香港中文大學(深圳)
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Introduction to Computer Science: Programming Methodology

Lecture 9
Recursion, Stack and Queue

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线性递归

Linear Recursion

每次调用的函数体

- If a recursive function is designed so that each invocation of the body makes at most one new recursive call, this is known as linear recursion ☆

二分查找

- Finding the smallest number and binary search are both linear recursive algorithms

Practice: Sum of a list

- Given a list of numbers, write a program to calculate the sum of this list using recursion

Solution:

a list

len(L)

递归函数

```
def linearSum(L, n):
    ① if n==0:
        return 0
    ② else:
        return linearSum(L, n-1)+L[n-1]
```

return 0 后终止.

previous sum + last letter

main() 函数

```
def main():
    L = [1, 2, 3, 4, 5, 9, 100, 46, 7]
    print('The sum is:', linearSum(L, len(L)))
```

9.

```python
linearSum([1,100,7], 3)
```

```python
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return linearSum(L,n-1)+L[n-1]
```

n = 3

```python
linearSum([1,100,7], 2)
```

```python
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return linearSum(L,n-1)+L[n-1]
```

n = 2

```python
linearSum([1,100,7], 1)
```

```python
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return linearSum(L,n-1)+L[n-1]
```

n = 1

```python
linearSum([1,100,7], 0)
```

```python
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return linearSum(L,n-1)+L[n-1]
```

n = 0

108

```
linearSum([1,100,7], 3)
```

```
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return 101+L[n-1]
```

n = 3

101

```
linearSum([1,100,7], 2)
```

```
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return 1+L[n-1]
```

n = 2

1

```
linearSum([1,100,7], 1)
```

```
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return 0+L[n-1]
```

n = 1

0

```
linearSum([1,100,7], 0)
```

```
def linearSum(L,n):
    if n==0:
        return 0
    else:
        return linearSum(L,n-1)+L[n-1]
```

n = 0

Practice: Power function

- Write a program to calculate the power function $f(x, n) = x^n$ using Recursion. The time complexity of the program should be O(logn)

→ 和二分法类似!!

$$\underbrace{x*x*x* \ldots *x}_{\text{(n-1) is O(n)}}$$

A better recursive definition of power function

☆类比二分法 ⇒ $O(\log_2 n)$

floor division

$$power(x,n) = \begin{cases} 1 & \text{if } n = 0 \\ x \cdot \left(power\left(x, \left\lfloor \frac{n}{2} \right\rfloor\right)\right)^2 & \text{if } n > 0 \text{ is odd} \\ \left(power\left(x, \left\lfloor \frac{n}{2} \right\rfloor\right)\right)^2 & \text{if } n > 0 \text{ is even} \end{cases}$$

$x^n = (x^{\frac{n}{2}})^2$

$n=5$

$x \cdot (x^{5//2})^2 = x^4 \cdot x = x^5$

$m \sim n$.

write the code following

Solution: the formula.

例: x n

myPower(3,2)

p1 = mp(3,1) = 1

↓ ②

p2 = mp(3,0) = 1.

r = p_1^2 = 1 ①

⇒ 1%2 = 1

⇒ r = 3×1 = 3. ③

```
def myPower(x, n):
    if n==0:
        return 1
    else:
        partial = myPower(x, n//2)
        result = partial * partial
        if n%2==1:
            result = result * x
        return result
```

n≠0, 首先计算x的(n//2)次方.

⇒将中间结果平方.

☆ 若n为奇

$(x^{\lfloor \frac{n}{2} \rfloor})^2 = x^n$

$p=3,\ r=q,\ 2\%2!=1 \Rightarrow r=q\times r=q.$

Multiple re

- When a functi ... rsive calls, we say that it
- Drawing the E ... ursion program

```
def draw_line(tickLen, tickLabel=''):
    line = '-'*tickLen
    if tickLabel:
        line+=' '+tickLabel
    print(line)

def draw_interval(centerLen):
    if centerLen>0:
        draw_interval(centerLen-1)
        draw_line(centerLen)
        draw_interval(centerLen-1)

def draw_ruler(numInch, majorLen):
    draw_line(majorLen, '0')

    for j in range(1, 1+numInch):
        draw_interval(majorLen-1)
        draw_line(majorLen, str(j))
```

Practice: Binary sum

- Write a function binarySum() to calculate the sum of a list of numbers. Inside binarySum() two recursive calls should be made

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	5	7	8	9	12	14	17	19	22	25	27	28	33	37

Practice: Binary sum

分成两部分.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	5	7	8	9	12	14	17	19	22	25	27	28	33	37

```
binarySum(L,start,mid)+binarySum(L,mid,stop)
```

