



香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen



Ack: Prof. Jignesh Patel @ CMU
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CSC3170

7: Hash Tables

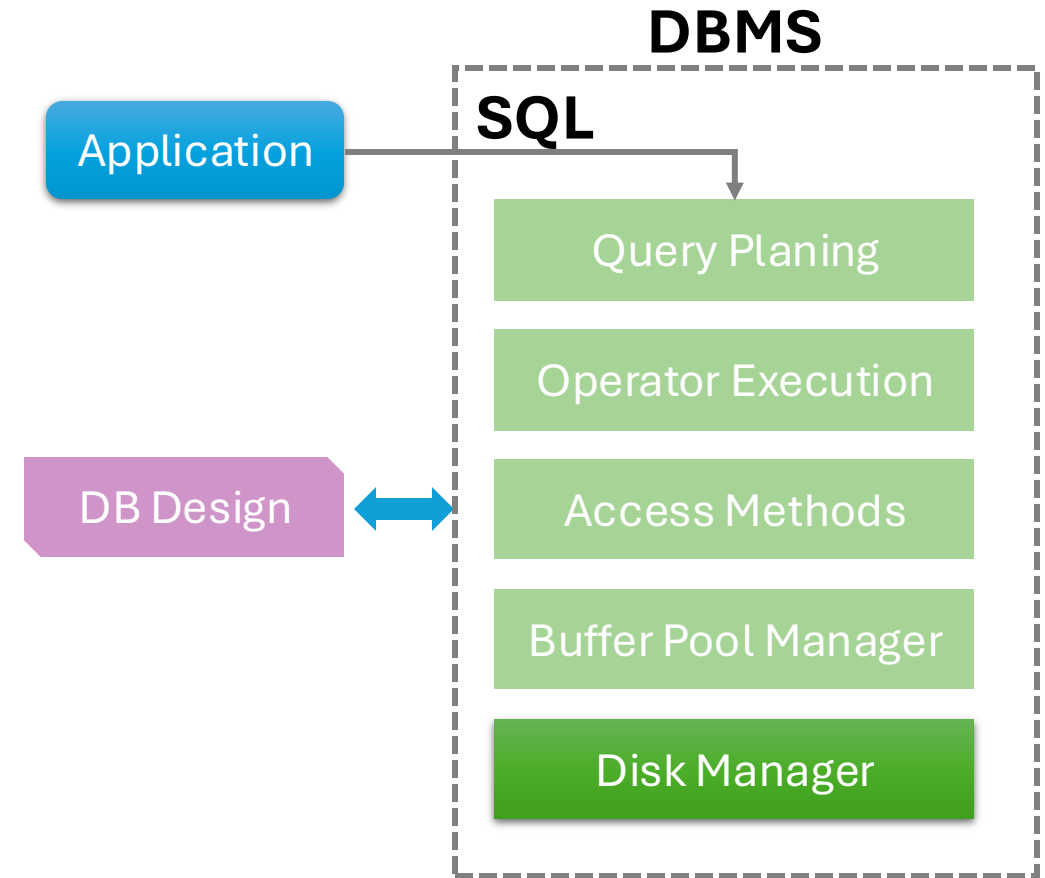
Chenhao Ma

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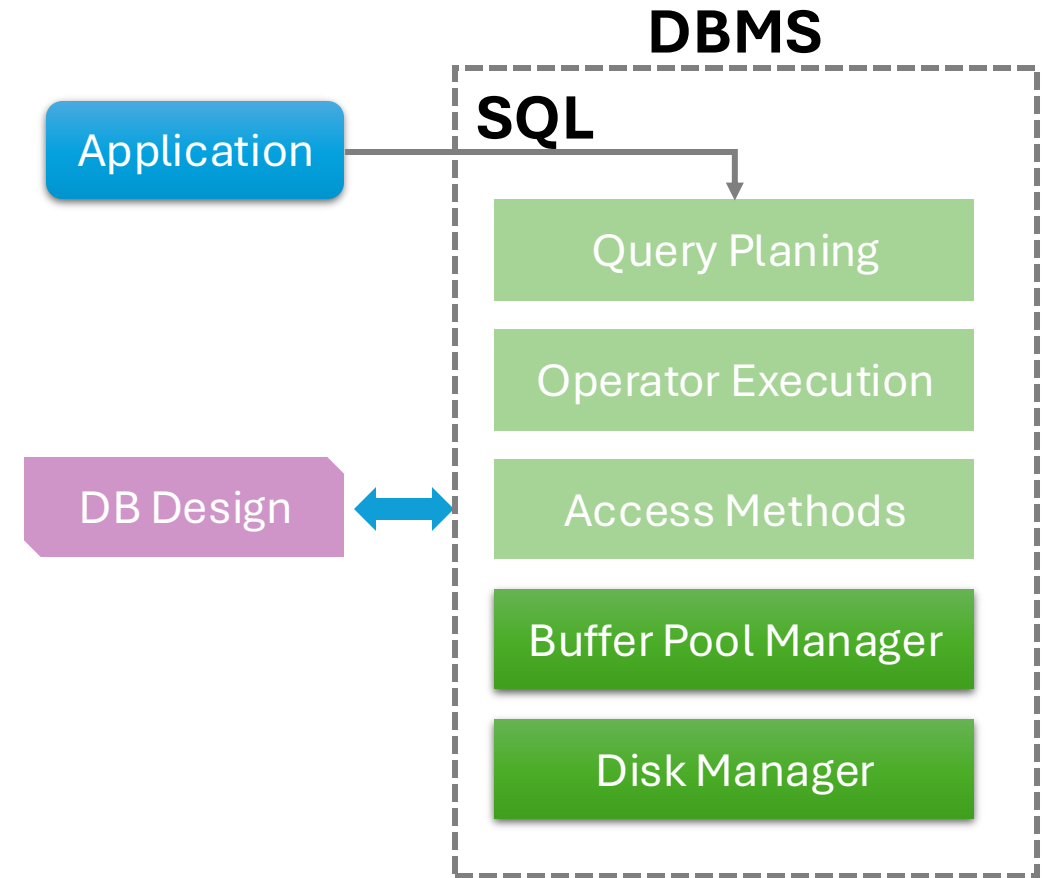
Course Status

- We are now going to talk about how to support the DBMS's execution engine to read/write data from pages.
- Two types of data structures:
 - Hash Tables (Unordered)
 - Trees (Ordered)



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Data Structures

- Internal Meta-data
- Core Data Storage
- Temporary Data Structures
- Table Indexes

Design Decisions

- **Data Organization**

- How we layout data structure in memory/pages and what information to store to support efficient access.

- **Concurrency**

- How to enable multiple threads to access the data structure at the same time without causing problems.

Hash Tables

- A **hash table** implements an unordered associative array that maps keys to values.
- It uses a **hash function** to compute an offset into this array for a given key, from which the desired value can be found.
- Space Complexity: $O(n)$
Time Complexity:
 - Average: $O(1)$
 - Worst: $O(n)$

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← *Databases care about constants!*

Static Hash Table

- Allocate a giant array that has one slot for every element you need to store.
- To find an entry, mod the key by the number of elements to find the offset in the array.

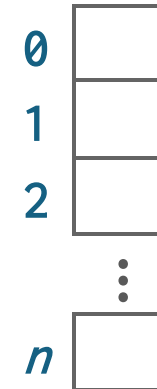
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Static Hash Table

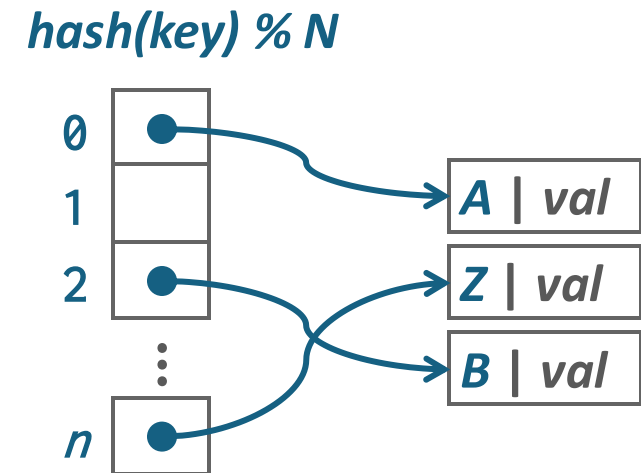
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$hash(key) \% N$

0	A
1	∅
2	B
⋮	
n	Z

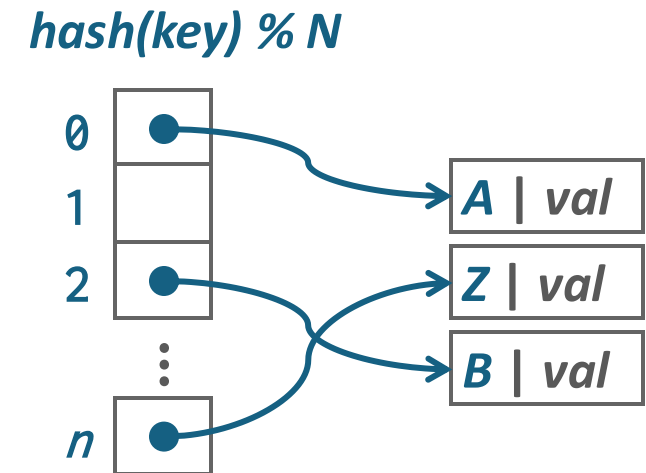
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Unrealistic Assumptions

- **Assumption #1:** Number of elements is known ahead of time and fixed.
- **Assumption #2:** Each key is unique.
- **Assumption #3:** Perfect hash function guarantees no collisions.
 - If $\text{key1} \neq \text{key2}$, then $\text{hash}(\text{key1}) \neq \text{hash}(\text{key2})$



Hash Table

- **Design Decision #1: Hash Function**

- How to map a large key space into a smaller domain.
- Trade-off between being fast vs. collision rate.

- **Design Decision #2: Hashing Scheme**

- How to handle key collisions after hashing.
- Trade-off between allocating a large hash table vs. additional instructions to get/put keys.

This Lecture

- Hash Functions
- Static Hashing Schemes
- Dynamic Hashing Schemes

Hash Functions

Hash Functions

- For any input key, return an integer representation of that key.
- We do not want to use a cryptographic hash function for DBMS hash tables (e.g., [SHA-2](#)).
- We want something that is fast and has a low collision rate.

Hash Functions

- **CRC-64 (1975)**
 - Used in networking for error detection.
- **MurmurHash (2008)**
 - Designed as a fast, general-purpose hash function.
- **Google CityHash (2011)**
 - Designed to be faster for short keys (<64 bytes).
- **Facebook XXHash (2012)**
 - From the creator of zstd compression.
- **Google FarmHash (2014)**
 - Newer version of CityHash with better collision rates.

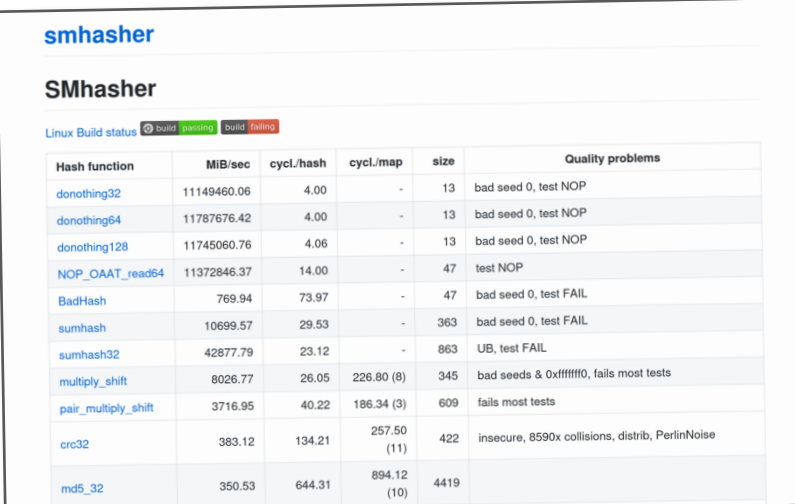
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smhasher

SMhasher

Linux Build status: build passing build testing

Hash function	MiB/sec	cycl./hash	cycl./map	size	Quality problems
donthing32	11149460.06	4.00	-	13	bad seed 0, test NOP
donthing64	11787676.42	4.00	-	13	bad seed 0, test NOP
donthing128	11745060.76	4.06	-	13	bad seed 0, test NOP
NOP_OAAT_read64	11372846.37	14.00	-	47	test NOP
BadHash	769.94	73.97	-	47	bad seed 0, test FAIL
sumhash	10699.57	29.53	-	363	bad seed 0, test FAIL
sumhash32	42877.79	23.12	-	863	UB, test FAIL
multiply_shift	8026.77	26.05	226.80 (8)	345	bad seeds & 0xffffffff0, fails most tests
pair_multiply_shift	3716.95	40.22	186.34 (3)	609	fails most tests
crc32	383.12	134.21	257.50 (11)	422	insecure, 8590x collisions, distrib, PerlinNoise
md5_32	350.53	644.31	894.12 (10)	4419	

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NOP_OAAT_read64				47	test NOP
BadHash					
sumhash					
sumhash32					
multiply_shift					
pair_multiply_shift					
crc32					
md5_32					

Summary

I added some SSE assisted hashes and fast intel/arm CRC32-C, AES and SHA HW variants. See also the old <https://github.com/aappleby/smhasher/wiki>, the improved, but unmaintained fork <https://github.com/demerphq/smhasher>, and the new improved version SMHasher3 <https://gitlab.com/wojcik/smhasher3>.

So the fastest hash functions on x86_64 without quality problems are:

- xxh3low
- wyhash
- ahash64
- t1ha2_atonce
- komihash
- FarmHash (not portable, too machine specific: 64 vs 32bit, old gcc, ...)
- halftime_hash128
- Spooky32
- pengyhash
- nmhash32
- mx3
- MUM/mir (different results on 32/64-bit archs, lots of bad seeds to filter out)
- fasthash32

Static Hashing Schemes

- **Approach #1: Linear Probe Hashing**
- **Approach #2: Cuckoo Hashing**
- There are several other schemes (not covered in this course):
 - Robin Hood Hashing
 - Hopscotch Hashing
 - Swiss Tables

Static Hashing Schemes

- **Approach #1: Linear Probe Hashing**
- **Approach #2: Cuckoo Hashing**

Both are members of a broader class called “**Open Addressing**”.



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 - Hopscotch Hashing
 - Swiss Tables

Linear Probe Hashing

Static Hashing Schemes

Linear Probe Hashing

- Single giant table of slots.
- Resolve collisions by linearly searching for the next free slot in the table.
 - To determine whether an element is present, hash to a location in the index and scan for it.
 - Must store the key in the index to know when to stop scanning.
 - Insertions and deletions are generalizations of lookups.
- **Example:** Google's [absl::flat_hash_map](#)

