§ 4. *Use versus Mention*

IN THE literature on the logic of statements, and in other founda-tional studies of mathematics as well, confusion and controversy have resulted from failure to distinguish clearly between an object and its name. Ordinarily the failure to maintain this distinction is not to be attributed to any close resemblance between the object and the name, even if the object happens to be a name in turn; for even the discrimination between one name and another is a visual operation of an elementary kind. The trouble comes rather in forgetting that a statement about an object must contain a name of the object rather than the object itself. If the object is a man or a city, physical circumstances prevent the error of using it instead of its name; when the object is a name or other expression in turn, however, the error is easily committed.

As an illustration of the essential distinction, consider these three statements:

(1)   Boston is populous,
(2)   Boston is disyllabic,
(3)   'Boston' is disyllabic.

The first two are incompatible, and indeed (1) is true and (2) false. Boston is a city rather than a word, and whereas a city may be populous, only a word is disyllabic. To say that the place-name in question is disyllabic we must use, not that name itself, but a name of it. The name of a name or other expression is commonly formed by putting the named expression in single quotation marks; the whole, called a *quotation*, denotes its interior. This device is used in (3), which, like (1), is true. (3) contains a name of the di-syllabic word in question, just as (1) contains a name of the populous city in question. (3) is *about* a word which (1) *contains;* and (1) is about no word at all, but a city. In (1) the place-name is *used*, and in this way the city is *mentioned;* in (3) a quotation is used, and the place-name is mentioned. We mention x by using a name of x; and a statement about x contains a name of x.[1]

[1] By these considerations, the first sentence of the present paragraph might be

The foregoing treatment of (1)–(3) is itself replete with mention of expressions, yet free from quotations. These were avoided by circumlocution. As an exercise in quotation marks, however, it may be useful now to add a few comments involving them. 'Boston is populous' is about Boston and contains 'Boston'; ' 'Boston' is disyllabic' is about 'Boston' and contains ' 'Boston' '. ' 'Boston' ' designates 'Boston', which in turn designates Boston. To mention Boston we use 'Boston' or a synonym, and to mention 'Boston' we use ' 'Boston' ' or a synonym. ' 'Boston' ' contains six letters and just one pair of quotation marks; 'Boston' contains six letters and no quotation marks; and Boston contains some 800,000 people.

Such examples as (3), or:

(4)   'Boston' has six letters,
(5)   'Boston' is a noun,
(6)   'Boston' occurs in Walt Whitman's *Chants Democratic*,

must not be thought of as exhausting the kinds of things that can be said about an expression. These four statements ascribe properties to 'Boston' which might be classed respectively as phonetic, morphological, grammatical, and literary; roughly speaking, they have nothing to do with meaning. But an expression which has use in language will also have *semantic* properties, or properties which arise from the meaning of the expression. Such properties are ascribed to 'Boston' by these statements:

(7)   'Boston' designates Boston,
(8)   'Boston' designates a populous city,
(9)   'Boston' designates the capital of Massachusetts,
(10)   'Boston' is synonymous with 'the capital of Massachusetts'.

(7)–(10) are just as genuinely statements about 'Boston' as are (3)–(6), and omission of the quotation marks from (7)–(10) would give results no less objectionable than (2); for it is only

criticized for failure to enclose the whole statements (1)–(3) in quotation marks. But it is clearer to avoid quotation, in such cases, by agreeing to regard the colon as equivalent to quotation when followed by displayed text (text centered in a new line).

expressions, not places, that can designate or be synonymous. A statement about an expression may depend for its verification upon considerations of sound or shape or literary locale, or even upon considerations of population or other extra-linguistic matters of fact with which the expression is indirectly connected by its use; but so long as the statement is about the expression it must contain a name of the expression.

Lack of care in thus distinguishing the name from the named is common in mathematical writings. The following passage, from a widely used textbook on the differential calculus, is fairly typical:

The expression $D_x y \Delta x$ is called the *differential* of the function and is denoted by dy:

$$dy = D_x y \Delta x.$$

The third line of this passage, an equation, is apparently supposed to reproduce the sense of the first two lines. But actually, whereas the equation says that the entities dy and $D_x y \Delta x$ (whatever these may be) are the same, the preceding two lines say rather that the one is a name of the other. And the first line of the passage involves further difficulties; taken literally it implies that the exhibited expression '$D_x y \Delta x$' constitutes a name of some other, unexhibited *expression* which is known as a differential. But all these difficulties can be removed by a slight rephrasing of the passage: drop the first two words and change 'and is denoted by' to 'or briefly'.

