香港中文大學(深圳)
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CSC3100 Data Structures
Lecture 13: Tree and binary tree

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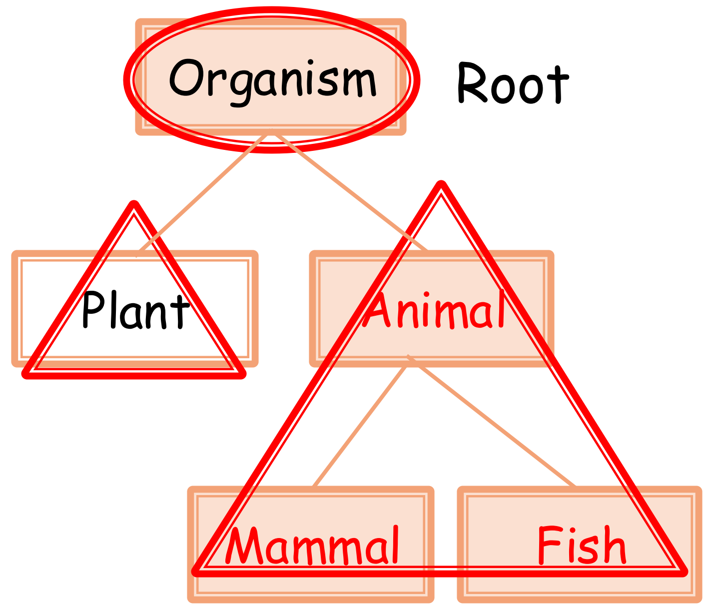
Outline

- In this lecture, we will learn
  - Basic concept of trees
  - Binary tree ADT and implementations
  - Traversal of binary trees
  - Reconstruction of binary trees

Tree definition

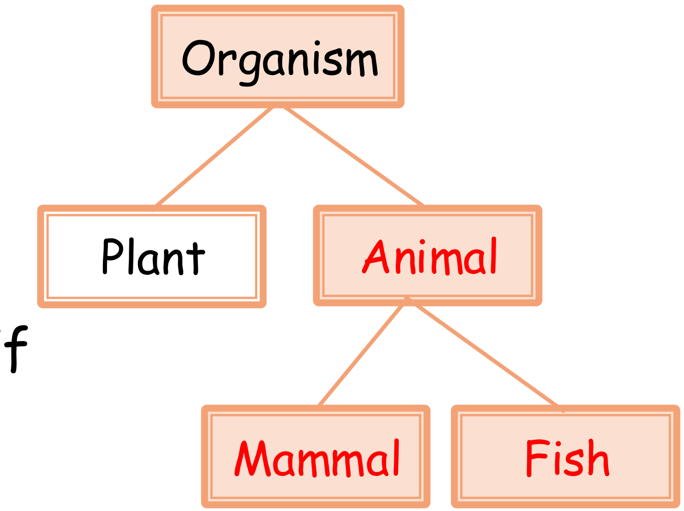
- A tree is a finite set of one or more nodes such that
  - Each node stores an element
  - There is a special node called the root
  - The remaining nodes are partitioned into $n \geq 0$ disjoint sets $T_1, \ldots, T_n$ where each of these sets is a tree
  - We call $T_1, \ldots, T_n$ the subtrees of the root



- Note
  - A tree with N nodes has N-1 edges
  - Every node in the tree is the root of some subtree (recursive definition)

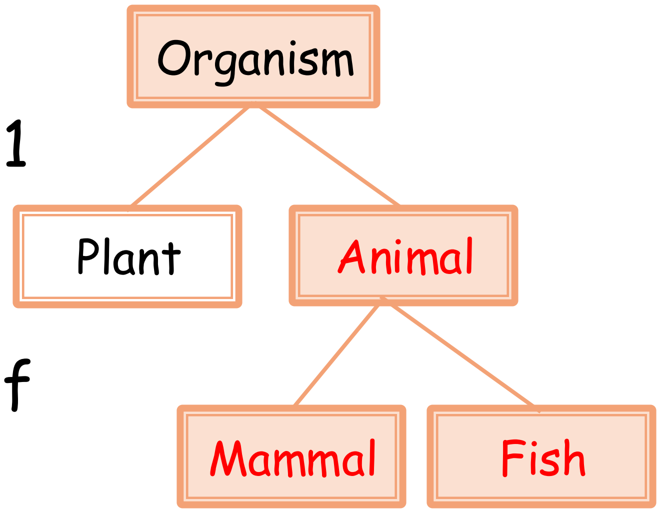
Definitions

- Parent
  - Node A is the parent of node B if B is the root of the left or right sub-tree of A
- Left (right) child
  - Node B is the left (right) child of node A if A is the parent of B
- Sibling
  - Node B and node C are siblings if they have the same parent
- Leaf
  - A node is called a leaf if it has no children



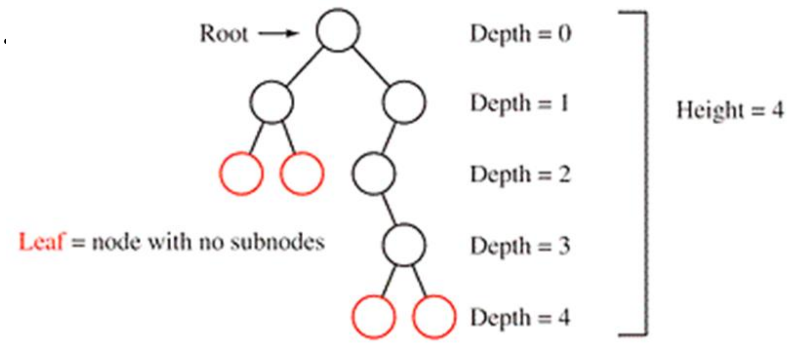
Definitions

- A path from node n_1 to n_k
  - A sequence of nodes $n_1, n_2, \ldots, n_k$ such that n_i is the parent of n_{i+1} for $1 \leq i < k-1$
- Length of a path
  - The length of this path is the number of edges on the path, namely k-1
  - Notice that in a tree, there is exactly only one path from the root to each node



