SQIsign

Krijn Reijnders
Crypto Working Group
June 21, 2024

Radboud University

SQIsign
SQIsign2

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SQIsign
SQIsign2
SQIsignHD

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SQIsign
SQIsign2
SQIsignHD
AprèsSQI

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SQIsign
SQIsign2
SQIsignHD
AprèsSQI
SQIsign2D-West
SQIsign2D-East

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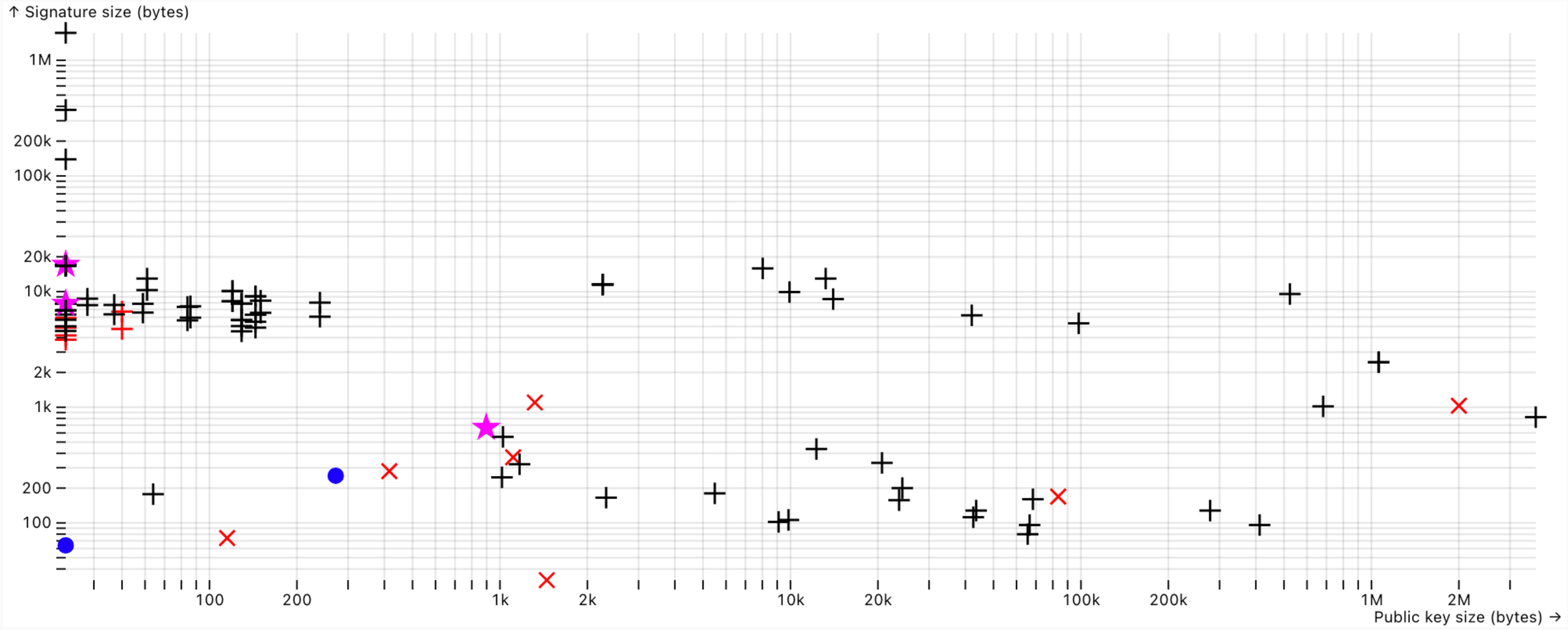
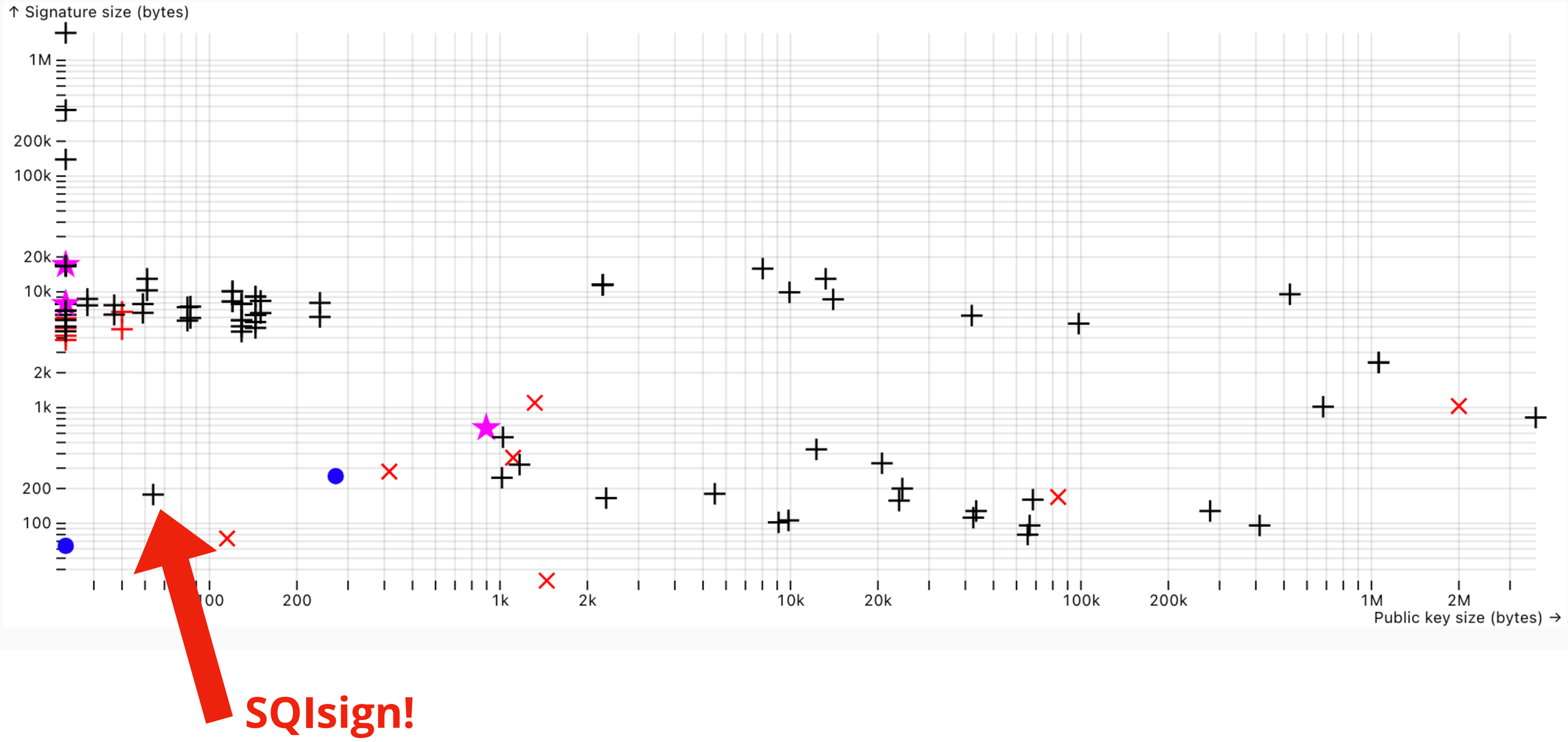
SQIsign
SQIsign2
SQIsignHD
AprèsSQI
SQIsign2D-West
SQIsign2D-East
SQIPrime

Krijn Reijnders
Crypto Working Group
June 21, 2024

(by Diego Aranha)


↑ Signature size (bytes)
1M
200k
100k
20k
10k
2k
1k
200
100
100
200
1k
2k
10k
20k
100k
200k
1M
2M
Public key size (bytes) →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1k
2k
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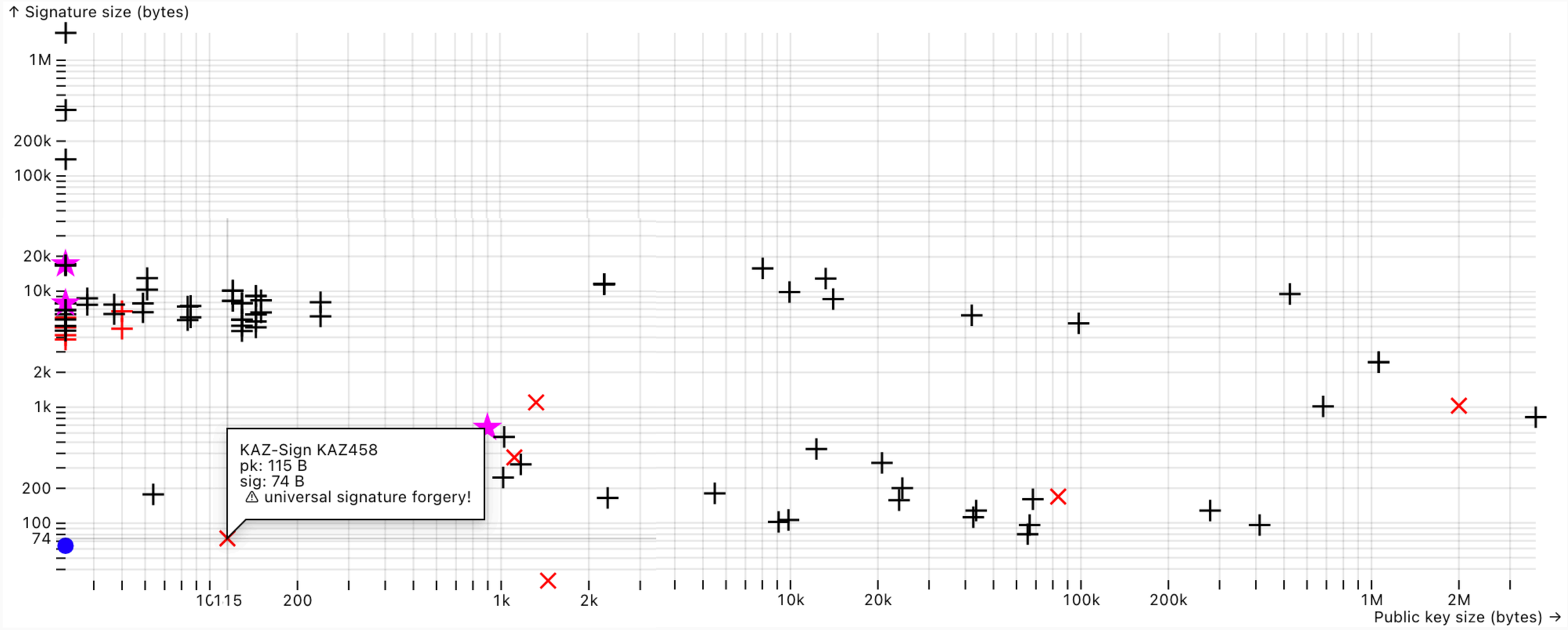
20k
100k
200k
1M
2M
Public key size (bytes) →
SQIsign!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1M
200k
100k
20k
10k
2k
1k
200
100
74
KAZ-Sign KAZ458
pk: 115 B
sig: 74 B
⚠ universal signature forgery!
100
115
200
1k
2k
10k
20k
100k
200k
1M
2M
Public key size (bytes) →

100,000,000,000
1,000,000,000
10,000,000
100,000
10,000
100,000
1,000,000
10,000,000
100,000,000
1,000,000,000
Signing
Verifying
SLH-DSA (SPHINCS+)
SQIsign
FuLeeca
Ascon-Sign
SPHINCS-alpha
Wave
QR-UOV
LESS
MiRitH
SLH-DSA (SPHINCS+)
Ascon-Sign
Raccoon
MOOM
PERK
SNOVA
SDitH
RSA
CROSS
ALTEQ
AIMer
HuFu
RYDE
Squirrels
Enhanced pqsigRM
Falcon
MAYO
VOX
TUOV
UOV
HAWK
eMLE-Sig 2.0
EdDSA


SQIsign!
100,000,000,000
1,000,000,000
10,000,000
100,000
Signing
10,000
100,000
1,000,000
10,000,000
100,000,000
1,000,000,000
Verifying
SQIsign
MiRitH
SLH-DSA (SPHINCS+)
Ascon-Sign
FuLeeca
SPHINCS-alpha
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LESS
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Falcon
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VOX
TUOV
UOV
HAWK
eMLE-Sig 2.0
EdDSA
CROSS
ALTEQ
AIMer

The Landscape of SQIsign

an overview in five acts

Krijn Reijnders
Crypto Working Group
June 21, 2024

Radboud University

PART 0: Maths

PART 1

SQIsign

WARNING!