Expository confusions of this sort have persisted because, in most directions of mathematical inquiry, they have not made themselves felt as a practical obstacle. They do give rise to minor perplexities, indeed, even at the level of elementary arithmetic. A student of arithmetic may wonder, e.g., how 6 can be the denominator of $\frac{4}{6}$ and not of $\frac{2}{3}$ when $\frac{4}{6}$ is $\frac{2}{3}$; this puzzle arises from failure to observe that it is the *fractions* '$\frac{4}{6}$' and '$\frac{2}{3}$' that have denominators, whereas it is the designated *ratios* $\frac{4}{6}$ and $\frac{2}{3}$ that are identical. But it is primarily in mathematical logic that carelessness over these distinctions is found to have its more serious effects. At the level of the logic of statements, one effect is obliteration of the distinction between predicates of statements and

composition of statements — a distinction which will be con-
sidered in the next section.

Scrupulous use of quotation marks is the main practical measure
against confusing objects with their names. But it has already
been suggested that this particular method of naming expressions
is not theoretically essential. E.g., using elaborately descriptive
names of 'Boston', we might paraphrase (3) in either of the follow-
ing ways:

> The word composed successively of the second, fifteenth,
> nineteenth, twentieth, fifteenth, and fourteenth letters of the
> alphabet is disyllabic.

> The 4354th word of *Chants Democratic* is disyllabic.

Quotation is the more graphic and convenient method, but it has
a certain anomalous feature which calls for special caution: from
the standpoint of logical analysis each whole quotation must be
regarded as a single word or sign, whose parts count for no more
than serifs or syllables. A quotation is not a *description*, but a
hieroglyph; it designates its object not by describing it in terms of
other objects, but by picturing it. The meaning of the whole does
not depend upon the meanings of the constituent words. The
personal name buried within the first word of the statement:

> (11)   'Cicero' has six letters,

e.g., is logically no more germane to the statement than is the verb
'let' which is buried within the last word. Otherwise, indeed, the
identity of Tully with Cicero would allow us to interchange these
personal names, in the context of quotation marks as in any other
context; we could thus argue from the truth (11) to the falsehood:

> 'Tully' has six letters.

Frege seems to have been the first logician to recognize the importance of scru-
pulous use of quotation marks for avoidance of confusion between use and mention
of expressions (cf. *Grundgesetze*, vol. 1, p. 4); but unfortunately his counsel and good
example in this regard went unheeded by other logicians for some thirty years.
For further discussion of this topic see Carnap, *Syntax*, pp. 153-160. Concerning
the necessity of treating a whole quotation as a single sign, see also § 1 of Tarski's
"Wahrheitsbegriff".

§ 5. *Statements about Statements*

TO SAY that a city or a word has a given property, e.g. populousness or disyllabism, we attach the appropriate predicate to a name of the city or word in question (cf. § 4). To say that a statement has a given property, e.g. the phonetic property of being a hexameter or the semantic property of truth or falsehood, we attach the appropriate predicate to a name of the statement in question — not to the statement itself. Thus, to attribute truth to:

(1)   Jones is ill

we write:

(2)   'Jones is ill' is true,

and to attribute falsehood we write:

(3)   'Jones is ill' is false.

Equivalently, we may write:

(4)   (1) is true,
(5)   (1) is false;

but never:

(6)   Jones is ill is true,
(7)   Jones is ill is false,

on the analogy of:

(8) $\sim$ Jones is ill.

(2)–(5) are about the statement (1), but (8) is not; it, like (1), is about Jones. 'Is true' and 'is false' attach to names of statements precisely because, unlike '$\sim$', they are predicates by means of which we speak *about* statements. Whereas statement connectives ('$\sim$', '.', 'v', '$\supset$', '$\equiv$') attach to statements to form statements, a predicate is an expression which attaches to names to form statements. Grammar alone is enough to condemn (6) and (7), since each occurrence of 'is' should have a noun as subject. Confusion over this matter results in the view that the suffix 'is true' is

vacuous, and that the suffix 'is false' is the English translation of the prefix '∼'; the view, in other words, that (6) is equivalent to (1) and (7) to (8).