Linear Probe Hashing

hash(key) % N

A

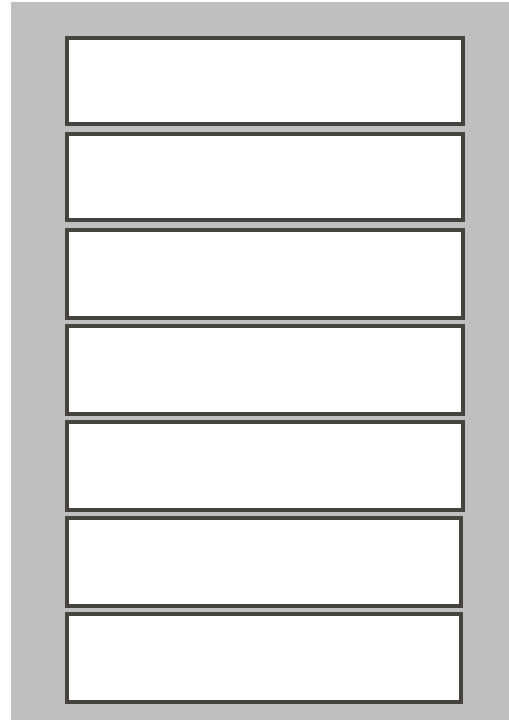
B

C

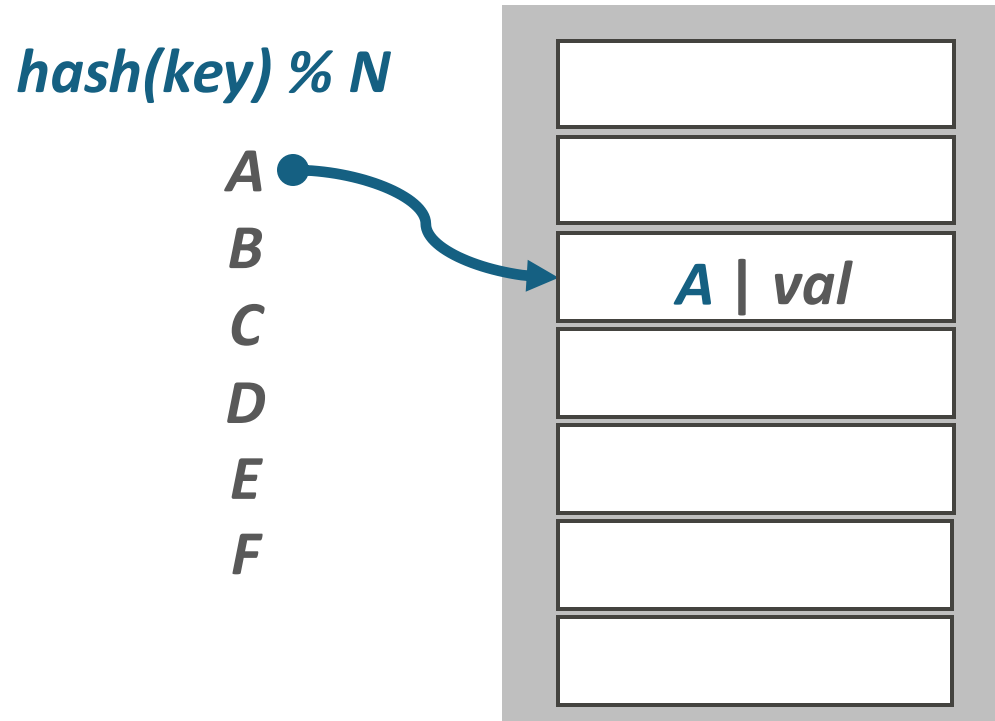
D

E

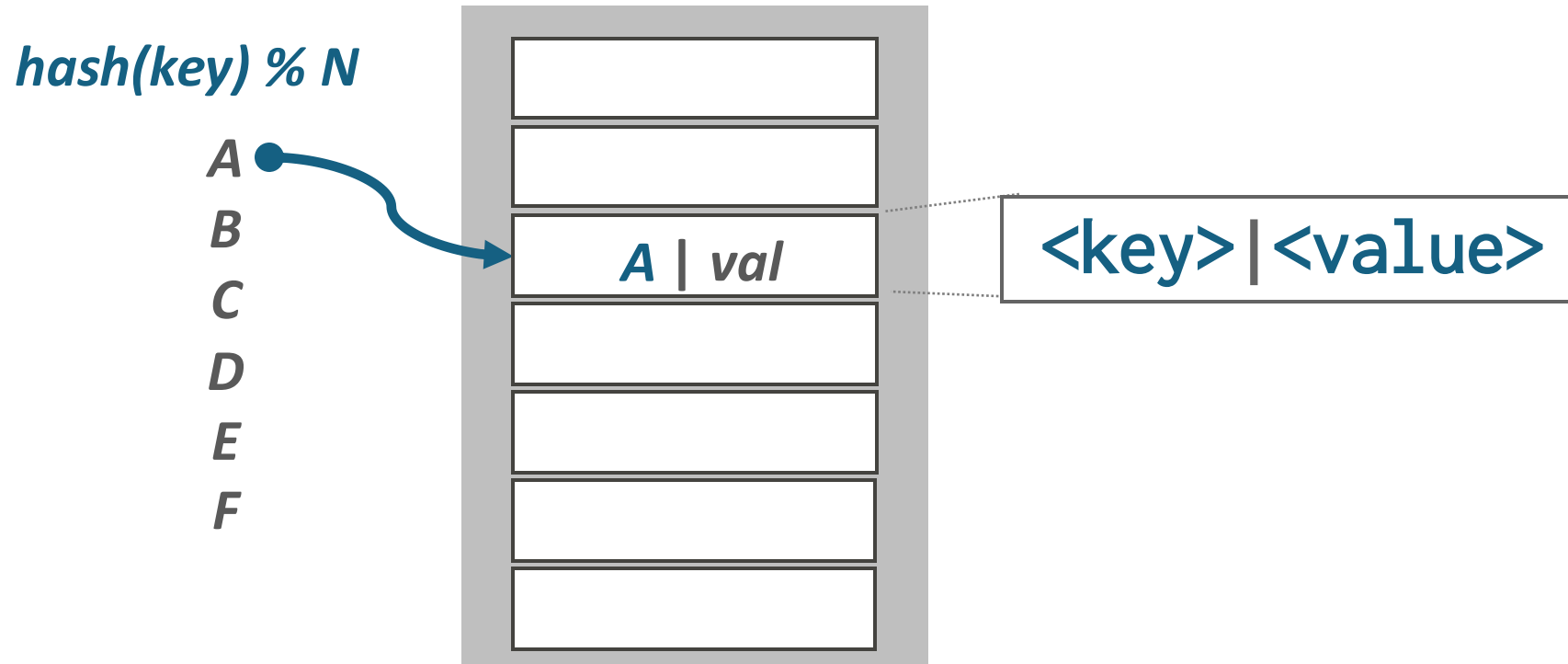
F



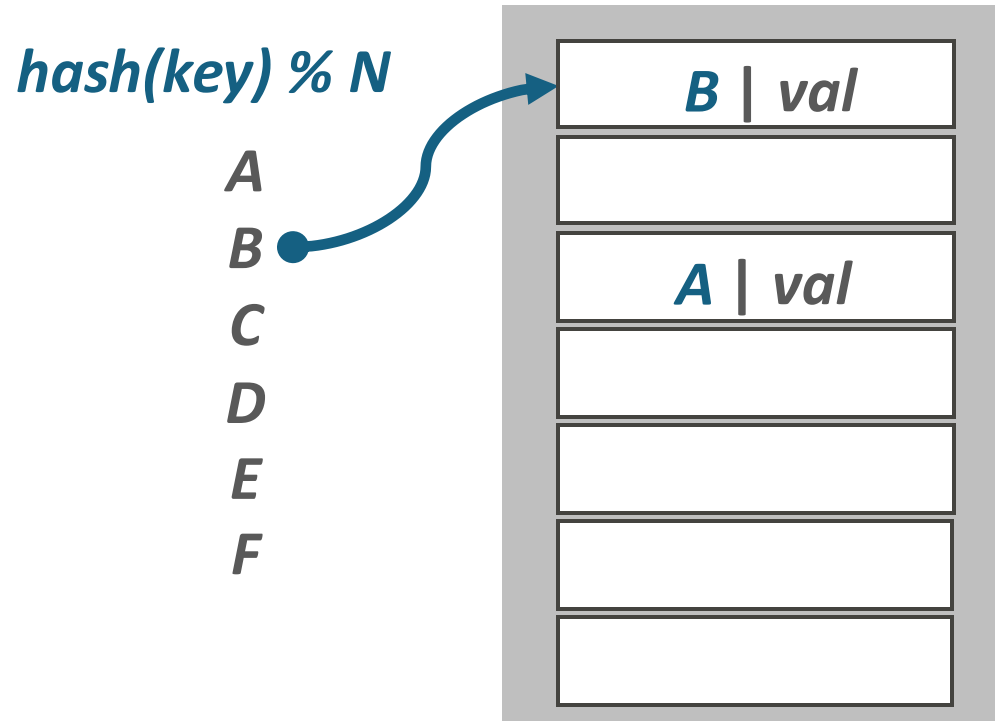
Linear Probe Hashing



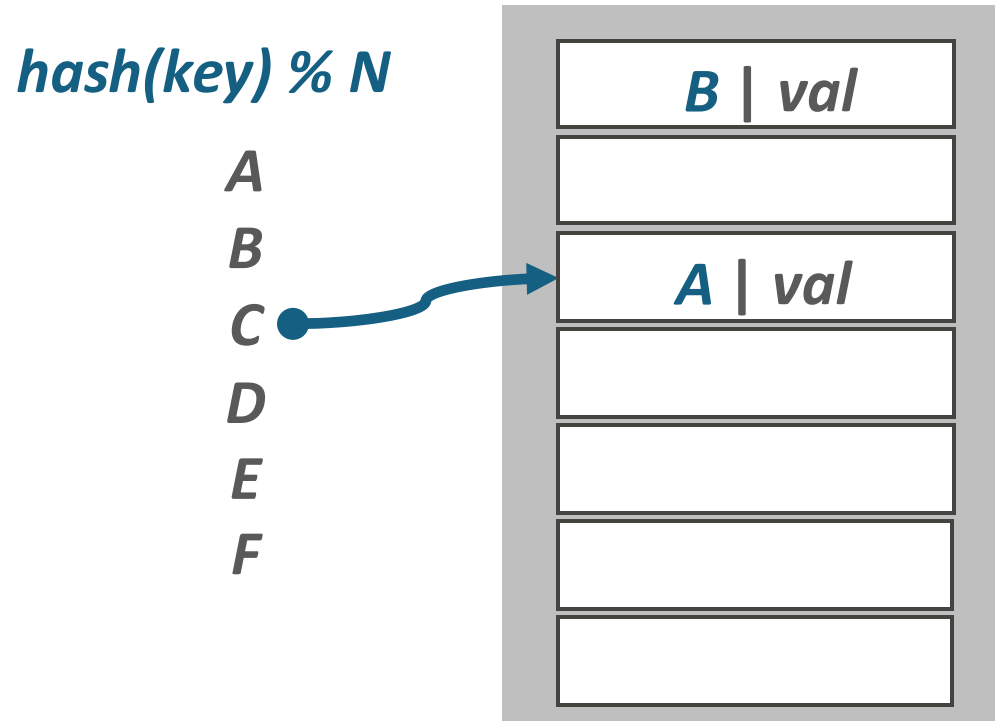
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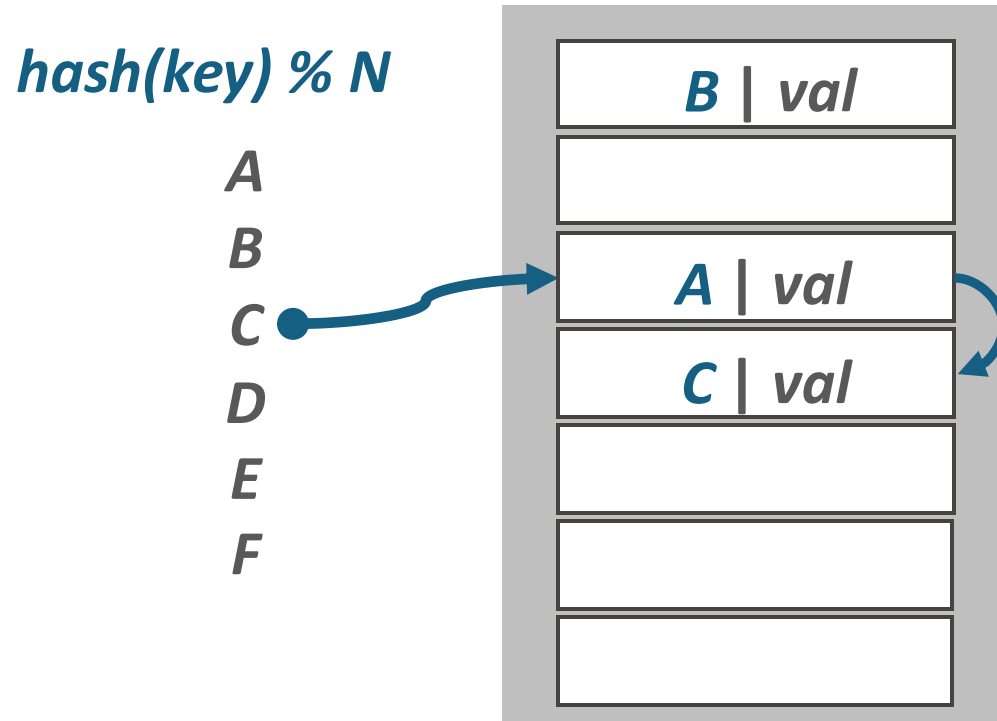
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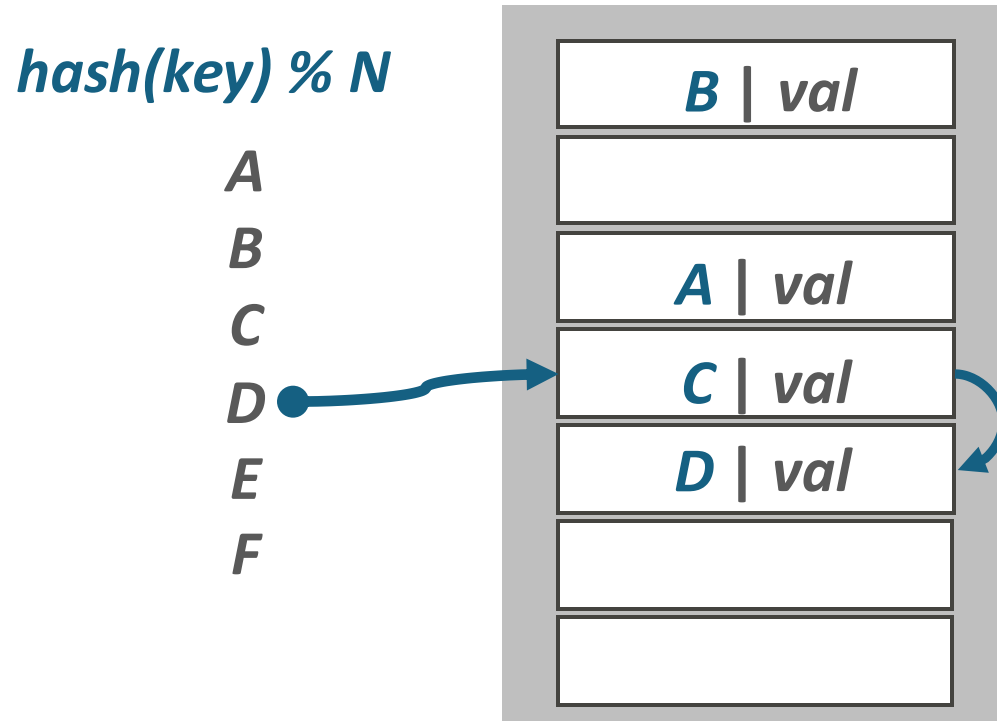
Linear Probe Hashing



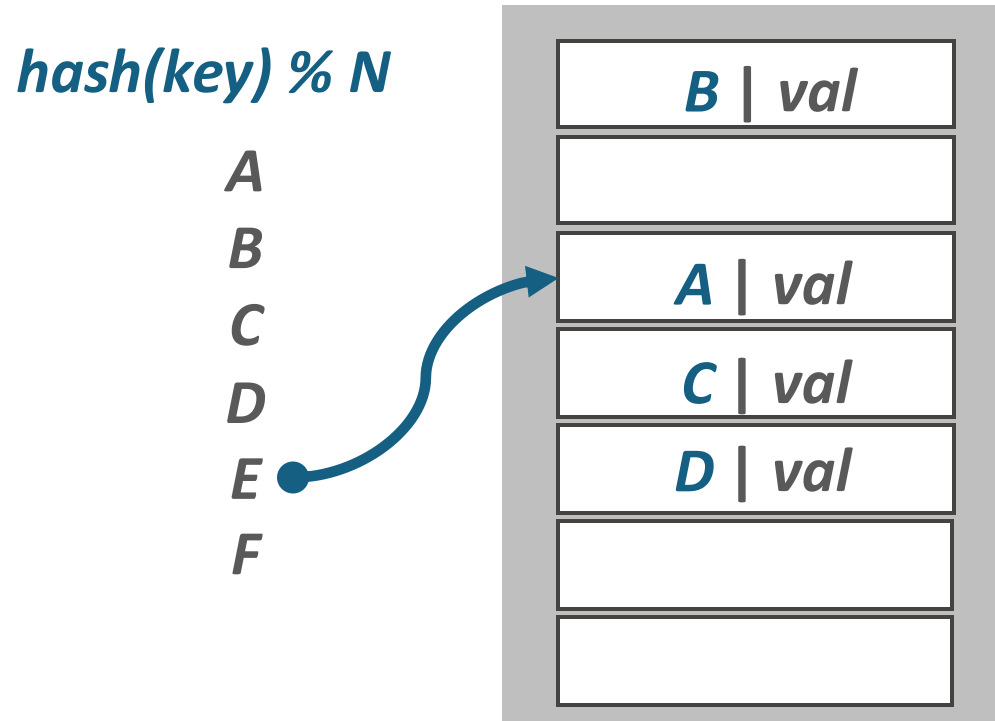
Linear Probe Hashing



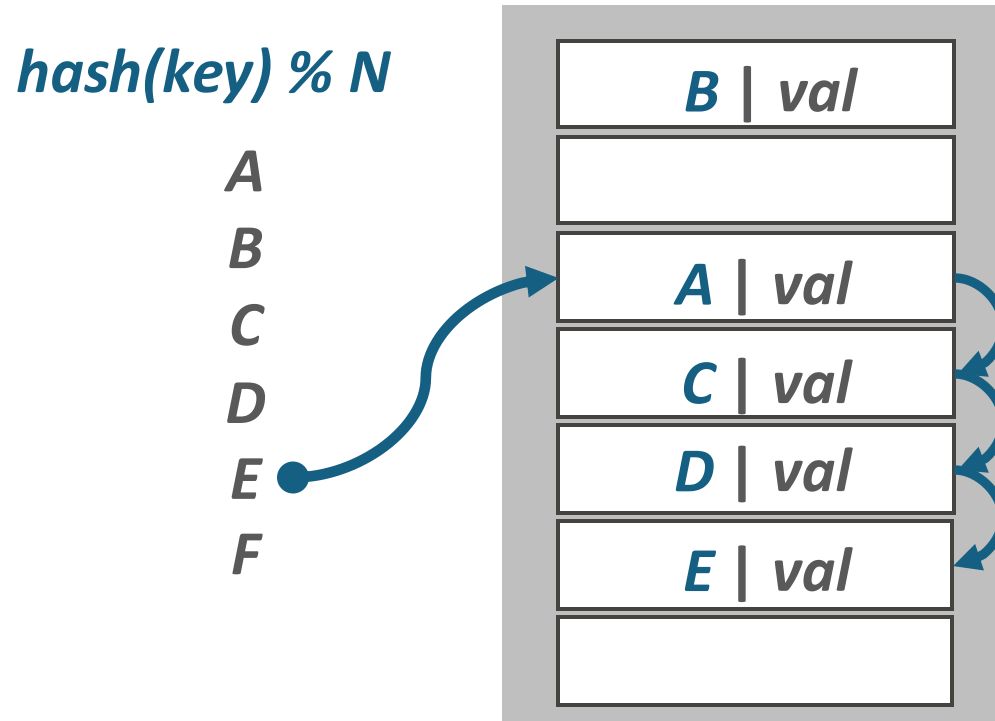
Linear Probe Hashing



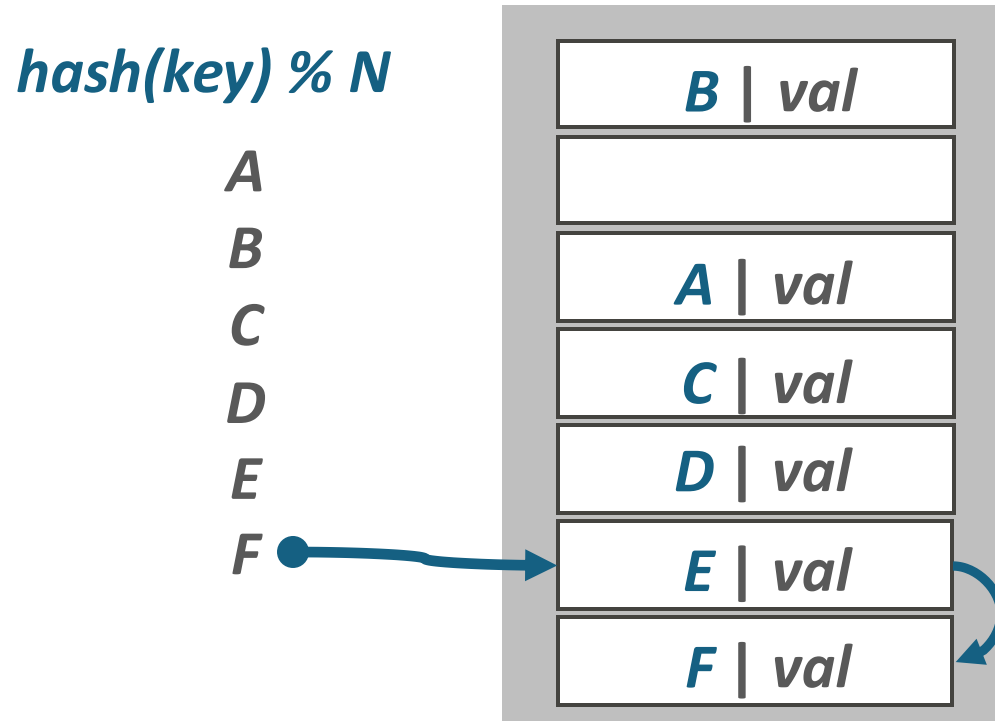
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Linear Probe Hashing



Linear Probe Hashing



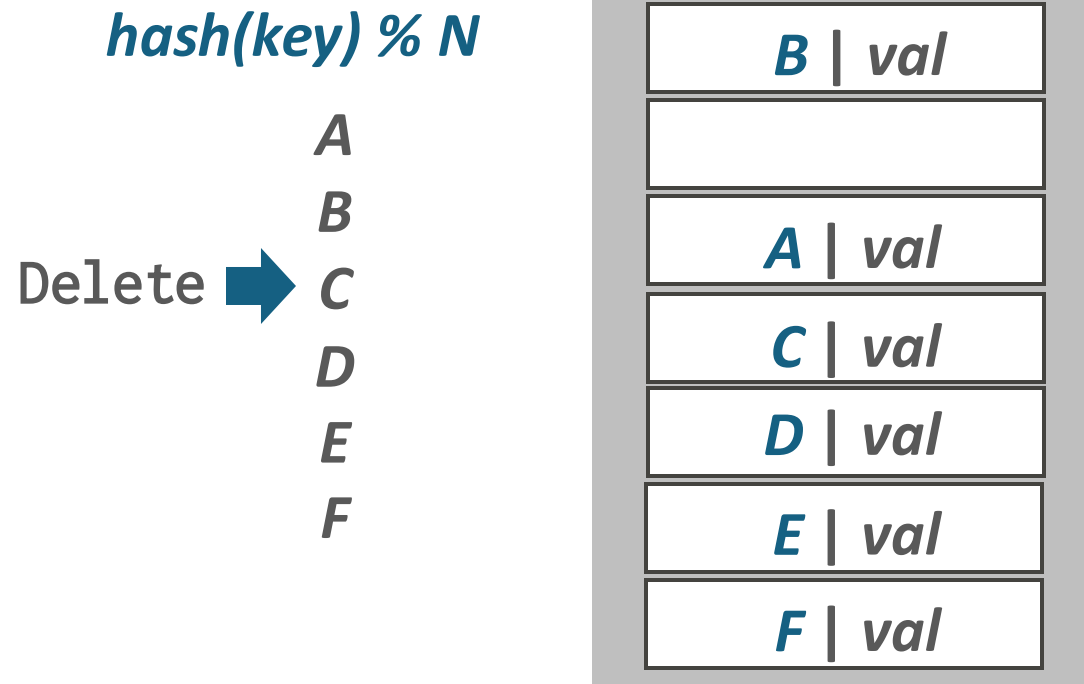
Linear Probe Hashing - Deletes

hash(key) % N

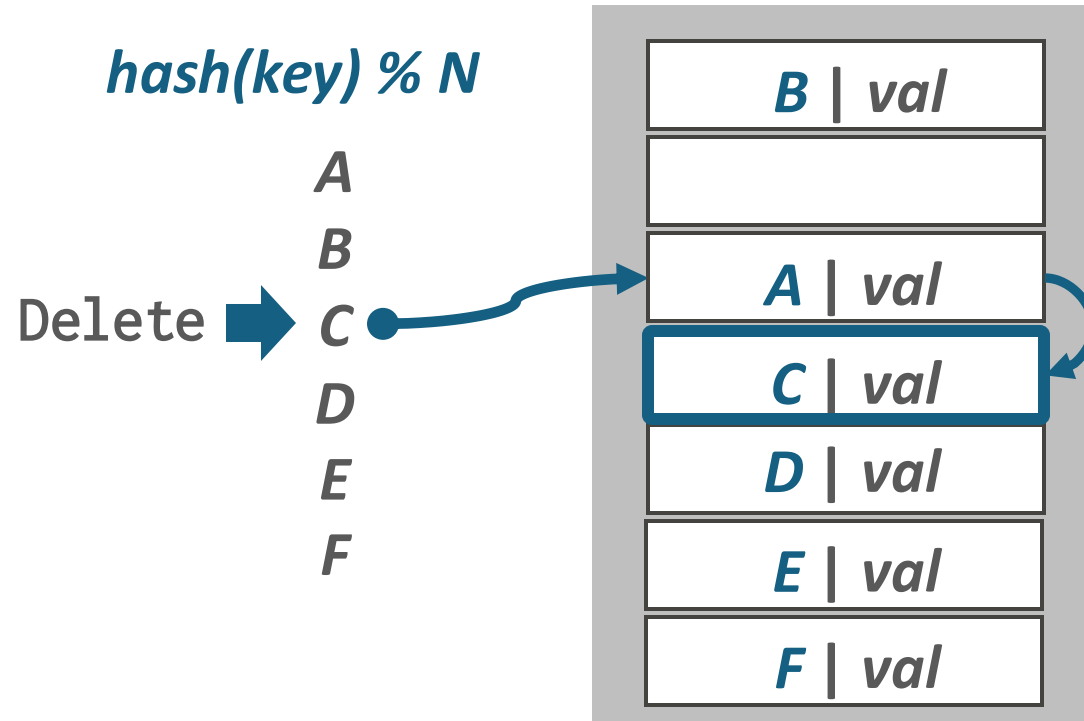
A
B
C
D
E
F

<i>B</i> <i>val</i>
<i>A</i> <i>val</i>
<i>C</i> <i>val</i>
<i>D</i> <i>val</i>
<i>E</i> <i>val</i>
<i>F</i> <i>val</i>

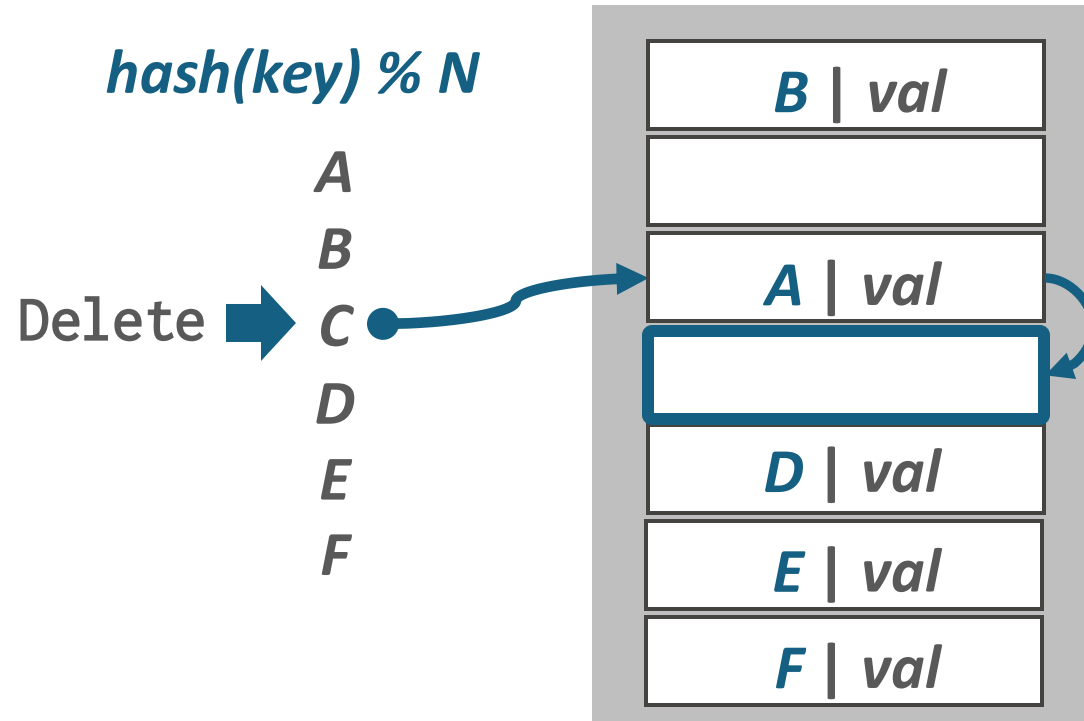
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Linear Probe Hashing - Deletes

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A
B
C
D
E
F

<i>B</i> <i>val</i>
<i>A</i> <i>val</i>
<i>D</i> <i>val</i>
<i>E</i> <i>val</i>
<i>F</i> <i>val</i>

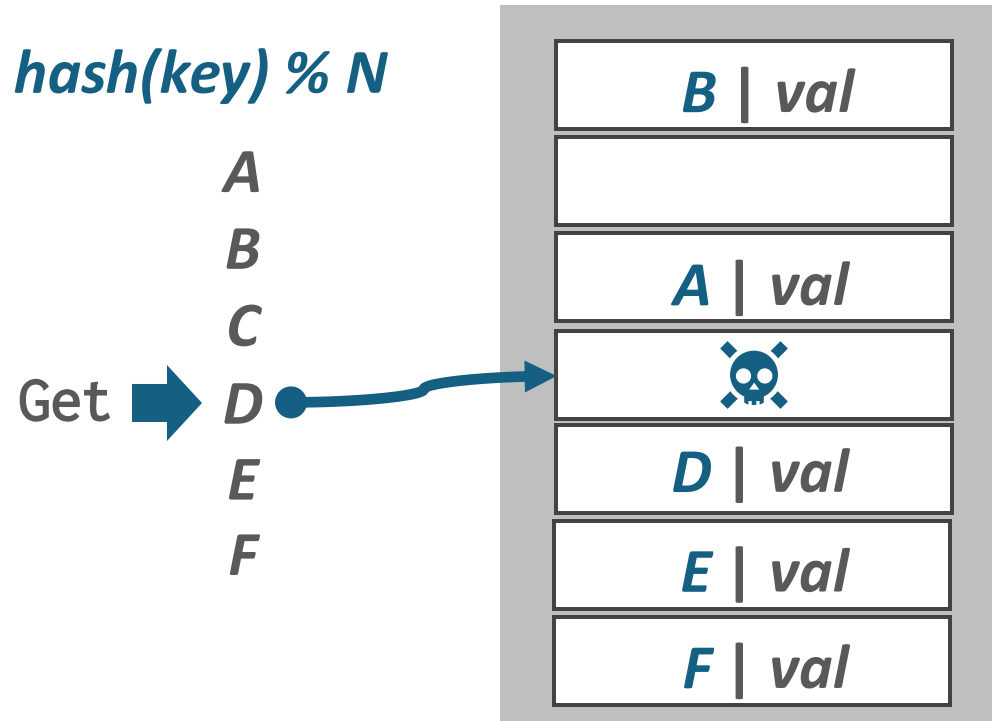
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hash(key) % N

A
B
C
Get ➡ D
E
F

<i>B val</i>
<i>A val</i>
<i>D val</i>
<i>E val</i>
<i>F val</i>

Linear Probe Hashing - Deletes



Linear Probe Hashing - Deletes

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A
B
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Get ➡ D
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Linear Probe Hashing - Deletes

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A
B
C
Get ➡ D
E
F

B val
A val
D val
E val
F val

- **Approach #1:
Movement**

- Rehash keys until you find the first empty slot.

