Contents

Hashing Algorithms

Simple Tabulation Hashing

Theory. Tabulation hashing is a hashing scheme which combines table lookups and `xor` operations in order to calculate the hash value. It views a key x as a vector of c characters $x_1, ..., x_c$, and relies on one totally random table T_i for each of the c character positions. The hash function simply performs a lookup for each character in the corresponding table, and `xor` the lookup-results together,

$$h(x) = T_1[x_1] \oplus, ..., \oplus T_c[x_c]$$

The tables are initialized prior to execution of the hashing function, which makes the complexity of the algorithm only depend on the speed of the table-lookups and the `xor`'ing (assumed *O(1)*). By constructing the table small enough to fit in fast cache, the table lookups very efficient, making algorithm is very fast while also being straightforward to implement.

If drawing keys from a universe of size u, we see that we have $O(u^{1/c})$ entries in each table, making the total amount of entries become $O(cu^{1/c})$. Since the c in the exponent outweights the factor c, the total space required for the tables is decreased as the amount of characters is increased. Thus, in order to ensure that the tables can fit in fast cache (optimally `L1`-cache), the size of the character could be decreased.

However, decreasing the size of the character yields more characters, which in turn yields more lookups and more `xor` operations, thus increasing the runtime of the algorithm. The decision of a good character-size is therefore a trade-off between having small enough characters for the tables to fit in fast cache, while not having too small characters to avoid too many computations.

The algorithm is by no means a new form of hashing, as the first instances of tabulashing was published in 1970 by Albert Zobrist [1], and was later rediscovered in greater generality by Carter & Wegman in 1979 [2]. The simple tabulation hashing is 3-independant, while better implementations of tabulation-based methods can supply 5-independance [3]. Pătraşcu and Thorup has recently shown that the low independance of the simple tabulation hashing can be shown to be non-fatal for many key applications [4], yielding strong hashing-properties for a very simple hashing method.

Implementation. The algorithm has been implemented in the `tabulation_hash.[h|cpp]`. It contains the `tabulation_hash` class, which holds the tables needed for the hashing, and only exposes one functions, namely `tabulation_hash::get_hash()`, which calculates the hash value based on the input string.

```
value_t tabulation_hash::get_hash(std::string key)
```

The type of the hash-values has been defined as `value_t`, which is set to `uint32_t`. By setting the character-size to 1B, there will be 256 entries in each table. Since each entry contains the hash-value for the given key, they will be of size 4B, making the total size of each table become |256*4B = 1kB|. Thus, to be able to hash a string of e.g. length 8, the memory needed to hold the tables will be 8kB. By allow character-sizes of 2B, amount of tables needed would be reduced by a factor of 2, but it would increase the amount of entries in each table by a factor 2^8, thus increasing the total size needed by a factor 2^7. Thus, for the same length 8 string, the memory required would become $2^{16} * 4 * (8/2)\text{B} = 1\text{MB}$, making the tables too large for fast cache. Therefore, the character-size has been fixed to 1B.

As for the generation of the random values in the tables, the suggestion of taking truly random values from `random.org` has been adhered to to some extend, as 16 tables of 256 entries each has been generated and hardcoded into the implementation. However, if more than 16 tables are needed, the tables are generated using a mersenne-twister in the constructor. This constructor takes as templating argument the maximum length of any key, which the hashing should be able to process, as this sets a limit to how many tables are needed, thus limiting the amount of space used.

The actual hashing works by splitting the key string into 8-bit chunks (i.e. `chars`), and using the 8 bits as an index in the table corresponding to the given chunk. The results of all these are simply `xor`'ed together.

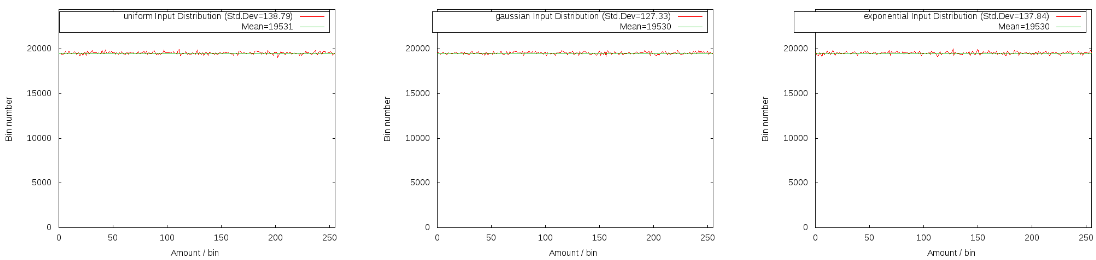
Experiments. To test the performance of the implementation, three kinds of experiments has been implemented:

