, consider equation (1). Then, one can propose the following Generalized Proportional Integral (GPI) controller for asymptotic and robust tracking to the desired position trajectory $\theta^*(t)$ for the movable platform position and velocity, which employs only linear position measurements of the platform. For more details on GPI control see [13-16].

Consider the perturbed system as

$$\ddot{x}=u+\xi \tag{4}$$

were u is a new control input, given by

$$u=\frac{F_x}{m_1+m_2}-\frac{b}{m_1+m_2}\dot{x},\quad \xi=\frac{-P_1}{m_1+m_2} \tag{5}$$

We propose the following GPI controller for asymptotic and robust tracking to the desired trajectory planning $x_d(t)$ for the linear position on X axis:

$$\begin{aligned} u = \ddot{x}_d - k_2\left(\hat{\dot{x}}-\dot{x}_d\right)-k_1(x-x_d) \\ - k_0\int(x-x_d)dt \end{aligned} \tag{6}$$
$$F_x=u\left(m_1+m_2\right)+b\dot{x}$$

where $\hat{\dot{x}}$ is an integral reconstructor of the linear velocity of the carriage on X axis, which is given by

$$\hat{\dot{x}}=\frac{1}{m_1+m_2}\int_0^t F(\tau)\,d\tau-\frac{b}{m_1+m_2}x \tag{7}$$

The use of the GPI controller (6) yields the following closed-loop dynamics for the trajectory tracking error, $e=x-x_d$:

$$\dddot{e}+k_2\ddot{e}+k_1\dot{e}+k_0e=0 \tag{8}$$

Therefore, selecting the design parameters $\{k_0,k_1,k_2\}$ such that the associated characteristic polynomial for (8) be Hurwitz one guarantees that the error dynamics be globally asymptotically stable. The GPI controller (6) employs only carriage position measurement of the linear guide.

A trajectory tracking task was adopted to have the controlled movable platform position θ track by means of the linear position of the carriage, a Bézier polynomial smoothly interpolating between zero and a final position located at 0.1m from the initial rest position in approximately 8s.

Desired position trajectory is given by the following Bézier polynomial:

$$
\begin{aligned}
&x_d(t) = x_i + (x_f - x_i)\sigma(t, t_i, t_f)\mu_p^5,\\
&\sigma(t, t_i, t_f) = \gamma_1 - \gamma_2\mu_p + \gamma_3\mu_p^2 - \cdots + \gamma_6\mu_p^5,\\
&\mu_p = \frac{t - t_i}{t_f - t_i},
\end{aligned}
\tag{9}
$$

where $x_i = x_d(t_i)$ and $x_f = x_d(t_f)$ are initial and final desired positions, so that the basis of rehabilitation starts from an initial position and go to a final position with a smooth change, such that:

$$
x_d(t) = \begin{cases} 0 & 0 \le t < t_i \\ \sigma(t, t_i, t_f)x_f & t_i \le t < t_f. \\ x_f & t > t_f \end{cases} \tag{10}
$$

Parameters of the polynomial function $x_d(t)$ are γ_1=252, γ_2=1050, γ_3=1800, γ_4=1575, γ_5=700, γ_6=126.

3. Simulation Results

3.1. Using the mathematical model

Table 2 shows the simulation parameters obtained from the virtual prototype and used with the GPI control law obtained in (6) to tracking trajectories (9).

TABLE 2. Simulation parameters

b=0 Ns/m	Viscous damping coefficient
m_1=1.06Kg	Mass of the linear guide on X axis
m_2=0.275Kg	Mass of the linear guide on Y axis
P_1=5N	Disturbance force on X axis
P_2=5N	Disturbance force on Y axis

Fig. 7 shows the response for the real and desired linear position on the X axis and corresponding φ angle, and control force required for the movement of abduction using the mathematical model (1-2) and the PID controller (6). Note that the movement obtained is smooth from 0 to 0.1m, using the Bezier polynomial (9). The Therapist using an interface can program both the amplitude and the linear velocity required, based on the rehabilitation and improvement of the affected part. Fig. 8 shows the response for the dorsiflexion motion (Y axis). Also, it has a smooth tracking but exits an error between the real and desired trajectory due to the foot and movable platform weights.