Linear Probe Hashing - Deletes

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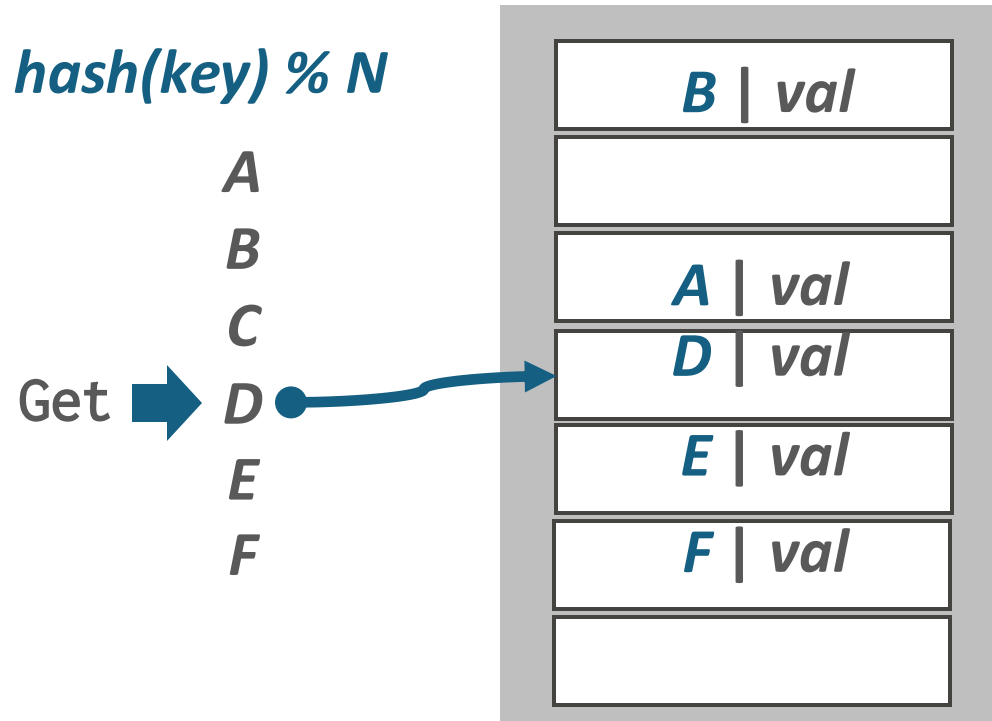
Get ➡
A
B
C
D
E
F

<i>B</i> <i>val</i>
<i>A</i> <i>val</i>
<i>D</i> <i>val</i>
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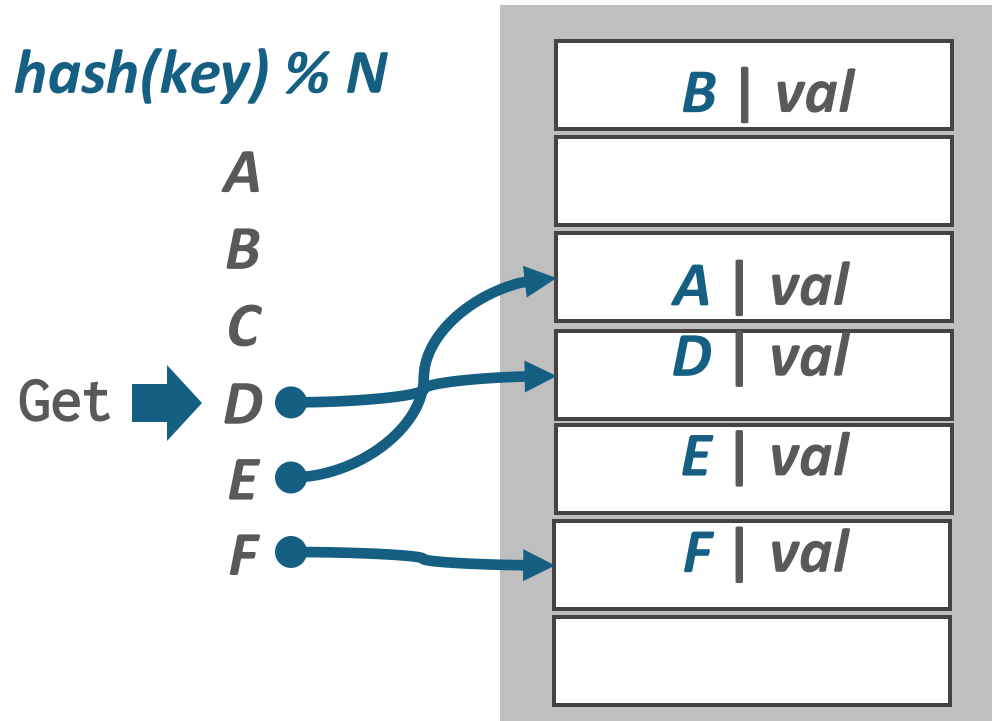
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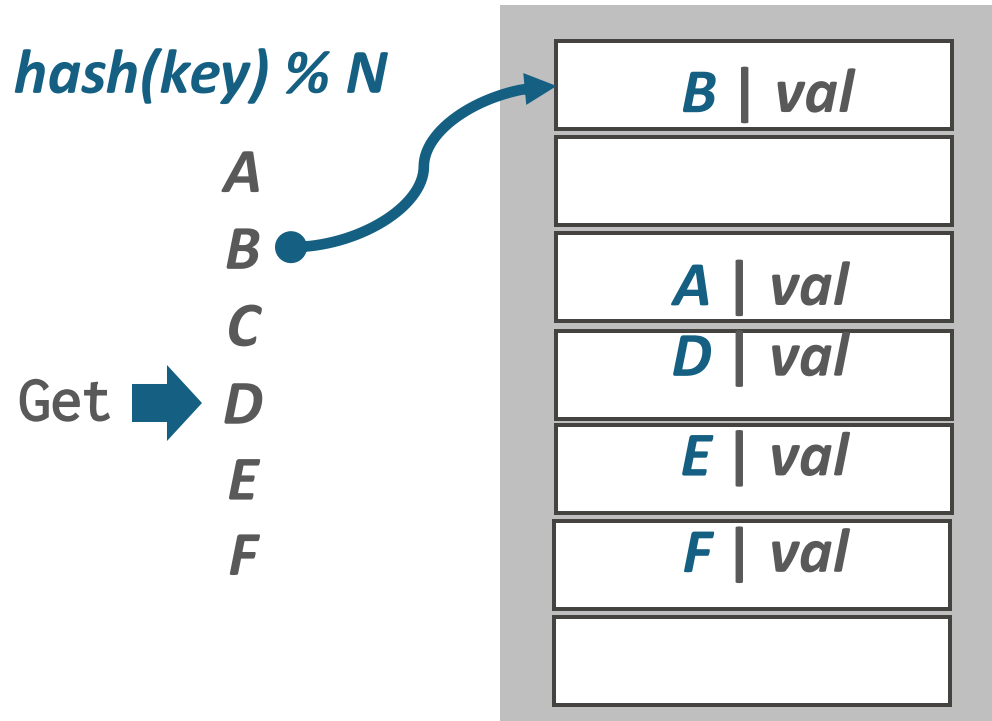
A
B
C
Get → D
E
F

B val
A val
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E val
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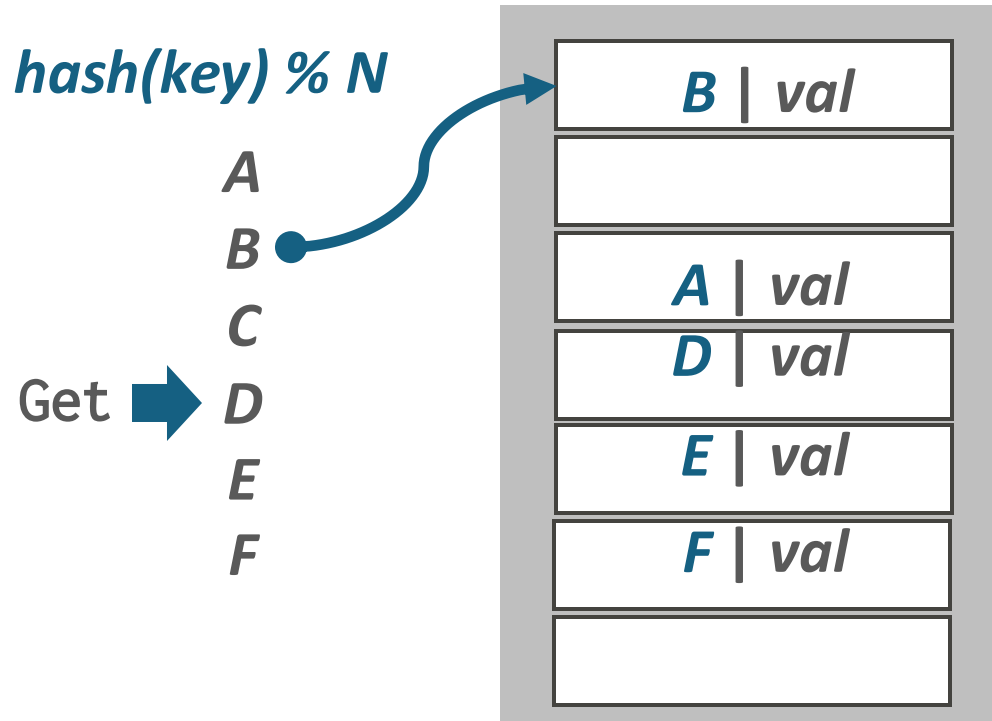
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- **Approach #1: Movement**

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Linear Probe Hashing - Deletes



- **Approach #1: Movement**

- Rehash keys until you find the first empty slot.
- Expensive! May need to reorganize the entire table.
- No DBMS does this.

Linear Probe Hashing - Deletes

hash(key) % N

A
B
C
D
E
F

<i>B val</i>
<i>A val</i>
<i>C val</i>
<i>D val</i>
<i>E val</i>
<i>F val</i>

- **Approach #2:
Tombstone**

- Set a marker to indicate that the entry in the slot is logically deleted.
- Reuse the slot for new keys.
- May need periodic garbage collection.

Linear Probe Hashing - Deletes

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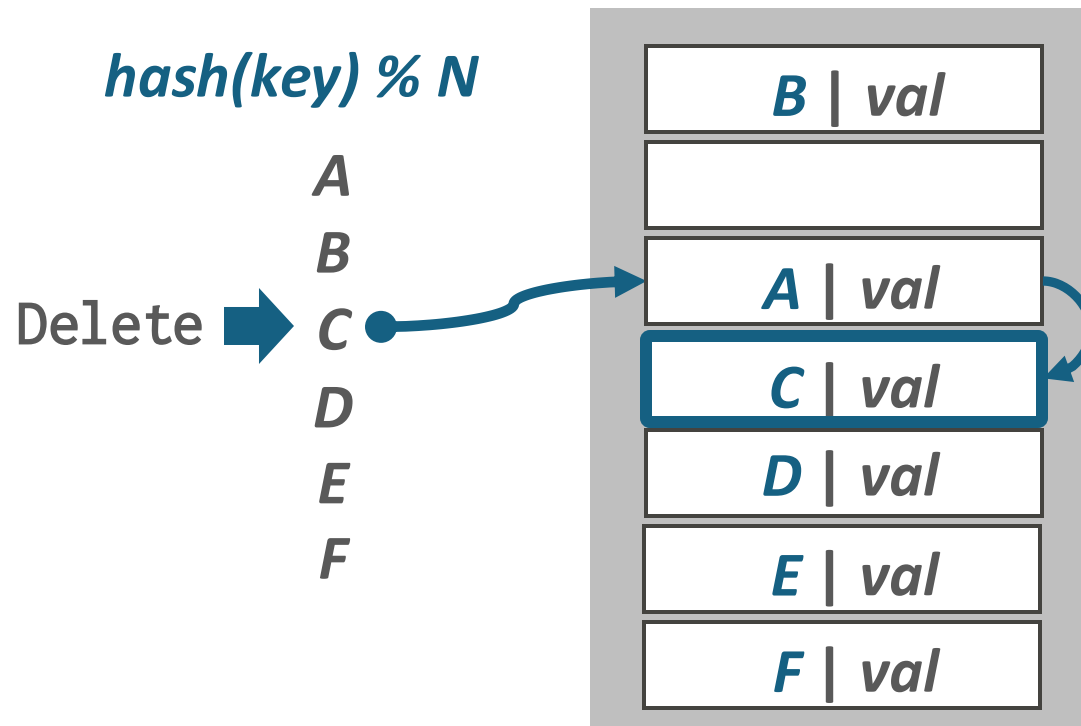
A
B
Delete → C
D
E
F

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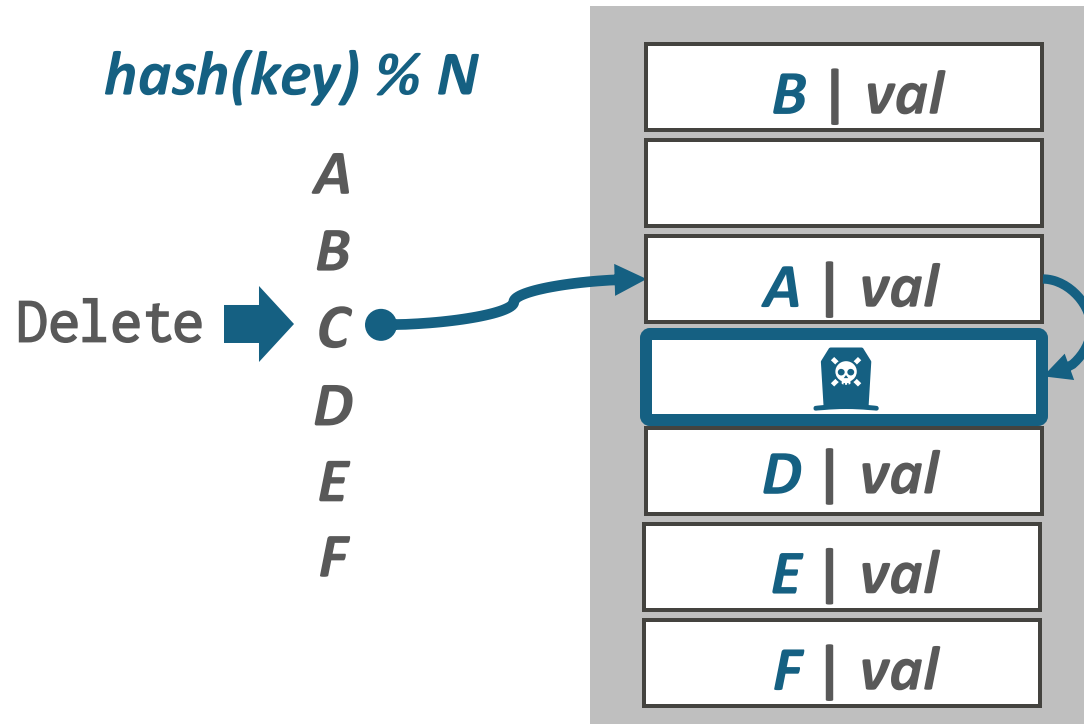
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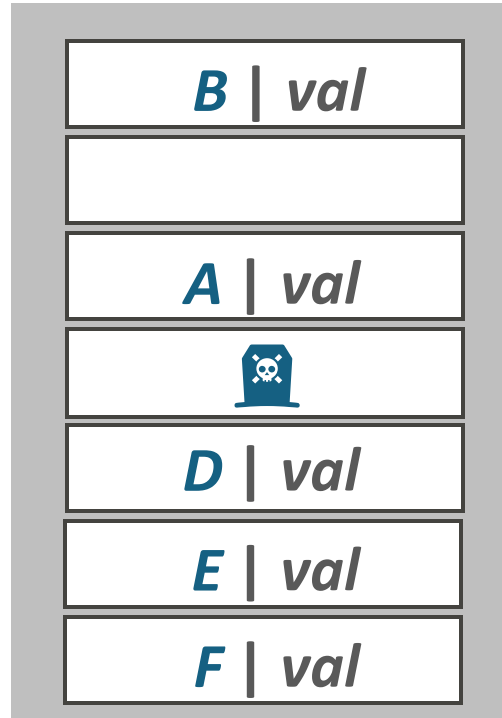
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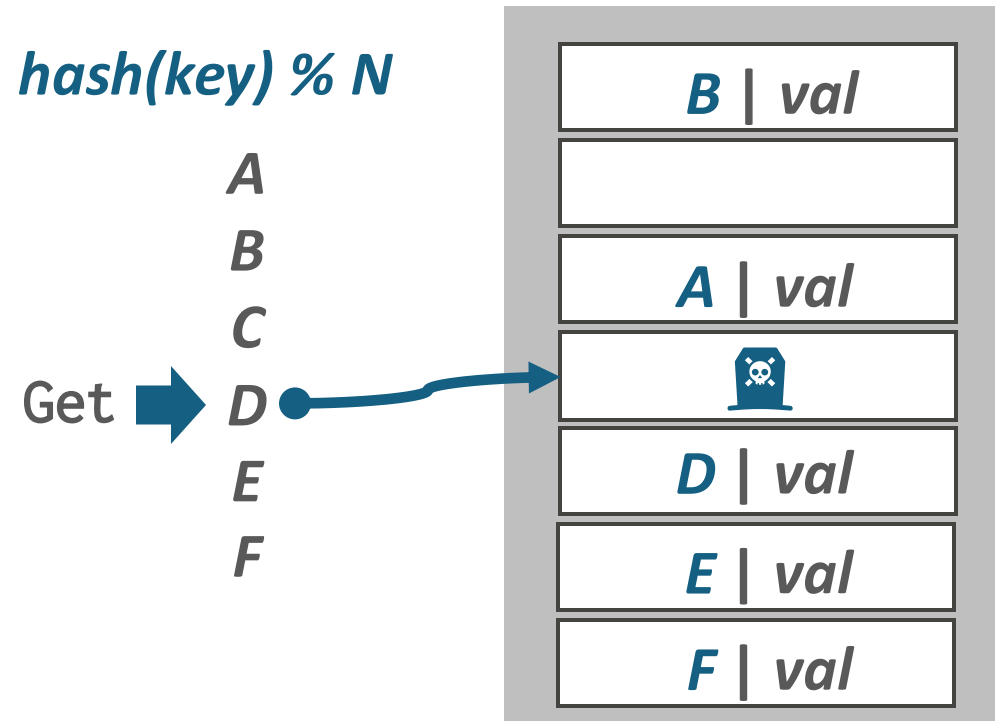
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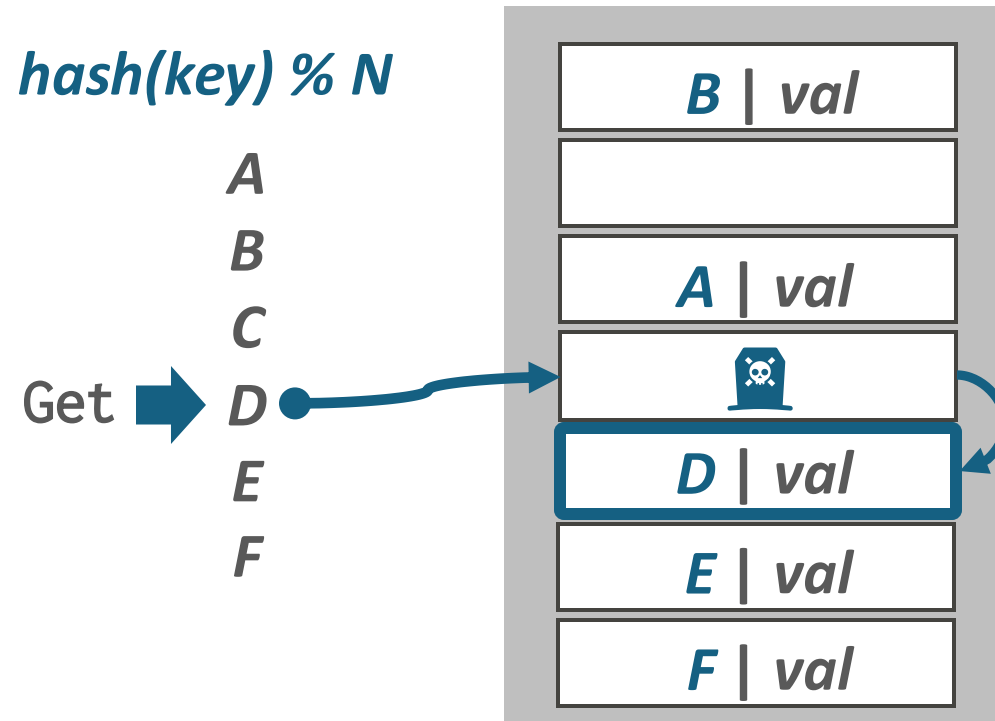
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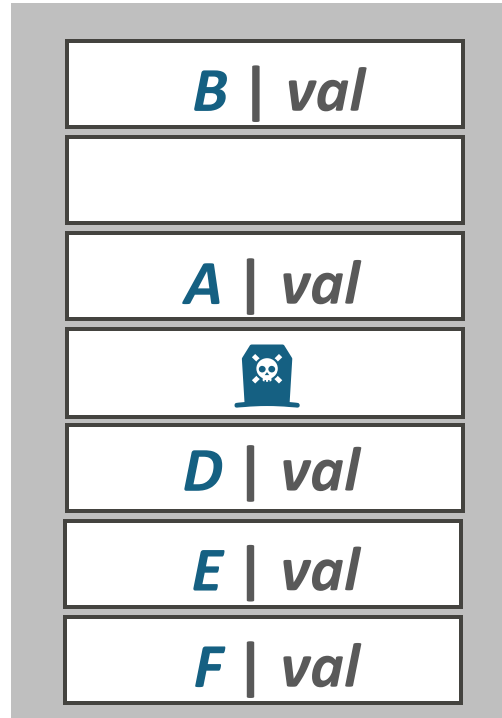
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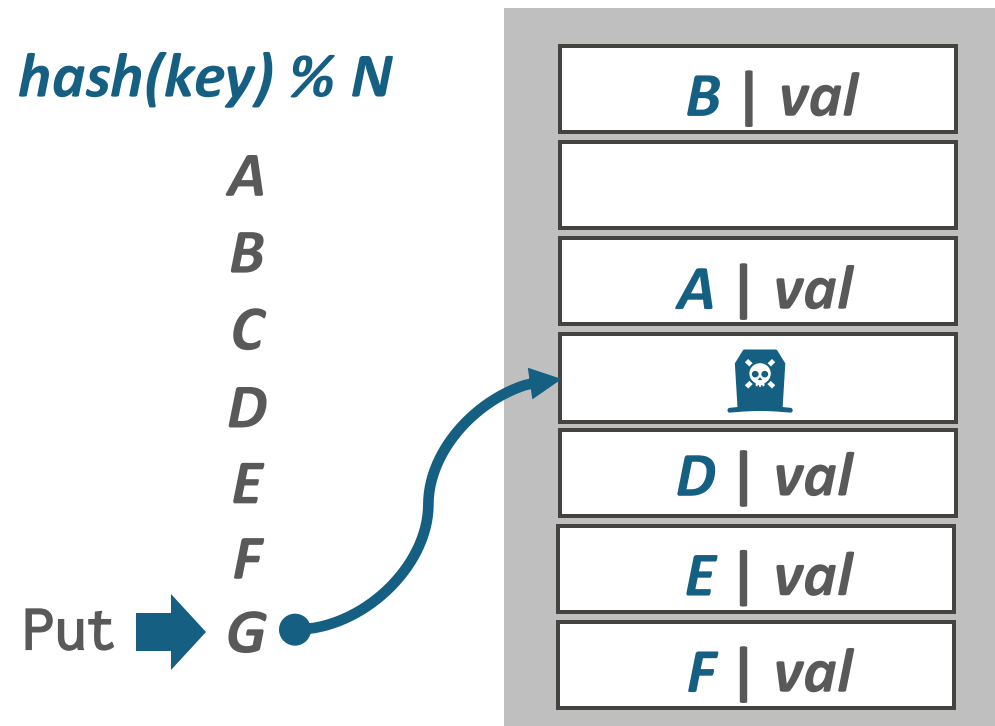
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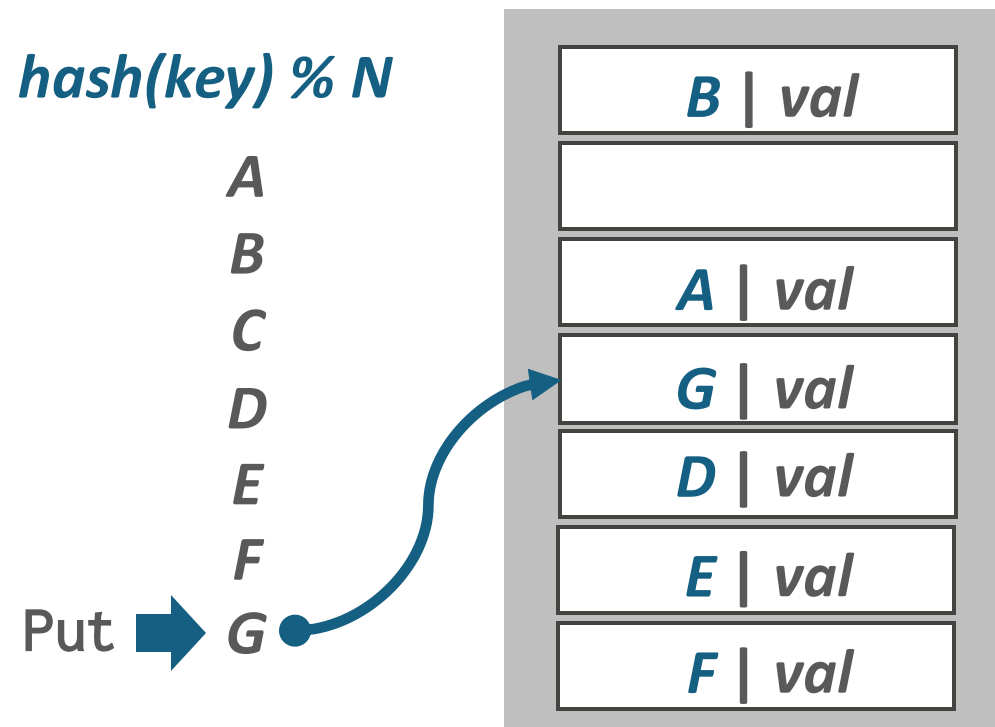
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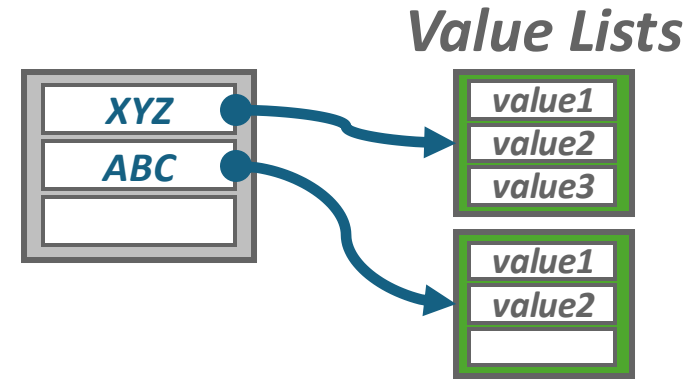
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Non-Unique Keys