In order to say that two objects stand in a given relation, e.g. hate, or remoteness, one puts an appropriate binary predicate (transitive verb) between names of the objects thus: 'Roosevelt hates Hitler', 'Berlin is far from Washington'. To say that two statements stand in a given relation, whether the phonetic relation of rhyming or the semantic relation of implication, we put the appropriate binary predicate between names of the statements — not between the statements themselves. We may write:

(9) 'All men are mortal' implies 'all white men are mortal',

(10) The third statement of the book implies the seventh,

but never:

(11) All men are mortal implies all white men are mortal

on the analogy of:

(12) If all men are mortal then all white men are mortal,

(13) All men are mortal ⊃ all white men are mortal.

The verb 'implies' belongs between names of statements precisely because, unlike '⊃' or 'if-then', it expresses a relation between statements; it is a binary predicate by means of which we talk *about* statements. (9) and (10) are about statements, while (12) and (13) are about men.

The relation of implication in one fairly natural sense of the term, viz. *logical implication*, is readily described with help of the auxiliary notion of *logical truth*. A statement is logically true if it is not only true but remains true when all but its logical skeleton is varied at will; in other words, if it is true and contains only logical expressions essentially, any others vacuously (cf. Introduction). Now one statement may be said logically to imply another when the truth-functional conditional which has the one statement as antecedent and the other as consequent is logically true. Thus (9), so construed, is equivalent to:

(13) is logically true.

A trivial analogue, *material implication*, may be said to hold whenever the truth-functional conditional which has the one statement as antecedent and the other as consequent is true. Thus one statement materially implies another provided merely that the first is false or the second true. This relation is so broad as not to deserve the name of implication at all except by analogy. But — and this is the point usually missed — 'materially implies' is still a binary predicate, not a binary statement connective. It stands to '⊃' precisely as 'is false' stands to '∼'. Insertion of the connective '⊃' between statements as in (13) amounts to inserting the verb 'materially implies', not between the statements themselves as in (11), but between their names as in (9).

With a few trivial exceptions such as material implication, any relation between statements will depend on something more than the truth values of the statements related. Such is the case, e.g., with the phonetic relation of rhyming. The same holds for the semantic relation of logical implication described above, and for any other relation which has (unlike material implication) a serious claim to the name of implication. Such relations are quite consonant with a policy of shunning non-truth-functional modes of statement composition (cf. § 1), since a relation of statements is not a mode of statement composition. On this account, the policy of admitting none but truth-functional modes of statement composition is not so restrictive as might have at first appeared; what could be accomplished by a subjunctive conditional or other non-truth-functional mode of statement composition can commonly be accomplished just as well by talking *about* the statements in question, thus using an implication relation or some other strong relation of statements instead of the strong mode of statement composition. Instead of saying:

If Perth were 400 miles from Omaha then Perth would be in America

one might say:

'Perth is 400 miles from Omaha' implies 'Perth is in America',

in some appropriate sense of implication.

Much of what has been said regarding implication applies equally to other semantic relations of statements, e.g. *equivalence* and *compatibility*. Statements are logically equivalent when they logically imply each other, and logically compatible when one does not imply the other's denial. Or, what comes to the same thing, statements are logically equivalent when the biconditional formed from them is logically true, and they are logically compatible except when the conjunction formed from them is logically false, i.e., except when the denial of the conjunction is logically true. Trivial analogues, material equivalence and compatibility, are similarly determined: statements are materially equivalent when they materially imply each other, and materially compatible when one does not materially imply the other's denial. Or, what comes to the same thing, statements are materially equivalent whenever their biconditional is true, and materially compatible except when their conjunction is false, hence whenever their conjunction is true. Material equivalence is agreement in truth value, and material compatibility is joint truth. Equivalence and compatibility, even in this degenerate sense, must be distinguished from the biconditional and conjunction; insertion of ' ≡ ' or '.' between statements amounts to inserting 'is materially equivalent to' or 'is materially compatible with', not between the statements themselves, but between their names.

