



Constant-time  
Gauss' algorithm?

## Finite field world

**Q:** Given  $\mathbb{F}_q$  find generator  $\zeta$  for  $\mathbb{F}_q^*$

**A:**

### GAUSS' ALGORITHM

1. Take random  $\zeta \in \mathbb{F}_q$ , compute  $t = \text{Order}(\zeta)$
2. If  $t = q - 1$ , **stop**,
3. **else** take random  $\beta \in \mathbb{F}_q^*$  and compute  $s = \text{Order}(\beta)$ 
  - a. if  $s = q - 1$ , **stop**
  - b. **else** find coprime  $d \mid t$  and  $e \mid s$  with  $d \cdot e = \text{lcm}(t, s)$
  - c. set  $\zeta \leftarrow \zeta^{t/d} \cdot \beta^{s/e}$  and  $t \leftarrow d \cdot e$  and **repeat** from 2.

## Curve world

Given curve  $E$  over  $\mathbb{F}_p$ ,  
find full torsion point  $P$



Take  $P$  and  $Q$ ,  
Compute their torsion.  
If  $P$  not full torsion,  
take right multiple  $Q$   
set  $P \leftarrow P + Q$  to fill  
missing torsion in  $P$   
repeat until full torsion



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