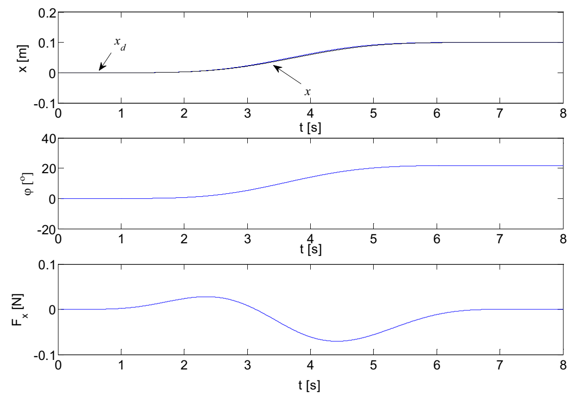


Figure 7. Closed loop simulation response of abduction motion for the controlled movable platform.

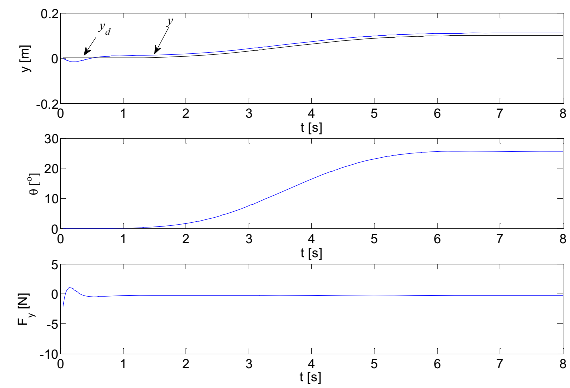


Figure 8. Closed loop simulation response of dorsiflexion motion for the controlled movable platform.

3.2. Using the virtual prototype

Some simulations were performed with the virtual prototype in MD ADAMS (Automatic Dynamic Analysis of Mechanical Systems) software to verify the performance of the GPI controller. Fig. 9 shows the reality model of the ankle rehabilitation machine simulated under the ADAMS environment.

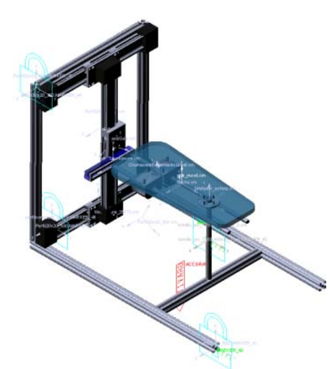


Figure 9. ARM based on an XY table in ADAMS software environment.

Fig. 10 shows the response for the real and desired angle, and force required for the movement of abduction, using the virtual prototype and the GPI controller (2). Note that the movement obtained is smooth from 0 to 0.1m, using the Bezier polynomial (4).

For this simulation only contact forces are considering in the strut-movable platform and linear guide coupling - movable platform. The gravity acceleration is not considered because the screw is self-locking and compensate the weights of the ankle and movable platform.

The control force applied (4) to carry the movable platform from the rest to the final desired position (0.1m), using the Bezier soft trajectory, is shown in Fig. 7.

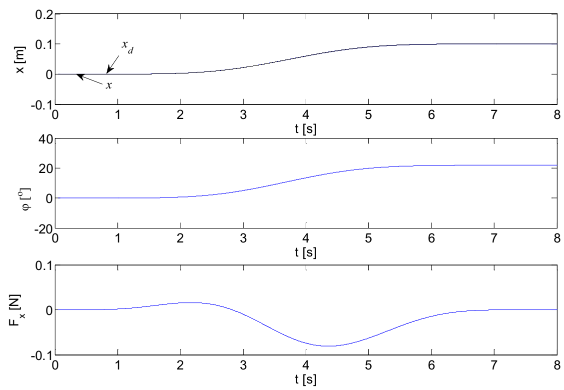


Figure 10. Closed loop simulation response of abduction motion for the controlled movable platform.

Figure 11 shows the real and desired linear movement on X axis from 0 to 0.1m. The control force required is minimal due to the self-locking force that is presented on the linear guide and is opposed to the foot and movable platform weights. The gravity is not considered.