- **Choice #1: Separate Linked List**
 - Store values in separate storage area for each key.
 - Value lists can overflow to multiple pages if the number of duplicates is large.

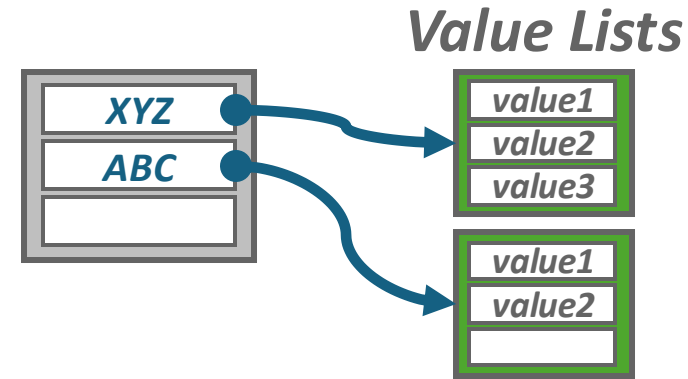
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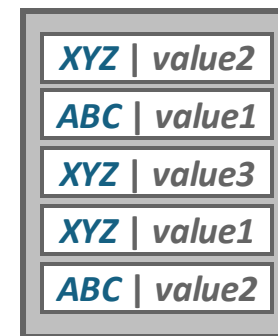
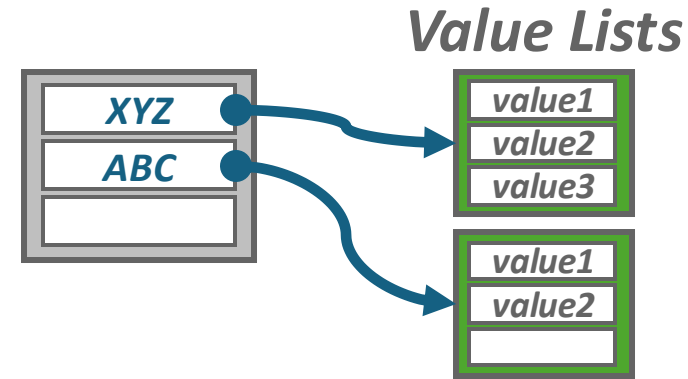
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Optimizations

- Specialized hash table implementations based on key type(s) and sizes.
 - Example: Maintain multiple hash tables for different string sizes for a set of keys.
- Store metadata separate in a separate array.
 - Packed bitmap tracks whether a slot is empty/tombstone.
- Use table + slot versioning metadata to quickly invalidate all entries in the hash table.
 - Example: If table version does not match slot version, then treat the slot as empty.

Source: [Maksim Kita](#)

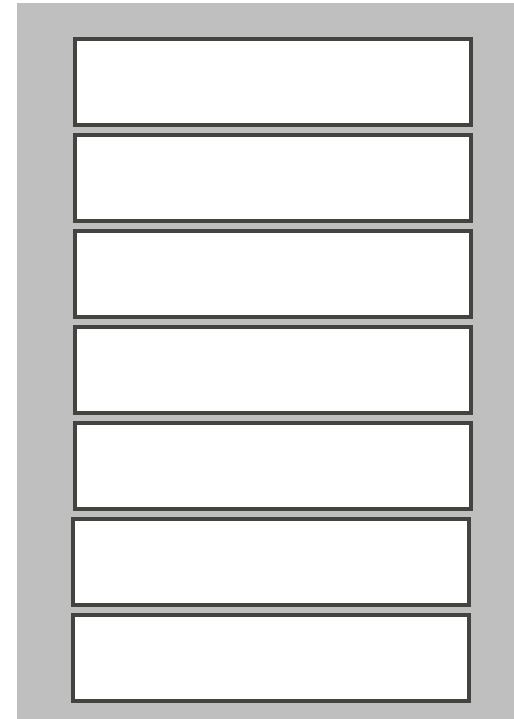
Cuckoo Hashing

Static Hashing Schemes

Cuckoo Hashing

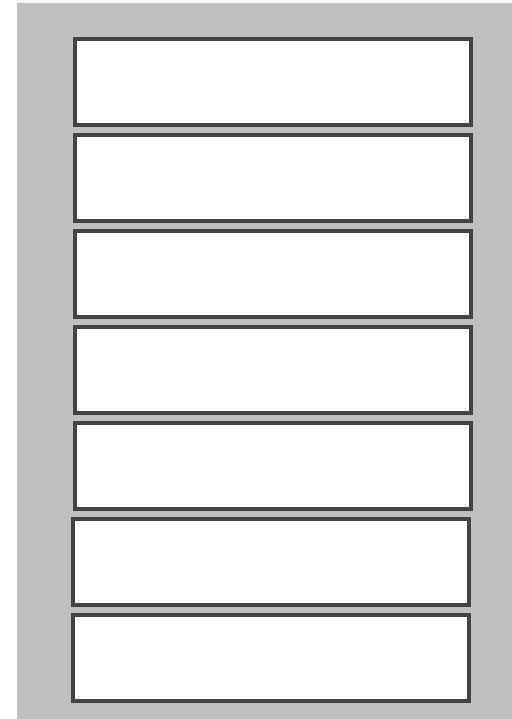
- Use multiple hash functions to find multiple locations in the hash table to insert records.
 - On insert, check multiple locations and pick the one that is empty.
 - If no location is available, evict the element from one of them and then re-hash it find a new location.
- Look-ups and deletions are always $O(1)$ because only one location per hash table is checked.
- Best [open-source implementation](#) is from CMU.

