MEDS

Matrix Equivalence Digital Signature

Tung Chou, Ruben Niederhagen, Edoardo Persichetti,
Lars Ran, Tovohery Hajatiana Randrianarisoa,
Krijn Reijnders, Simona Samardjiska, Monika Trimoska

Slides by: Krijn Reijnders

Lars Ran
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Radboud University

MEDS: a new code-based signature scheme

1

Matrix Code Equivalence

2

From MCE to MEDS

3

Performance

Matrix Code Equivalence

1

Matrix Code Equivalence

matrix code

A k-dimensional subspace $\mathscr{C} \subseteq \mathbb{F}_q^{m \times n}$ equipped with the *rank metric*

$$d(C_1, C_2) = \mathrm{Rank}(C_1 - C_2) \qquad C_1, C_2 \in \mathscr{C}$$

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$$d(C_1, C_2) = \mathrm{Rank}(C_1 - C_2) \qquad C_1, C_2 \in \mathscr{C}$$

Two matrix codes $\mathscr{C}$ and $\mathscr{D}$ are *equivalent* if we have a linear map $\mu : \mathscr{C} \to \mathscr{D}$ that preserves the metric (isometry): $\mathrm{Rank}\,\mu(C) = \mathrm{Rank}\,C, \quad \forall C \in \mathscr{C}$

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Matrix Code Equivalence

$$A = \begin{bmatrix} 0 & 0 & 5 & 7 \\ 5 & 1 & 2 & 7 \\ 0 & 4 & 4 & 0 \\ 4 & 3 & 7 & 7 \end{bmatrix} \in \mathrm{GL}_m(q)$$

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we get $ACB \in \mathscr{D}$ for all $C \in \mathscr{C}$

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✓ we get $ACB \in \mathscr{D}$ for all $C \in \mathscr{C}$

✓ the map $\mu = (A, B)$ preserves rank!

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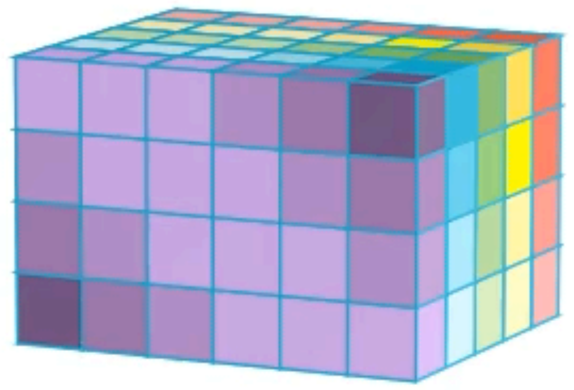
Matrix Code Equivalence

3-tensor

Can think of a matrix code as a 3-tensor over $\mathbb{F}_q$

Equivalence then becomes *tensor isomorphism*

$$\mathscr{C} \subseteq \mathbb{F}_q^{m\times n\times k}$$

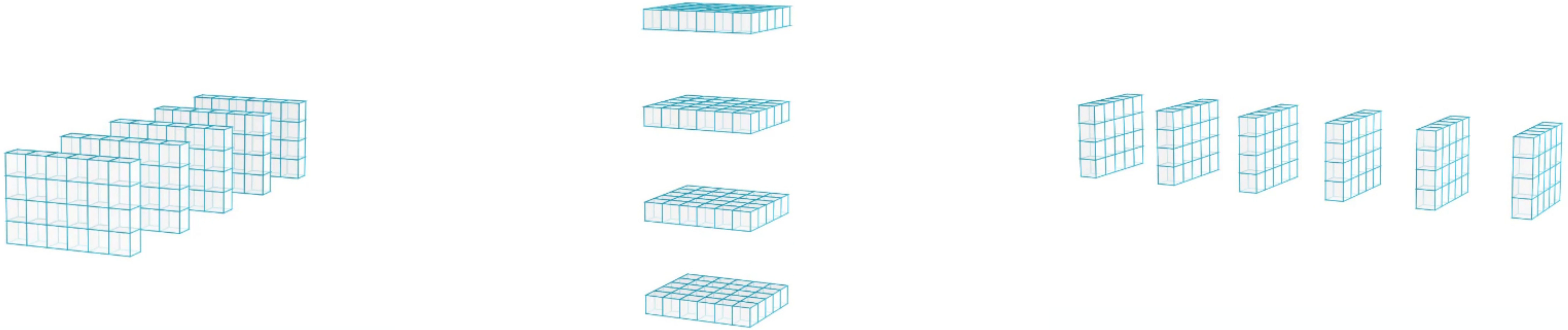


Matrix Code Equivalence

symmetry

Viewed as a 3-tensor, we can see $\mathscr{C}$ using three orientations

- a k-dimensional code in $\mathbb{F}_q^{m \times n}$
- an m-dimensional code in $\mathbb{F}_q^{n \times k}$
- an n-dimensional code in $\mathbb{F}_q^{m \times k}$



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Matrix Code Equivalence

combinatorial

Attacks using isometry-invariant substructures

Example: *find low-rank codewords in both codes and construct collisions using the birthday paradox*

- Graph-based algorithm
- Leon's like algorithm

$$\tilde{\mathcal{O}}(q^{\min(n,m,k)})$$

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algebraic

Attacks reducing MCE to solving a system of polynomial equations using Gröbner basis techniques

Example: *use the tensor isomorphism formulation to get a trilinear system* ***or***, *consider transformed codewords AC_iB as dual to the dual code $\mathscr{D}^\perp$*

