Tutorial 1

Presented by Jiawei Xu

jiaweixu1@link.cuhk.edu.cn

The Chinese University of Hong Kong, Shenzhen

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Recap: Propositional Logic

Conjunction	$\wedge$	"AND"	$p \wedge q$
Disjunction	$\vee$	"OR"	$p \vee q$
Negation	$\neg$	"NOT"	$\neg q$
Implication	$\rightarrow$	"IF, Then"	$p \rightarrow q$
Biconditional	$\leftrightarrow$	"IF AND ONLY IF"	$p \leftrightarrow q$

Translating Propositional Logic Statements

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p : I go to the store.

q : I go to the movies.

r : I do my homework.

Then we can have $(p \vee q) \to \neg r$.

Exercise

I can get a free sandwich on Thursday, if I buy a sandwich or a cup of soup.

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p : I buy a sandwich.

q : I buy a cup of soup.

r : I get a free sandwich on Thursday.

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Then we can have $(p \vee q) \rightarrow r$.

Exercise

I can get a free sandwich on Thursday, **only if** you I a sandwich or a cup of soup.

p : I buy a sandwich.

q : I buy a cup of soup.

r : I get a free sandwich on Thursday.

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p : I buy a sandwich.

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r : I get a free sandwich on Thursday.

Then we can have $r \to (p \lor q)$.

Exercise

1. You get an A on the final exam and do not complete every exercise in the textbook. Nevertheless, you still obtain an A for the class.
2. You get an A on the final exam and complete every exercise in the textbook, which helps you obtain an A for the class.

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p : You get an A on the final exam.

q : You complete every exercise in the textbook.

r : You obtain an A for the class.

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$$p \wedge \neg q \wedge r$$

Exercise

1. You get an A on the final exam and do not complete every exercise in the textbook. Nevertheless, you still obtain an A for the class.

$$p \land \neg q \land r$$

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$$(p \land q) \to r$$

Solving Logic Puzzles

An island has two kinds of inhabitants, knights, who always tell the truth, and knaves, who always lie. You go to the island and meet A and B, A says "B is a knight.". B says "The two of us are of opposite types." What are A and B?

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p : A is a knight.

q : B is a knight.

Solving Logic Puzzles

A says "B is a knight."

B says "The two of us are of opposite types."

$$p : \text{A is a knight.}$$

$$q : \text{B is a knight.}$$

Four combinations:

$$p \land q,\ \ p \land \neg q,\ \ \neg p \land q,\ \ \neg p \land \neg q$$

Solving Logic Puzzles

p : A is a knight, q : B is a knight.

Possibilities		A says B is a knight		B says they are opposite types	
p	q	p	q	p	q
T	T				
T	F				
F	T				
F	F				

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~~T~~	~~F~~	~~T~~	~~T~~		~~F~~
F	T	F	F		T
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p : A is a knight, q : B is a knight.

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p	q	p	q	p	q
T	T	T	T	F	T
T	F	T	T		F
F	T	F	F		T
F	F	F	F	F	F

Recap: Logical Equivalences

Commutative Laws

$$p \wedge q \equiv q \wedge p$$

$$p \vee q \equiv q \vee p$$

Associative Laws

$$(p \wedge q) \wedge r \equiv q \wedge (p \wedge r)$$

$$(p \vee q) \vee r \equiv q \vee (p \vee r)$$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Distributive Laws

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Conditional-Disjunction Law $p \to q \equiv \neg p \vee q$

Exercise

Prove $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$.

Exercise

Prove $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$.

$$
\begin{aligned}
& (p \to q) \lor (p \to r) \\
\equiv \quad & (\neg p \lor q) \lor (\neg p \lor r) \\
\equiv \quad & \neg p \lor q \lor \neg p \lor r \\
\equiv \quad & \neg p \lor (q \lor r) \\
\equiv \quad & p \to (q \lor r)
\end{aligned}
$$

Exercise

Prove $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$.

Exercise

Prove $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$.

$$
\begin{aligned}
& \neg(p \vee (\neg p \wedge q)) \\
\equiv\quad & \neg p \wedge \neg(\neg p \wedge q) \\
\equiv\quad & \neg p \wedge (p \vee \neg q) \\
\equiv\quad & (\neg p \wedge p) \vee (\neg p \wedge \neg q) \\
\equiv\quad & F \vee (\neg p \wedge \neg q) \\
\equiv\quad & \neg p \wedge \neg q
\end{aligned}
$$

Recap: Rules of Inference

Modus Ponens	$p \to q$ p $\therefore q$	Modus Tollens	$p \to q$ $\neg q$ $\therefore \neg p$
Addition	p $\therefore p \vee q$	Simplification	$p \wedge q$ $\therefore q$
Conjunction	p q $\therefore p \wedge q$	Resolution	$\neg p \vee r$ $p \vee q$ $\therefore q \vee r$
Transitivity	$p \to q$ $q \to r$ $\therefore p \to r$	Elimination	$p \vee q$ $\neg p$ $\therefore q$

Exercise

Prove the premises "John works hard", "If John works hard then he isn't having any fun" and "if john isn't having any fun, then he won't make any friends" imply the conclusion "John will not make any friends".

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p : John works hard.

q : John is having any fun.

r : John is making friends.

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p : John works hard.

q : John is having any fun.

r : John is making friends.

Assumption: p, $p \to \neg q$, $\neg q \to \neg r$.

Conclusion: $\neg r$.

Exercies

Assumption: p, $p \to \neg q$, $\neg q \to \neg r$.

Conclusion: $\neg r$.

Prove:

1. p assumption
2. $p \to \neg q$ assumption
3. $\neg q$ modus ponens
4. $\neg q \to \neg r$ assumption
5. $\neg r$ modus ponens

Exercise

Show that the argument with premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, $\neg s$, q and conclusion $q \to r$ is valid.

Exercise

Prove:

1. q assumption
2. $q \rightarrow (u \land t)$ assumption
3. $u \land t$ modus ponens
4. u simplification
5. t simplification
6. $u \rightarrow p$ assumption
7. p modus ponens
8. $p \land t$ conjunction
9. $(p \land t) \rightarrow (r \lor s)$ assumption
10. $(r \lor s)$ modus ponens
11. $\neg s$ assumption
12. r elimination
13. $q \rightarrow r$ implication

Conclusion

- Translate statements into logic.
- Use truth tables to solve logic puzzles.
- Apply rules of inference and logical equivalences.

Thank You

Thank you for your attention!