Cuckoo Hashing

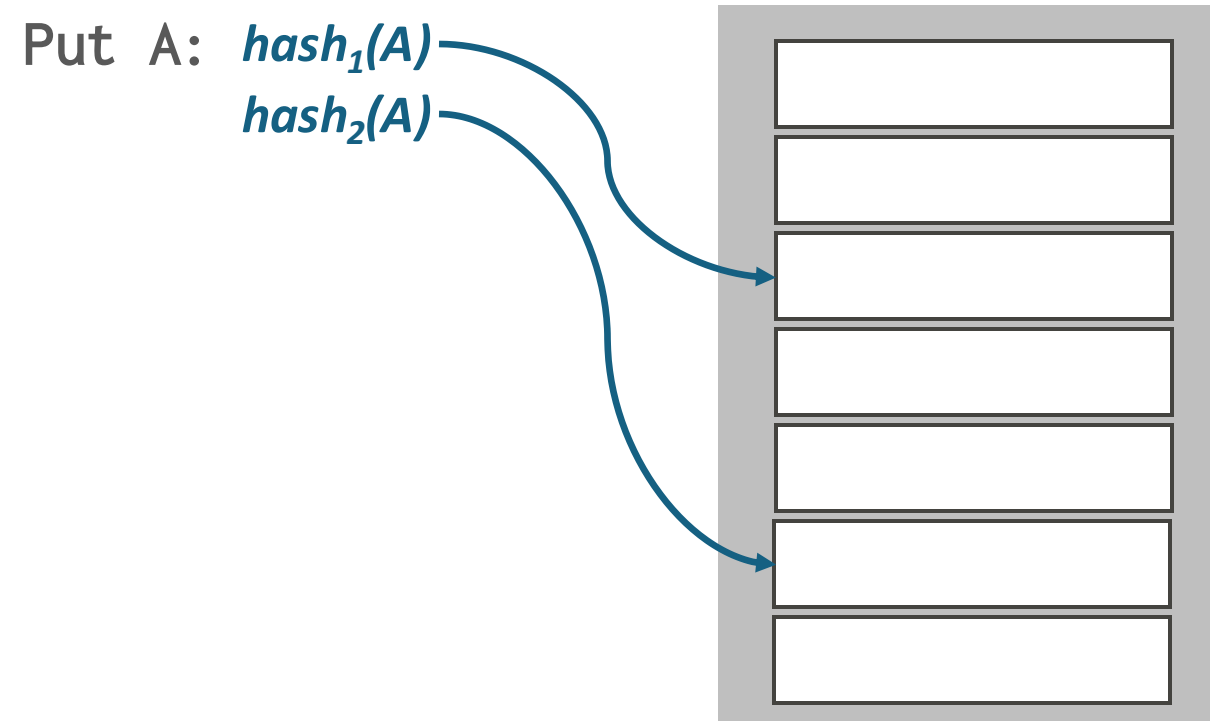


Cuckoo Hashing

Put A: $hash_1(A)$
 $hash_2(A)$

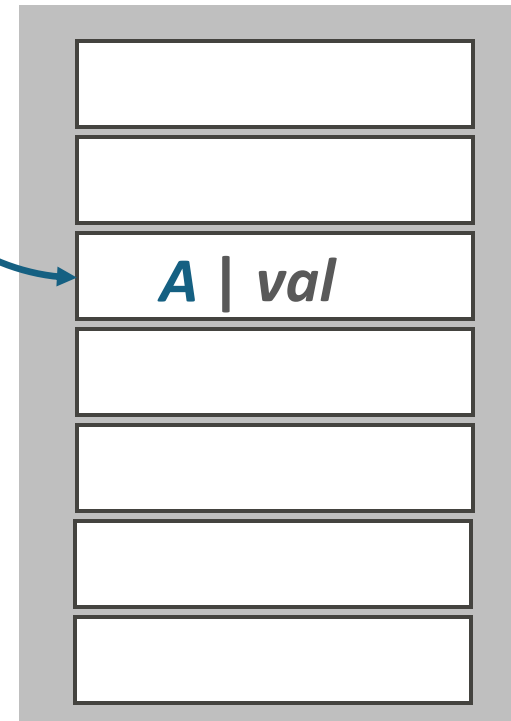


Cuckoo Hashing



Cuckoo Hashing

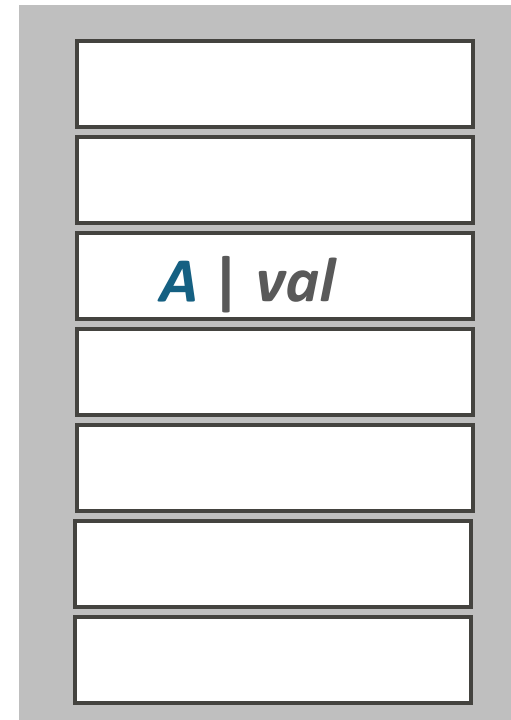
Put A: $hash_1(A)$
 $hash_2(A)$



Cuckoo Hashing

Put A: $hash_1(A)$
 $hash_2(A)$

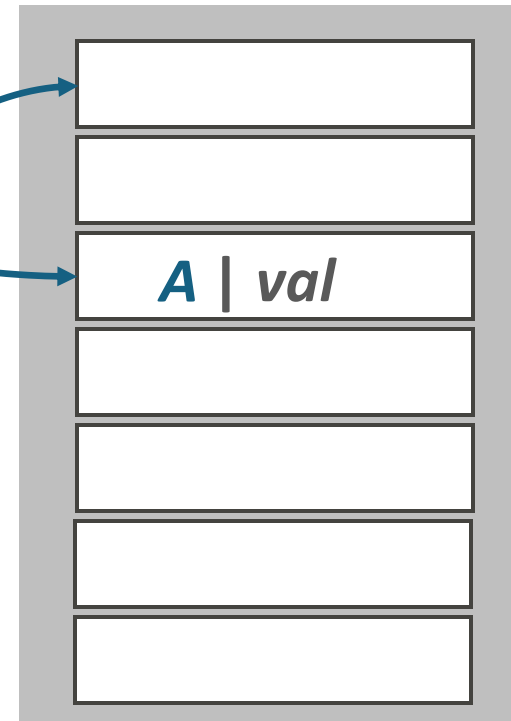
Put B: $hash_1(B)$
 $hash_2(B)$



Cuckoo Hashing

Put A: $hash_1(A)$
 $hash_2(A)$

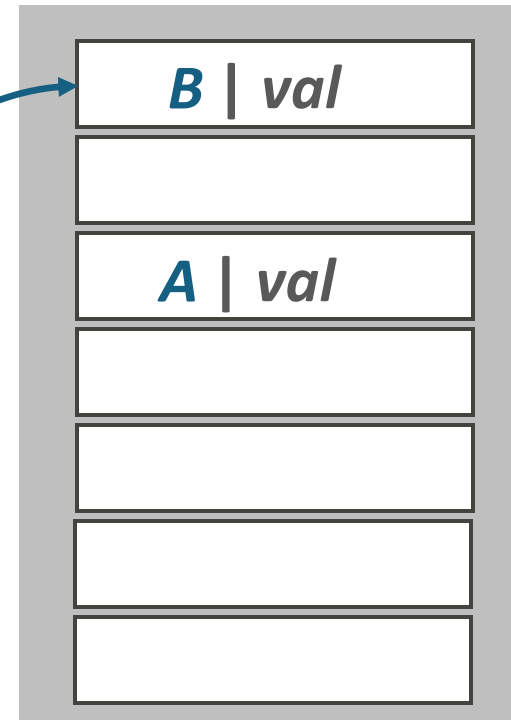
Put B: $hash_1(B)$
 $hash_2(B)$



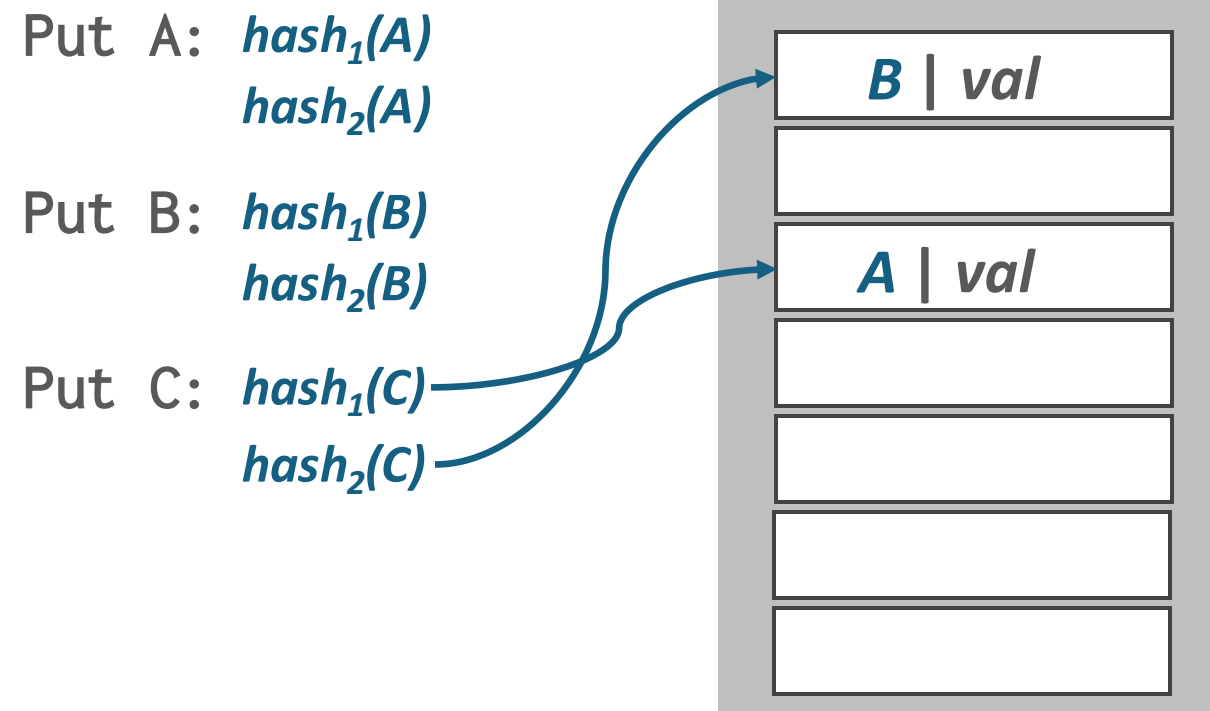
Cuckoo Hashing

Put A: $hash_1(A)$
 $hash_2(A)$

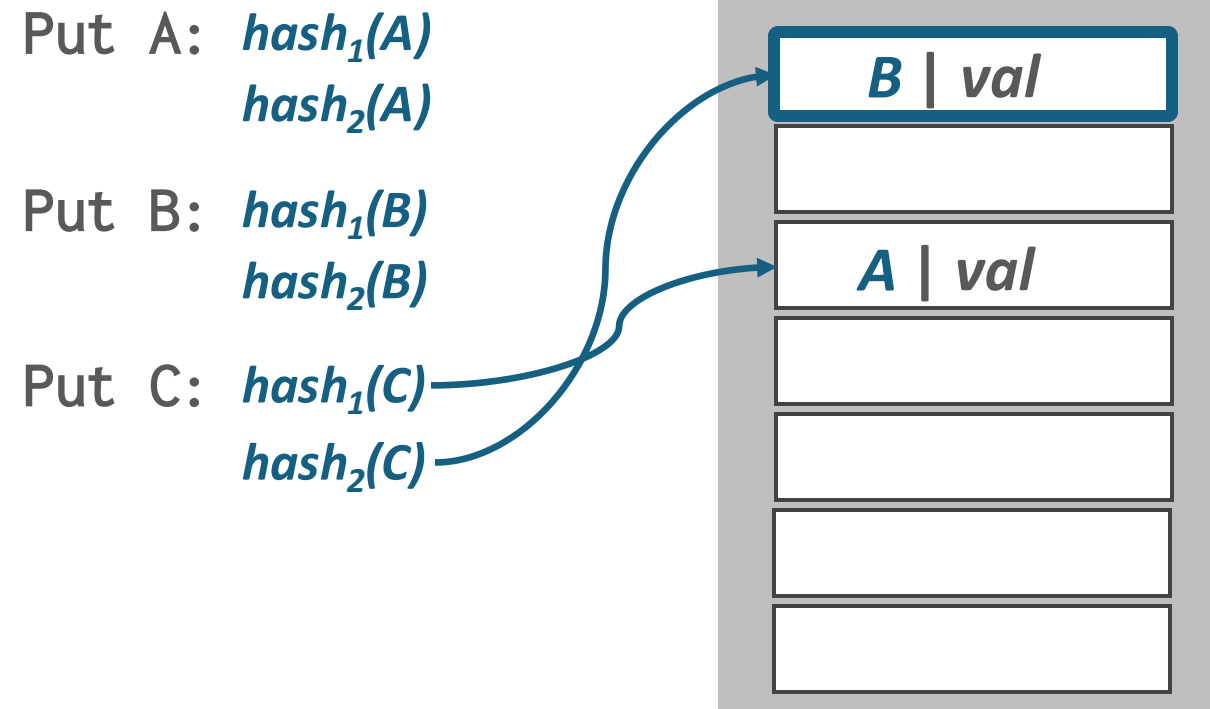
Put B: $hash_1(B)$
 $hash_2(B)$



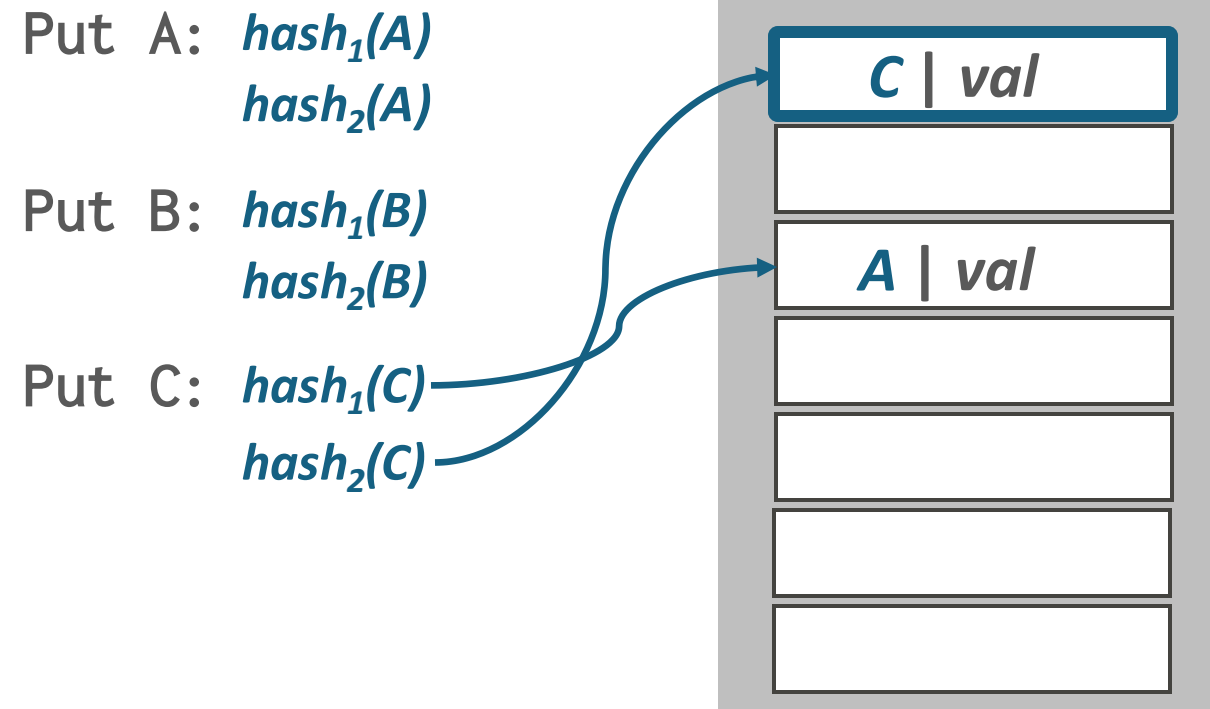
Cuckoo Hashing



Cuckoo Hashing



Cuckoo Hashing

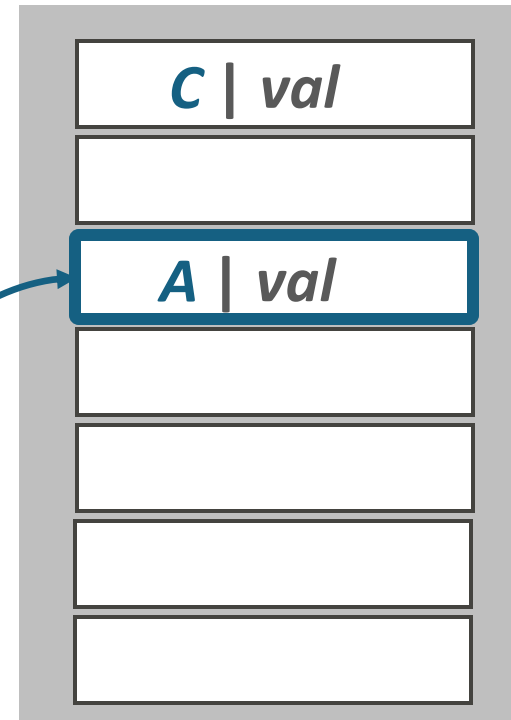


Cuckoo Hashing

Put A: $hash_1(A)$
 $hash_2(A)$

Put B: $hash_1(B)$
 $hash_2(B)$

Put C: $hash_1(C)$
 $hash_2(C)$
 $hash_1(B)$

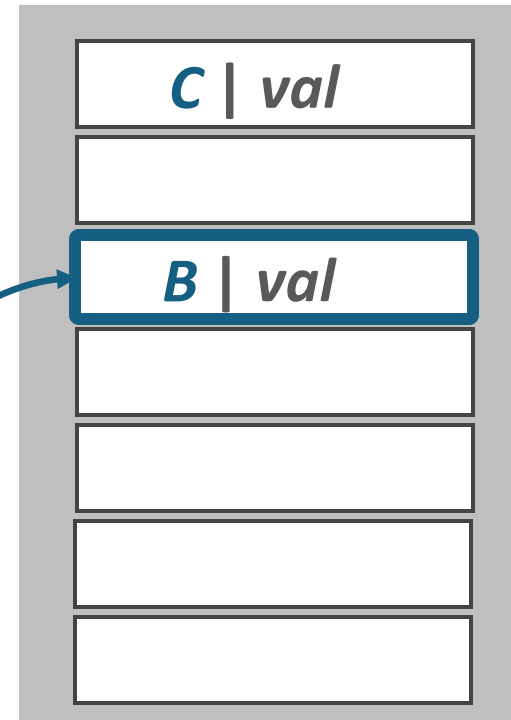


Cuckoo Hashing

Put A: $hash_1(A)$
 $hash_2(A)$

Put B: $hash_1(B)$
 $hash_2(B)$

Put C: $hash_1(C)$
 $hash_2(C)$
 $hash_1(B)$

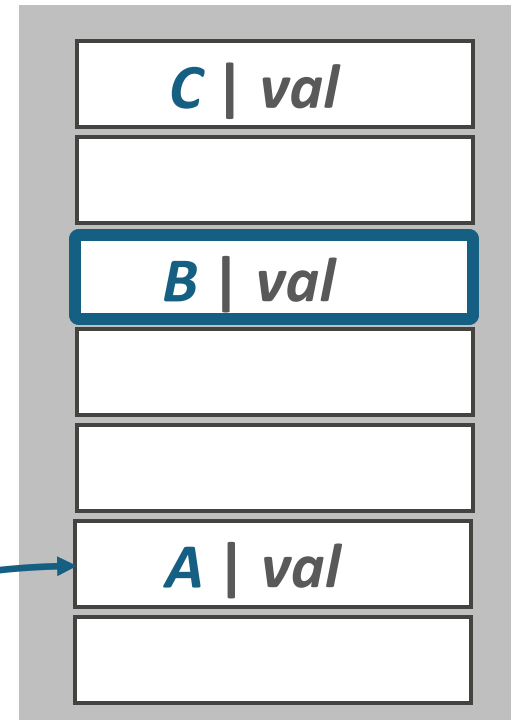


Cuckoo Hashing

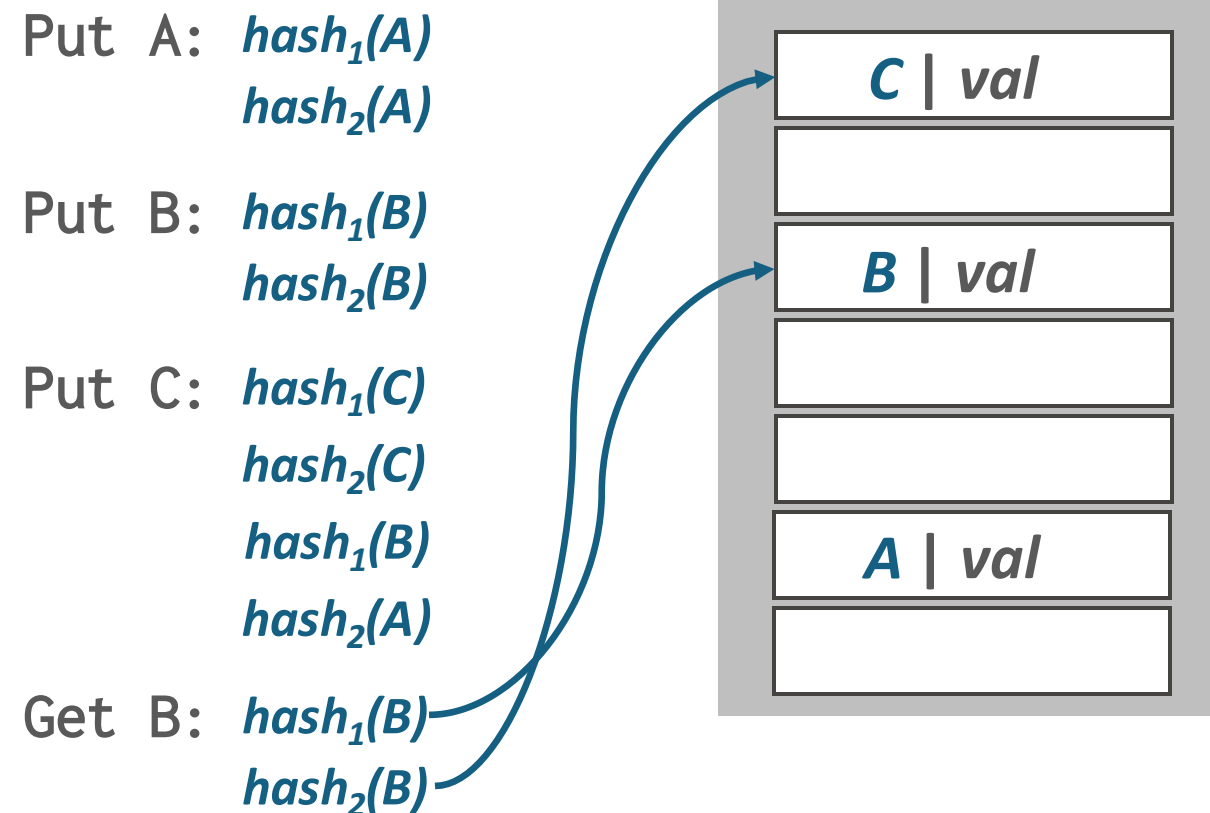
Put A: $hash_1(A)$
 $hash_2(A)$

Put B: $hash_1(B)$
 $hash_2(B)$

Put C: $hash_1(C)$
 $hash_2(C)$
 $hash_1(B)$
 $hash_2(A)$



Cuckoo Hashing



Chained Hash Table

Dynamic Hashing Schemes

Observation

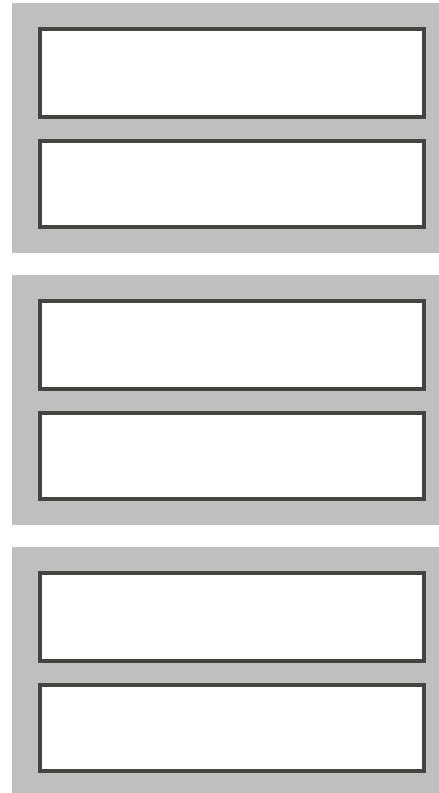
- The previous hash tables require the DBMS to know the number of elements it wants to store.
 - Otherwise, it must rebuild the table if it needs to grow/shrink in size.
- Dynamic hash tables incrementally resize themselves as needed.
 - Chained Hashing
 - Extendible Hashing
 - Linear Hashing

Chained Hashing

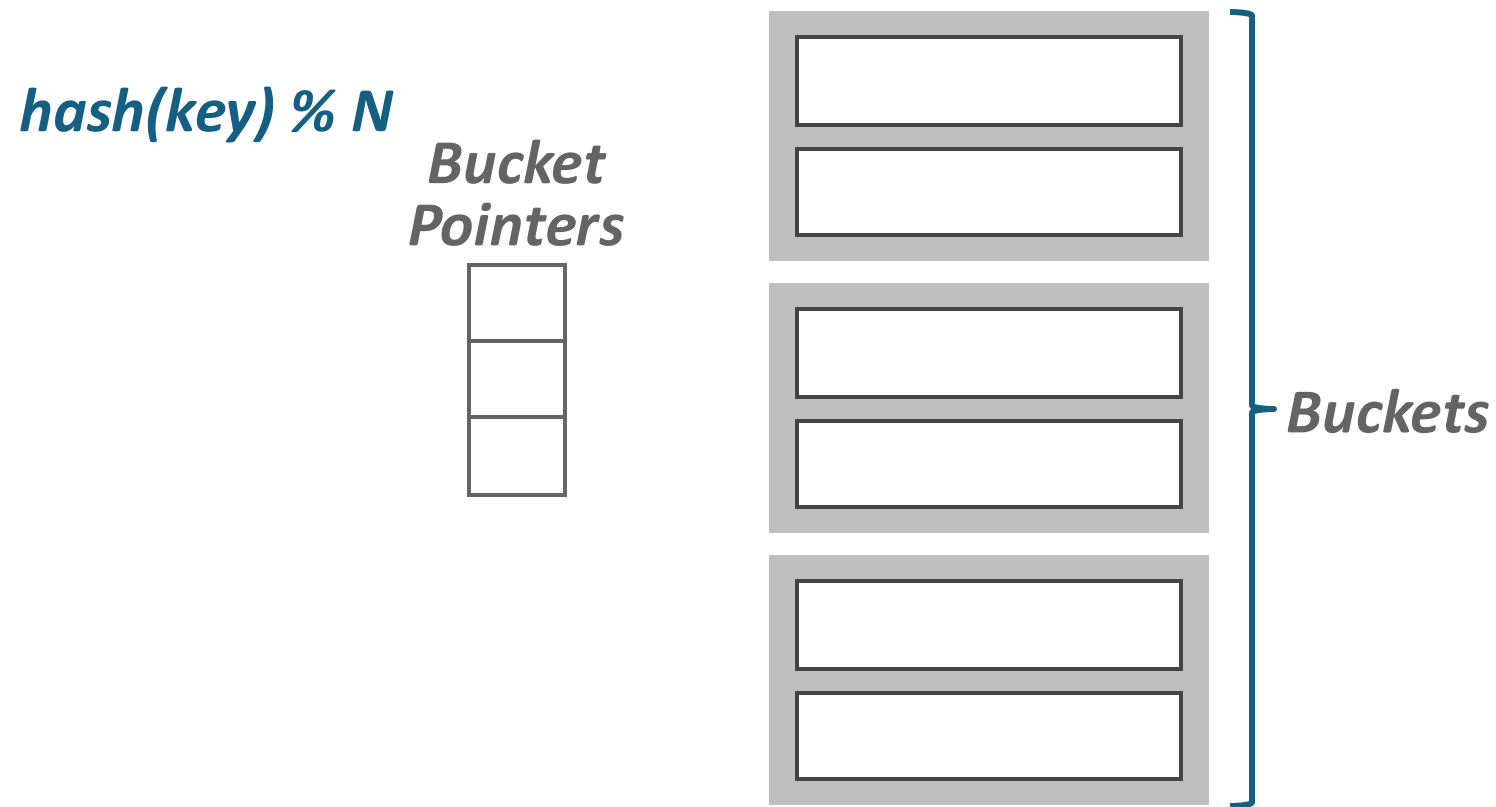
- Maintain a linked list of buckets for each slot in the hash table.
- Resolve collisions by placing all elements with the same hash key into the same bucket.
 - To determine whether an element is present, hash to its bucket and scan for it.
 - Insertions and deletions are generalizations of lookups.