- SQIsign is a **difficult** scheme, especially signing
- To keep this talk “down to earth”, I will **simplify** a lot
- This will increase clarity and intuition by being **hand-wavy**, at the cost of rigor

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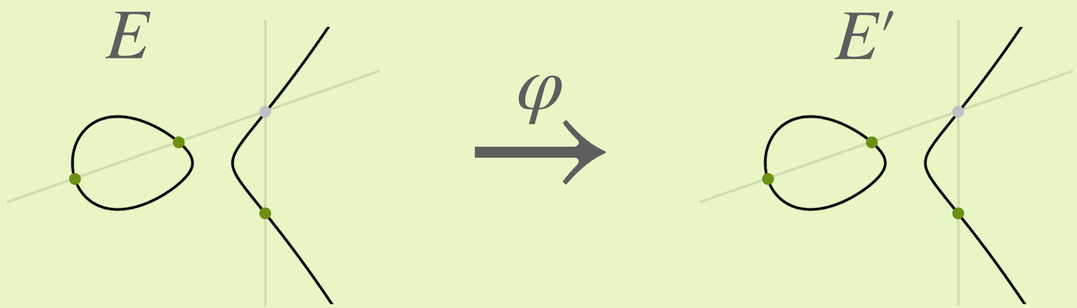
isogenies



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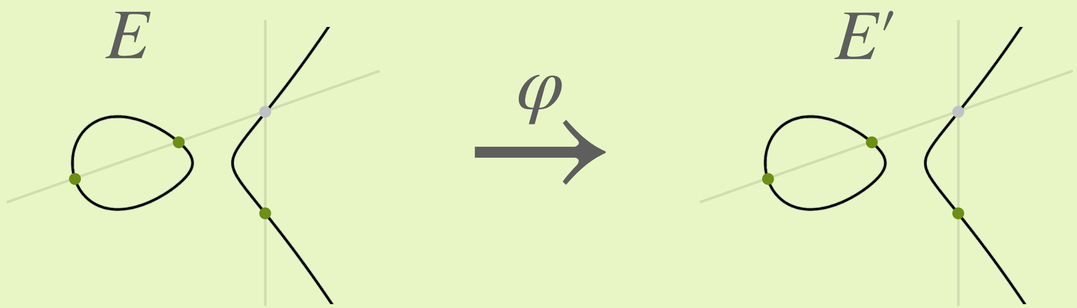

Isogeny

- "nice" map φ (group homomorphism) between elliptic curves $E \to E'$
- given by rational functions: a point $(x, y) \in E$ is mapped to $(f_1(x,y)/f_2(x,y) \ , \ g_1(x,y)/g_2(x,y))$
- size of $\ker \varphi$ is same as degree of φ!

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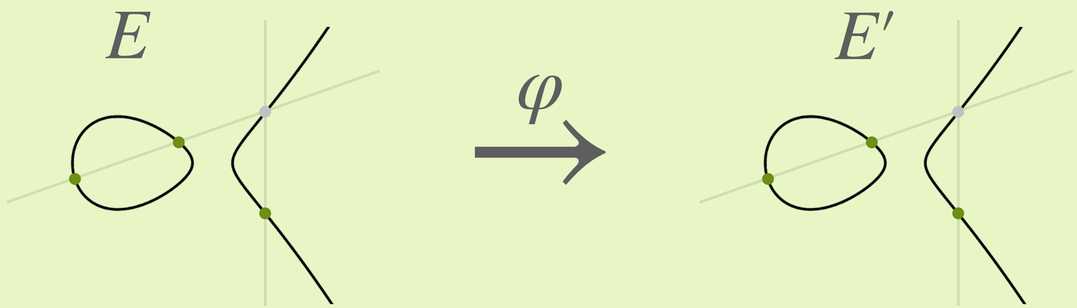

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toy example

$$E : y^2 = x^3 + x \xrightarrow{\varphi} E' : y^2 = x^3 + 5$$

$$(x, y) \mapsto \left(\frac{x^3 + x^2 + x + 2}{(x-5)^2}, \frac{y(x^3 - 4x^2 + 2)}{(x-5)^3} \right) \quad \text{over } \mathbb{F}_{11}$$

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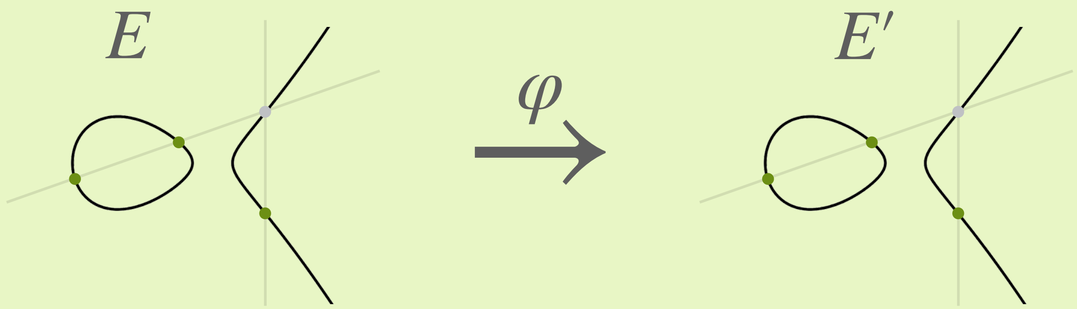

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Can check

- this is a group homomorphism: $\varphi(\mathcal{O}) = \mathcal{O}'$ and $\varphi(P + Q) = \varphi(P) + \varphi(Q)$
- kernel: $\varphi(P) = \mathcal{O}'$ when $P = \mathcal{O}$ or $x_P = 5$, so $P = (5,3)$ and $P = (5, -3)$
- so φ is of degree 3 and we can say E and E' are 3-isogenous

PART 1

SQIsign

Question

given E and E', can we find an isogeny $\varphi : E \to E'$?

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easy

easy to verify that *some isogeny* exists, e.g. that E and E' are **isogenous**

hard

actually giving an isogeny $\varphi : E \to E'$ or some way to compute this

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SQIsign

Question

given E and E', can we find an isogeny $\varphi : E \to E'$?

easy

easy to verify that *some isogeny* exists, e.g. that E and E' are **isogenous**

intermediate

what if we additionally know some points $P, Q \in E$ and their images $\varphi(P), \varphi(Q) \in E'$

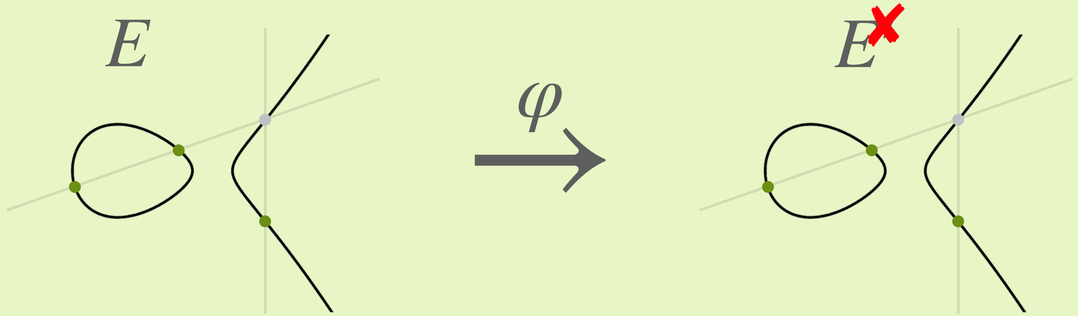
hard

actually giving an isogeny $\varphi : E \to E'$ or some way to compute this

PART 1

SQIsign

endomorphism



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PART 1

SQIsign

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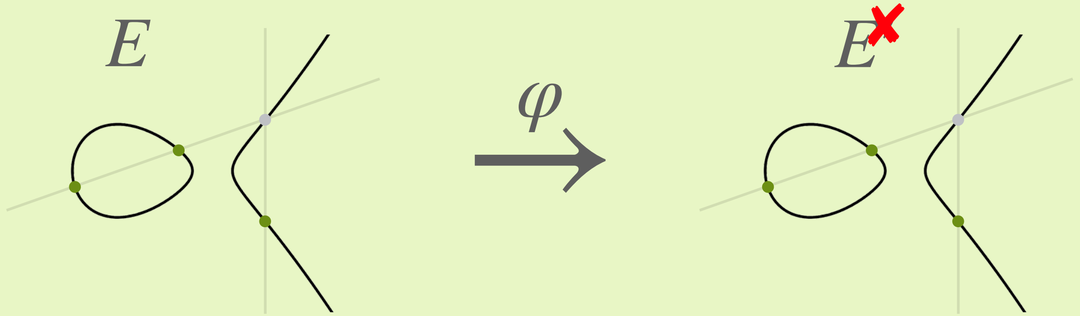

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$$E : y^2 = x^3 + x \xrightarrow{\varphi} E : y^2 = x^3 + x$$

$$(x, y) \mapsto \left(\frac{x^4 - 2x^2 + 1}{4(x^3 + x)} : \frac{x^6 y + 5x^4 y - 5x^2 y - y}{8(x^6 + 2x^4 + x^2)} \right) \quad \text{over } \mathbb{F}_{11}$$

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SQIsign

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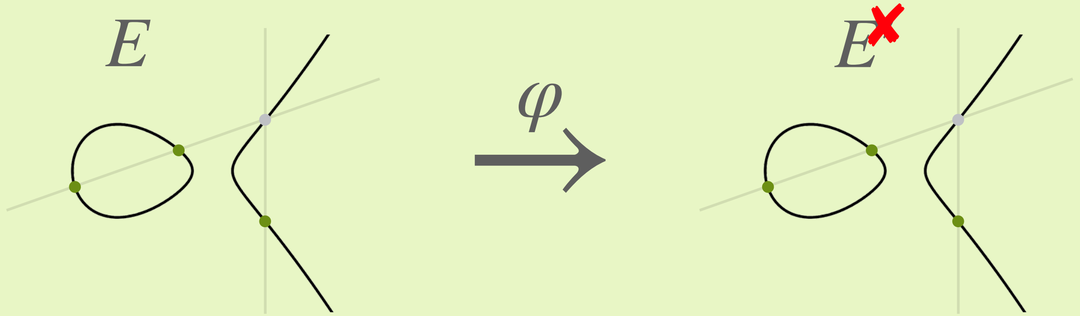

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- looks difficult... but actually this just the map $[2] : P \mapsto P + P$
- so [2] has kernel $\mathcal{O}, (0,0), (8 + 7i, 0), (3 + 4i, 0)$, degree [2] is $4 = 2^2$

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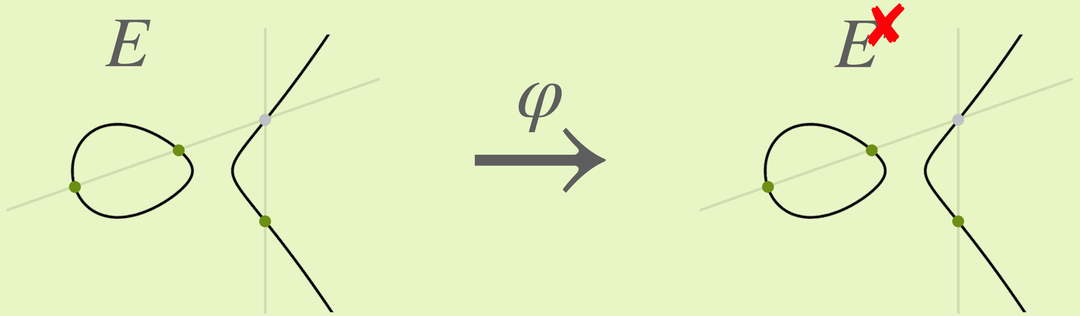

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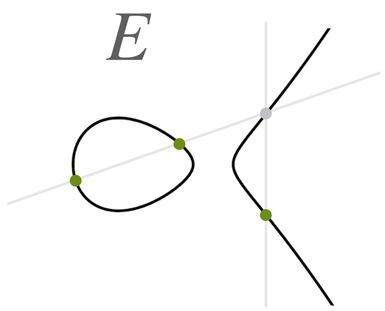
second toy example

Frobenius map. $\pi : (x, y) \mapsto (x^q, y^q)$ **always** an endomorphism for E over $\mathbb{F}_q$

PART 1

SQIsign



Given just any E over $\mathbb{F}_{q'}$ we just saw the endomorphisms

- multiplication-by-n, so $[n] : P \mapsto P + \ldots + P$ for any $n \in \mathbb{Z}$
- Frobenius π and easily also $[n] \cdot \pi$ for any $n \in \mathbb{Z}$

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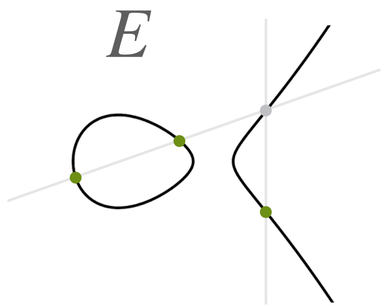

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Note: applying π twice gives $\pi^2 = [-p]$, so no "new" endom.

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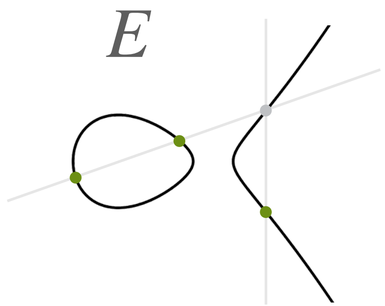

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endomorphism ring

- we can "add together" different endomorphisms

$$(\varphi + \psi)(P) = \varphi(P) + \psi(P)$$

- we can "multiply" endomorphisms by composition

$$(\varphi \cdot \psi)(P) = \varphi(\psi(P))$$

- so, we get a ring structure $\mathrm{End}(E)$, by our examples dimension is at least 2

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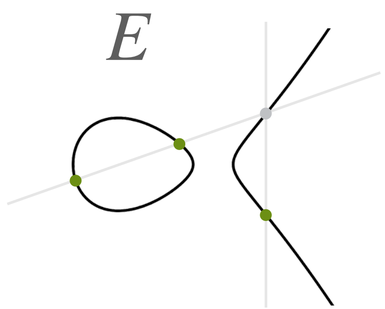

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E is **ordinary**

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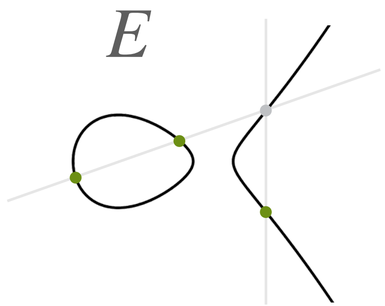

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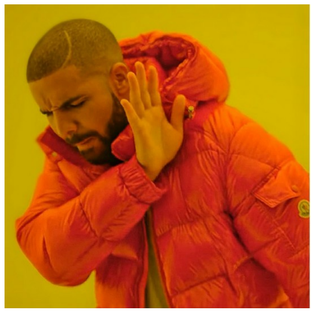

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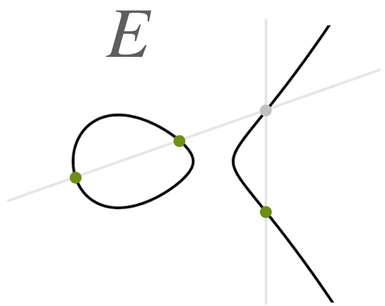

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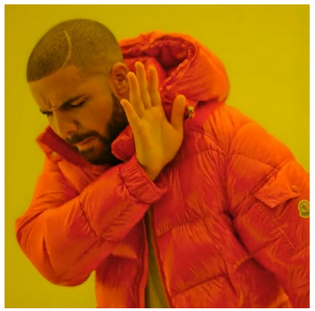
if dim 4

E is
✨✨ **supersingular** ✨✨
(weird funky maps!!)