Practice: Binary sum

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	5	7	8	9	12	14	17	19	22	25	27	28	33	37

```
elif start==stop-1:
    return L[start]
```

练

Solution:

```
def binarySum(L, start, stop):
    if start>=stop:
        return 0
    elif start==stop - 1:
        return L[start]
    else:
        mid = (start+stop)//2
        return binarySum(L, start, mid)+binarySum(L, mid, stop)

def main():
    L = [1, 2, 3, 4, 5, 6, 7]
    print(binarySum(L, 0, len(L)))
```

→ 只有一个元素

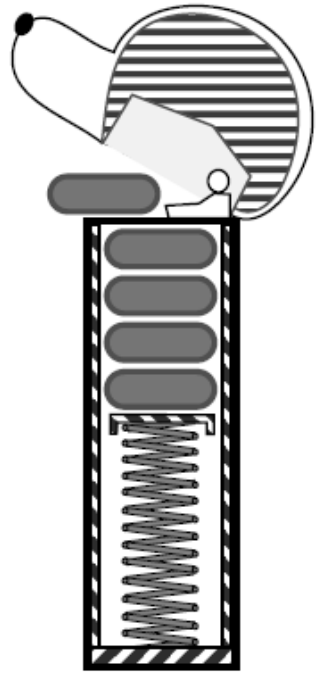
Stack

- A stack is a collection of objects that are inserted and removed according to the last-in, first-out (LIFO) principle

后进先出

- A user may insert objects into a stack at any time, but may only access or remove the most recently inserted object that remains (at the so-called "top" of the stack)

只能访问/删除最近插入的对象



Example: Web Browser 浏览器

- Internet Web browsers store the addresses of recently visited sites in a stack. Each time a user visits a new site, that site's address is "pushed" onto the stack of addresses. The browser then allows the user to "pop" back to previously visited sites using the "back" button.

Example: Text editor 文本编辑器

- Text editors usually provide an “undo” mechanism that cancels recent editing operations and reverts to former states of a document. This undo operation can be accomplished by keeping text changes in a stack.

撤消操作

The stack class

- Generally, a stack may contain the following methods:

S.push(e): Add element e to the top of stack S.

S.pop( ): Remove and return the top element from the stack S; an error occurs if the stack is empty.

S.top( ): Return a reference to the top element of stack S, without removing it; an error occurs if the stack is empty.

S.is_empty( ): Return True if stack S does not contain any elements.

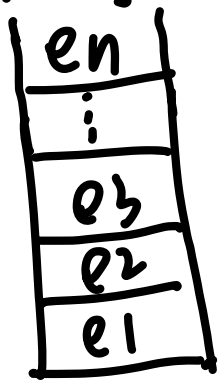
len(S): Return the number of elements in stack S; in Python, we implement this with the special method __len__.

也__len__(self)，必须
不可改!! 命名规则!!

是两条下划线

The Code of Stack Class

simplify version




```
class ListStack:

    def __init__(self):
        self.__data = list()

    def __len__(self):
        return len(self.__data)

    def is_empty(self):
        return len(self.__data) == 0

    def push(self, e):
        self.__data.append(e)

    def top(self):
        if self.is_empty():
            print('The stack is empty.')
        else:
            return self.__data[self.__len__()-1]

    def pop(self):
        if self.is_empty():
            print('The stack is empty.')
        else:
            return self.__data.pop()
```

创建个空列表.

return True / False

判断双等号

add 'e' to the top

(without removing)

不要漏掉小括号

last element

取出,对检查是否为空

(remove)

list.pop()

The code to use stack class

}.

True

```
def main():
    s = ListStack()
    print('The stack is empty? ',s.is_empty())
    s.push(100)
    s.push(200)
    s.push(300)
    print(s.top())
    print(s.pop())
    print(s.top())
```

Practice: Reverse a list using stack

- Write a program to reverse the order of a list of numbers using the stack class

先push再pop

Solution:

```
from stack import ListStack

def reverse_data(oldList):
    s = ListStack()
    newList = list()

    for i in oldList:
        s.push(i)

    while (not s.is_empty()):
        mid = s.pop()
        newList.append(mid)

    return newList

def main():
    oldList = [1, 2, 3, 4, 5]
    newList = reverse_data(oldList)
    print(newList)
```

create an object of stack

push i into the stack

not empty, enter the loop.

应用 is_empty() 方法.

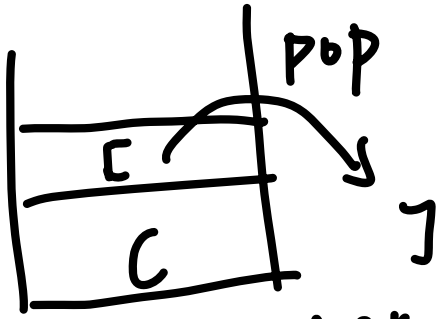
Practice: Brackets match checking

- In correct arithmetic expressions, the opening brackets must match the corresponding closing brackets. Write a program to check whether all the opening brackets have matched closing brackets.

