The Chinese University of Hong Kong, Shenzhen
School of Data Science

CSC3001 · Homework 5

Due: evening (11:59pm), Dec 6

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a PDF file. The file name should be in the format **{last name}-{first name}-hw5**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (10 pts). Give a combinatorial proof of the identity $1n + 2(n-1) + 3(n-2) + \cdots + (n-1)2 + n1 = \binom{n+2}{3}$.

Problem 2 (10 pts). How many bit strings of length 19 contain at least nine 1's and at least nine 0's? You may leave your answer as an equation.

Problem 3 (10 pts). Alice is going to choose a selection of 12 chocolates. There are 25 different brands of them and she can have as many as she wants of each brand (but can only choose 12 pieces). How many ways can she make this selection? You may leave your answer as an equation.

Problem 4 (10 pts). You are getting 10 ice cream sandwiches for 10 students. There are 4 types: Mint, Chocolate, Resse's, and Plain. If there are only 2 Mint ice cream sandwiches and only 3 Plain (and plenty of the other two), how many different ways could you select the ice cream sandwiches for students?

Problem 5 (10 pts). Suppose a CUHKSZ party has six students. Consider any two of them. They might be meeting for the first time—in which case we will call them mutual strangers, or they might have met before—in which case we will call them mutual acquaintances. Shows: In any party of six students, at least three of them are (pairwise) mutual strangers or mutual acquaintances.

Problem 6 (10pts). Suppose another CUHKSZ party has ten or more students, there are either four mutual acquaintances or three mutual strangers.

Problem 7 (10 pts). Count the number of triples (x, y, z) from $\{1, 2, ..., n+1\}^3$ with $z > \max(x, y)$.

Problem 8 (10 pts). Count the number of triples (x, y, z) from $\mathbb{Z}^3$ with $x, y, z \geq 0$ and $x+y+z = 9$.

Problem 9 (10pts). Count the number of triples (x, y, z) from $\mathbb{Z}^3$ with $x, y, z \geq 0$ and $x+y+z = 9$ in condition of $3 \nmid x$.

Problem 10 (10 pts). One plays poker with a deck of 52 cards, which come in 4 suits (hearts, clubs, spades, diamonds) with 13 values per suit (A, 2, 3, ..., 10, J, Q, K).

Problem 10.1 In a game of 3-card poker, a "three of a kind" is a set of three cards of the same rank (value). What is the probability of being dealt three of a kind?

Problem 10.2 In a game of 5-card poker, a "flush" is a set of five cards of the same suit. The order in which one holds the cards in one's hand is immaterial. A "straight" consists of five cards with values forming a string of five consecutive values (with no "wrap around"). For example, 45678, A2345, and 10JQKA are considered straight, but KQA23 is not. How many flushes are possible in poker, and how many of them are straight flushes?

Problem 10.3 In a game of 7-card poker, calculate the probability of being dealt:

1. A hand with no pairs (all seven cards have different ranks).
2. A hand with exactly one pair (one pair and the remaining five cards have different ranks).
3. A hand with exactly two pairs (two pairs and the remaining three cards have different ranks).

You may leave your answer as an equation.