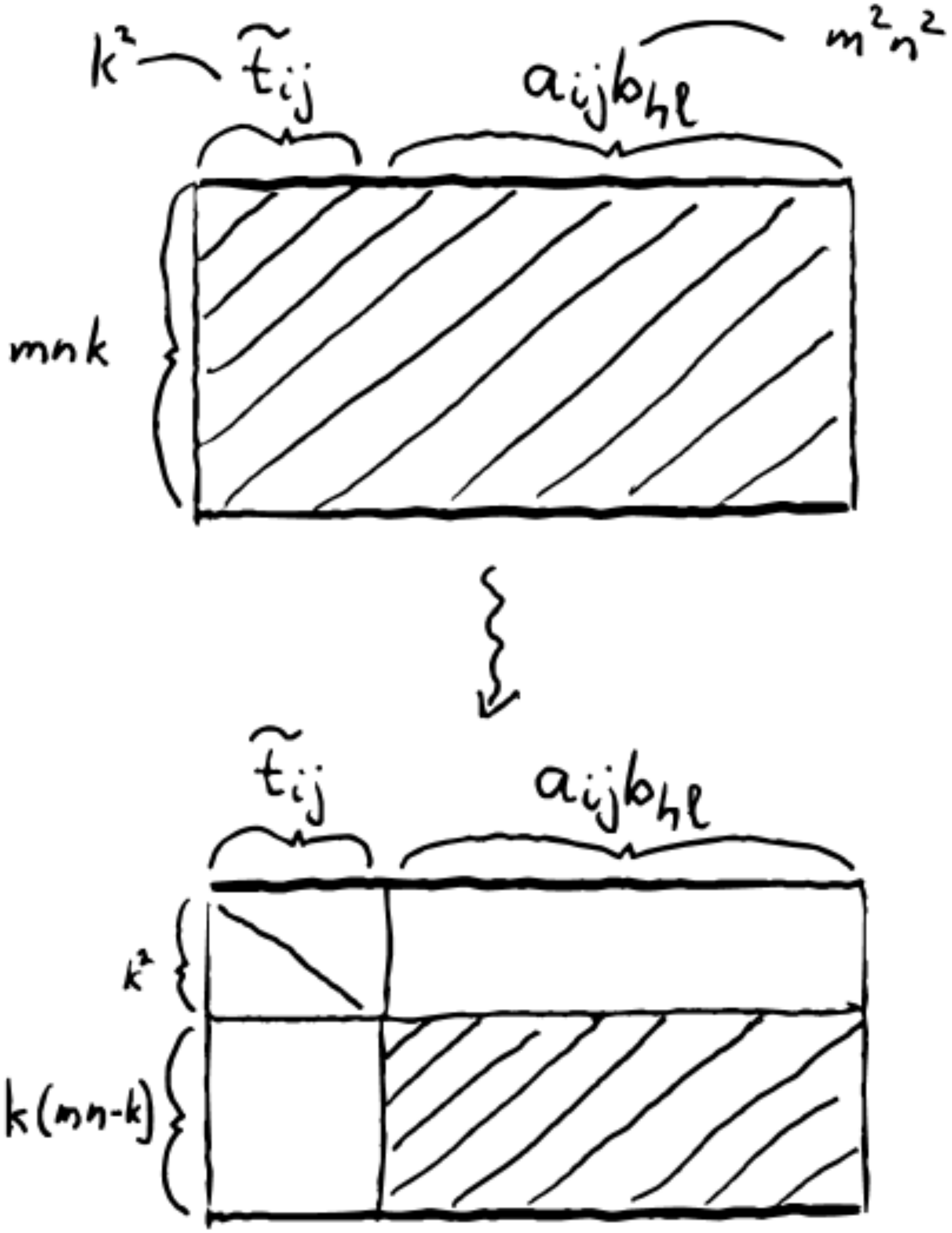
- direct modelling
- minor's modelling
- *improved* modelling

$$\mathcal{O}\left(n^{\omega\frac{n}{4}}\right)$$

Matrix Code Equivalence

equations

$$\mathscr{C}(Ax, By, z) = \mathscr{D}(x, y, T^{-1}z)$$



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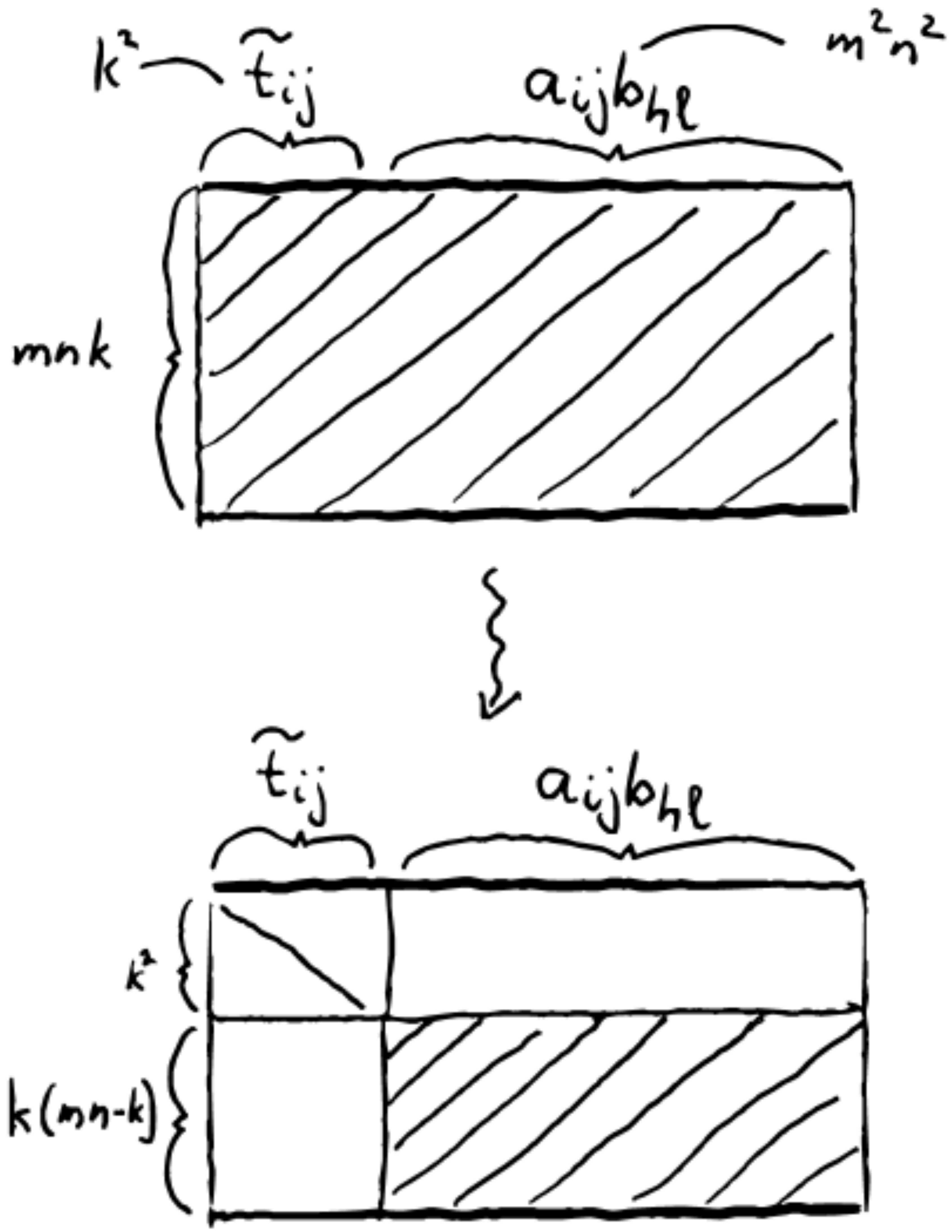