Definitions

- Depth of a node
  - The depth of a node n_i is the length of the unique path from the root to n_i
  - The root is at depth 0



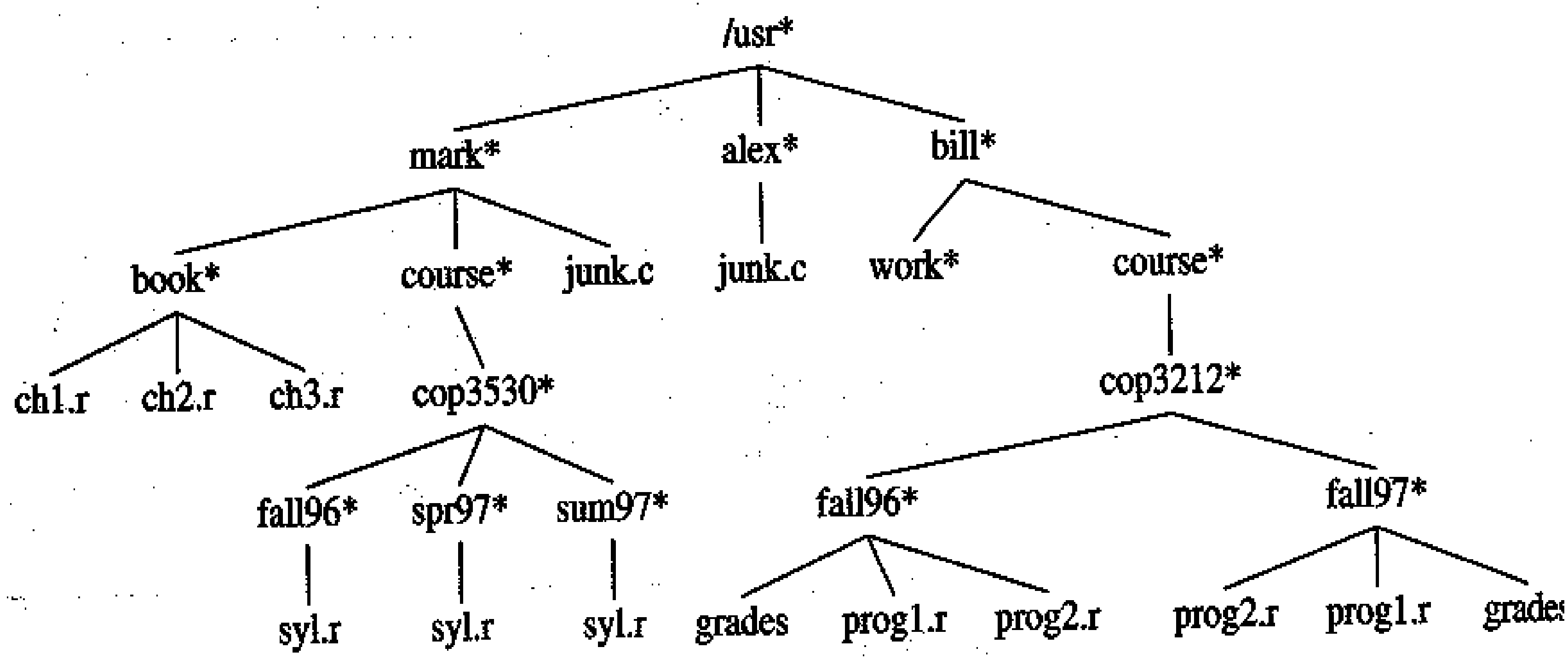
- Height of a node
  - The height of a node n_i is the length of the longest path from n_i to a leaf
  - All leaves are at height 0

Note 1: The height of a tree is equal to the height of the root

Note 2: The depth of a tree is equal to the depth of the deepest leaf

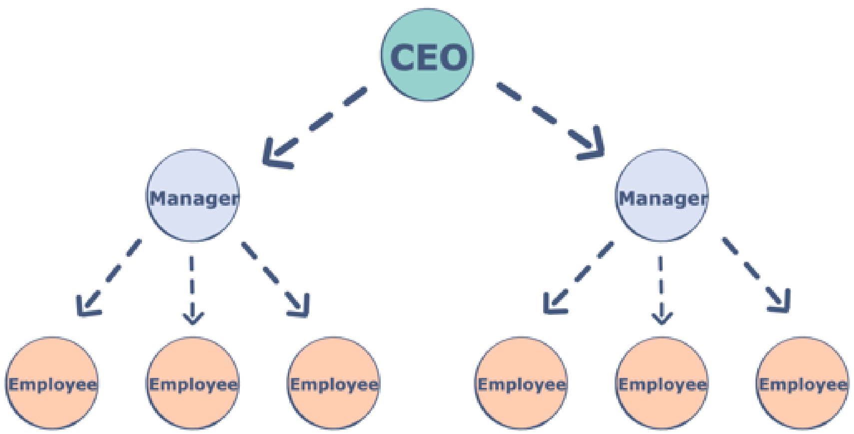
Note 3: The maximum height is equal to the maximum depth