Note that 'is true', 'is false', 'implies', 'is equivalent to', etc. do not admit of iterated application as do the statement connectives. The expressions which '∼', '.', '⊃', etc. govern and the expressions which they produce are homogeneously statements; the expressions produced can hence be so governed in turn, and thus we obtain statements of the forms:

$$\sim \sim \text{———,} \qquad \text{———} \supset (\text{———} \supset \text{———}), \qquad (\sim \text{———} \,.\, \text{———}) \supset \sim \text{———,}$$

etc. But 'is false', 'implies', etc. govern names and produce statements; hence the expressions produced cannot be so governed in turn. Formally the predicates 'is false' and 'implies' resemble the predicates 'is negative' and ' ⩽ ' of arithmetic rather than the statement connectives '∼' and '⊃'. Just as it is true for all numbers x, y, z that

$$(x \leqslant y \,.\, y \leqslant z) \supset x \leqslant z$$

and $(y$ is negative $.\, x \leqslant y) \supset x$ is negative,

so it is true for all statements ϕ, ψ, χ that

$(\phi$ implies $\psi \,.\, \psi$ implies $\chi) \supset \phi$ implies χ

and $(\psi$ is false $.\, \phi$ implies $\psi) \supset \phi$ is false;

on the other hand the contexts:

$(\phi$ implies $\psi \,.\, \psi$ implies $\chi)$ implies $(\phi$ implies $\chi)$

and: $(\psi$ is false $.\, \phi$ implies $\psi)$ implies $(\phi$ is false$)$

make no more sense than:

$$(x \leqslant y \,.\, y \leqslant z) \leqslant (x \leqslant z),$$
$$(y \text{ is negative} \,.\, x \leqslant y) \leqslant (x \text{ is negative}).$$

In Whitehead and Russell's exposition and terminology the distinction between predicate and statement connective is blurred. The notation '— $\supset$ —' is explained indiscriminately in the sense of the truth-functional conditional and in the sense of material implication. It is translated not only thus:

(14)   If — then —

but also thus:

(15)   If — is true then — is true,

(16)   — is false or — is true,

(17)   — implies —.

Similarly '$\equiv$' is explained both in the sense of the truth-functional biconditional and in the sense of material equivalence, and '$\sim$' is explained both in the sense of denial and in the sense of falsehood. The authors even adopt 'implication' or 'material implication' as their regular terminology in connection with '$\supset$', and 'equivalence' in connection with '$\equiv$'. Actually, as we have seen, the blanks in (14) admit only statements whereas those in (15)–(17) admit only names of statements. In the construction of examples, indeed, grammatical sense leads Whitehead and Russell to fill the blanks of '— is true', '— is false', and '— implies —' with quotations rather than statements; [1] but the distinction is straightway obliterated in the discussion.

Once having noted the discrepancy between (14) and the other proposed translations of '— $\supset$ —', one need not delay in making his choice. In all technical developments the expressions which Whitehead and Russell adjoin to the sign

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[1] "'x wrote Waverly' is true" (vol. 1, p. 68); "'the author ... was a poet' is false" (ibid.); "'Socrates is a man' implies 'Socrates is mortal'" (ibid., pp. 20, 138).

'⊃' have the form of statements rather than names. The mere fact of its iteration indeed, e.g. in the manner·

$$- \supset (- \supset -),$$

is enough to determine the sign as a statement connective rather than a predicate about statements. In short, the versions (15)–(17) do not operate in Whitehead and Russell's work beyond the level of unfortunate exposition and nomenclature. The English idiom which '— ⊃ —' supplants in practice is not (15), (16), or (17), but (14). The case is similar with '≡' and '∿'.

On the topic of implication Whitehead and Russell have many critics, who rightly object that the trivial relation of material implication expressed in (16) is too weak to constitute a satisfactory version of (17). But it is seldom observed that this objection does not condemn the truth-functional conditional '— ⊃ —' as a version of 'if — then —'. Lewis, Smith, and others have undertaken systematic revision of '⊃' with a view to preserving just the properties appropriate to a satisfactory relation of implication; but what the resulting systems describe are actually modes of statement composition — revised conditionals of a non-truth-functional sort — rather than implication relations between statements.

