Radbound University

Cryptography using Matrix Code Equivalence.

A post-quantum cryptographic group action?

Krijn Reijnders (joint work with **Simona Samardjiska** and **Monika Trimoska**)
Radboud University, Nijmegen

CBCrypto, May 30th, 2022

- **Matrix Code Equivalence (MCE):**
  Given two k-dimensional codes $\mathcal{C}$ and $\mathcal{D}$ of $m \times n$ matrices over a finite field $\mathbb{F}_q$,
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- In this talk:
  - **The hardness of MCE**
  - **MCE as cryptographic group actions**

Matrix Code Equivalence (MCE)

The Matrix Code Equivalence Problem

Matrix code $\mathcal{C}$: a subspace of $\mathcal{M}_{m\times n}(\mathbb{F}_q)$ of dimension k endowed with **rank metric**

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Matrix Code Equivalence (MCE) problem [Berger, 2003]

MCE$(k, n, m, \mathcal{C}, \mathcal{D})$:

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Known: Any isometry $\mu:\mathcal{C}\to\mathcal{D}$ can be written, for some $\mathrm{A}\in\mathrm{GL}_m(q),\mathrm{B}\in\mathrm{GL}_n(q)$, as

$$\mathrm{C}\mapsto\mathrm{ACB}\in\mathcal{D}$$

Known results [Couvreur, Debris–Alazard & Gaborit, 2021]

$$\mu : \mathsf{C} \mapsto \mathsf{ACB} \in \mathcal{D}, \quad \text{with } \mathsf{A} \in \mathsf{GL}_m(q) \text{ and } \mathsf{B} \in \mathsf{GL}_n(q)$$

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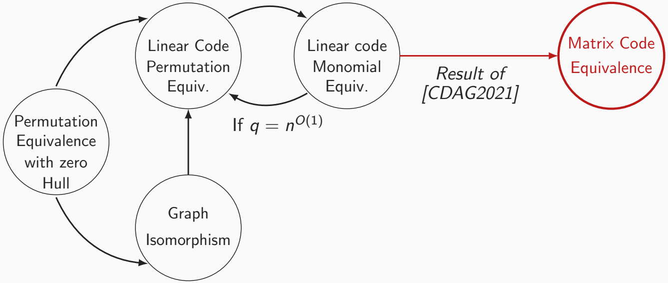

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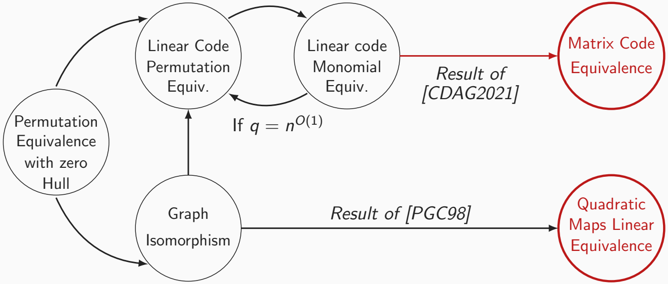

What is QMLE?

Multivariate crypto basics

- systems of multivariate polynomials $\mathcal{P} = (p_1, p_2, \ldots, p_k)$, every p_s polynomial in N variables $x_1, \ldots, x_N$

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$$p_s(x_1, \ldots, x_N) = \sum \gamma_{ij}^{(s)} x_i x_j + \sum \beta_i^{(s)} x_i + \alpha^{(s)}, \qquad \alpha^{(s)}, \beta_i^{(s)}, \gamma_{ij}^{(s)} \in \mathbb{F}_q$$

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Quadratic Maps Linear Equivalence (QMLE) problem

QMLE($N, k, \mathcal{F}, \mathcal{P}$):

Input: Two k-tuples of quadratic maps

$\mathcal{F} = (f_1, f_2, \ldots, f_k),\ \mathcal{P} = (p_1, p_2, \ldots, p_k) \in \mathbb{F}_q[x_1, \ldots, x_N]^k$

Question: Find – if any – $\mathrm{S} \in \mathrm{GL}_N(q), \mathrm{T} \in \mathrm{GL}_k(q)$ such that

$$\mathcal{P}(\mathrm{x}) = \mathcal{F}(\mathrm{xS}) \cdot \mathrm{T}$$

Quadratic maps as matrix codes

$$p_s = \sum \gamma_{ij}^{(s)} x_i x_j \quad = \quad (x_1, \ldots, x_N) \underbrace{\begin{pmatrix} \gamma_{11} & \cdots & \frac{\gamma_{1N}}{2} \\ & & \\ \frac{\gamma_{N1}}{2} & \cdots & \gamma_{NN} \end{pmatrix}}_{P^{(s)} \in \mathcal{M}_{N\times N}(\mathbb{F}_q)} \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

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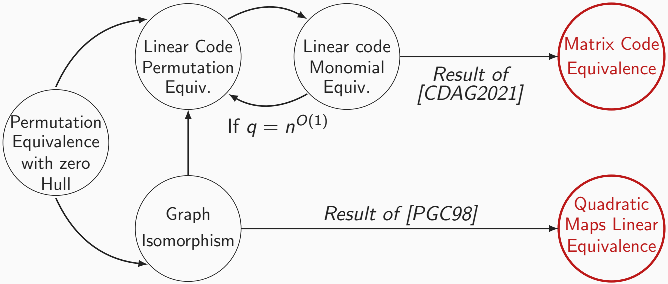
Main idea