- *Distribution Test*, where the distribution of many hash values is tested, for different input-distributions.
- *Key-Length Test*, where the run-time of the hashing algorithm is tested for various key lengths.
- *Multicore Test*, where the throughput of the hashing algorithm is tested for various amounts of cores.

Distribution Test. To test the distribution of the hashed output-values, three different input-distributions have been used:

- Uniform Distribution
- Gaussian Distribution
- Exponential Distribution

For each of the three distributions, 5.000.000 random 8-byte key-strings have been generated by generating a random integer using the given distribution, and interpreting it as a string. The keys have then been hashed, and the outputs have been categorized into 256 evenly sized bins. The distributions can been seen on Figure 1.



(a) Uniform Input Distr. (b) Gaussian Input Distr. (c) Exponential Input Distr.

Figure 1: Output Distributions of Tabulation Hashing

It can be seen, that for all three input distribution, the output distribution is very close to uniform, thus showing that tabulation hashing will map roughly the same amount of inputs to each hash value. Since the cost of hashing based methods increases drastically as more collisions occurs, a uniform output distribution will yield the best performance.

Key-Length Test. Testing of the speed of the algorithm on a single core is done by hashing keys of different length, and calculating the average runtime of each hashing. This test shows how well the implementation scales over key lengths.

To avoid filling the L1 cache with keys, we've chosen to only generate a small amount of strings (i.e. 5) for each of the lengths from 1-64 bytes. The strings are then hashed in groups of their lengths, from which the average run-time for each length can be calculated. To reduce the noise of the low amount of strings, the hashing of the groups are repeated a high amount of times. The average run-time for each length can be seen on figure 2. It contains four data-sets, representing hashing using 1, 8, 32 and 64 tabulation tables, respectively.