If we were willing to reconstrue statements as names of some sort of entities, we might take implication as a relation between those entities rather than between the statements themselves; and correspondingly for equivalence, compatibility, etc. This procedure would dissolve the distinction between material implication and the truth-functional conditional, and likewise between other sorts of implication and other sorts of conditionals. 'Implies' would come to enjoy simultaneously the status of a binary predicate and the status of a binary statement connective. Expressions such as (11) would be legitimized; and so also would the iterated use of implication, characteristic of Lewis and Smith. For thus construing statements as names some slight support can be adduced, indeed, by appeal to substantive clauses. The statement 'All men are mortal' might be held to designate that abstract entity, whatever it is, which we ordinarily designate by the substantive 'that all men are mortal'. A deterring consideration, however, is the obscurity of these alleged entities. What are they like? and under what circumstances may the entities designated by two statements be said to be the same or different entities? Certain entities which are perhaps less obscure than these but no less abstract will indeed be countenanced at a later point (§ 22), viz. classes or properties, if only through ignorance of how to get on without them; but entities designated by statements are happily dispensable.[1] It thus seems well to adhere to the common-sense view that statements are not names at all, though they may contain names along with verbs and adverbs and the rest. A statement remains meaningful, but meaningful by virtue of its structure together with the meanings of the constituent names and other words; its meaningfulness does not consist in its being a name of something.

Conceding that 'implies' belongs between names of statements as in (9), rather than between statements, one might still urge that such a relation of implication produces a derivative mode of composition of the statements themselves — namely,

[1] See my "Ontological Remarks," "Logistical Approach," "Designation."

a mode which consists notationally of compounding the statements by means of 'implies' *and* the two pairs of quotation marks.[1] If implication is construed as going beyond questions of truth value, this derivative mode of statement composition will not be truth-functional. Implication thus construed would then seem, after all, to interfere with a policy of admitting none but truth-functional modes of statement composition. By the same argument, indeed, a purely morphological or phonetic relation such as containing or rhyming would interfere similarly. Actually, however, derivation of modes of statement composition from relations in the suggested fashion involves abuse of quotation. The statements buried in the quotations in (9) cannot be treated in turn as constituents of (9), for a quotation figures as a single irreducible word. Similar abuse of quotation was seen in § 4 to lead from ' 'Cicero' has six letters' to ' 'Tully' has six letters'.[2]

These latter remarks serve only to show that we can construe implication as going beyond questions of truth value without *thereby* committing ourselves to any form of conditional which goes beyond questions of truth value. The need for some such strong form of conditional might still be urged on other grounds. Certainly not all uses of the subjunctive conditional submit to the easy method of paraphrase illustrated in the case of Perth and America. When this fails we may look to other devices, e.g. Carnap's method of reduction sentences ("Testability," pp. 439–453); but if any really useful cases prove to resist all such methods of analysis, then we shall perhaps have to choose pragmatically between the usefulness of those cases and the convenience and clarity of the truth functional kind of statement composition. Mathematics itself gives rise to no such recalcitrant cases; and any which seem to arise beyond the bounds of mathematics should be critically regarded.[3]

§ 6. *Quasi-Quotation*

IN DISCUSSING the modes of statement composition we are having continually to talk of expressions. Quotation suffices for the mention of any specific expression, such as 'v' or ' ≡ ' or 'Jones is away', but is not available when we want to speak generally of an unspecified expression of such and such kind. On such occasions use has been made of general locutions such as 'a conditional', 'the first component', etc.; and more difficult cases have been managed indirectly by introducing a blank '——' from time to

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[1] Analogous reasoning appears in Huntington's "Note on a Recent Set", p. 11.

[2] See also Tarski, "Wahrheitsbegriff," § 1.

[3] For further discussion and references see Carnap, *Syntax*, §§ 67-71.

time. But the developments to follow call for a more elastic method of referring to unspecified expressions.

For the beginnings of such a method, the use of letters in algebra provides us with an adequate model. In algebra 'x', 'y', etc. are used as names of unspecified numbers; we may suppose them replaced by names of any specific numbers we choose. Analogously, Greek letters other than 'ϵ', 'ι', 'λ'[1] will now be used as names of unspecified *expressions;* we may suppose them replaced by names (e.g. quotations) of any specific expressions we choose.

