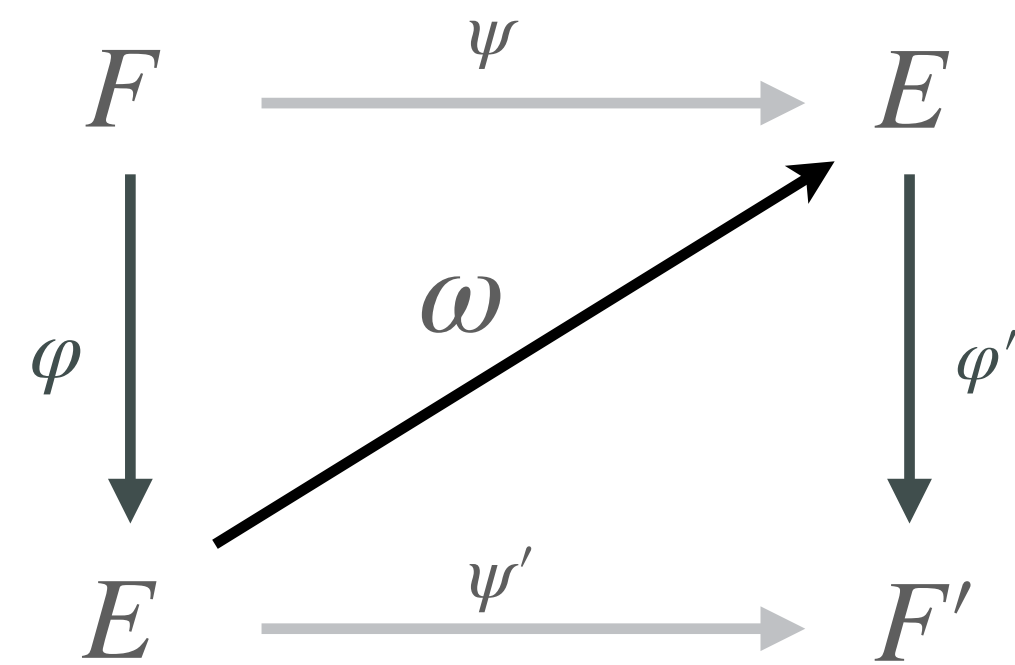


## PART 4 2D Future

### Nakagawa - Onuki trick (2023)



say we want to create such  
a square, but we only have  $E$   
and some  $\omega \in \text{End}(E)$   
of degree  $q(2^a - q)$

we can find a suitable isogeny  
 $\varphi: F \rightarrow E$  using Kani!!!



1

If the square above existed,  
then Kani's lemma should apply

should give 2D isogeny  
 $\Phi: E \times E \rightarrow F \times F'$   
of degree  $2^a$

2

(ignoring some isogeny maths)  
then the kernel of  $\Phi$  should be  
given by  $[q]P, \omega(P)$  for  $P \in E[2^a]$

But we know these!!  
We can compute 2D  $\Phi$  using Kani

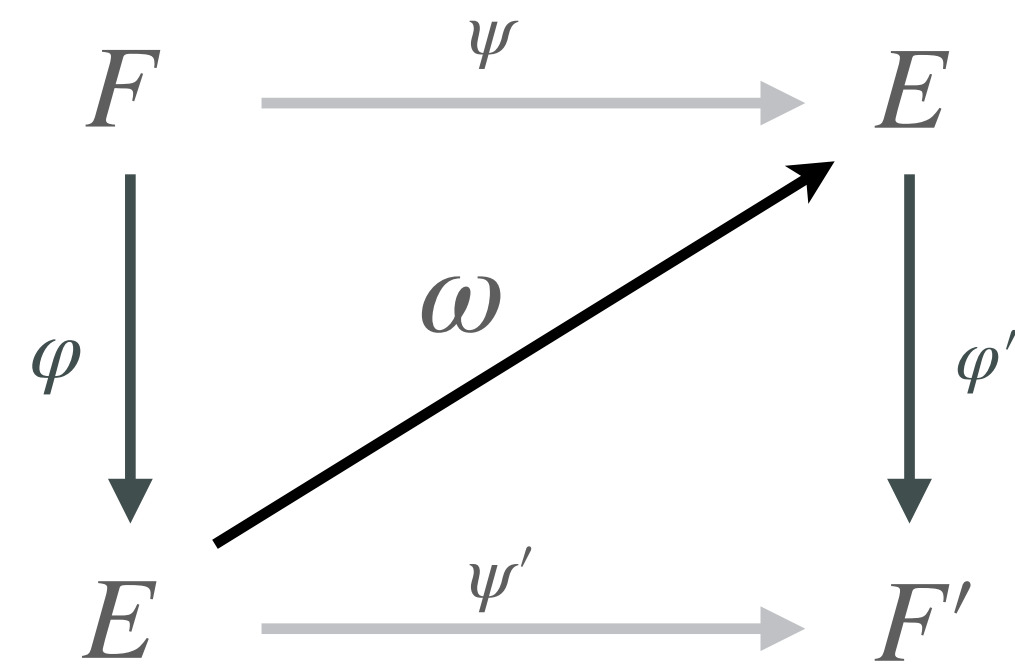
3

So we can also compute  
 $\varphi: F \rightarrow E, \psi: F \rightarrow E$

that is, we can factor  $\omega$   
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So we can also compute  $\varphi : F \rightarrow E, \psi : F \rightarrow E$

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Clapoti(s)

apply this trick to translate ideal  $I$  to suitable 2D isogenies