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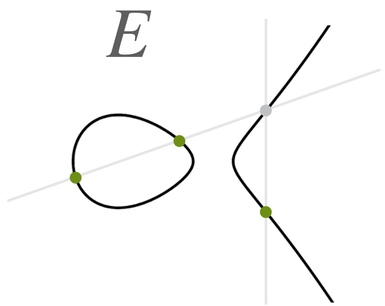

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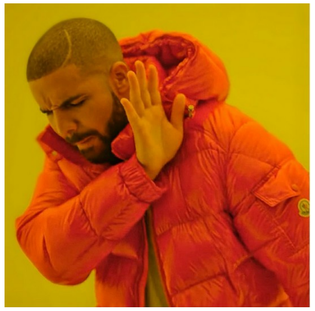
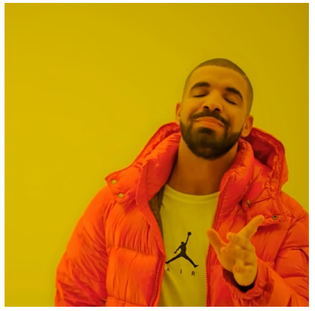


if dim 4

E is
✨✨ **supersingular** ✨✨
(weird funky maps!!)



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PART 1

SQIsign

Question

given supersingular E, can we find
weird, funky endomorphisms $\omega \in \mathrm{End}(E)$?

PART 1

SQIsign

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easy

easy to verify that
such endoms exists,
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hard

actually giving an
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or some way to
compute this

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SQIsign

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easy

easy to verify that
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e.g. that E
is **supersingular**

surprisingly easy

we know $\mathrm{End}(E_0)$ for the
specific curve $E_0 : y^2 = x^3 + x$
and for any $E_0 \to E_A$,
we can then compute $\mathrm{End}(E_A)$
(knowledge of endom. ring is contagious)

hard

actually giving an
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PART 1

SQIsign

Question

given supersingular E, can we find weird, funky endomorphisms $\omega \in \mathrm{End}(E)$?

hard

actually giving an endom. $\omega \in \mathrm{End}(E)$ or some way to compute this

Question

given E and E', can we find an isogeny $\varphi : E \to E'$?

hard

actually giving an isogeny $\varphi : E \to E'$ or some way to compute this

PART 1

SQIsign

Question

given supersingular E, can we find weird, funky endomorphisms $\omega \in \mathrm{End}(E)$?

hard

actually giving an endom. $\omega \in \mathrm{End}(E)$ or some way to compute this

equivalent!!!

Question

given E and E', can we find an isogeny $\varphi : E \to E'$?

hard

actually giving an isogeny $\varphi : E \to E'$ or some way to compute this

Best Paper ASIACRYPT 2020

Best Paper award

YouTube Chat

Chair: Shiho Moriai & Huaxiong Wang

Finding Collisions in a Quantum World: Quantum Black-Box Separation of Collision-Resistance and One-Wayness

Show abstract ›

Akinori Hosoyamada, Tak[illegible] Yasuda

S[illegible] PRESTO, Laboratories / Nagoya Uni[illegible]ty / NTT [illegible] Research, Inc.

Media:

PART 1: SQIsign

New results on Gimli: full-permutation distinguishers and improved collisions

Show abstract ›

Antonio Flórez Gutiérrez, Gaëtan Leurent, María Naya-Plasencia, Léo Perrin, André Schrottenloher, Ferdinand Sibleyras

Inria, France

Media:

SQISign: Compact Post-Quantum signatures from Quaternions and Isogenies

Show abstract ›

Luca De Feo, David Kohel, Antonin Leroux, Christophe Petit, Benjamin Wesolowski

IBM Research, Zurich; Université Aix-Marseilles; DGA, Ecole Polytechnique; University of Birmingham; Université de Bordeaux, CNRS

Media:

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A new isogeny-based signature scheme, with **high soundness**

2020 | 2021 | 2022 | 2023 | 2024

PART 1

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Key Generation

- **System parameters:** prime p, starting curve E_0
- **Secret Key:** isogeny $\varphi_A : E_0 \to E_A$, and then also $\mathrm{End}(E_A)$
- **Public Key:** the curve $E_A : y^2 = X^3 + Ax^2 + x$, with $A \in \mathbb{F}_q$

everyone knows $\mathrm{End}(E_0)$

E_0

PART 1

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everyone knows $\mathrm{End}(E_0)$

only **you** know φ_A and $\mathrm{End}(E_A)$

PART 1

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Identification protocol

- **Commitment:** random isogeny $\varphi_{\text{com}} : E_0 \to E_1$
- **Challenge:** semi-random isogeny $\varphi_{\text{chall}} : E_1 \to E_2$
- **Response:** "matching" isogeny $\varphi_{\text{resp}} : E_A \to E_2$

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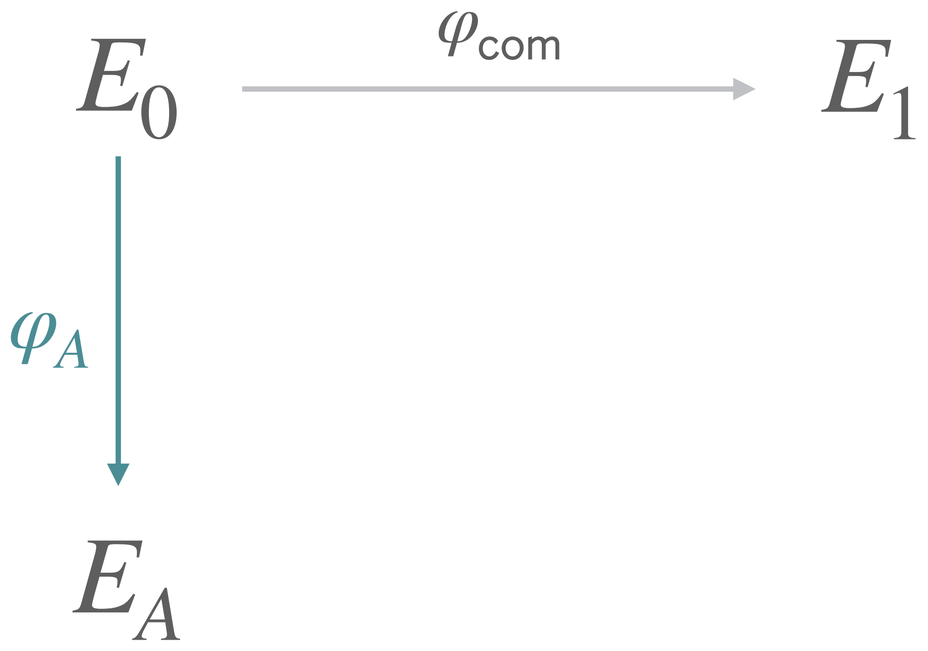

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PART 1
SQIsign

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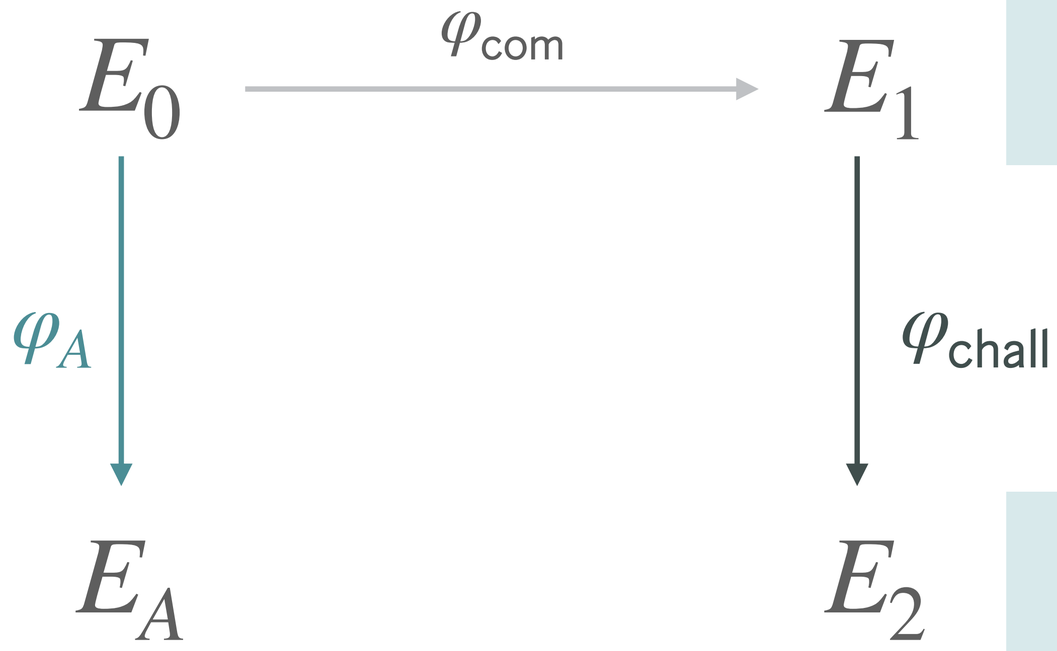

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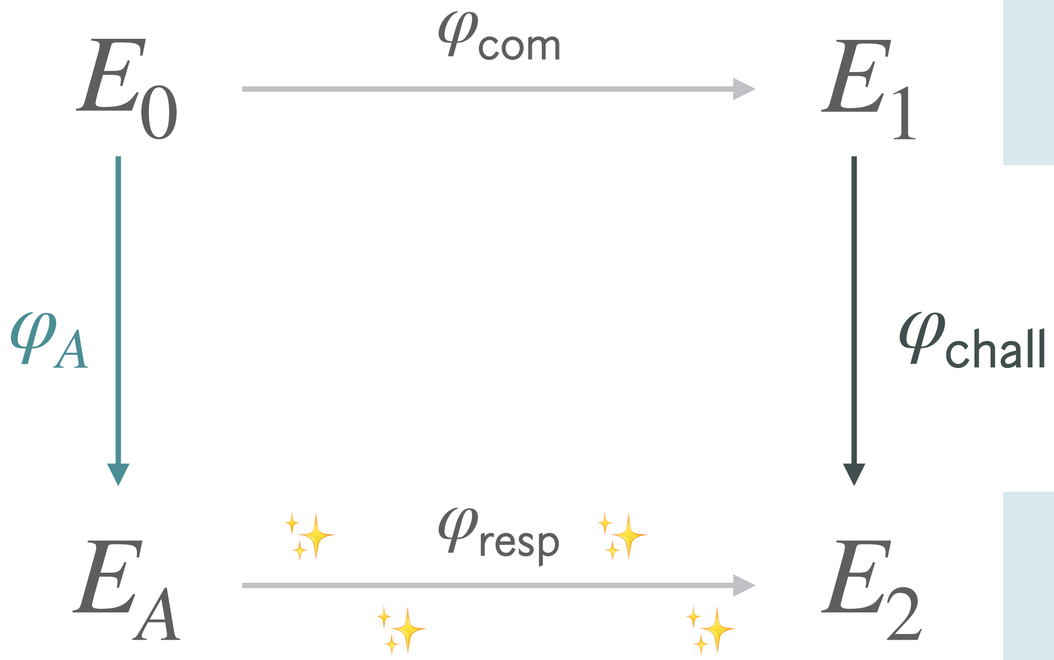

only **you** know φ_A and $\text{End}(E_A)$

only **you** know $\text{End}(E_2)$

Fact: ONLY, given $\text{End}(E_a)$ and $\text{End}(E_2)$ you can compute a proper response