system

Three bilinear systems:

$$\mathscr{C}(Ax, By, z) = \mathscr{D}(x, y, T^{-1}z)$$

$$\mathscr{C}(Ax, y, Tz) = \mathscr{D}(x, B^{-1}y, z)$$

$$\mathscr{C}(x, By, Tz) = \mathscr{D}(A^{-1}x, y, z)$$

Equations:
$k(nm - k) + m(kn - m) + n(mk - n)$

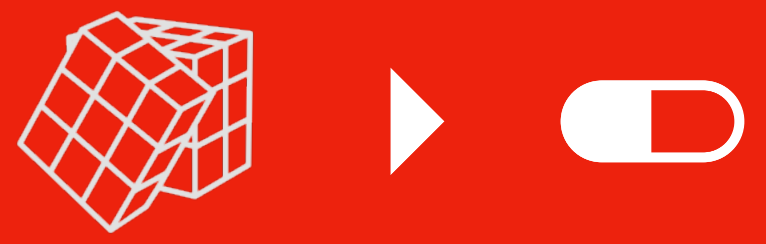
Variables:
$n^2 + m^2 + k^2$

From MCE to MEDS

From MCE to MEDS

1 equivalence relation → 2 zero knowledge identification scheme → 3 signature scheme!

2



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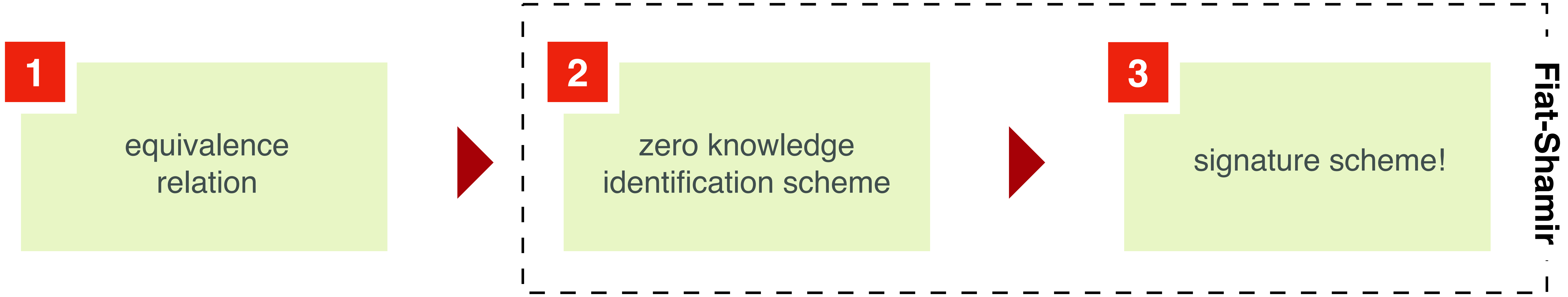

From MCE to MEDS

1 equivalence relation → 2 zero knowledge identification scheme → 3 signature scheme!

Fiat-Shamir

1 → 2

SETUP

- Assume parameter set $q, n, m, k.$ and "starting" code $\mathscr{C}$
- Generate **secret key** $A \in \mathrm{GL}_{\mathrm{m}}(q)$, $B \in \mathrm{GL}_n(q)$
- Generate **public key** $\mathscr{D} = A\mathscr{C}B$

$$
\begin{tikzcd}
\mathscr{C} \arrow[d, "{(A, B)}"'] \\
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$$\begin{tikzcd} \mathscr{C} \arrow[r, "(\tilde{A}, \tilde{B})"] \arrow[d, "(A, B)"'] & \tilde{\mathscr{C}} \\ \mathscr{D} & \end{tikzcd}$$

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CHALLENGE

- Pick a bit $b \in \{0,1\}$

RESPONSE

- if $b = 0$, reply with $(\tilde{A}, \tilde{B})$
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RESPONSE

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- if $b = 1$, reply with $(\tilde{A} \cdot A^{-1}, B^{-1} \cdot \tilde{B})$

$$
\begin{tikzcd}
\mathscr{C} \arrow[r, "(\tilde{A}, \tilde{B})"] \arrow[d, "(A, B)"'] & \tilde{\mathscr{C}} \\
\mathscr{D} \arrow[ur, "(\tilde{A} \cdot A^{-1}, B^{-1} \cdot \tilde{B})"] &
\end{tikzcd}
$$

