CSC3001 Discrete Mathematics

Homework 1

1. Let A and B be propositions. Use truth tables to prove the De Morgan's rules. (8 marks)

   (1) $\neg(A \wedge B) \equiv (\neg A \vee \neg B)$
   (2) $\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

2. Show that each of these conditional statements is a tautology by using truth tables. (8 marks)

   (1) $\neg p \to (p \to q)$
   (2) $\neg(p \to q) \to \neg q$

3. For each of these compound propositions, use the conditional-disjunction equivalence to find an equivalent compound proposition that does not involve conditionals. (10 marks)

   (1) $p \to \neg q$
   (2) $(p \to q) \to r$

4. Determine whether each of these statements is true or false. (7 marks)

   (1) $0 \in \emptyset$
   (2) $\emptyset \in \{0\}$
   (3) $\{0\} \subset \emptyset$
   (4) $\emptyset \subset \{0\}$
   (5) $\{0\} \in \{0\}$
   (6) $\{0\} \subset \{0\}$
   (7) $\{\emptyset\} \subseteq \{\emptyset\}$

5. Show that $(\exists x(P(x) \to Q(x))) \leftrightarrow (\forall x P(x) \to \exists x Q(x))$ is a tautology. (10 marks)

6. Let M be a set and let $A, B \subset M$. Prove $M - (A \cup B) = (M - A) \cap (M - B)$. (10 marks)

7. Show that

$$\begin{array}{c} \forall x(P(x) \to (Q(x) \wedge S(x)) \\ \forall x((P(x) \wedge R(x)) \\ \hline \therefore \forall x(R(x) \wedge S(x)) \end{array} \tag{1}$$

   is a valid argument. (11 marks)

8. Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \cdots$ (12 marks)

   (1) Find a_0, a_1, a_2, a_3 and a_4.
   (2) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.
   (3) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

9. Prove or disprove that: (12 marks)

   (1) $(A - B) - (C - D) = (A - C) - (B - D)$.
   (2) $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$.

10. Prove that there is no positive integers n such that $n^2 + n^3 = 100$. (12 marks)