E
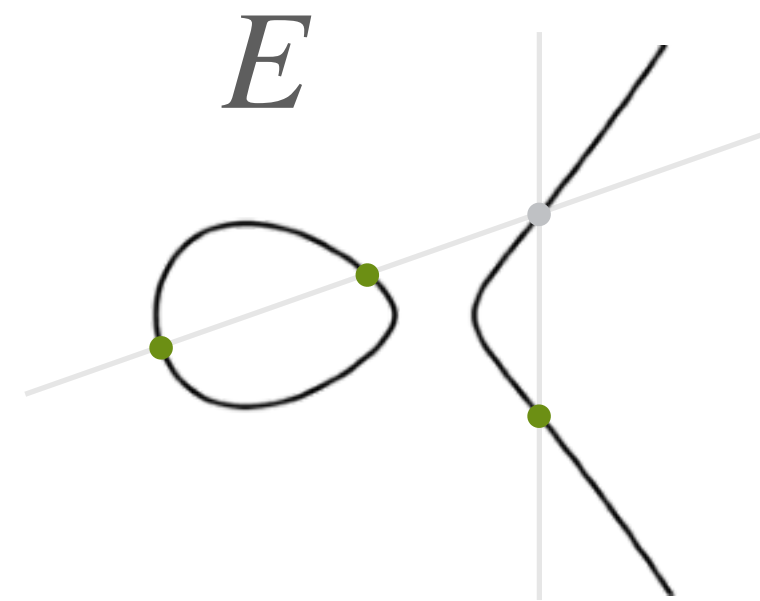
Given just any E over $\mathbb{F}_{q'}$, we just saw the endomorphisms

- multiplication-by-n, so $[n] : P \mapsto P + \ldots + P$ for any $n \in \mathbb{Z}$
- Frobenius π and easily also $[n] \cdot \pi$ for any $n \in \mathbb{Z}$
- we write this as $\mathbb{Z} + \pi\mathbb{Z} \subseteq \mathrm{End}(E)$

Note: applying π twice gives $\pi^2 = [-p]$, so no "new" endom.

endomorphism ring

- we can "add together" different endomorphisms
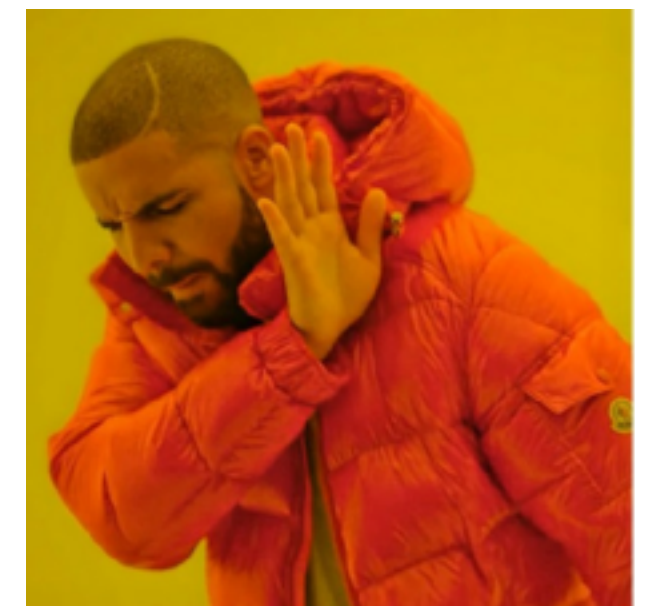
$$(\varphi + \psi)(P) = \varphi(P) + \psi(P)$$

- we can "multiply" endomorphisms by composition

$$(\varphi \cdot \psi)(P) = \varphi(\psi(P))$$

- so, we get a ring structure $\mathrm{End}(E)$, by our examples dimension is at least 2
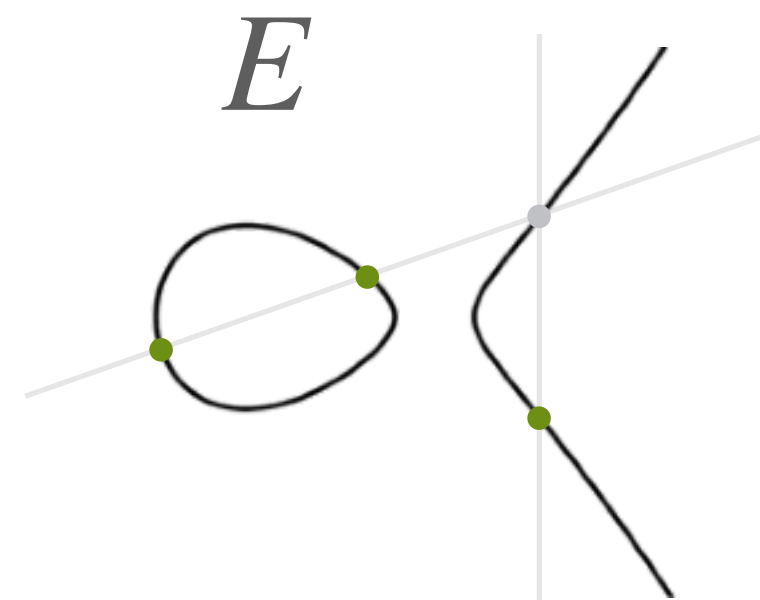
if dim 2

E is **ordinary**

E

Given just any E over $\mathbb{F}_{q'}$ we just saw the endomorphisms

· multiplication-by-n, so $[n] : P \mapsto P + \ldots + P$ for any $n \in \mathbb{Z}$

· Frobenius π and easily also $[n] \cdot \pi$ for any $n \in \mathbb{Z}$

· we write this as $\mathbb{Z} + \pi\mathbb{Z} \subseteq \mathrm{End}(E)$

Note: applying π twice gives $\pi^2 = [-p]$, so no "new" endom.

endomorphism ring

· we can "add together" different endomorphisms

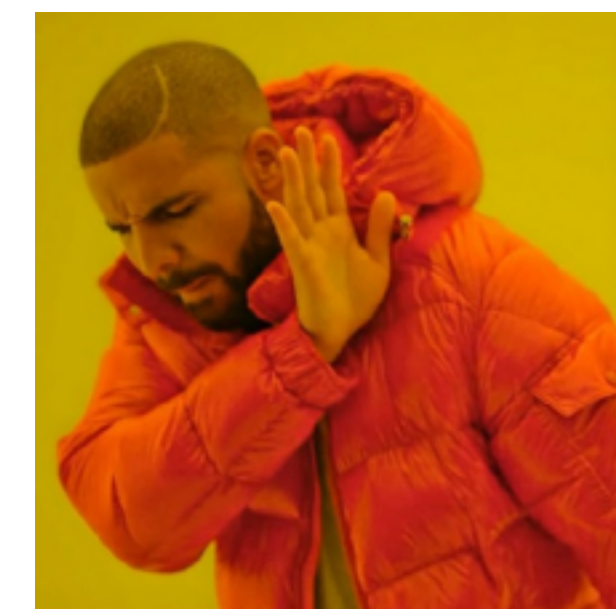$$(\varphi + \psi)(P) = \varphi(P) + \psi(P)$$

· we can "multiply" endomorphisms by composition

$$(\varphi \cdot \psi)(P) = \varphi(\psi(P))$$

· so, we get a ring structure $\mathrm{End}(E)$, by our examples dimension is at least 2

if dim 2

E is **ordinary**

if dim 4

E is
✨✨ **supersingular** ✨✨
(weird funky maps!!)