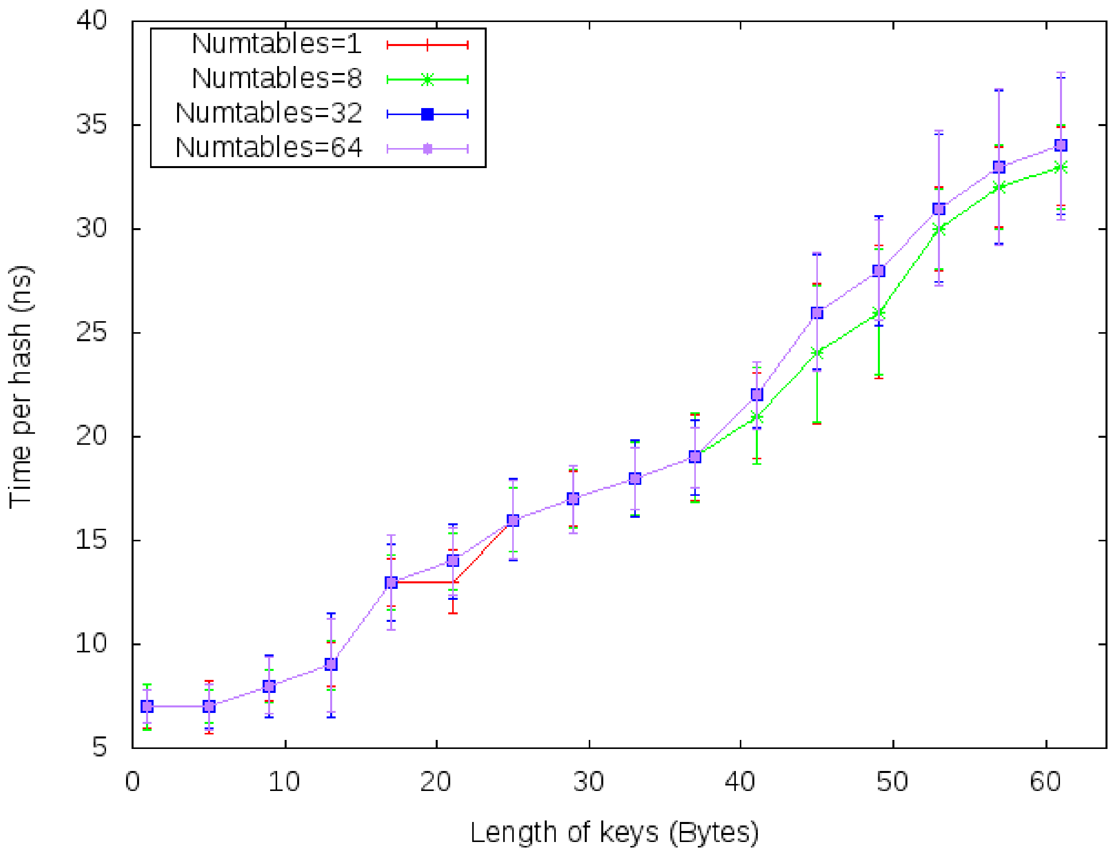


Figure 2: Run-times for different input string lengths.

It should first be noted that we see a close to linear increase in run-time as the length of the string is increased, in al of the four scenarios. In general, we've found that hashing a string of length 1 byte takes roughly 5ns, while each additional byte in the string adds half of a nanosecond. The low cost of the bytes following the first can be explained by that the reads of these bytes can be overlapped with the read of the previous bytes, where the first byte has to perform the entire read anyway.

Secondly, it should be noted that the four scenarios yields very similar results. Specifically, we can see that for short strings, the runtime is the same regardless of the amounts of tables. However, as the length of the strings is increased, we see that using a large amount of tables yields slower run-times than when using fewer tables. This is caused by the L1 cache being more contested by the high amount of tables and bytes in the strings.

Multicore Test To test the scalability across multiple cores, we've run a test similar to the key-length test on multiple threads, and measured the throughput of each thread, as well as the total througput. To do this, we've initialized one `tabulation_hash` object, which is used by all the threads in their computations. However, since the `tabulation_hash` object is never modified after initialization, a linear increase in throughput would be expected, as more cores are being utilized.

On Figure 3 the average throughput per thread, as well as the total throughput, is shown. Additionally, a linear function with slope equal to the throughput found when using just one thread has been added, which shows the optimal throughput scaling.