PART 1
SQIsign

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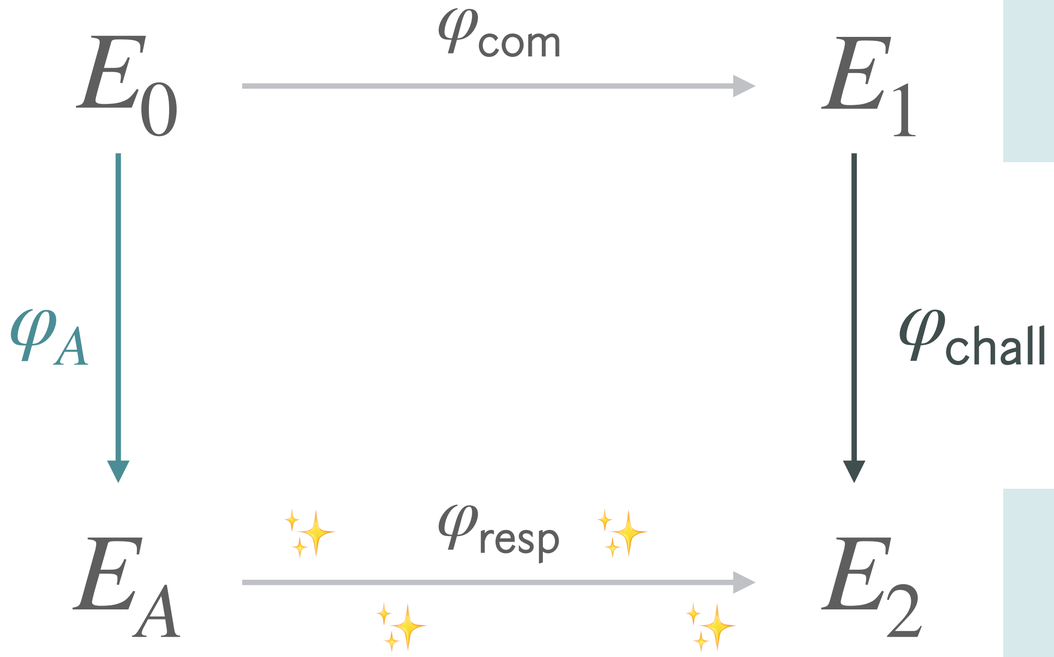
signature scheme

replace semi-random φ_{chall}
by a challenge isogeny generated
from SHAKE256(msg || E_1)

everyone knows
$\text{End}(E_0)$

only **you** know
φ_{com} and $\text{End}(E_1)$



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PART 1

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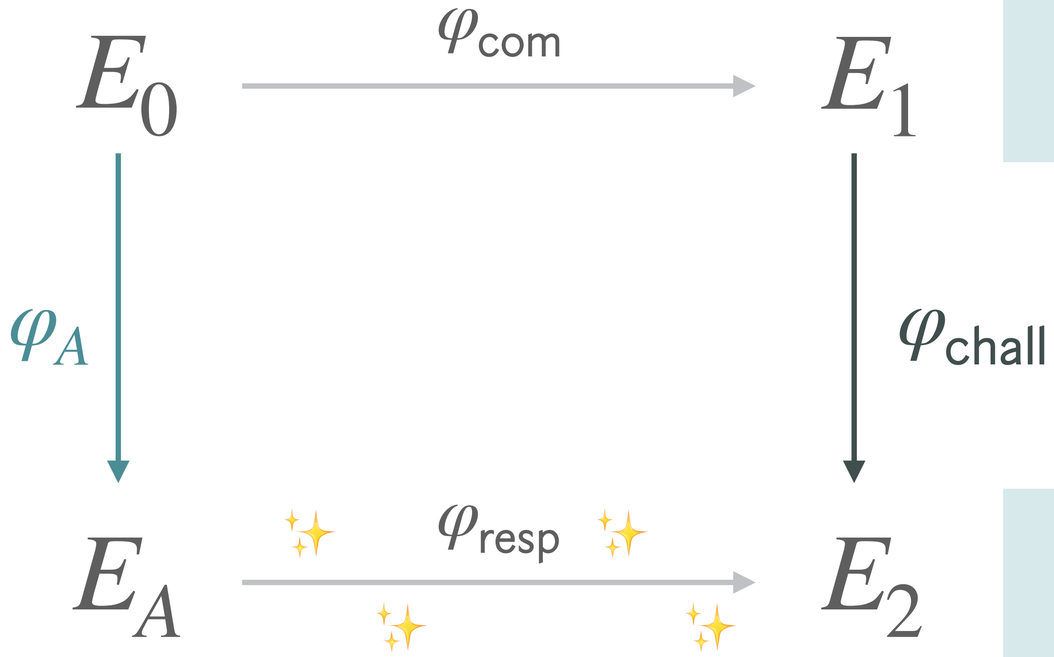

only **you** know
φ_A and $\text{End}(E_A)$

only **you** know
$\text{End}(E_2)$

WARNING!

with this approach
the response will
be **large**, degree 2^{1000}

Fact: ONLY, given $\text{End}(E_a)$ and $\text{End}(E_2)$
you can compute a proper response

To learn more about verification in detail:
see tutorial at https://vodice.post-quantum-crypto.com
by Lorenz Panny & me

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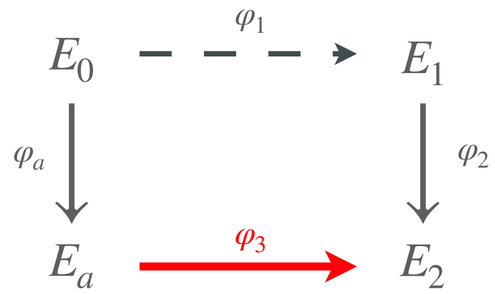
PART 1

SQIsign

computing the signature

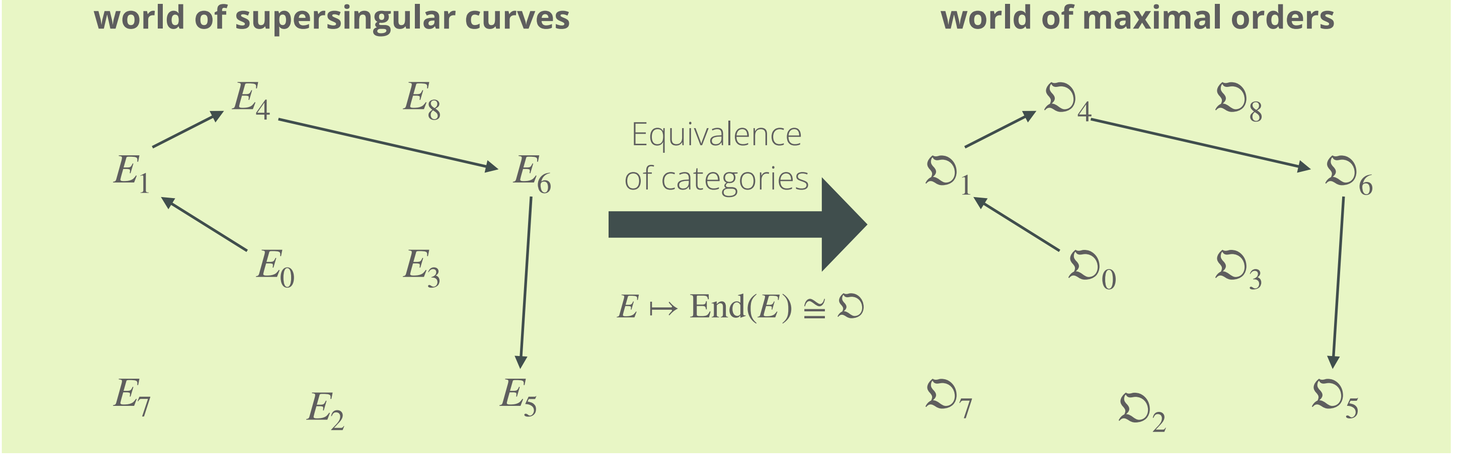


Fact: Given $\mathrm{End}(E_a)$ and $\mathrm{End}(E_2)$ you can compute $\varphi_3 : E_a \to E_2$

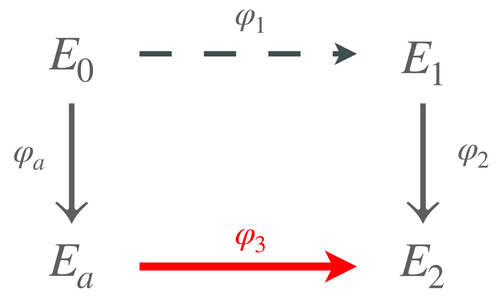
PART 1

SQIsign

Deuring correspondence



computing the signature



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PART 1

SQIsign

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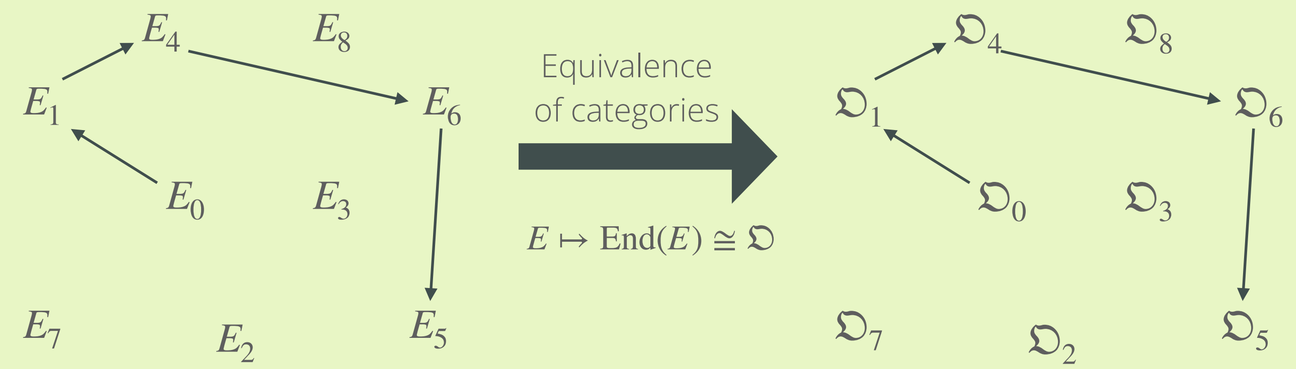

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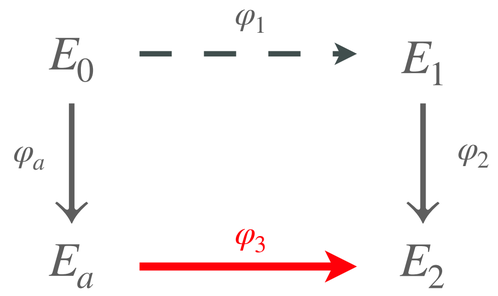

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curve-order dictionary

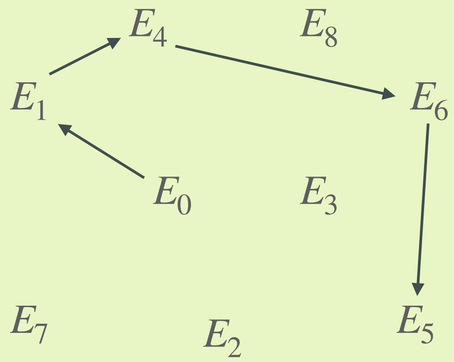
supersingular curves	quaternion orders
curve E (up to Galois conjugacy)	maximal order $\mathfrak{O}$ (up to isomorphism)

PART 1

SQIsign

Deuring correspondence

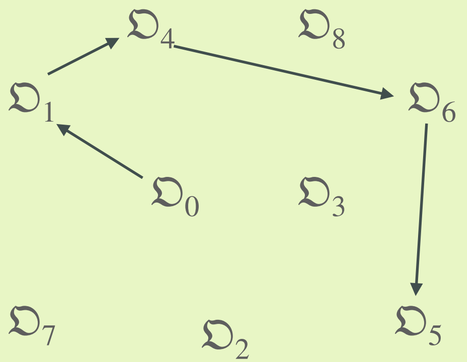
world of supersingular curves



Equivalence of categories

$E \mapsto \mathrm{End}(E) \cong \mathfrak{O}$

world of maximal orders



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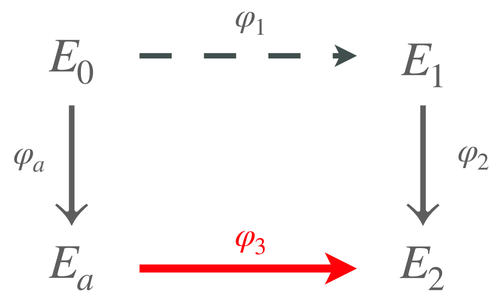

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PART 1

SQIsign

Deuring correspondence

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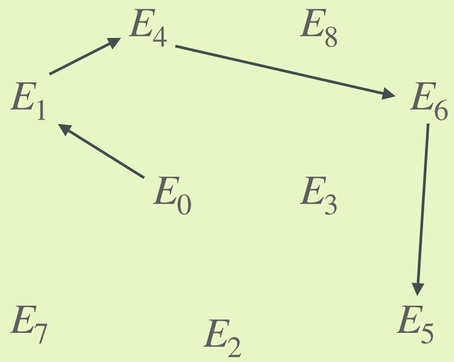

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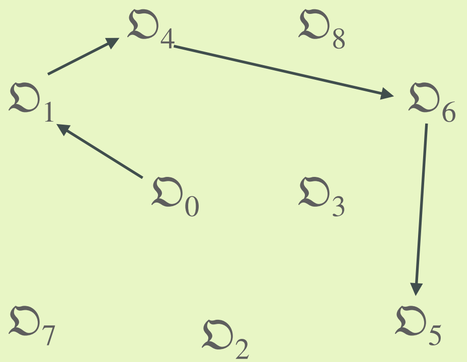

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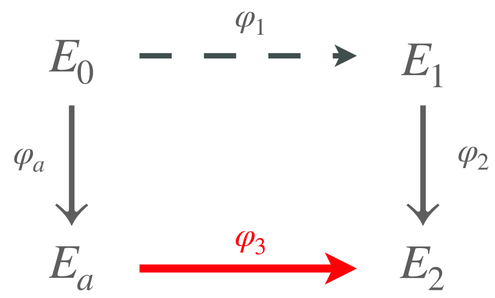

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and this continues for the *degree*, the *dual*, *equivalence*, *composition*...	and this continues for the *norm*, the *dual*, *equivalence*, *multiplication*...