Applications: Unix file system



More applications

- HR system



- Java data types

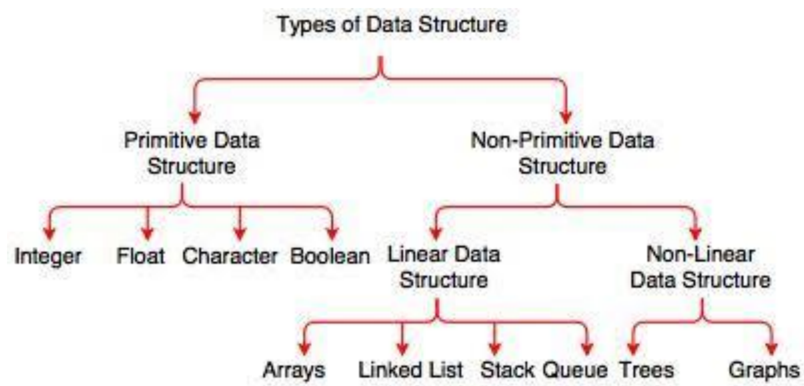
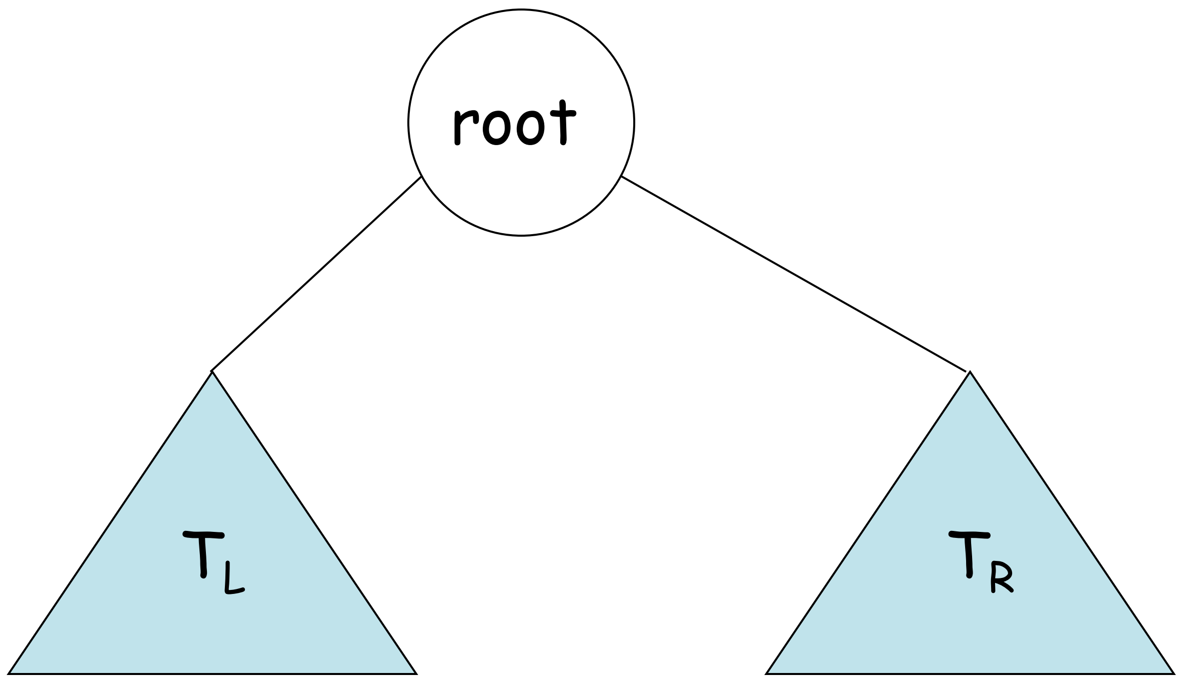


Fig. Types of Data Structure

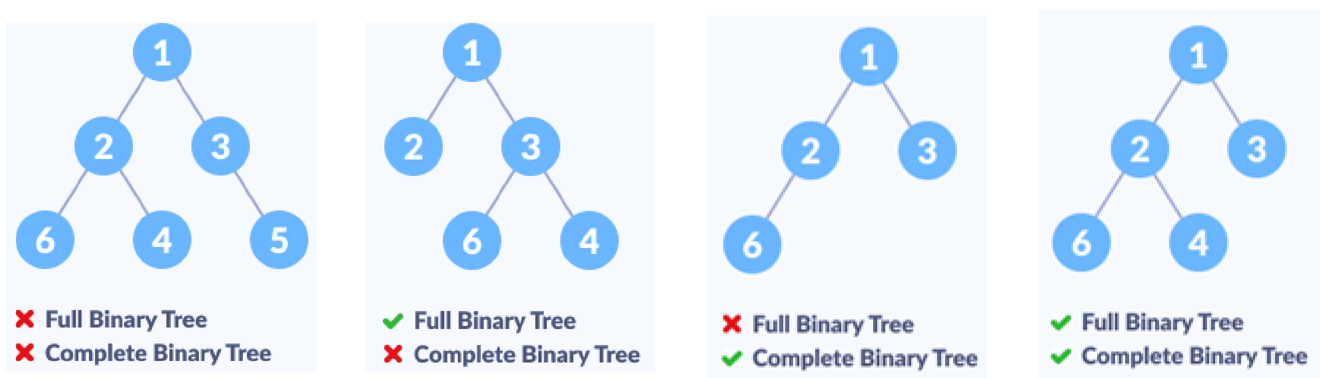
Binary tree

- A binary tree is a tree, in which
  - No node can have more than two children (subtrees): T_L and T_R, both of which could possibly be empty



Binary tree

- Full binary tree
  - A binary tree where all the nodes have either two or no children
- Complete binary tree
  - A binary tree where all the levels are completely filled except possibly the lowest one, which is filled from the left



Binary tree ADT

- Operations:
  - Create(bintree): creates an empty binary tree
  - Boolean IsEmpty(bintree): if bintree is empty return TRUE else FALSE
  - MakeBT(bintree1,element,bintree2): return a binary tree whose left subtree is bintree1 and right subtree is bintree2, and whose root node contains the data element

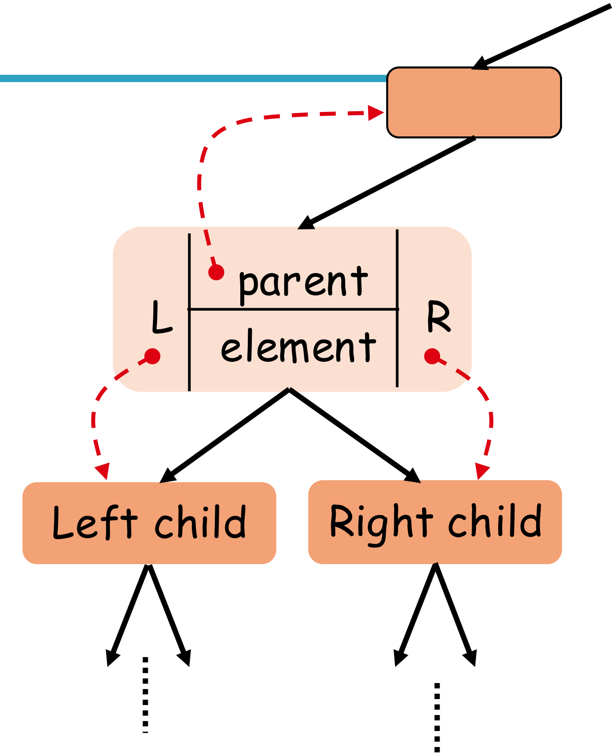
  - Lchild(bintree): if bintree is empty return error else return the left subtree of bintree
  - Rchild(bintree): if bintree is empty return error else return the right subtree of bintree
  - Data(bintree): if bintree is empty return error else return the element data stored in the root node of bintree

