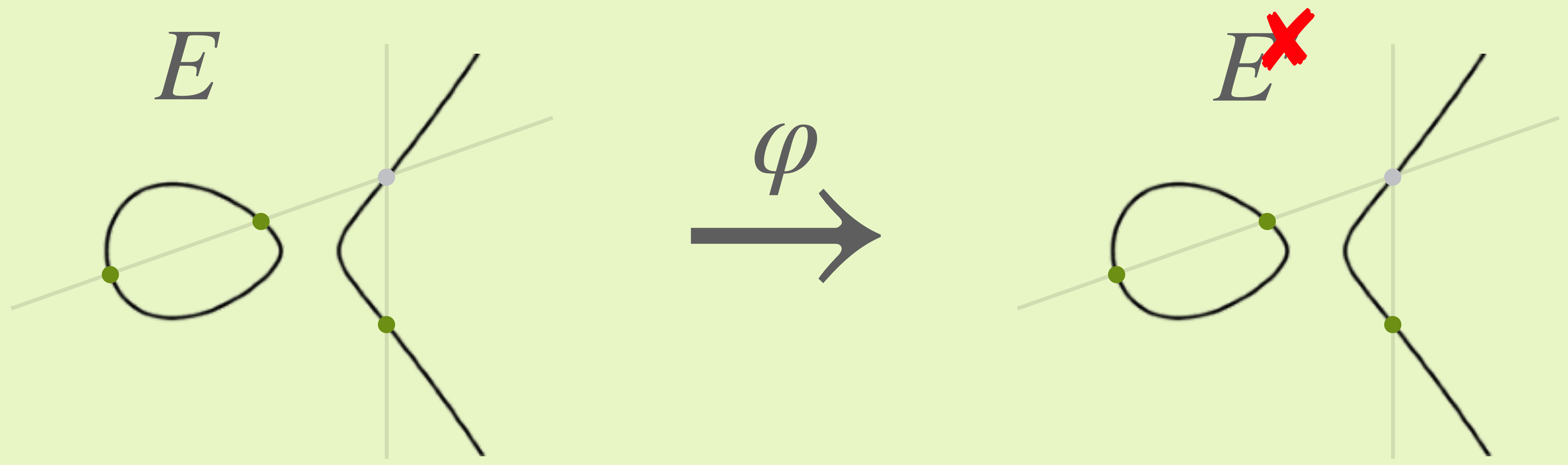
PART 1

SQIsign

endomorphism



Iso~~ge~~ny Endomorphism

- "nice" map φ (group homomorphism) between elliptic curves $E \to E$
- given by rational functions: a point $(x, y) \in E$ is mapped to ($f_1(x,y)/f_2(x,y)$, $g_1(x,y)/g_2(x,y)$)
- size of $\ker \varphi$ is same as degree of φ!

toy example

$$E : y^2 = x^3 + x \xrightarrow{\varphi} E : y^2 = x^3 + x$$

$$(x, y) \mapsto \left(\frac{x^4 - 2x^2 + 1}{4(x^3 + x)} : \frac{x^6 y + 5x^4 y - 5x^2 y - y}{8(x^6 + 2x^4 + x^2)} \right) \quad \text{over } \mathbb{F}_{11}$$

Can check

- this is a group homomorphism: $\varphi(\mathcal{O}) = \mathcal{O}$ and $\varphi(P + Q) = \varphi(P) + \varphi(Q)$
- looks difficult… but actually this just the map $[2] : P \mapsto P + P$
- so $[2]$ has kernel $\mathcal{O}, (0,0), (8 + 7i, 0), (3 + 4i, 0)$, degree $[2]$ is $4 = 2^2$

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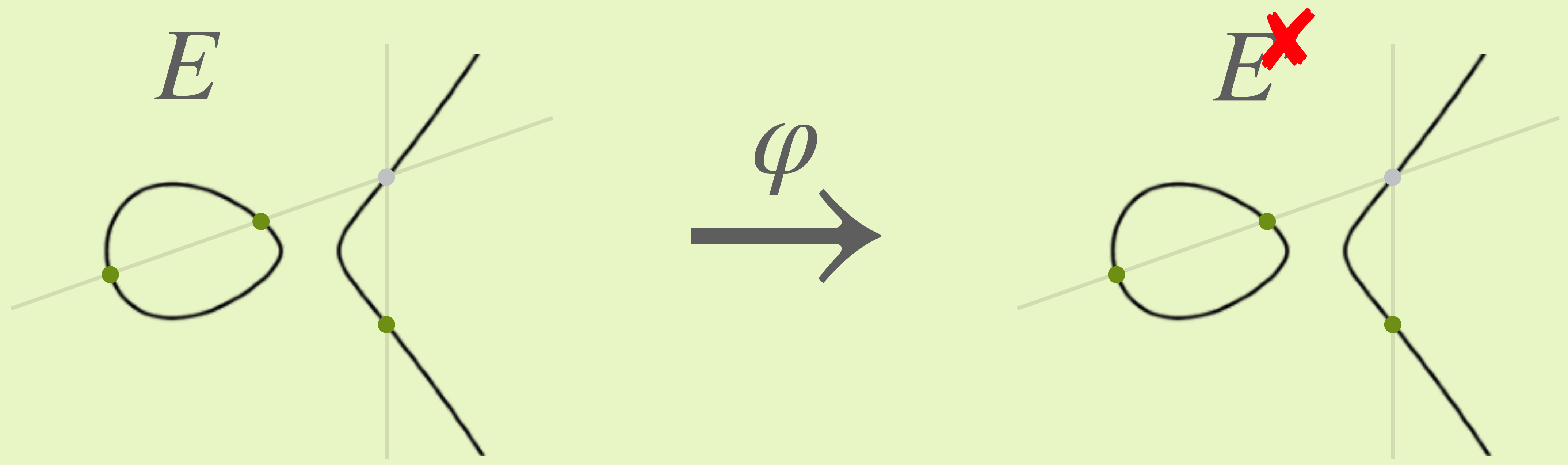

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second toy example

Frobenius map. $\pi : (x, y) \mapsto (x^q, y^q)$ **always** an endomorphism for E over $\mathbb{F}_q$