PART 1

SQIsign

SQIsign

A new isogeny-based signature scheme, with **high soundness.**

SQIsign2

A new algorithm to translate ideals to isogenies.

2020 2021 2022 2023 2024

PART 1

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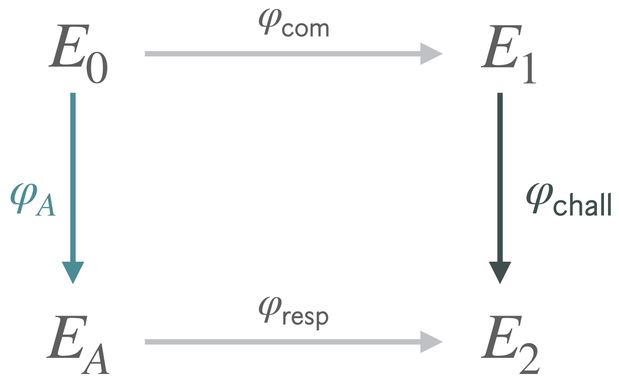
Elliptic curve world



PART 1

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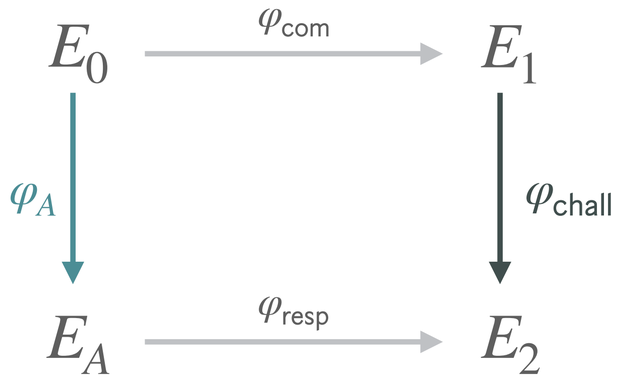
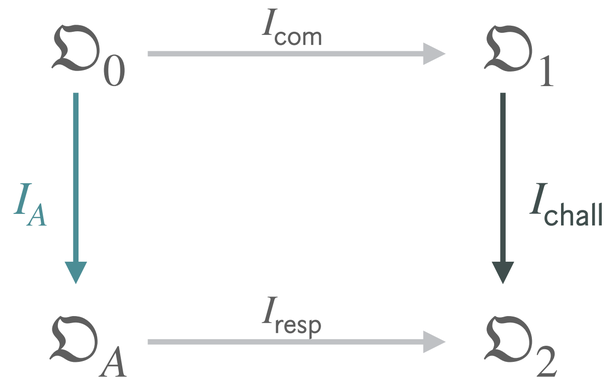
Elliptic curve world



Quaternion world



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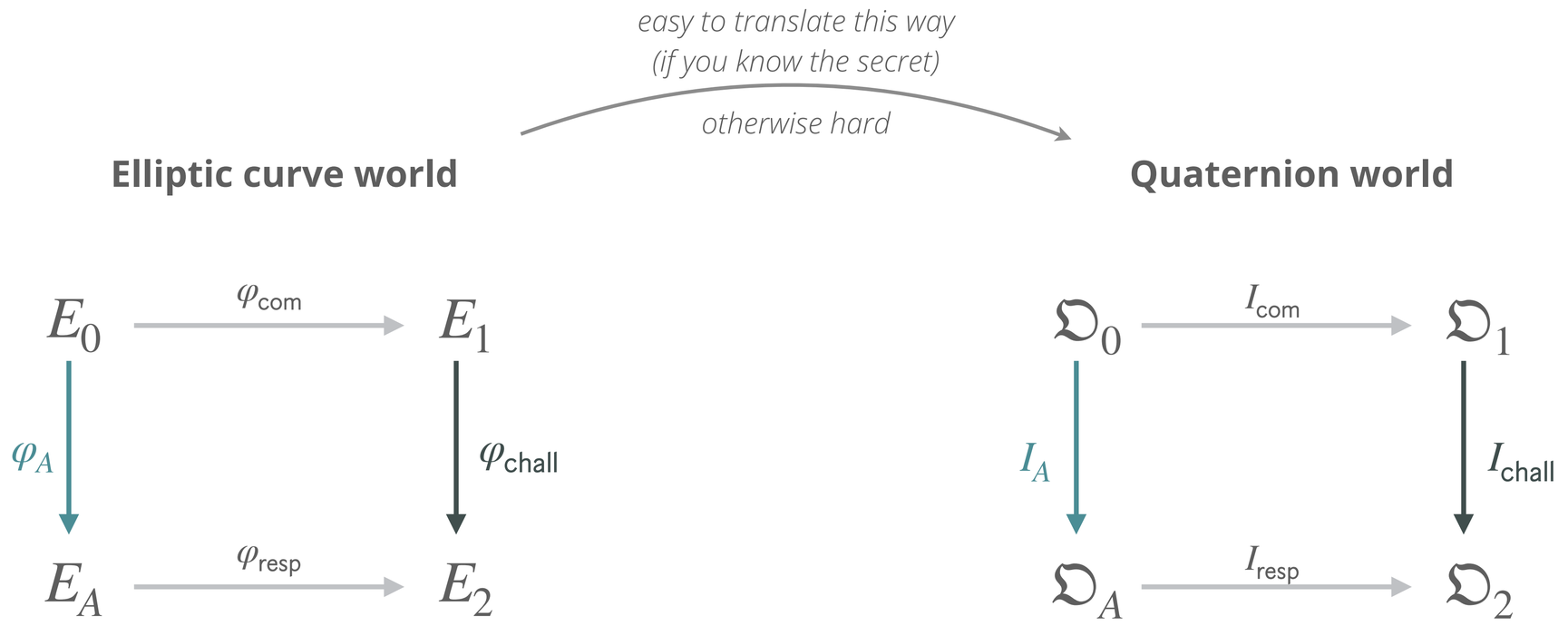

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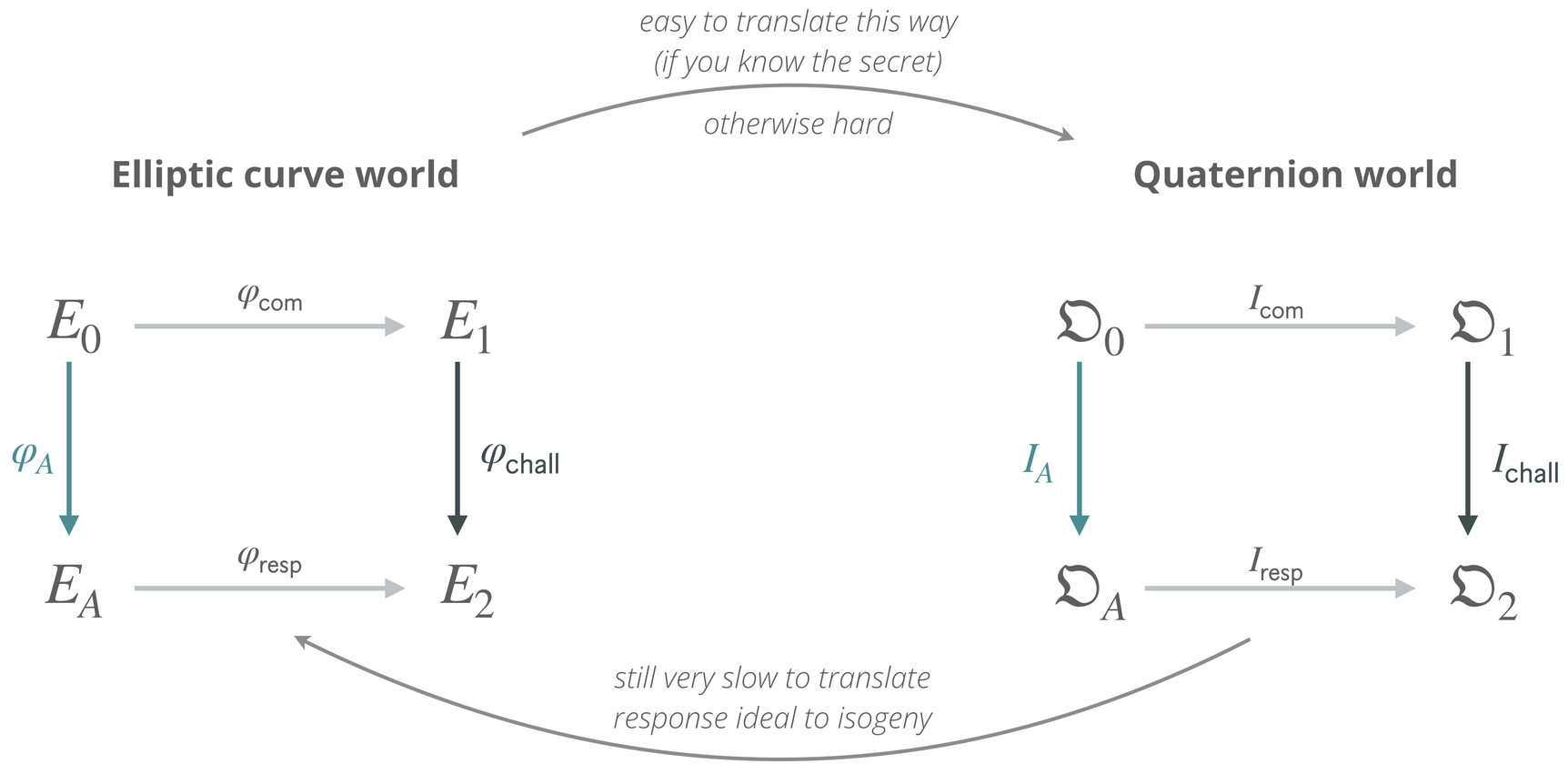

PART 1

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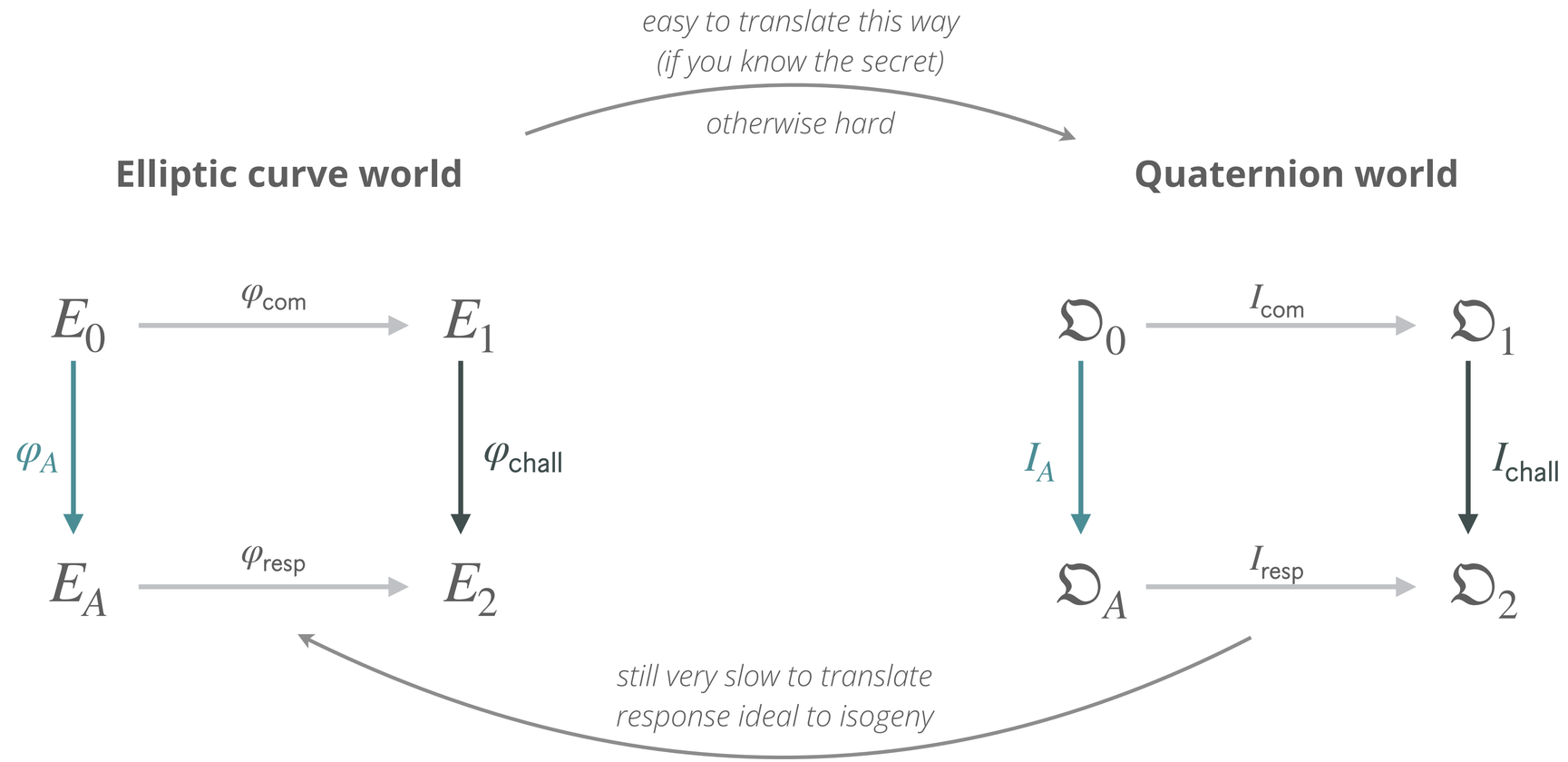