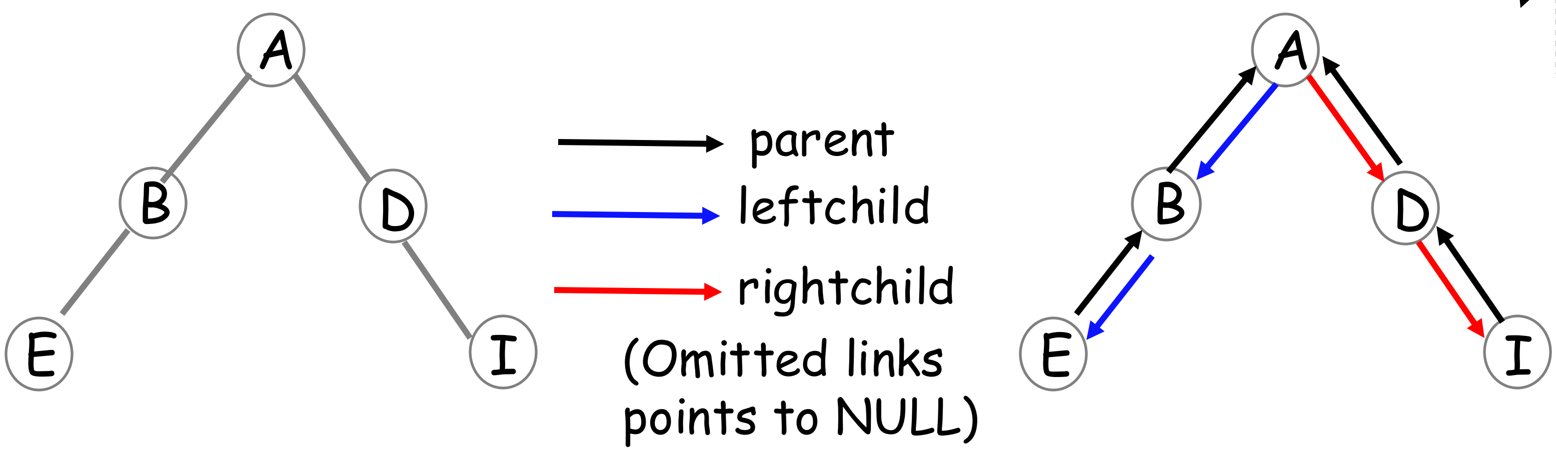
Binary tree design

- Two solutions
  - Using pointers
    - More intuitive solution
    - We will see the pseudo-codes
  - Using array
    - Need more complicated design, and cannot efficiently handle all operations (thus will omit its implementations for each operation)
    - Will be used for heap, a special type of complete binary tree

Binary tree design (i)

- A pointer representation
  - For each node node, we maintain
    - node.parent: store the address of its parent,
    - node.leftchild: store the address of its left child,
    - node.rightchild: store the address of its right child
    - node.element: store the values





Binary tree: pointer implementation

- Create(bintree)

Algorithm: create(bintree)

```
bintree = NULL
return bintree
```

- isEmpty(bintree)

Algorithm: isEmpty(bintree)

```
return bintree == NULL
```

- MakeBT(bintree1, element, bintree2)

Algorithm: MakeBT(bintree1, element, bintree2)

```
rootNode <- allocate new memory
rootNode.element = element
rootNode.parent = NULL
rootNode.leftchild = bintree1
rootNode.rightchild = bintree2
if bintree1 != NULL
  bintree1.parent = rootNode
if bintree 2 != NULL
  bintree2.parent = rootNode
return  rootNode
```

Binary tree: pointer implementation

- Lchild(bintree)

Algorithm: **Lchild(bintree)**

```
if bintree == NULL
  error "empty tree"
return bintree.leftchild
```

- Rchild(bintree)

Algorithm: **Rchild(bintree)**

```
if bintree == NULL
  error "empty tree"
return bintree.rightchild
```

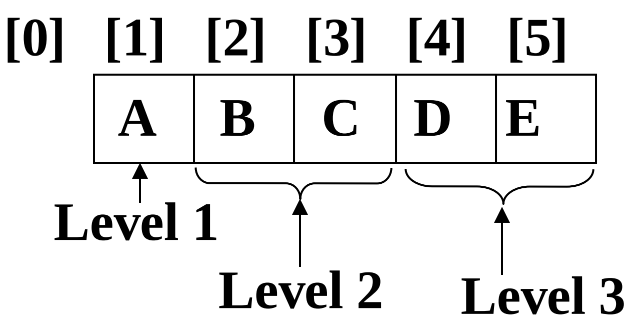
- Data(bintree)

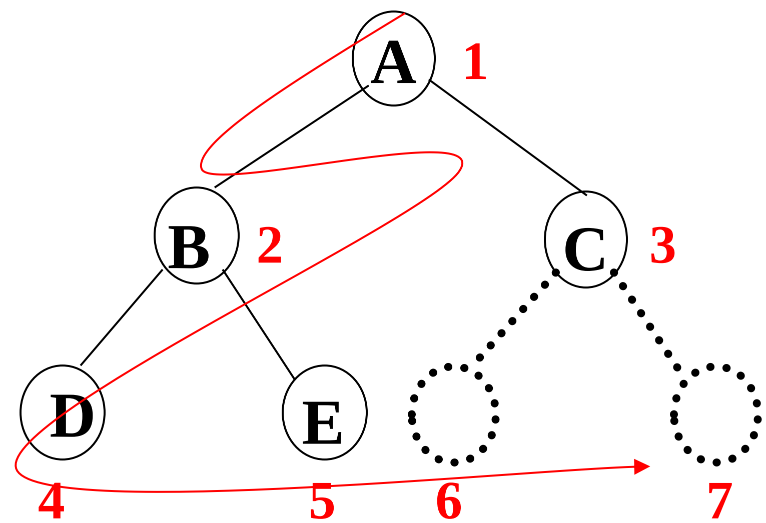
Algorithm: **Data(bintree)**

```
if bintree == NULL
  error "empty tree"
return bintree.element
```

