



香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen



Ack: Prof. Jignesh Patel @ CMU
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CSC3170

7: Hash Tables

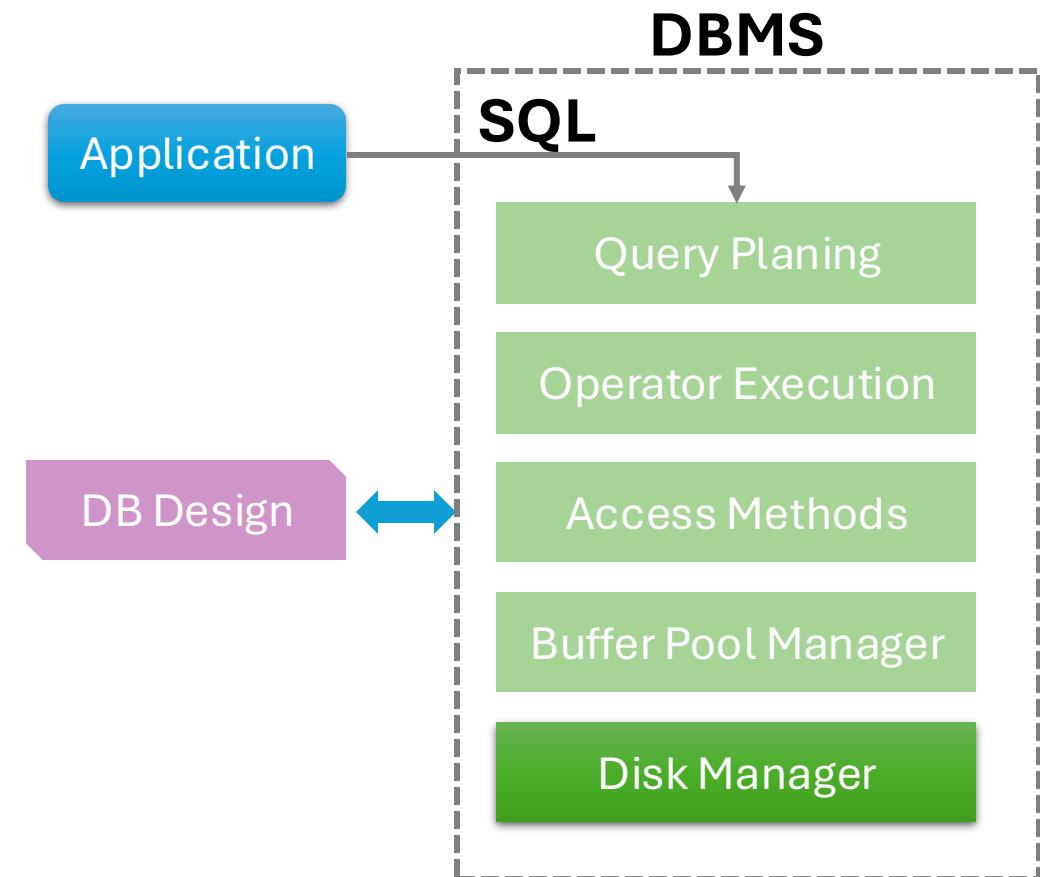
Chenhao Ma

School of Data Science

The Chinese University of Hong Kong, Shenzhen

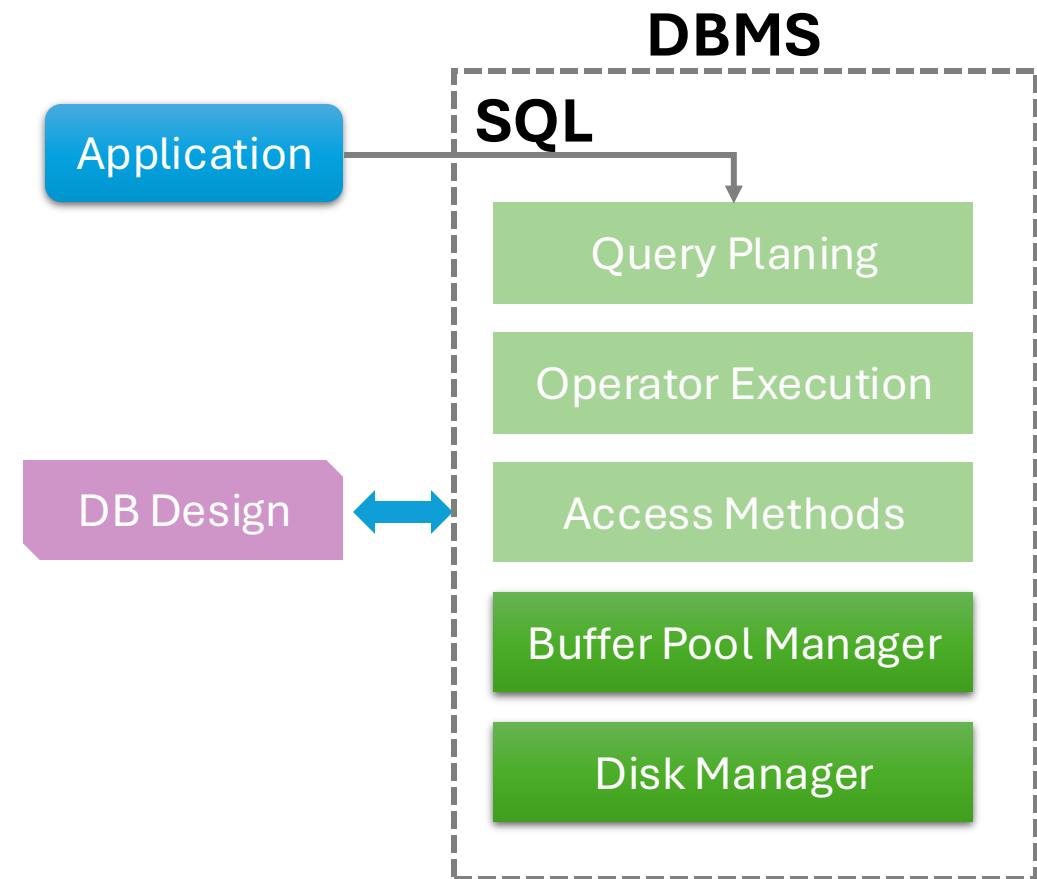
Course Status

- We are now going to talk about how to support the DBMS's execution engine to read/write data from pages.
- Two types of data structures:
 - Hash Tables (Unordered)
 - Trees (Ordered)



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Data Structures

- Internal Meta-data
- Core Data Storage
- Temporary Data Structures
- Table Indexes

Design Decisions

- **Data Organization**
 - How we layout data structure in memory/pages and what information to store to support efficient access.
- **Concurrency**
 - How to enable multiple threads to access the data structure at the same time without causing problems.

Hash Tables

- A **hash table** implements an unordered associative array that maps keys to values.
- It uses a **hash function** to compute an offset into this array for a given key, from which the desired value can be found.
- Space Complexity: $\mathcal{O}(n)$
Time Complexity:
 - Average: $\mathcal{O}(1)$
 - Worst: $\mathcal{O}(n)$

Hash Tables

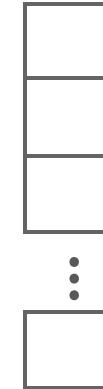
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← *Databases care about constants!*

Static Hash Table

- Allocate a giant array that has one slot for every element you need to store.
- To find an entry, mod the key by the number of elements to find the offset in the array.

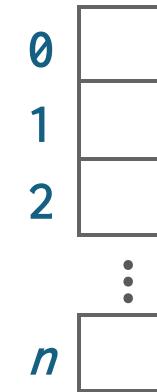
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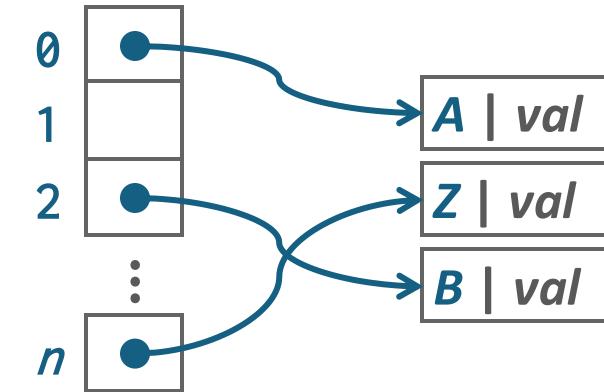
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0	A
1	Ø
2	B
⋮	⋮
<i>n</i>	Z

Static Hash Table

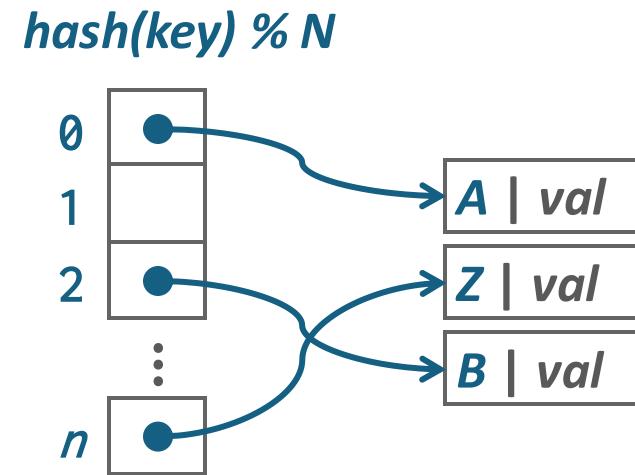
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Unrealistic Assumptions

- **Assumption #1:** Number of elements is known ahead of time and fixed.
- **Assumption #2:** Each key is unique.
- **Assumption #3:** Perfect hash function guarantees no collisions.
 - If $\text{key}_1 \neq \text{key}_2$, then $\text{hash}(\text{key}_1) \neq \text{hash}(\text{key}_2)$



Hash Table

- **Design Decision #1: Hash Function**
 - How to map a large key space into a smaller domain.
 - Trade-off between being fast vs. collision rate.
- **Design Decision #2: Hashing Scheme**
 - How to handle key collisions after hashing.
 - Trade-off between allocating a large hash table vs. additional instructions to get/put keys.

This Lecture

- Hash Functions
- Static Hashing Schemes
- Dynamic Hashing Schemes

Hash Functions

Hash Functions

- For any input key, return an integer representation of that key.
- We do not want to use a cryptographic hash function for DBMS hash tables (e.g., SHA-2).
- We want something that is fast and has a low collision rate.

Hash Functions

- **CRC-64** (1975)
 - Used in networking for error detection.
- **MurmurHash** (2008)
 - Designed as a fast, general-purpose hash function.
- **Google CityHash** (2011)
 - Designed to be faster for short keys (<64 bytes).
- **Facebook XXHash** (2012)
 - From the creator of zstd compression.
- **Google FarmHash** (2014)
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← State-of-the-art

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smhasher

SMhasher

Linux Build status [build](#) [passing](#) [build failing](#)

Hash function	MiB/sec	cycl./hash	cycl./map	size	Quality problems
donothing32	11149460.06	4.00	-	13	bad seed 0, test NOP
donothing64	11787676.42	4.00	-	13	bad seed 0, test NOP
donothing128	11745060.76	4.06	-	13	bad seed 0, test NOP
NOP_OAAT_read64	11372846.37	14.00	-	47	test NOP
BadHash	769.94	73.97	-	47	bad seed 0, test FAIL
sumhash	10699.57	29.53	-	363	bad seed 0, test FAIL
sumhash32	42877.79	23.12	-	863	UB, test FAIL
multiply_shift	8026.77	26.05	226.80 (8)	345	bad seeds & 0xffffffff, fails most tests
pair_multiply_shift	3716.95	40.22	186.34 (3)	609	fails most tests
crc32	383.12	134.21	257.50 (11)	422	insecure, 8590x collisions, distrib, PerlinNoise
md5_32	350.53	644.31	894.12 (10)	4419	

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NOP_OAAT_read64				47	test NOP
BadHash					
sumhash					
sumhash32					
multiply_shift					
pair_multiply_shift					
crc32					
md5_32					

Summary

I added some SSE assisted hashes and fast intel/arm CRC32-C, AES and SHA HW variants. See also the old <https://github.com/aappleby/smhasher/wiki>, the improved, but unmaintained fork <https://github.com/demerphq/smhasher>, and the new improved version SMHasher3 <https://gitlab.com/twojciek/smhasher3>.

So the fastest hash functions on x86_64 without quality problems are:

- xxh3_low
- wyhash
- ahash64
- t1ha2_atonce
- komihash
- FarmHash (*not portable, too machine specific: 64 vs 32bit, old gcc, ...*)
- halftime_hash128
- Spooky32
- pengyhash
- nmhash32
- mx3
- MUM/mir (*different results on 32/64-bit archs, lots of bad seeds to filter out*)
- fasthash32

Static Hashing Schemes

- **Approach #1: Linear Probe Hashing**
- **Approach #2: Cuckoo Hashing**
- There are several other schemes (not covered in this course):
 - Robin Hood Hashing
 - Hopscotch Hashing
 - Swiss Tables

Static Hashing Schemes

- **Approach #1: Linear Probe Hashing**
- **Approach #2: Cuckoo Hashing**

Both are members of a broader class called “[Open Addressing](#)”.



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Linear Probe Hashing

Static Hashing Schemes

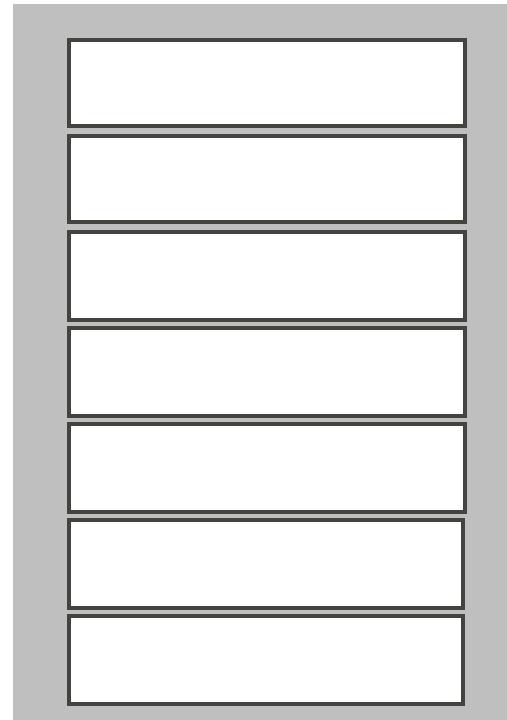
Linear Probe Hashing

- Single giant table of slots.
- Resolve collisions by linearly searching for the next free slot in the table.
 - To determine whether an element is present, hash to a location in the index and scan for it.
 - Must store the key in the index to know when to stop scanning.
 - Insertions and deletions are generalizations of lookups.
- **Example:** Google's [absl::flat_hash_map](#)

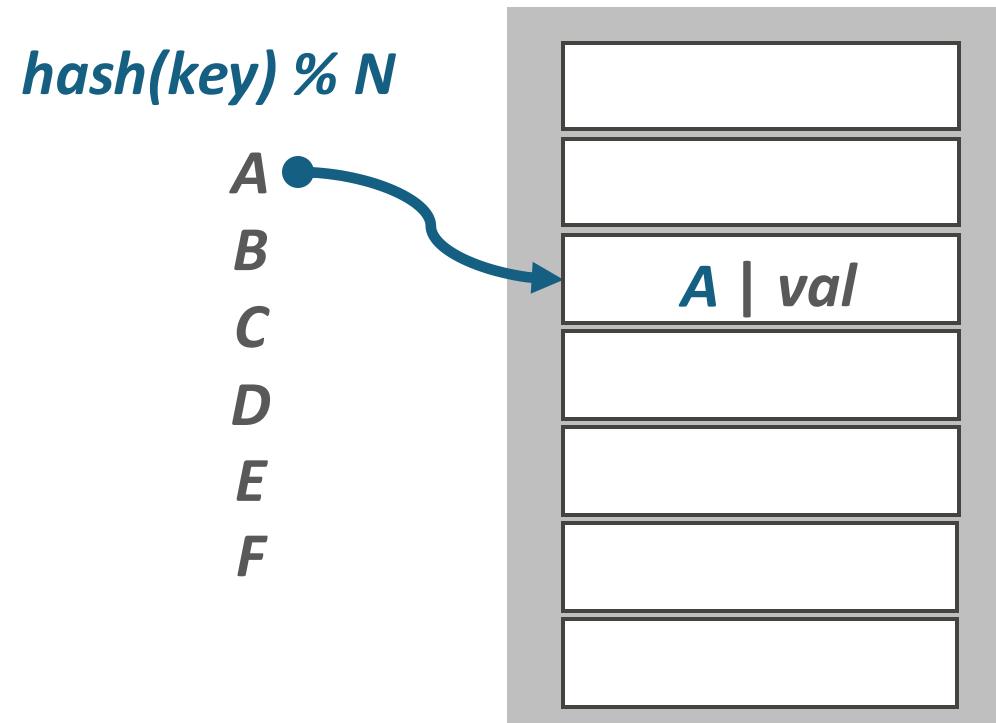
Linear Probe Hashing

$hash(key) \% N$

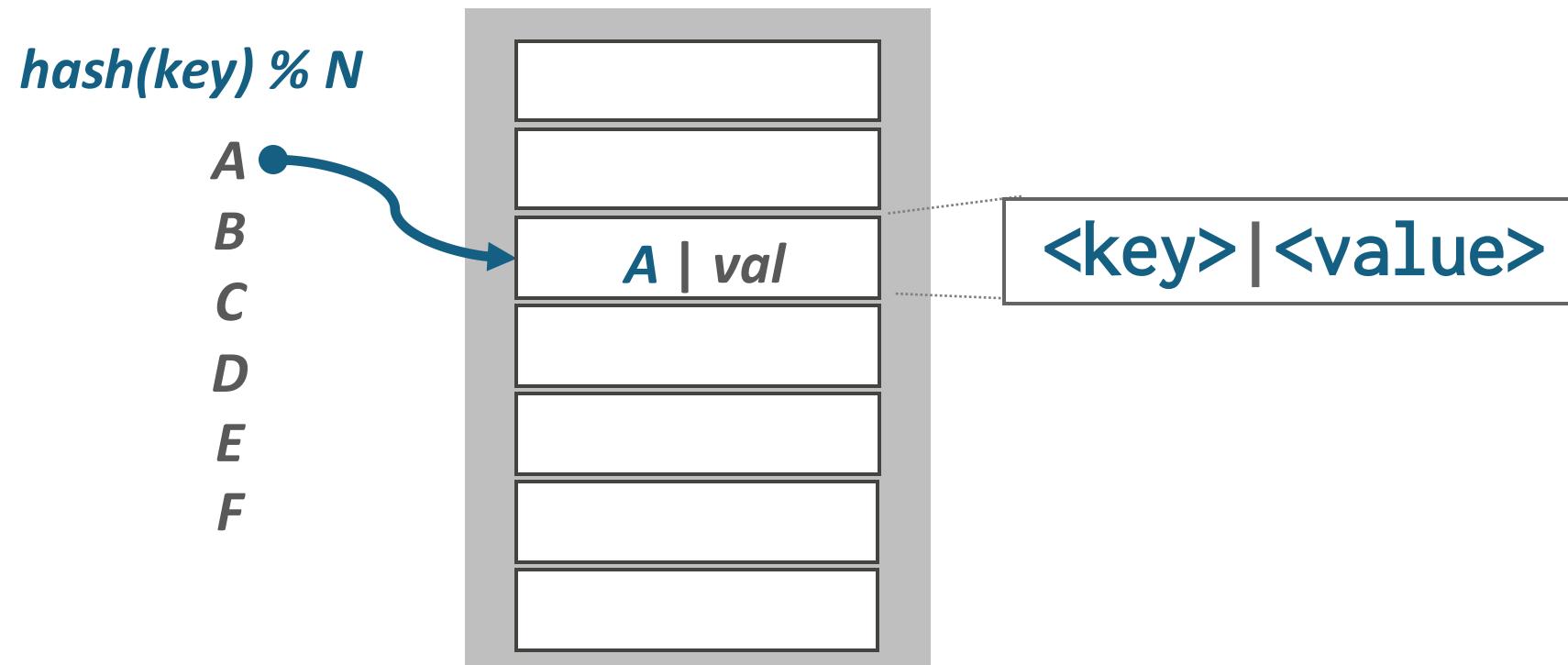
A
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C
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E
F



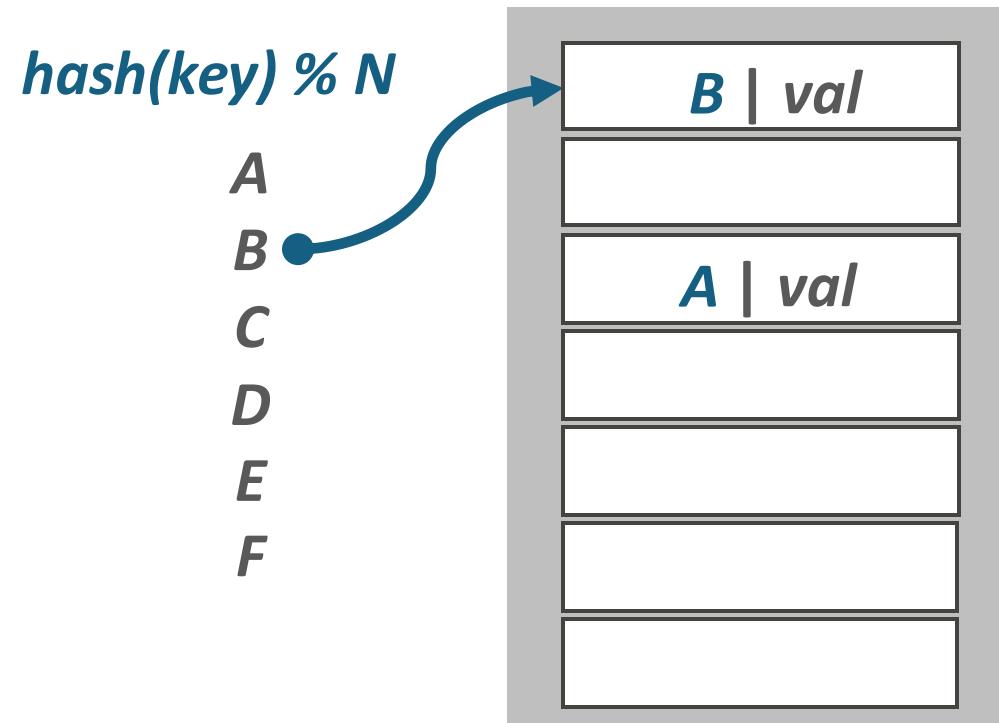
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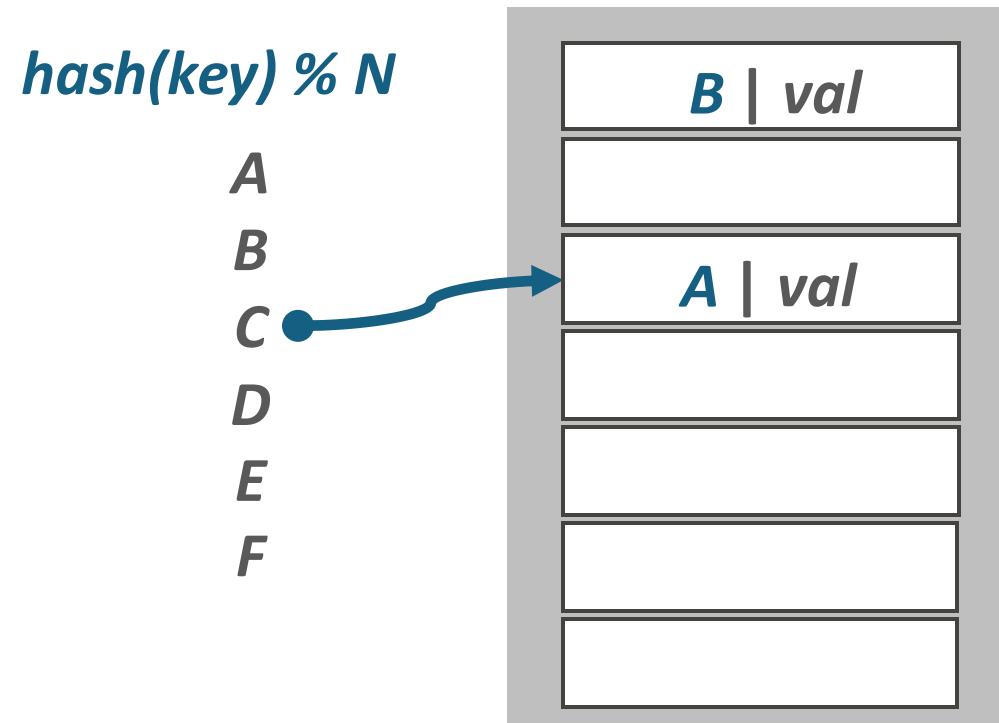
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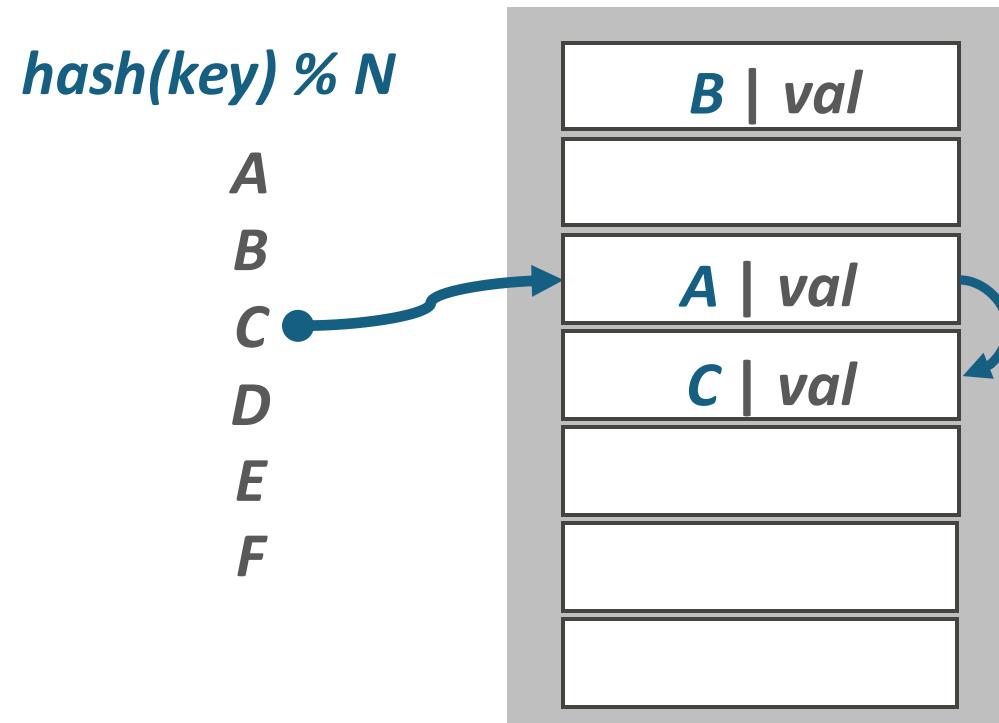
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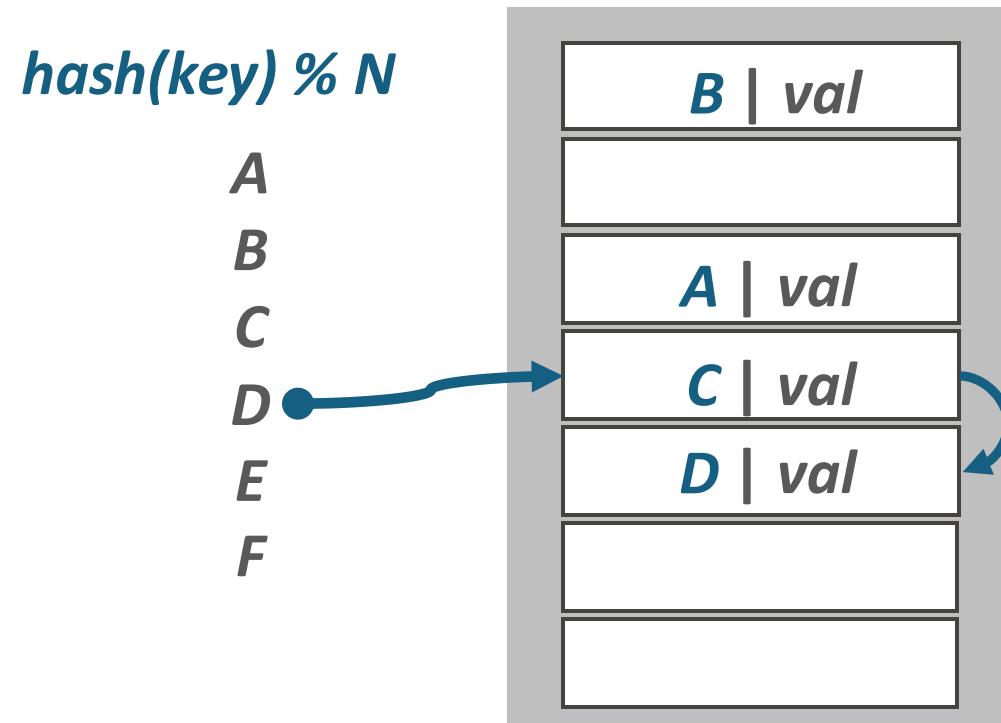
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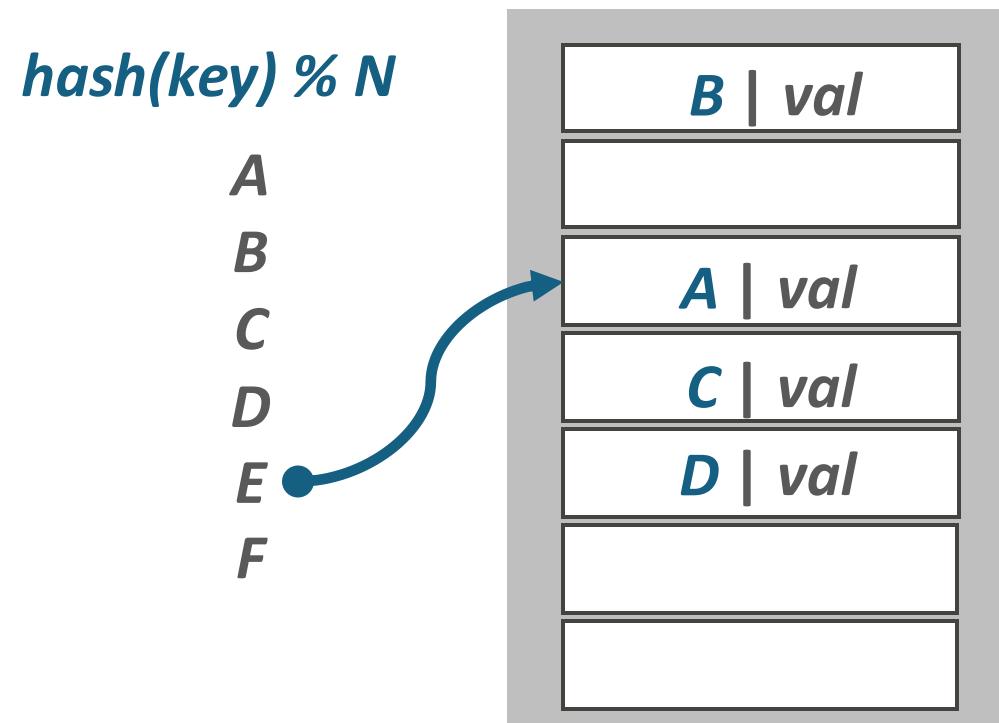
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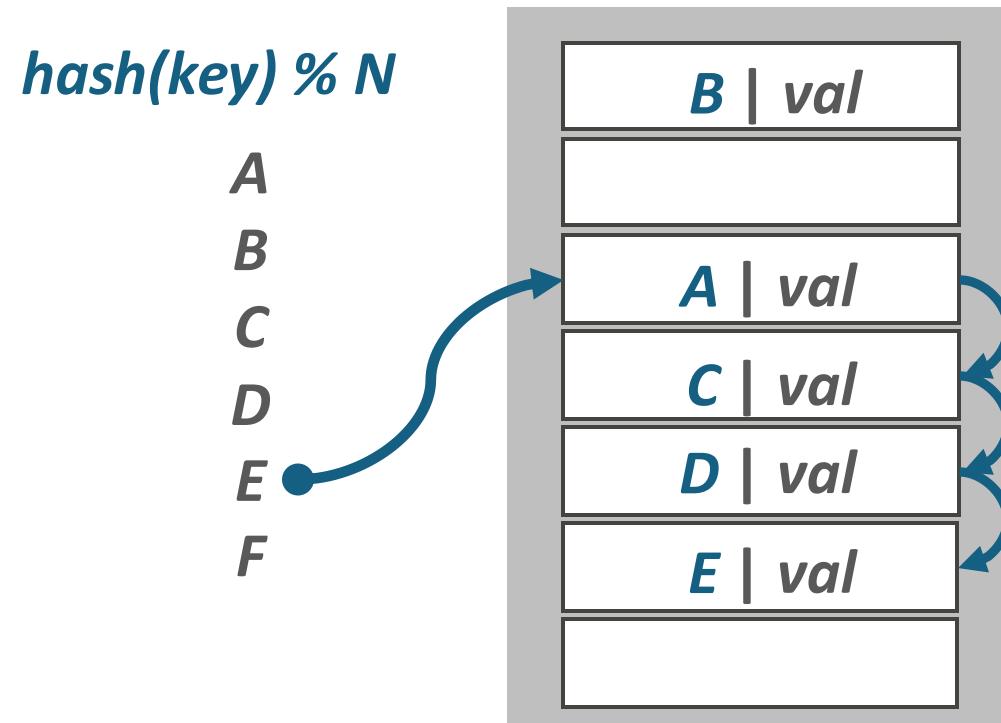
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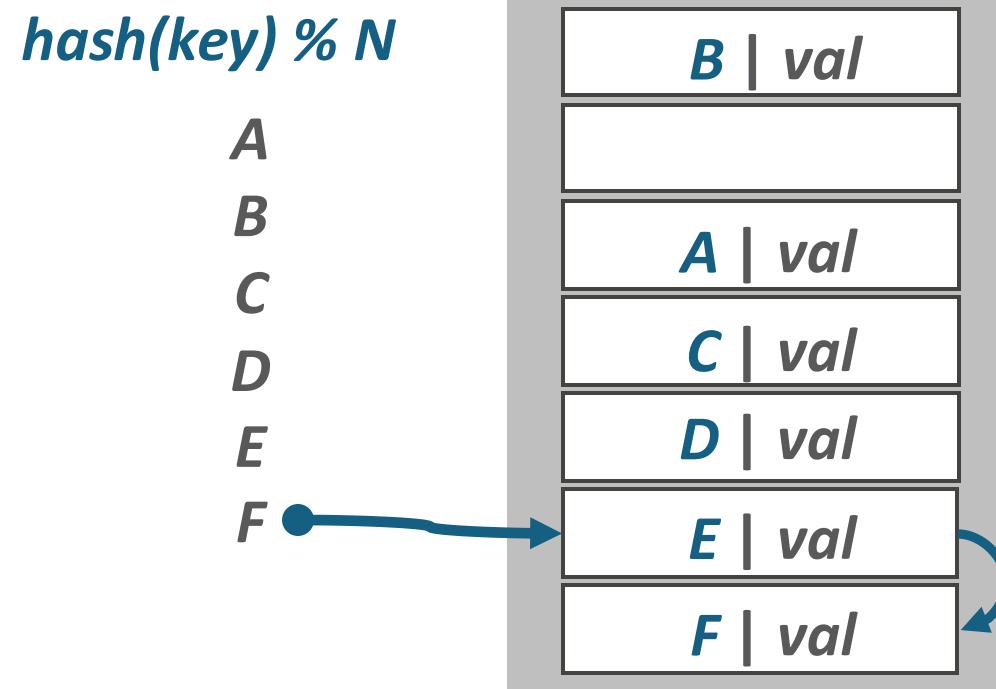
Linear Probe Hashing



Linear Probe Hashing



Linear Probe Hashing



Linear Probe Hashing - Deletes

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A

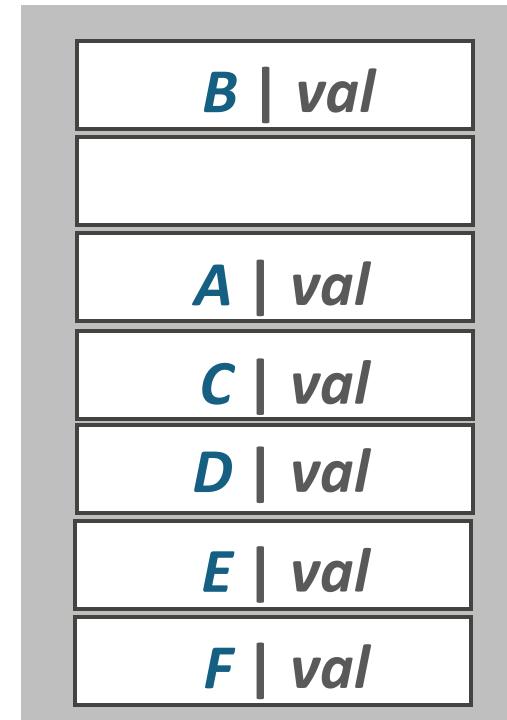
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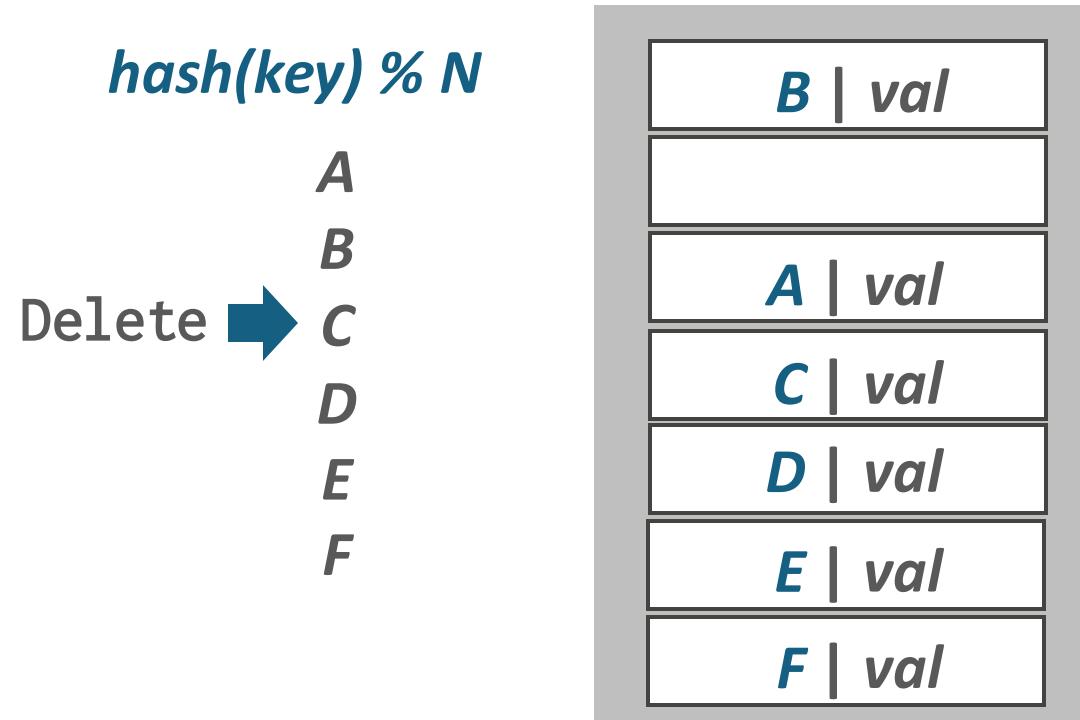
D