problem

need to break up $E_A \to E_2$ into smaller blocks

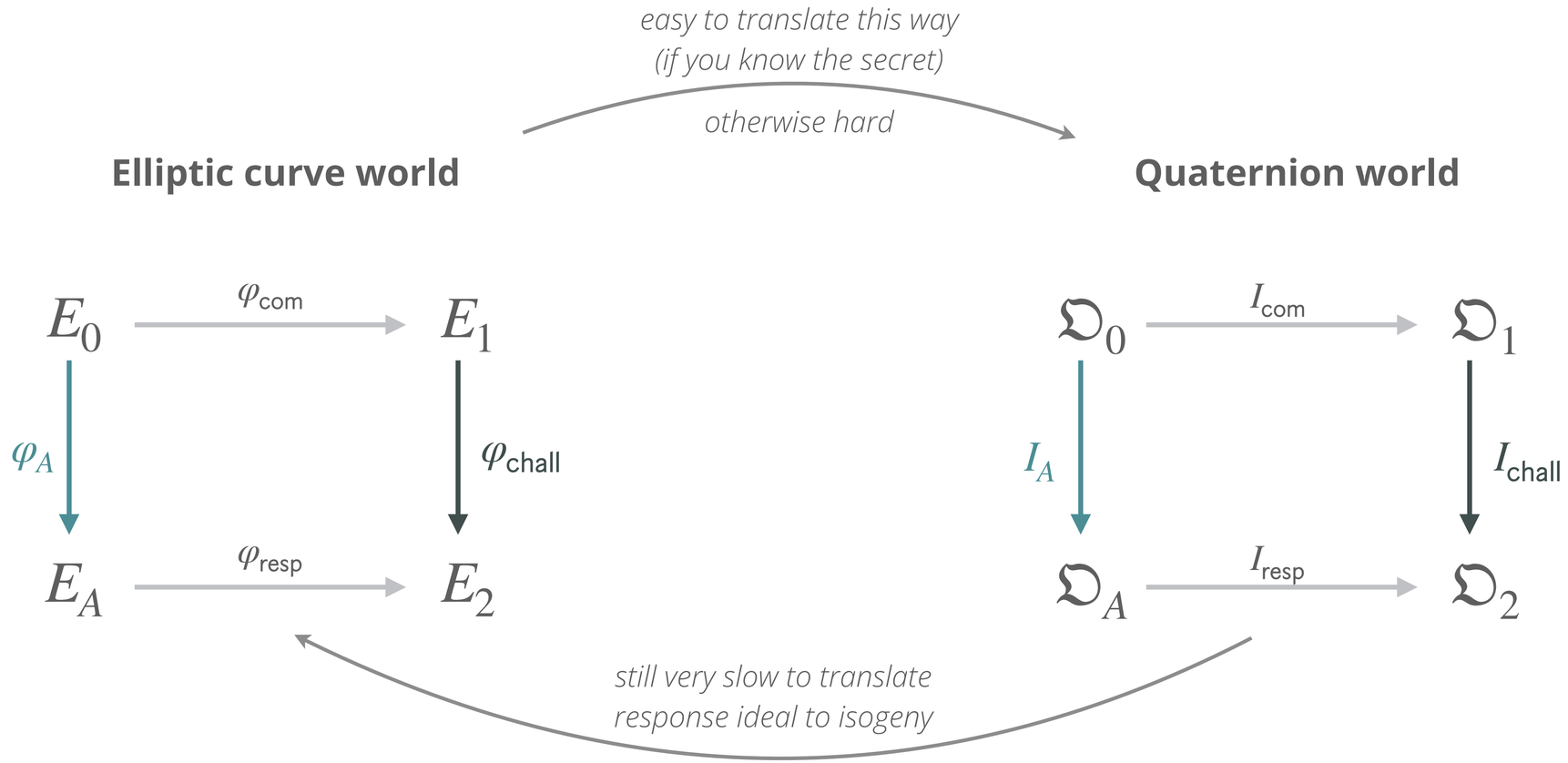
$E_A \to E^{(1)} \to E^{(2)} \to \ldots \to E^{(n-1)} \to E^{(n)} = E_2$

translating to the right blocks is very slow...

(NIST SQIsign has 13 blocks)

PART 1

SQIsign



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SQIsign2

among others, a much better way
to translate I_{resp} back to φ_{resp}
improving speed **per block**

3 Best Papers EUROCRYPT 2023

Lyon Congress Center - Plenary - Auditorium Lumière

Session chair: Joppe Bos

YouTube

An Efficient Key Recovery Attack on SIDH

Best Paper Award

Wouter Castryck, Thomas Decru

KU Leuven

Speaker(s): Thomas Decru

Show abstract ›

(paper #409) Media:

PART 2: The BREAK.

Breaking SIDH in Polynomial Time

Honourable Mention

Damien Robert

Inria Bordeaux

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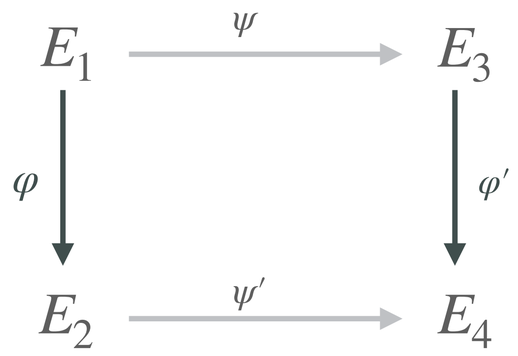
(paper #96) Media:

PART 2

The BREAK

Kani's Lemma (1997)



if $\deg \varphi = \deg \varphi'$
and $\deg \psi = \deg \psi'$
then this square of
1-dimensional isogenies

is associated to

a **2-dimensional isogeny**
$\Phi : E_2 \times E_3 \to E_1 \times E_4$

PART 2

The BREAK

Kani's Lemma (1997)

$$\begin{array}{ccc} E_1 & \xrightarrow{\psi} & E_3 \\ {\scriptstyle\varphi}\downarrow & & \downarrow{\scriptstyle\varphi'} \\ E_2 & \xrightarrow{\psi'} & E_4 \end{array}$$

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1D isogeny

if we know $\ker\varphi$,
then we can compute
$\varphi : E \to E'$ and $\varphi(P)$

PART 2

The BREAK

Kani's Lemma (1997)

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1D isogeny

if we know $\ker\varphi$,
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2D kernel

the kernel of 2D-iso Φ
is given by images $\varphi(P), \psi(P)$
for $P \in E_1$ of order $\deg\varphi + \deg\psi$

PART 2

The BREAK

Kani's Lemma (1997)

$$\begin{array}{ccc} E_1 & \xrightarrow{\psi} & E_3 \\ {\scriptstyle\varphi}\downarrow & & \downarrow{\scriptstyle\varphi'} \\ E_2 & \xrightarrow{\psi'} & E_4 \end{array}$$

if $\deg\varphi = \deg\varphi'$
and $\deg\psi = \deg\psi'$
then this square of
1-dimensional isogenies

is associated to

a **2-dimensional isogeny**
$\Phi : E_2 \times E_3 \to E_1 \times E_4$

1D isogeny

if we know $\ker\varphi$,
then we can compute
$\varphi : E \to E'$ and $\varphi(P)$

2D kernel

the kernel of 2D-iso Φ
is given by images $\varphi(P), \psi(P)$
for $P \in E_1$ of order $\deg\varphi + \deg\psi$

2D isogeny

if we know $\deg\varphi$ and $\deg\psi$
and we know these $\varphi(P)$, $\psi(P)$,
compute $\Phi : E_2 \times E_3 \to E_1 \times E_4$

PART 2

The BREAK

SQIsign

A new isogeny-based signature scheme, with **high soundness.**

SQIsign2

A new algorithm to translate ideals to isogenies.

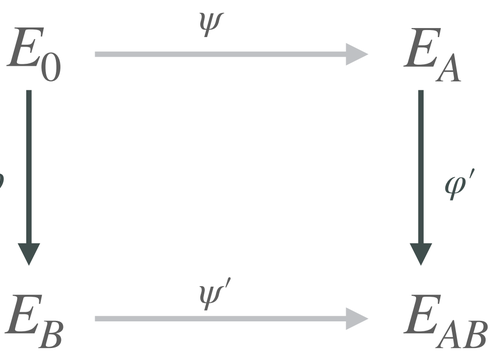
The SIKE breaks

In a series of three papers, SIKE was destroyed using **HD isogenies** in the summer of 2022.

PART 2

The BREAK

Castryck & Decru (2022)



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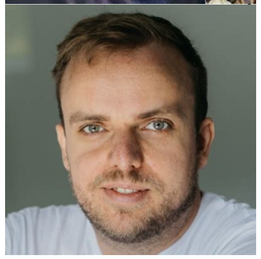

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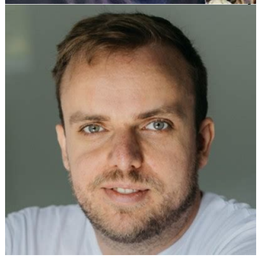

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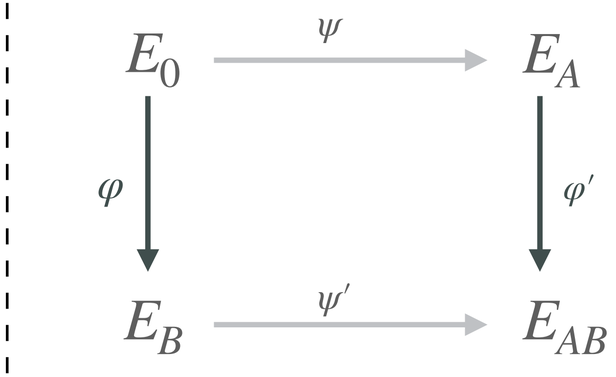

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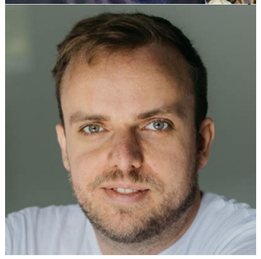

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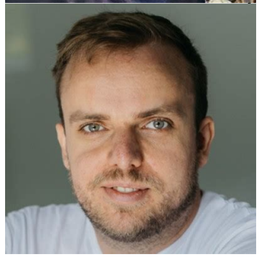

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making Φ difficult/impossible
to compute in practice...

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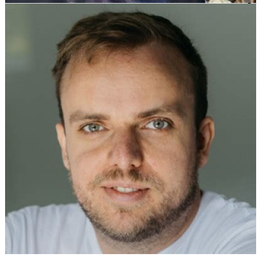

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use knowledge of $\mathrm{End}(E_0)$ to modify the square so that Φ is of degree 2^n, then compute Φ easily

PART 2

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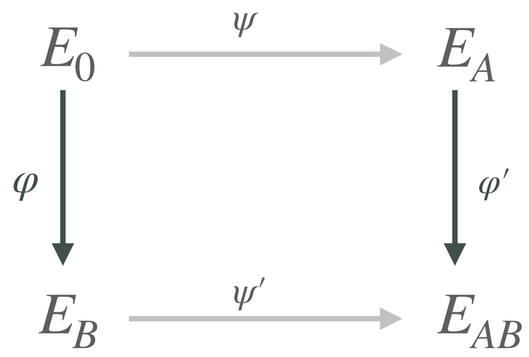

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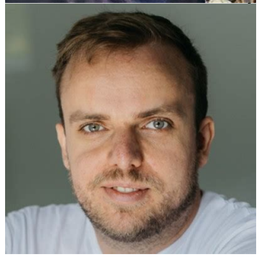

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Robert (2022)

generalize Kani's lemma:
don't just embed 1D into 2D,
embed into 4D or 8D!
Then Φ easy to compute
and we don't need $\mathrm{End}(E_0)$

2 Best Papers EUROCRYPT 2024

Awarded Papers

Kongresssaal

Marc Joye and Gregor Leander

Tight Indistinguishability Bounds for the XOR of Independent Random Permutations by Fourier Analysis

Itai Dinur

Ben-Gurion University

Speaker(s): Itai Dinur

(paper #326)

Show abstract ›

PART 3: New Dimensions

SQIsignHD: New Dimensions in Cryptography

Pierrick Dartois, Antonin Leroux, Damien Robert, Benjamin Wesolowski

INRIA, IMB, DGA-MI, ENS de Lyon, CNRS, UMPA

Speaker(s): Pierrick Dartois

(paper #149)

Show abstract ›

AprèsSQI: Extra Fast Verification for SQIsign Using Extension-Field Signing

Maria Corte-Real Santos, Jonathan Komada Eriksen, Michael Meyer, Krijn Reijnders

University College London, NTNU, University of Regensburg, Radboud University Nijmegen

Speaker(s): Jonathan Komada Eriksen

(paper #86)

Show abstract ›

PART 3

New Dimensions

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A new isogeny-based signature scheme, with **high soundness.**

SQIsign2

A new algorithm to translate ideals to isogenies.

2020 | 2021 | 2022 | 2023 | 2024

The SIKE breaks

In a series of three papers, SIKE was destroyed using **HD isogenies** in the summer of 2022.

SQIsignHD

Use the SIKE attacks! Represent the response as a **HD isogeny**. Required 4/8-dimensions.