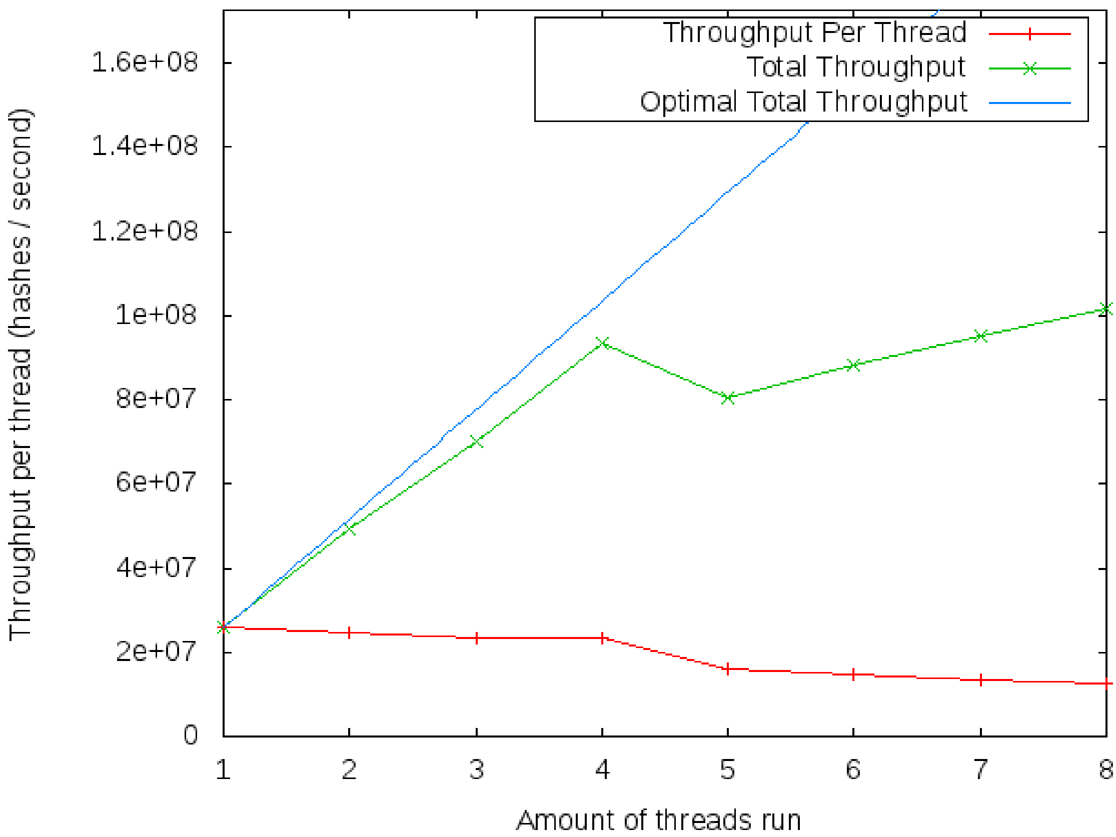


Figure 3: Throughput using different amounts of threads.

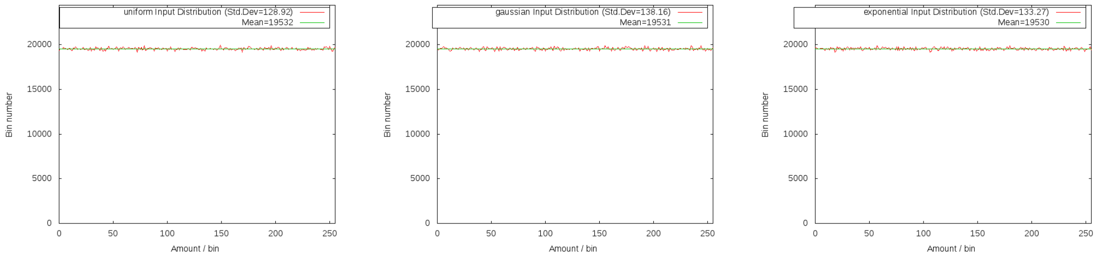
On the data-set for the average throughput per thread, we can see that for the first four cores, the throughput for each thread is almost as high as the throughput found when using just one thread. However, when running more threads than 4, the average throughput of each thread decreases. This can also be seen from the total throughput, which for the first 4 threads is close to the optimal throughput, while using more threads doesn't increase the total throughput. This could be explained by that the computer on which the tests has been run only has 4 physical cores.

Comparison to other hashing algorithms. Recently, a thorough comparison of hashing methods has been performed by Richter et al. [5], in which they compare four hash functions, namely

- Multiply-Shift Hashing

- Multiply-Add-Shift Hashing
- Tabulation Hashing
- Murmur Hashing

Since the first two only have implementations for fixed-length keys, we've chosen to focus on the comparison to murmur hashing.



(a) Uniform Input Distr. (b) Gaussian Input Distr. (c) Exponential Input Distr.

Figure 4: Output Distributions of Murmur Hashing

Distributions for other algorithms.

References

[1] Zobrist, A. *A New Hasahing Method with Application for Game Playing.* http://research.cs.wisc.edu/techreports/1970/TR88.pdf

[2] Carter, J.; Wegman, Mark. *Universal classes of hash functions.* Journal of Computer and System Sciences, 143-154: `http://ac.els-cdn.com/0022000079900448/1-s2.0-0022000079900448-main.pdf?_tid=f6ef3296-dbb5-11e5-996c-00000aacb35f&acdnat=1456401208_4e624f31178f38bc62b0b994c4b534b4`

[3] Thorup, M.; Zhang, Y. *Tabulation based 5-universal hashing and linear probing.* In Proc. 12th Workshop on Algorithm Engineering and Experiments (ALENEX), 2009

[4] Pătraşcu, M.; Thorup, M. *The Power of Simple Tabulation Hashing* http://arxiv.org/abs/1011.5200

[5] Ritcher, S.; Alvarez, V.; Dittrich, J. *A seven Dimensional Analysis of Hashing Methods and its Implications on Query Processing* http://www.vldb.org/pvldb/vol9/p96-richter.pdf