soundness 1/2

From MCE to MEDS

naive approach

$$\begin{tikzcd}
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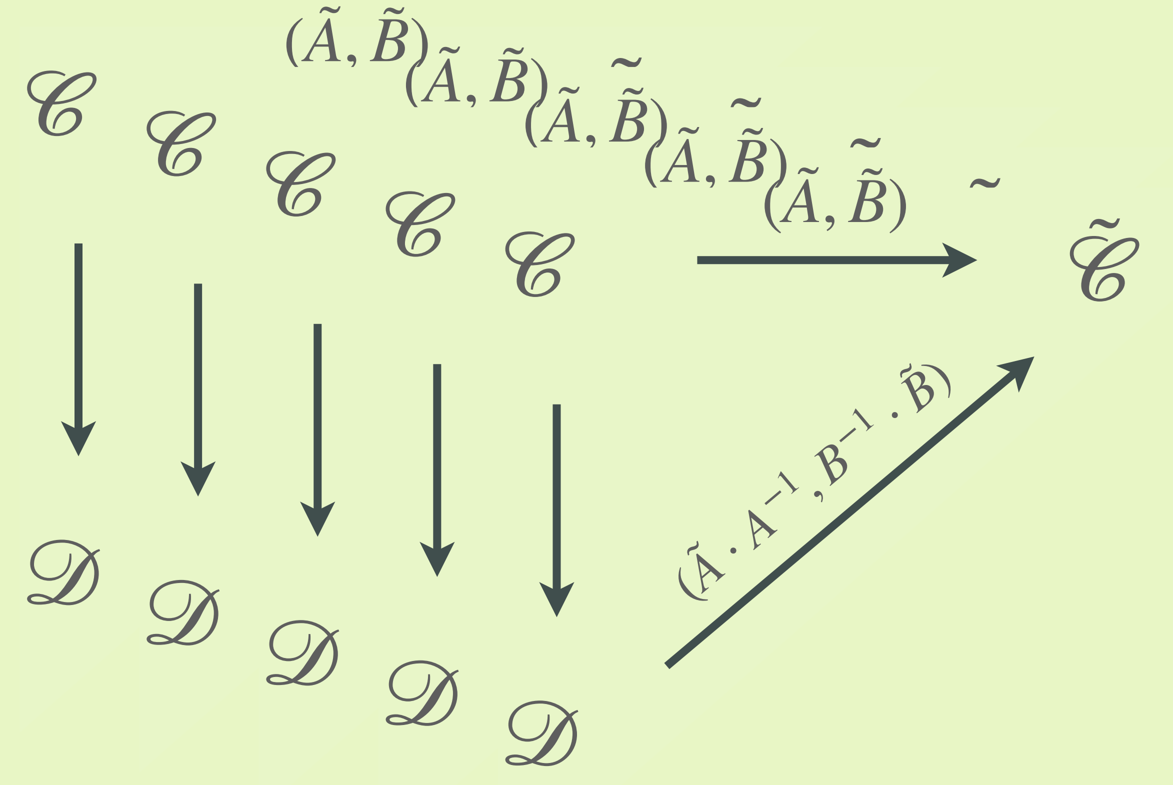
repeat t times



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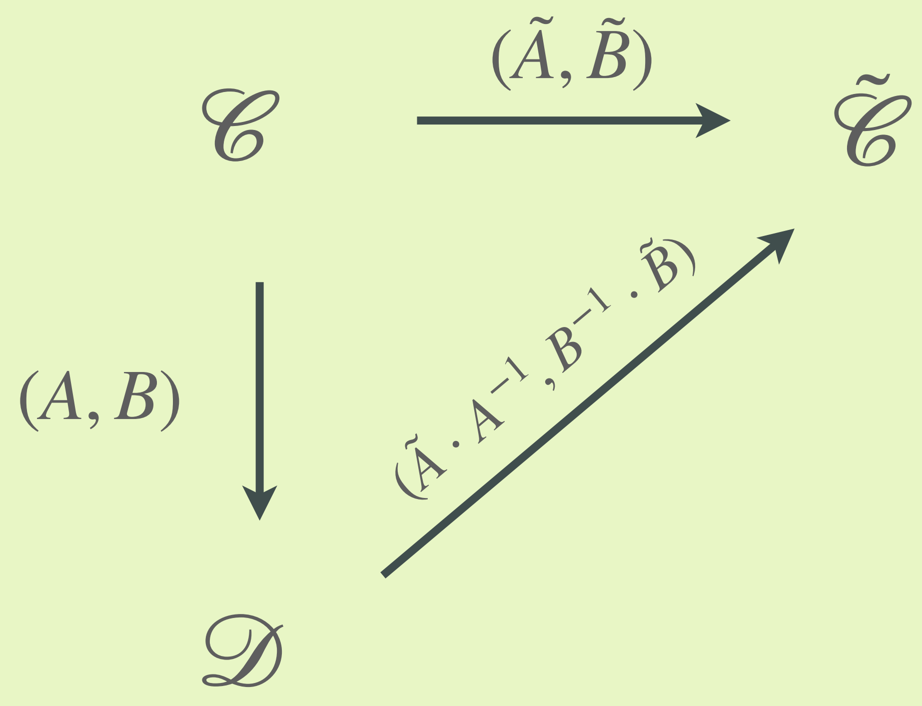

