PART 2
The Tate Profile

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Definition 4. Assume $E[m] \subseteq E(\mathbb{F}_q)$ and let (P_1, P_2) be a basis for $E[m]$. Then, the *m-Tate profile* is the map

$$t_{[m]} : E(\mathbb{F}_q) \to \mu_m^2, \qquad Q \mapsto \big(t_2(P_1, Q),\, t_2(P_2, Q) \big).$$

For $Q \in E(\mathbb{F}_q)$, we say that $t_{[m]}(Q)$ is the *m-profile* of Q. When $t_{[m]}(Q) = (1,1)$, we say the profile is *trivial.*

1

is linear.

If the Tate pairing

is bilinear, then the Tate profile

t_m

$$t_{[m]}$$

2

with

is non-degenerate, then

Thus,

is trivial if and only if there is an

If the Tate pairing

t_m

$$\ker t_{[m]} = [m]E(\mathbb{F}_q)$$

$$t_{[m]}(Q)$$

$$R \in E(\mathbb{F}_q)$$

$$[m]R = Q$$

3

Together, $t_{[m]}$ gives us isomorphisms

$$E[m] \cong E(\mathbb{F}_q)/[m]E(\mathbb{F}_q) \cong \mu_m^2.$$

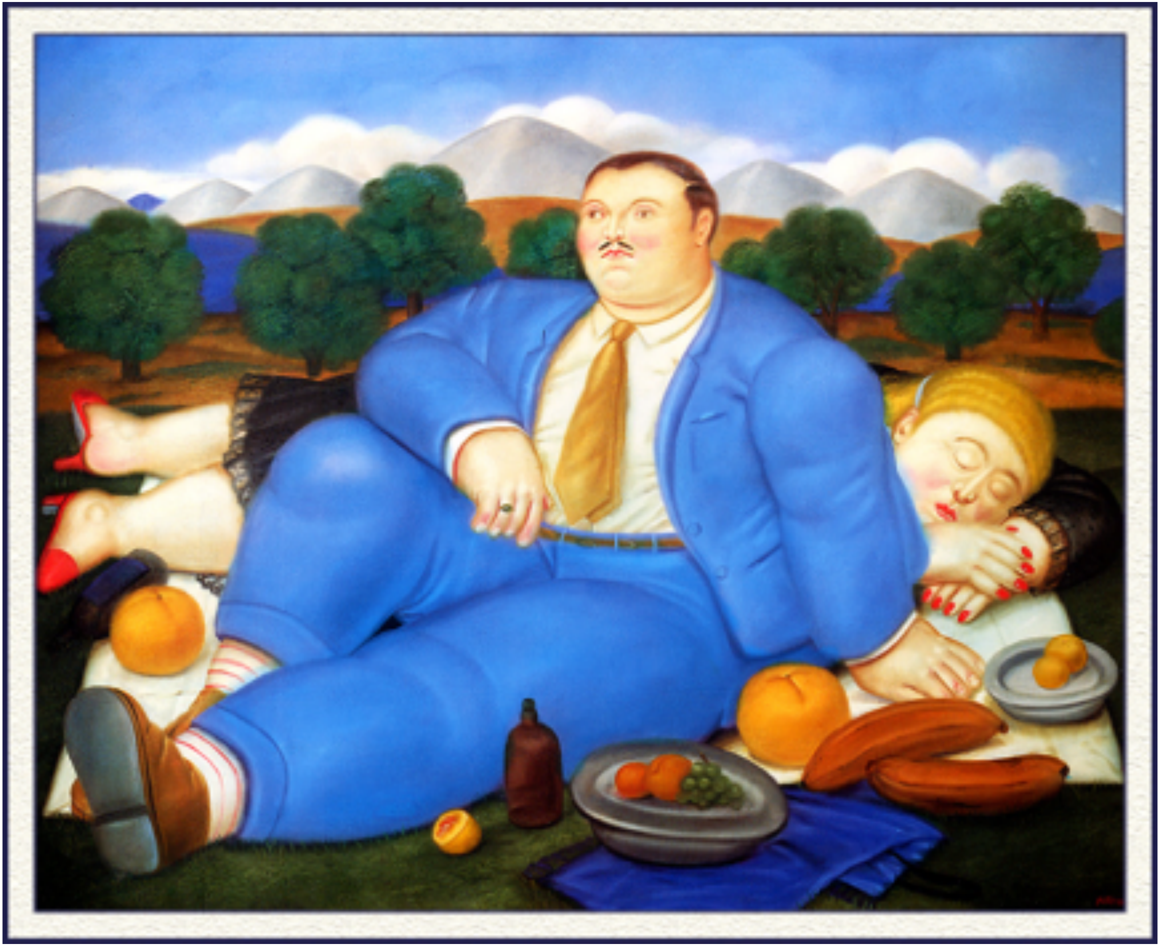
Thus, the basis (P_1, P_2) together with $t_{[m]}$ gives us *coordinates*.

Example 2

Theorem 4. Let $E : y^2 = x^3 + Ax^2 + x$ be a Montgomery curve over $\mathbb{F}_{p^2}$ with $2^n \mid p+1$ and 2-torsion $L_1 = (0,0)$, $L_2 = (\alpha,0)$, and $L_3 = (\frac{1}{\alpha},0)$. Then, for $P \in E[2^n]$ we have $[2^{n-1}]P = L_i$ if and only if

$$t_2(L_i, P) = 1 \text{ and } t_2(L_j, P) \neq 1 \text{ for } j \neq i.$$

Theorem 4. For $P \in E[2^n]$, the profile $t_{[2]}(P)$ determines $[2^{n-1}]P$.



La Siesta (1982)

Example 1

Theorem 3. For an elliptic curve $E : y^2 = (x - r_1)(x - r_2)(x - r_3)$ with $r_i \in \mathbb{F}_p$, we have $P \in [2]E(\mathbb{F}_p)$ if and only if $(x_P - r_1)$, $(x_P - r_2)$, and $(x_P - r_3)$ are squares.

Consequently, $P \in E(\mathbb{F}_p) \ \setminus \ [2]E(\mathbb{F}_p)$ if any $(x_P - r_i)$ is a non-square.

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