**The Chinese University of Hong Kong, Shenzhen**
**School of Data Science**

**CSC3001 · Homework 4**

Due: evening (11:59pm), Dec 1

**Instructions:**

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a PDF file. The file name should be in the format **{last name}-{first name}-hw4**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

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**Problem 1 (10pts).** How many edges do the following graphs have:

(a) $P_n$ - a path through $n$ vertices;

(b) $C_n$ - a cycle through $n$ vertices;

(c) $K_n$ - a complete graph on $n$ vertices;

(d) $K_{m,n}$ - a complete bipartite graph with $m$ vertices in one component and $n$ vertices in the other.

**Problem 2 (10pts).** The complement of a simple graph $G = (V, E)$ is the graph $(V, \{(x, y) : x, y \in V, x \neq y\} \setminus E)$. A graph is **self-complementary** if it is isomorphic to its complement. Find an example of **self-complementary** simple graph with 4 vertices, and an example for 5 vertices.

**Problem 3 (10pts).** Let $G$ be a simple graph with $n$ vertices. Show that if the degree of any vertex of $G$ is $\geq \frac{(n-1)}{2}$, then $G$ must be connected.

**Problem 4 (10pts).** Let $G$ be a simple graph with $n$ vertices. Show that if $G$ has more than $\frac{(n-1)(n-2)}{2}$ edges, then $G$ must be connected.

**Problem 5 (10pts).** For which positive integers $n$ does $K_n$ have an

(a) Eulerian cycle.

(b) Eulerian path.

**Problem 6 (10pts).** Find the number of perfect matchings in $K_{n,n}$.

**Problem 7 (10pts).** Find the number of perfect matchings in $K_{2n}$.

**Problem 8 (10pts).** The *chromatic number* of a graph is the least number of colors needed for the coloring of this graph. What is the chromatic number for the following graphs?

(a) $P_n$ - a path through $n$ vertices;

(b) $C_n$ - a cycle through $n$ vertices;

(c) $K_n$ - a complete graph on $n$ vertices;

(d) $K_{m,n}$ - a complete bipartite graph with $m$ vertices in one component and $n$ vertices in the other.

**Problem 9 (10pts).** Schedule the final exams for CSC3001, CSC3002, CSC3003, CSC3004, CSC3005, CSC3006, CSC3007, and CSC3008, using the fewest number of different time slots, if there are **no students** taking both CSC3001 and CSC3008, both CSC3002 and CSC3008, both CSC3004 and CSC3005, both CSC3004 and CSC3006, both CSC3001 and CSC3002, both CSC3001 and CSC3003, and both CSC3003 and CSC3004, but there are students in every other pair of courses.

**Problem 10 (10pts).** Show that if $G$ is a simple graph with at least 11 vertices, then either $G$ or its complement graph $\overline{G}$, the complement of $G$, is nonplanar.