Binary tree design (ii)

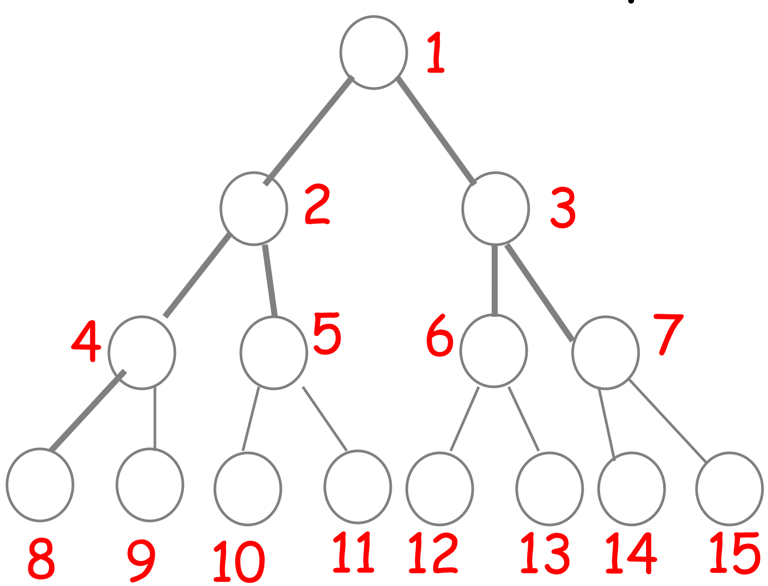
- An array representation
  - Given a complete binary tree with n nodes, for any i-th node, $1 \leq i \leq n$,
    - parent(i) is $\lfloor i/2 \rfloor$
    - leftChild(i) is at $2i$ if $2i \leq n$; otherwise, i has no left child
    - rightChild(i) is at $2i+1$ if $2i+1 \leq n$; otherwise, i has no right child



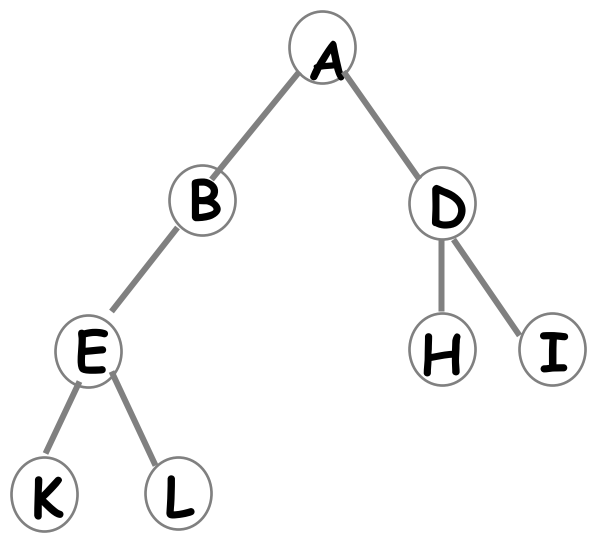


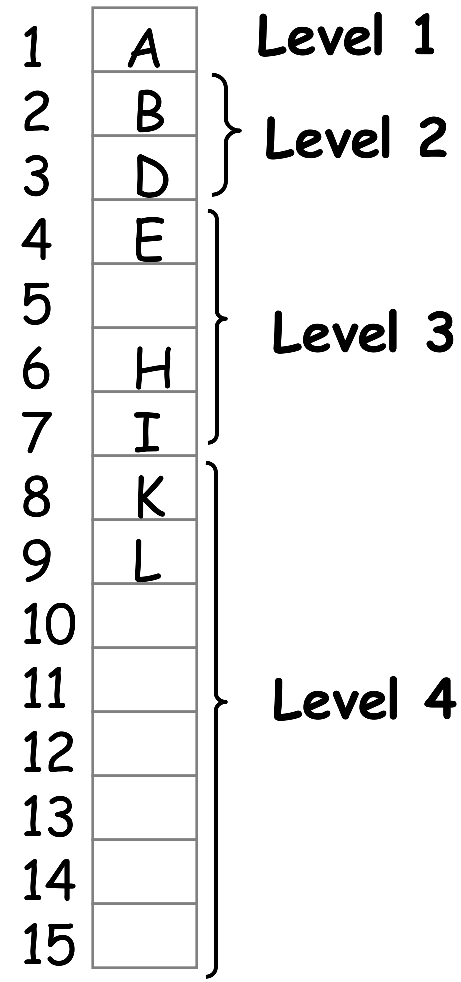
Binary tree design (ii)

- An array representation
  - Generalize to all binary trees
  - Efficient for complete binary trees
  - But inefficient for skewed binary trees
  - Inefficient to implement the ADT