E

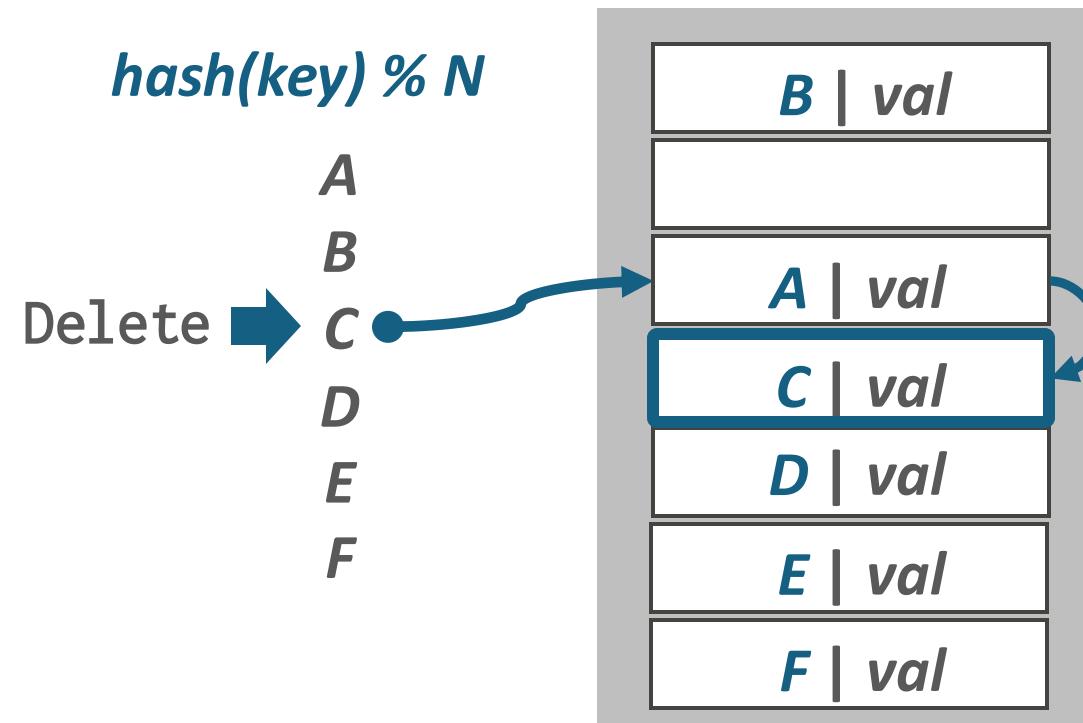
F



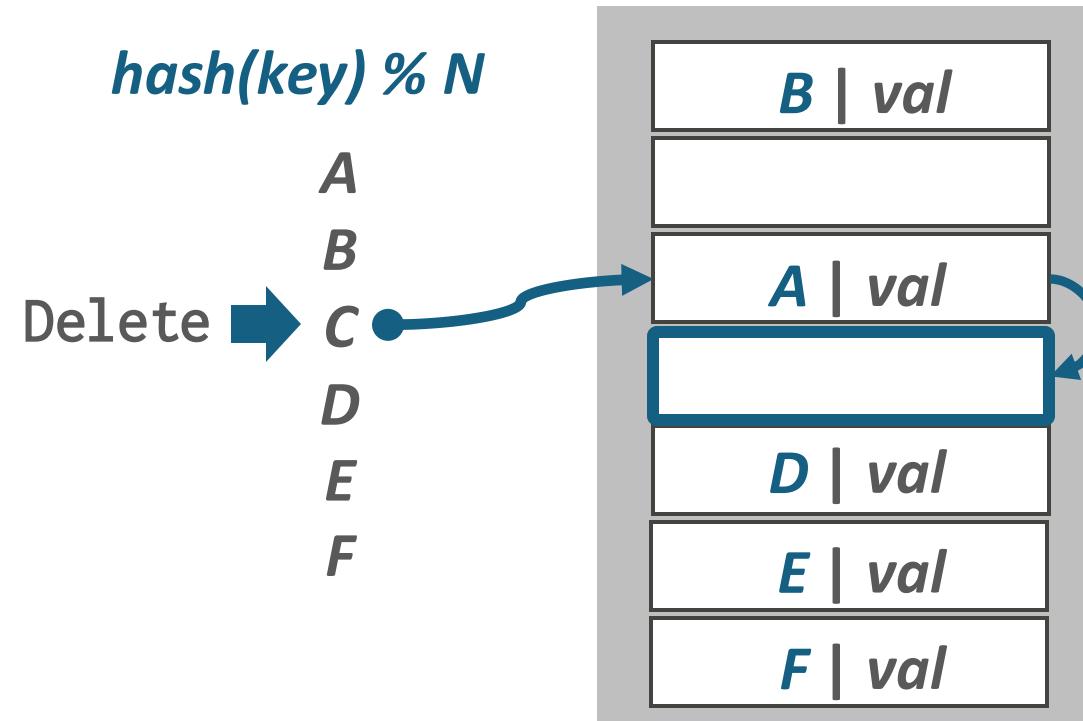
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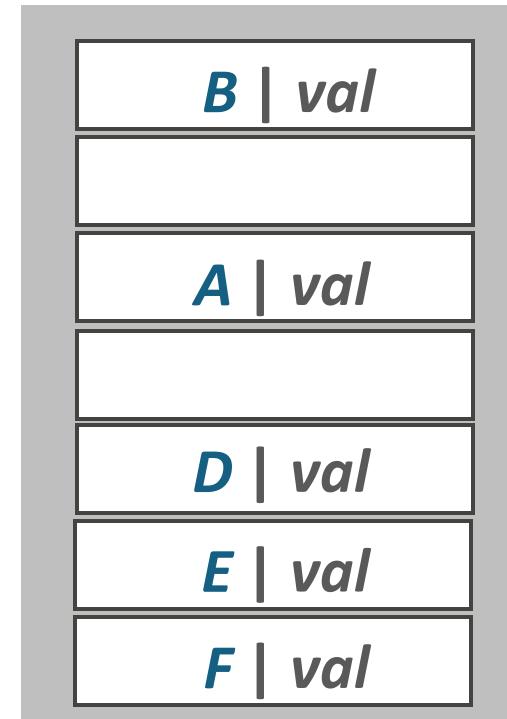
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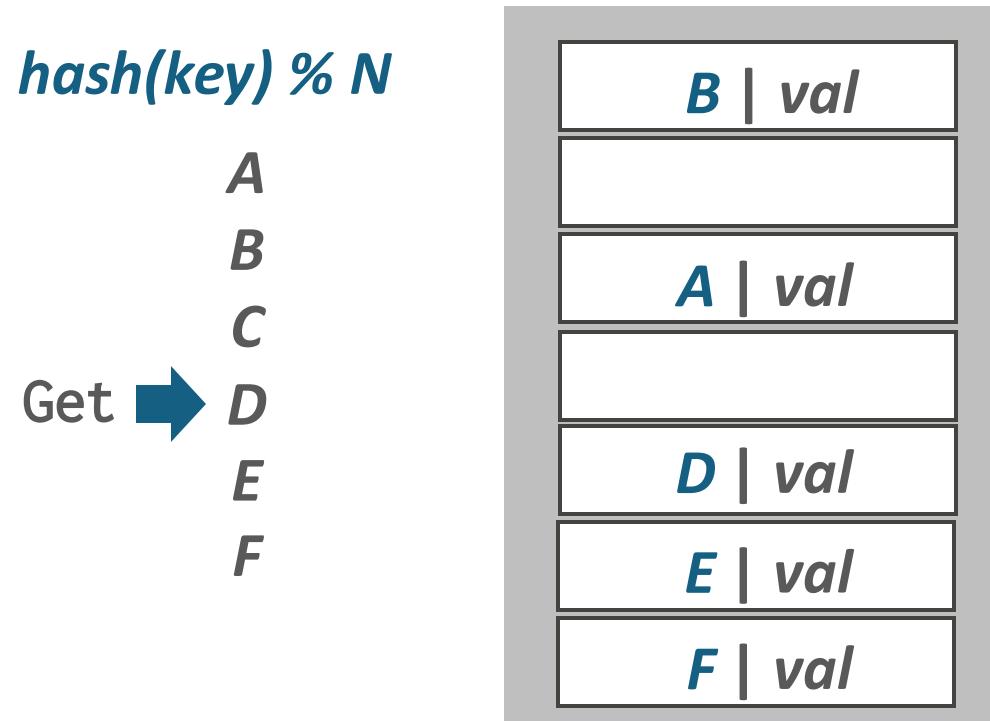
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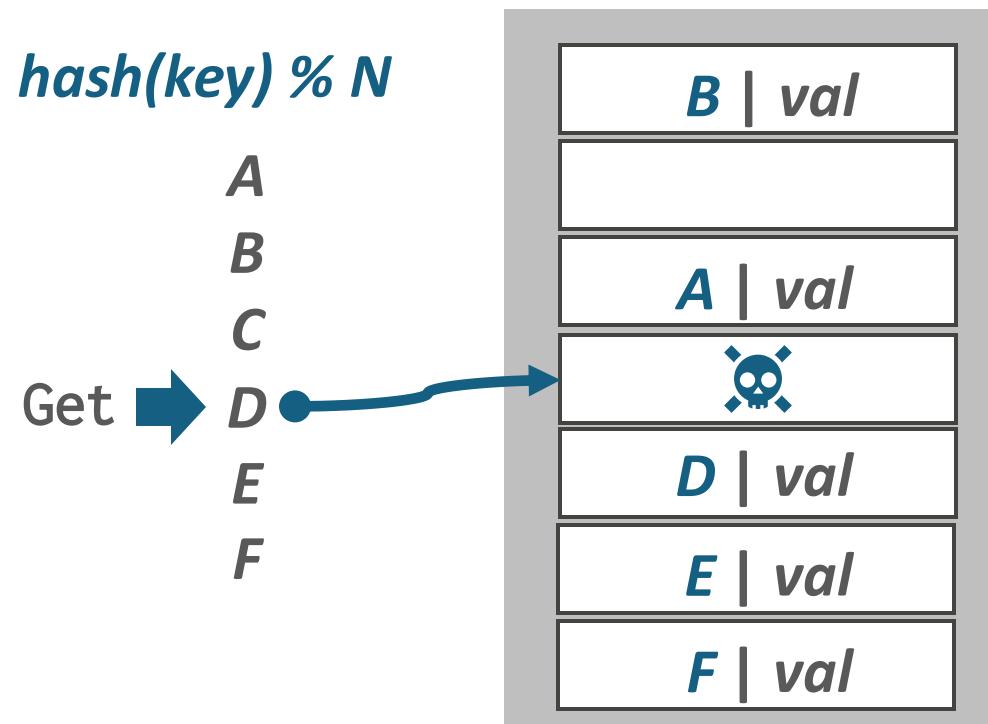
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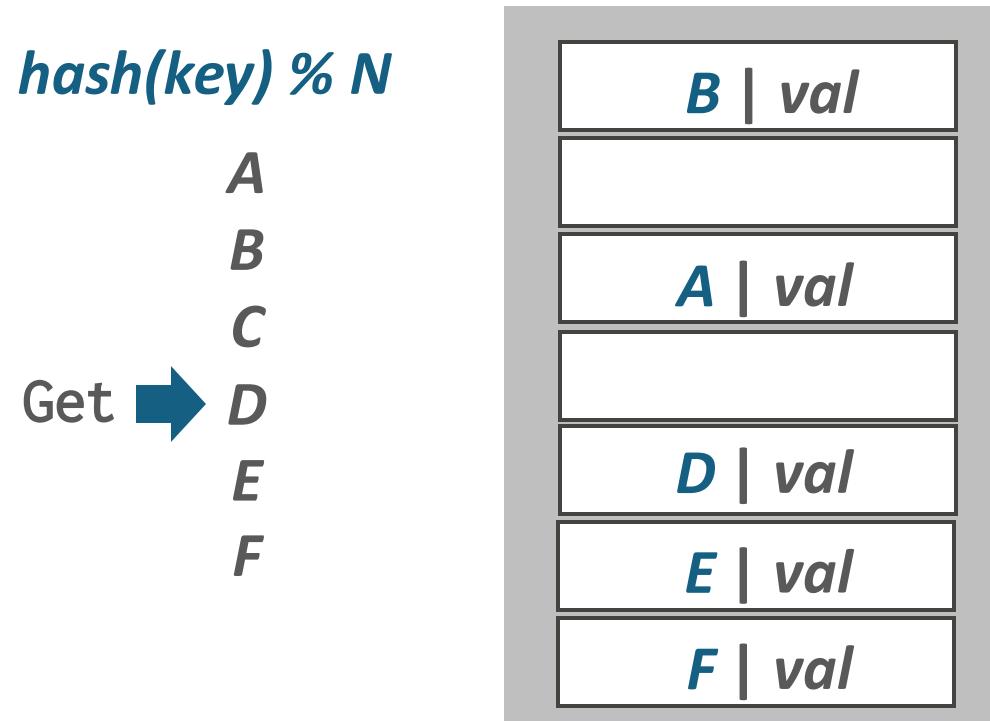
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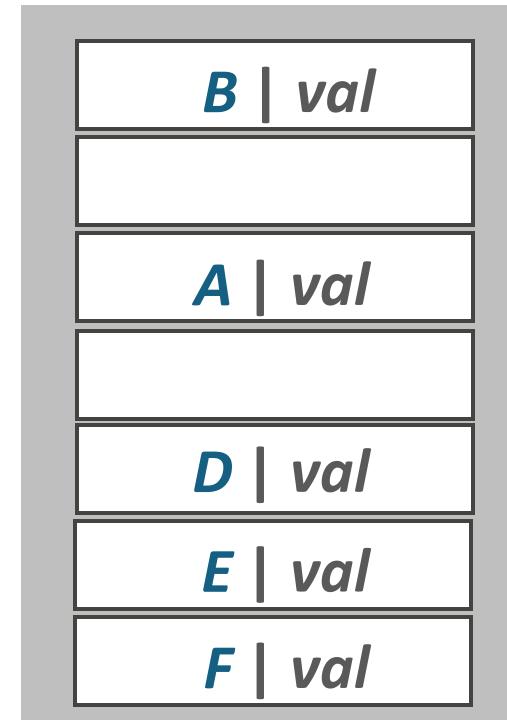
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Linear Probe Hashing - Deletes

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A
B
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Get \rightarrow D
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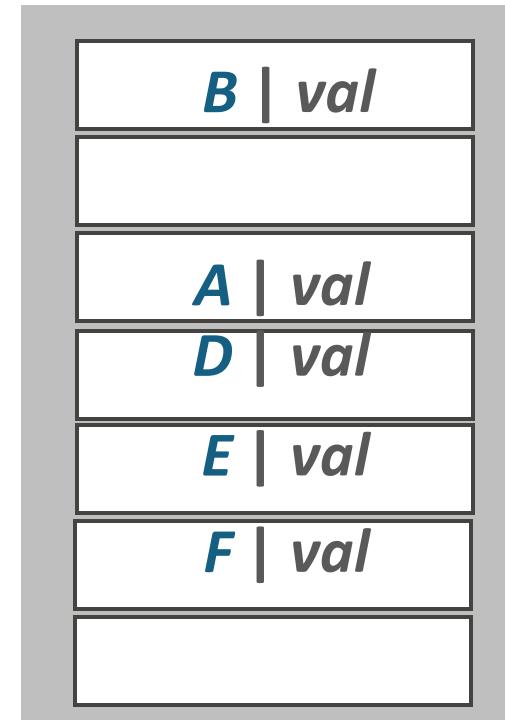
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- Rehash keys until you find the first empty slot.

Linear Probe Hashing - Deletes

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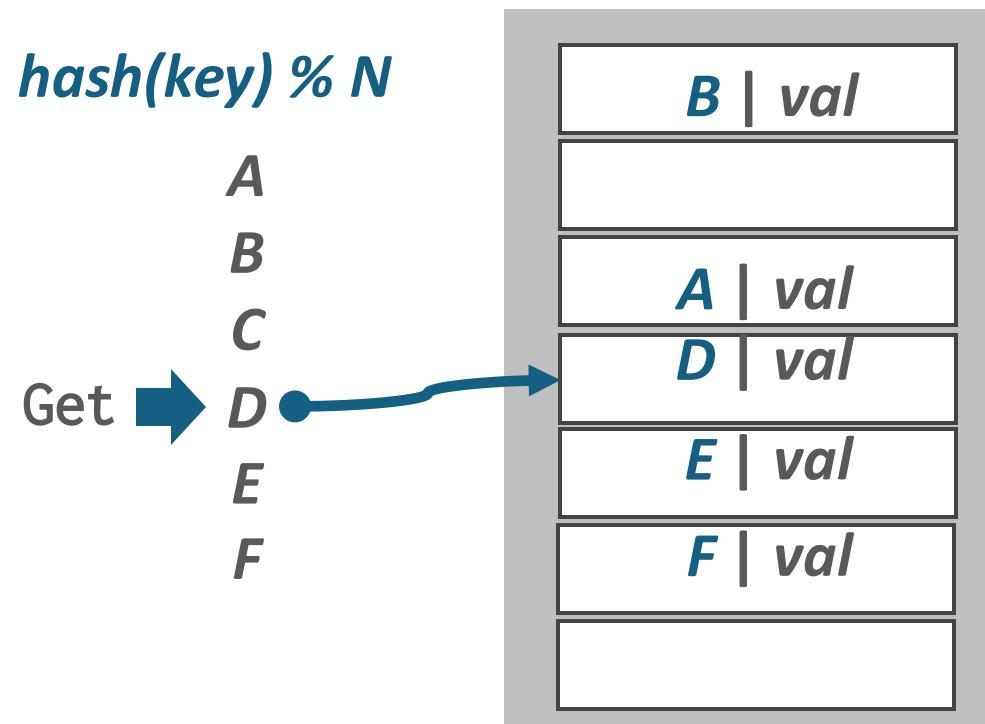
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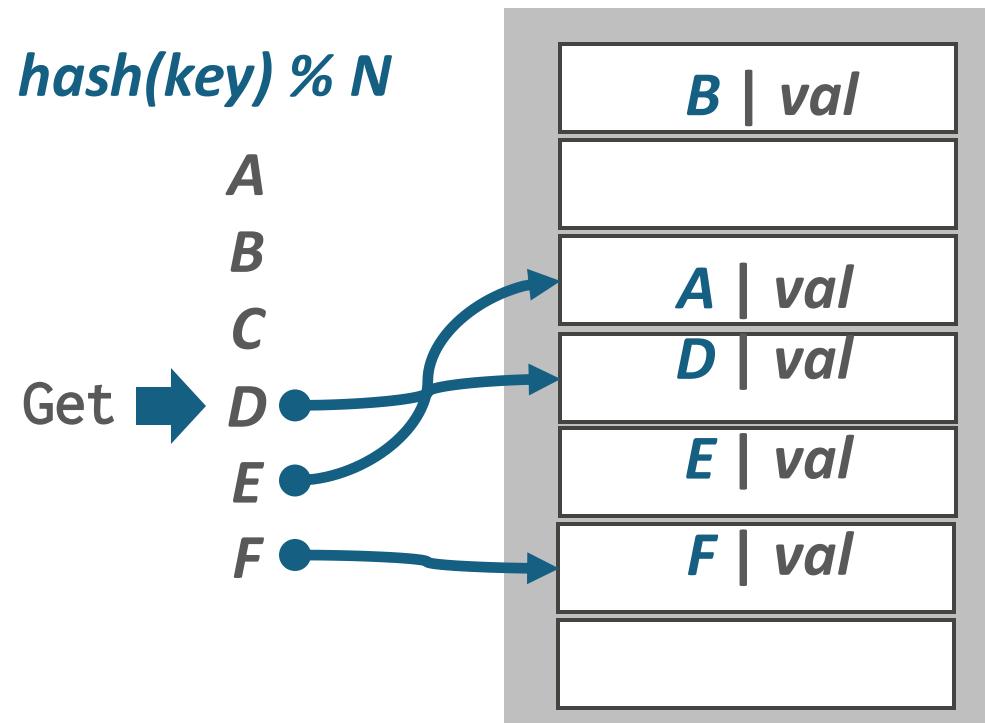
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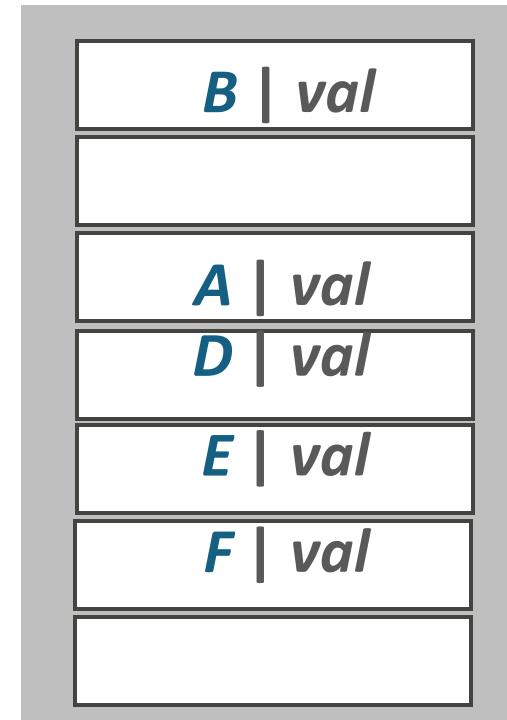
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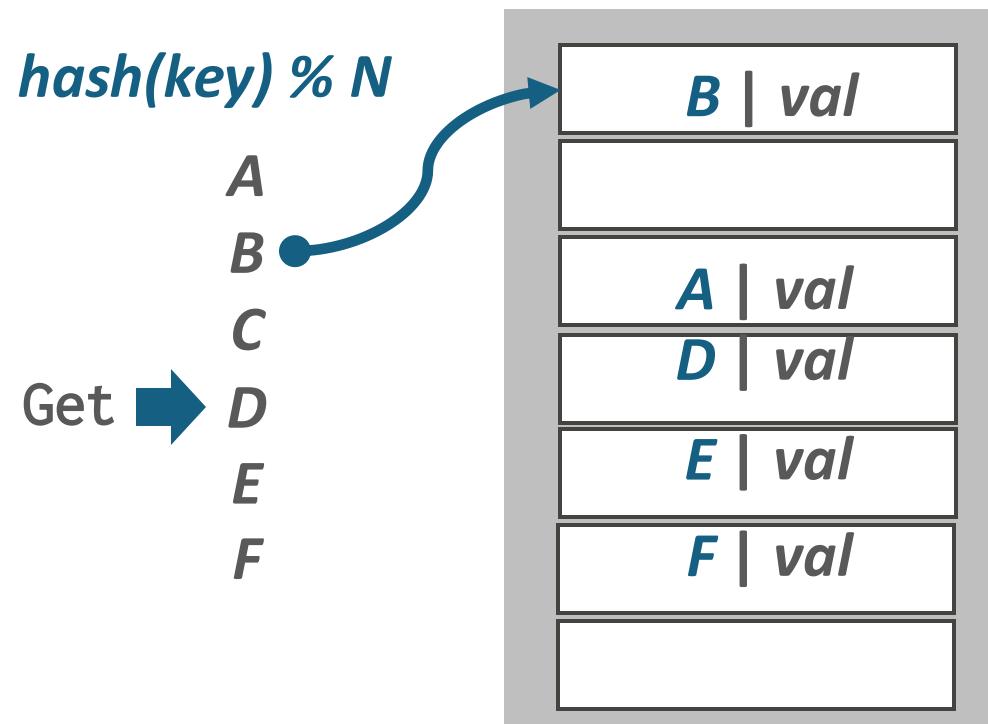
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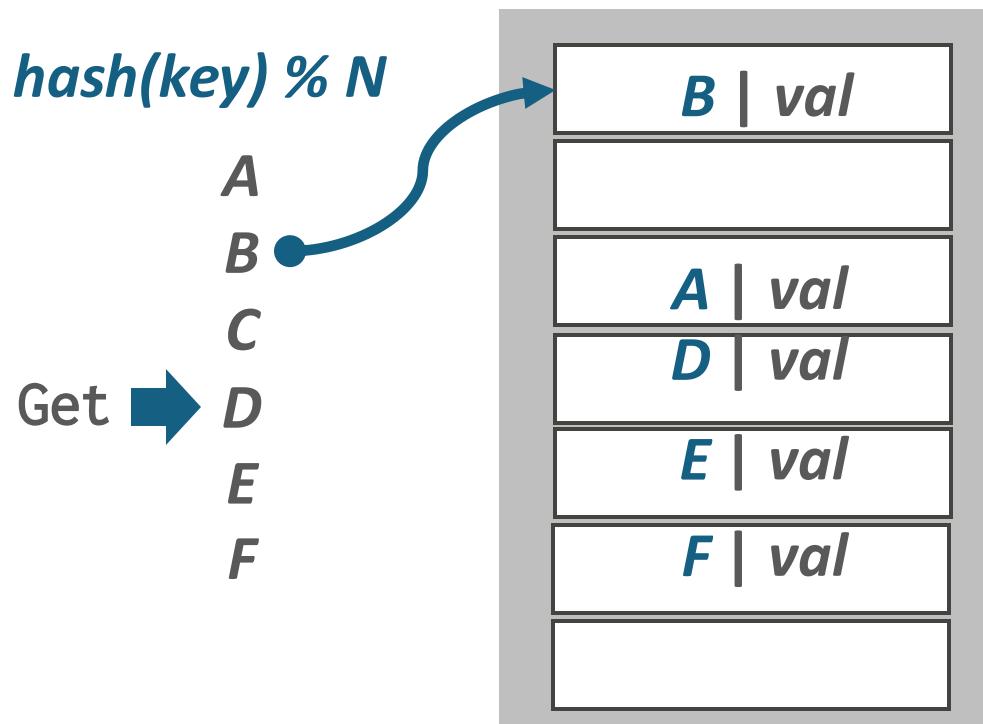
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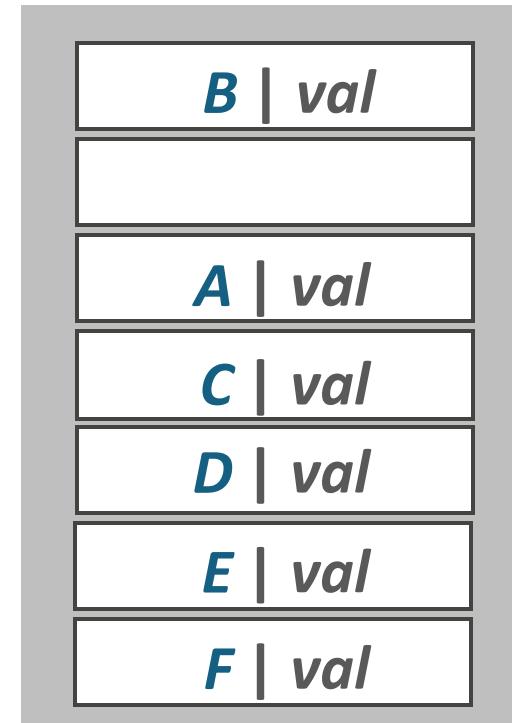
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- Expensive! May need to reorganize the entire table.
- No DBMS does this.

Linear Probe Hashing - Deletes

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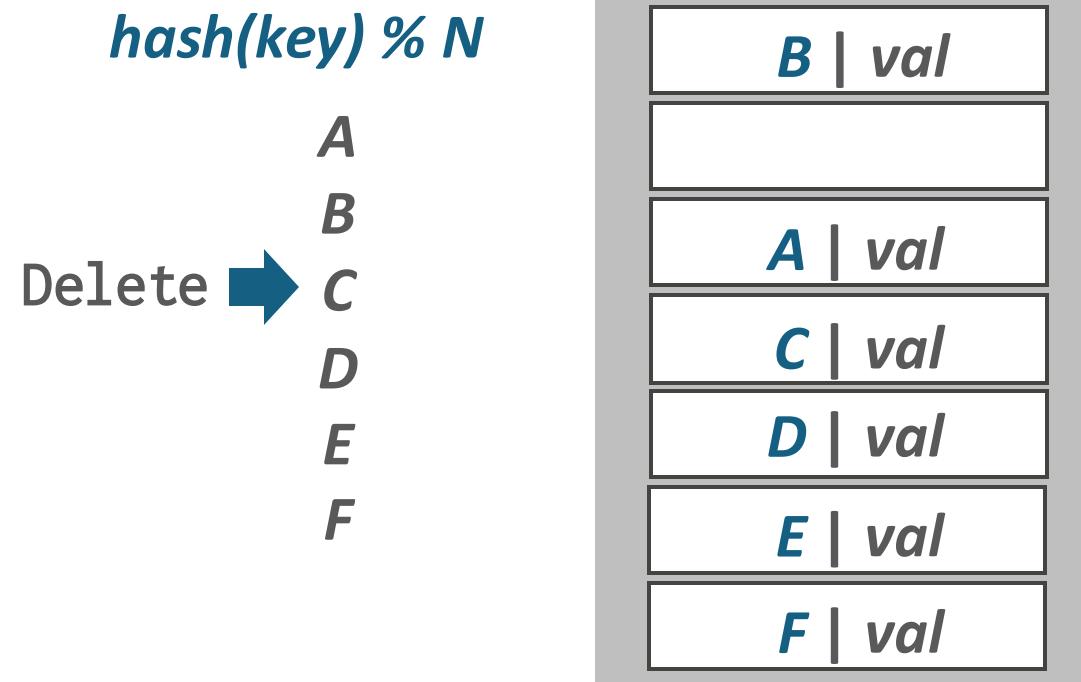
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- Set a marker to indicate that the entry in the slot is logically deleted.
- Reuse the slot for new keys.
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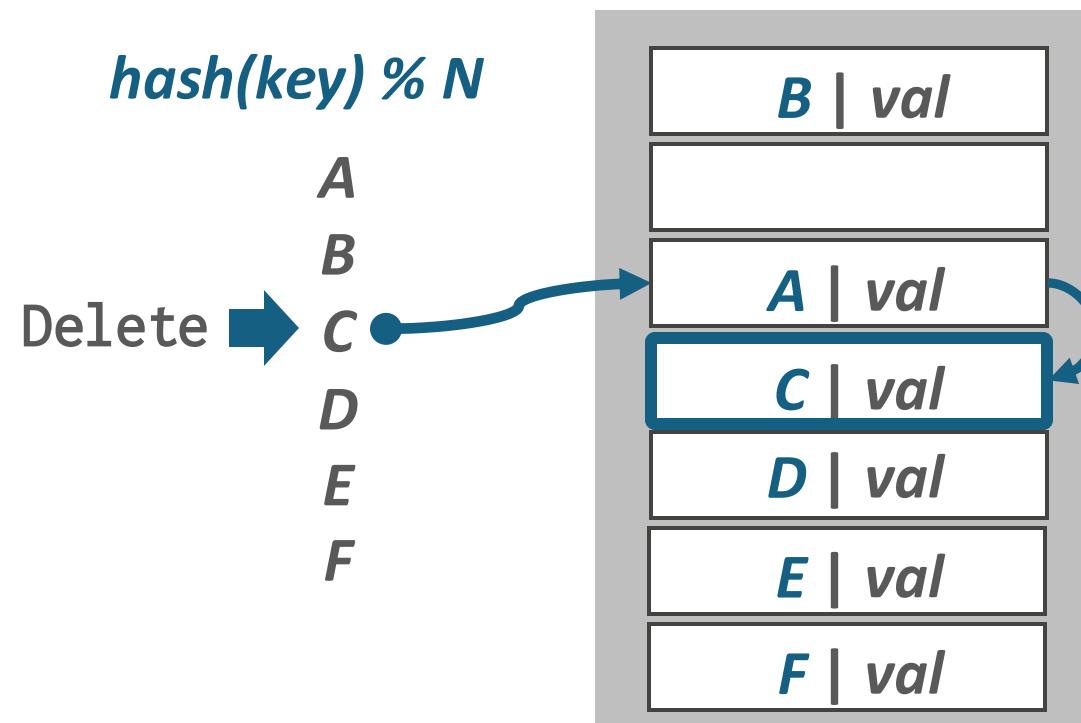
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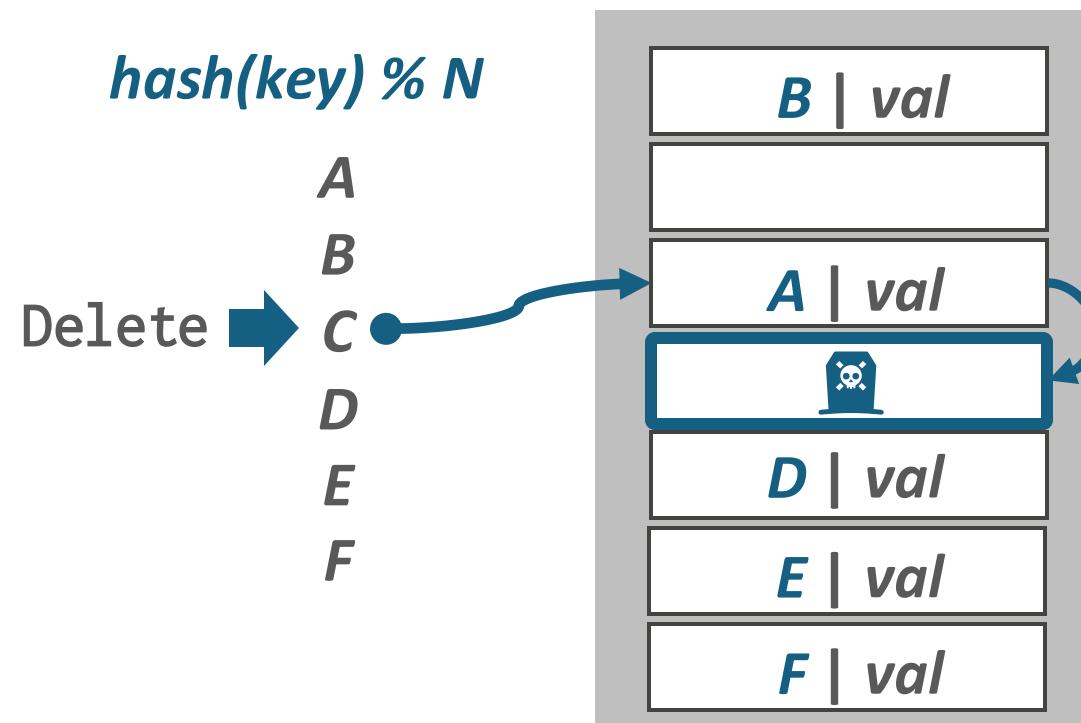
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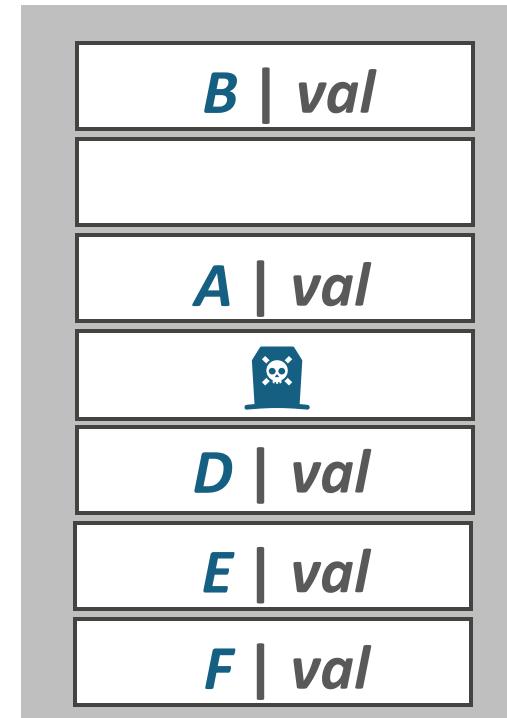
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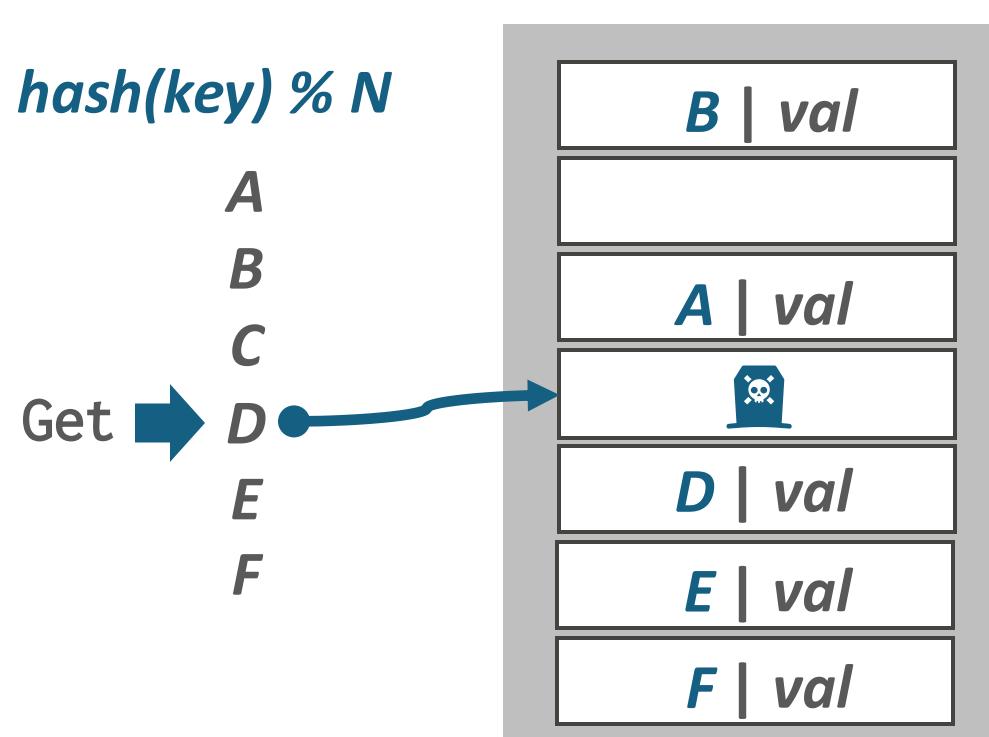
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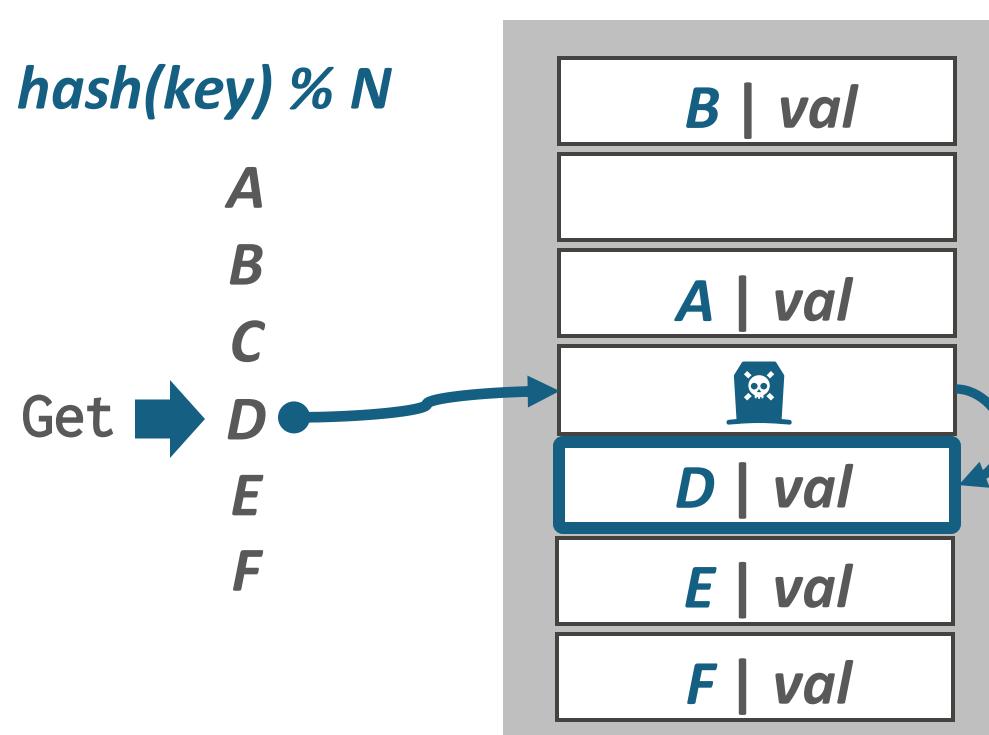
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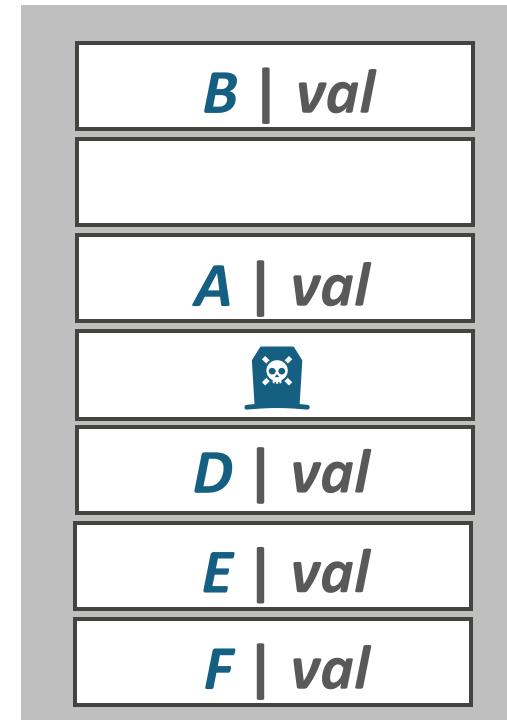
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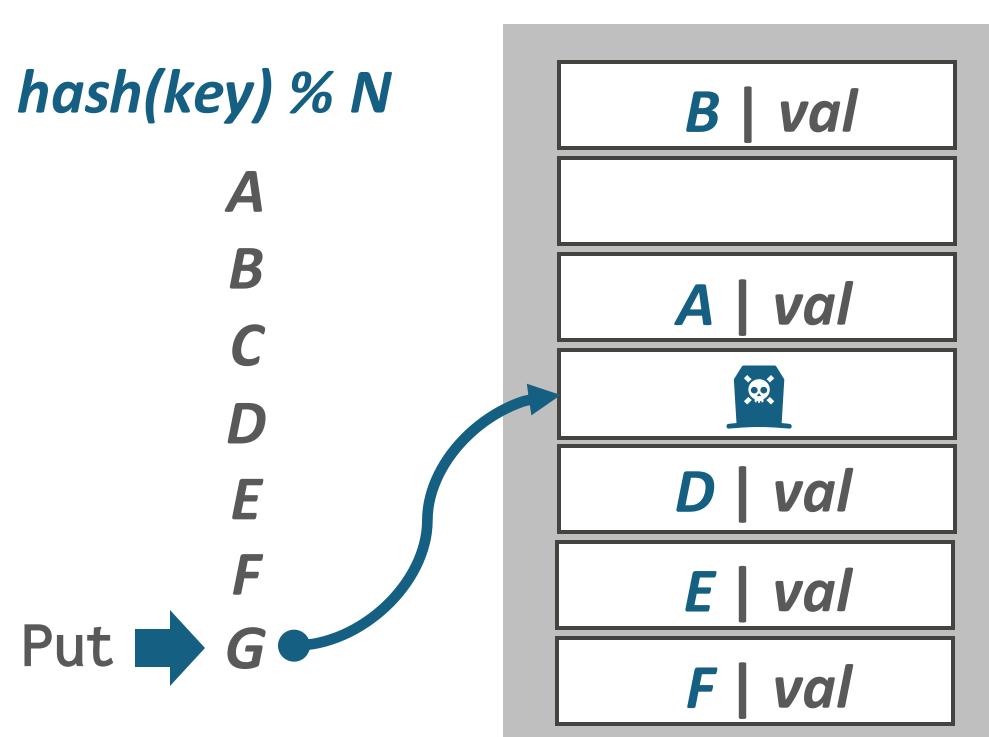
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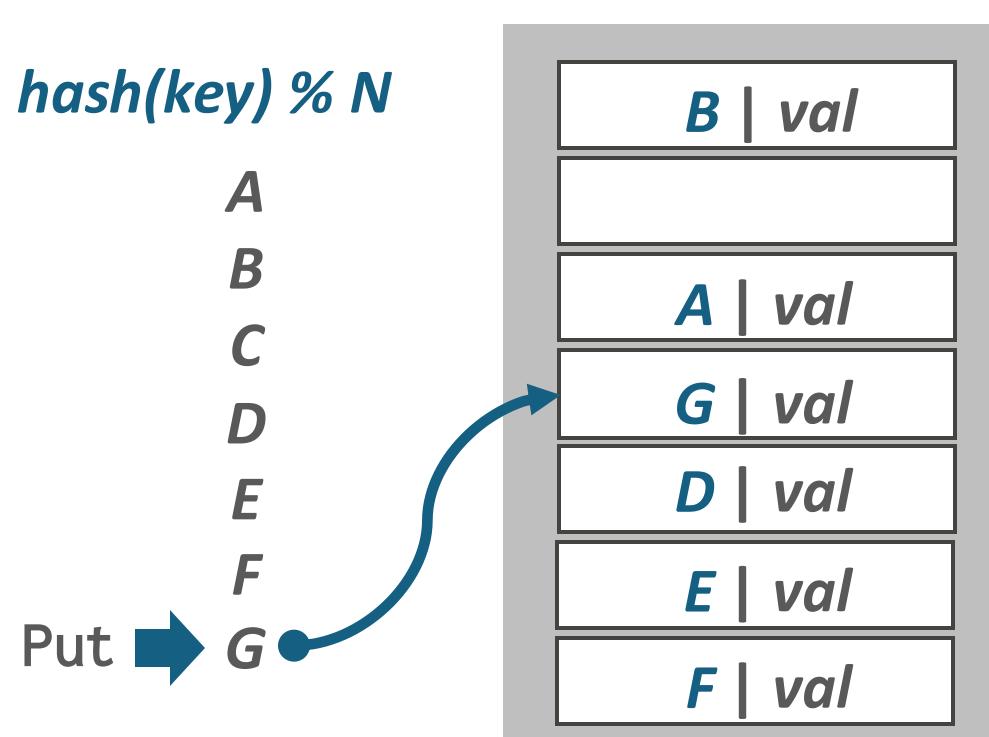
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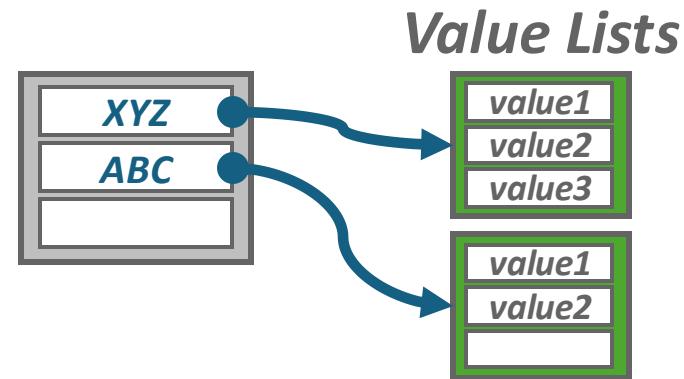
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Non-Unique Keys