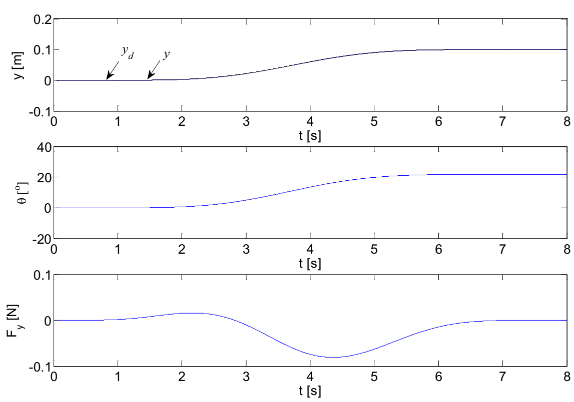


Figure 11. Closed loop simulation response of dorsiflexion motion using the mathematical model.

6. Experimental results using the physical prototype

The prototype of the ankle rehabilitation machine based on an XY table is shown in Fig. 11. The ARM is capable to provide the full range of dorsi/plantarflexion and abduction/adduction motions.

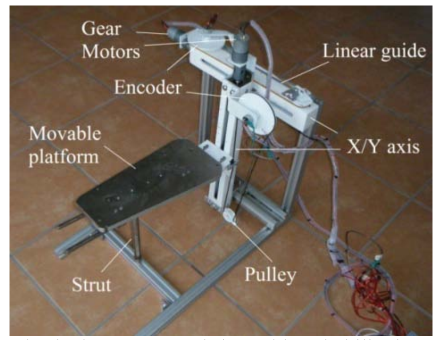


Fig. 11. Physical prototype of the ankle rehabilitation machine based on an XY table.

Fig. 12 shows the response for the real and desired linear position, and Force required for the movement of abduction. Fig. 13 shows the force required for the movement of abduction (Y axis) and is similar to the obtained in the virtual prototype, Fig. 11.

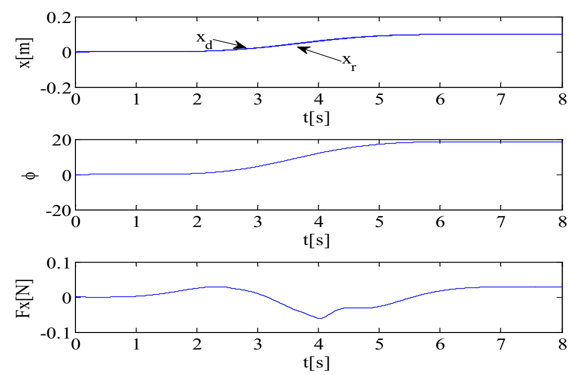


Figure 12. Closed loop simulation response of abduction motion using the physical prototype.

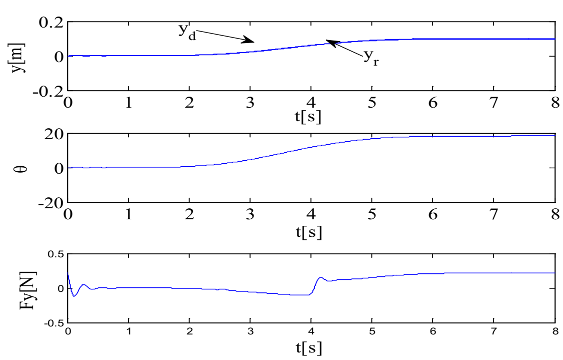


Figure 13. Experimental response of dorsiflexion motion using the physical prototype.

7. Conclusion

This paper presents the virtual prototype of an ankle rehabilitation machine based on an XY table that covers the full range of dorsi/plantarflexion and abduction/adduction motions. The ankle rehabilitation machine provides smooth and controlled movements of rehabilitation using a GPI controller. The simulation results are obtained from the mathematical model and the virtual prototype simulated under the ADAMS environment. These simulations show the effectiveness of the proposed controller. Also, some experimental results are presented showing a similar response to those obtained in the simulation results.

Future work will include the use of algebraic identification of some parameters of the system like the variable stiffness force in the ankle of the patients. Consequently, the rehabilitation machine will be evaluated through experiments with patients affected by ankle injuries or other sorts of impairments.

ACKNOWLEDGMENT

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