The Chinese University of Hong Kong, Shenzhen
School of Data Science

CSC3001 · Homework 3

Due: evening (11:59pm), Nov 19

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a pdf file with codes. The file name should be in the format **last name-first name-hw3**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (10pts). Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.

Problem 2 (10pts). Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n.

Problem 3 (10pts). Find the coefficient of x^{10} in the power series of each of these functions.

(a) $1/(1 + x)^2$

(b) $1/(1 - x)^3$

(c) $1/(1 + 2x)^4$

(d) $x^4/(1 - 3x)^3$

.

Problem 4 (10pts). Use generating functions to solve the recurrence relation $a_k = 4a_{k-1} - 4a_{k-2} + k^2$ with initial conditions $a_0 = 2$ and $a_1 = 5$.

Problem 5 (10pts). Use the Euclidean algorithm to find

(a) $gcd(12, 18)$

(b) $gcd(111, 201)$

(c) $gcd(1001, 1331)$

(d) $gcd(12345, 54321)$

.

Problem 6 (10pts). Suppose that a and b are integers, $a \equiv 4(mod\ 13)$, and $b \equiv 9(mod\ 13)$. Find the integer c with $0 \leq c \leq 12$ such that

(a) $c \equiv 9a(mod\ 13)$.

(b) $c \equiv 11b(mod\ 13)$.

(c) $c \equiv a + b(mod\ 13)$.

(d) $c \equiv 2a + 3b(mod\ 13)$.

.

Problem 7 (10pts). Find the integer a such that

(a) $a \equiv -15(mod\ 27)$ and $-26 \leq a \leq 0$.

(b) $a \equiv 24(mod\ 31)$ and $-15 \leq a \leq 15$.

(c) $a \equiv 99(mod\ 41)$ and $100 \leq a \leq 140$.

.

Problem 8 (10pts). Find an inverse of a modulo m for each of these pairs of relatively prime integers

(a) $a = 4, m = 9$.

(b) $a = 19, m = 141$.

(c) $a = 55, m = 89$.

(d) $a = 89, m = 232$.

.

Problem 9 (10pts). Solve each of these congruences using the modular inverses found in parts (b), (c), and (d) of Problem 8.

(a) $19x \equiv 4(mod\ 141)$.

(b) $55x \equiv 34(mod\ 89)$.

(c) $89x \equiv 2(mod\ 232)$.

.

Problem 10 (10pts). Find all solutions, if any, solutions to the system

$$x \equiv 1\ (mod\ 2)$$
$$x \equiv 2\ (mod\ 3)$$
$$x \equiv 3\ (mod\ 5)$$
$$x \equiv 4\ (mod\ 11)$$