The Chinese University of Hong Kong, Shenzhen
School of Data Science

CSC 3001 · Assignment 1
Due: 23:59, September 30th, 2024

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must independently complete each assignment.
- You must submit your assignment in Blackboard with all necessary supplemental material.
- Late submission will not be graded.

Question 1 (10 marks)

Show that each of these conditional statements is a tautology by **using truth tables**.

(1) $\neg p \to (p \to q)$

(2) $\neg(p \to q) \to \neg q$

Question 2 (10 marks)

Using a truth table to obtain the principal conjunctive normal form of $(p \leftrightarrow q) \to r$.
Hint: Principal conjunctive normal form is the logical expression using only AND ($\wedge$), OR ($\vee$), and NOT ($\neg$) operators.

Question 3 (10 marks)

Prove $p \to (q \to r)$ and $p \wedge q \to r$ are logically equivalent.

Question 4 (10 marks)

Determine whether each of these statements is true or false.

(1) $0 \in \emptyset$

(2) $\emptyset \in \{0\}$

(3) $\{0\} \subset \emptyset$

(4) $\emptyset \subset \{0\}$

(5) $\{0\} \in \{0\}$

(6) $\{0\} \subset \{0\}$

(7) $\{\emptyset\} \subseteq \{\emptyset\}$

(8) $\emptyset \in \emptyset$

(9) $\emptyset \in \{\emptyset\}$

(10) $\{a, b\} \in \{a, b, \{a, b, c\}\}$

Question 5 (10 marks)

Let M be a set and let $A, B \subset M$. Prove that $M - (A \cup B) = (M - A) \cap (M - B)$.

Question 6 (10 marks)

Let A, B, and C be sets such that $A \cap B = A \cap C$ and $\bar{A} \cap B = \bar{A} \cap C$. Prove that $B = C$.

Question 7 (10 marks)

Let $Q(x, y, z)$ denote the statement $x+y = z$. Determine the truth value of the following statements and provide explanations for your answers. The domain of all variables is $\mathbb{R}$.

(1) $\forall x \forall y \exists z\, Q(x, y, z)$

(2) $\exists z \forall x \forall y\, Q(x, y, z)$

Question 8 (10 marks)

Show that

$$\begin{array}{l} \forall x(P(x) \vee (Q(x)) \\ \forall x(\neg Q(x) \vee S(x)) \\ \forall x(R(x) \to \neg S(x)) \\ \exists x \neg P(x) \\ \hline \therefore \exists x \neg R(x) \end{array}$$

is a valid argument.

Question 9 (10 marks)

Prove that when x and y are integers of opposite parity, $x^2 - xy - y^2$ is an odd integer.

Question 10 (10 marks)

Prove or disprove that there exists a rational number x and an irrational number y such that x^y is irrational.