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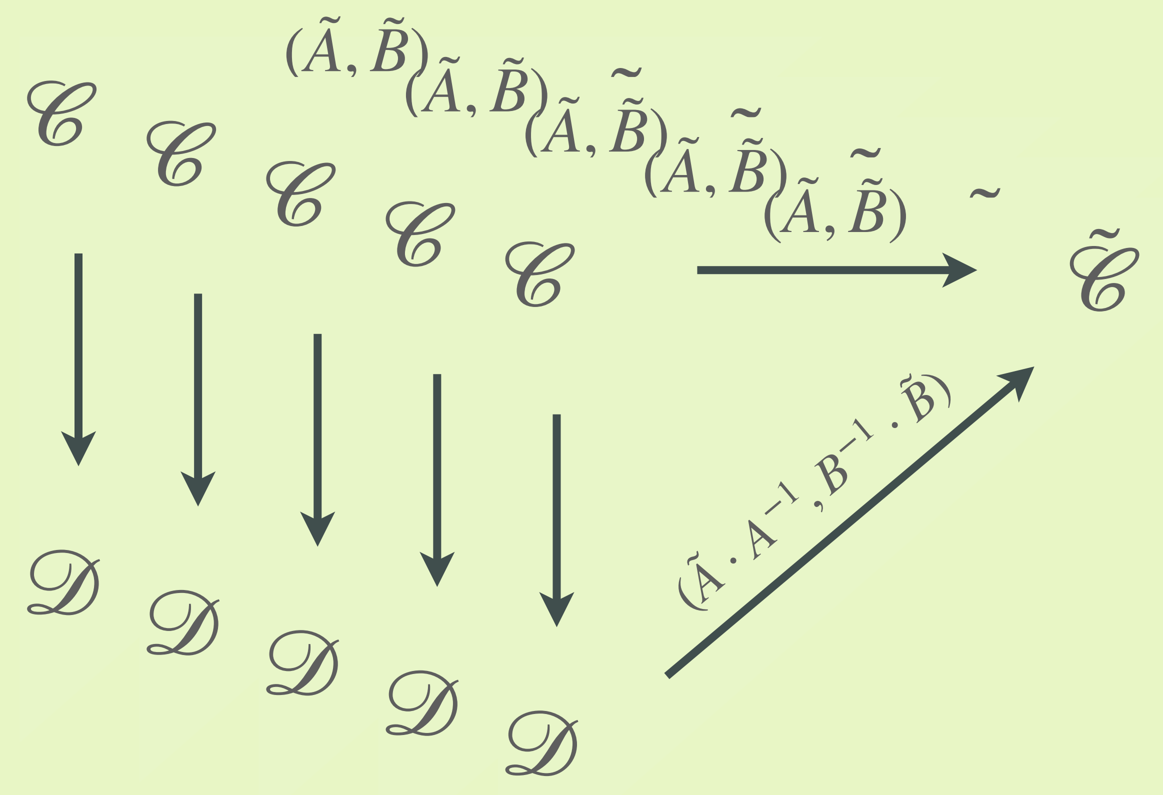
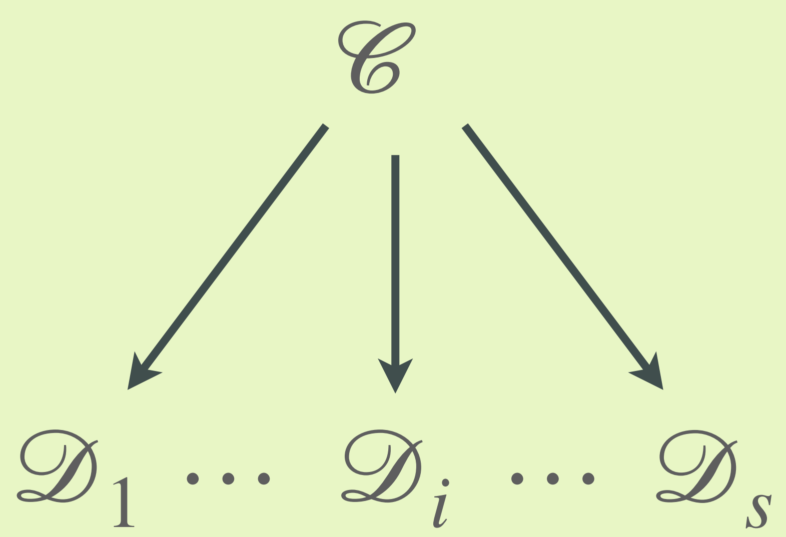


1

multiple pk

[1]



provide s public keys,
$b \in \{1,\ldots,s\}$
response is an isometry
$\mathscr{D}_b \to \tilde{\mathscr{C}}$ or $\mathscr{C} \to \tilde{\mathscr{C}}$

[1] L. De Feo and S. D. Galbraith. SeaSign: Compact isogeny signatures from class group actions. EUROCRYPT 2019.
[2] W. Beullens, S, Katsumata, and F. Pintore. Calamari and Falafl: Logarithmic (linkable) ring signatures from isogenies and lattices. ASIACRYPT 2020.

2

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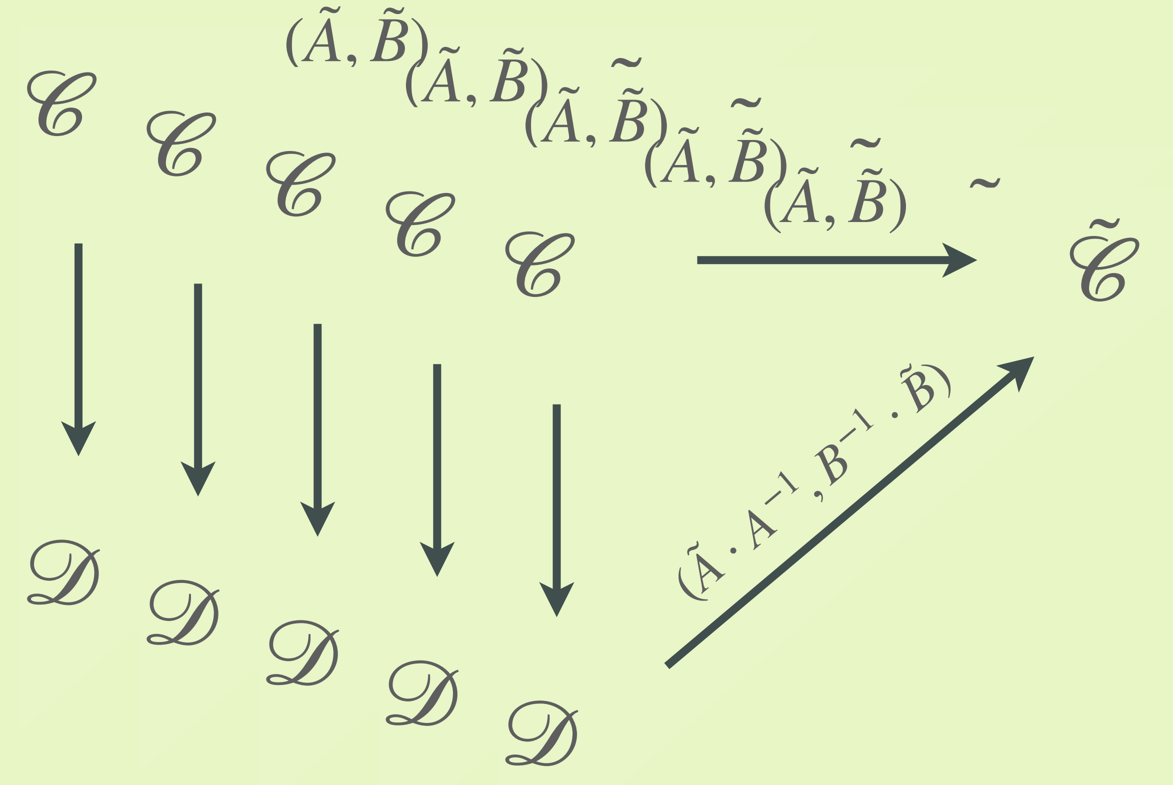