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 - Store values in separate storage area for each key.
 - Value lists can overflow to multiple pages if the number of duplicates is large.

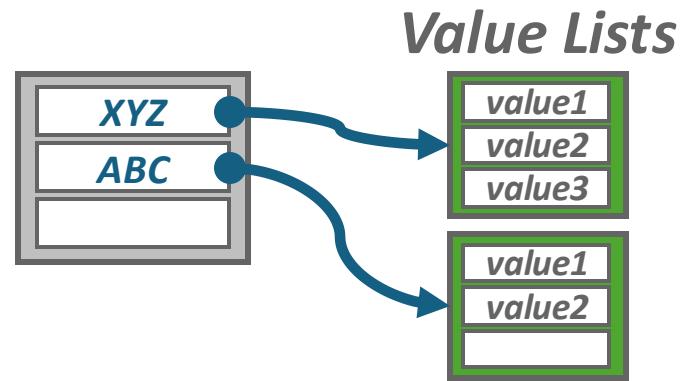
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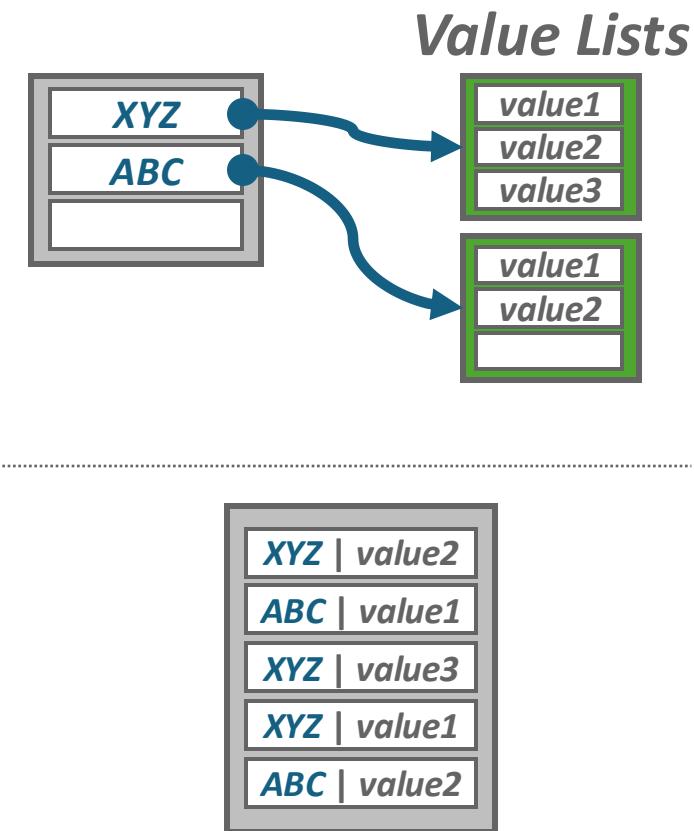
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Optimizations

- Specialized hash table implementations based on key type(s) and sizes.
 - Example: Maintain multiple hash tables for different string sizes for a set of keys.
- Store metadata separate in a separate array.
 - Packed bitmap tracks whether a slot is empty/tombstone.
- Use table + slot versioning metadata to quickly invalidate all entries in the hash table.
 - Example: If table version does not match slot version, then treat the slot as empty.

Source: [Maksim Kita](#)

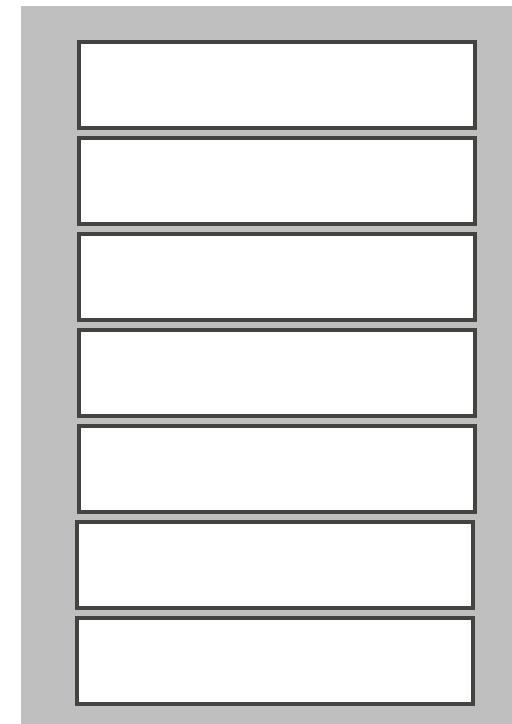
Cuckoo Hashing

Static Hashing Schemes

Cuckoo Hashing

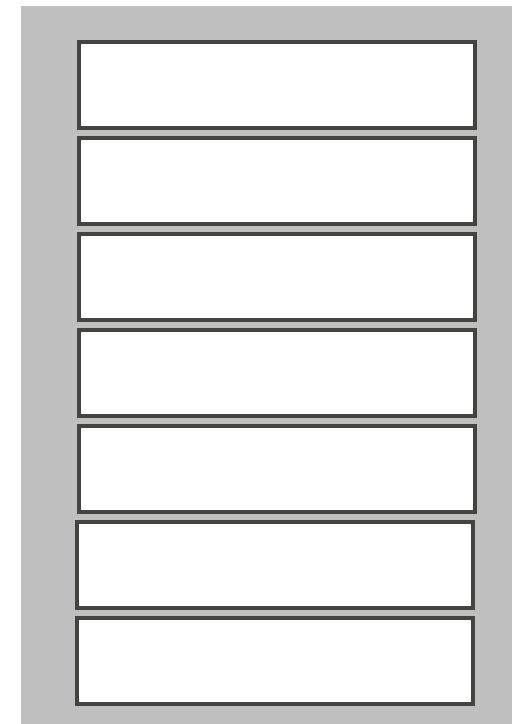
- Use multiple hash functions to find multiple locations in the hash table to insert records.
 - On insert, check multiple locations and pick the one that is empty.
 - If no location is available, evict the element from one of them and then rehash it find a new location.
- Look-ups and deletions are always $O(1)$ because only one location per hash table is checked.
- Best open-source implementation is from CMU.

Cuckoo Hashing

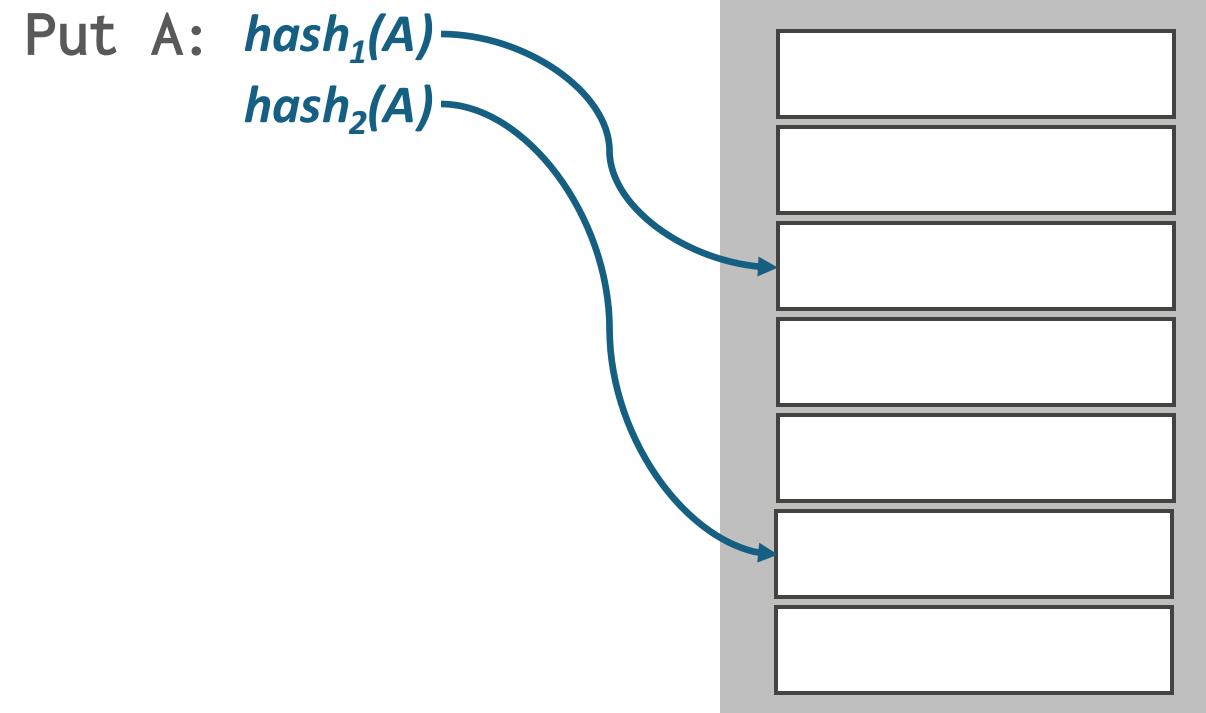


Cuckoo Hashing

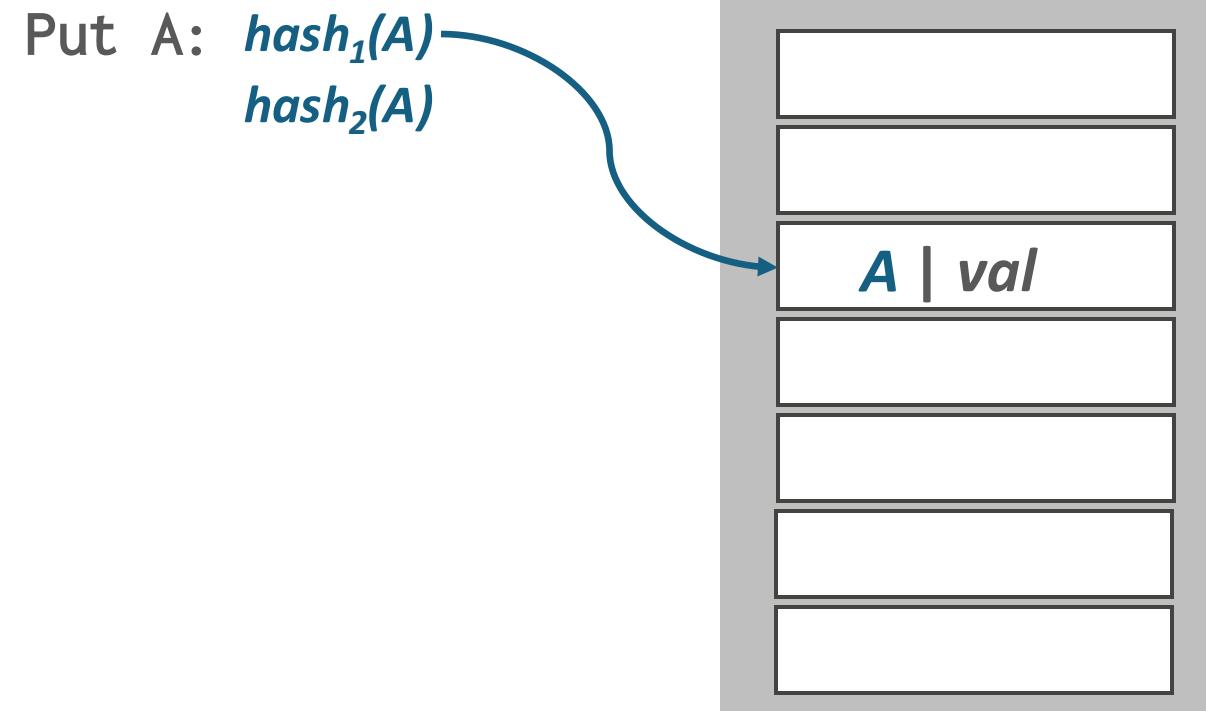
Put A: $hash_1(A)$
 $hash_2(A)$



Cuckoo Hashing



Cuckoo Hashing



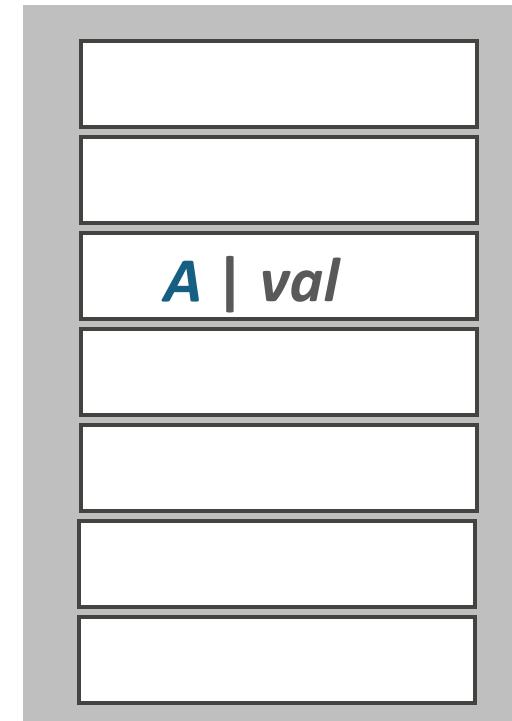
Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$



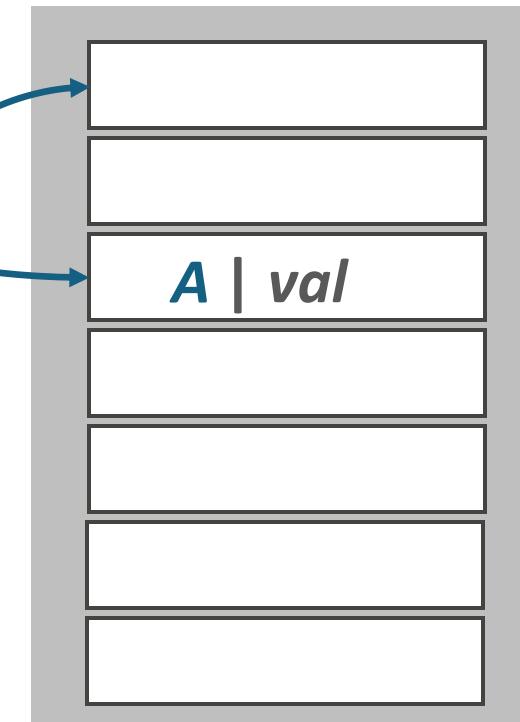
Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$



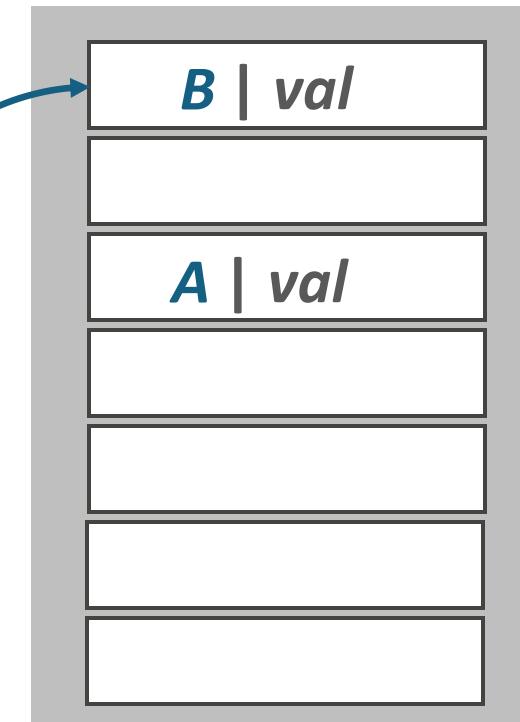
Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$



Cuckoo Hashing

Put A: $hash_1(A)$

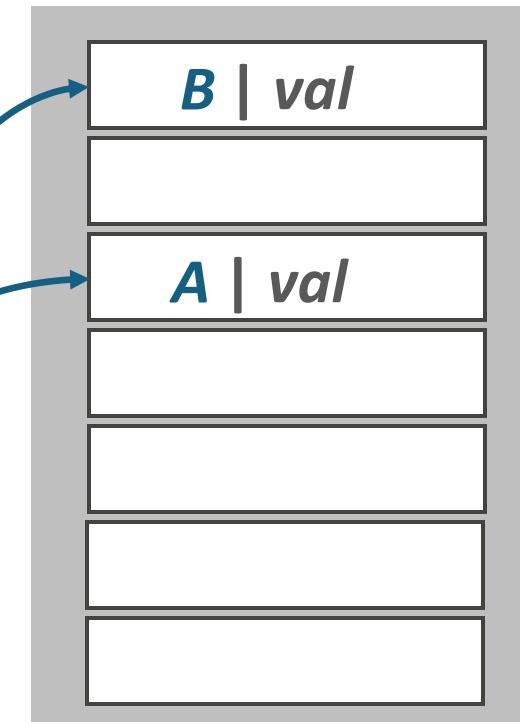
$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$

Put C: $hash_1(C)$

$hash_2(C)$



Cuckoo Hashing

Put A: $hash_1(A)$

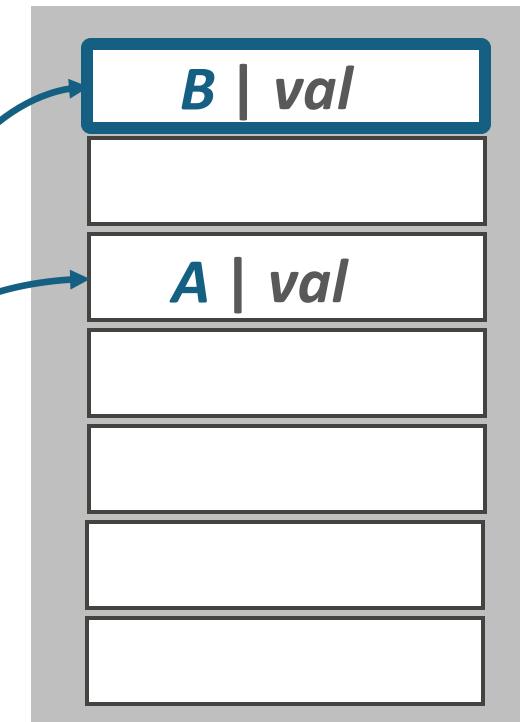
$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$

Put C: $hash_1(C)$

$hash_2(C)$



Cuckoo Hashing

Put A: $hash_1(A)$

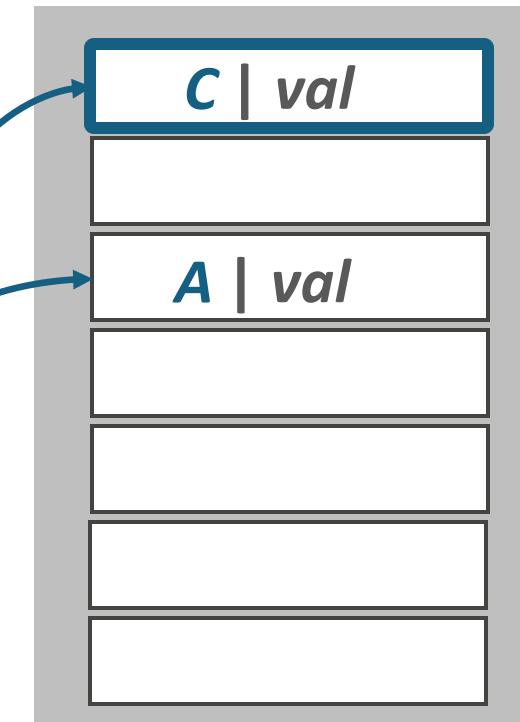
$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$

Put C: $hash_1(C)$

$hash_2(C)$



Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

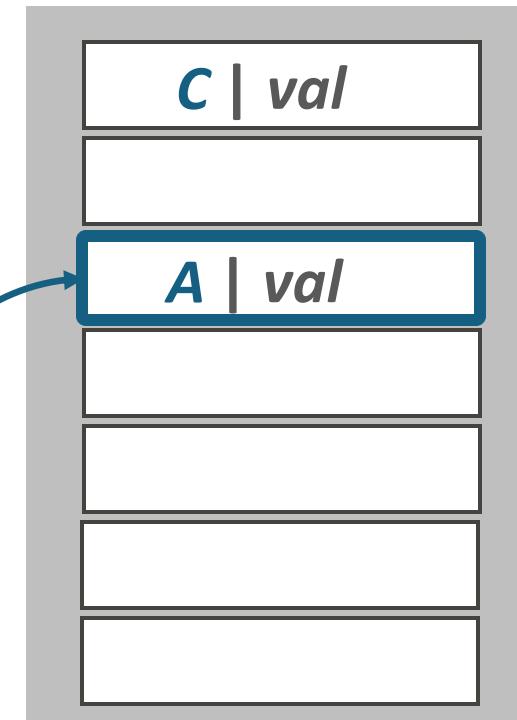
Put B: $hash_1(B)$

$hash_2(B)$

Put C: $hash_1(C)$

$hash_2(C)$

$hash_1(B)$



Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

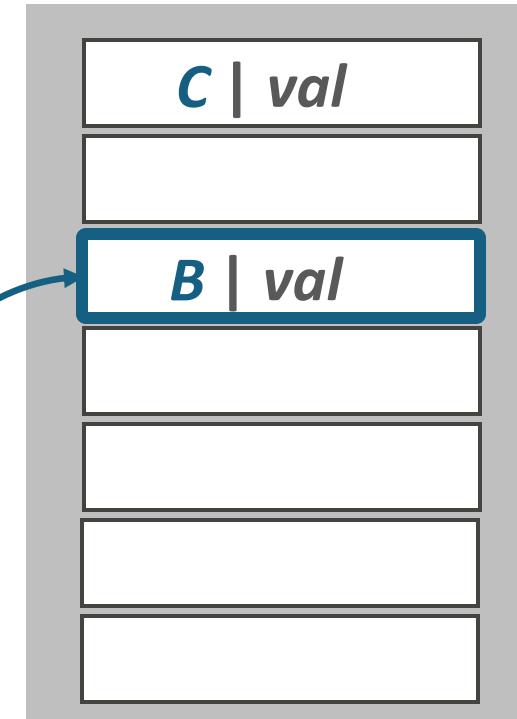
Put B: $hash_1(B)$

$hash_2(B)$

Put C: $hash_1(C)$

$hash_2(C)$

$hash_1(B)$



Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

Put B: $hash_1(B)$

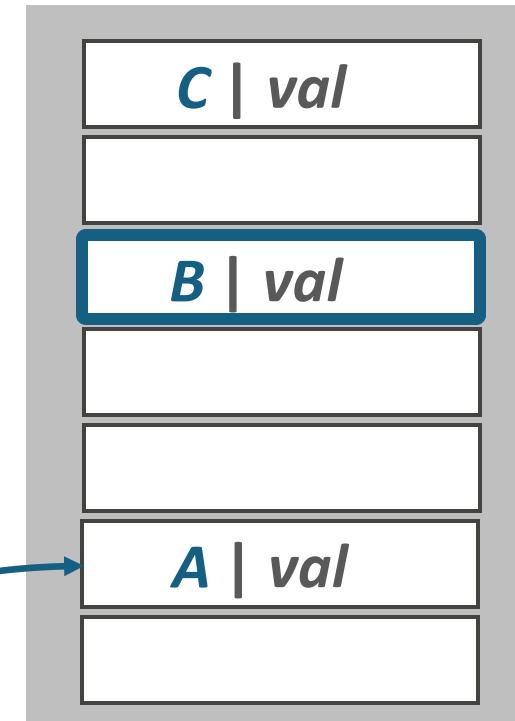
$hash_2(B)$

Put C: $hash_1(C)$

$hash_2(C)$

$hash_1(B)$

$hash_2(A)$



Cuckoo Hashing

Put A: $hash_1(A)$

$hash_2(A)$

Put B: $hash_1(B)$

$hash_2(B)$

Put C: $hash_1(C)$

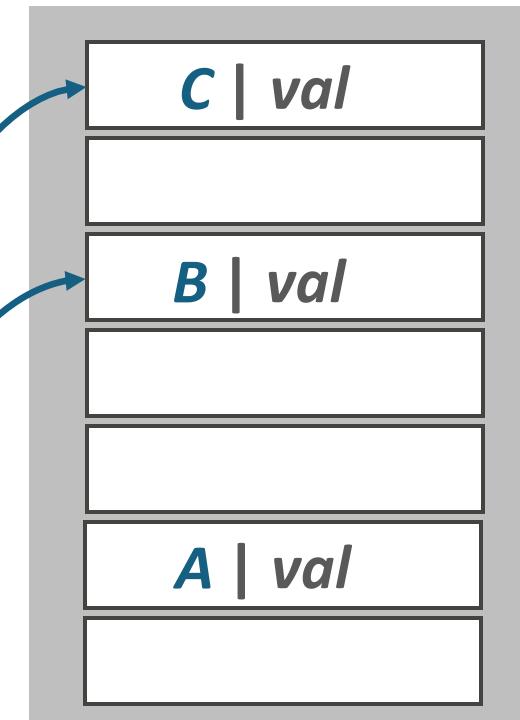
$hash_2(C)$

$hash_1(B)$

$hash_2(A)$

Get B: $hash_1(B)$

$hash_2(B)$



Chained Hash Table

Dynamic Hashing Schemes

Observation

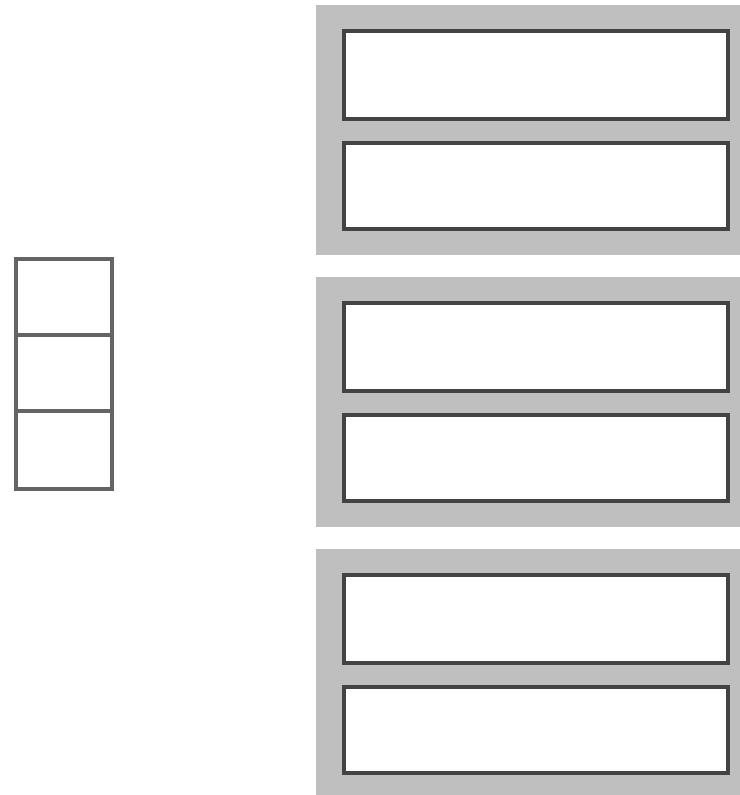
- The previous hash tables require the DBMS to know the number of elements it wants to store.
 - Otherwise, it must rebuild the table if it needs to grow/shrink in size.
- Dynamic hash tables incrementally resize themselves as needed.
 - Chained Hashing
 - Extendible Hashing
 - Linear Hashing

Chained Hashing

- Maintain a linked list of buckets for each slot in the hash table.
- Resolve collisions by placing all elements with the same hash key into the same bucket.
 - To determine whether an element is present, hash to its bucket and scan for it.
 - Insertions and deletions are generalizations of lookups.