检查所有左括号是否匹配右括号

Solution:

e.g.



check whether it match


```python
from stack import ListStack

def is_matched(expr):
    lefty = '([{'
    righty = ')]}'
    s = ListStack()

    for c in expr:

        if c in lefty:
            s.push(c)
        elif c in righty:
            if s.is_empty():
                return False
            if righty.index(c)!=lefty.index(s.pop()):
                return False
    return s.is_empty()

def main():
    expr = '1+2*(3+4)-[5-6]'
    print(is_matched(expr))
    expr = '(((){}]'
    print(is_matched(expr))
```

一一对应

push it into the stack.

所无任何lefty可对应.

0!=0

是否对应.

s.pop()

返回并删除

如果没有进入循环.

∴说明expr为空.即stack的空.

Practice: Matching Tags in HTML Language

- HTML is the standard format for hyperlinked documents on the Internet

- In an HTML document, portions of text are delimited by HTML tags. A simple opening HTML tag has the form “<name>” and the corresponding closing tag has the form “</name>”

HTML Tags

- Commonly used HTML tags that are used in this example include

  - body: document body
  - h1: section header
  - center: center justify
  - p: paragraph
  - ol: numbered (ordered) list
  - li: list item

An example of HTML document

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine.  The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

(a)

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

(b)

☆

Solution:

· 对于 a.find(b).
若b不存在于a,
将返回-1.

☆会先把所有
不带/的<...>
都放进stack中.

```
from stack import ListStack

def is_matched_html(raw):
    s = ListStack()
    j = raw.find('<')
    while j!=-1:
        k = raw.find('>', j+1)
        if k==-1:
            return False
        tag = raw[j+1:k]
        if not tag.startswith('/'):
            s.push(tag)
        else:
            if s.is_empty():
                return False
            if tag[1:]!=s.pop():
                return False
        j = raw.find('<',k+1)
    return s.is_empty()

def main():
    fhand = open('sampleHTML.txt','r')
    raw = fhand.read()
    print(raw)
    print(is_matched_html(raw))
```

→ 找到了<

<tag>

→ '>'不存在

'>'存在.

False.

开头/.

与上题
思路相同

keep looping.

找到结尾
的</>.

这里开头与
结尾.

若'<'不存在,
不进入while loop.
直接return
False.

找不对<...>
</tag>

I
a
g

按下多个 <...>

file.read()
读取整个文件的内容.

Queue

- Queue is another fundamental data structure

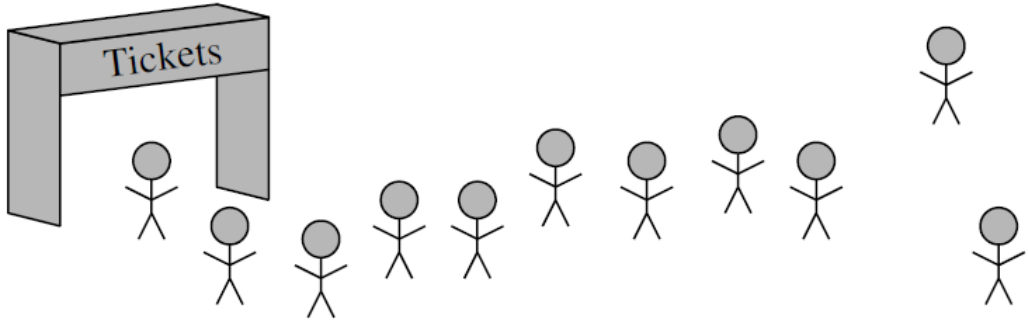
- A queue is a collection of objects that are inserted and removed according to the first-in, first-out (FIFO) principle

先进,先出

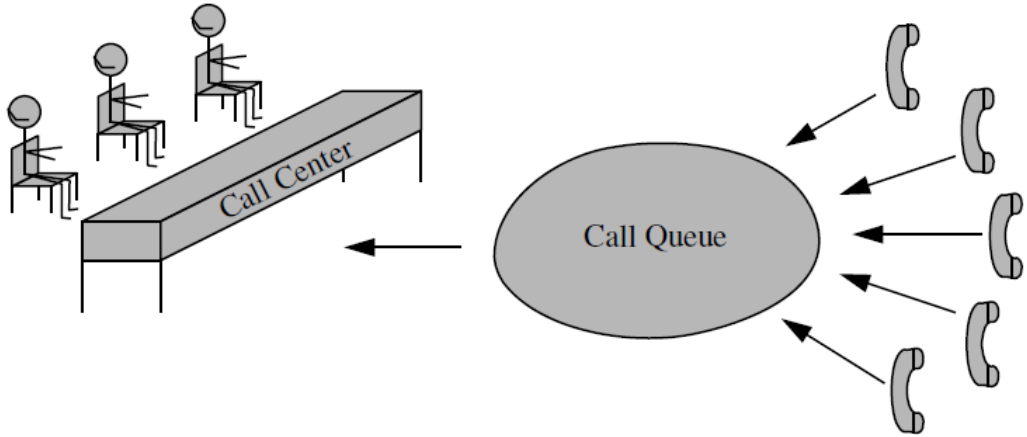
- Elements can be inserted at any time, but only the element that has been in the queue the longest can be next removed