1 **multiple pk** [1]

$$\begin{tikzcd} & \mathscr{C} \arrow[dl] \arrow[d] \arrow[dr] & \\ \mathscr{D}_1 \cdots & \mathscr{D}_i & \cdots \mathscr{D}_s \end{tikzcd}$$

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2 **fix weight** [2]

- generate $\mathscr{C} \to \tilde{\mathscr{C}}$ from seed
- respond to $b = 0$ with seed
- response much cheaper!

adjust probability so that
$b = 0$ appears more

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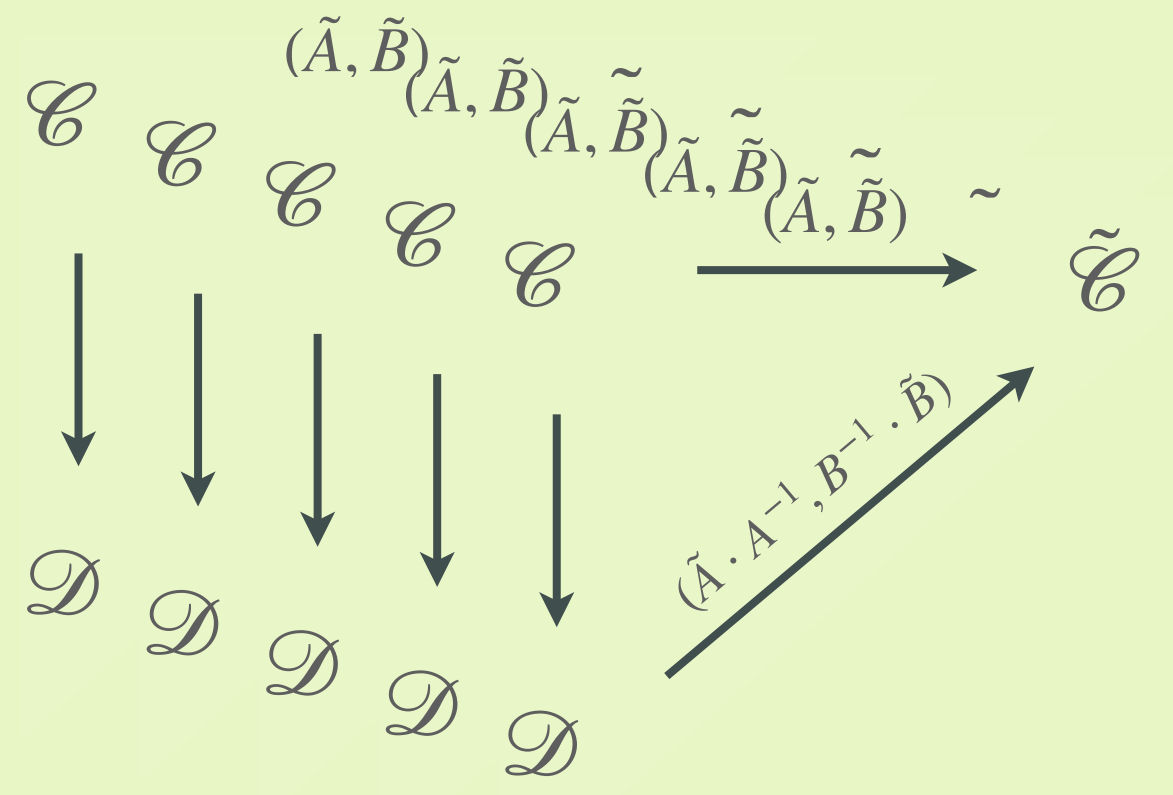

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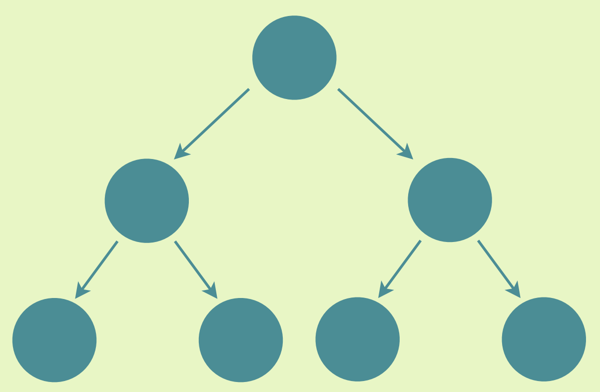
3 seed tree [2]

instead of sending t seeds, send tree



to reveal nodes $N_1, \ldots, N_w$,
communicate $N_1 \ldots, N_w$ and for the
$t - w$ remaining nodes only appropriate
parent nodes

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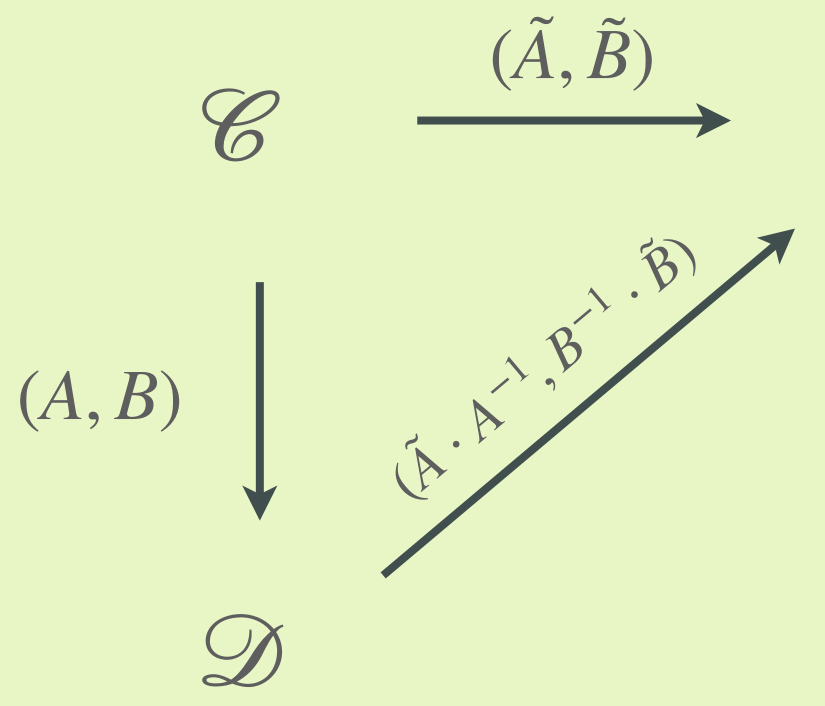