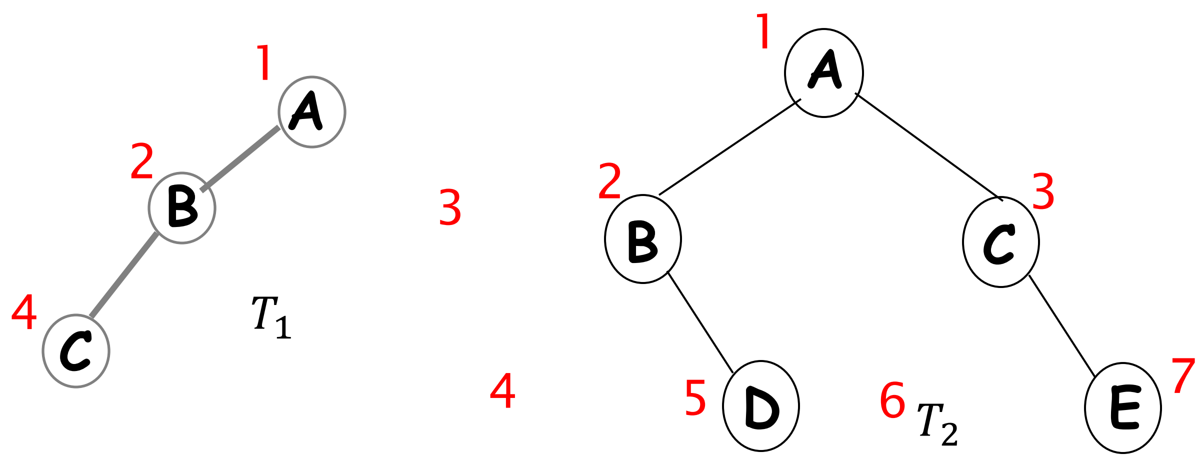
full binary tree





Practice

- What are the array representation of the following binary trees?
  - Show the content in the array
  - Hint: first obtain the ID for each node



	[1]	[2]	[3]	[4]	[5]	[6]	[7]
arr							

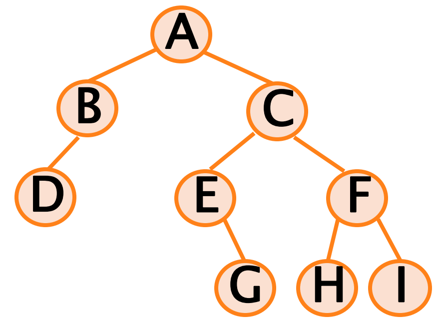
Traversing strategies

- Preorder (depth-first)
  - Visit the node
  - Traverse the left subtree in preorder
  - Traverse the right subtree in preorder
- Inorder
  - Traverse the left subtree in inorder
  - Visit the node
  - Traverse the right subtree in inorder
- Postorder
  - Traverse the left subtree in postorder
  - Traverse the right subtree in postorder
  - Visit the node

Traversing binary tree

When the binary tree is empty, it is "traversed" by doing nothing, otherwise:

Example:



preorder traversal

Visit the root

Traverse the left subtree

Traverse the right subtree

A B D C E G F H I

Result:

= A (A's left) (A's right)
= A B **(B's left)** (B's right=NULL) (A's right)
= A B D (D's left=NULL) (D's right=NULL) (B's right=NULL) (A's right)
= A B D (A's right)
= A B D C **(C's left)** (C's right)
= A B D C E **(E's left=NULL) (E's right)** (C's right)
= A B D C E **(E's right)** (C's right)
= A B D C E G **(G's left=NULL) (G's right = NULL)** (C's right)
= A B D C E G (C's right)
= A B D C E G F (F's left) (F's right)
= A B D C E G F H (H's left=NULL) (H's right =NULL) (F's right)
= A B D C E G F H (F's right)
= A B D C E G F H I (I's left=NULL) (I's right =NULL)
= A B D C E G F H I