Chained Hashing

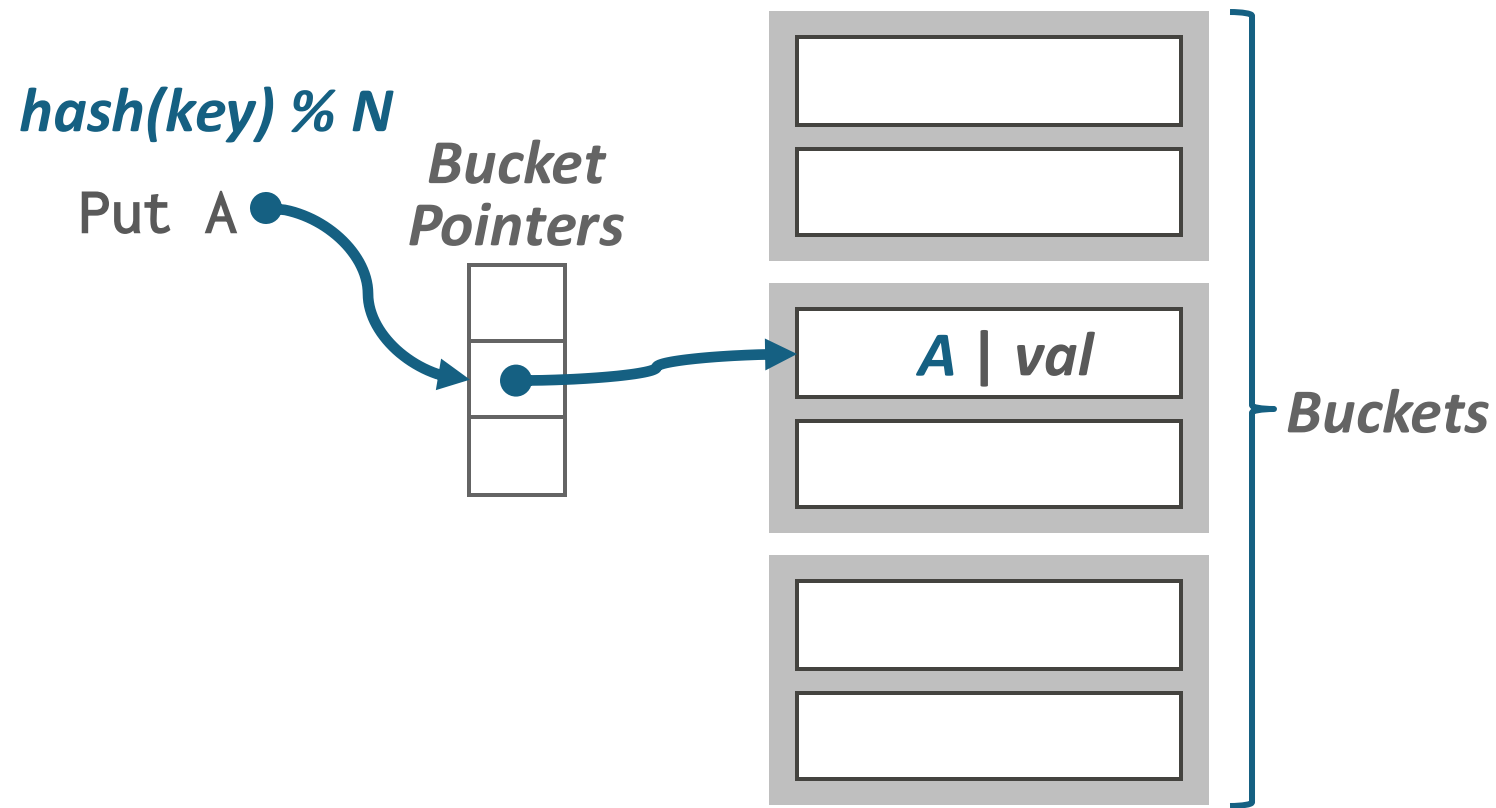
hash(key) % N



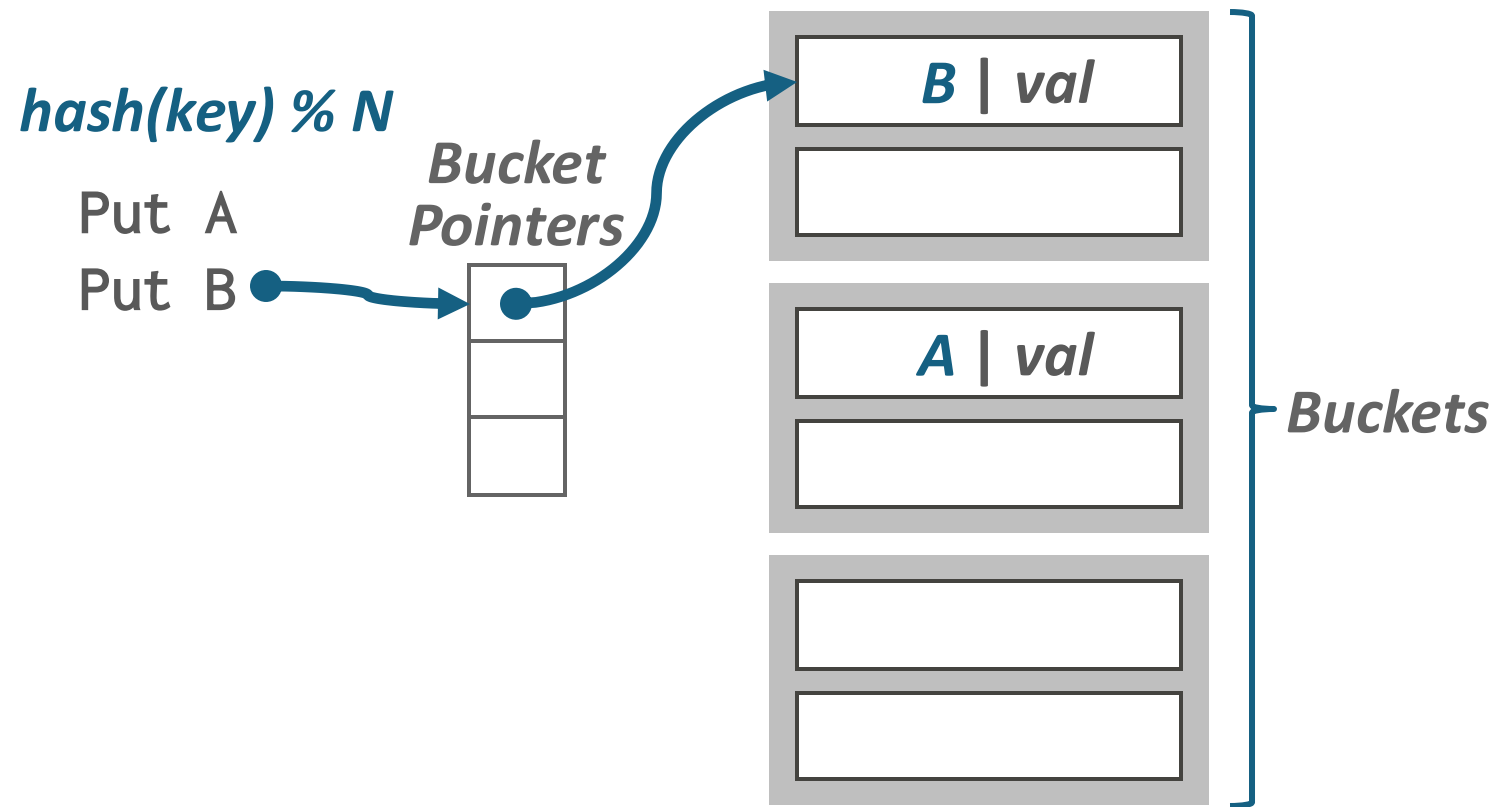
Chained Hashing



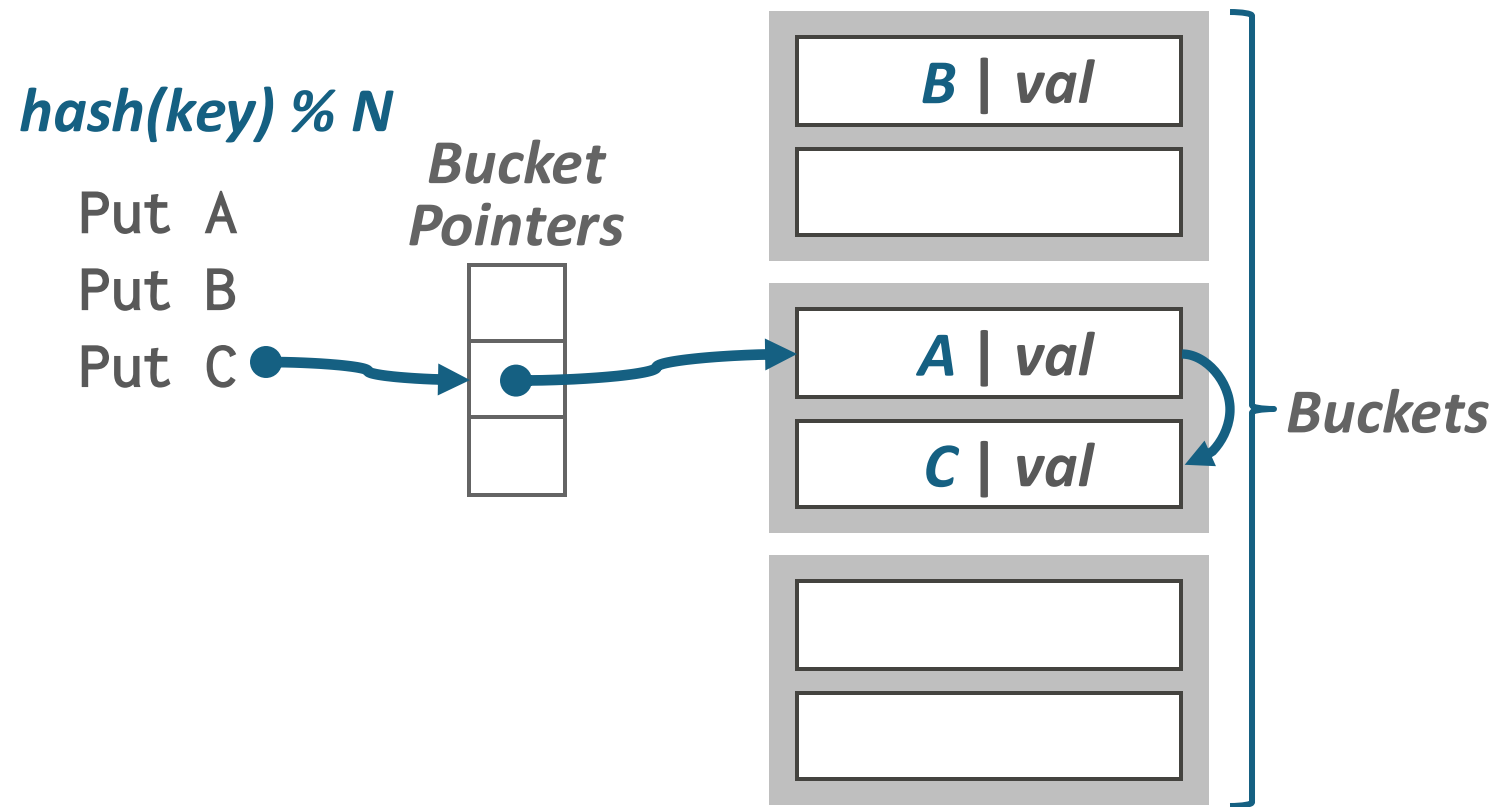
Chained Hashing



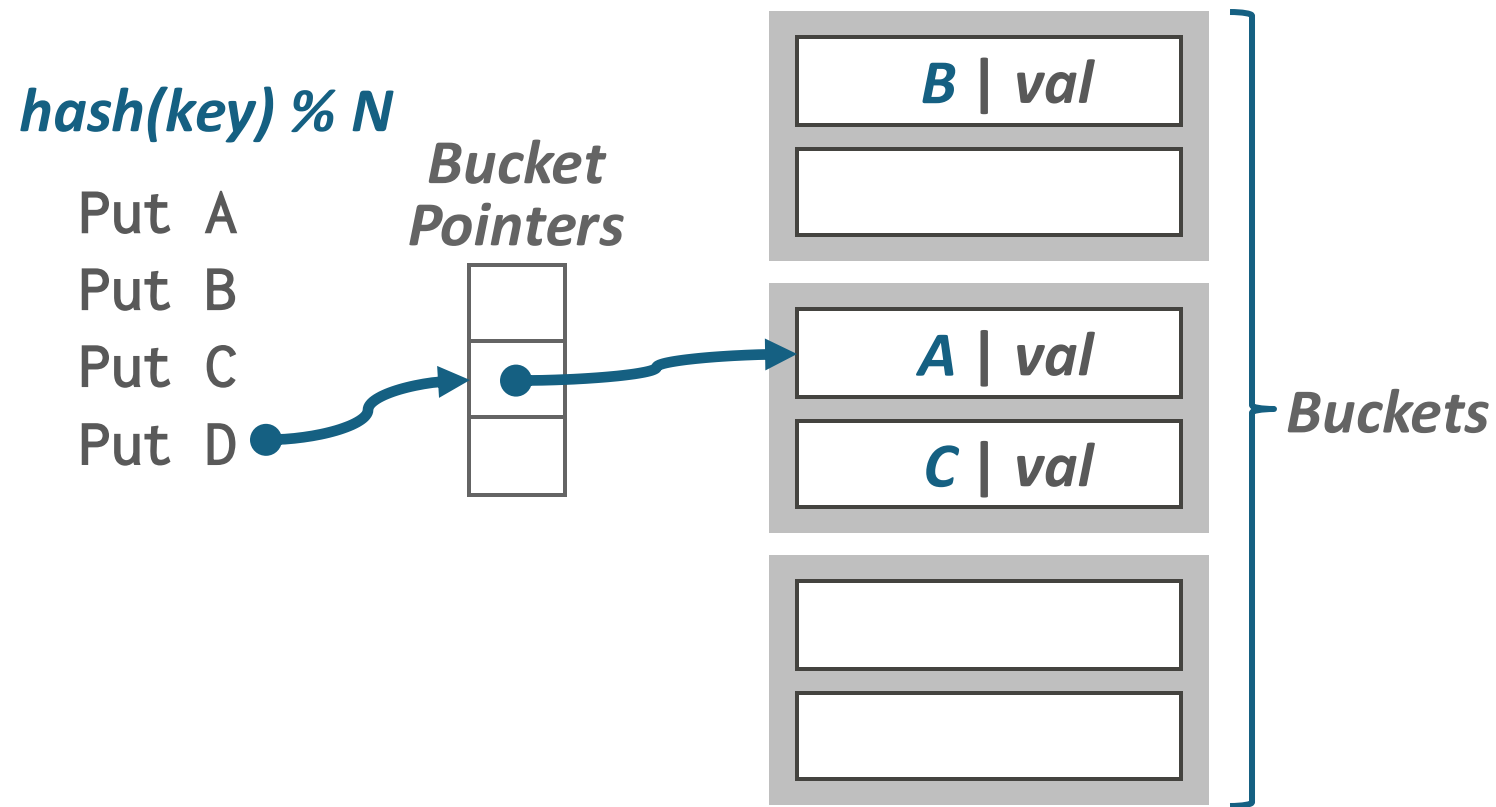
Chained Hashing



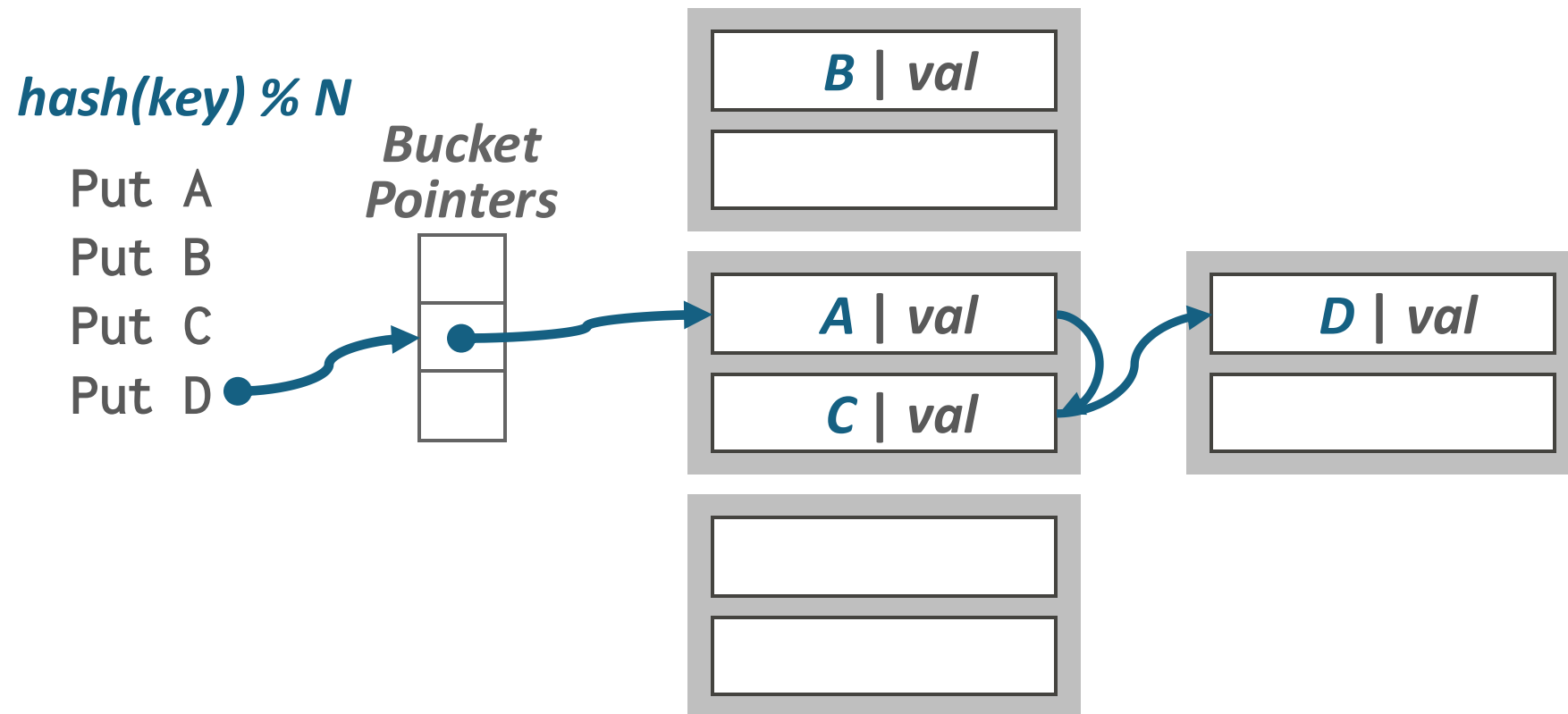
Chained Hashing



Chained Hashing



Chained Hashing



Chained Hashing

hash(key) % N

Put A

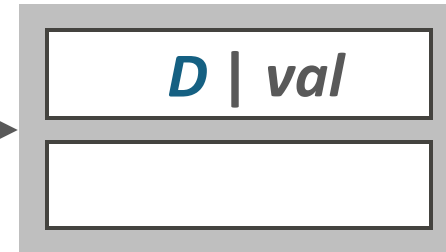
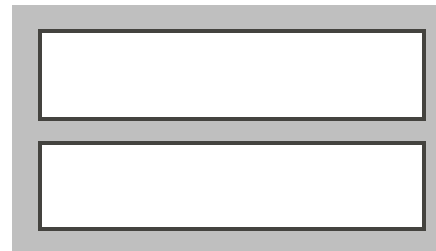
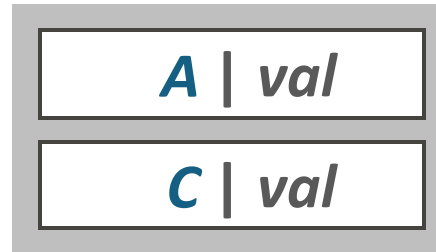
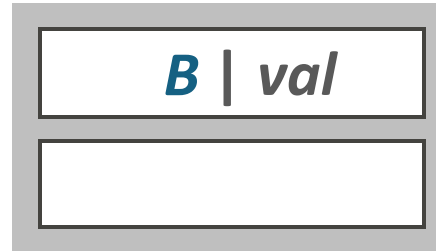
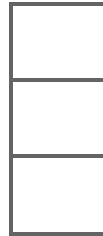
Put B