PART 3

New Dimensions

HD representations

instead of describing 1D isogeny $\varphi : E \to E'$ by its kernel $\ker \varphi$,
we can also describe it by $E, P_1, \ldots, P_n, \varphi(P_1), \ldots, \varphi(P_n)$, for enough points $P_i \in E$

then, with Kani's lemma & improvements, compute $\varphi(Q)$ for any other $Q \in E$

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In the words of the HD master

"If we know the value of $\varphi : E \to E'$ on enough nice points, then we know how to efficiently evaluate it everywhere"

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isogeny embedding (rough sketch)

We want to embed the 1-dimensional isogeny $\varphi : E \to E'$ and
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Assume for the moment that $\deg \varphi = 2^n - x^2$ for some $x \in \mathbb{Z}$

$$E \xrightarrow{\varphi} E'$$

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We know and can compute $[x]$ easily! So we can apply Kani's!

$$\Phi : E \times E' \to E \times E'$$

Radboud University

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As Φ of degree 2^n, easy to compute and we can use Φ to compute $\varphi(Q)$ for any other point $Q \in E$

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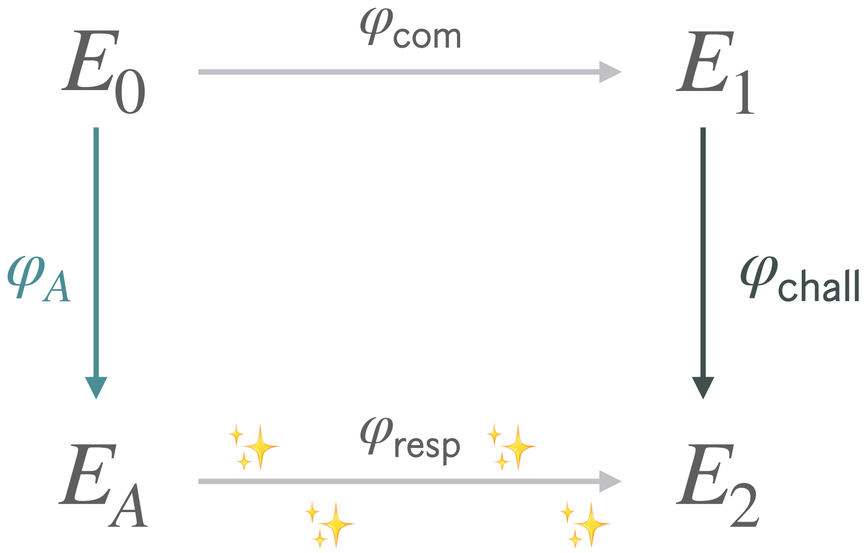

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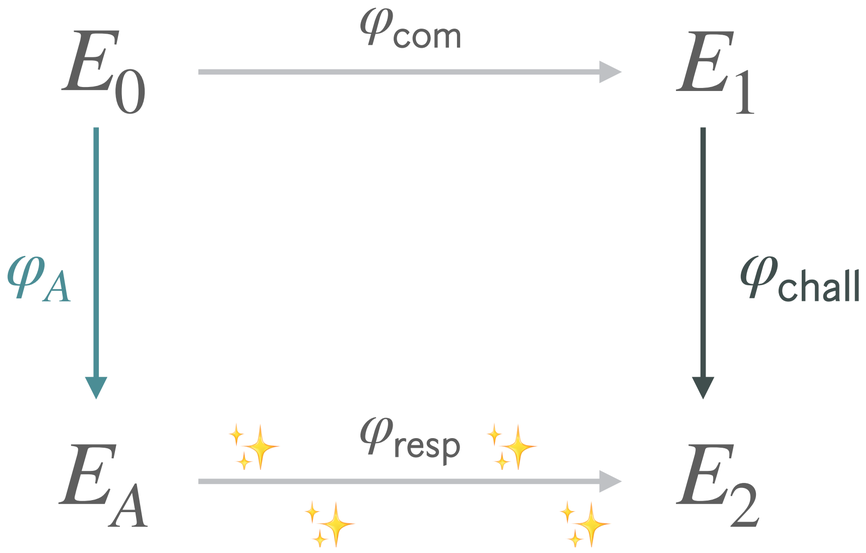

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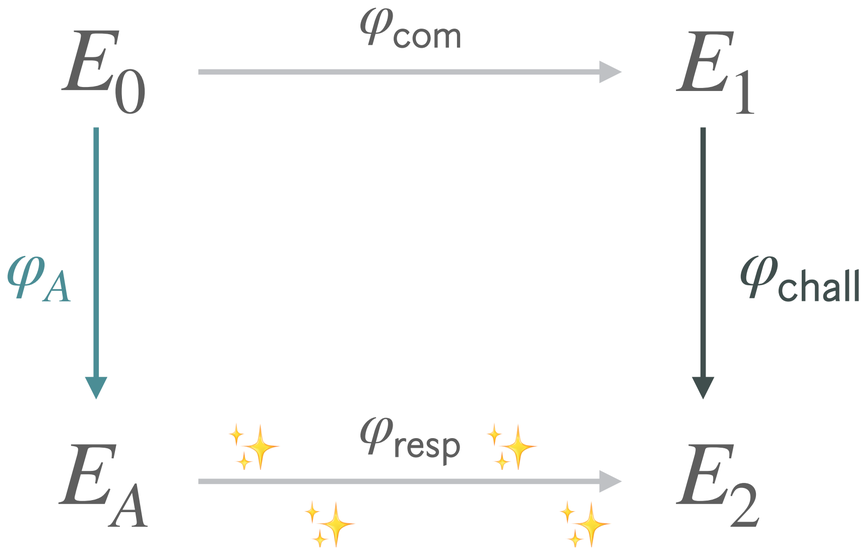

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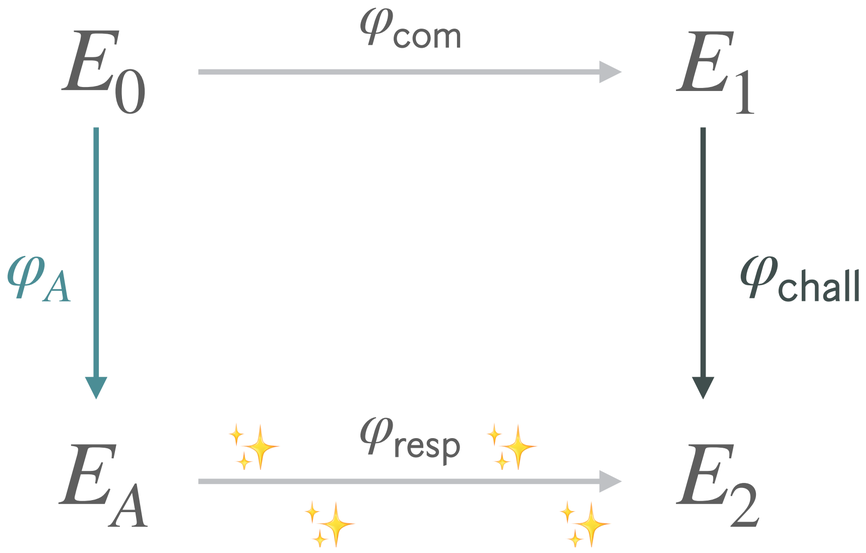

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✗ **verification is now a 4D- or 8D-isogeny... difficult, complex, and rather slow**

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A new isogeny-based signature scheme, with **high soundness.**

SQIsign2

A new algorithm to translate ideals to isogenies.

AprèsSQI

Signing will be slow... We push verification to the limits using extension fields.

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Use the SIKE attacks! Represent the response as a **HD isogeny**. Required 4/8-dimensions.

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extension fields

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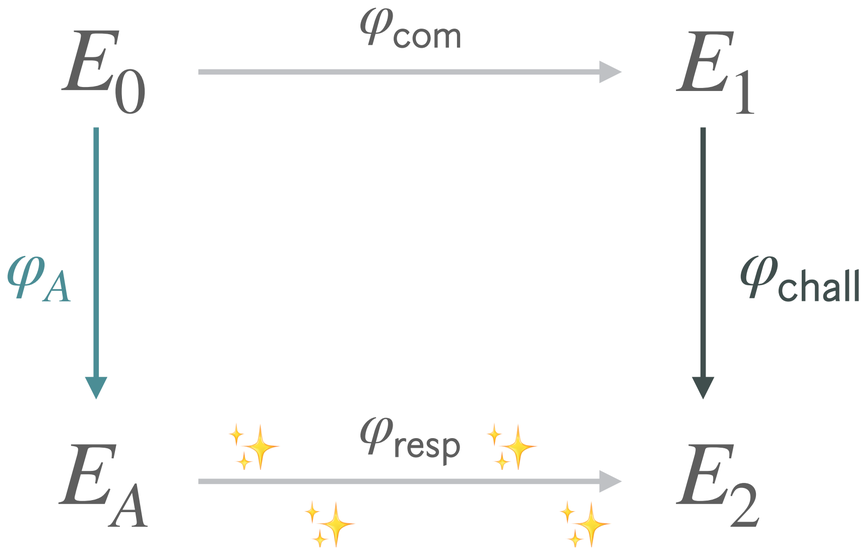
Idea: signing is slow anyway, what if we work over $\mathbb{F}_{p^{2k}}$ during signing, and push verification speeds to the absolute limits?



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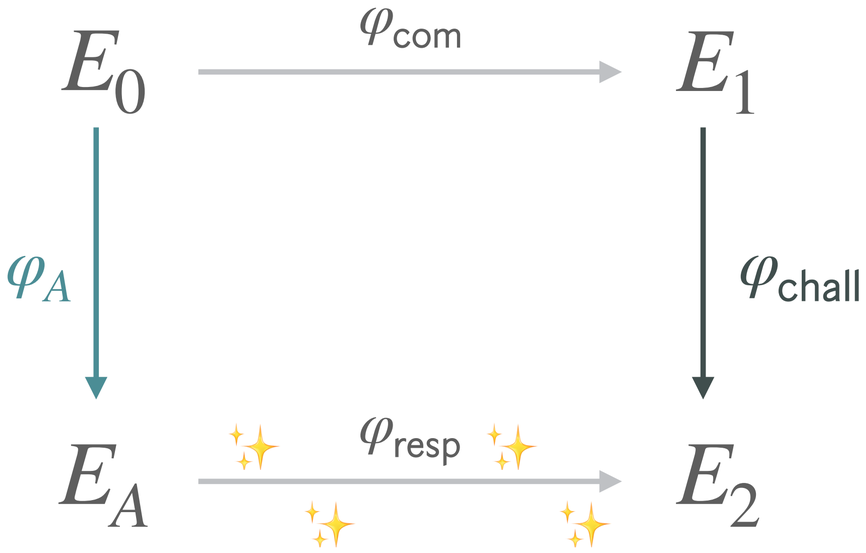

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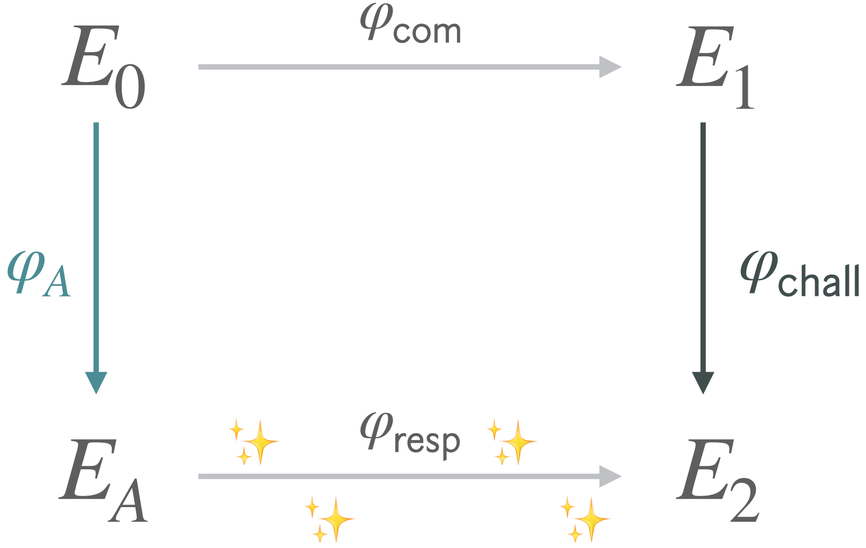

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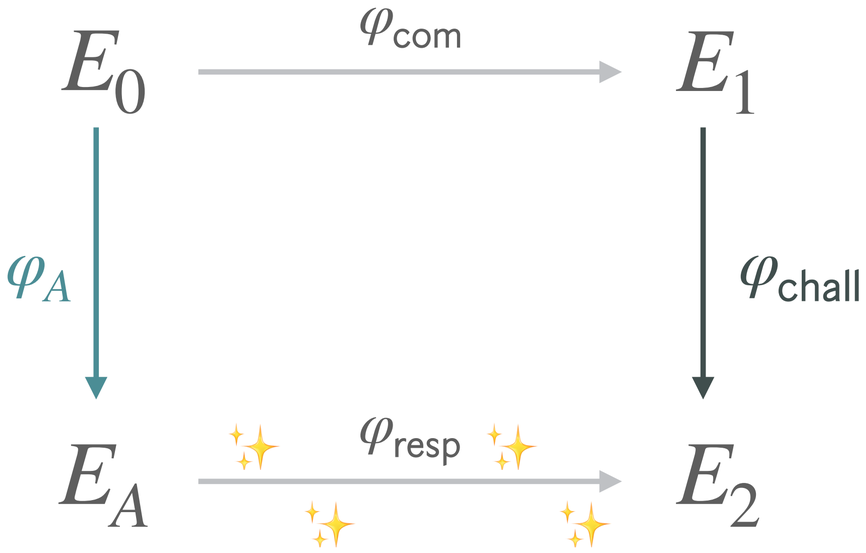

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3 Best Papers ASIACRYPT 2024?