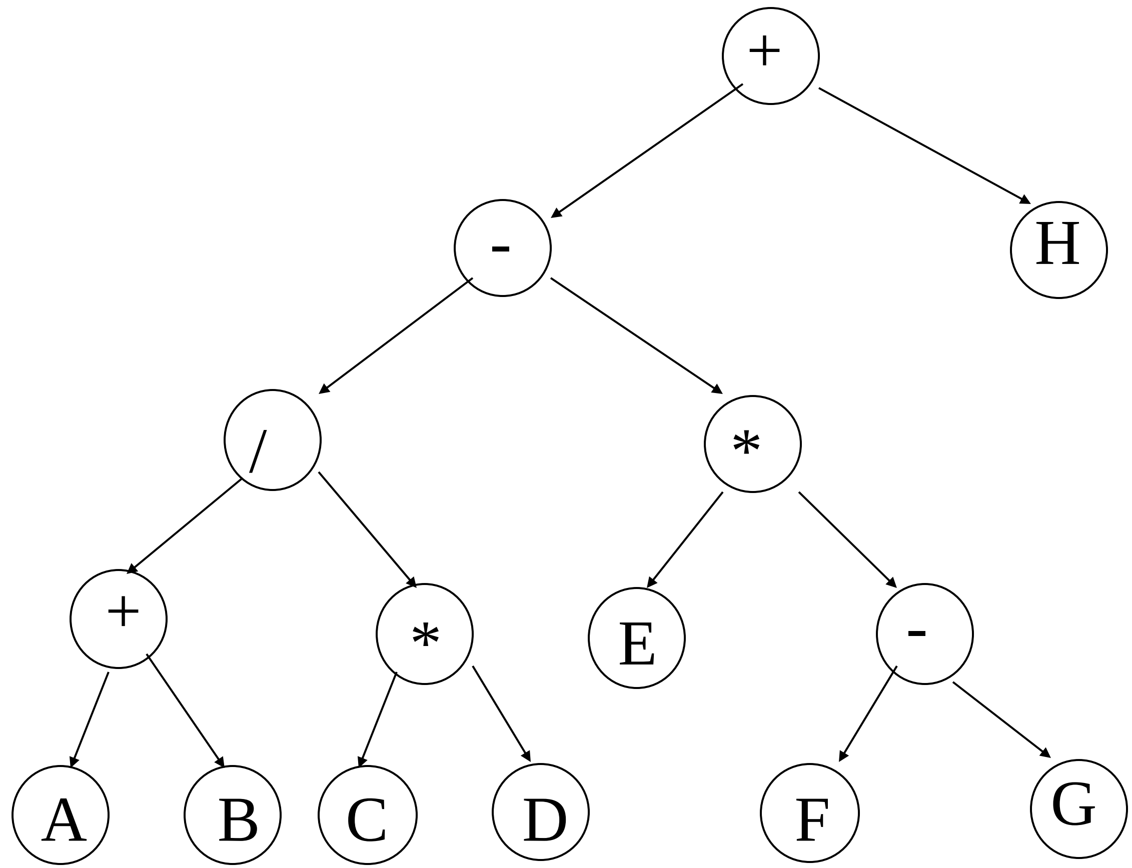
Exercise

(A+B)/(C*D)-E*(F-G)+H

Preorder:
+-/+AB*CD*E-FGH

Inorder :
A+B/C*D-E*F-G+H

Postorder:
AB+CD*/EFG-*-H+

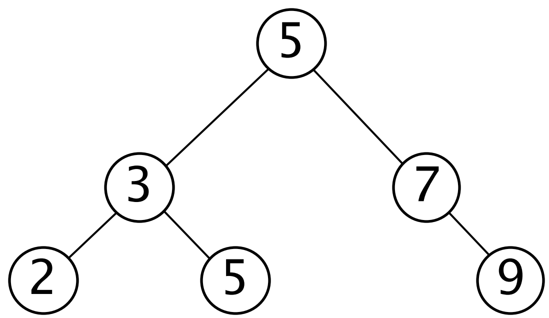


Given an expression, what is the relationship between its postfix and postorder?

Implementation

```
INORDER-TREE-WALK(x)
    if x ≠ NIL
       then INORDER-TREE-WALK ( left [x] )
            print key [x]
            INORDER-TREE-WALK ( right [x] )
```

E.g.:

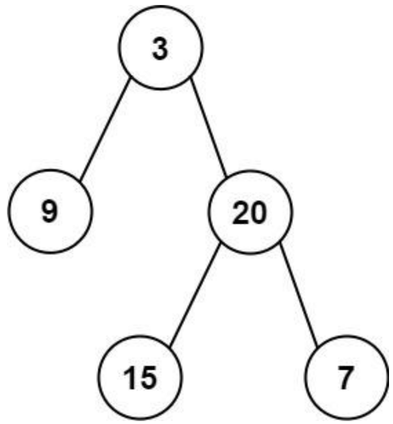


Output: 2 3 5 5 7 9

- Running time:
  - $\Theta(n)$, where n is the size of the tree rooted at x

Exercise

- Given a binary tree, show its preorder, inorder, and postorder



  - preorder=[3, 9, 20, 15, 7]
  - inorder=[9, 3, 15, 20, 7]
  - postorder=[9, 15, 7, 20, 3]

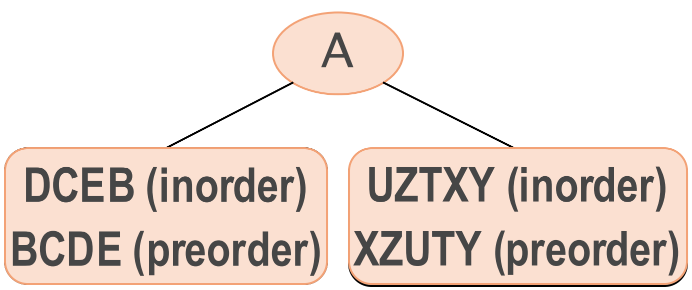
Binary tree reconstruction

Reconstruction of a binary tree from its preorder and inorder sequences	**Example:** Given the following sequences, find the corresponding binary tree: inorder : DCEBAUZTXY preorder : ABCDEXZUTY

Looking at the whole tree:

- "preorder : ABCDEXZUTY"
  ==> A is the root
- Then, "inorder : DCEBAUZTXY"

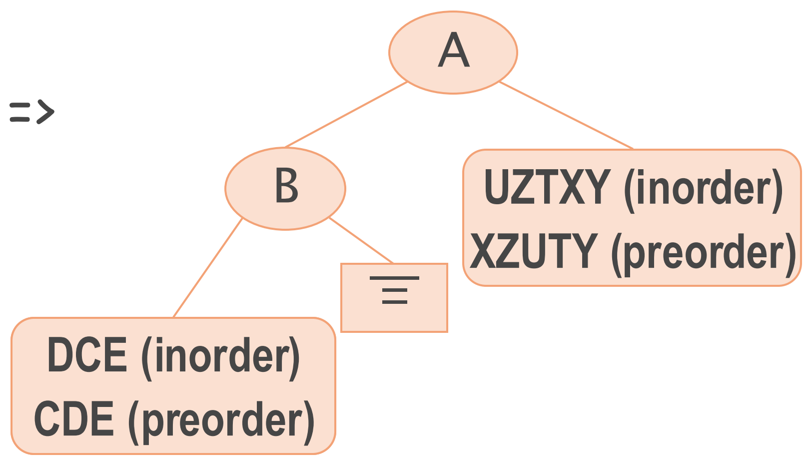
==>



Looking at the left subtree of A:

- "preorder : BCDE"
  ==> B is the root
- Then, "inorder: DCEB"

=>



Binary tree reconstruction

Reconstruction of a binary tree from its preorder and inorder sequences

Example: Given the following sequences, find the corresponding binary tree:

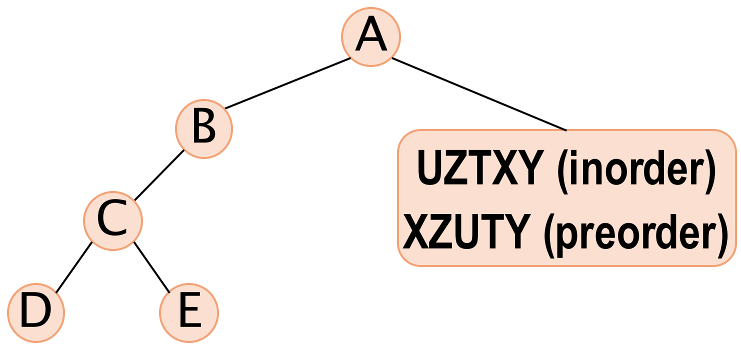
inorder : DCEBAUZTXY

preorder : ABCDEXZUTY

Looking at the left subtree of B:

- "preorder : CDE"
  ==> C is the root
- Then, "inorder: D<u>C</u>E"