Chained Hashing

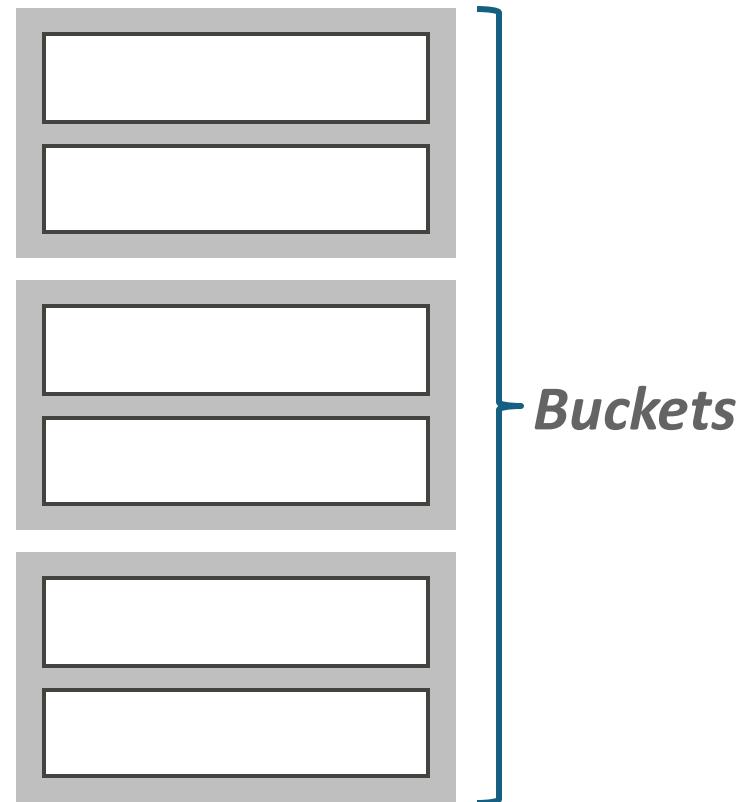
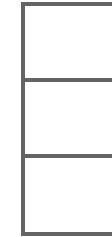
hash(key) % N



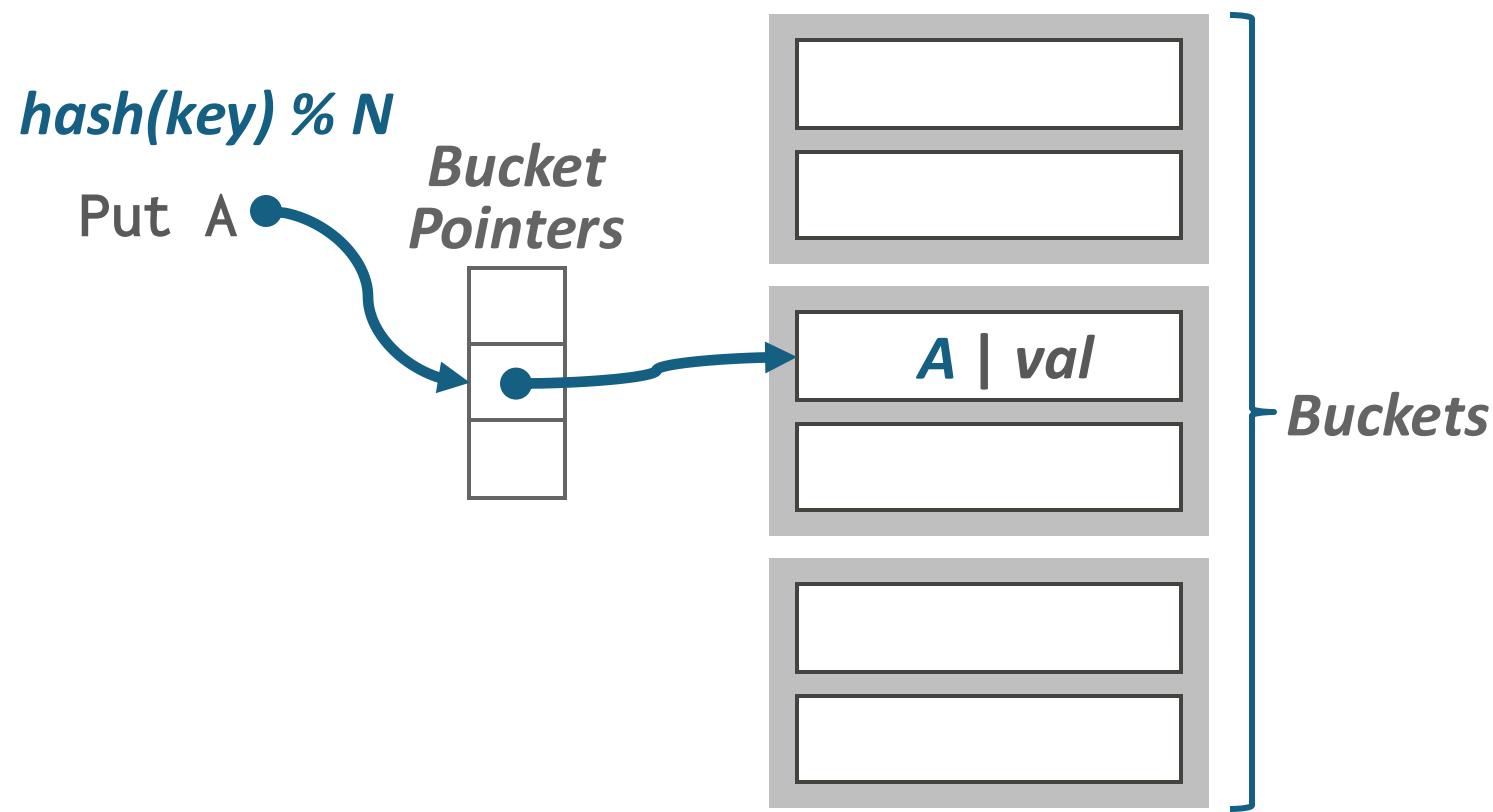
Chained Hashing

$hash(key) \% N$

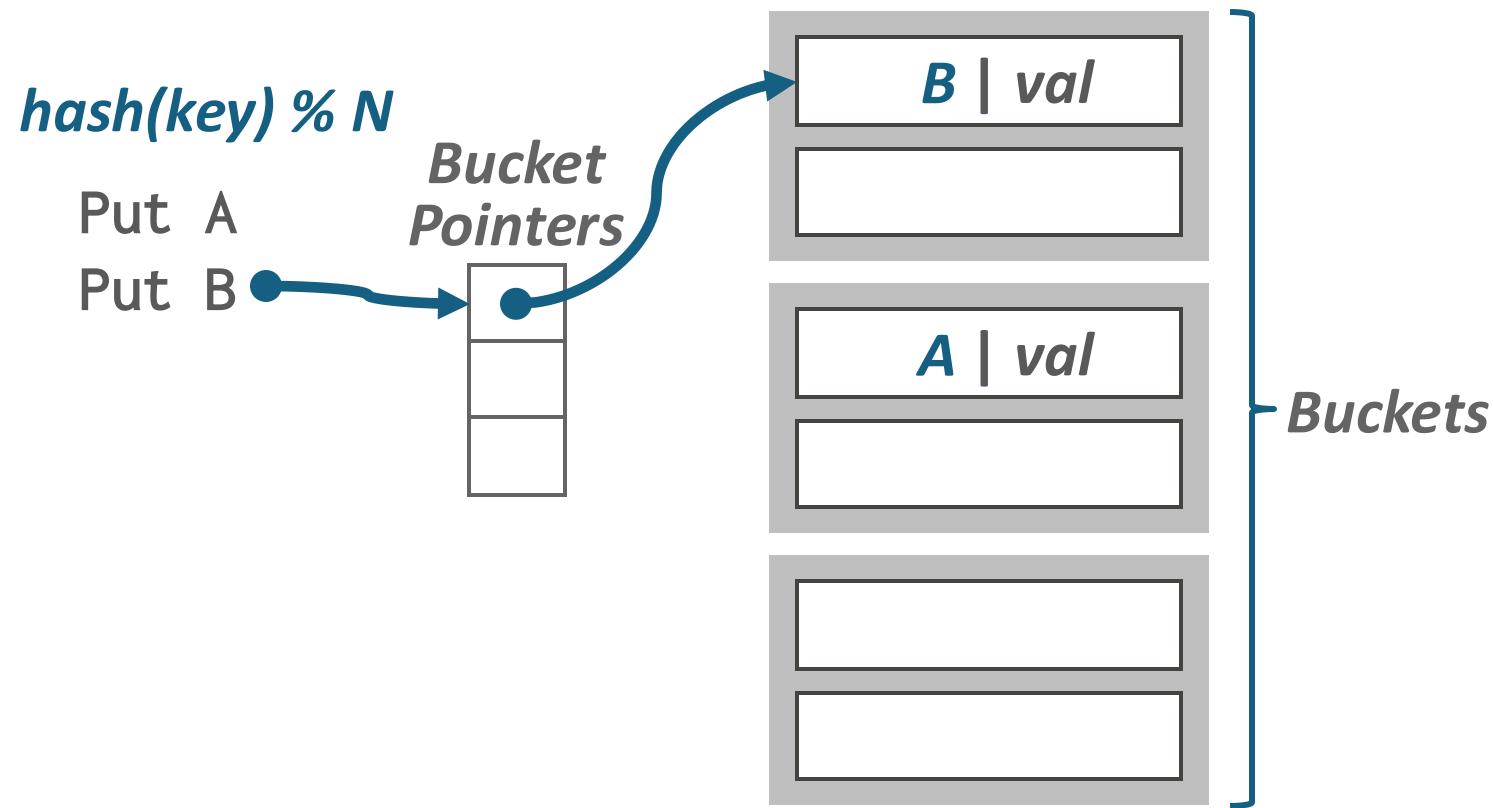
*Bucket
Pointers*



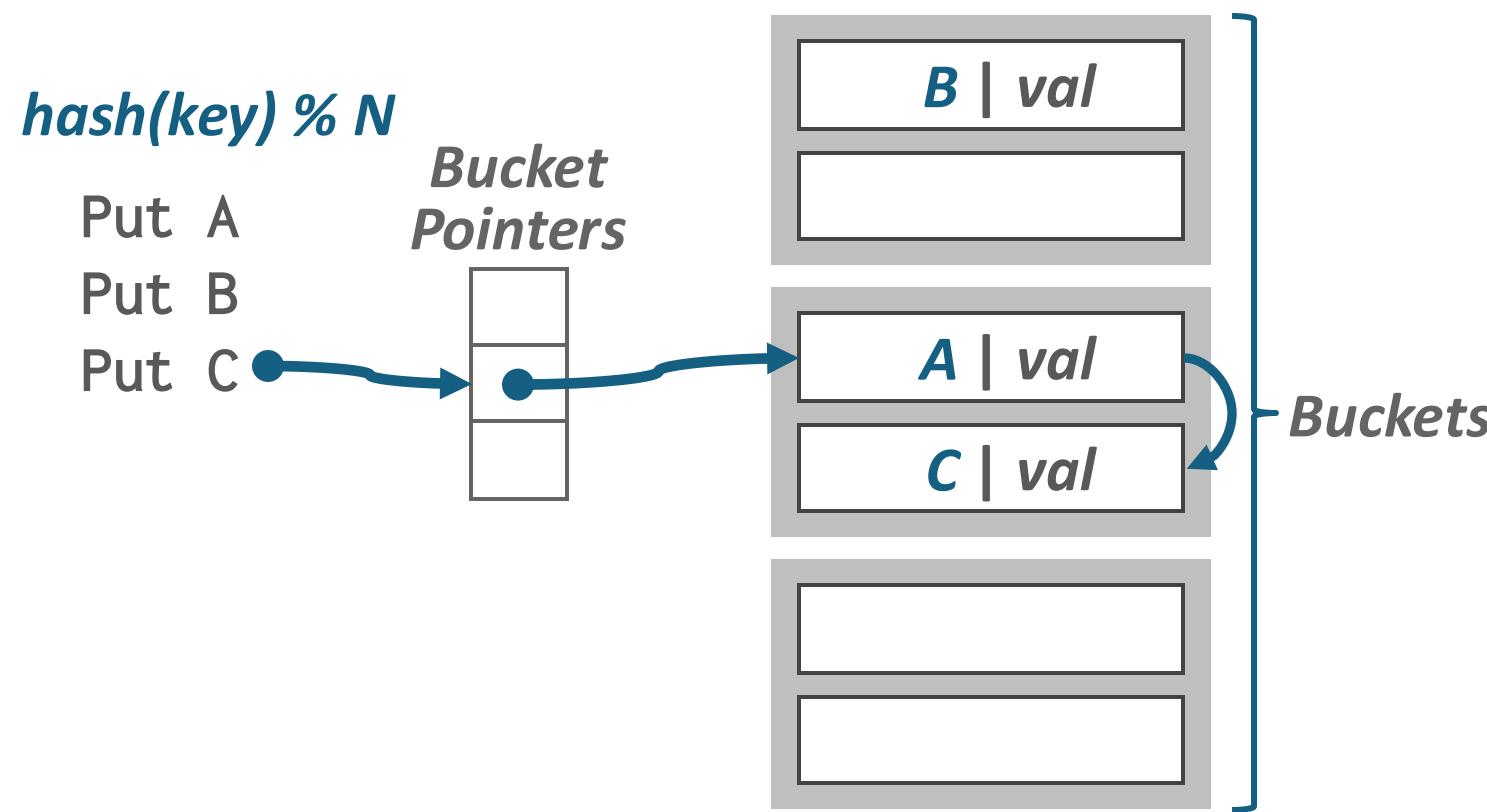
Chained Hashing



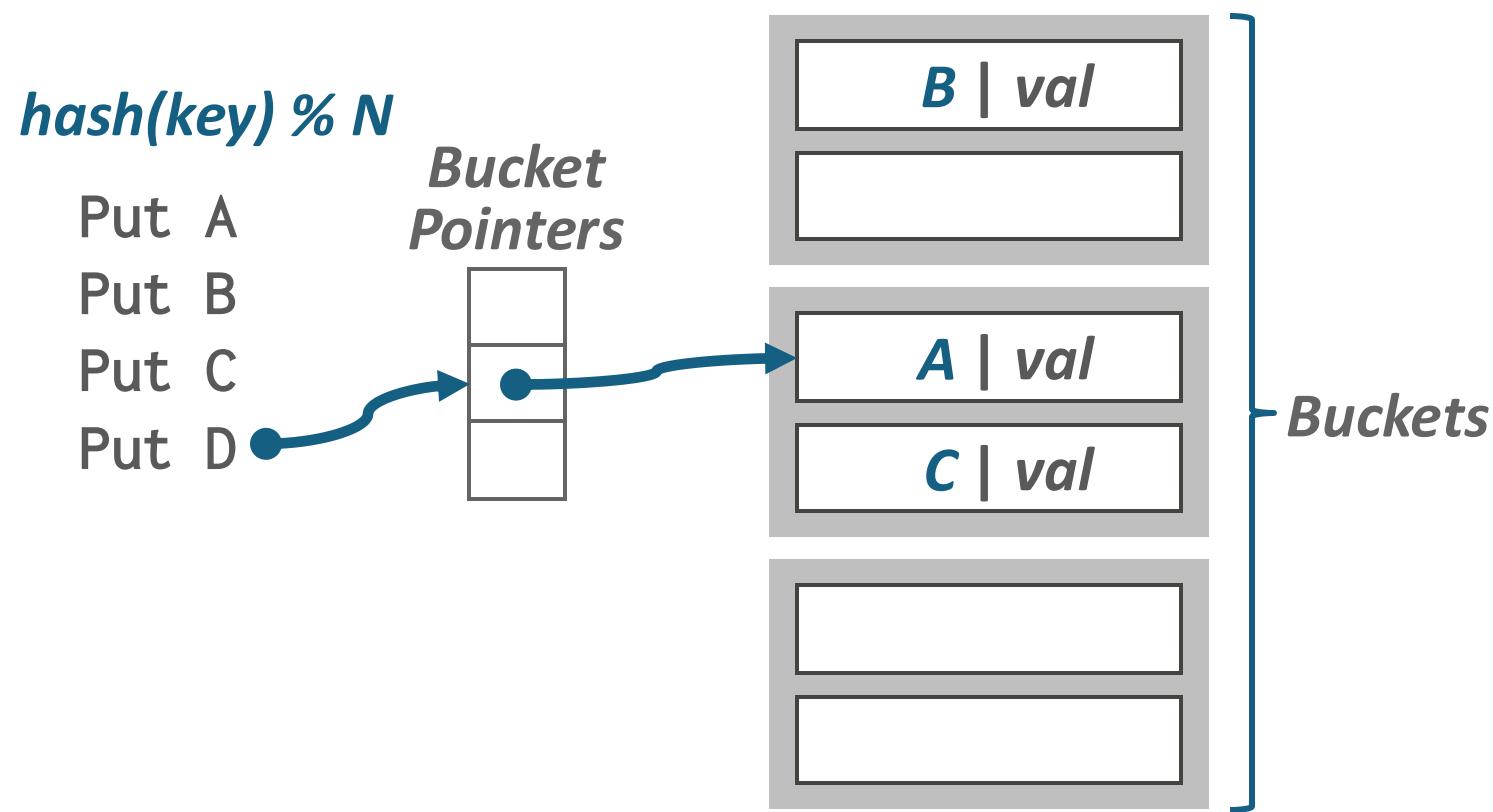
Chained Hashing



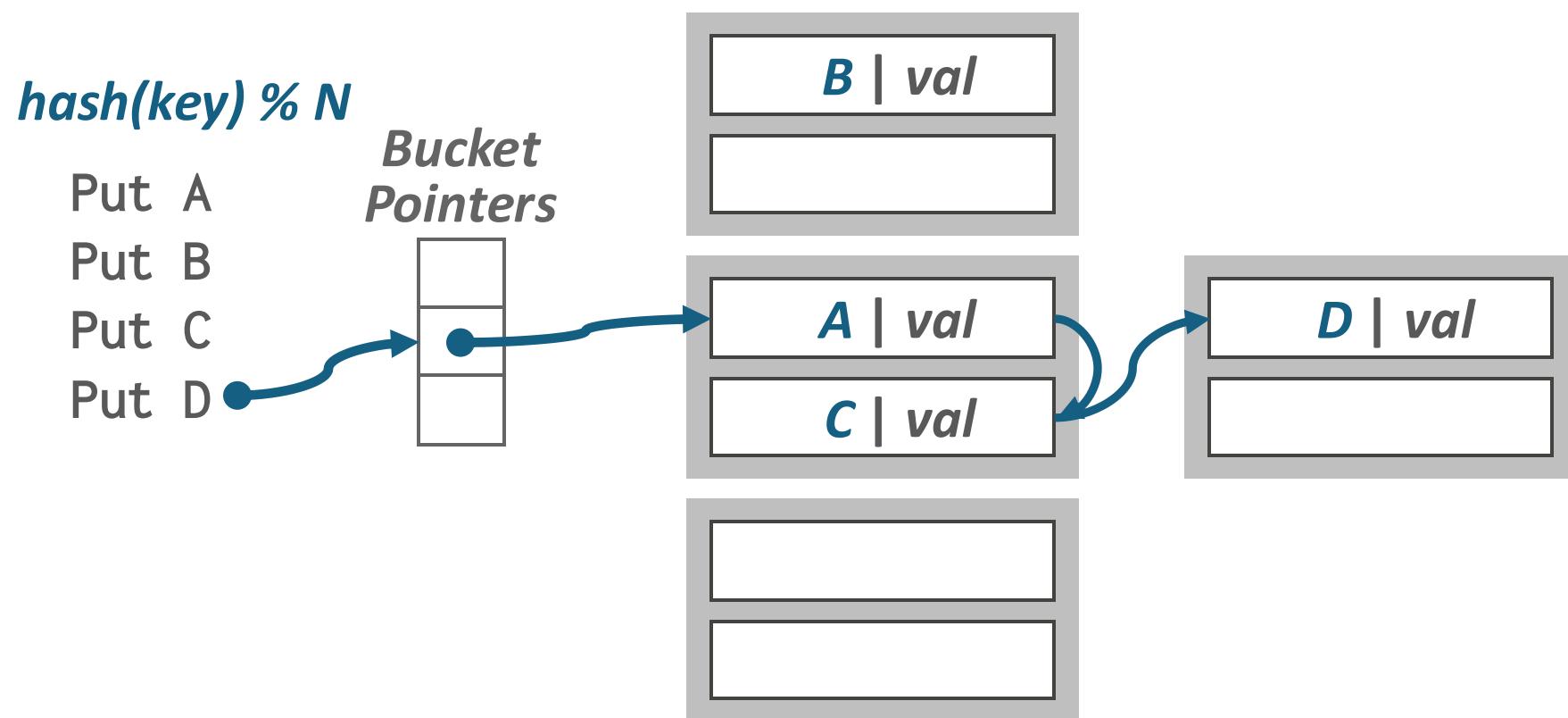
Chained Hashing



Chained Hashing



Chained Hashing

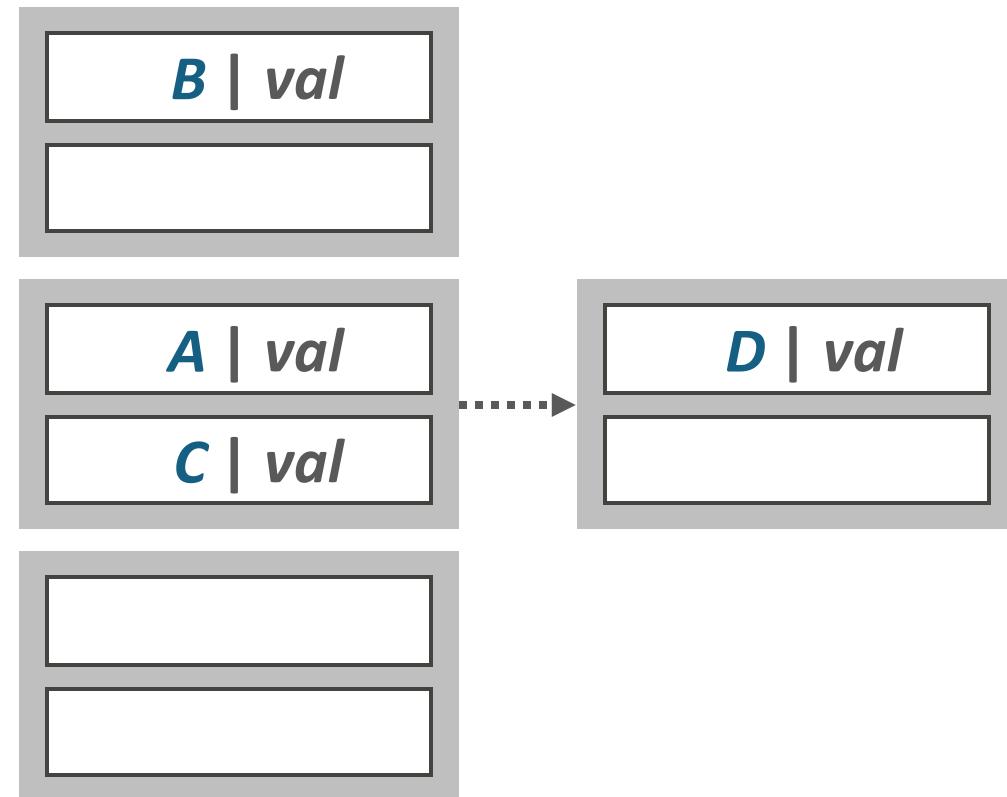
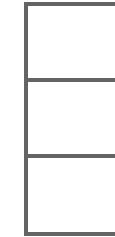


Chained Hashing

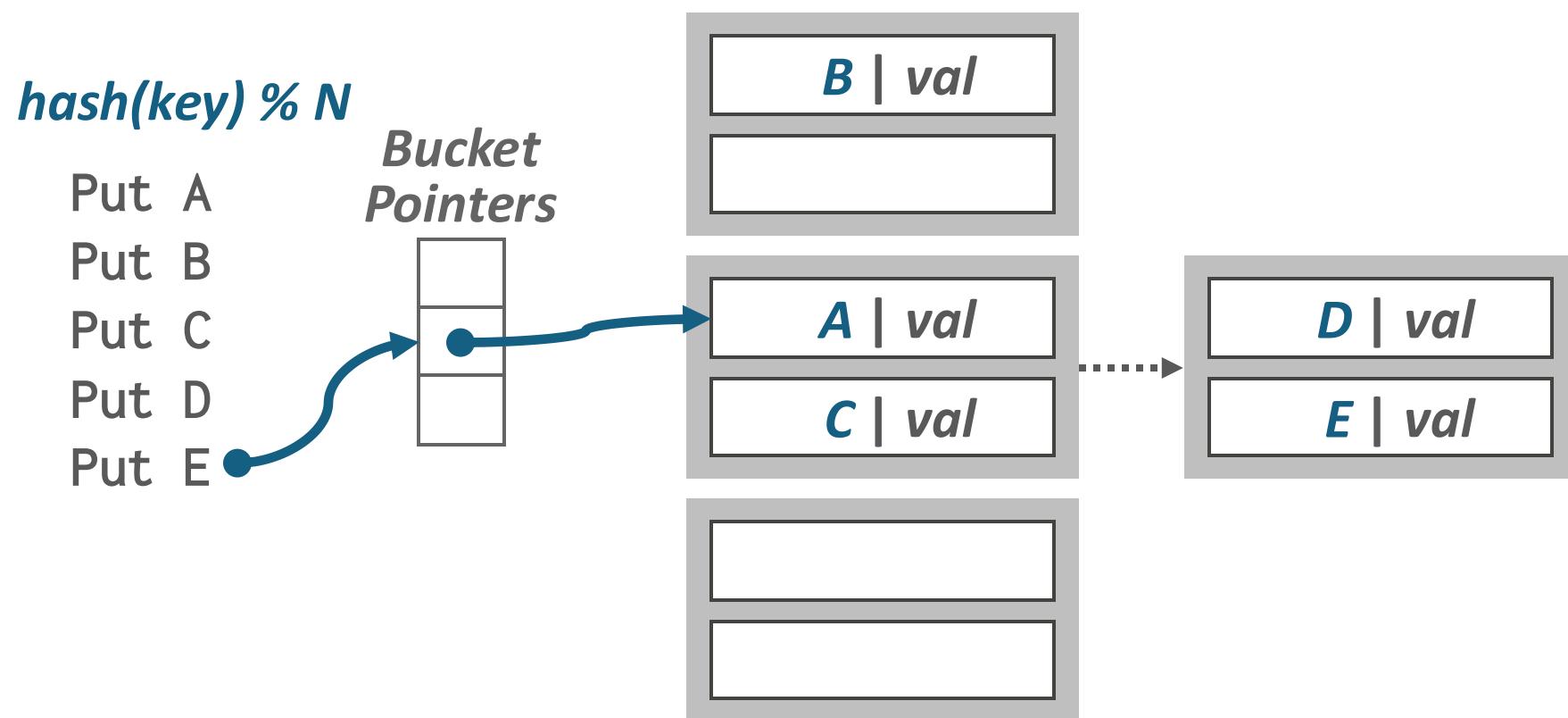
$\text{hash}(\text{key}) \% N$

Put A
Put B
Put C
Put D
Put E

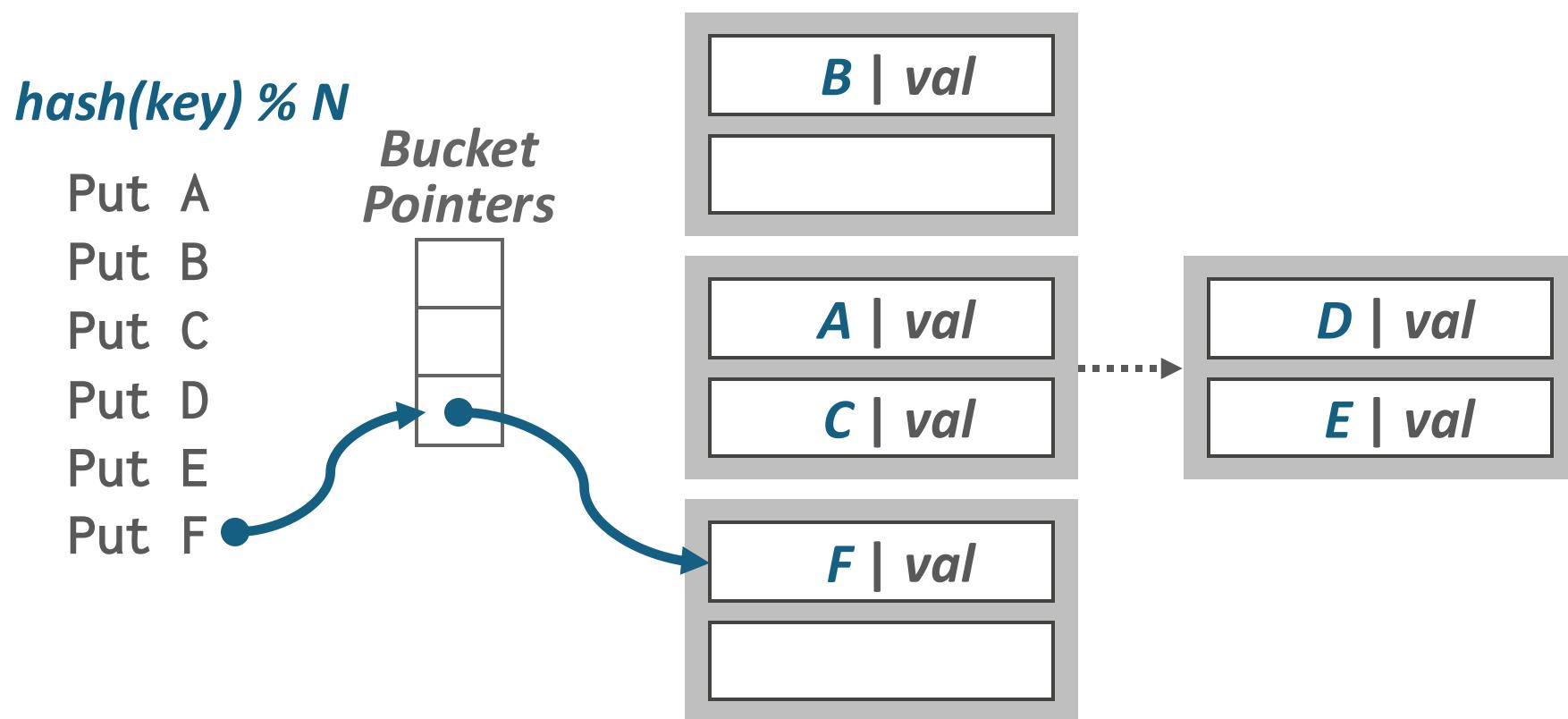
*Bucket
Pointers*



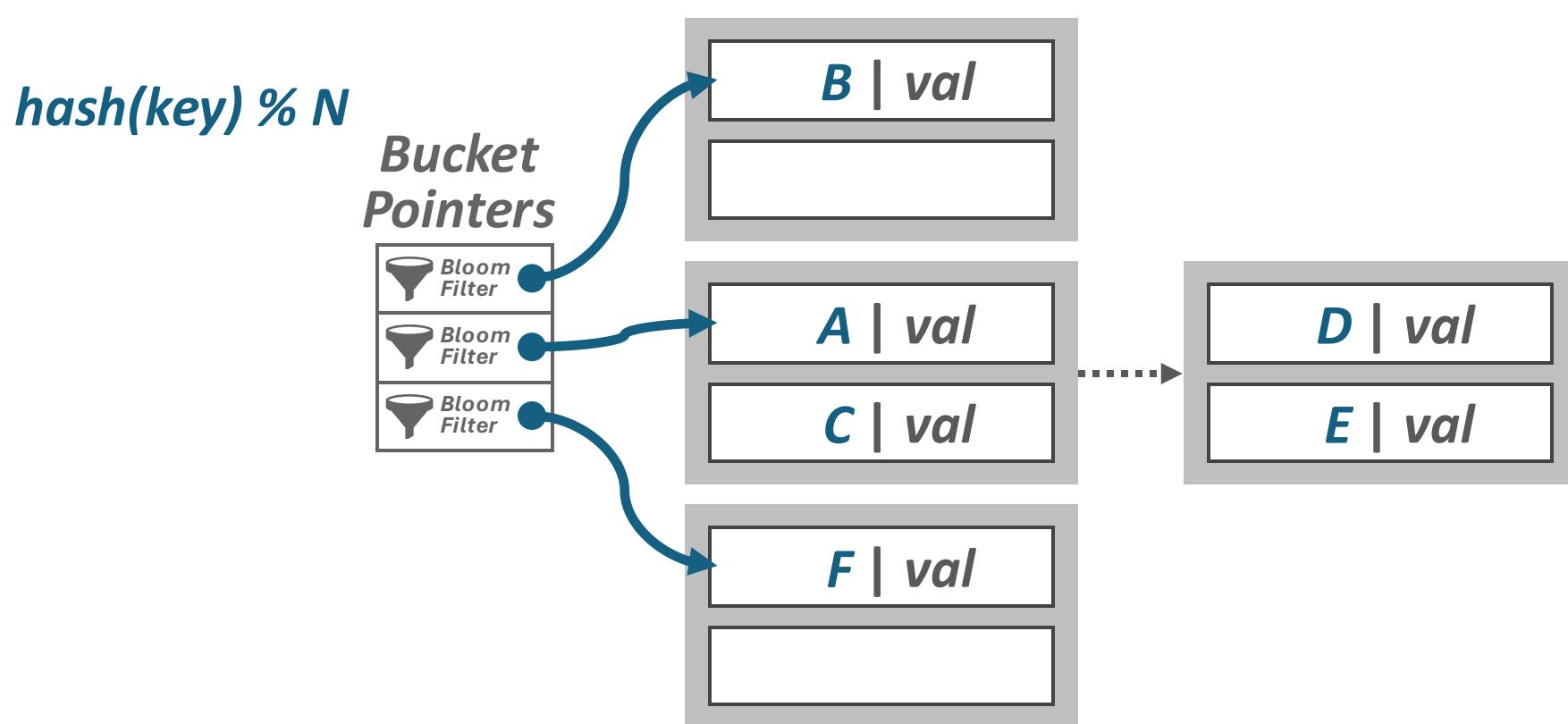
Chained Hashing



Chained Hashing



Chained Hashing



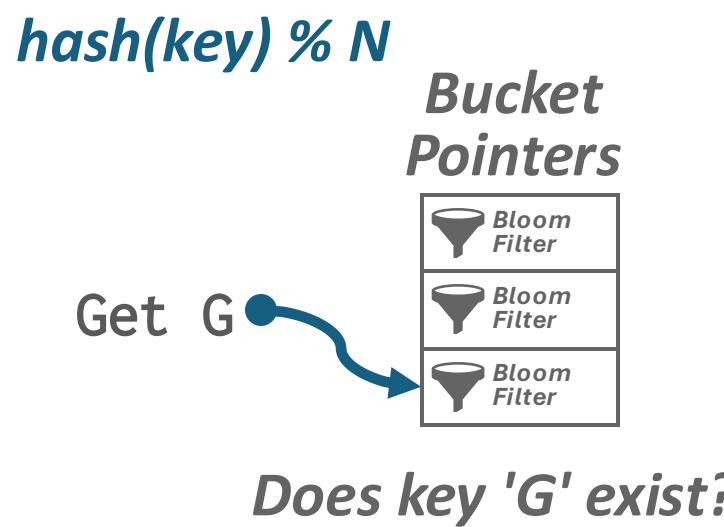
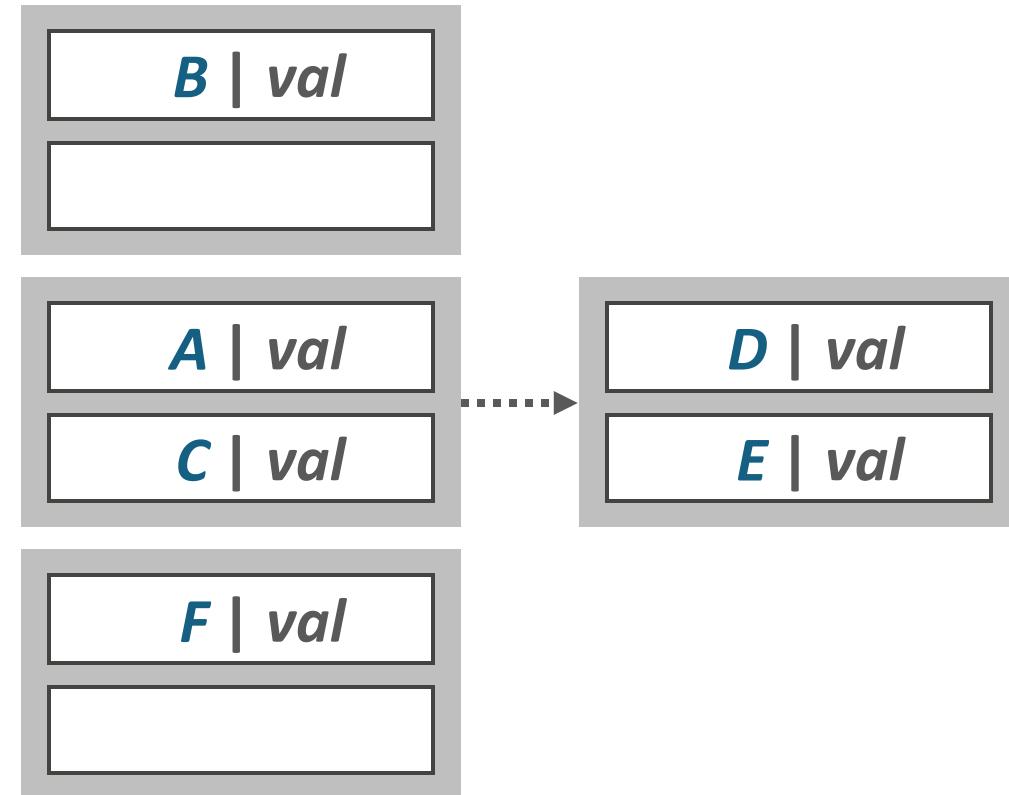
Chained Hashing

$\text{hash}(\text{key}) \% N$

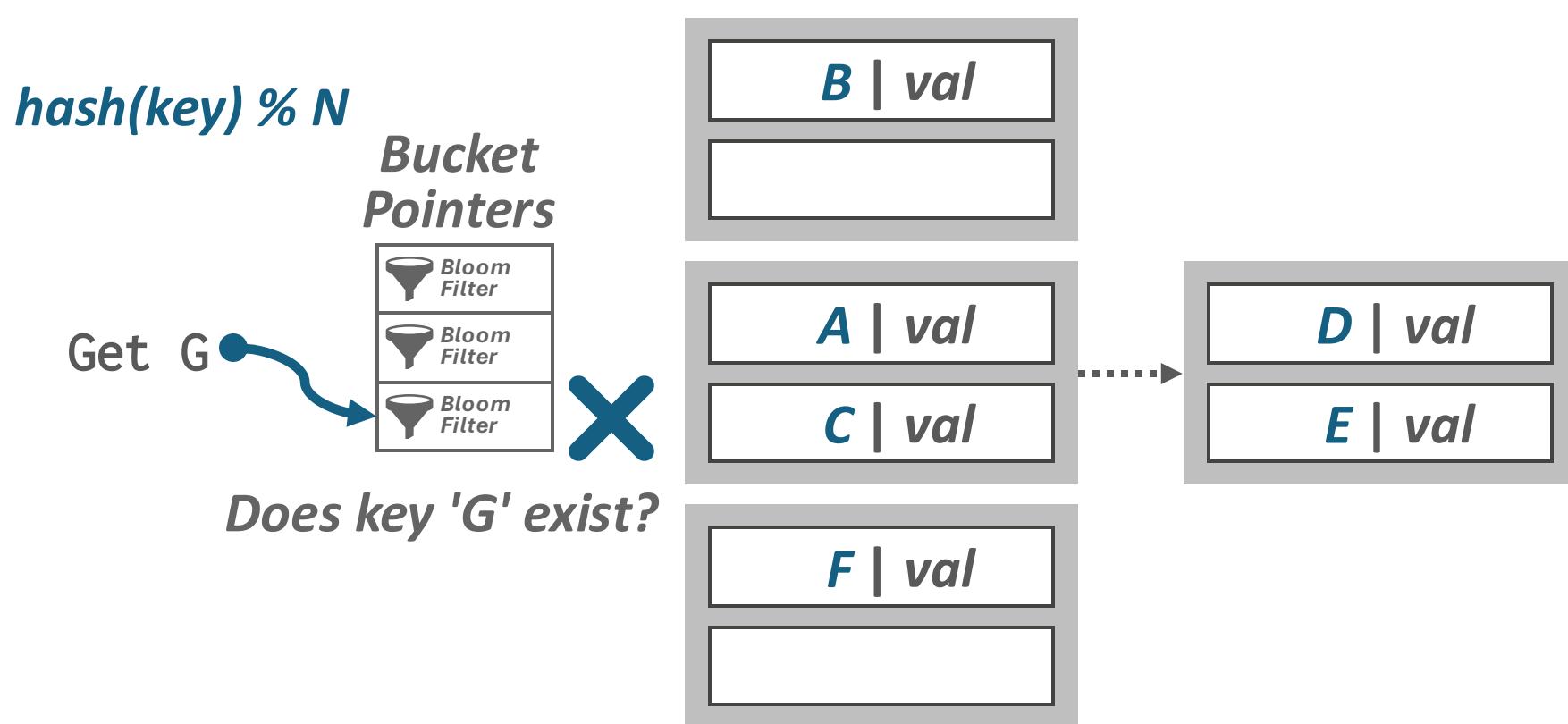
Get G

Bucket Pointers

Does key 'G' exist?