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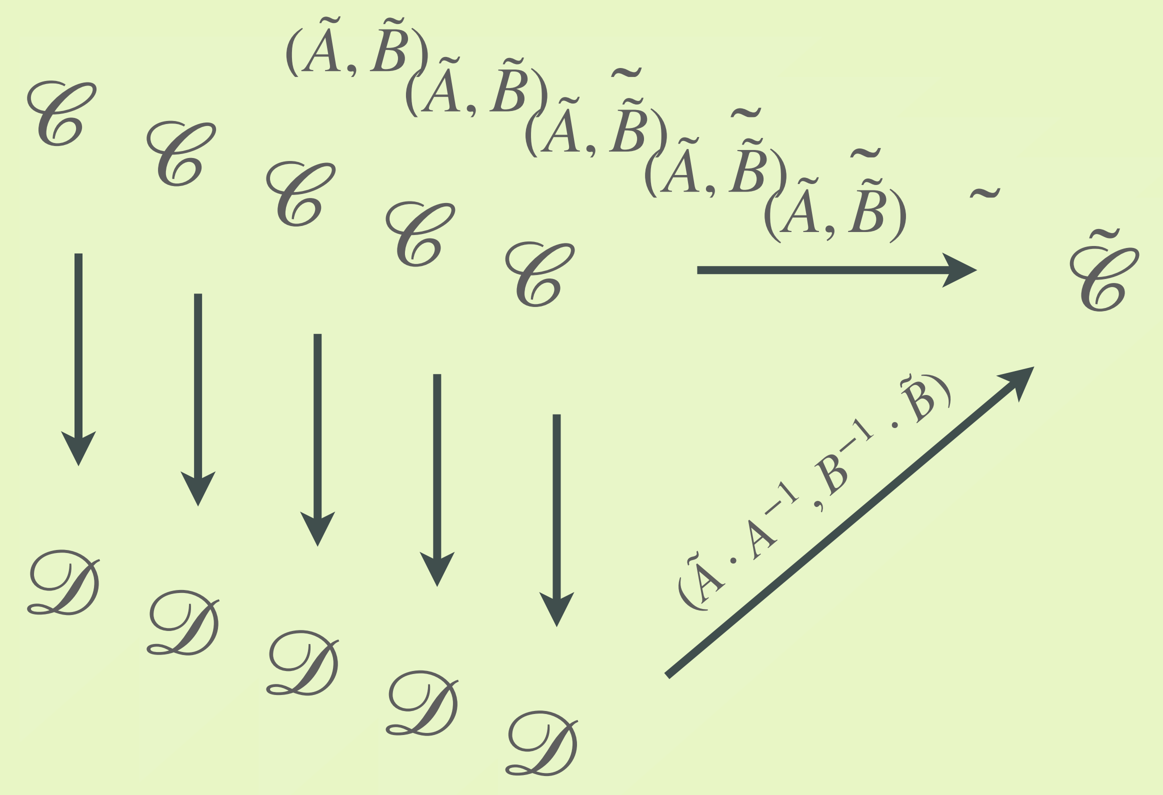

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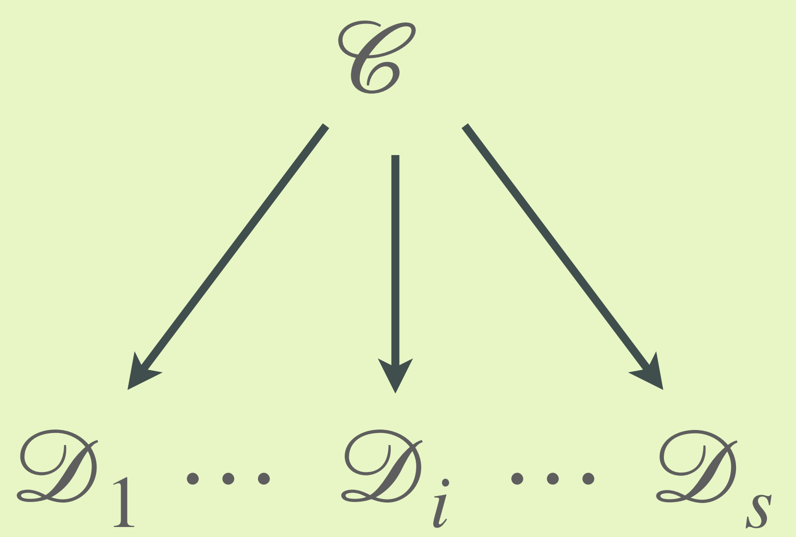

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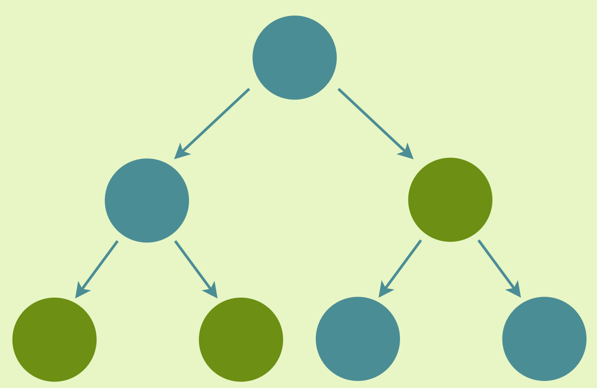