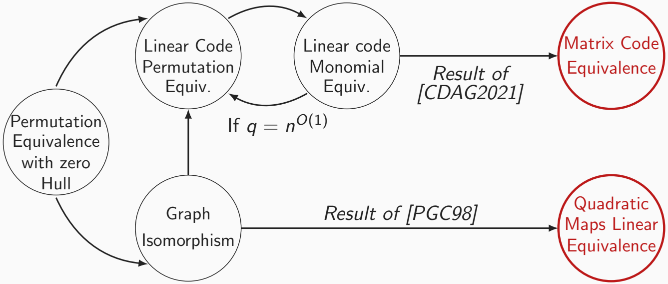
turn QMLE-instance $\mathcal{P}(\mathrm{x}) = \mathcal{F}(\mathrm{xS}) \cdot \mathrm{T}$ into MCE-instance $\tilde{\mathcal{F}} \to \tilde{\mathcal{P}} : \mathrm{F}^{(s)} \mapsto \mathrm{AF}^{(s)}\mathrm{B}$

and vice versa!

Main result


Linear Code Permutation Equiv.
Linear code Monomial Equiv.
Matrix Code Equivalence
Result of [CDAG2021]
If $q = n^{O(1)}$
Permutation Equivalence with zero Hull
Graph Isomorphism
Result of [PGC98]
Quadratic Maps Linear Equivalence

Main result



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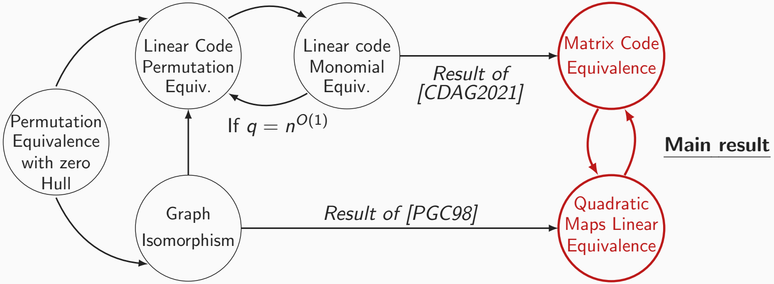

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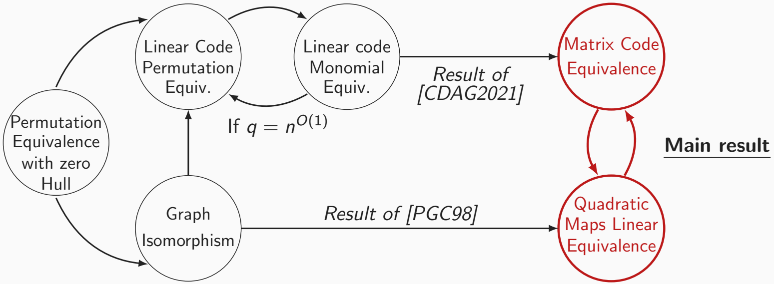

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- Main result of our work: MCE **is equivalent to** QMLE
- Gives **improved upper bound** to complexity of solving MCE (w.l.o.g. assume $m \leqslant n$)
  - solvable in $\mathcal{O}^*(q^{2/3(m+n)})$ time, when $k \leqslant n + m$ can be improved to $\mathcal{O}^*(q^m)$
  - previous upper bound $\mathcal{O}^*(q^{m^2})$ time: brute force smallest side, then solve MCRE

Code equivalence:
a cryptographic group action?

MCE as cryptographic group action.

$$\mu : \mathcal{C} \rightarrow \mathcal{D}$$
$$\mathsf{C} \mapsto \mathsf{ACB}$$

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- So: **Let's use it as a primitive!**

Building crypto from group actions

Cryptographic Group Action: $G \times X \to X$

Given x_1 and x_2, find (if any) an element g s.t. $x_2 = g \cdot x_1$

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What can we do with it?

- **Zero-Knowledge Interactive Proof of knowledge**
  - Zero-Knowledgness
  - soundness
  - can be used as identification scheme (IDS)

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- **Digital Signature via Fiat-Shamir transform**
  - F-S is a common strategy for PQ signatures
    - Dilithium, MQDSS, Picnic in NIST competition
  - From cryptographic group actions
    - Patarin's signature, LESS-FM, CSIDH, SeaSign ...

Zero-Knowledge Interactive Proof of knowledge from group actions

Let g be an element s.t. $x_1 = g \cdot x_0$.

Given x_0, x_1, the prover $\mathcal{P}$ wants to prove to the verifier $\mathcal{V}$ knowledge of g without revealing any information about it

$$\begin{tikzcd} x_0 \arrow[d, dashed, "g"'] \\ x_1 \end{tikzcd}$$

$\mathcal{P}(x_0, x_1, g)$	$\mathcal{V}(x_0, x_1)$

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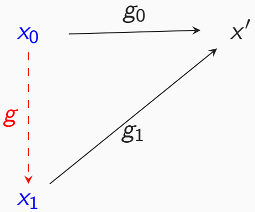

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$$x_0 \xrightarrow{g} x_1 \qquad x'$$

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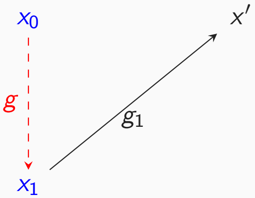

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\mathsf{resp} \leftarrow g_{\mathsf{ch}} & & \\
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\end{array}$$

Matrix Code Equivalence as a cryptographic primitive!

Advertising MCE as a cryptographic primitive

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(4) (mathematically very interesting part of coding theory!)

Thank you for listening!