Applications of Queue

first in,
first served.



(a)



(b)

The queue class

- The queue class may contain the following methods:

Q.enqueue(e): Add element e to the back of queue Q.

Q.dequeue( ): Remove and return the first element from queue Q; an error occurs if the queue is empty.

Q.first( ): Return a reference to the element at the front of queue Q, without removing it; an error occurs if the queue is empty.

Q.is_empty( ): Return True if queue Q does not contain any elements.

len(Q): Return the number of elements in queue Q; in Python, we implement this with the special method __len__.

key 分清次序 (对 dequeue 不改变end, enqueue 不改变front)

The code of queue class

```
class ListQueue:
    default_capacity = 5 (upper limit)

    def __init__(self):
        self.__data = [None]*ListQueue.default_capacity
        self.__size = 0
        self.__front = 0
        self.__end = 0

    def __len__(self):
        return self.__size

    def is_empty(self):
        return self.__size ==0

    def first(self):
        if self.is_empty():
            print('Queue is empty.')
        else:
            return self.__data[self.__front]
```

列表 x 数字[N,N,N,N,N]

初始: create a list

return without remove (head of the queue)

```
    def dequeue(self):
        (不改变end的索引位置, end可为None)
        if self.is_empty():
            print('Queue is empty.')
            return None
```

→ nothing to dequeue

return and remove

```
        answer = self.__data[self.__front]
        self.__data[self.__front] = None
        self.__front = (self.__front+1) \
                       % ListQueue.default_capacity
        self.__size -=1
        return answer
```

取走第一位

update

为什么求余:

0	1	2	3	4
x	a	1	2	3

end

next, 4+1=5 (X).

⇒ %, 5%5=0 return to the head (0).

add: (不改变front位置)

```
    def enqueue(self, e):
        if self.__size == ListQueue.default_capacity:
            print('The queue is full.')
            return None

        self.__data[self.__end] = e
        self.__end = (self.__end+1) \
                     % ListQueue.default_capacity
        self.__size += 1
```

添加元素

incase it has moved to the

```
    def outputQ(self):
        print(self.__data)
```

end of the queue

Practice: Simulating a web service

- An online video website handles service requests in the following way:
  1) It maintains a service queue which stores all the unprocessed service requests.
  2) When a new service request arrives, it will be saved at the end of the service queue.
  3) The server of the website will process each service request on a "first-come-first-serve" basis.

- Write a program to simulate this process. The processing time of each service request should be randomly generated.

Solution

```
from ListQueue import ListQueue
from random import random
from math import floor

class WebService():
    default_capacity = 5
    def __init__(self):
        self.nameQ = ListQueue()
        self.timeQ = ListQueue()

    def taskArrive(self, taskName, taskTime):
        if self.nameQ.__len__() < WebService.default_capacity:
            self.nameQ.enqueue(taskName)
            self.timeQ.enqueue(taskTime)
            print('A new task 《'+taskName+'》 has arrived and is waiting for processing...')
        else:
            print('The service queue of our website is full, the new task is dropped.')

    def taskProcess(self):
        if (self.nameQ.is_empty() == False):
            taskName = self.nameQ.dequeue()
            taskTime = self.timeQ.dequeue()
            print('Task 《'+taskName+'》 has been processed, it costs '+str(taskTime)+' seconds.')
```

☆必须检查queue是否满

Solution

```
def main():
    ws = WebService()
    taskNameList = ['Dark knight','X-man','Kungfu','Shaolin Soccer','Matrix','Walking in the clouds'\
                    ,'Casino Royale','Bourne Supremacy','Inception','The Shawshank Redemption']

    print('Simulation starts...')
    print('-------------------------------------')
    for i in range(1,31):
        rNum = random()
        if rNum<=0.6:
            taskIndex = floor(random()*10)
            taskTime  = floor(random()*1000)/100
            ws.taskArrive(taskNameList[taskIndex],taskTime)
        else:
            ws.taskProcess()
    print('-------------------------------------')
    print('Simulation finished.')
```