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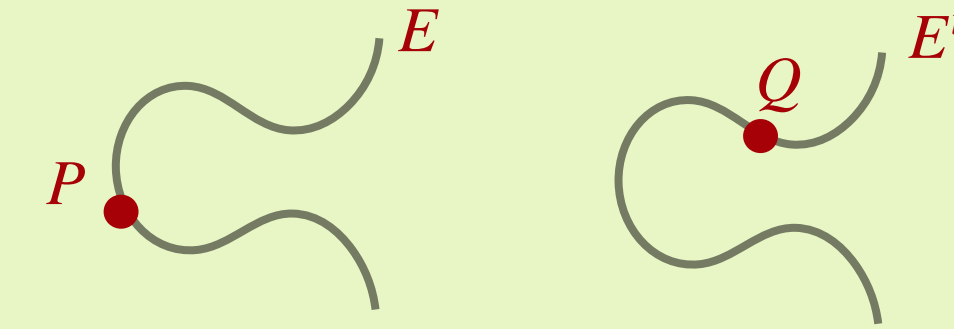


Isogenies & Pairings

the twist of E

Twist over \mathbb{F}_p of supersingular curve E

- a curve E' with $p + 1$ points over \mathbb{F}_p
- isomorphic to a specific subset of $E(\mathbb{F}_{p^2})$
- used in CSIDH to “move backwards” in graph
- want $P \in E(\mathbb{F}_p)$ and $Q \in E'(\mathbb{F}_p)$, both full order



1

consider P and Q as

$$P = P_0 + P_1 + \dots + P_n$$

$$Q = Q_0 + Q_1 + \dots + Q_n$$

2

let $r = p + 1$

Tate pairing $e_r(P, Q)$ captures
where **both** $P_i, Q_i \neq \mathcal{O}$

crucial lemma

Let $P \in E(\mathbb{F}_p)$, $Q \in E'(\mathbb{F}_p)$, and $r = p + 1$. Let $\zeta = e_r(P, Q) \in \mathbb{F}_{p^2}$.

Then ζ is an r -th root of unity, whose order is precisely
gcd of order of P , order of Q

example

If P and Q both full torsion,
then ζ has order $r = p + 1$

example

If P has order 5, and Q has
order 15, then ζ has order 5

1

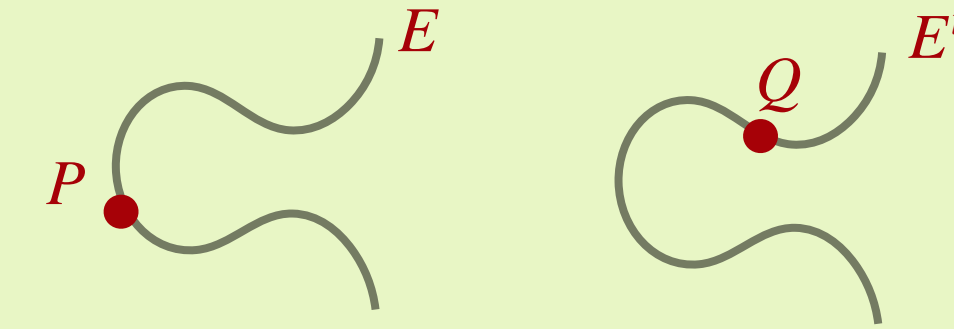


Isogenies & Pairings

the twist of E

Twist over \mathbb{F}_p of supersingular curve E

- a curve E^t with $p + 1$ points over \mathbb{F}_p
- isomorphic to a specific subset of $E(\mathbb{F}_{p^2})$
- used in CSIDH to “move backwards” in graph
- want $P \in E(\mathbb{F}_p)$ and $Q \in E^t(\mathbb{F}_p)$, both full order



1

consider P and Q as

$$P = P_0 + P_1 + \dots + P_n$$

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let $r = p + 1$

Tate pairing $e_r(P, Q)$ captures
where **both** $P_i, Q_i \neq \mathcal{O}$

crucial lemma

Let $P \in E(\mathbb{F}_p)$, $Q \in E^t(\mathbb{F}_p)$, and $r = p + 1$. Let $\zeta = e_r(P, Q) \in \mathbb{F}_{p^2}$.

Then ζ is an r -th root of unity, whose order is precisely
gcd of order of P , order of Q

example

If P and Q both full torsion,
then ζ has order $r = p + 1$

example

If P has order 5, and Q has
order 15, then ζ has order 5

!

notice

Curve arithmetic is slow!
Field arithmetic is fast!!
(more than factor 6)