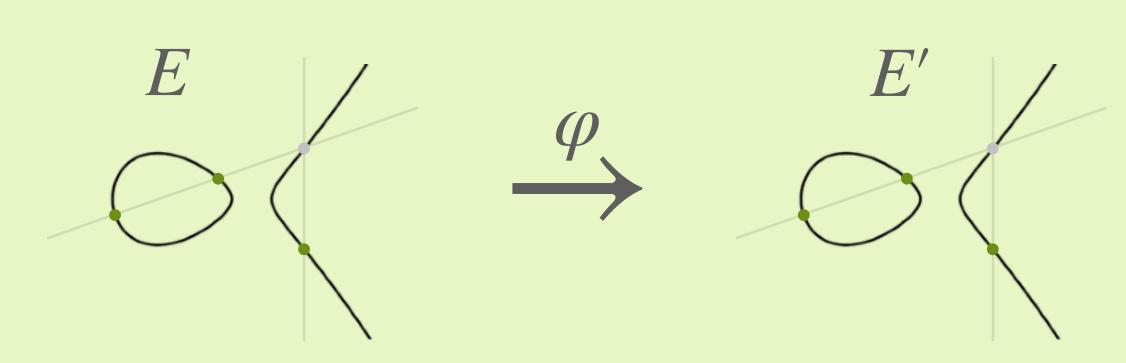
PART 1 SQIsign

WARNING!

- SQIsign is a difficult scheme, especially signing
- To keep this talk "down to earth", I will **simplify** a lot
- This will increase clarity and intuition by being **hand-wavy**, at the cost of rigor

isogenies



Isogeny

- "nice" map φ (group homomorphism) between elliptic curves $E \to E'$
- given by rational functions: a point $(x,y) \in E$ is mapped to $(f_1(x,y)/f_2(x,y), g_1(x,y)/g_2(x,y))$
- size of $\ker \varphi$ is same as degree of φ !

toy example

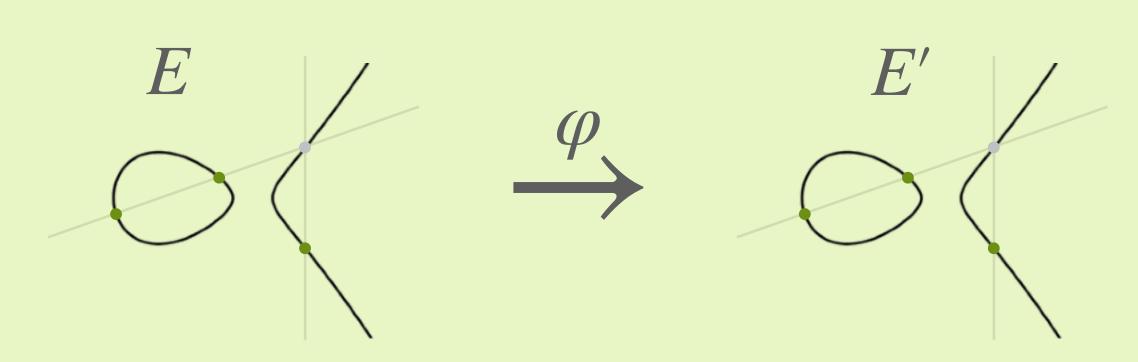
$$E: y^{2} = x^{3} + x \qquad \varphi \qquad E': y^{2} = x^{3} + 5$$

$$(x,y) \mapsto \left(\frac{x^{3} + x^{2} + x + 2}{(x-5)^{2}}, \frac{y(x^{3} - 4x^{2} + 2)}{(x-5)^{3}}\right) \quad \text{over } \mathbb{F}_{11}$$

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$$(x,y) \mapsto \left(\frac{x^3 + x^2 + x + 2}{(x-5)^2}, \frac{y(x^3 - 4x^2 + 2)}{(x-5)^3}\right)$$
 over \mathbb{F}_{11}

Can check

- this is a group homomorphism: $\varphi(\mathcal{O}) = \mathcal{O}'$ and $\varphi(P+Q) = \varphi(P) + \varphi(Q)$
- kernel: $\varphi(P) = \mathscr{O}'$ when $P = \mathscr{O}$ or $x_P = 5$, so P = (5,3) and P = (5,-3)
- so φ is of degree 3 and we can say E and E' are 3-isogenous