The Chinese University of Hong Kong, Shenzhen
School of Data Science

CSC3001 · Homework 4 (Solution)

Due: evening (11:59pm), Nov 22

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a PDF file. The file name should be in the format **{last name}-{first name}-hw4**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (10pts). How many edges do the following graphs have:

(a) P_n - a path through n vertices:

(b) C_n - a cycle through n vertices:

(c) K_n - a complete graph on n vertices:

(d) $K_{m,n}$ - a complete bipartite graph with m vertices in one component and n vertices in the other:

Problem 2 (10pts). The complement of a simple graph $G = (V, E)$ is the graph $(V, \{(x, y) : x, y \in V, x \neq y\} \setminus E)$. A graph is **self-complementary** if it is isomorphic to its complement. Find an example of **self-complementary** simple graph with 4 vertices, and an example for 5 vertices.

Problem 3 (10pts). Let G be a simple graph with n vertices. Show that if the degree of any vertex of G is $\geq \frac{(n-1)}{2}$, then G must be connected.

Problem 4 (10pts). Let G be a simple graph with n vertices. Show that if G has more than $\frac{(n-1)(n-2)}{2}$ edges, then G must be connected.

Problem 5 (10pts). For which positive integers n does K_n have an

(a) Eulerian cycle.

(b) Eulerian path.

Problem 6 (10pts). Prove that any planar graph must have a vertex of degree 5 or less.

Problem 7 (10pts). Find the number of perfect matchings in K_{2n}.

Problem 8 (10pts). The *chromatic number* of a graph is the least number of colors needed for the coloring of this graph. What is the chromatic number for the following graphs?

(a) P_n - a path through n vertices;

(b) C_n - a cycle through n vertices;

(c) K_n - a complete graph on n vertices;

(d) $K_{m,n}$ - a complete bipartite graph with m vertices in one component and n vertices in the other.

Problem 9 (10pts). Prove that any (simple) graph G with at least two vertices always has two vertices of the same degree.

Problem 10 (10pts). Show that if G is a simple graph with at least 11 vertices, then either G or its complement graph $\overline{G}$, the complement of G, is nonplanar.