Awarded Papers

Kongresssaal

Marc Joye and Gregor Leander

Tight Indistinguishability Bounds for the XOR of Independent Random Permutations by Fourier Analysis

Speaker(s): Itai Dinur

(paper #326)

Show abstract ›

PART 4: 2D Future?

SQIsign2D-West: The Fast, the Small, and the Safer

Andrea Basso,Pierrick Dartois,Luca De Feo,Antonin Leroux,Luciano Maino,Giacomo Pope,Damien Robert, Benjamin Wesolowski

SQIsign2D-East

Kohei Nakagawa, Hiroshi Onuki

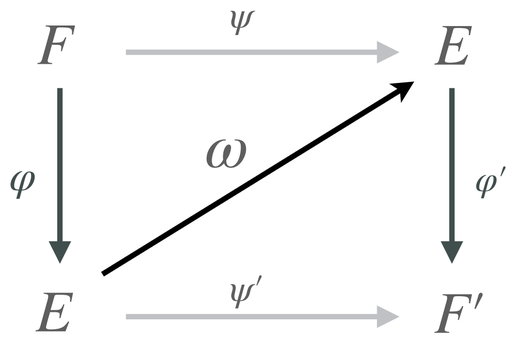
SQIPrime: a Dimension 2 Variant of SQIsignHD with Non-Smooth Challenge Isogenies

Max Duparc, Tako Boris Fouotsa

PART 4

2D Future

Nakagawa - Onuki trick (2023)



say we want to create such
a square, but we only have E
and some $\omega \in \mathrm{End}(E)$
of degree $q(2^a - q)$

we can find a suitable isogeny
$\varphi : F \to E$ using Kani!!!

PART 4

2D Future

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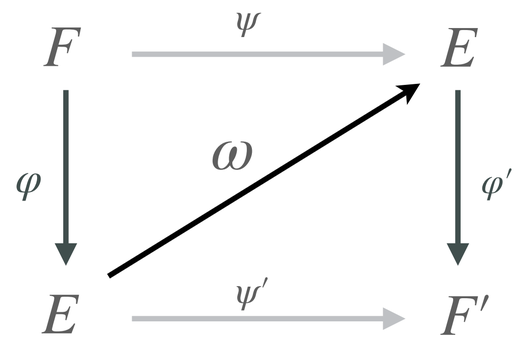

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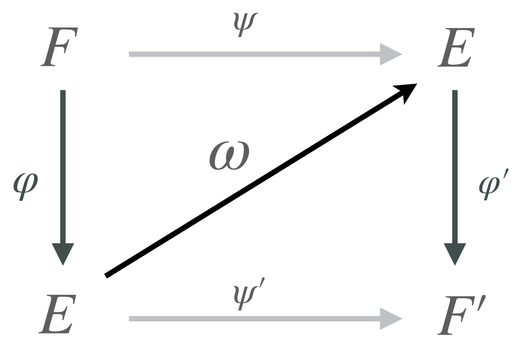

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If the square above existed, then Kani's lemma should apply

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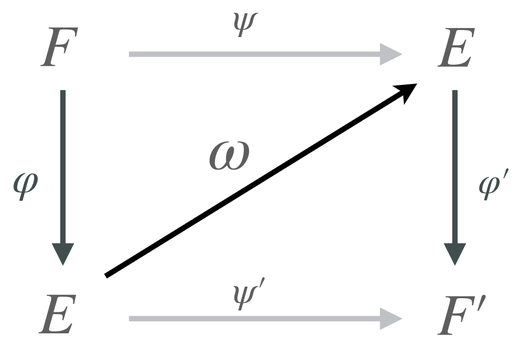
PART 4

2D Future

Nakagawa - Onuki trick (2023)



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(ignoring some isogeny maths) then the kernel of Φ should be given by $[q]P, \omega(P)$ for $P \in E[2^a]$

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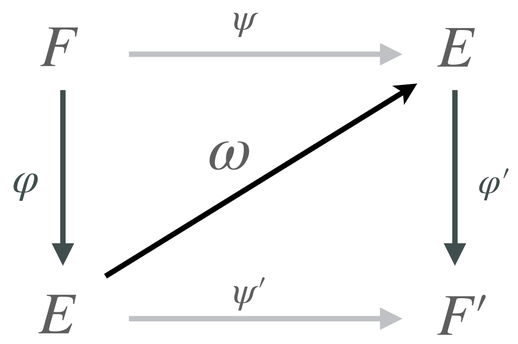

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So we can also compute $\varphi : F \to E, \psi : F \to E$

that is, we can factor ω using Kani's lemma

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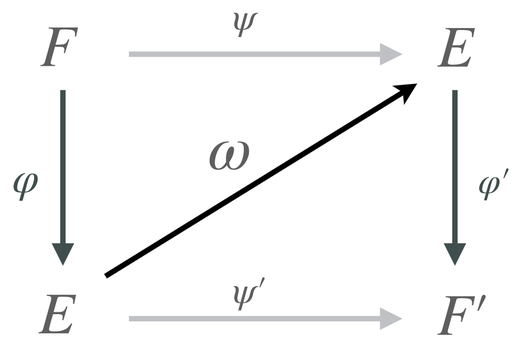

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Clapoti(s)

apply this trick to translate ideal I to suitable 2D isogenies

PART 4

2D Future

SQIsign

A new isogeny-based signature scheme, with **high soundness.**

SQIsign2

A new algorithm to translate ideals to isogenies.

AprèsSQI

Signing will be slow... We push verification to the limits using extension fields.

The SIKE breaks

In a series of three papers, SIKE was destroyed using **HD isogenies** in the summer of 2022.

SQIsignHD

Use the SIKE attacks! Represent the response as a **HD isogeny**. Required 4/8-dimensions.

Going 2D

Simultaneously, three works adapted SQIsignHD to enable verification with **2D isogenies**

PART 4

2D Future

SQIsign2D

Don’t do “slow” translation of ideal into blocks of 1D-isogenies (SQIsign, AprèsSQI)
Don’t do “fast” translation of ideal into slow 4D/8D isogenies (SQIsignHD)

Do use the previous section to translate ideal into 2D isogenies

PART 4

2D Future

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Do use the previous section to translate ideal into 2D isogenies

1. SQIsign2D-West

&

2. SQIsign2D-East

&

3. SQIPrime

PART 4

2D Future

- ✓ **faster primes!**
- ✓ **FASTER signing!**
- ✓ **THE BEST security!**
- ✓ **FAST verification!**

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1. SQIsign2D-West **&**
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concrete numbers

NIST SQIsign Level I		SQIsign2D-West Level I	
• **Public Key:**	64 bytes	• **Public Key:**	66 bytes
• **Signature :**	177 bytes	• **Signature :**	148 bytes
• **Signing:**	2,400 MCycles	• **Signing:**	160 MCycles
• **Verification:**	39 MCycles	• **Verification:**	9 MCycles

PART 4

2D Future

From here on: wishful thinking

More Best Papers??

Best Paper Session

Location: South Hall 3

Session Chairs: Diego F. Aranha, Marcel Medwed

PART 5: A 1D Miracle?

YouTube

Ideal-to-isogeny Algorithm using 2-dimensional Isogenies and its Application to SQIsign

Hiroshi Onuki, Kohei Nakagawa

Optimized Implementation of SQIsign Verification on Intel and M4

Anonymous

log(p) KLPT

Winner of next Field medal

PART 5

A 1D Miracle?

2020

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2021

2022

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2023

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Signing will be slow... We push verification to the limits using extension fields.

SQIsignHD

Use the SIKE attacks! Represent the response as a **HD isogeny**. Required 4/8-dimensions.

2024

More 1D?

Recent works seem to allow improved signing for Après-primes, *(and more...)*

Going 2D

Simultaneously, three works adapted SQIsignHD to enable verification with **2D isogenies**

PART 5

A 1D Miracle?

Remember this slide?

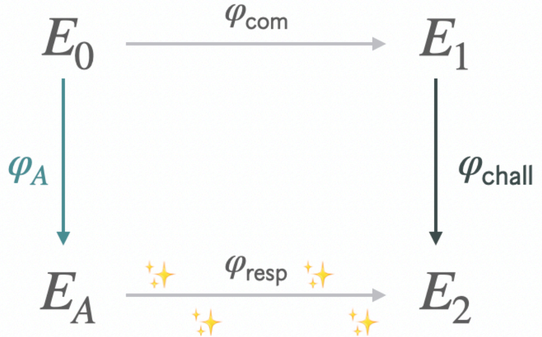
PART 3

New Dimensions

extension fields

in signing, we want to keep working over $\mathbb{F}_{p^2}$ for efficiency reasons

Idea: signing is slow anyway, what if we work over $\mathbb{F}_{p^{2k}}$ during signing, and push verification speeds to the absolute limits?



1. instead of (slow) translation of I_{resp} to φ_{resp} in 13 blocks....

2. slower translation using $\mathbb{F}_{p^{2k}}$ arithmetic but only 4 blocks!

✗ **signing is now even slower, using extension fields, takes literal seconds**

✓ **faster primes!**

✓ **fewer blocks!**

✓ **FAST verification!**

Radboud University

[2024/778]: Hiroshi Onuki, Kohei Nakagawa *"Ideal-to-isogeny algorithm using 2-dimensional isogenies and its application to SQIsign"*

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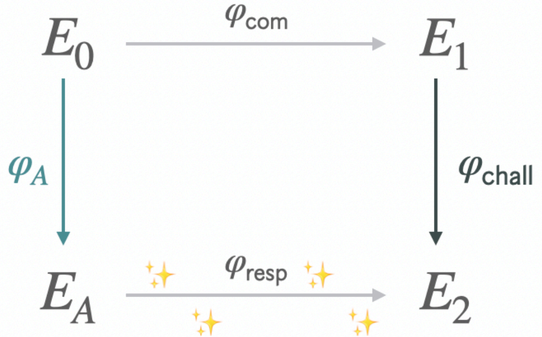

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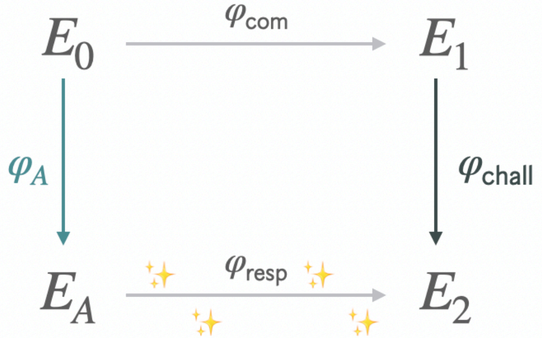

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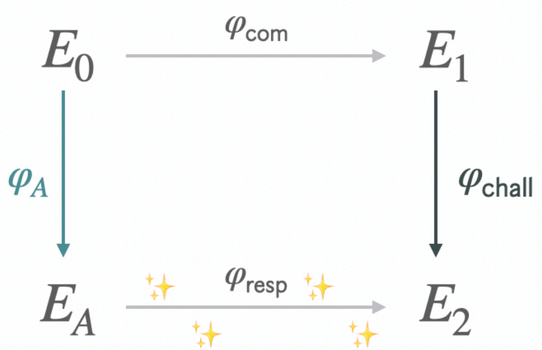

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2. Ongoing work shows highly-optimised verification for 1D verification “very likely” outperforms 2D verification

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[2024/778]: Hiroshi Onuki, Kohei Nakagawa *“Ideal-to-isogeny algorithm using 2-dimensional isogenies and its application to SQIsign”*

CONCLUSION

2D SQIsign is the “way to go” for now!

1D SQIsign is not dead! However, requires a breakthrough to achieve similar results

Want best paper awards? Do SQIsign research.

BONUS PART: THE BEAST

From here on:
“this way madness lies”

PART 6

THE BEAST

Remember that I said verification is relatively easy?

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THE BEAST

Remember that I said verification is relatively easy?

1D SQIsign

Verification recomputes a 2^{1000} isogeny

$$\varphi_{\text{resp}} : E_A \to E_2$$

in a number of blocks

$$\varphi_i : E^{(i)} \to E^{(i+1)}$$

All of this is done over $\mathbb{F}_{p^2}$ and requires a few essential building blocks that we know for a long time now.

- isogeny-evaluation formulas
- pairing-based techniques
- efficient basis sampling
- point compression
- curve normalisation

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2D SQIsign

Verification recomputes a 2^{128} isogeny

$$E_1 \times E_2 \to F_1 \times F_2$$

in a single block.

All of this is done over $\mathbb{F}_{p^2}$, and for such "short" 2D-isogenies, we essentially only need formulas to evaluate the isogeny.

These have recently been studied by Dartois, Maino, Pope, Robert using theta-models.

(If you ever heard of Richelot isogenies between hyperelliptic curves, they are essentially the same, but different...)

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however

1

Scholten has shown in 2003 that each elliptic curve over $\mathbb{F}_{p^2}$ has a "friend" in dimension 2 over $\mathbb{F}_p$, using Weil restriction.

2

Costello has shown in 2018 that the same holds for 2-isogenies between curves! They become 2D-isogenies.

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Requires *tons of work* as we now don't do a single "short" 2D-isogeny, but a number of blocks.

So, we developed:

- pairing-based techniques
- efficient basis sampling
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Remember that I said verification is relatively easy?

Return of the Kummer:
a toolbox for genus 2 cryptography

Maria Corte-Real Santos[1] and Krijn Reijnders[2]

[1] University College London
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THANK YOU!

ANY QUESTIONS?

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