Chained Hashing



Bloom Filters

- Probabilistic data structure (bitmap) that answers set membership queries.
 - False negatives will **never occur**.
 - False positives can **sometimes occur**.
 - See [Bloom Filter Calculator](#).
- **Insert(x):**
 - Use k hash functions to set bits in the filter to 1.
- **Lookup(x):**
 - Check whether the bits are 1 for each hash function.

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0

- Insert('RZA')

Bloom Filters

Bloom Filter

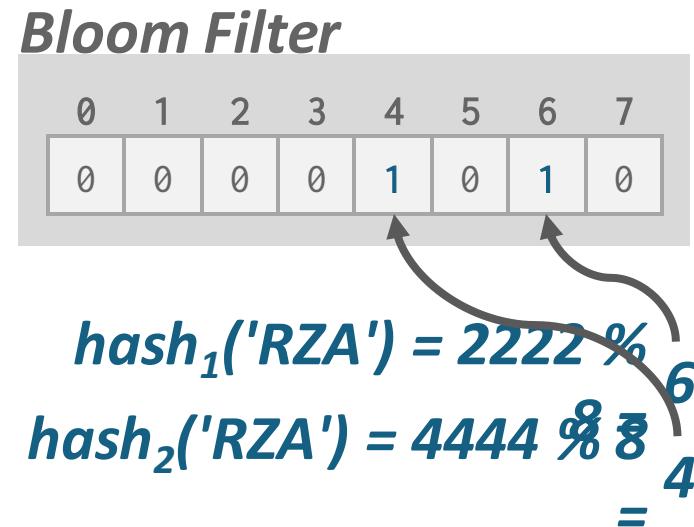
0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0

- Insert('RZA')

$$\text{hash}_1(\text{'RZA'}) = 2222 \%$$

$$\begin{aligned}\text{hash}_2(\text{'RZA'}) &= 4444 \% \\ &\quad \begin{matrix} 8 & 8 \\ 6 & 4 \end{matrix} \\ &= \end{aligned}$$

Bloom Filters



- Insert('RZA')

Bloom Filters

Bloom Filter

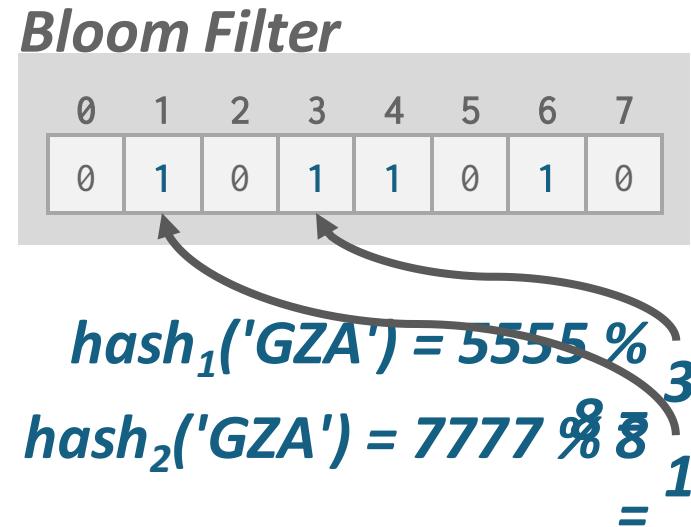
0	1	2	3	4	5	6	7
0	0	0	0	1	0	1	0

- Insert('RZA')
- Insert('GZA')

$$\text{hash}_1(\text{'GZA'}) = 5555 \% \quad 3$$

$$\text{hash}_2(\text{'GZA'}) = 7777 \% \quad 8 \\ = 1$$

Bloom Filters



- Insert('RZA')
- Insert('GZA')

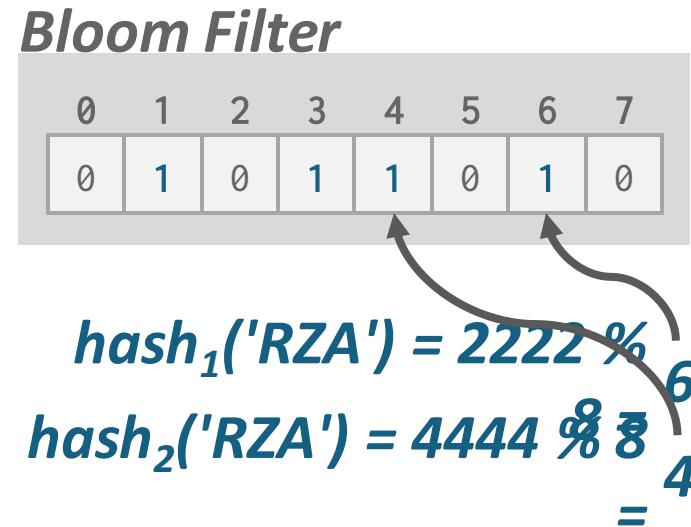
Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

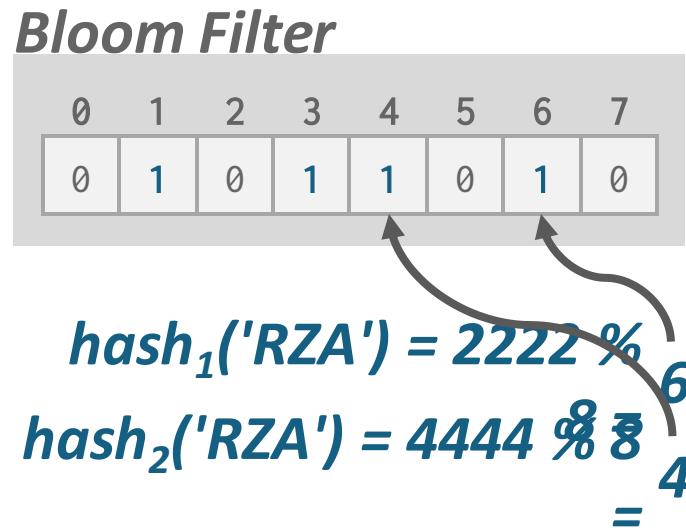
- Insert('RZA')
- Insert('GZA')
- Lookup('RZA')

Bloom Filters



- Insert('RZA')
- Insert('GZA')
- Lookup('RZA')

Bloom Filters



- Insert('RZA')
 - Insert('GZA')
 - Lookup('RZA') → **TRUE**

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon')

Bloom Filters

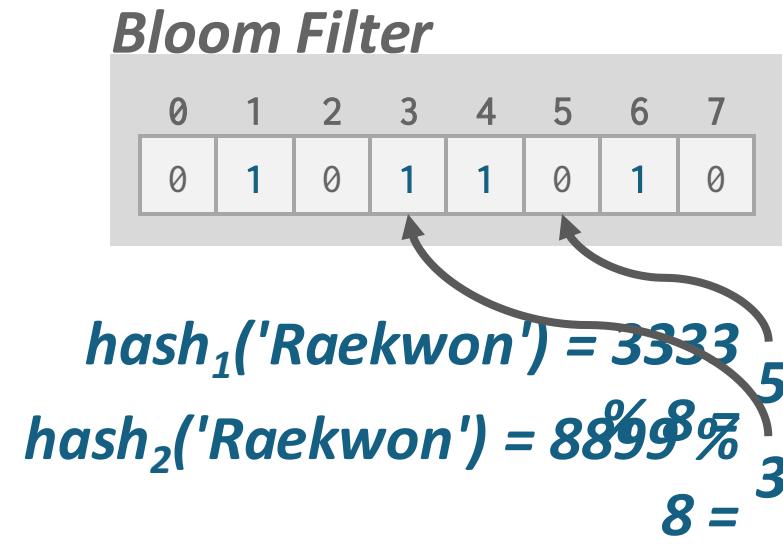
Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$hash_1('Raekwon') = 3333 \quad 5$
 $hash_2('Raekwon') = 88998\% \quad 3$
 $8 = \underline{3}$

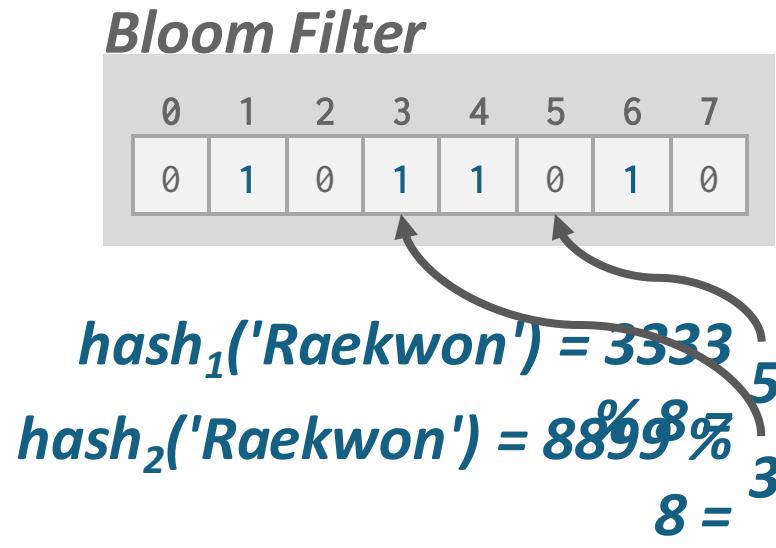
- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon')

Bloom Filters



- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon')

Bloom Filters



- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**

Bloom Filters

Bloom Filter

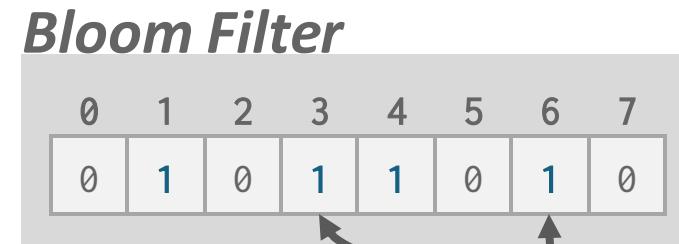
0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$\text{hash}_1('ODB') = 6699 \% 8 = 3$$

$$\text{hash}_2('ODB') = 9966 \% 8 = 6$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**
- Lookup('ODB')

Bloom Filters

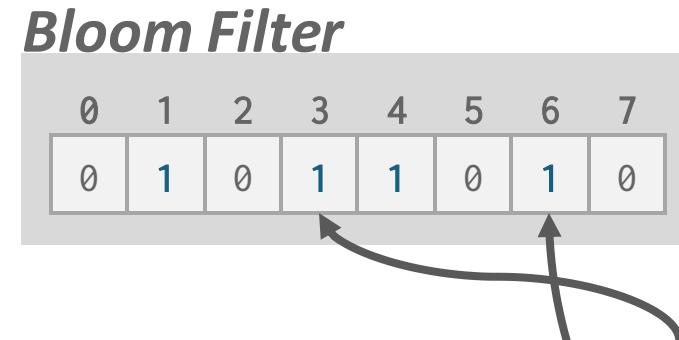


$$\text{hash}_1('ODB') = 6699 \% 8 = 3$$

$$\text{hash}_2('ODB') = 9966 \% 8 = 6$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**
- Lookup('ODB')

Bloom Filters



$$\text{hash}_1('ODB') = 6699 \% 8 = 3$$

$$\text{hash}_2('ODB') = 9966 \% 8 = 6$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**
- Lookup('ODB') → **TRUE (false positive)**

Extendible Hashing

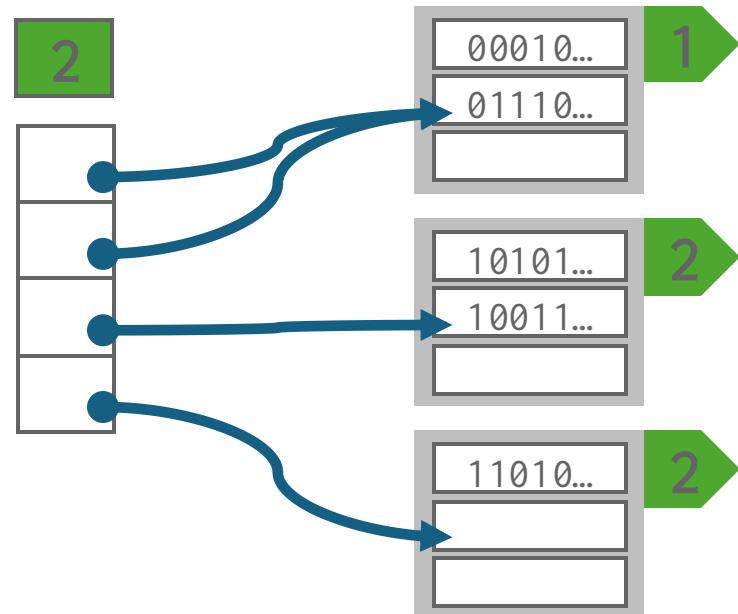
Dynamic Hashing Schemes

Extendible Hashing

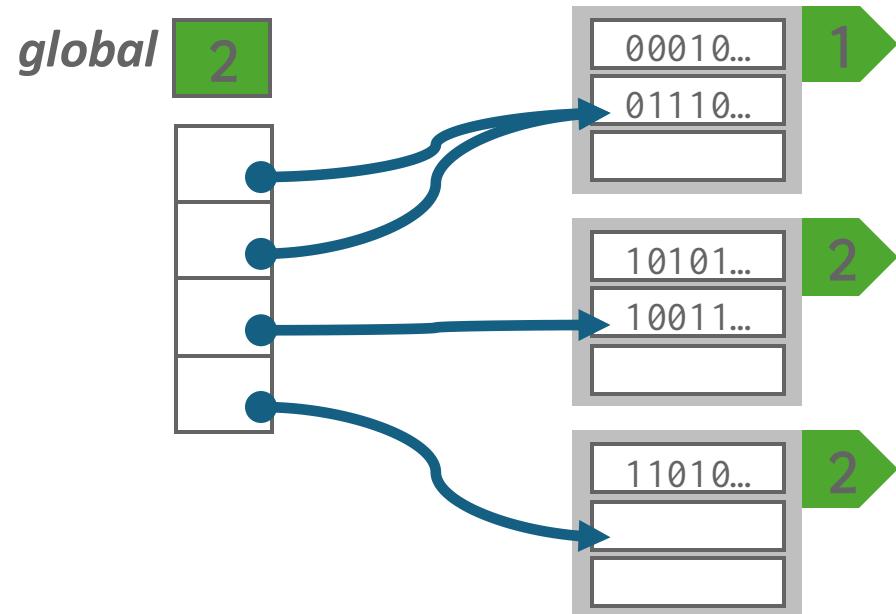
- Extendible-hashing approach that splits buckets incrementally instead of letting the linked list grow forever.
- Multiple slot locations can point to the same bucket chain.
- Reshuffle bucket entries on split and increase the number of bits to examine.
 - Data movement is localized to just the split chain.



