The Chinese University of Hong Kong, Shenzhen
School of Data Science

CSC3001 · Homework 5

Due: evening (11:59pm), Dec 15

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a PDF file. The file name should be in the format **{last name}-{first name}-hw5**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (10pts). Give a combinatorial proof of the identity $2 + 2 + 2 = 3 \cdot 2$.

Problem 2 (10pts). Give a combinatorial proof of the identity $1n + 2(n-1) + 3(n-2) + \cdots + (n-1)2 + n1 = \binom{n+2}{3}$.

Problem 3 (10pts). Give a formula for the coefficient of x^k in the expansion of $(x^2 - \frac{1}{x})^{100}$, where k is an integer.

Problem 4 (10pts). You are getting 10 ice cream sandwiches for 10 students. There are 4 types: Mint, Chocolate, Resse's, and Plain. If there are only 2 Mint ice cream sandwiches and only 3 Plain (and plenty of the other two), how many different ways could you select the ice cream sandwiches for students?

Problem 5 (10pts). Suppose a CUHKSZ party has six students. Consider any two of them. They might be meeting for the first time—in which case we will call them mutual strangers, or they might have met before—in which case we will call them mutual acquaintances. Shows: In any party of six students, at least three of them are (pairwise) mutual strangers or mutual acquaintances.

Problem 6 (10pts). Suppose another CUHKSZ party has ten or more students, there are either four mutual acquaintances or three mutual strangers.

Problem 7 (10pts). Count the number of triples (x, y, z) from $\{1, 2, ..., n+1\}^3$ with $z > \max(x, y)$.

Problem 8 (10pts). Count the number of triples (x, y, z) from $\mathbb{Z}^3$ with $x, y, z \geq 0$ and $x+y+z = 9$.

Problem 9 (10pts). Count the number of triples (x, y, z) from $\mathbb{Z}^3$ with $x, y, z \geq 0$ and $x+y+z = 9$ in condition of $3 \nmid x$.

Problem 10 (10pts). One plays poker with a deck of 52 cards, which come in 4 suits (hearts, clubs, spades, diamonds) with 13 values per suit (A, 2, 3, …, 10, J, Q, K). A "flush" is a set of five cards of the same suit. The order in which one holds the cards in one's hand is immaterial. A "straight" consists of five cards with values forming a string of five consecutive values (with no "wrap around"). For example, 45678, A2345, and 10JQKA are considered straight, but KQA23 is not. How many flushes are possible in poker, and how many of them are straight flushes?