3

Applying pairings in isogeny crypto

✔ **fast pairings**

Optimized pairing computation for the specific scenario $P \in E(\mathbb{F}_p), Q \in E^t(\mathbb{F}_p)$

&

✔ **core idea**

For $P \in E(\mathbb{F}_p)$ and $Q \in E^t(\mathbb{F}_p)$, don't use curve arithmetic but pairing $e(P, Q)$ to get overlap in orders!

Faster isogeny subroutines

verify full torsion P

In some CSIDH variants, we are given $P \in E(\mathbb{F}_p)$ and $Q \in E^t(\mathbb{F}_p)$.

Q: verify that both P and Q have order $p + 1$, e.g. full torsion points

A: compute $\zeta = e(P, Q)$ and check that order ζ is $p + 1$.

✔

speedup: -75%

compute full torsion P

In some CSIDH variants, we get E

Q: find $P \in E(\mathbb{F}_p)$ and $Q \in E^t(\mathbb{F}_p)$ of order $p + 1$, e.g. full torsion points

A: take random, P, Q, then find $\zeta = e(P, Q)$. Compute order ζ and apply Gauss' algorithm.

✔

speedup: case dependent, up to -75%

verify supersingularity

In some CSIDH variants, we get E

Q: is E even supersingular? verify that it is!

A: take random, P, Q, then find $\zeta = e(P, Q)$. Verify order $\zeta \geq 4\sqrt{p}$.

✔

speedup: -27% compared to CSIDH's

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3

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?

speedup: +2% compared to Doliskani

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