Put C

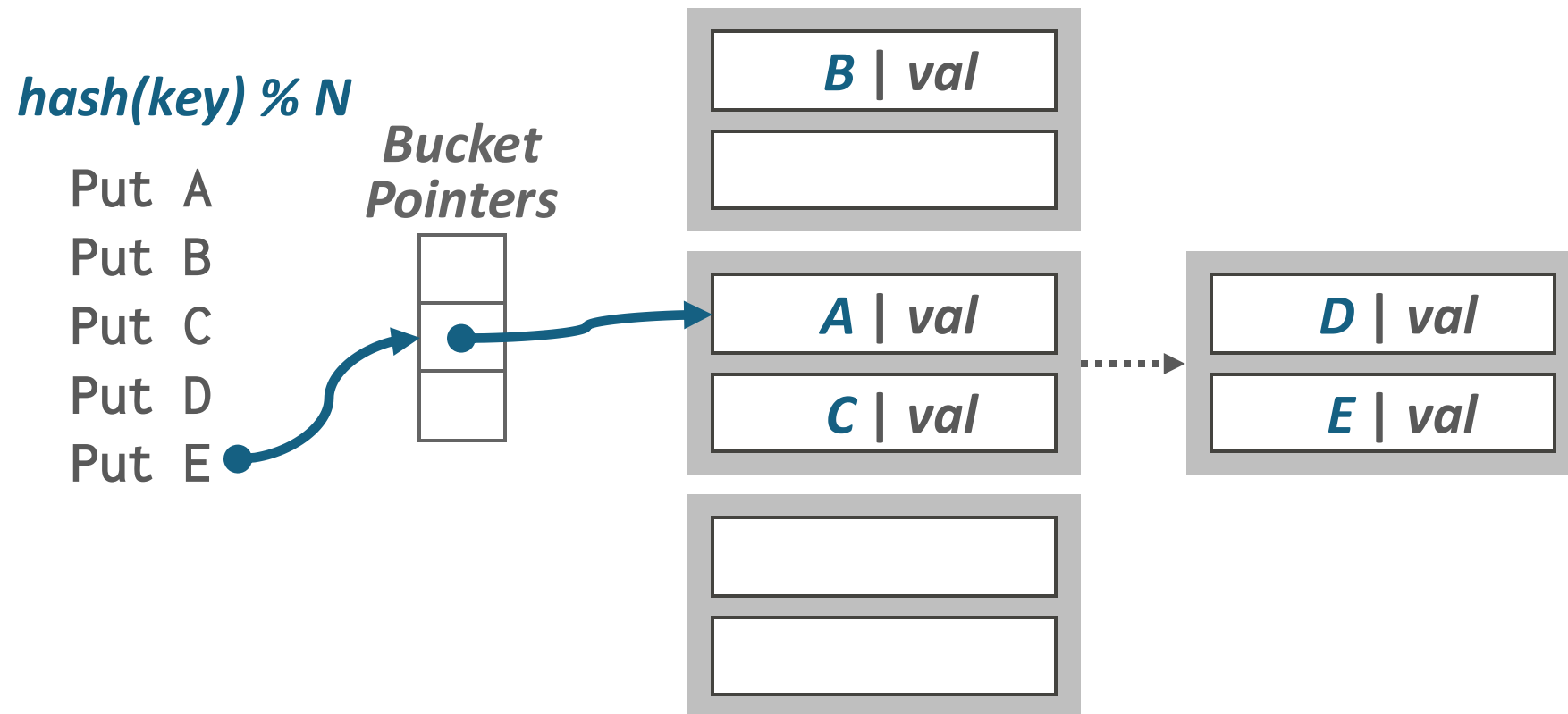
Put D

Put E

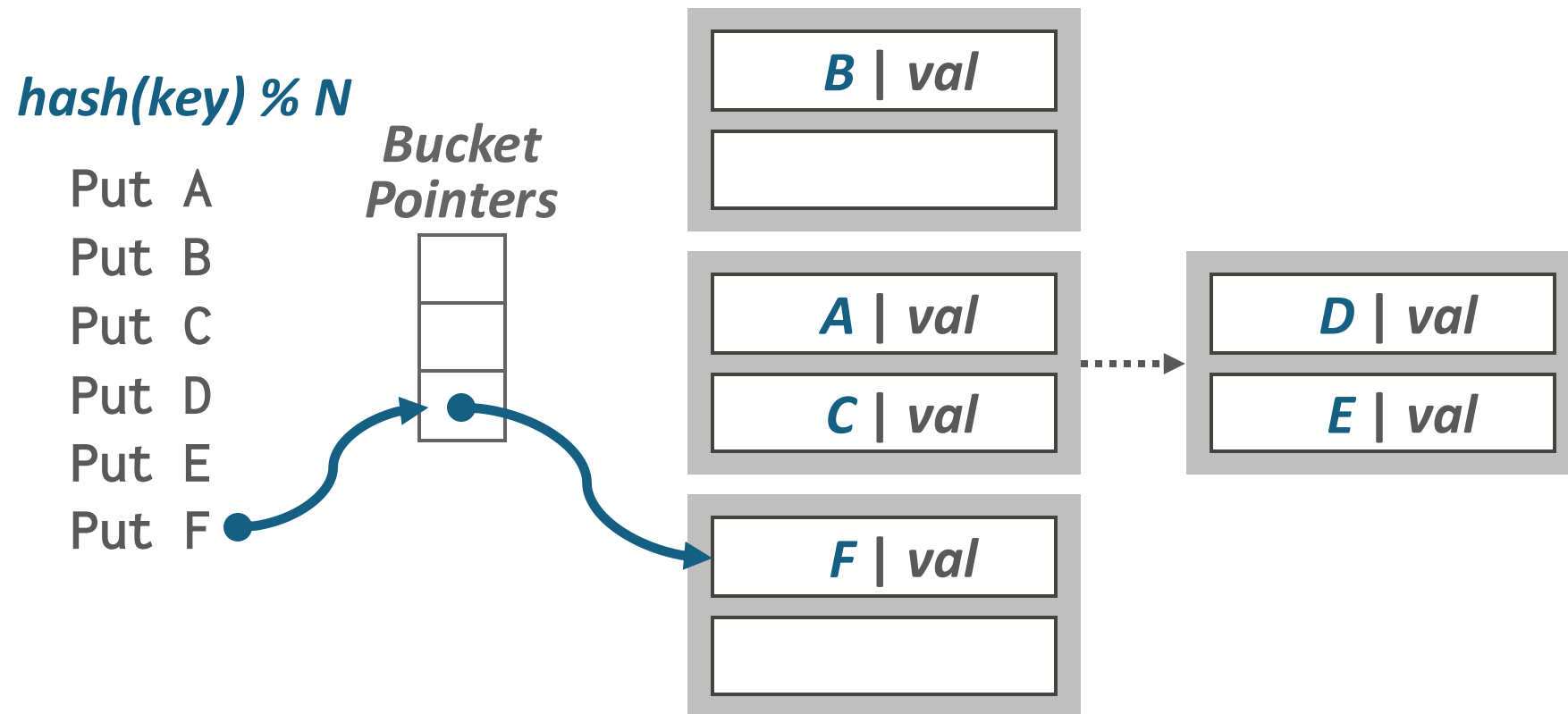
*Bucket
Pointers*



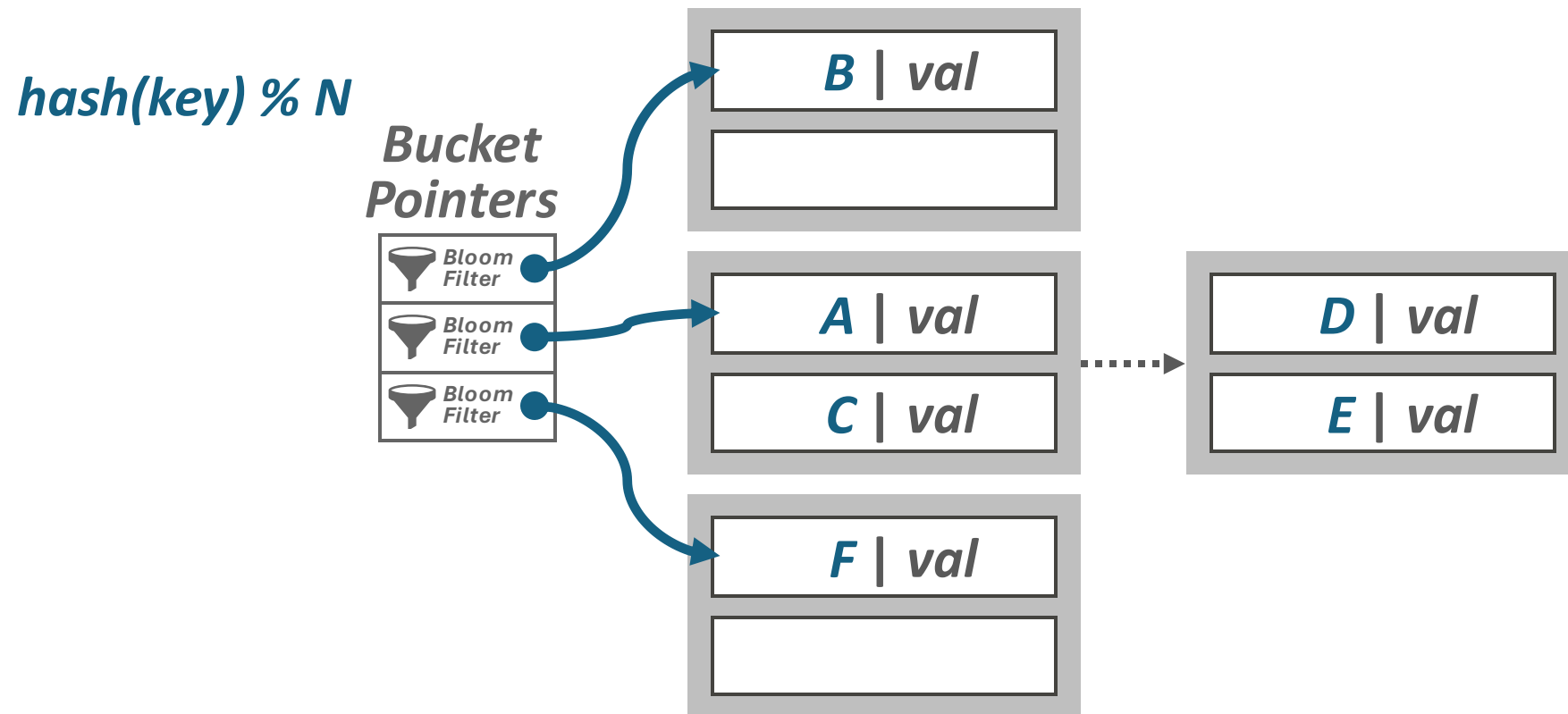
Chained Hashing



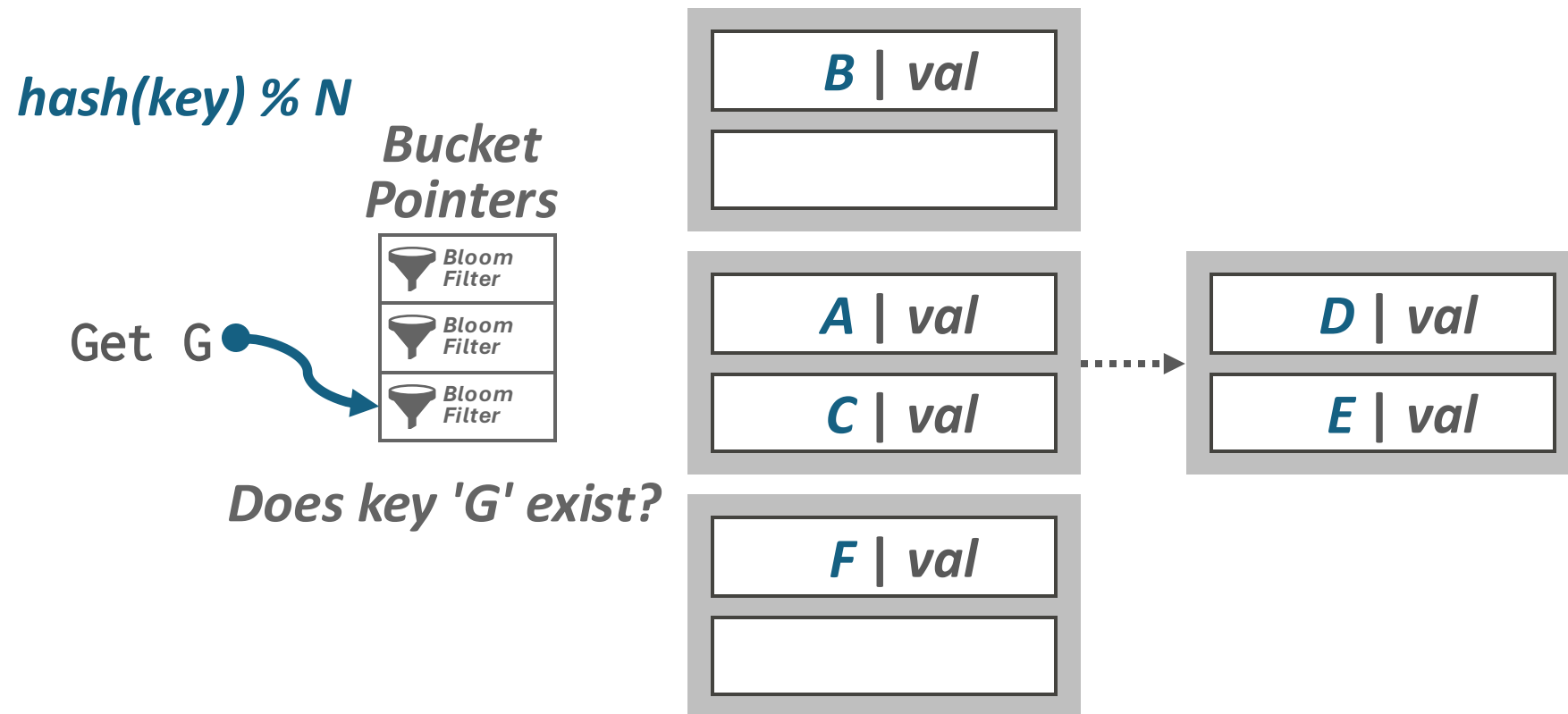
Chained Hashing



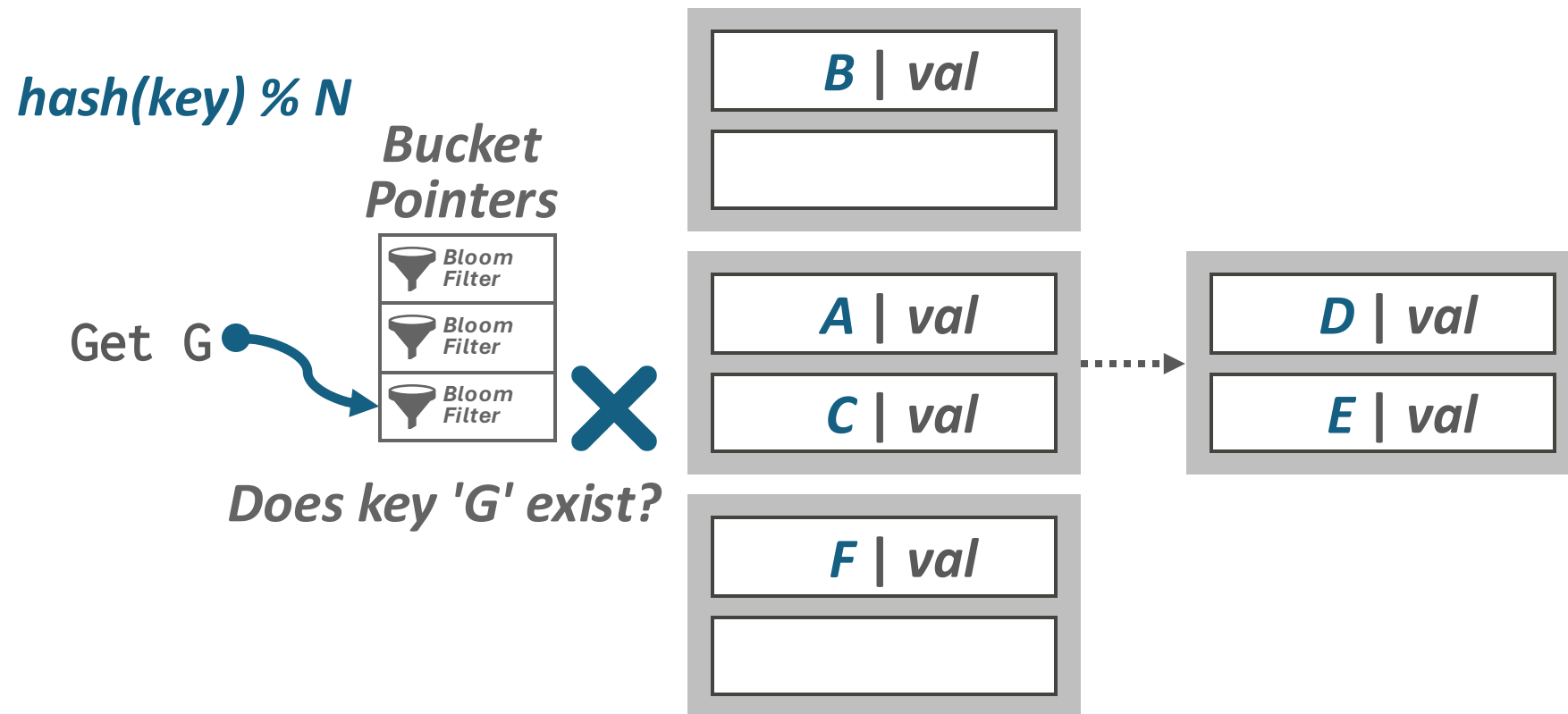
Chained Hashing



Chained Hashing



Chained Hashing



Bloom Filters

- Probabilistic data structure (bitmap) that answers set membership queries.
 - False negatives will **never occur**.
 - False positives can **sometimes occur**.
 - See [Bloom Filter Calculator](#).
- **Insert(x):**
 - Use k hash functions to set bits in the filter to 1.
- **Lookup(x):**
 - Check whether the bits are 1 for each hash function.

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0

- Insert('RZA')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0

- Insert('RZA')

$$hash_1('RZA') = 2222 \% 6$$

$$hash_2('RZA') = 4444 \% 8$$

$$= 4$$

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	1	0	1	0

$$hash_1('RZA') = 2222 \% 8 = 6$$

$$hash_2('RZA') = 4444 \% 8 = 4$$

- Insert('RZA')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	0	0	0	1	0	1	0

$$hash_1('GZA') = 5555 \% 8 = 3$$

$$hash_2('GZA') = 7777 \% 8 = 1$$

- Insert('RZA')
- Insert('GZA')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$hash_1('GZA') = 5555 \% 8 = 3$$

$$hash_2('GZA') = 7777 \% 8 = 1$$

- Insert('RZA')
- Insert('GZA')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

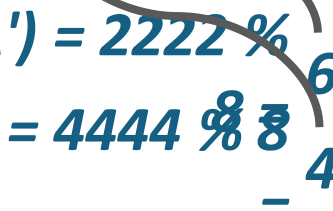
- Insert('RZA')
- Insert('GZA')
- Lookup('RZA')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$hash_1('RZA') = 2222 \% 8 = 6$$

$$hash_2('RZA') = 4444 \% 8 = 4$$


- Insert('RZA')
- Insert('GZA')
- Lookup('RZA')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$\begin{aligned} \text{hash}_1('RZA') &= 2222 \% 8 = 6 \\ \text{hash}_2('RZA') &= 4444 \% 8 = 4 \end{aligned}$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon')

Bloom Filters

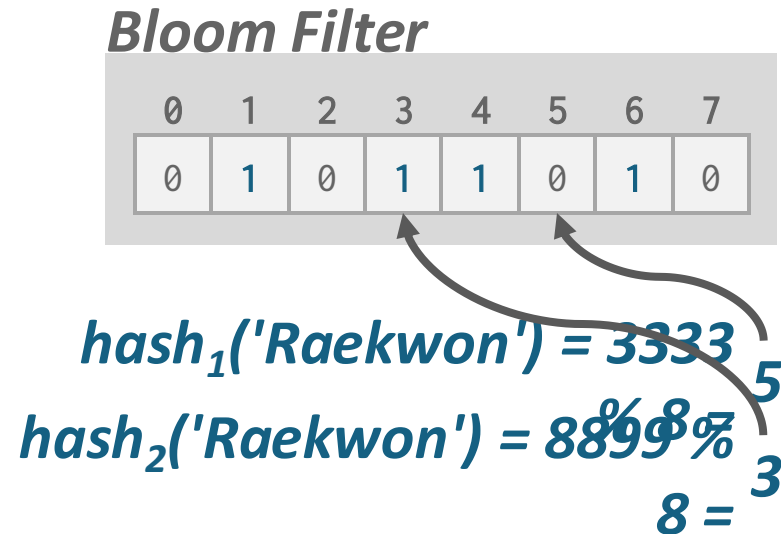
Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$\begin{aligned}
 \text{hash}_1(\text{'Raekwon'}) &= 3333 \\
 \text{hash}_2(\text{'Raekwon'}) &= 8899 \\
 8 &= 3
 \end{aligned}$$

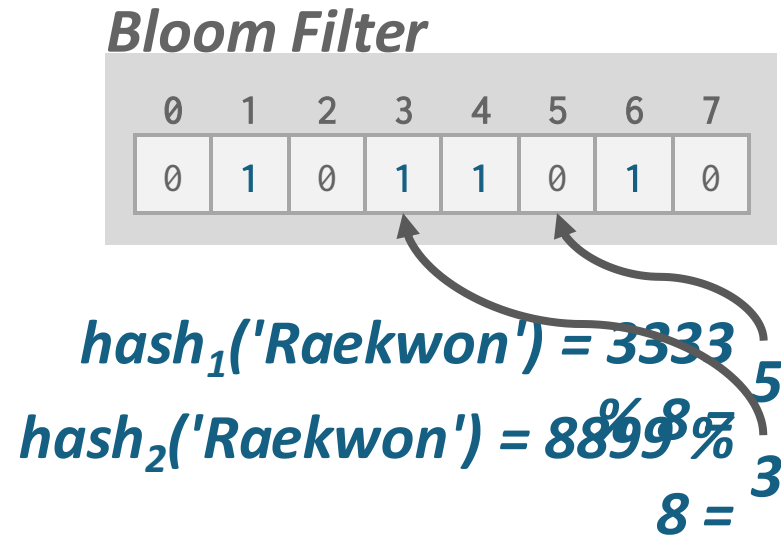
- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon')

Bloom Filters



- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon')

Bloom Filters



- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$\text{hash}_1('ODB') = 6699 \% 8 = 3$$

$$\text{hash}_2('ODB') = 9966 \% 8 = 6$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**
- Lookup('ODB')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$\text{hash}_1(\text{'ODB'}) = 6699 \% 8 = 3$$

$$\text{hash}_2(\text{'ODB'}) = 9966 \% 8 = 6$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**
- Lookup('ODB')

Bloom Filters

Bloom Filter

0	1	2	3	4	5	6	7
0	1	0	1	1	0	1	0

$$\text{hash}_1('ODB') = 6699 \% 8 = 3$$

$$\text{hash}_2('ODB') = 9966 \% 8 = 6$$

- Insert('RZA')
- Insert('GZA')
- Lookup('RZA') → **TRUE**
- Lookup('Raekwon') → **FALSE**
- Lookup('ODB') → **TRUE** (*false positive*)

Extendible Hashing

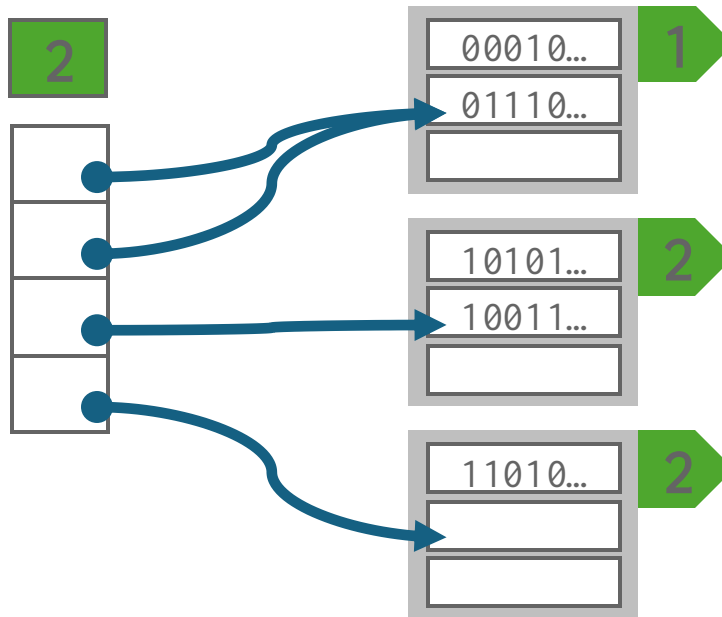
Dynamic Hashing Schemes

Extendible Hashing

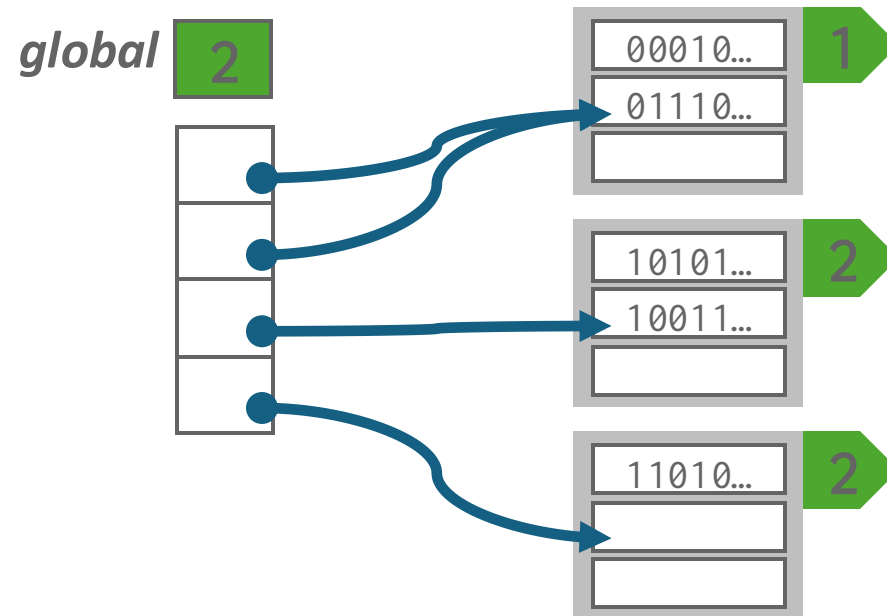
- Extendible-hashing approach that splits buckets incrementally instead of letting the linked list grow forever.
- Multiple slot locations can point to the same bucket chain.
- Reshuffle bucket entries on split and increase the number of bits to examine.
 - Data movement is localized to just the split chain.



