

Matrix Code Equivalence

matrix code

A k-dimensional subspace $\mathscr{C} \subseteq \mathbb{F}_q^{m \times n}$ equipped with the rank metric

$$d(C_1, C_2) = \operatorname{Rank}(C_1 - C_2) \qquad C_1, C_2 \in \mathscr{C}$$

$$q = 13, \quad m = 4, \quad n = 6, \quad k = 5$$

$$C = \lambda_{1} \cdot \begin{bmatrix} 2 & 8 & 10 & 4 & 5 & 7 \\ 1 & 11 & 7 & 9 & 6 & 12 \\ 3 & 0 & 13 & 5 & 4 & 8 \\ 9 & 6 & 3 & 2 & 10 & 11 \end{bmatrix} + \lambda_{2} \cdot \begin{bmatrix} 12 & 0 & 4 & 11 & 9 & 3 \\ 5 & 6 & 8 & 13 & 2 & 1 \\ 10 & 7 & 3 & 9 & 4 & 6 \\ 2 & 5 & 11 & 8 & 1 & 10 \end{bmatrix} + \lambda_{3} \cdot \begin{bmatrix} 5 & 2 & 9 & 11 & 4 & 8 \\ 3 & 7 & 1 & 10 & 12 & 0 \\ 6 & 9 & 2 & 13 & 11 & 8 \\ 1 & 5 & 6 & 3 & 10 & 7 \end{bmatrix} + \lambda_{4} \cdot \begin{bmatrix} 9 & 4 & 6 & 1 & 13 & 2 \\ 8 & 0 & 5 & 12 & 6 & 11 \\ 3 & 7 & 10 & 9 & 4 & 5 \\ 2 & 8 & 11 & 3 & 7 & 1 \end{bmatrix} + \lambda_{5} \cdot \begin{bmatrix} 7 & 10 & 4 & 6 & 8 & 3 \\ 1 & 5 & 2 & 11 & 9 & 0 \\ 13 & 7 & 6 & 4 & 12 & 2 \\ 8 & 3 & 1 & 9 & 5 & 10 \end{bmatrix}$$

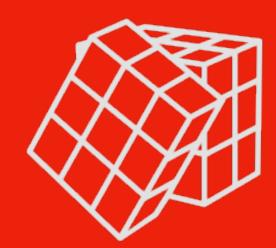
$$\lambda_{i} \in \mathbb{F}_{q}$$



$$D = \lambda_1 \cdot \begin{bmatrix} 4 & 12 & 9 & 9 & 12 & 12 \\ 6 & 3 & 2 & 2 & 5 & 7 \\ 5 & 7 & 12 & 12 & 0 & 6 \\ 12 & 3 & 7 & 12 & 2 & 7 \end{bmatrix} + \lambda_2 \cdot \begin{bmatrix} 0 & 1 & 12 & 9 & 1 & 9 \\ 11 & 2 & 0 & 11 & 5 & 6 \\ 9 & 6 & 9 & 10 & 11 & 0 \end{bmatrix} + \lambda_3 \cdot \begin{bmatrix} 1 & 1 & 3 & 9 & 3 & 7 \\ 9 & 5 & 12 & 9 & 1 & 1 \\ 4 & 3 & 7 & 12 & 10 & 7 \\ 7 & 4 & 9 & 3 & 2 & 4 \end{bmatrix} + \lambda_4 \cdot \begin{bmatrix} 2 & 12 & 2 & 3 & 4 & 5 \\ 12 & 9 & 10 & 6 & 12 & 1 \\ 3 & 3 & 11 & 11 & 11 & 2 \\ 9 & 6 & 0 & 12 & 11 & 7 \end{bmatrix} + \lambda_5 \cdot \begin{bmatrix} 10 & 2 & 12 & 8 & 9 & 9 \\ 2 & 10 & 2 & 11 & 1 & 11 \\ 9 & 2 & 9 & 10 & 3 & 6 \\ 9 & 11 & 7 & 10 & 11 & 6 \end{bmatrix}$$

$$\lambda_i \in \mathbb{F}_q$$





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Two matrix codes $\mathscr C$ and $\mathscr D$ are *equivalent* if we have a linear map $\mu:\mathscr C\to\mathscr D$ that preserves the metric (isometry): Rank $\mu(C)=\operatorname{Rank} C$, $\forall C\in\mathscr C$



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$$\lambda_{i} \in \mathbb{F}_{q}$$



$$D = \lambda_1 \cdot \begin{bmatrix} 4 & 12 & 9 & 9 & 12 & 12 \\ 6 & 3 & 2 & 2 & 5 & 7 \\ 5 & 7 & 12 & 12 & 0 & 6 \\ 12 & 3 & 7 & 12 & 2 & 7 \end{bmatrix} + \lambda_2 \cdot \begin{bmatrix} 0 & 1 & 12 & 9 & 1 & 9 \\ 11 & 2 & 0 & 11 & 5 & 6 \\ 9 & 6 & 9 & 10 & 11 & 0 \end{bmatrix} + \lambda_3 \cdot \begin{bmatrix} 1 & 1 & 3 & 9 & 3 & 7 \\ 9 & 5 & 12 & 9 & 1 & 1 \\ 4 & 3 & 7 & 12 & 10 & 7 \\ 7 & 4 & 9 & 3 & 2 & 4 \end{bmatrix} + \lambda_4 \cdot \begin{bmatrix} 2 & 12 & 2 & 3 & 4 & 5 \\ 12 & 9 & 10 & 6 & 12 & 1 \\ 3 & 3 & 11 & 11 & 11 & 2 \\ 9 & 6 & 0 & 12 & 11 & 7 \end{bmatrix} + \lambda_5 \cdot \begin{bmatrix} 10 & 2 & 12 & 8 & 9 & 9 \\ 2 & 10 & 2 & 11 & 1 & 11 \\ 9 & 2 & 9 & 10 & 3 & 6 \\ 9 & 11 & 7 & 10 & 11 & 6 \end{bmatrix}$$

$$\lambda_i \in \mathbb{F}_q$$