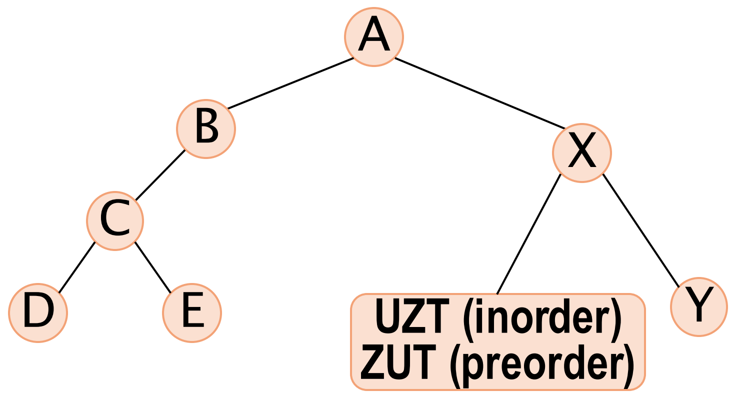
=>



Looking at the right subtree of A:

- "preorder : XZUTY"
  ==> X is the root
- Then, "inorder: UZT**<u>X</u>**Y"

=>



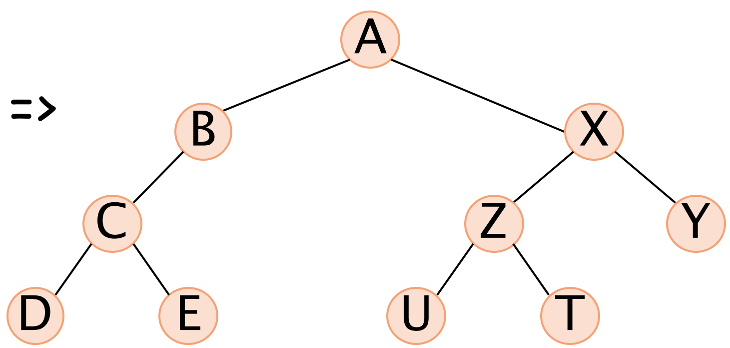
Binary tree reconstruction

Reconstruction of a binary tree from its preorder and inorder sequences	**Example:** Given the following sequences, find the corresponding binary tree: inorder : DCEBAUZTXY preorder : ABCDEXZUTY

Looking at the left subtree of X:

- "preorder : ZUT"
  ==> Z is the root
- Then, "inorder: UZT"

=>



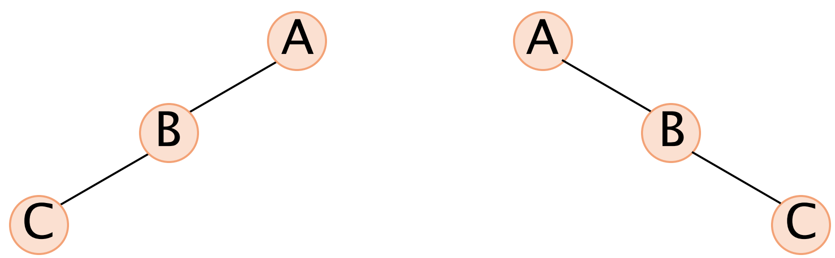
Binary tree reconstruction

But: A binary tree may not be uniquely defined by its preorder and postorder sequences

Example:

Preorder sequence: ABC
Postorder sequence: CBA

We can construct 2 different binary trees:



Java implementation codes

```java
class BTreeBuilder {
    private static int find(char[] array, char v){
        for (int i = 0;i < array.length; i ++){
            if (array[i] == v)  return i;
        }
        return -1;
    }

    public TreeNode build (char[] preorder, char[] inorder) {
        if (preorder.length == 0)  return null;

        //step 1: find the root key and create a root node
        char rootValue = preorder[0];
        TreeNode root = new TreeNode(rootValue);

        //step 2: find the index of root key in in-order
        int leftSize = find(inorder,rootValue);

        //step 3: build left and right sub-tree’s
        char[] leftPreorder = Arrays.copyOfRange(preorder, 1, 1+leftSize);
        char[] leftInorder = Arrays.copyOfRange(inorder, 0, leftSize);
        root.left = build (leftPreorder, leftInorder);

        char[] rightPreorder = Arrays.copyOfRange(preorder, 1+leftSize, preorder.length);
        char[] rightInorder = Arrays.copyOfRange(inorder, leftSize+1, preorder.length);
        root.right = build(rightPreorder, rightInorder);

        return root;
    }
}
```

```java
class TreeNode {
        char val;
        TreeNode left, right;

        TreeNode(char x) {
            val = x;
        }
}
```

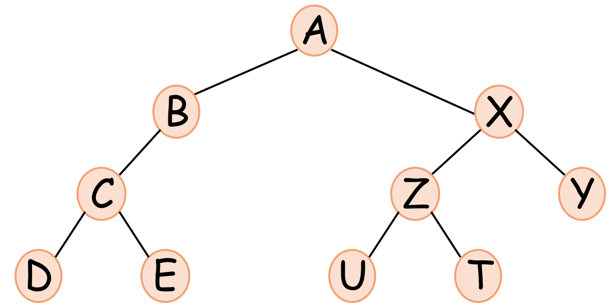
```java
public class Test{
      public static void main(String[] args) {
          char[] preOrder =
                  new char[]{'A','B','C','D','E'};
          char[] inOrder =
                  new char[]{'C','D','B','A','E'};

          BTreeBuilder b = new BTreeBuilder();
          TreeNode root = b.build(preOrder, inOrder);
          System.out.println("build successfully");
      }
}
```

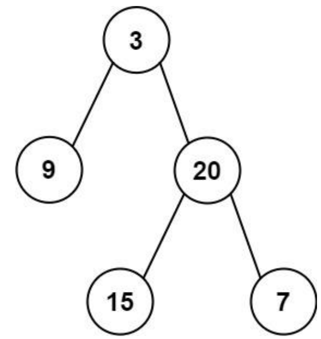
What’s the time complexity?

Exercises

- Show the results of traversing using preorder, inorder, and postorder respectively



- Construct a binary tree such that
  - preorder=[3,9,20,15,7]
  - inorder=[9,3,15,20,7]



- Is it possible to reconstruct a binary tree from its inorder and postorder?
  - If yes, how to do it? if no, why?

Recommended reading

- Reading this week
  - Chapter 12, textbook

- Next lecture
  - Binary search trees: chapter 12