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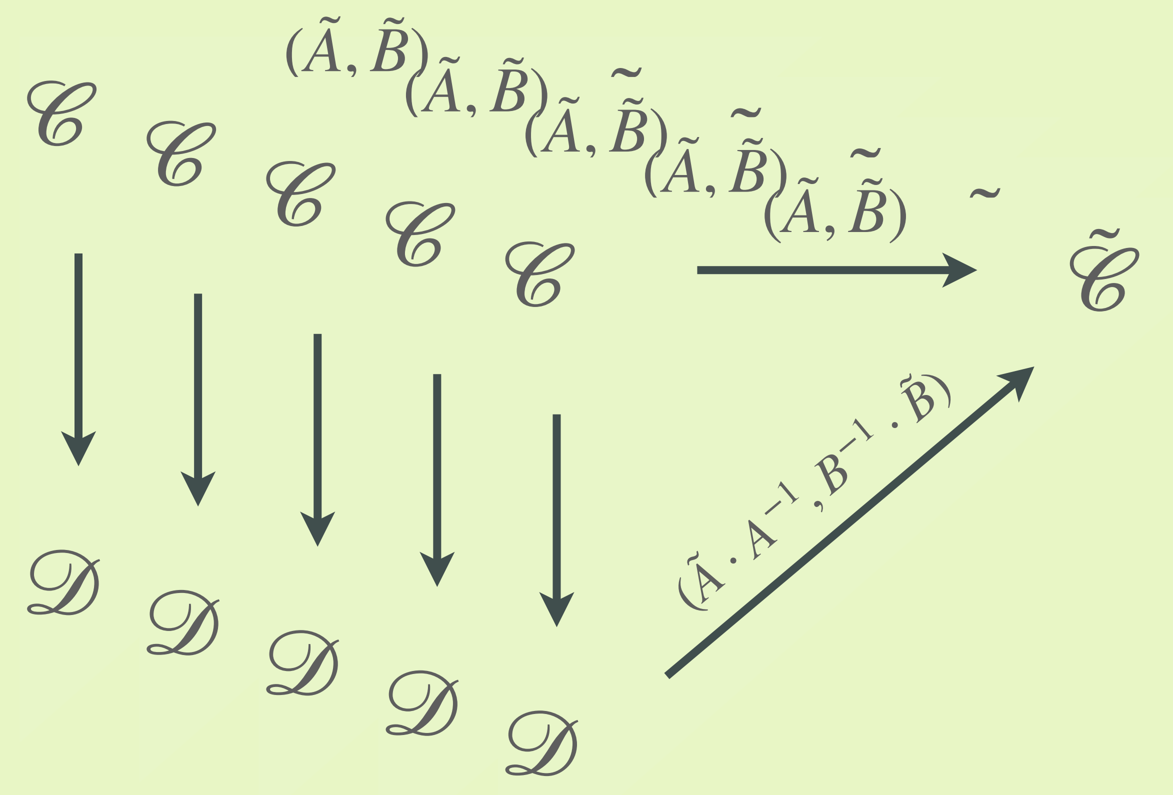

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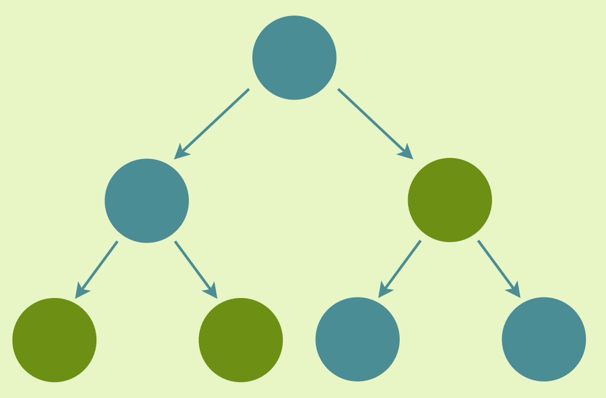

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4 compression

[3,4]

instead of generating A_i, B_i from seed and computing $\mathscr{D}_i = A_i \cdot \mathscr{C} \cdot B_i$

generate part of $\mathscr{D}_i$ from seed. compute appropriate A_i, B_i and rest of $\mathscr{D}_i$

Hint: this does not break MCE!

[3] J. Ding, M-S Chen, A. Petzoldt, D. Schmidt, B-Y. Yang, M. Kannwischer, and J. Patarin. Rainbow. NIST 2020.
[4] W. Beullens, M-S. Chen, S-H. Hung, M. Kannwischer, B. Peng, C-J. Shih, and B-Y. Yang. Oil and Vinegar: Modern parameters and implementations.

2

From MCE to MEDS

improved compression (ongoing work)

MCE can be solved efficiently with two full rank collisions

▼

present two collisions as proof of knowledge of the secret isometry

two collisions

$$AP_0B = R_0, \quad AP_1B = R_1$$

$$\Updownarrow$$

$$AP_0 = R_0B^{-1}, \quad AP_1 = R_1B^{-1}$$

isometry diagram

$$\begin{tikzcd}
P_0, P_1 \arrow[r, mapsto, "(\tilde{A}, \tilde{B})"] \arrow[d, mapsto, "(A, B)"'] & R_0, R_1 \\
AP_0B, AP_1B \arrow[ur, mapsto, "(\tilde{A}A^{-1}, \tilde{B}B^{-1})"'] &
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$$A P_0 B = R_0, \quad A P_1 B = R_1$$

$$\Downarrow$$

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challenge response

$$\begin{tikzcd}
\overset{b=0}{P_0, P_1} \arrow[r, mapsto] \arrow[d, mapsto] & R_0, R_1 \\
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\end{tikzcd}$$

Performance of MEDS

3

Performance

parameters	q	n = m = k	t (rounds)	s (no. of pk's)	w (seed tree)	Public Key (bytes)	Signature (bytes)
MEDS-9923	4093	14	1152	4	14	9923	9896
MEDS-13220	4093	14	192	5	20	13220	12976
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- single hardness assumption: **MCE**
- simple design and arithmetic
- great flexibility in sizes
- *generic*: room for improvements!

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- scaling to higher parameters
- needs more research on **MCE**
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advancing

- new technique to reduce sig. size
- MEDS-13220 to **2088** bytes (-84%)
- still analysing security of technique
- *explore*: potential for new ideas!

Thank you for your attention!

https://www.meds-pqc.org/