



**CSC3001 · Homework 2**

Due: evening (11:59pm), Nov 3

**Instructions:**

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a pdf file with codes. The file name should be in the format **last name-first name-hw2**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

**Problem 1 (8pts).** Find all sequences  $\{a_n\}_{n \in \mathbb{N}}$  satisfying

$$a_{n+2} - 2a_{n+1} + a_n = 2$$

**Problem 2 (10pts).** Prove that  $1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$  whenever  $n$  is a nonnegative integer.

**Problem 3 (10pts).** Prove that for every positive integer  $n$ ,

$$1 + 1/\sqrt{2} + 1/\sqrt{3} + \dots + 1/\sqrt{n} > 2(\sqrt{n+1} - 1)$$

**Problem 4 (12pts).** Find a closed form for the generating function for the sequence  $\{a_n\}$ , where

- (a)  $a_n = 3^n$  for all  $n = 0, 1, 2, \dots$
- (b)  $a_n = 2n + 3$  for all  $n = 0, 1, 2, \dots$
- (c)  $a_n = 2$  for all  $n = 3, 4, 5, \dots$  and  $a_0 = a_1 = a_2 = 0$

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**Problem 5 (10pts).** Use generating functions to solve the recurrence relation  $a_k = 3a_{k-1} + 2$  with the initial condition  $a_0 = 1$ .

**Problem 6 (10pts).** Use generating functions to solve the recurrence relation  $a_k = 5a_{k-1} - 6a_{k-2}$  with initial conditions  $a_0 = 6$  and  $a_1 = 30$ .

**Problem 7 (10pts).** Let  $a, b \in \mathbb{N}^+$ . Prove that

(a)  $\gcd(2^a - 1, 2^b - 1) = 2^{\gcd(a,b)} - 1$ .

(b)  $(2^a - 1) \bmod (2^b - 1) = 2^{a \bmod b} - 1$ .

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**Problem 8 (10pts).** Show steps to find

(a) the greatest common divisor of 1234567 and 7654321.

(b) the greatest common divisor of  $2^3 3^5 5^7 7^9 11$  and  $2^9 3^7 5^5 7^3 13$ .

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**Problem 9 (10pts).** Show that if  $2^n - 1$  is prime, then  $n$  is prime. [Hint: Use the identity  $2^{ab} - 1 = (2^a - 1) \times (2^{a(b-1)} + 2^{a(b-2)} + \dots + 2^a + 1)$ .]

**Problem 10 (10pts).** Find all solutions, if any, solutions to the system

$$x \equiv 5 \pmod{6}$$

$$x \equiv 3 \pmod{10}$$

$$x \equiv 8 \pmod{15}$$