A discussion of numbers may, for example, begin thus:

(1)   Let x be a factor of y.

Throughout the discussion thus prefaced, we are to think of x and y as any specific numbers we like which satisfy the condition (1) — say the numbers 5 and 15, or 4 and 32. We are to think of the letters 'x' and 'y' as if they were names of the numbers 5 and 15, or names of the numbers 4 and 32, etc. We are to imagine the letters 'x' and 'y' replaced by the numerals (expressions) '5' and '15', or by '4' and '32', etc.

Similarly a discussion of expressions might begin thus:

(2)   Let μ be part of ν.

Throughout the discussion thus prefaced, we are to think of μ and ν as any specific expressions we like which satisfy the condition (2) — say the expressions 'York' and 'New York', or '3' and '32'. We are to think of the letters 'μ' and 'ν' as if they were names of the expressions 'York' and 'New York', or names of the expressions '3' and '32', etc. We are to imagine the letters 'μ' and 'ν' replaced by the quotations ' 'York' ' and ' 'New York' ', or by ' '3' ' and ' '32' ', etc.

The reader is urged to compare the above short paragraph with the preceding one, word by word; also to review § 4. Roughly speaking, the letters 'x', 'y', etc. may be described as ambiguous numerals, ambiguous names of numbers, variables ambiguously designating numbers, or, in the usual technical phrase, variables taking numbers as their values. Correspondingly the letters

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[1] These three letters are reserved for later purposes; cf. §§ 22, 35, 41.

'μ', 'ν', etc. may be roughly described as ambiguous quotations, ambiguous names of expressions, variables ambiguously designating expressions, or variables taking expressions as their values. This does not mean simply that 'μ' and 'ν' take the place of expressions, or are replaceable by expressions, for this is true of 'x' and 'y' as well. Rather, the letters 'μ' and 'ν' take the place of quotations or other names of expressions, just as 'x' and 'y' take the place of numerals or other names of numbers.

Occasionally Greek letters will be used with accents or subscripts attached: 'μ'', 'μ'''', 'μ_1', 'μ_2', 'μ_n', etc. Such variants may be regarded simply as so many further Greek letters. Three Greek letters, 'ϕ', 'ψ', and 'χ', together with their accented and subscripted variants, will be limited in their use to those cases where the expression designated is intended to be a statement. They serve as names of unspecified statements, and are replaceable by statement quotations or other names of specific statements.[1]

There is need also of a convenient way of speaking of specific contexts of unspecified expressions: speaking, e.g., of the result of enclosing the unspecified expression μ in parentheses, or the result of joining the unspecified statements ϕ and ψ in that order by the sign '$\equiv$'. Note that quotation is not available here. The quotations:

$$'(\mu)', \qquad\qquad\qquad '\phi \equiv \psi'$$

designate only the specific expressions therein depicted, containing the specific Greek letters 'μ', 'ϕ', and 'ψ'. Reference to the intended contexts of the unspecified expressions μ, ϕ, and ψ will be accomplished by a new notation of *corners*, thus:

$$(3) \qquad\qquad \ulcorner(\mu)\urcorner, \qquad\qquad\qquad \ulcorner\phi \equiv \psi\urcorner.$$

Because of the close relationship which it bears to quotation, this device may be called *quasi-quotation*. It amounts to quoting the constant contextual backgrounds, '( )' and '$\equiv$', and imagining the unspecified expressions μ, ϕ, and ψ written in the blanks. If in particular we take the expression 'Jones' as μ,

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[1] The three letters have indeed already appeared in this use in § 5, where the sense intended was apparent.

'Jones is away' as ϕ, and 'Smith is ill' as ψ, then $\ulcorner(\mu)\urcorner$ is '(Jones)' and $\ulcorner\phi \equiv \psi\urcorner$ is 'Jones is away $\equiv$ Smith is ill'.

