1 $(\star)$

State the introduction and elimination rules for

1. $\times$-types

2. $\to$-types

3. $\prod$-types

2 $(\star)$

Fill in this proof tree:

$$
\cfrac{
\cfrac{
\cfrac{\rule{3cm}{0.4pt}}{a : A, b : B \vdash a : A}
\qquad
\cfrac{\rule{3cm}{0.4pt}}{}
}{
\vdash \hspace{6cm} : A \times B
}
}{
\cfrac{a : A \vdash \lambda(b : B).(a, b) :}{\cdot \vdash \lambda(a : A).\lambda(b : B).(a, b) : A \to B \to A \times B}
}
$$

3 $(\star\star)$

Write a proof tree ending with a term of type $A \times B \to B \times A$ in the empty context.

4 $(\star\star)$

For problems 2 and 3, what is the *logical* content of the proof tree? That is, under the "types are theorems" interpretation of Curry-Howard, what theorems have we proven?

Next, what is the *computational* content of the proof tree? That is, under the "programs are proofs" interpretation of Curry-Howard, what programs have we written?

5 $(\star\star\star)$

Define the **swap** function $\sigma_{A,B}$ of type

$$\sigma_{A,B} : \left(\prod_{x:A} \prod_{y:B} C(x,y) \right) \to \left(\prod_{y:B} \prod_{x:A} C(x,y) \right)$$

and show that $\sigma_{B,A} \circ \sigma_{A,B}$ is (definitionally) equal to the identity.