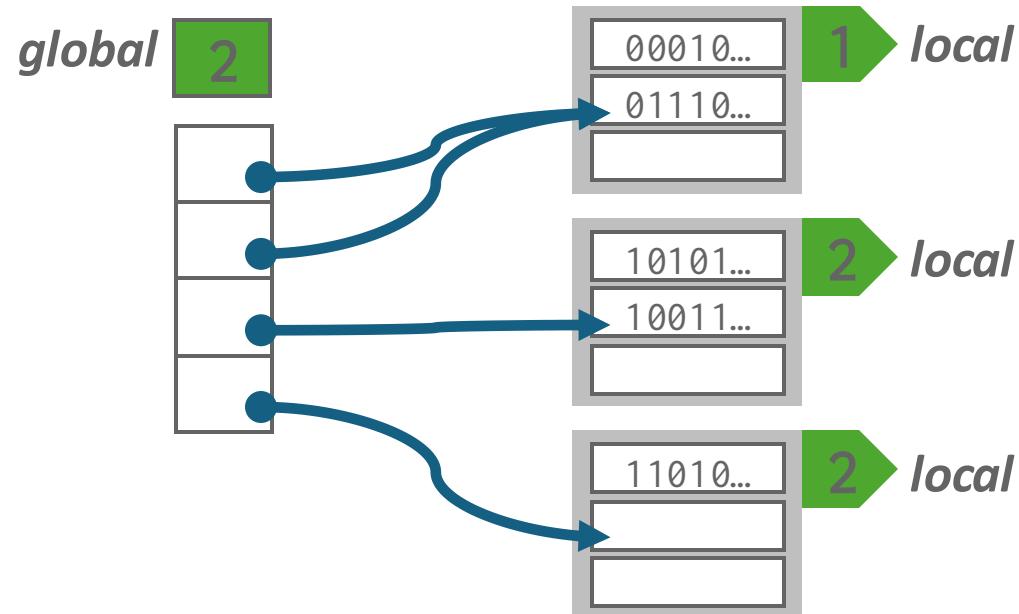
Extendible Hashing



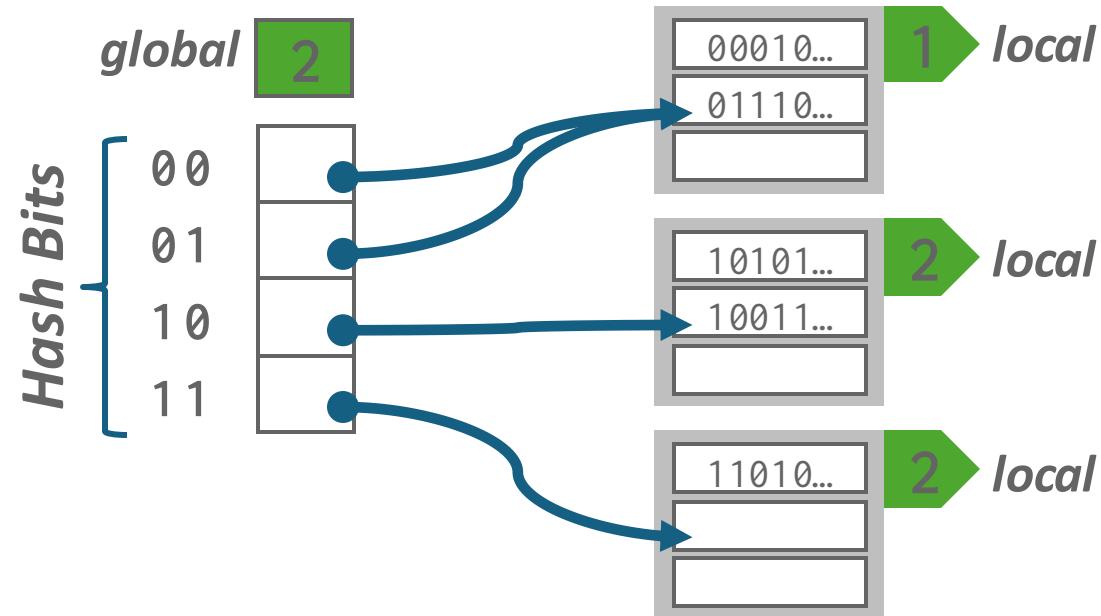
Extendible Hashing



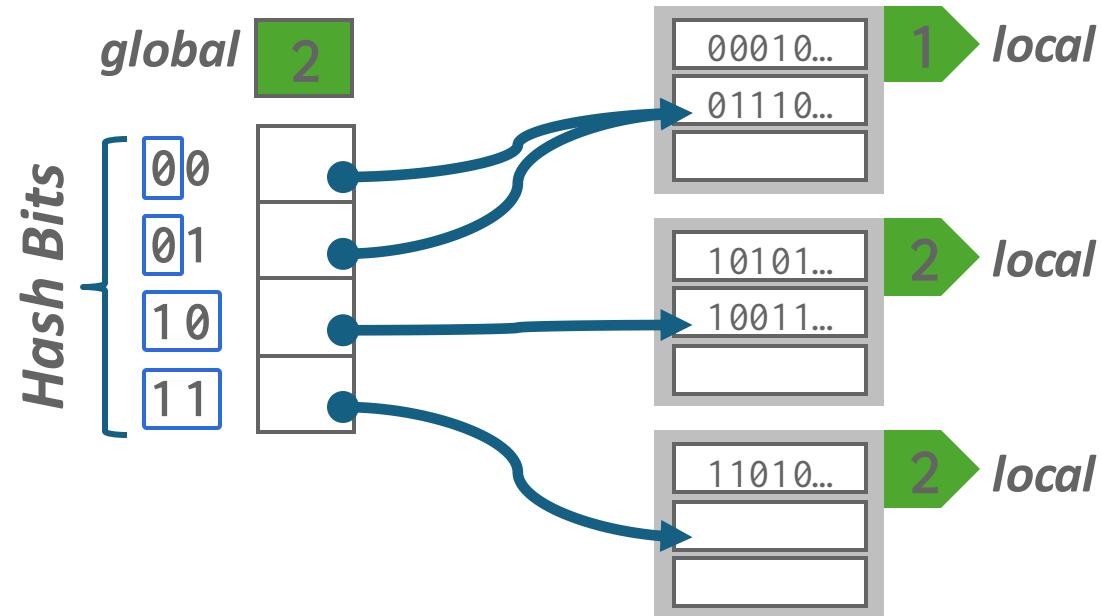
Extendible Hashing



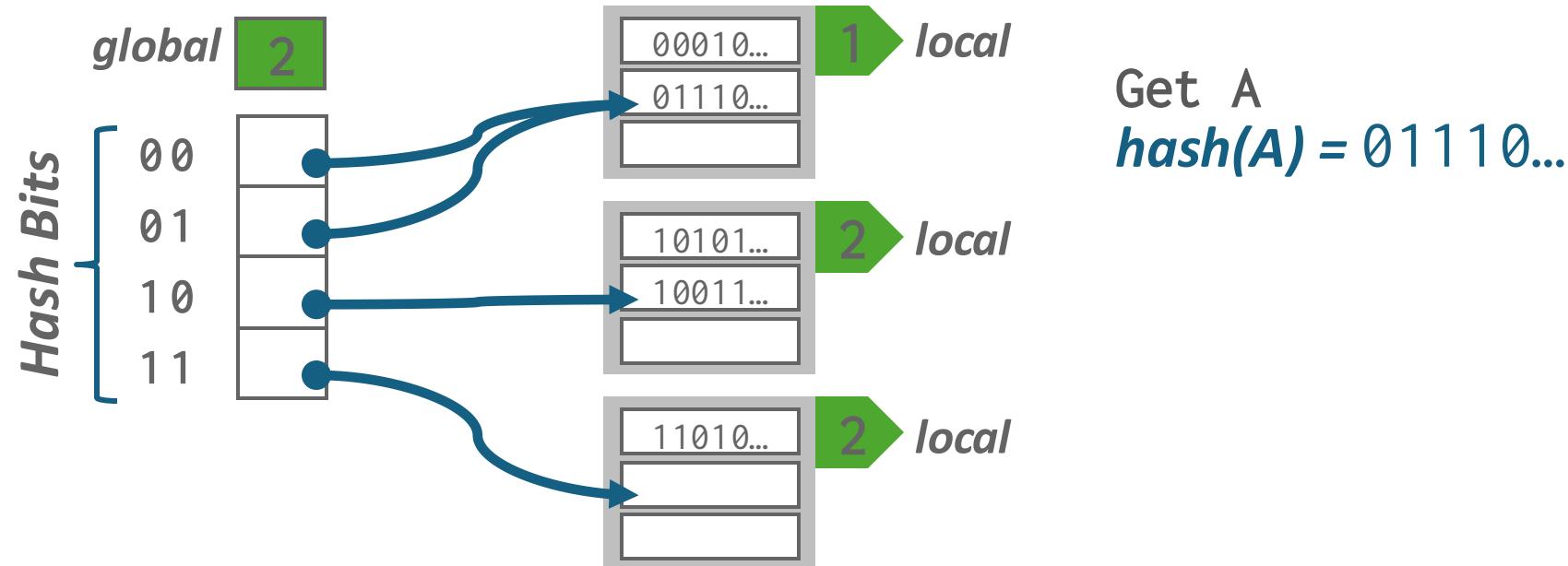
Extendible Hashing



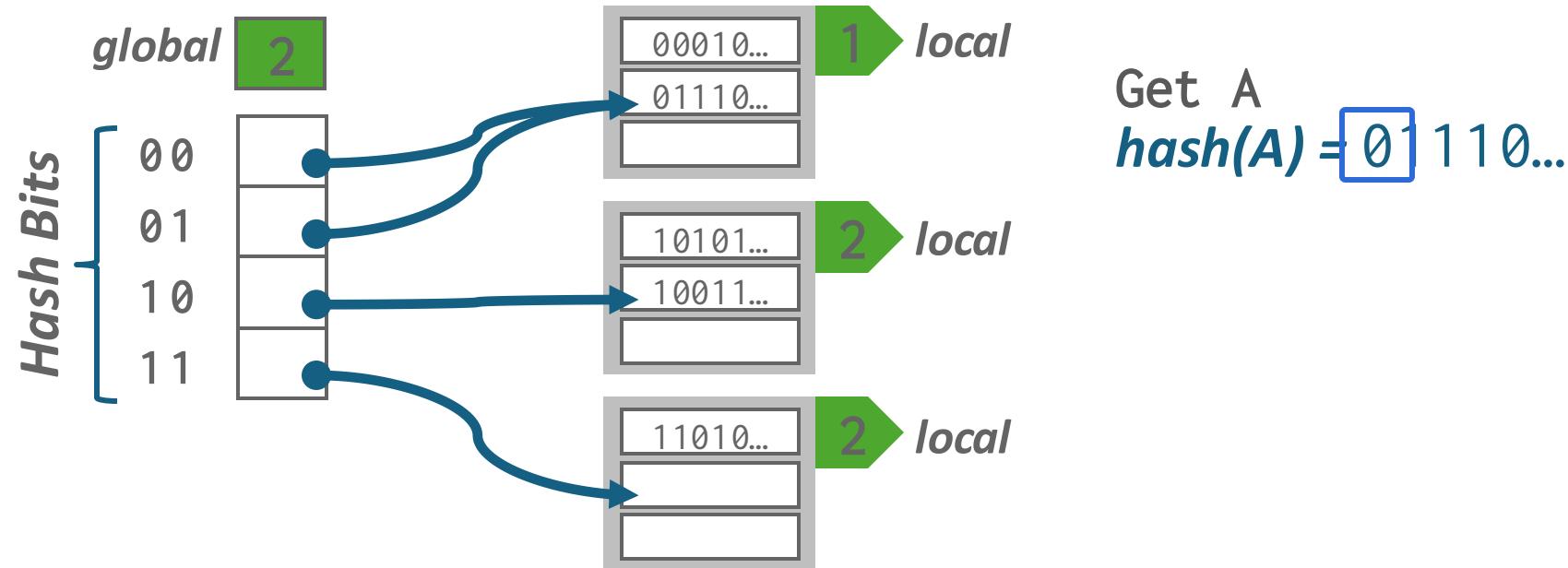
Extendible Hashing



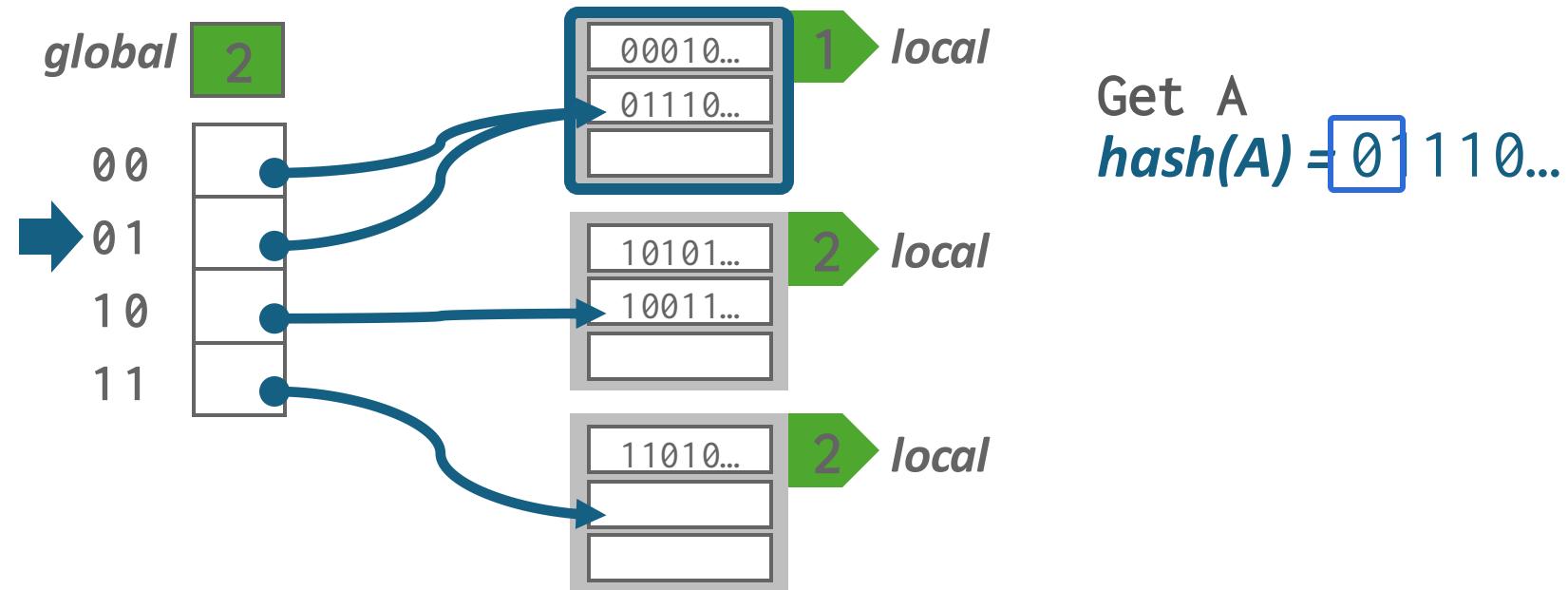
Extendible Hashing



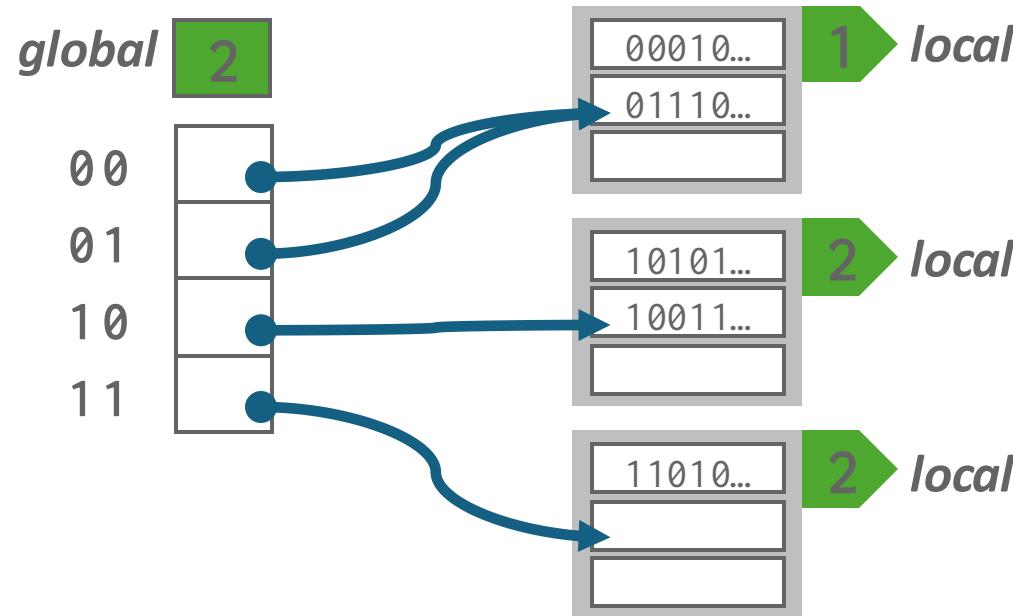
Extendible Hashing



Extendible Hashing



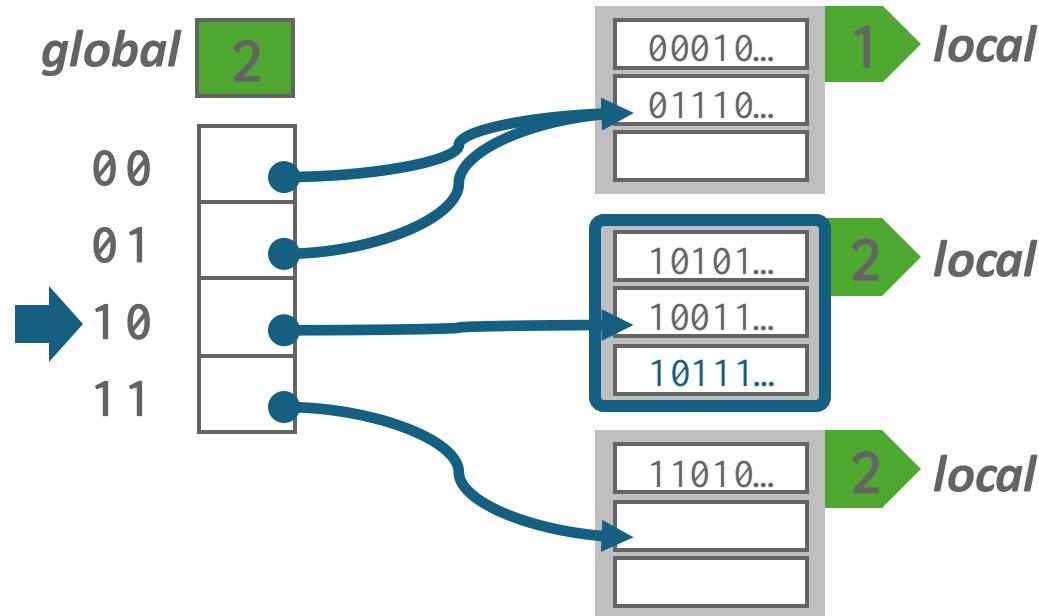
Extendible Hashing



Get A
 $\text{hash}(A) = 01110\dots$

Put B
 $\text{hash}(B) = 10111\dots$

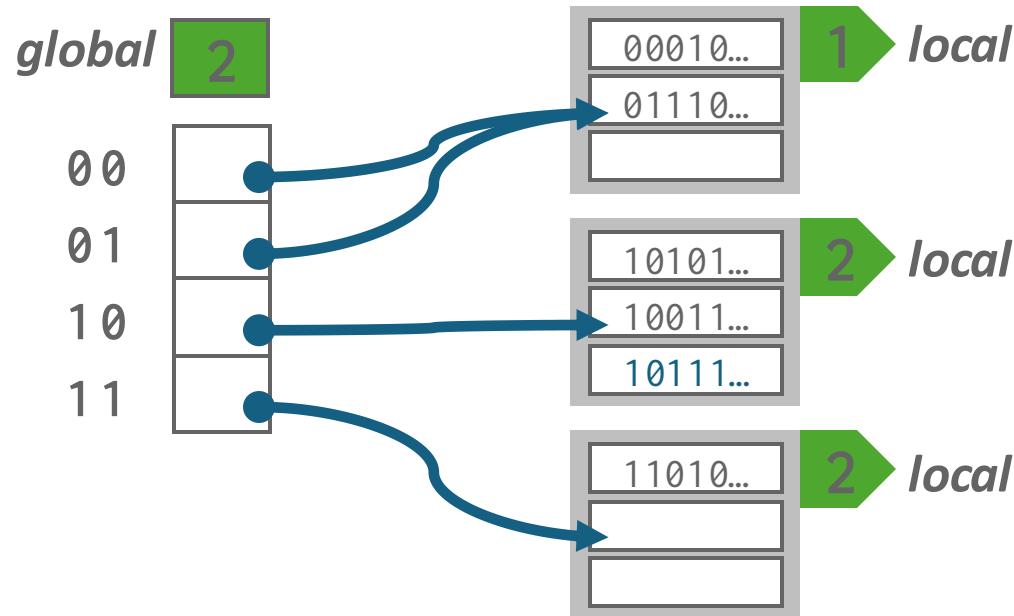
Extendible Hashing



Get A
 $\text{hash}(A) = 01110\dots$

Put B
 $\text{hash}(B) = \boxed{1}0111\dots$

Extendible Hashing

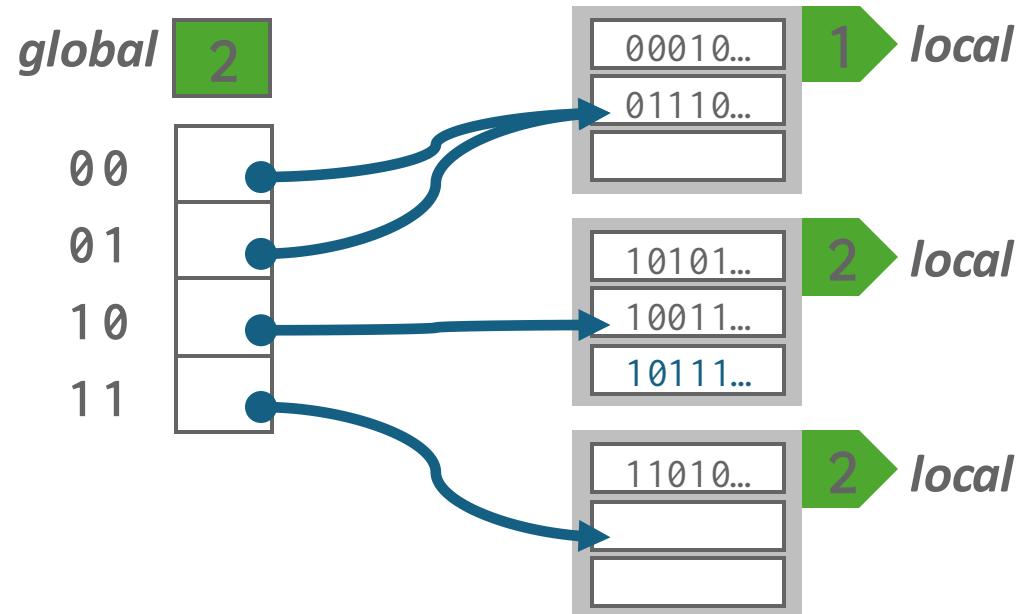


Get A
 $\text{hash}(A) = 01110\dots$

Put B
 $\text{hash}(B) = 10111\dots$

Put C
 $\text{hash}(C) = 10100\dots$

Extendible Hashing

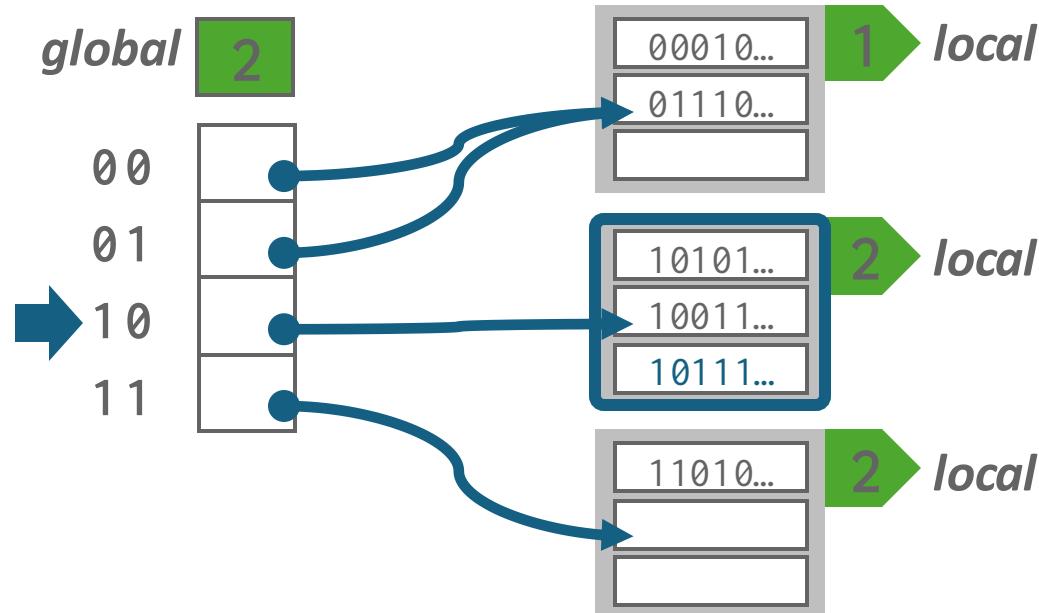


Get A
 $\text{hash}(A) = 01110\dots$

Put B
 $\text{hash}(B) = 10111\dots$

Put C
 $\text{hash}(C) = \boxed{10100\dots}$

Extendible Hashing

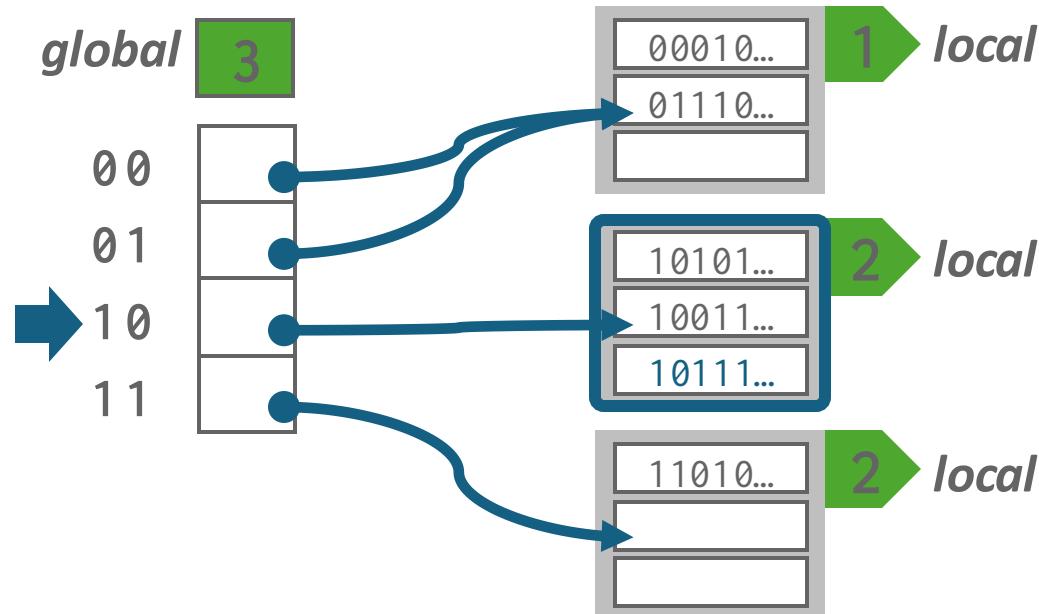


Get A
 $\text{hash}(A) = 01110\dots$

Put B
 $\text{hash}(B) = 10111\dots$

Put C
 $\text{hash}(C) = \boxed{10}100\dots$

Extendible Hashing

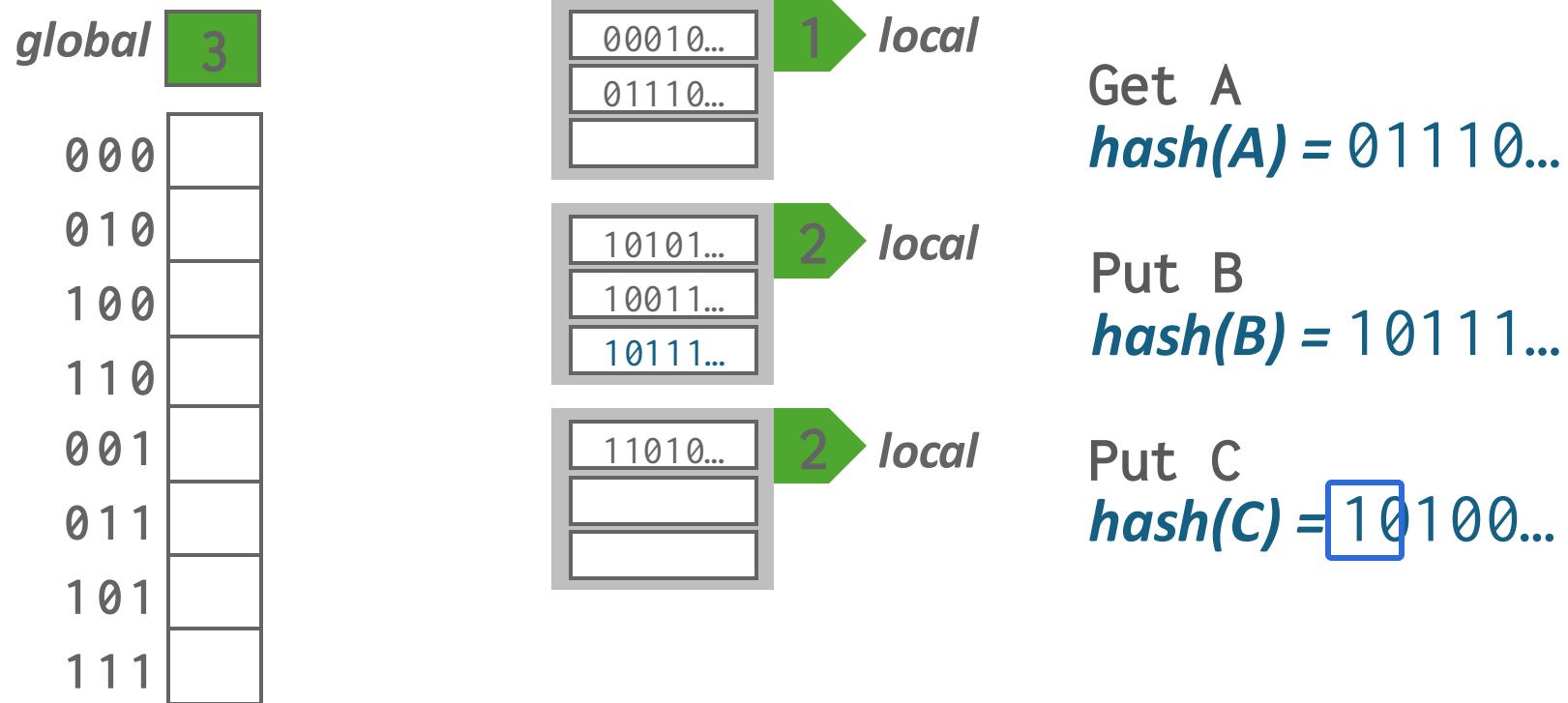


Get A
 $\text{hash}(A) = 01110...$

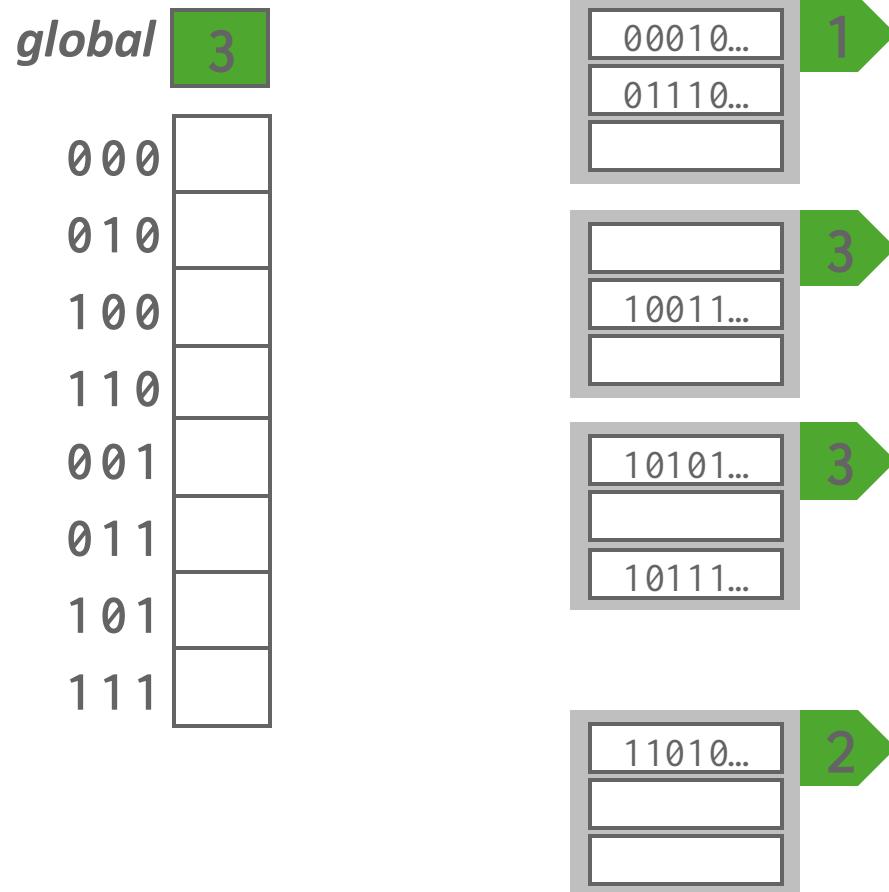
Put B
 $\text{hash}(B) = 10111...$

Put C
 $\text{hash}(C) = \boxed{10100...}$

Extendible Hashing



Extendible Hashing

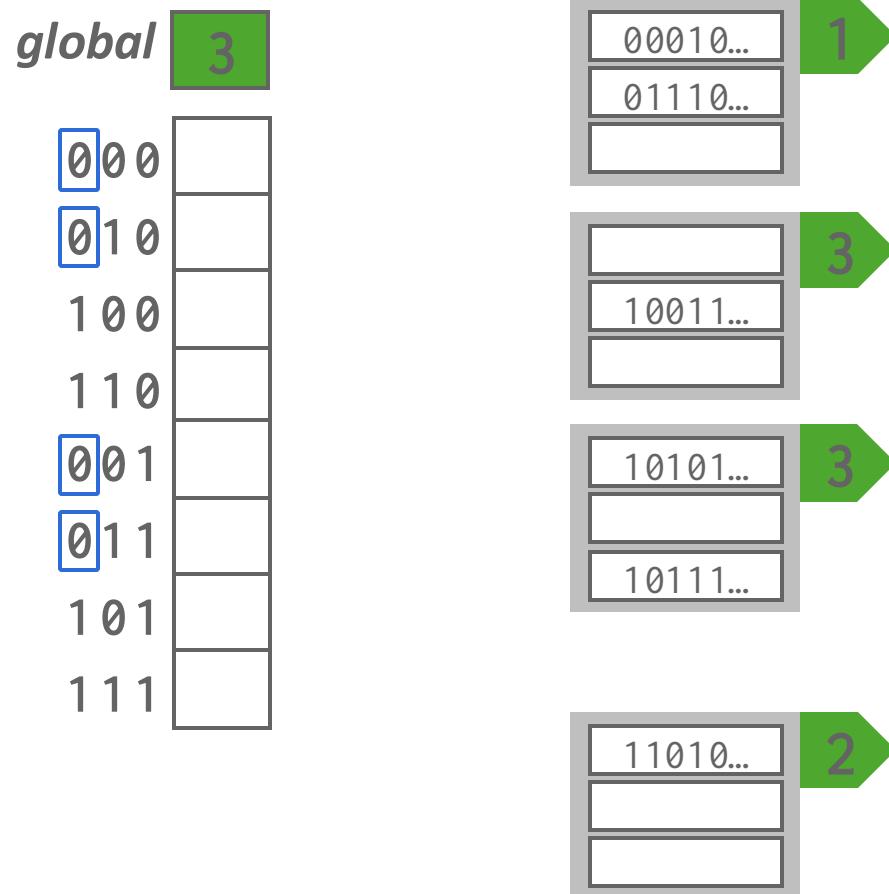


Get A
 $\text{hash}(A) = 01110...$

Put B
 $\text{hash}(B) = 10111...$

Put C
 $\text{hash}(C) = 10100...$

Extendible Hashing

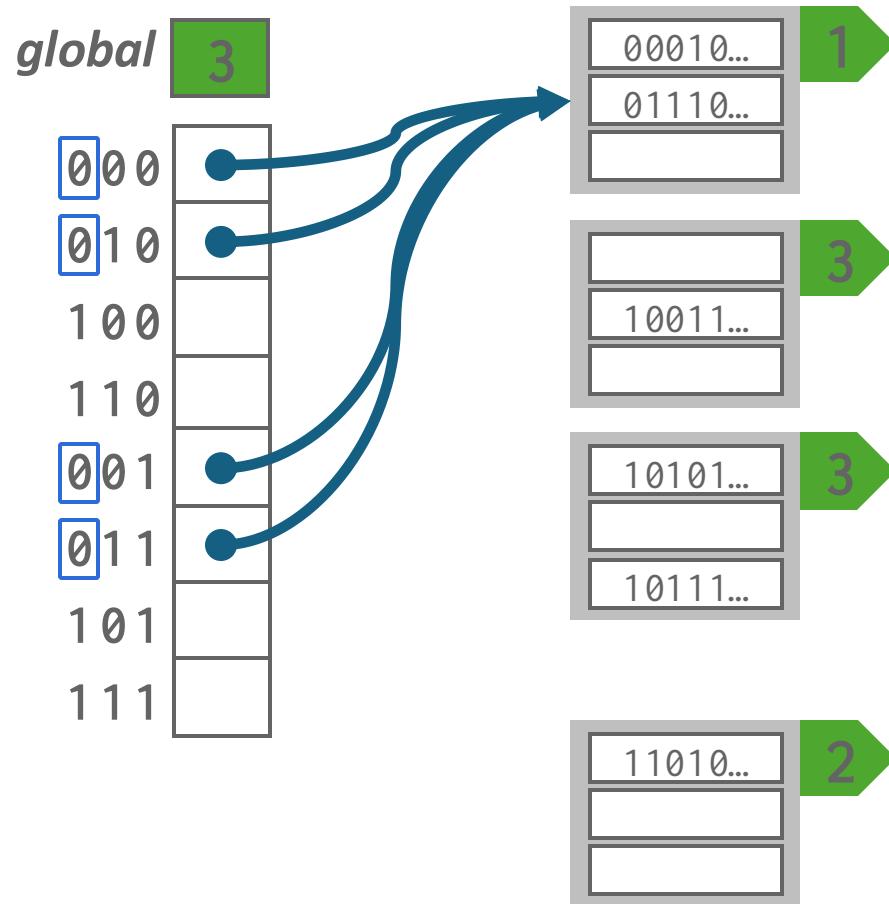


Get A
 $\text{hash}(A) = 01110...$

Put B
 $\text{hash}(B) = 10111...$

Put C
 $\text{hash}(C) = 10100...$

Extendible Hashing

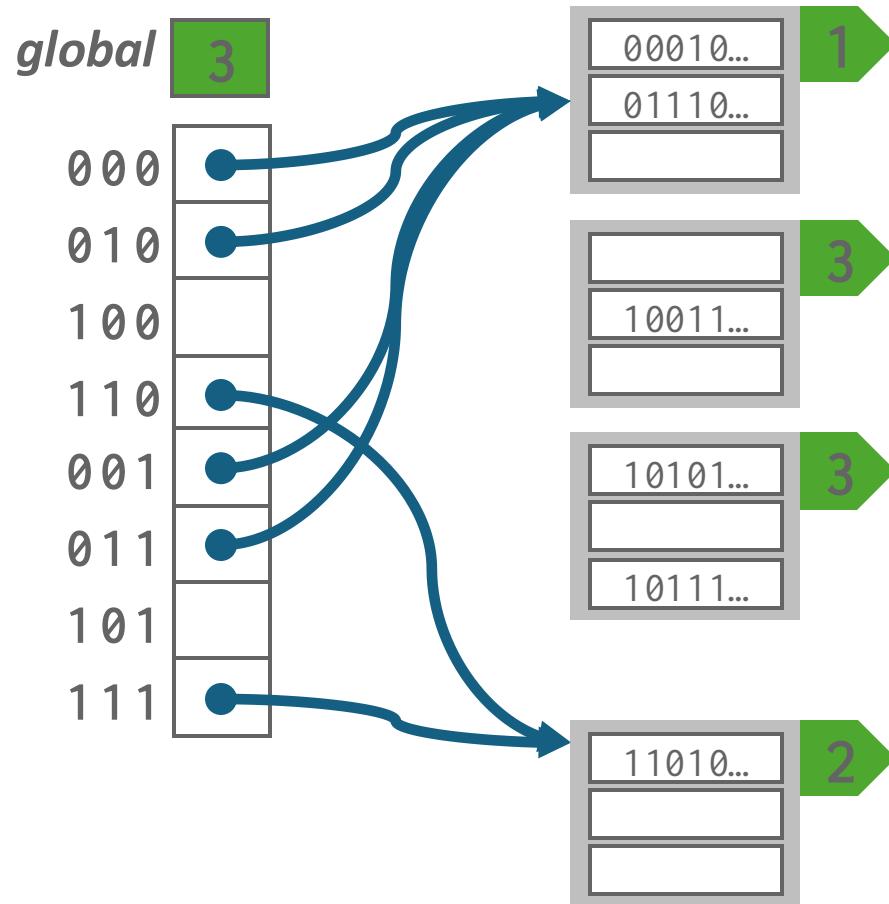


Get A
 $\text{hash}(A) = 01110\dots$

Put B
 $\text{hash}(B) = 10111\dots$

Put C
 $\text{hash}(C) = 10100\dots$

Extendible Hashing

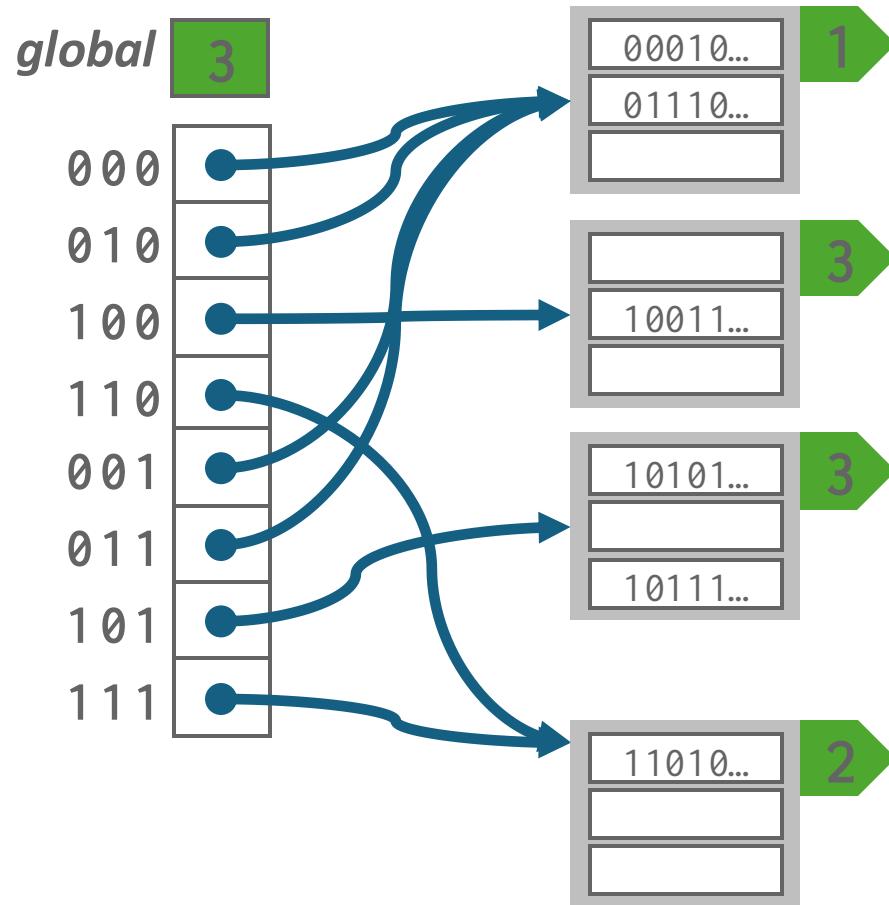


Get A
 $\text{hash}(A) = 01110...$

Put B
 $\text{hash}(B) = 10111...$

Put C
 $\text{hash}(C) = 10100...$

Extendible Hashing

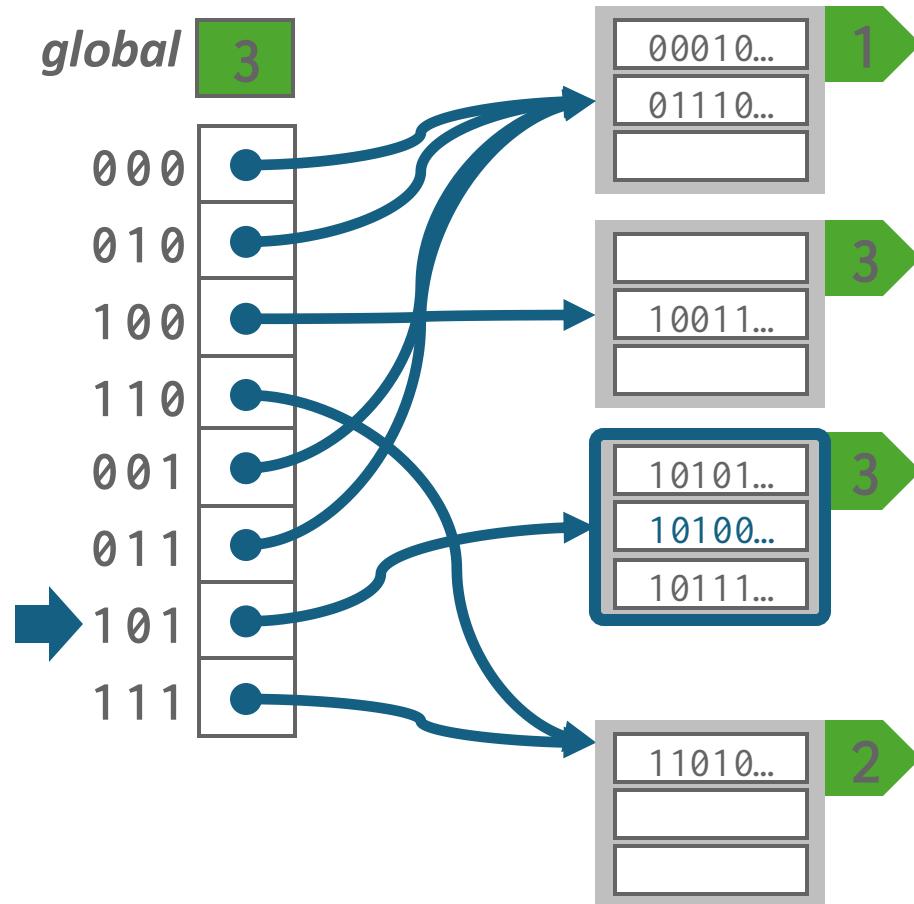


Get A
 $\text{hash}(A) = 01110...$

Put B
 $\text{hash}(B) = 10111...$

Put C
 $\text{hash}(C) = 10100...$

Extendible Hashing



Get A
 $\text{hash}(A) = 01110...$

Put B
 $\text{hash}(B) = 10111...$

Put C
 $\text{hash}(C) = \boxed{101}00...$

Linear Hashing

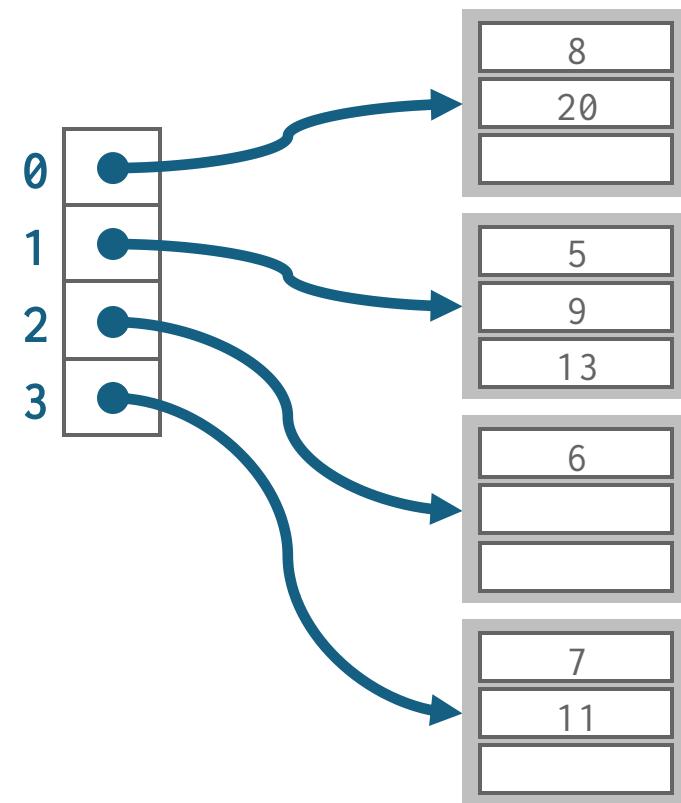
Dynamic Hashing Schemes

Linear Hashing

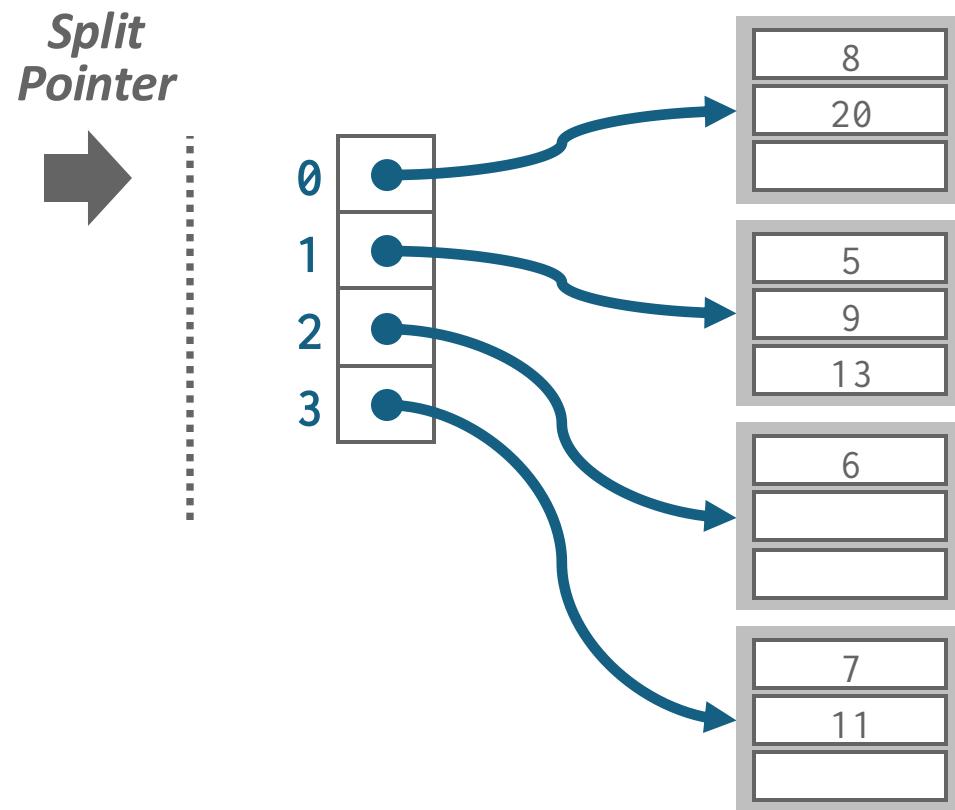
- The hash table maintains a pointer that tracks the next bucket to split.
 - When any bucket overflows, split the bucket at the pointer location.
- Use multiple hashes to find the right bucket for a given key.
- Can use different overflow criterion:
 - Space Utilization
 - Average Length of Overflow Chains



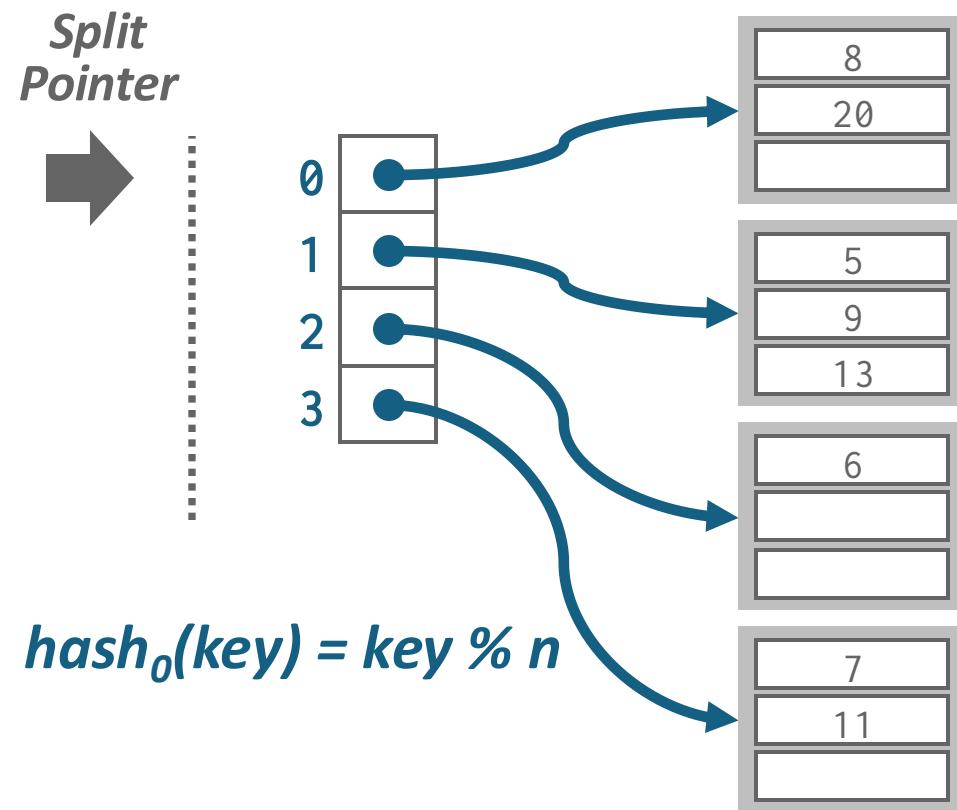
Linear Hashing



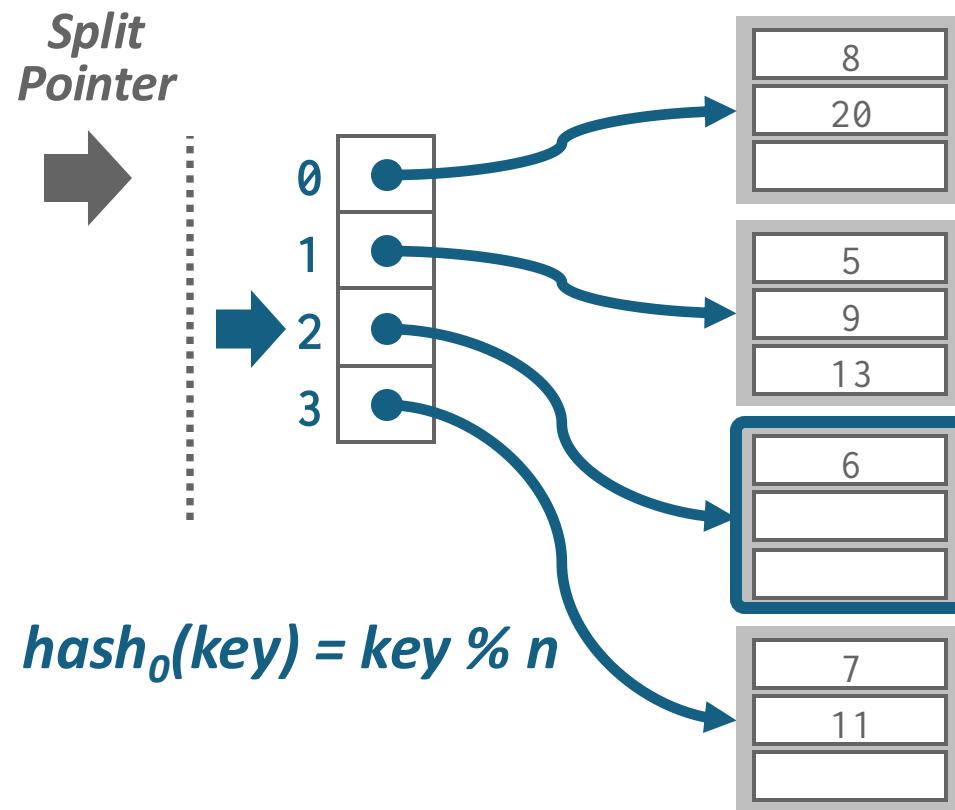
Linear Hashing



Linear Hashing

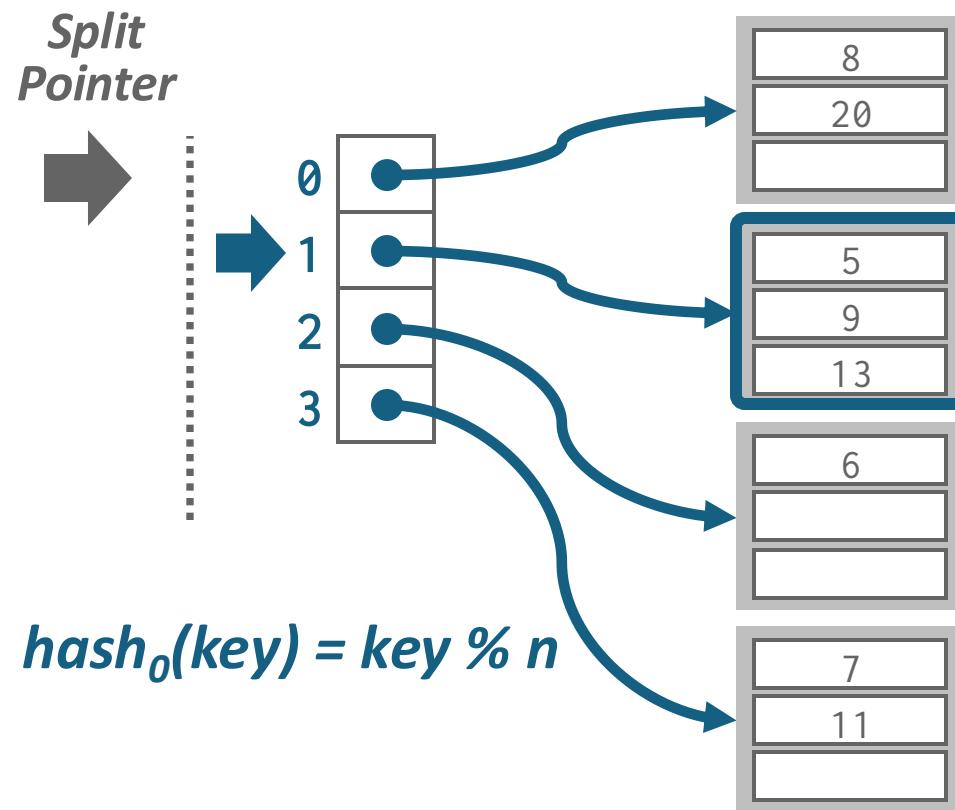


Linear Hashing



Get 6
 $hash_0(6) = 6 \% 4 = 2$

Linear Hashing



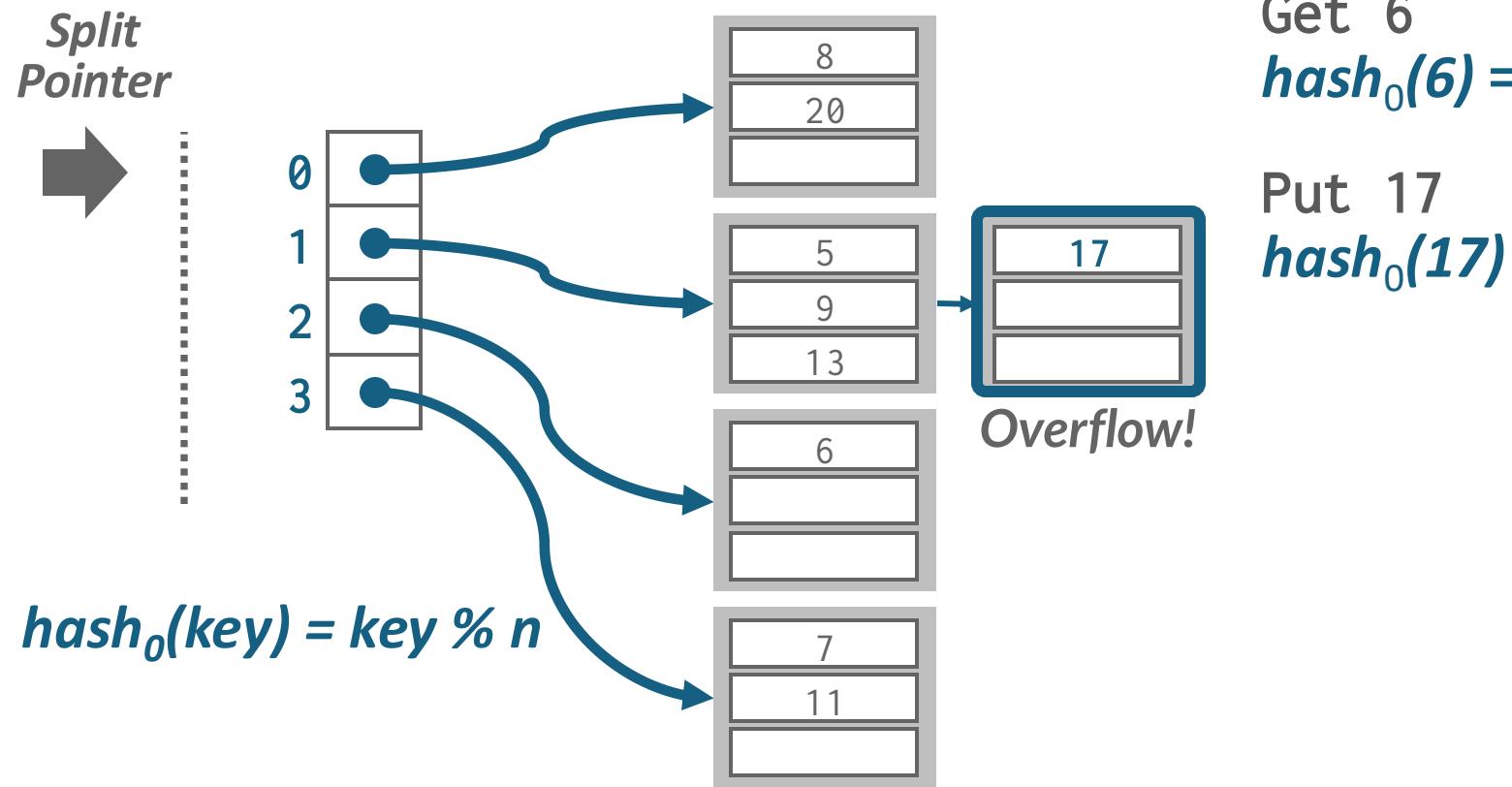
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