The quasi-quotations (3) are synonymous with the following verbal descriptions:

The result of writing '(' and then μ and then ')',
The result of writing ϕ and then ' $\equiv$ ' and then ψ;

or, equivalently:

The result of putting μ in the blank of '( )',
The result of putting ϕ and ψ in the respective blanks of ' $\equiv$ ';

or, equivalently:

The result of putting μ for 'μ' in '(μ)',
The result of putting ϕ for 'ϕ' and ψ for 'ψ' in '$\phi \equiv \psi$'.

We may translate any quasi-quotation:

$$\ulcorner\underline{\qquad}\urcorner$$

into words in corresponding fashion:

The result of putting μ for 'μ', ν for 'ν', ..., ϕ for 'ϕ', ψ for 'ψ', ... in '——'.

Described in another way: a quasi-quotation designates that (unspecified) expression which is obtained from the contents of the corners by replacing the Greek letters (other than 'ϵ', 'ι', 'λ') by the (unspecified) expressions which they designate.

When a Greek letter stands alone in corners, quasi-quotation is vacuous: $\ulcorner\mu\urcorner$ is μ. For, by the foregoing general description, $\ulcorner\mu\urcorner$ is the result of putting μ for 'μ' in 'μ'; $\ulcorner\mu\urcorner$ is what the letter 'μ' becomes when that letter itself is replaced by the (unspecified) expression μ; in other words, $\ulcorner\mu\urcorner$ is simply that expression μ.

Quasi-quotation would have been convenient at earlier points, but was withheld for fear of obscuring fundamentals with excess machinery. Now, however, it may be a useful exercise to recapitulate some sample points from §§ 1–5 in terms of this device. A conjunction $\ulcorner\phi . \psi\urcorner$ is true just in case ϕ and ψ are both true, and an alternation $\ulcorner\phi \vee \psi\urcorner$ is false just in case ϕ and ψ are both false. A conditional $\ulcorner\phi \supset \psi\urcorner$ is true if ϕ is false or ψ true, and false if ϕ is

true and ψ false. A biconditional $\ulcorner \phi \equiv \psi \urcorner$ is true just in case ϕ and ψ are alike in truth value. A denial $\ulcorner \sim \phi \urcorner$ is true just in case ϕ is false. ϕ logically implies ψ or is logically equivalent to ψ according as $\ulcorner \phi \supset \psi \urcorner$ or $\ulcorner \phi \equiv \psi \urcorner$ is logically true, and ϕ materially implies ψ or is materially equivalent to ψ according as $\ulcorner \phi \supset \psi \urcorner$ or $\ulcorner \phi \equiv \psi \urcorner$ is true.

§ 7. *Parentheses and Dots*

PARENTHESES, taken for granted thus far as an auxiliary nota-tion, are most simply construed as forming integral parts of the binary connectives. The notations of conjunction, alternation, the conditional, and the biconditional will no longer be regarded as $\ulcorner \phi . \psi \urcorner$, $\ulcorner \phi \vee \psi \urcorner$, $\ulcorner \phi \supset \psi \urcorner$, and $\ulcorner \phi \equiv \psi \urcorner$, but as $\ulcorner (\phi . \psi) \urcorner$, $\ulcorner (\phi \vee \psi) \urcorner$, $\ulcorner (\phi \supset \psi) \urcorner$, and $\ulcorner (\phi \equiv \psi) \urcorner$. But the notation of denial remains simply $\ulcorner \sim \phi \urcorner$. This formulation yields just the usage of parentheses which we have hitherto followed, except in one respect: a con-junction, alternation, conditional, or biconditional comes now to bear an outside pair of parentheses even when it stands apart from any further symbolic context. Such outside parentheses have hitherto been omitted.

When the binary modes of composition are thus construed, no auxiliary technique of grouping is needed. The syntactical sim-plicity thus gained proves useful in certain abstract studies (e.g. § 10; also later chapters). In applications, however, such sim-plicity is less important than facility of reading; and excess of parentheses is a hindrance, for we have to count them off in pairs to know which ones are mates. It is hence convenient in practice to omit the outside parentheses as hitherto, and furthermore to suppress most of the remaining parentheses in favor of a more graphic notation of *dots*.

Parentheses mark the outer limits of a binary compound; dots determine those same limits less directly. Roughly speaking, a group of dots placed beside 'v', '$\supset$', or '$\equiv$' indicates that the com-ponent on that side has its other end at the nearest larger group of