



Worksheet 1

HoTTEST Summer School 2022

The HoTTEST TAs, and
4th July 2022

1 (★)

State the introduction and elimination rules for

1. \times -types
2. \rightarrow -types
3. \prod -types

2 (★)

Fill in this proof tree:

$$\begin{array}{c}
 \frac{\frac{a : A, b : B \vdash a : A}{\vdash} \quad \frac{}{\vdash} : A \times B}{a : A \vdash \lambda(b : B).(a, b) :} \\
 \vdash \lambda(a : A).\lambda(b : B).(a, b) : A \rightarrow B \rightarrow A \times B
 \end{array}$$

3 (★★)

Write a proof tree ending with a term of type $A \times B \rightarrow B \times A$ in the empty context.

4 (★★)

For problems 2 and 3, what is the *logical* content of the proof tree? That is, under the "types are theorems" interpretation of Curry-Howard, what theorems have we proven?

Next, what is the *computational* content of the proof tree? That is, under the "programs are proofs" interpretation of Curry-Howard, what programs have we written?

5 (★ ★ ★)

Define the **swap** function $\sigma_{A,B}$ of type

$$\sigma_{A,B} : \left(\prod_{x:A} \prod_{y:B} C(x,y) \right) \rightarrow \left(\prod_{y:B} \prod_{x:A} C(x,y) \right)$$

and show that $\sigma_{B,A} \circ \sigma_{A,B}$ is (definitionally) equal to the identity.