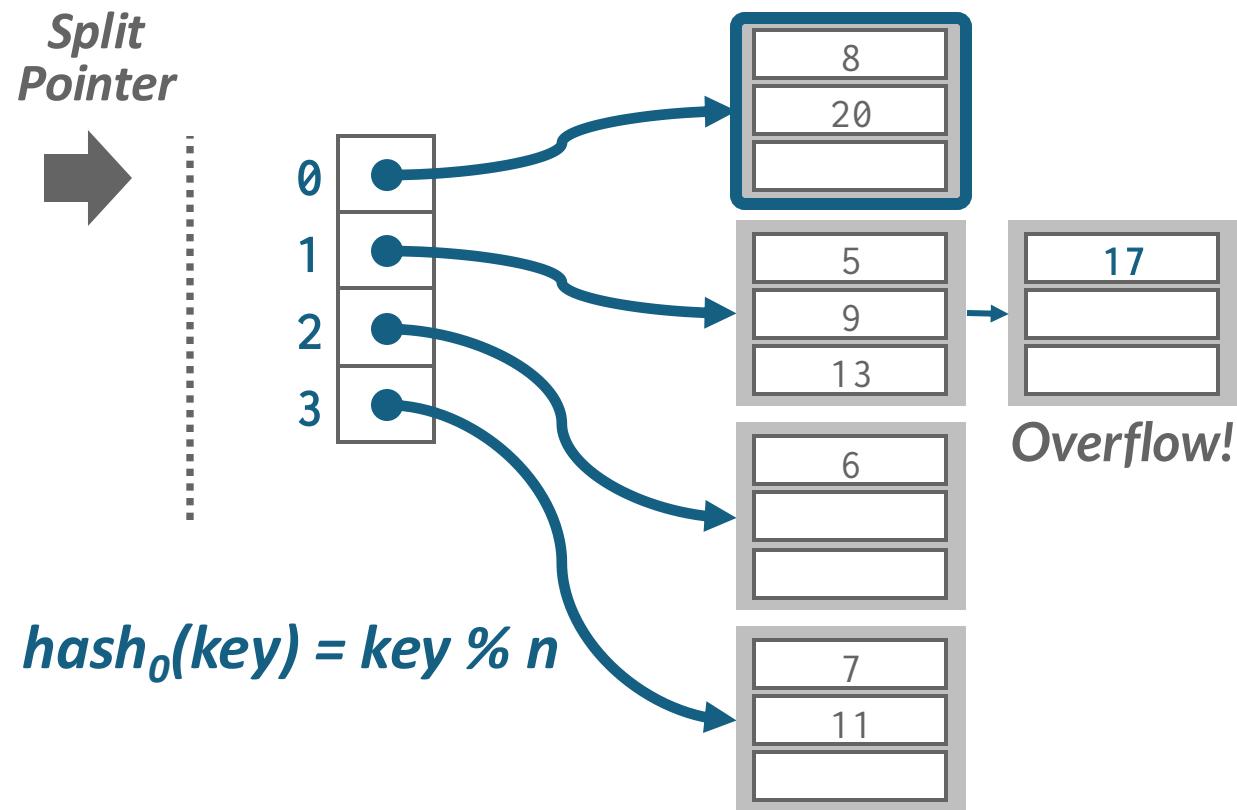
Put 17

$$hash_0(17) = 17 \% 4 = 1$$

Linear Hashing



Linear Hashing



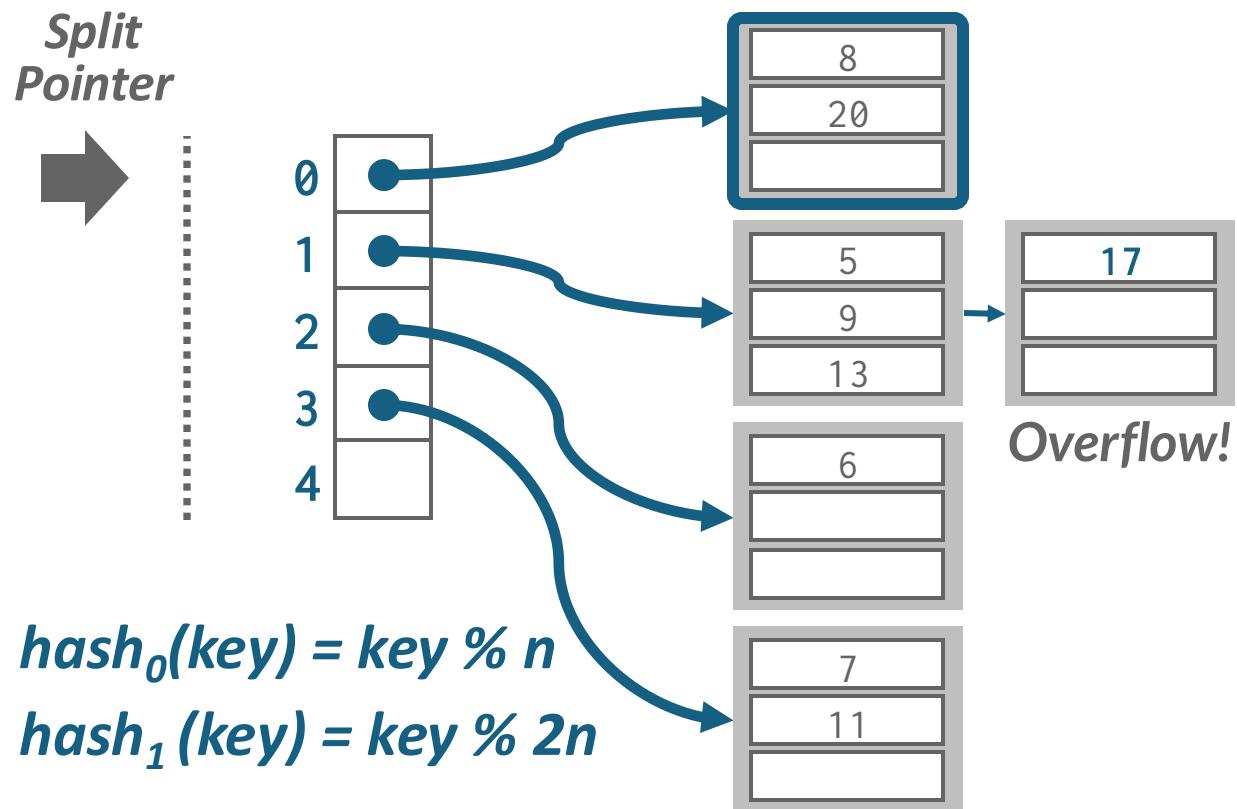
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

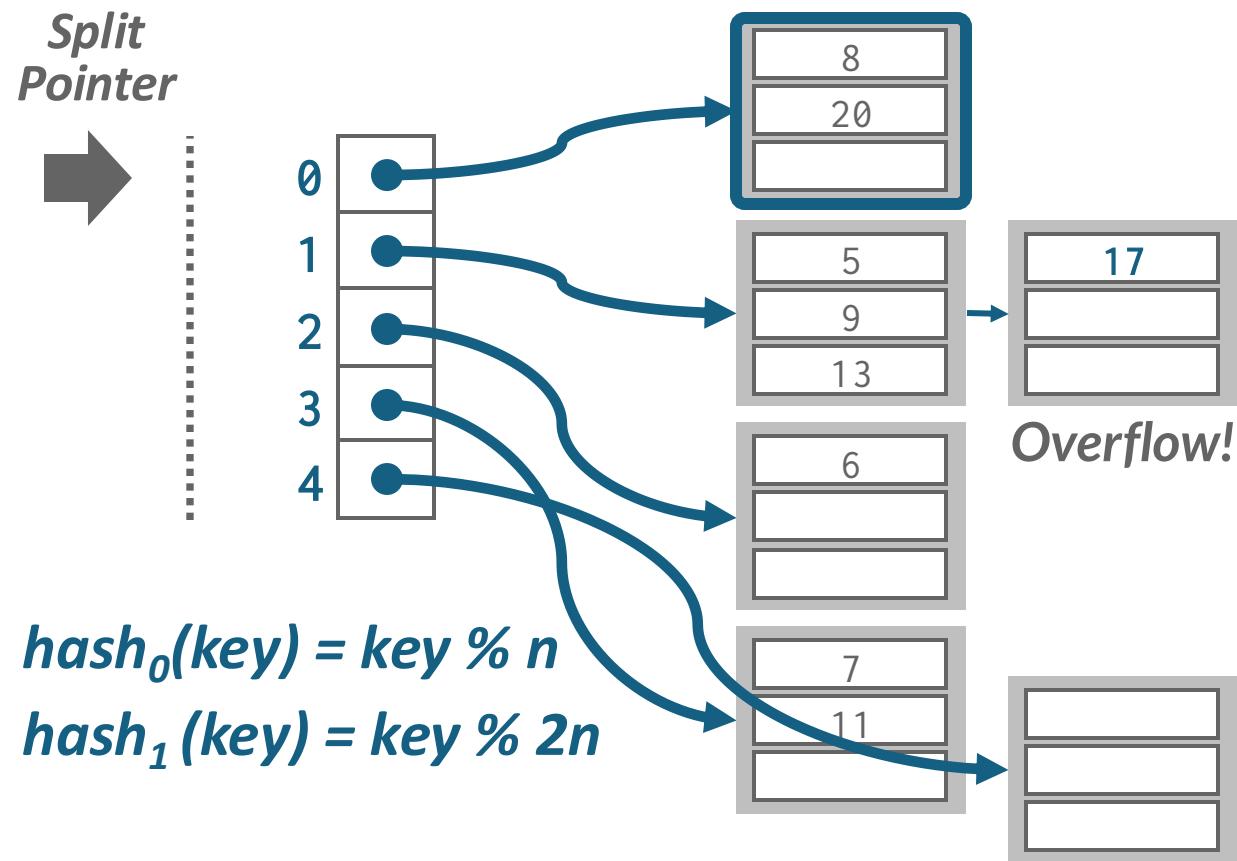
Linear Hashing



Get 6
 $hash_0(6) = 6 \% 4 = 2$

Put 17
 $hash_0(17) = 17 \% 4 = 1$

Linear Hashing

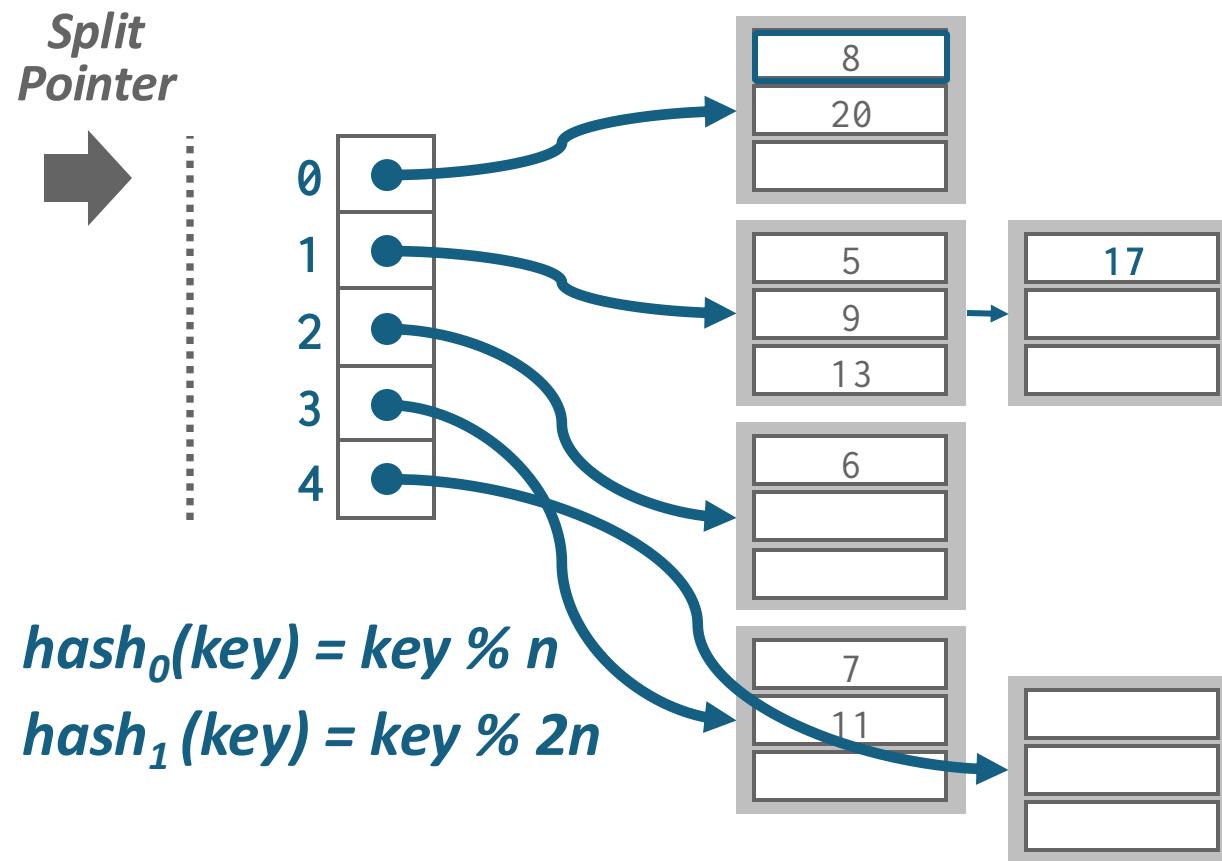


Get 6
 $\text{hash}_0(6) = 6 \% 4 = 2$

Put 17
 $\text{hash}_0(17) = 17 \% 4 = 1$

Overflow!

Linear Hashing



Get 6

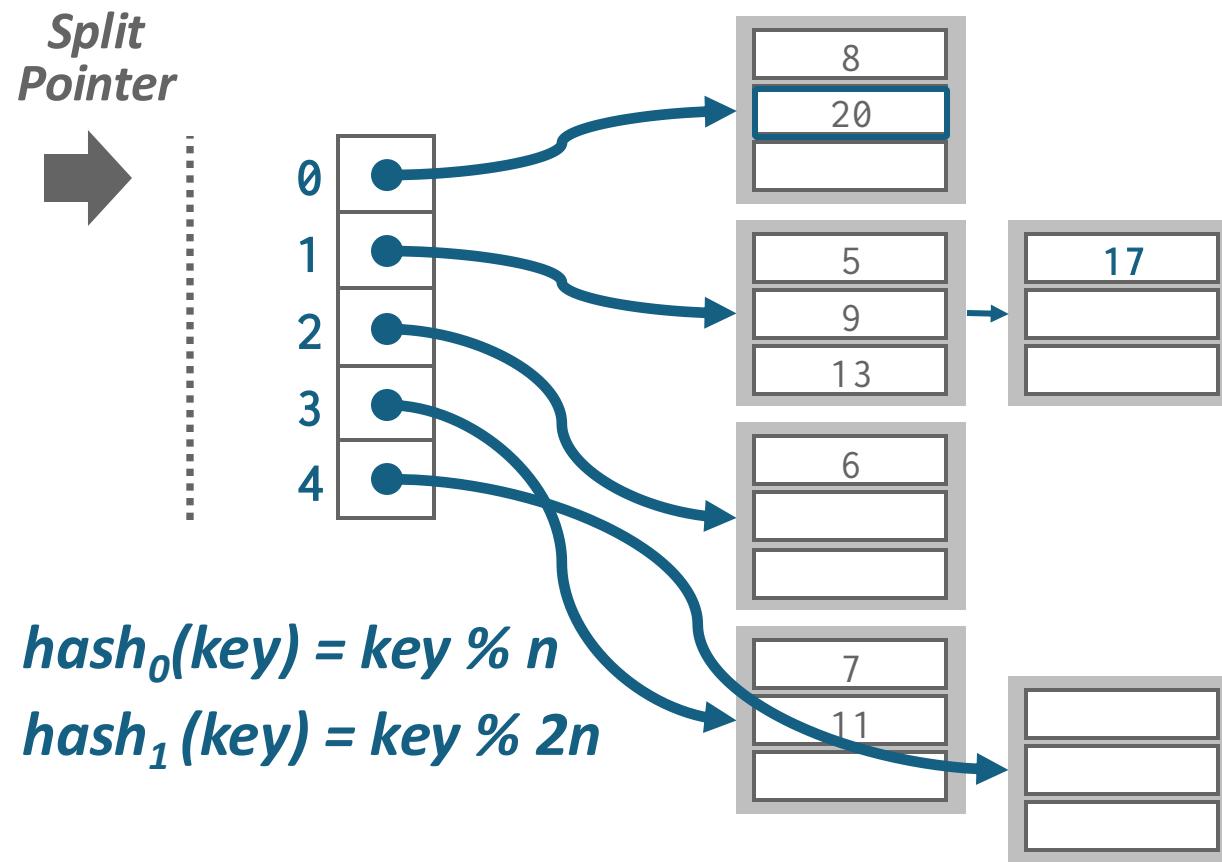
$$\text{hash}_0(6) = 6 \% 4 = 2$$

Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

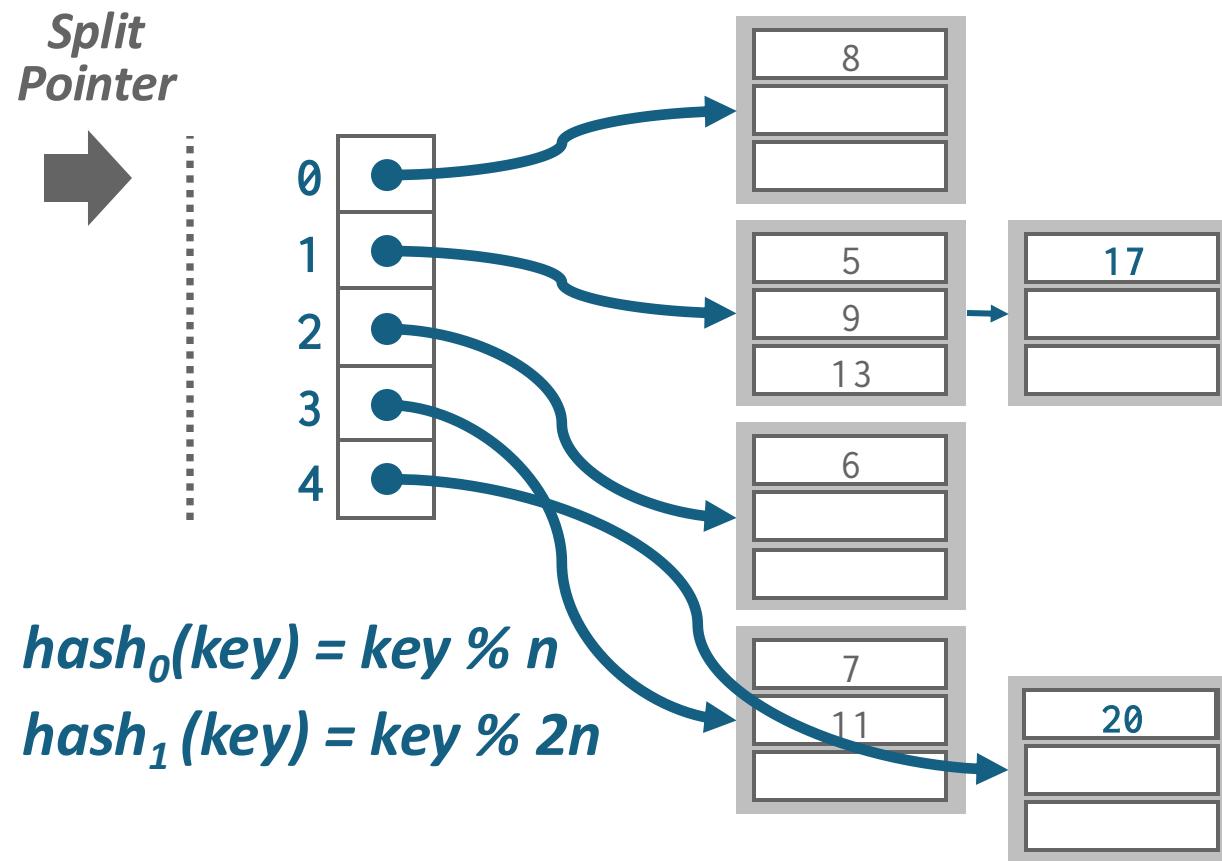
Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

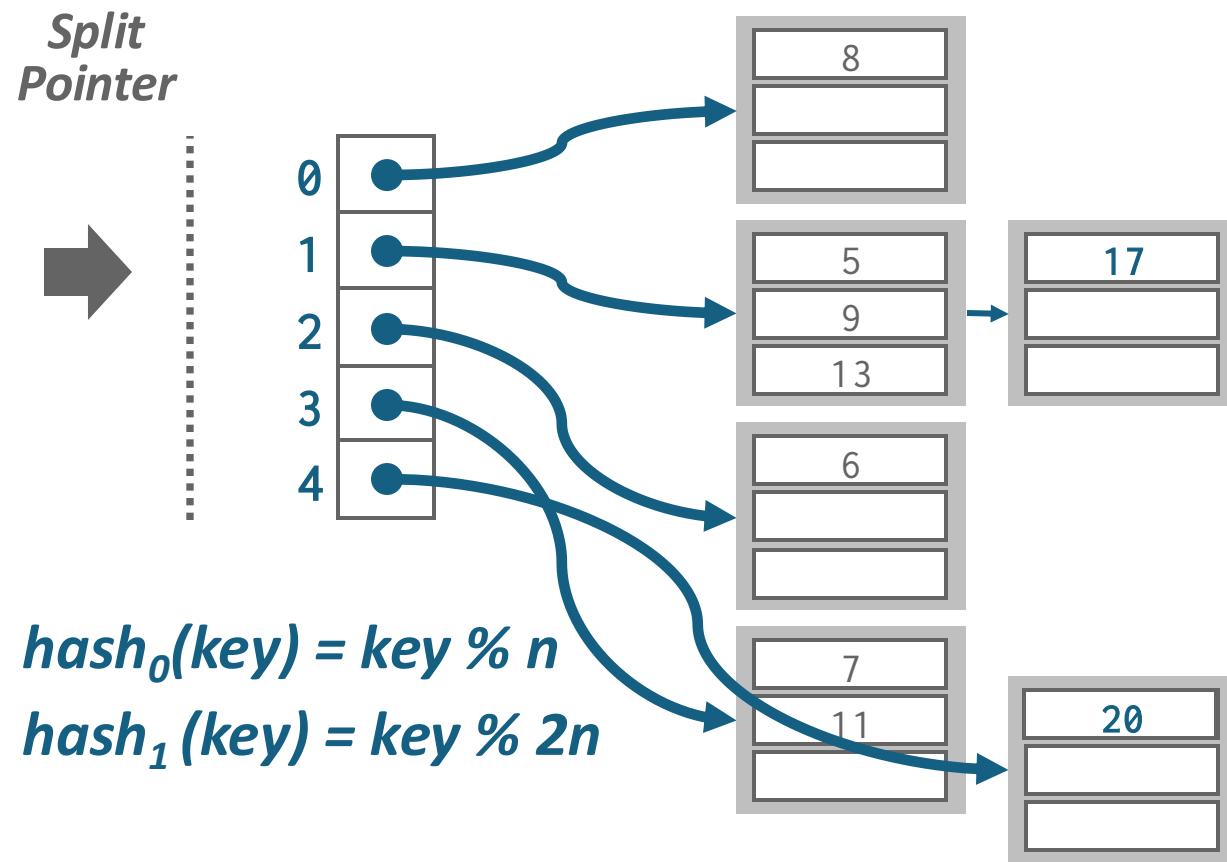
Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

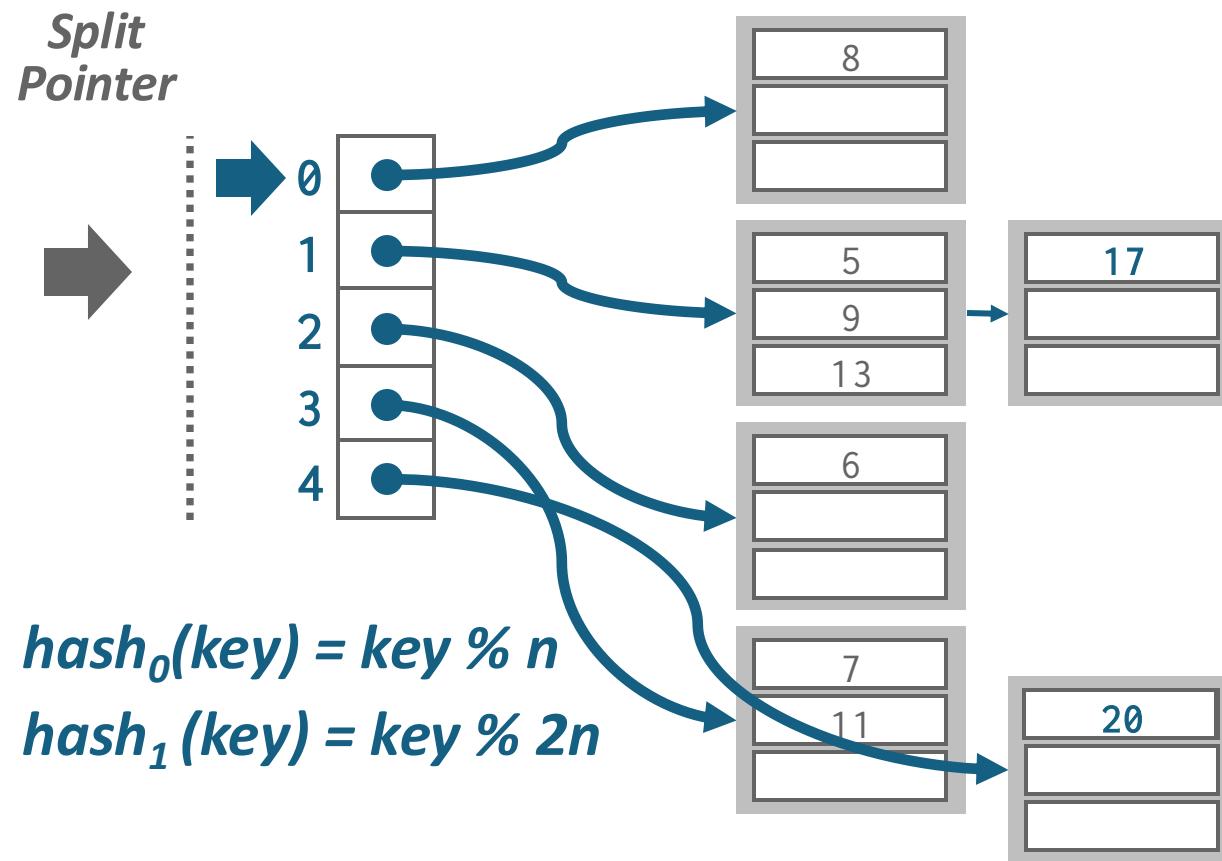
Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

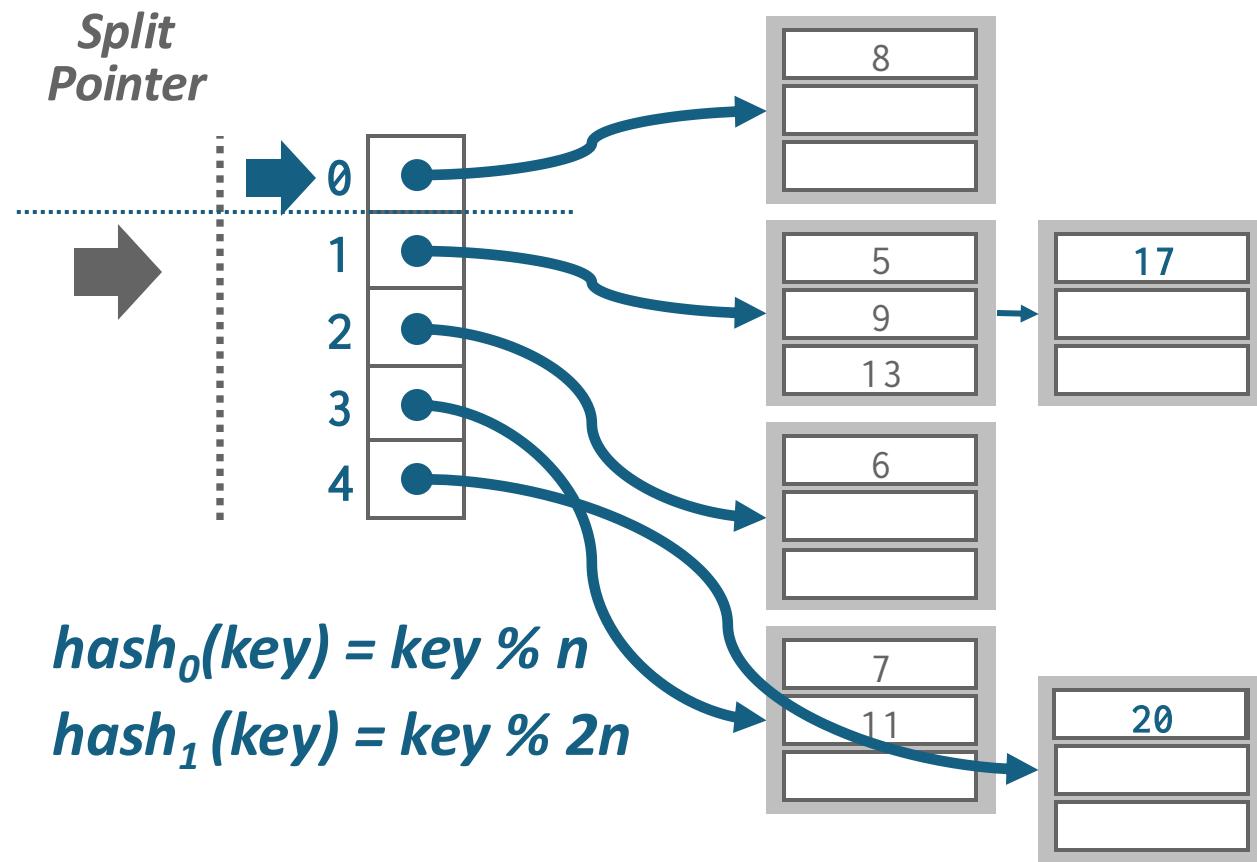
$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Get 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

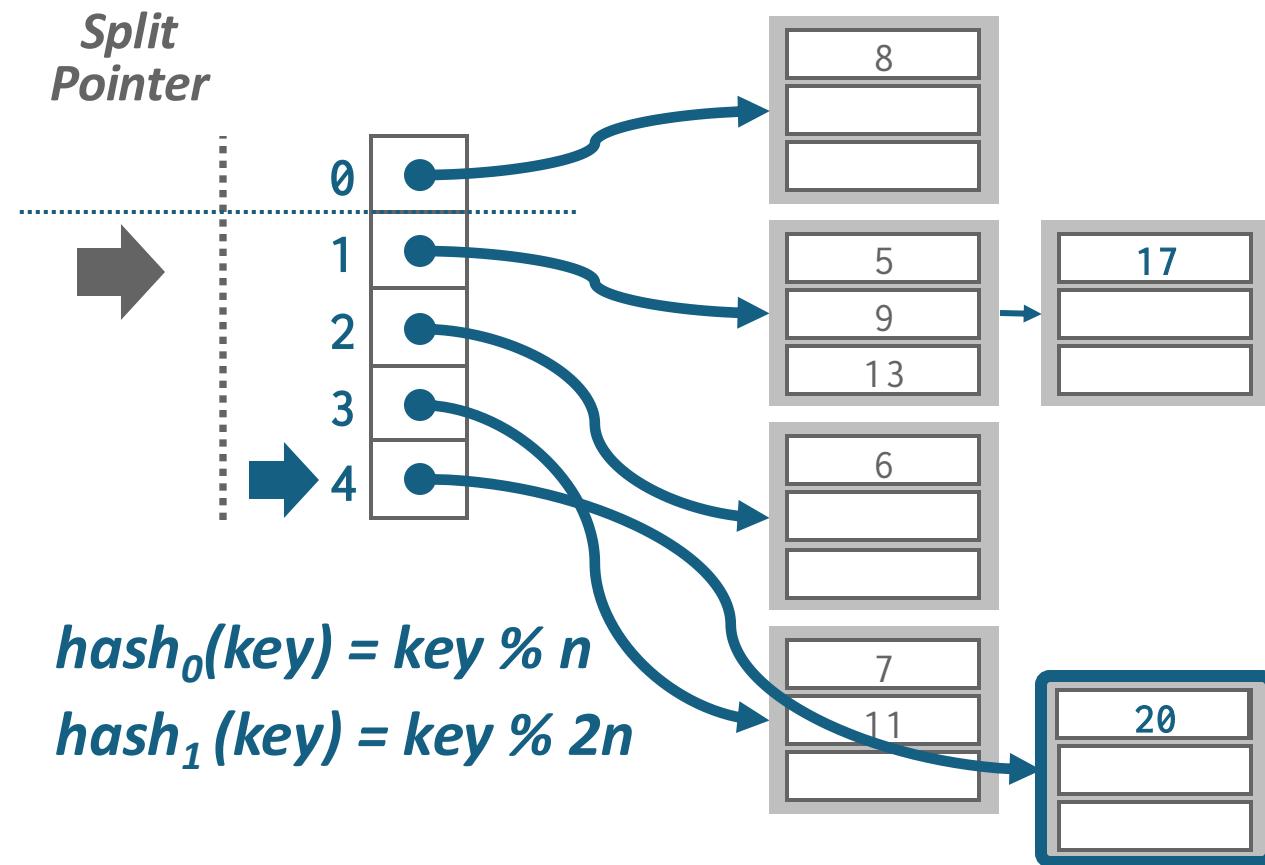
$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Get 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

