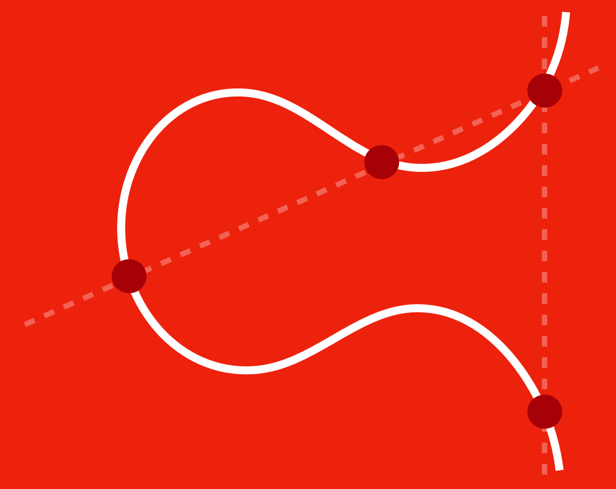
1

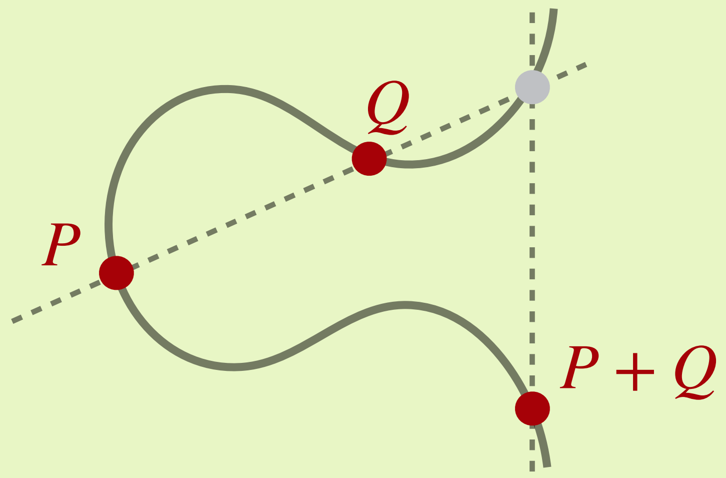


Isogenies & Pairings

elliptic curves in CSIDH

supersingular elliptic curve

- has $p+1$ points in $E(\mathbb{F}_p)$
- choose p so that $p+1 = 4 \cdot \ell_1 \cdot \ell_2 \cdot \ldots \cdot \ell_n$
- this implies the rational points on E have orders that divide $p+1$



$E : y^2 = x^3 + Ax^2 + x, \quad A \in \mathbb{F}_p$

the order of P is readable from the non-zero P_i's

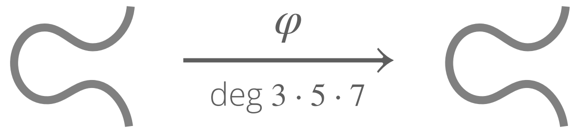
the torsion that P is *missing* are precisely the zero P_i's

full-torsion points

we call a point $P \in E(\mathbb{F}_p)$ a **full-torsion point** if the order is $p+1$, equivalently, all P_i are non-zero

torsion points and isogenies

1. Any* isogeny φ of degree N
   - given by kernel of size N
   - generated by point P of order N



2. Any* isogeny φ of degree $N = \prod \ell_i$
   - splits into sub-isogenies of degree ℓ_i
   - each generated by point P of order ℓ_i

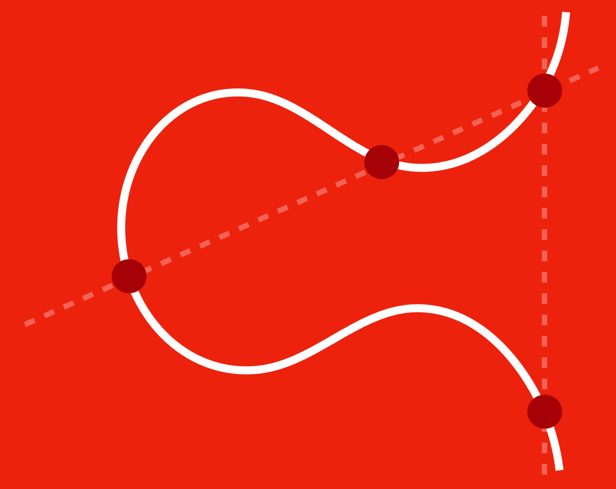
3. Any* isogeny φ of degree $N = \prod \ell_i$
   - computed using one **full-torsion** P
   - per ℓ_i, compute $[\frac{p+1}{\ell_i}]P$ to get $\ker(\varphi_i)$

$P = P_3 + P_5 + P_7 \in E(\mathbb{F}_p)$

$[5 \cdot 7]P = P_3' + \mathcal{O} + \mathcal{O} \in E(\mathbb{F}_p)$

$\varphi_1(P) = \mathcal{O} + P_5' + P_7' \in E'(\mathbb{F}_p)$

*cyclic, separable

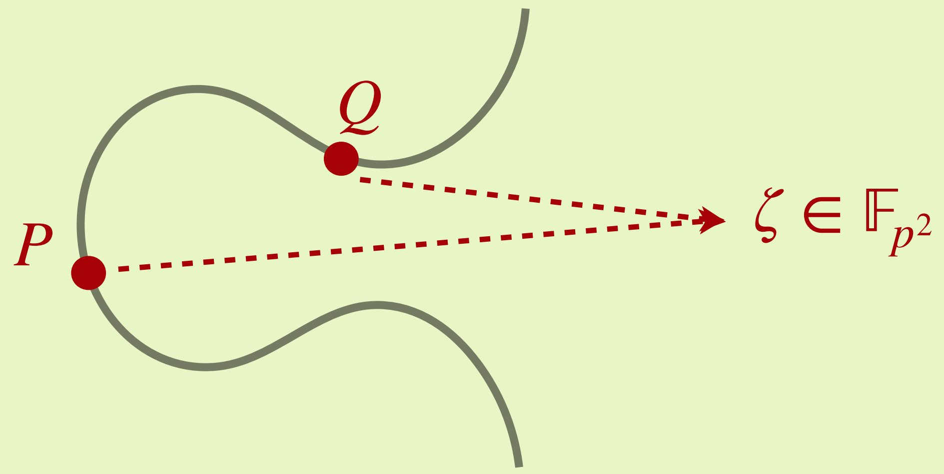


Isogenies & Pairings

the Tate pairing*

bilinear pairing from torsion groups to fields

- choose a degree r
- take point P of order r on E, that is $P \in E(\mathbb{F}_{p^2})[r]$
- take point Q on E such that $Q \in E(\mathbb{F}_{p^2})/rE(\mathbb{F}_{p^2})$
- then $e_r(P, Q) = \zeta \in \mu_r$



*reduced