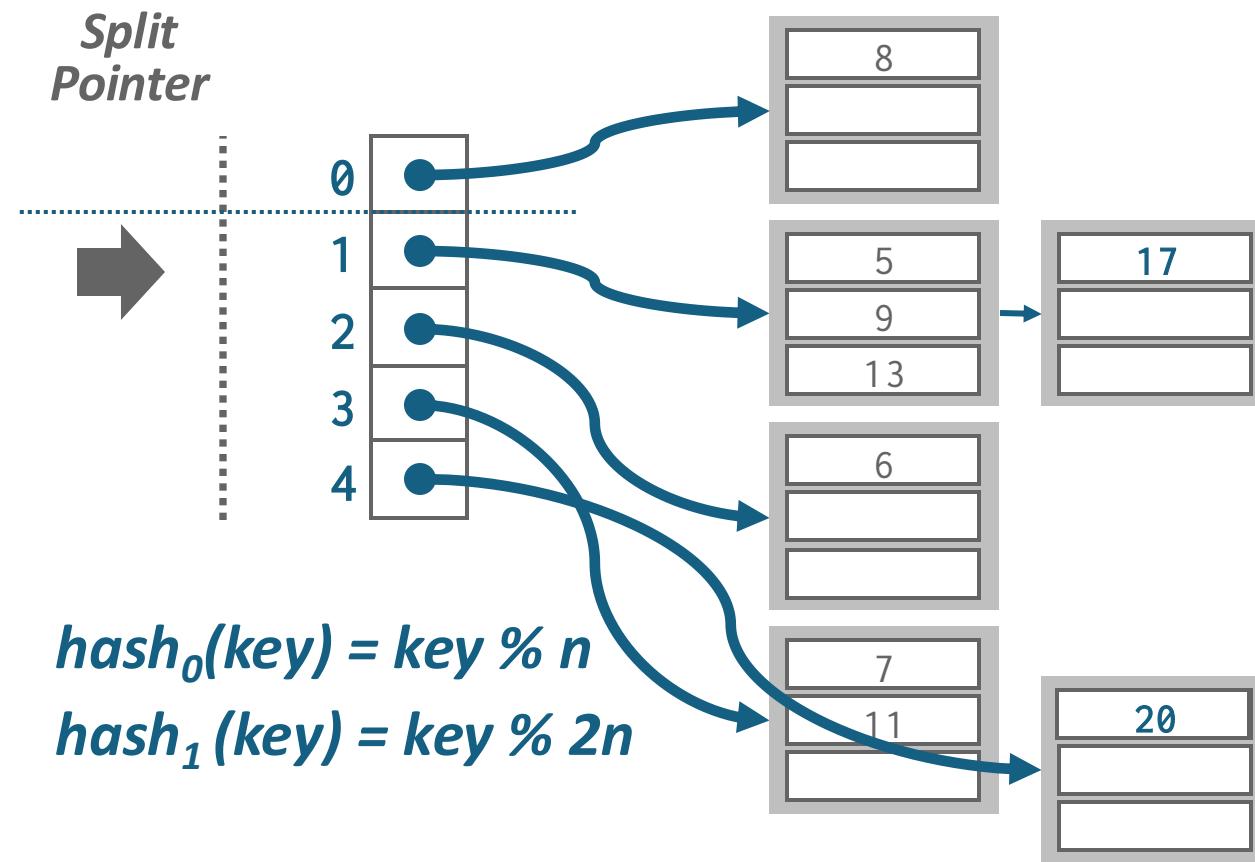
$$\text{hash}_1(20) = 20 \% 8 = 4$$

Get 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Get 20

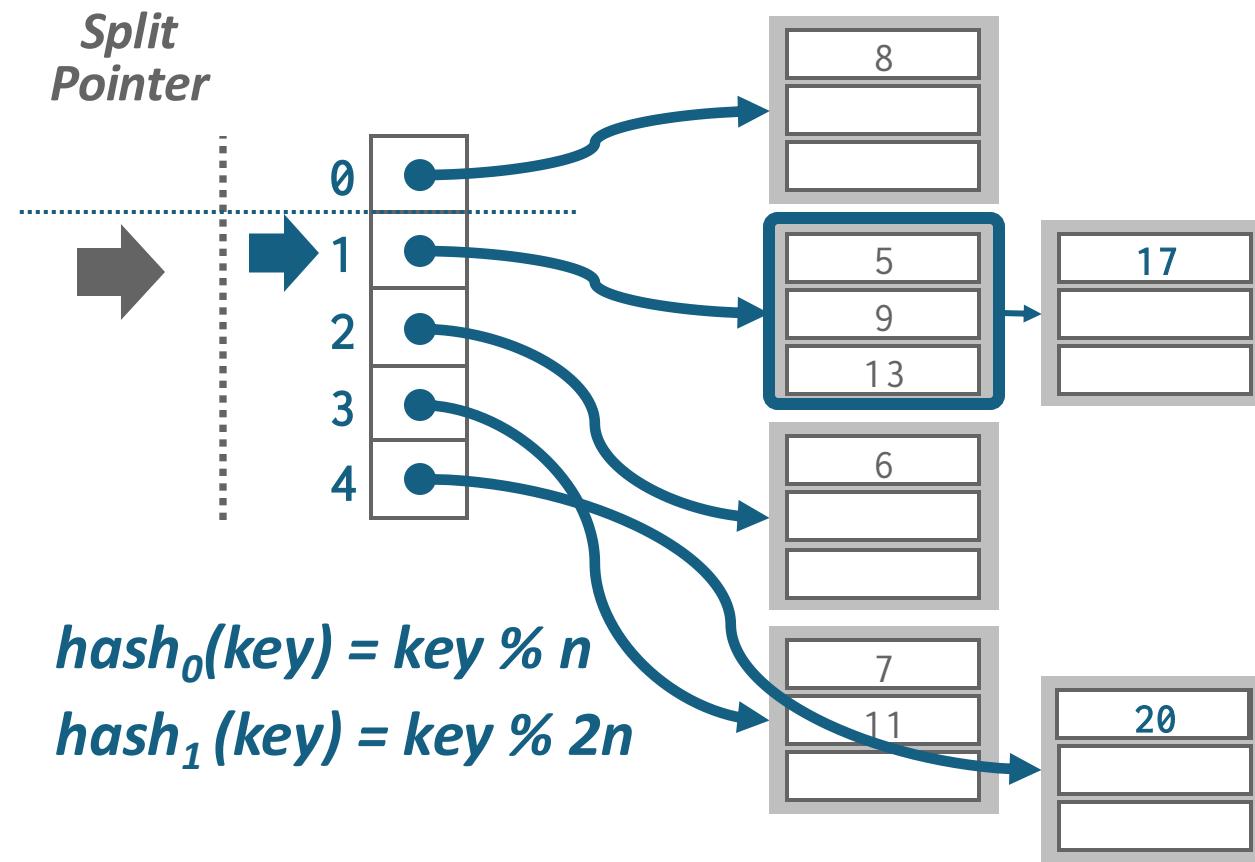
$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Get 9

$$\text{hash}_0(9) = 9 \% 4 = 1$$

Linear Hashing



Get 6

$$\text{hash}_0(6) = 6 \% 4 = 2$$

Put 17

$$\text{hash}_0(17) = 17 \% 4 = 1$$

$$\text{hash}_1(8) = 8 \% 8 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Get 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

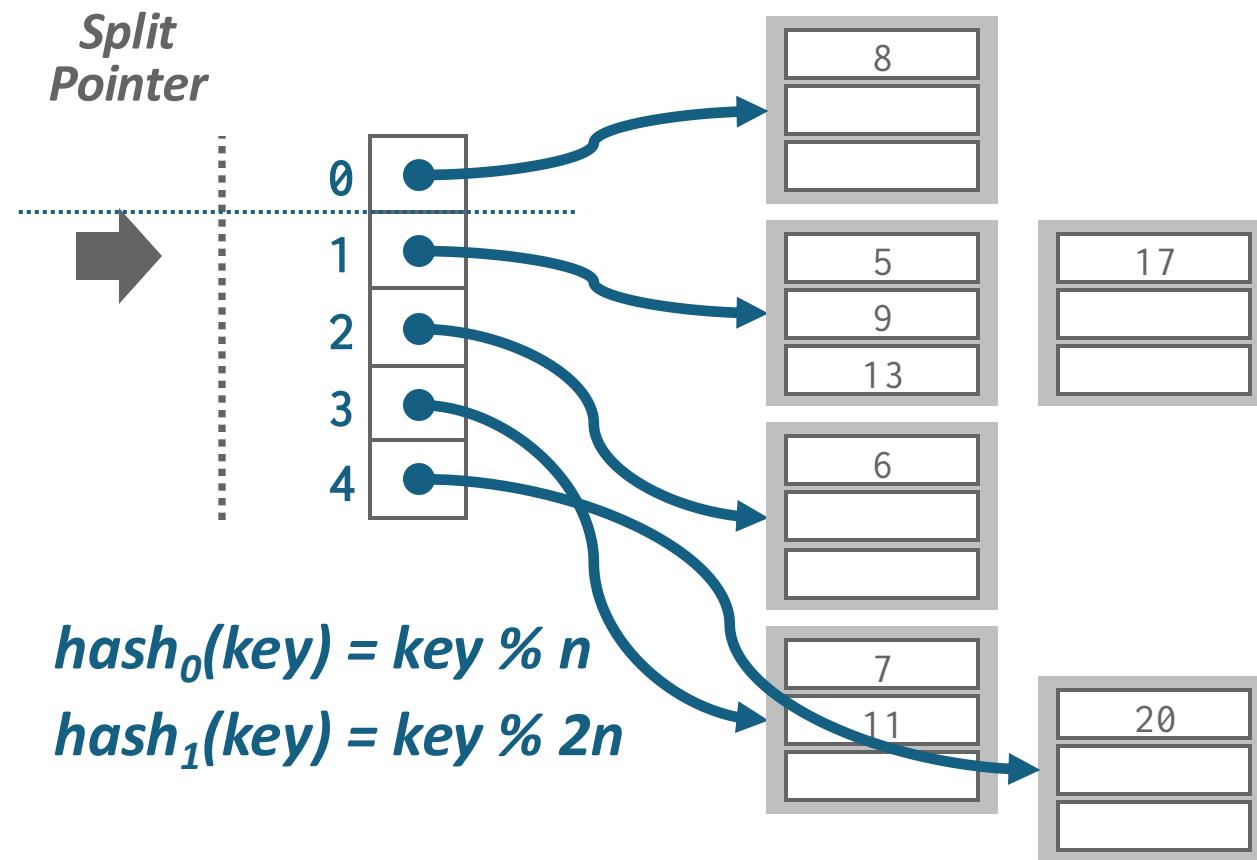
Get 9

$$\text{hash}_0(9) = 9 \% 4 = 1$$

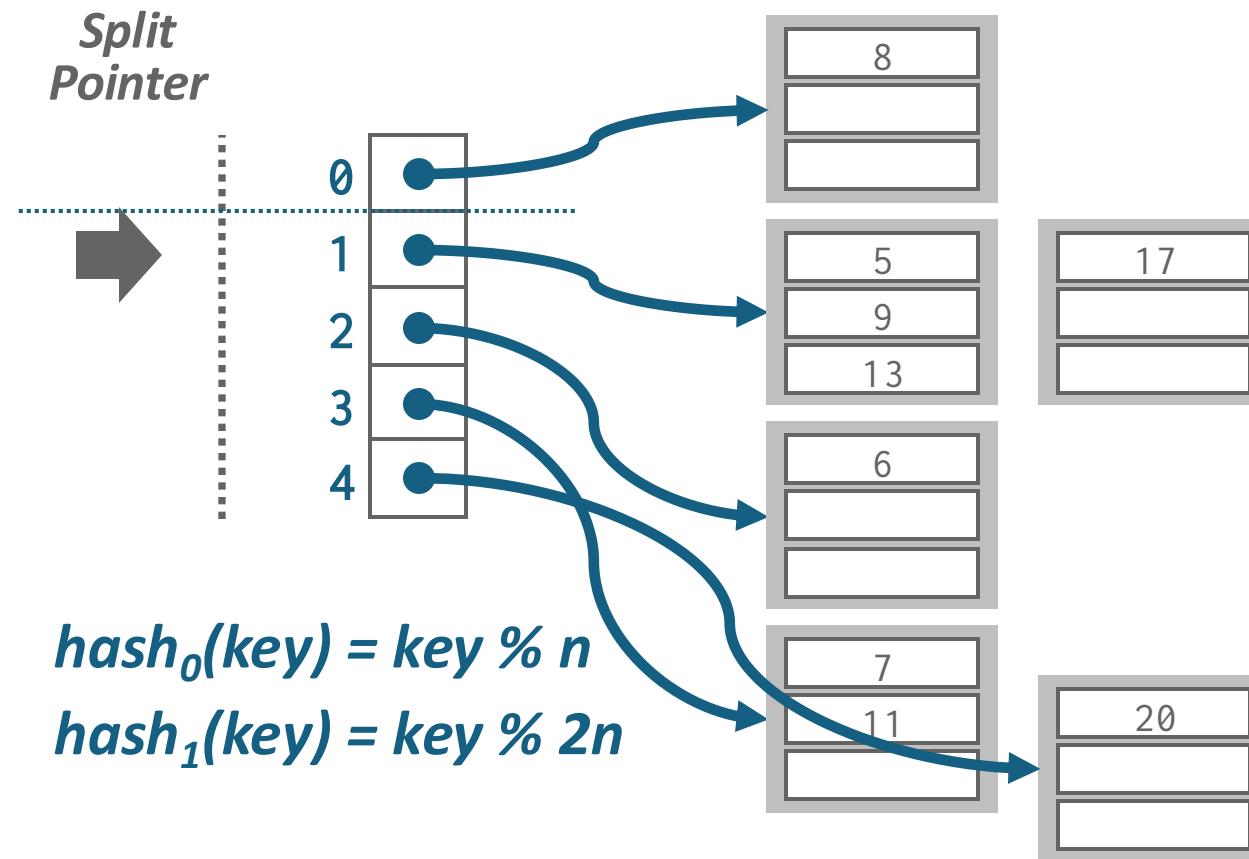
Linear Hashing - Resizing

- Splitting buckets based on the split pointer will eventually get to all overflowed buckets.
 - When the pointer reaches the last slot, remove the first hash function and move pointer back to beginning.
- If the “highest” bucket below the split pointer is empty, the hash table could remove it and move the splinter pointer in reverse direction.

Linear Hashing - Deletes

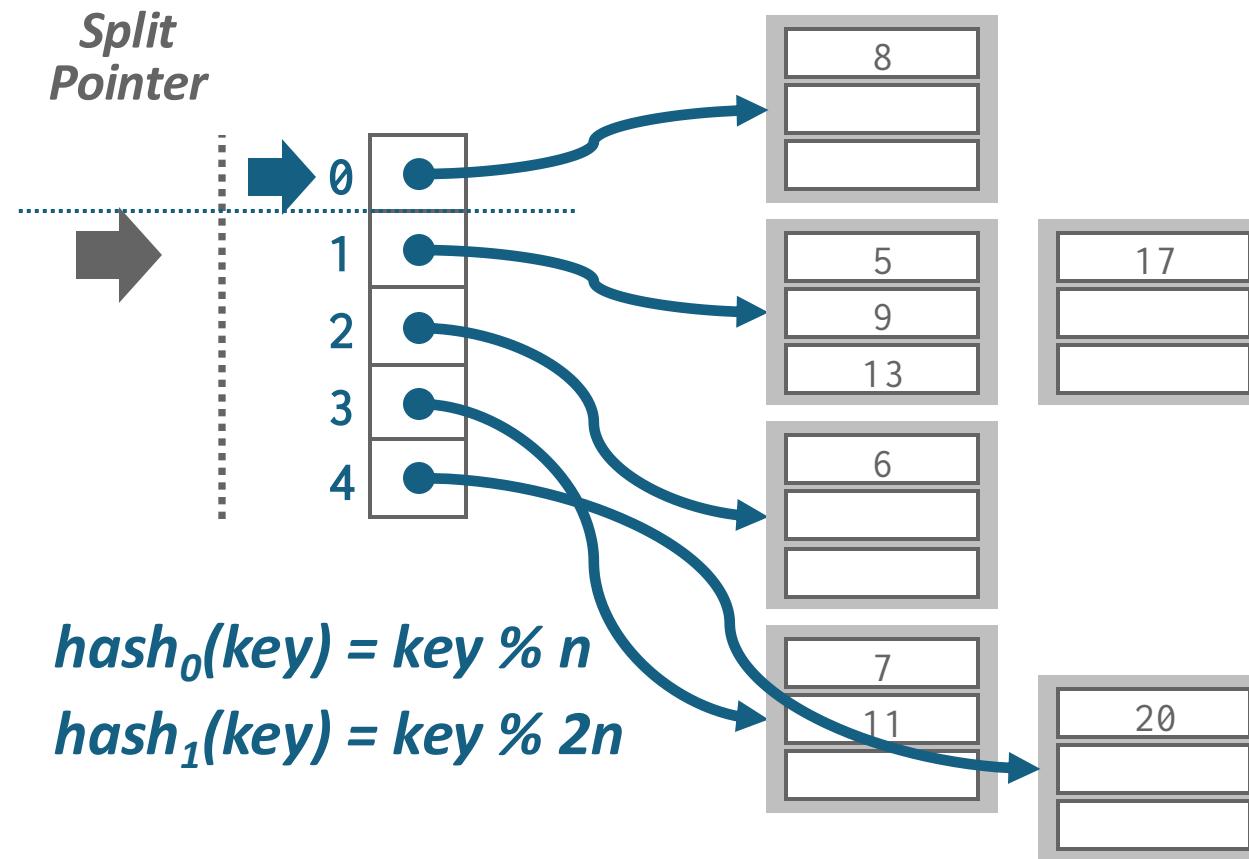


Linear Hashing - Deletes



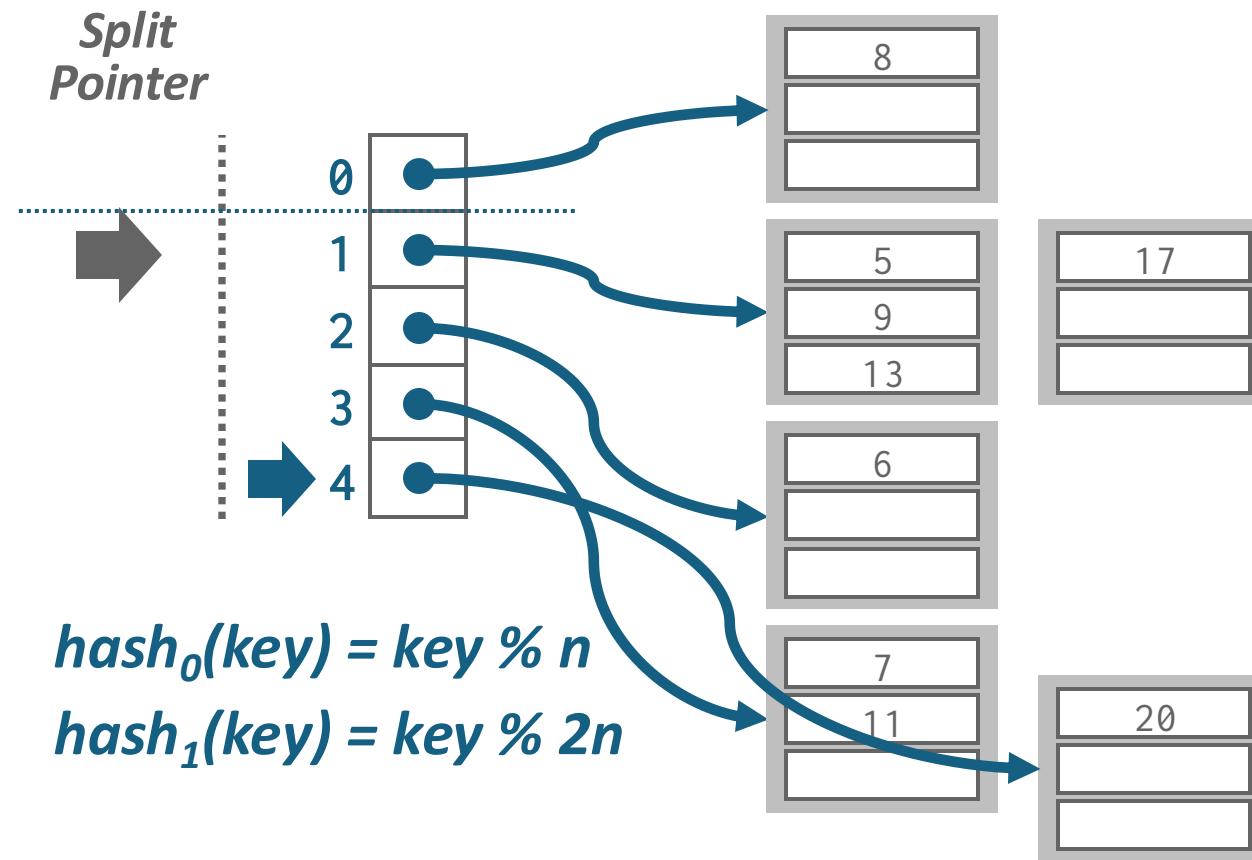
Delete 20
 $hash_0(20) = 20 \% 4 = 0$

Linear Hashing - Deletes



Delete 20
 $\text{hash}_0(20) = 20 \% 4 = 0$

Linear Hashing - Deletes

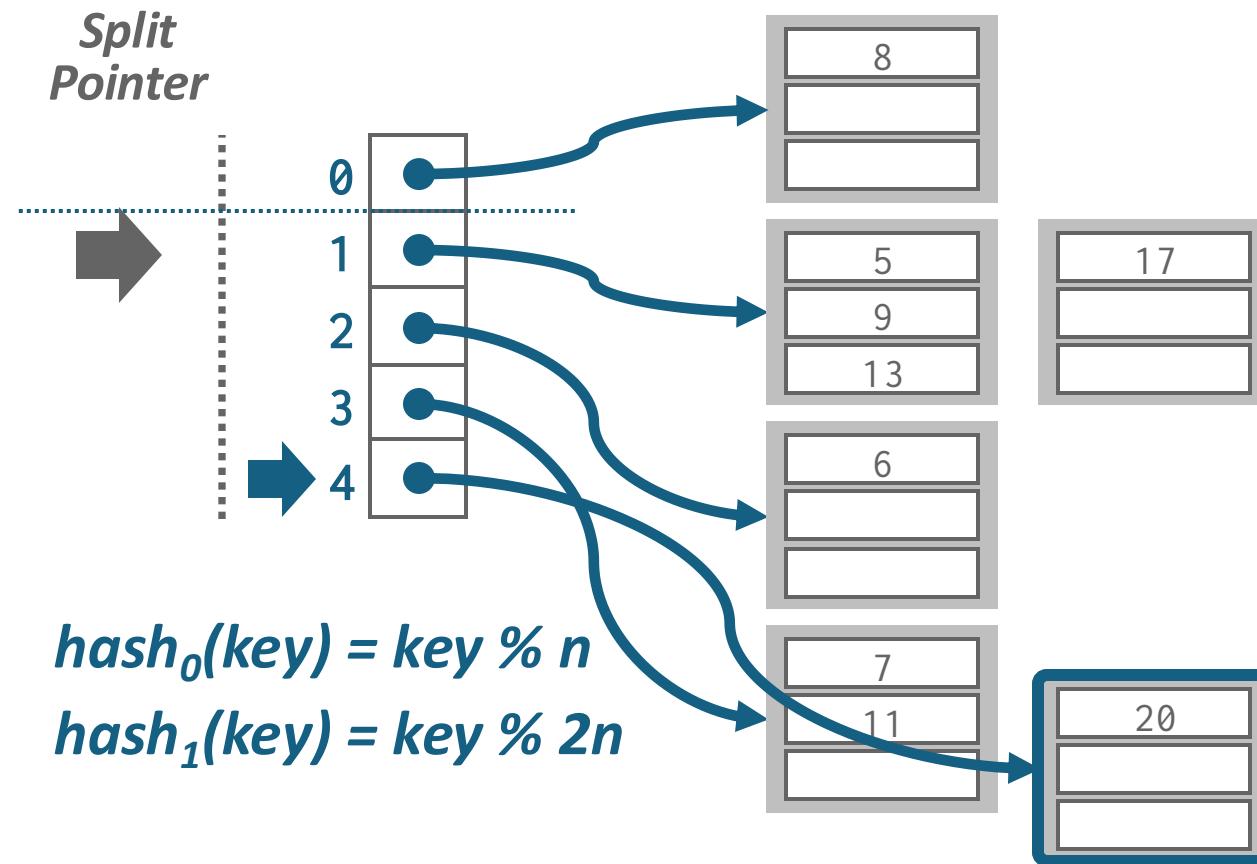


Delete 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

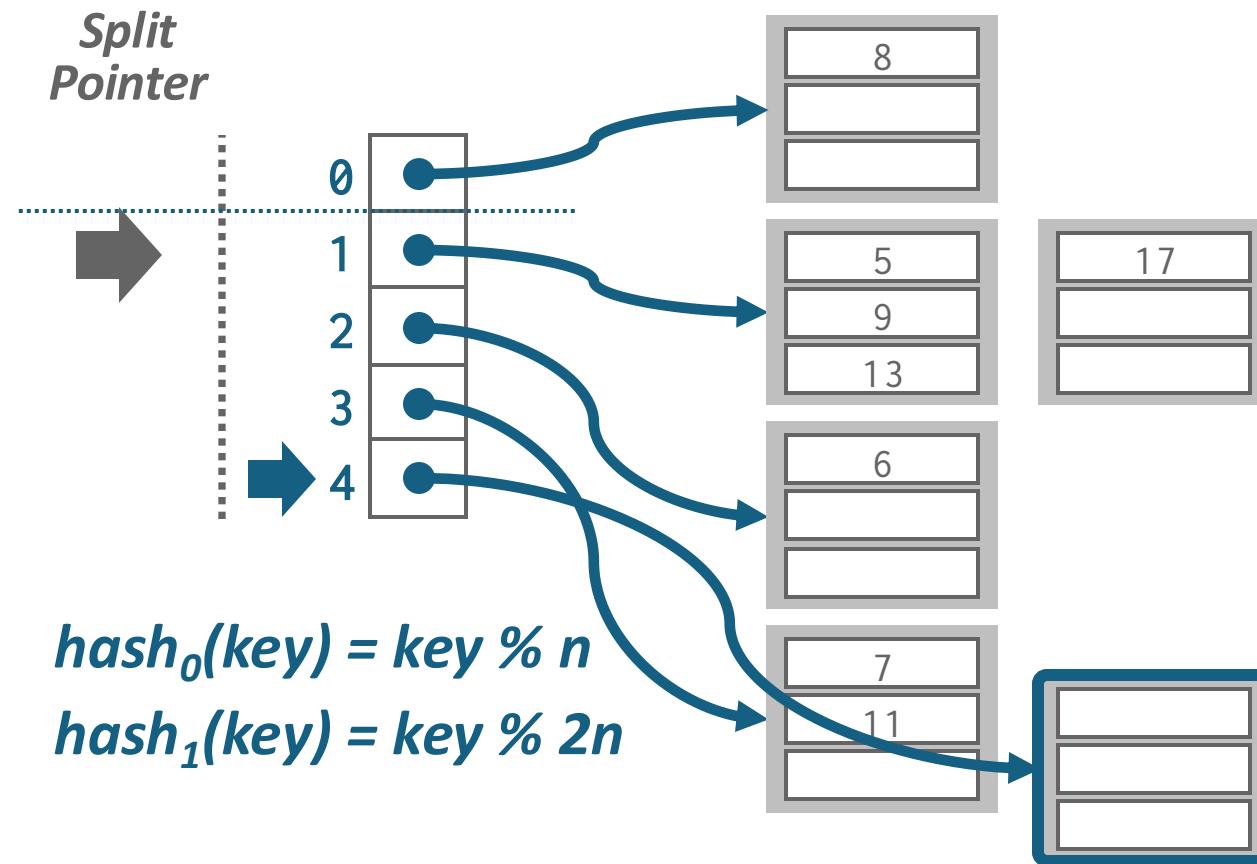


Delete 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

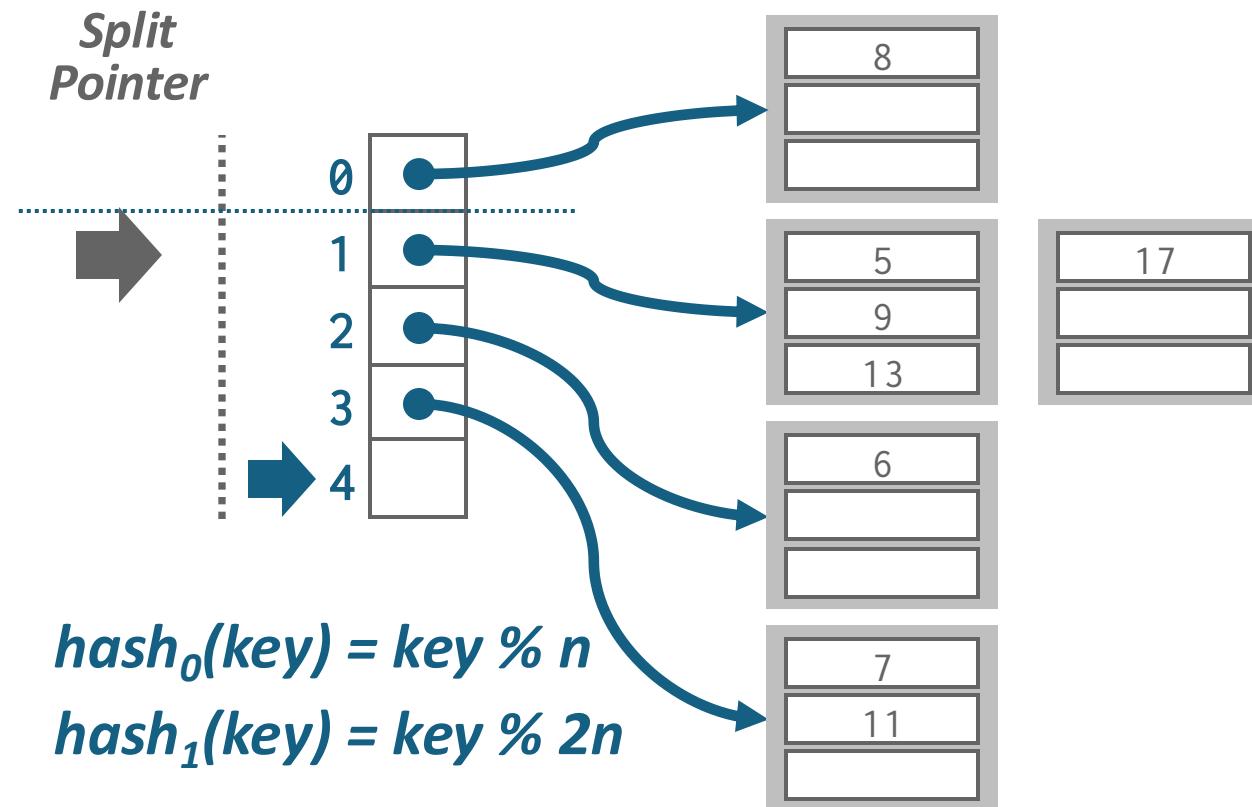


Delete 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

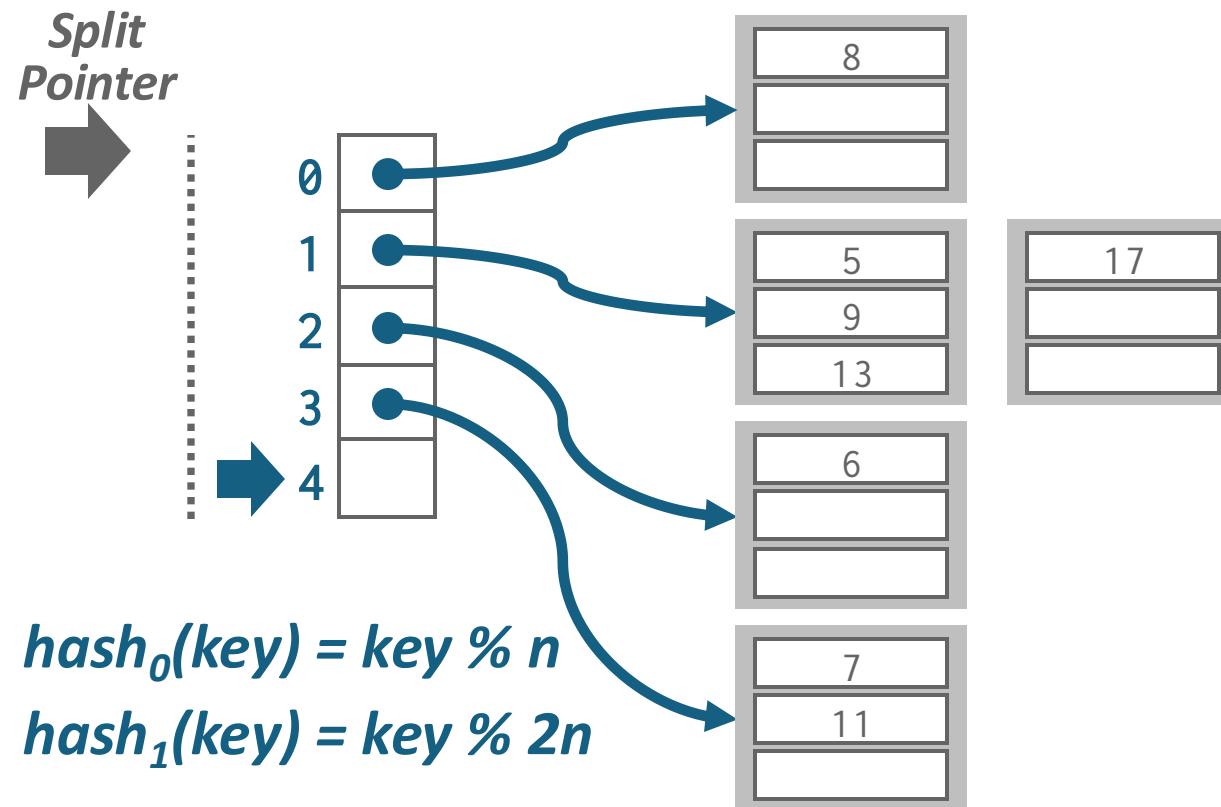


Delete 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

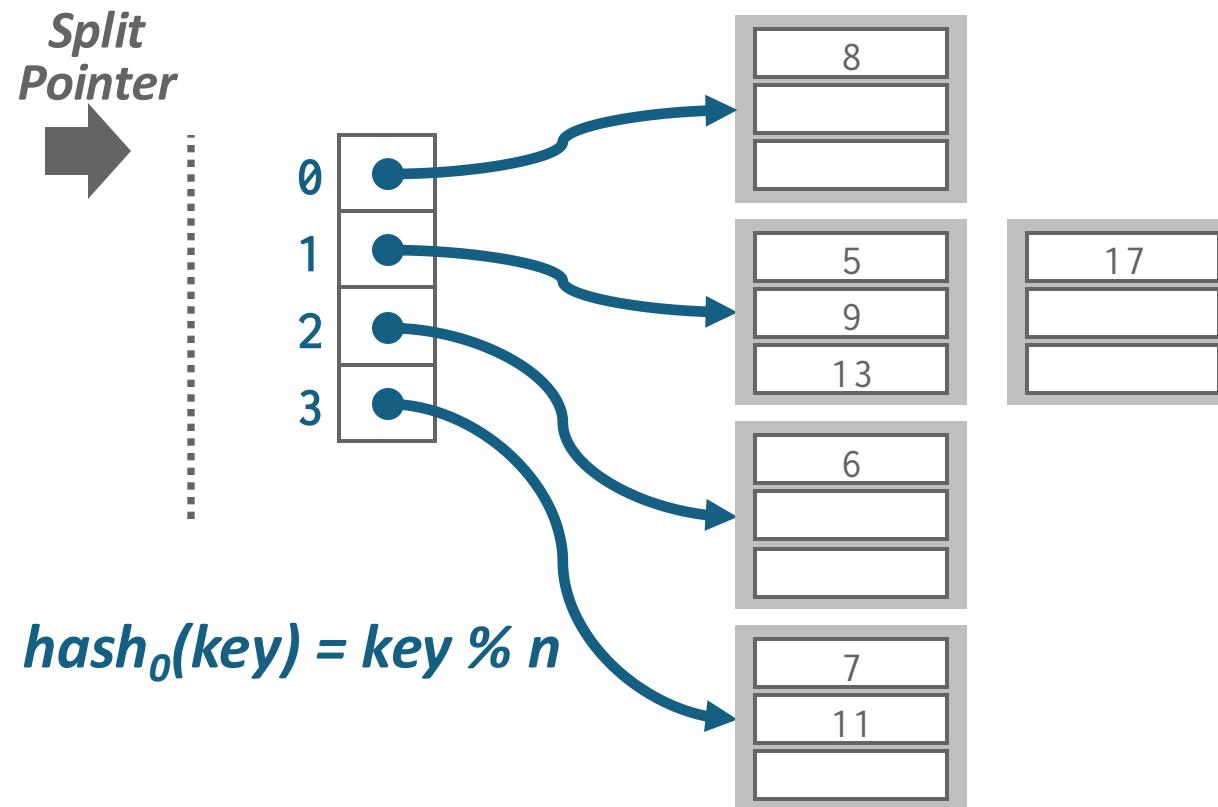


Delete 20

$$\text{hash}_0(20) = 20 \% 4 = 0$$

$$\text{hash}_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

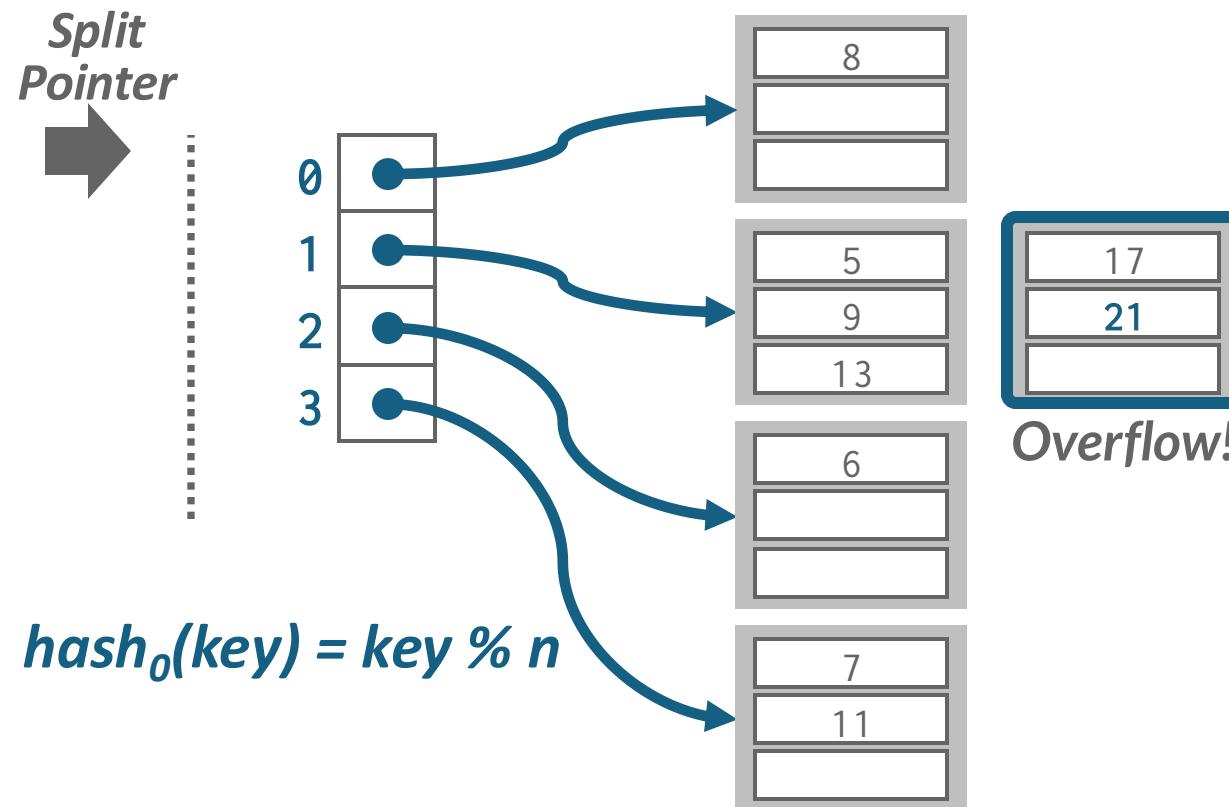


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes



Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Put 21

$$hash_0(21) = 21 \% 4 = 1$$

Conclusion

- Fast data structures that support $O(1)$ look-ups that are used all throughout DBMS internals.
 - Trade-off between speed and flexibility.
- Hashing schemes get used for both in-memory and on-disk.
 - Linear Probing, Cuckoo Hashing, Chained Hashing: Generally used for in-memory hash tables; e.g., in hash join and aggregate operators.
 - Extendible Hashing, Linear Hashing: for disk-based hash indexing.

Next Lecture

- **B+Trees**
 - aka “The Greatest Data Structure of All Time”