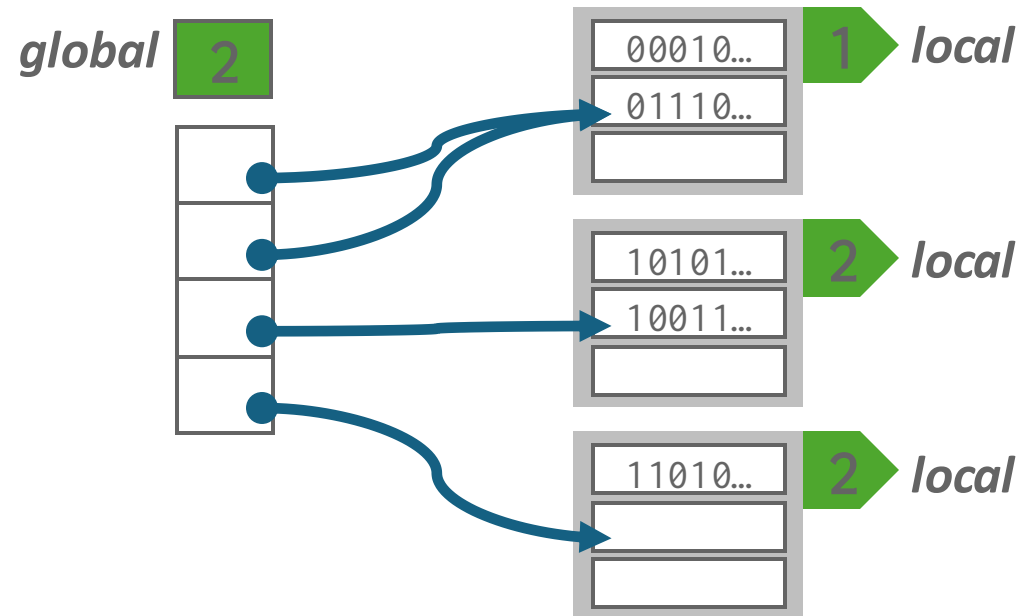
Extendible Hashing



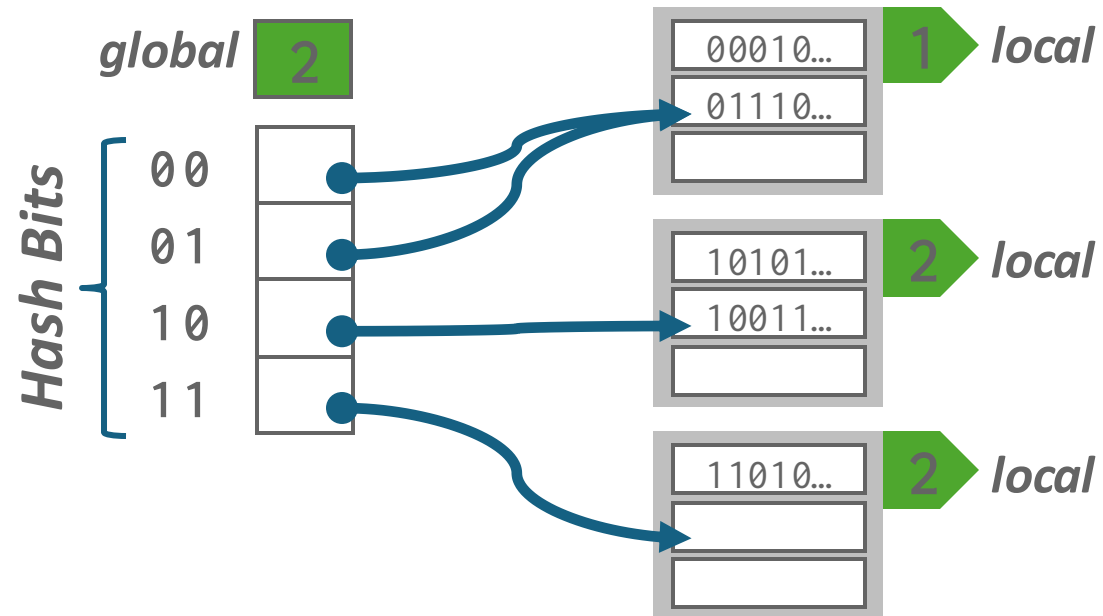
Extendible Hashing



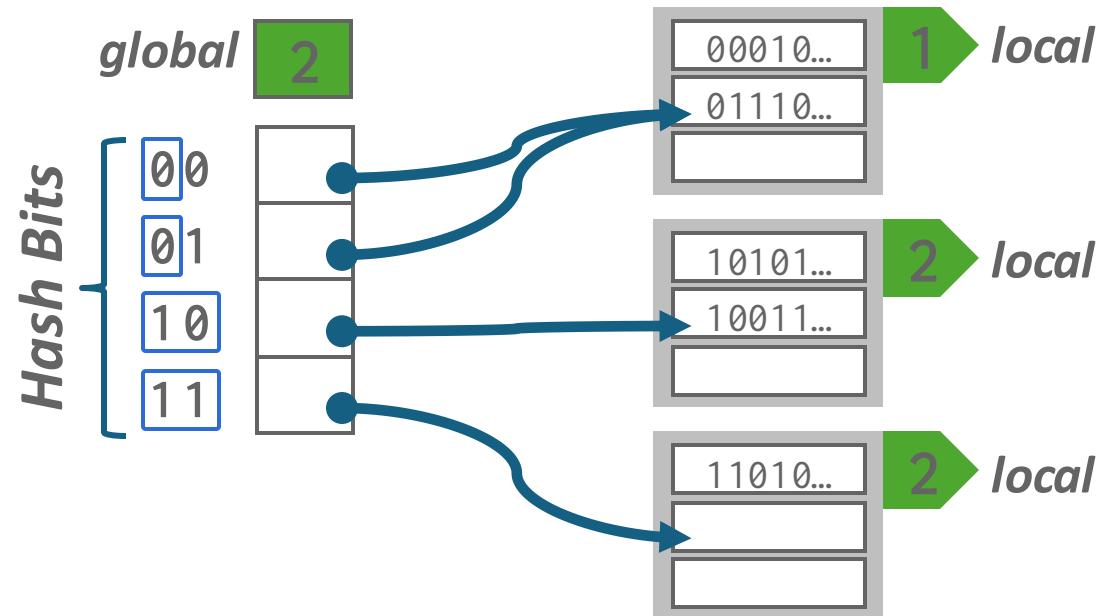
Extendible Hashing



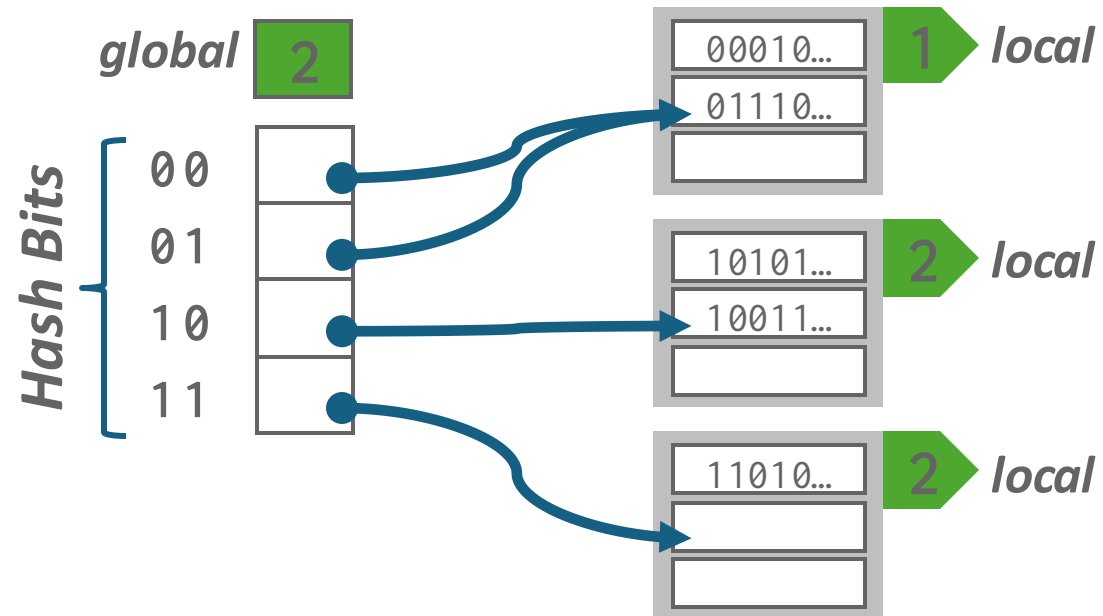
Extendible Hashing



Extendible Hashing

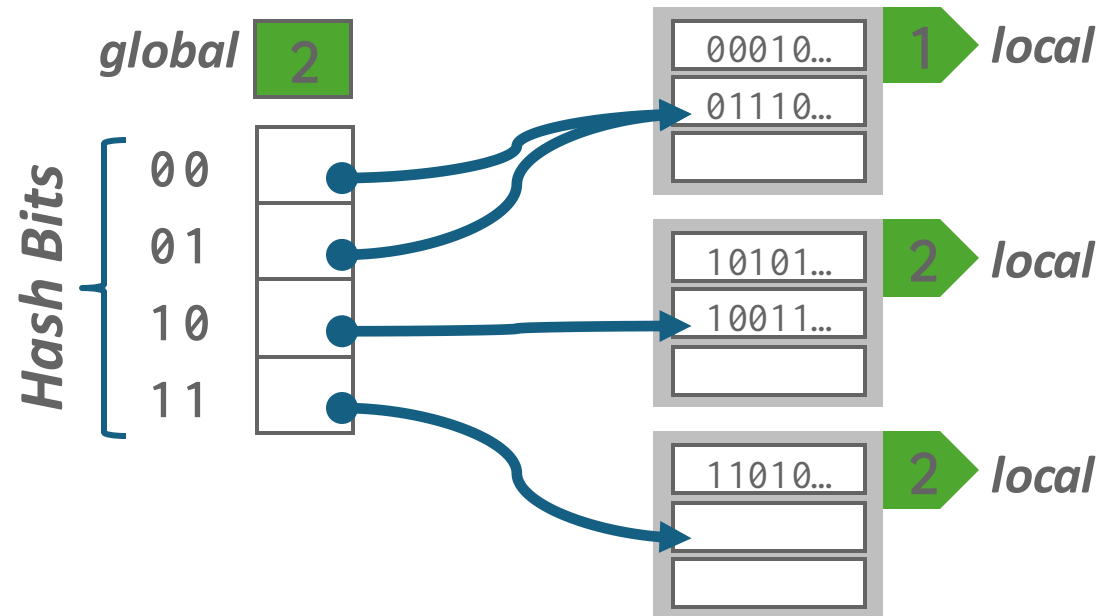


Extendible Hashing



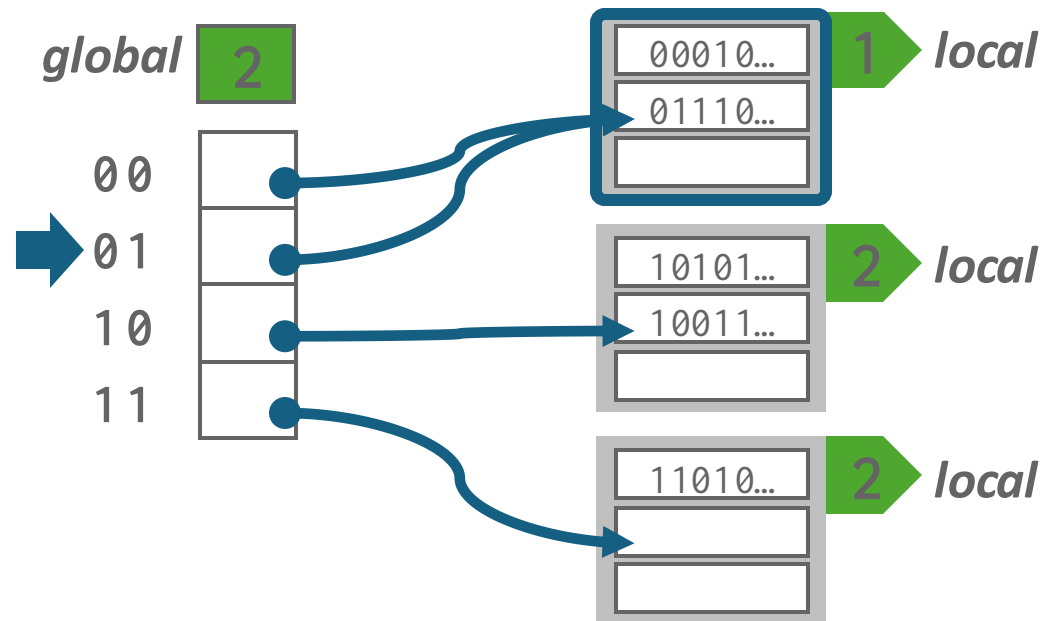
Get A
hash(A) = 01110...

Extendible Hashing



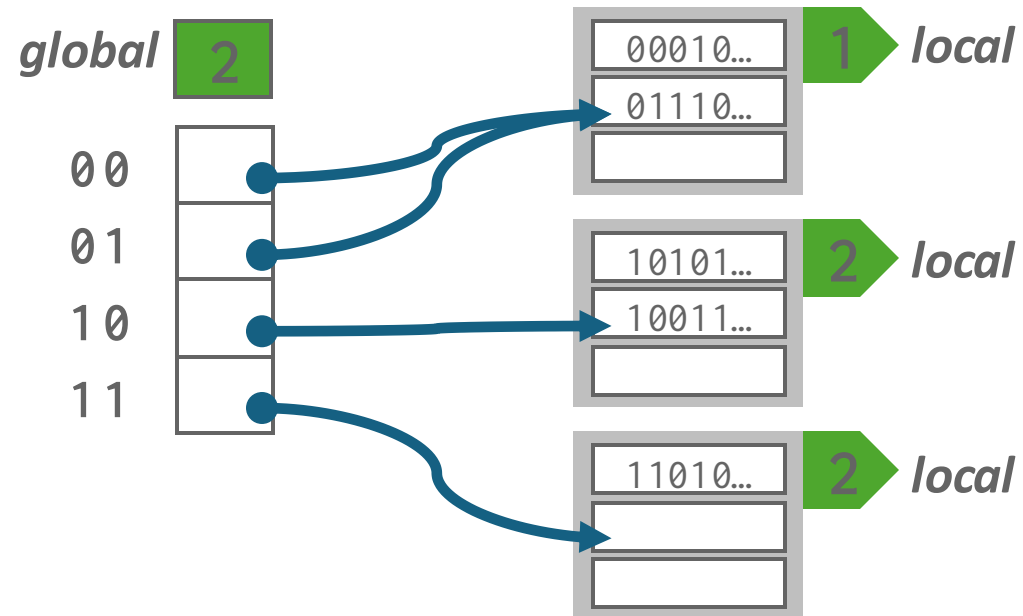
Get A
 $hash(A) = 0^1 110...$

Extendible Hashing



Get A
 $hash(A) = 01110...$

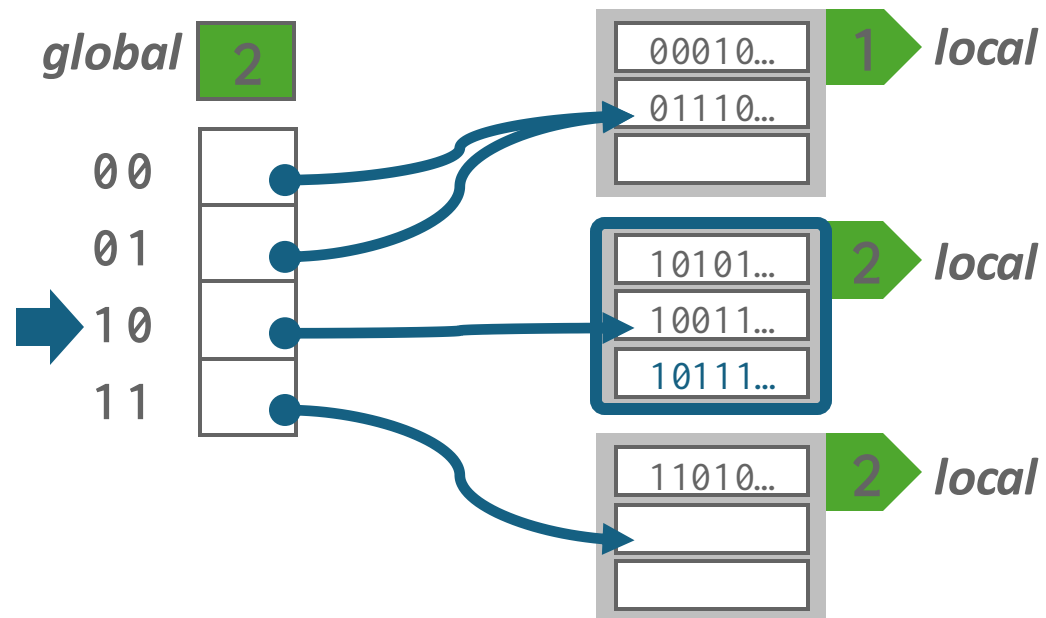
Extendible Hashing



Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

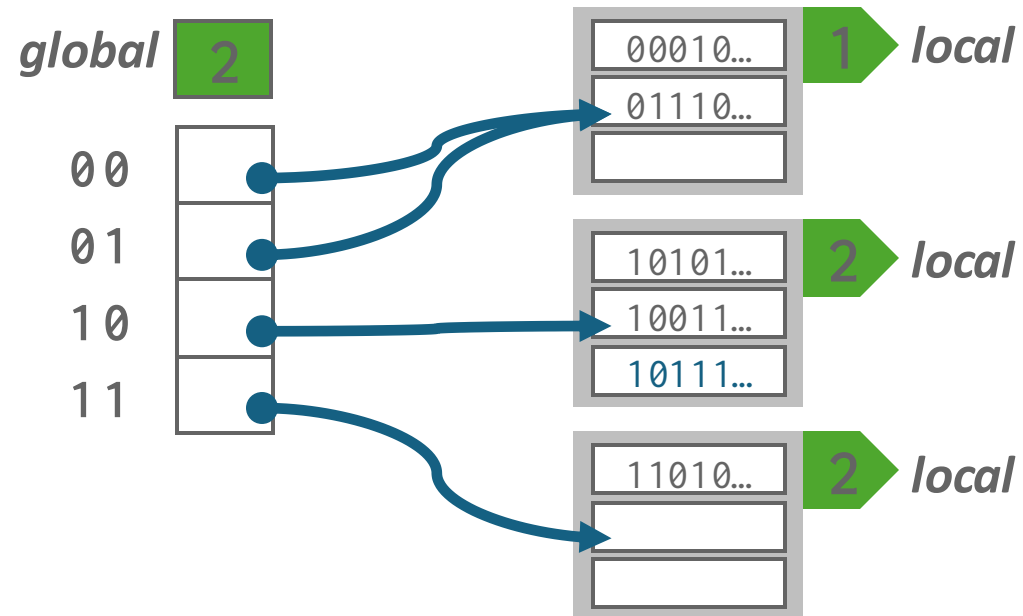
Extendible Hashing



Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Extendible Hashing

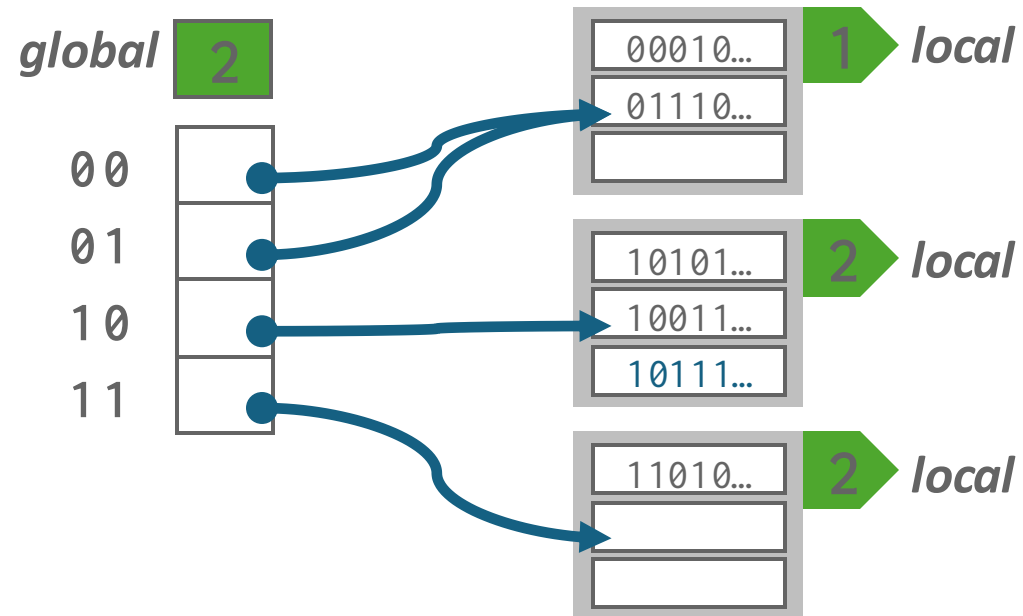


Get A
 $hash(A) = 01110\dots$

Put B
 $hash(B) = 10111\dots$

Put C
 $hash(C) = 10100\dots$

Extendible Hashing

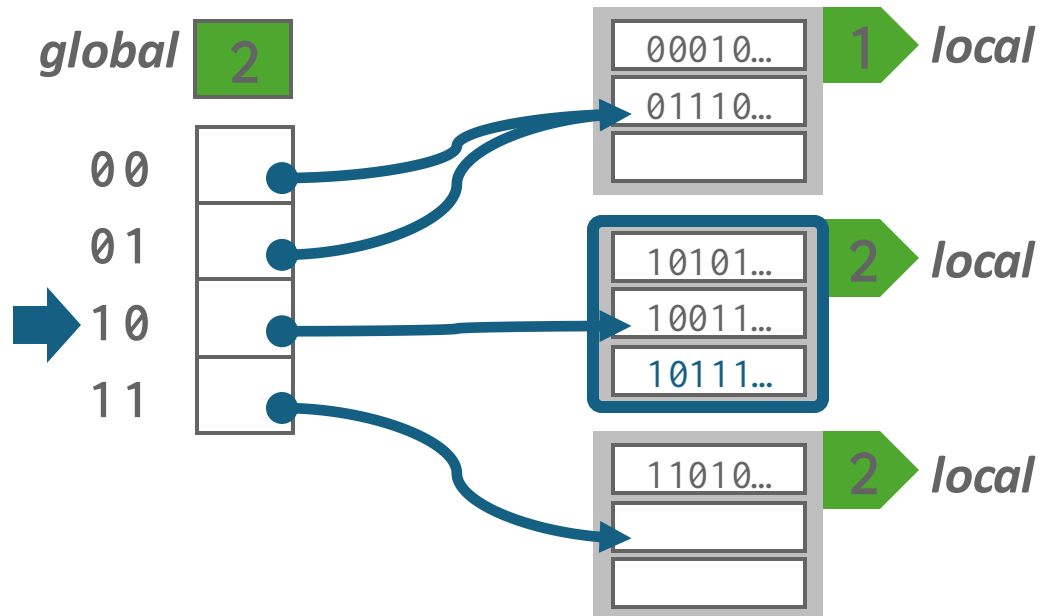


Get A
 $hash(A) = 01110\dots$

Put B
 $hash(B) = 10111\dots$

Put C
 $hash(C) = 10100\dots$

Extendible Hashing

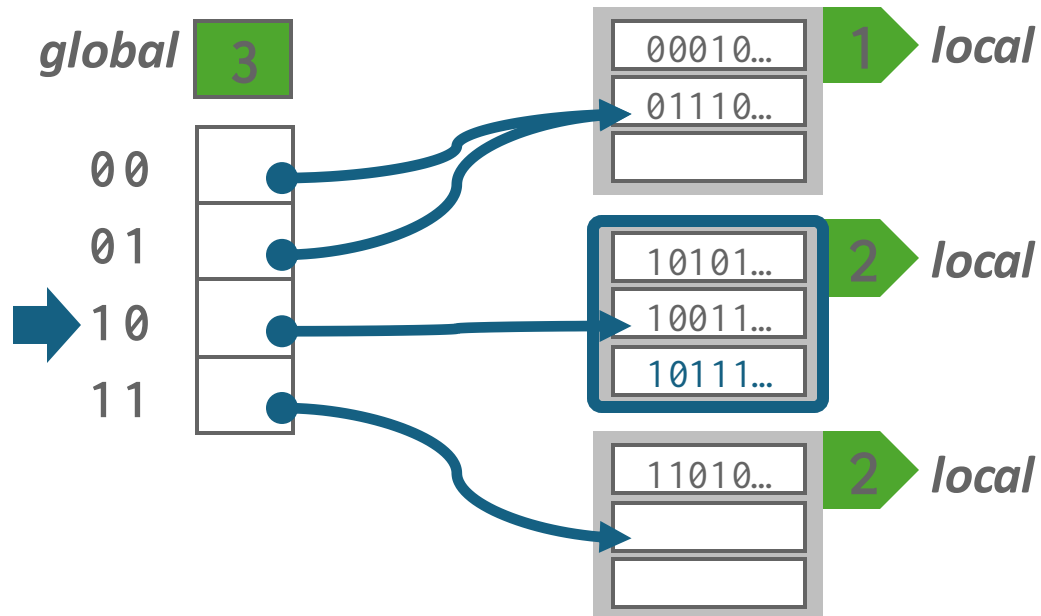


Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Put C
 $hash(C) = 10100...$

Extendible Hashing

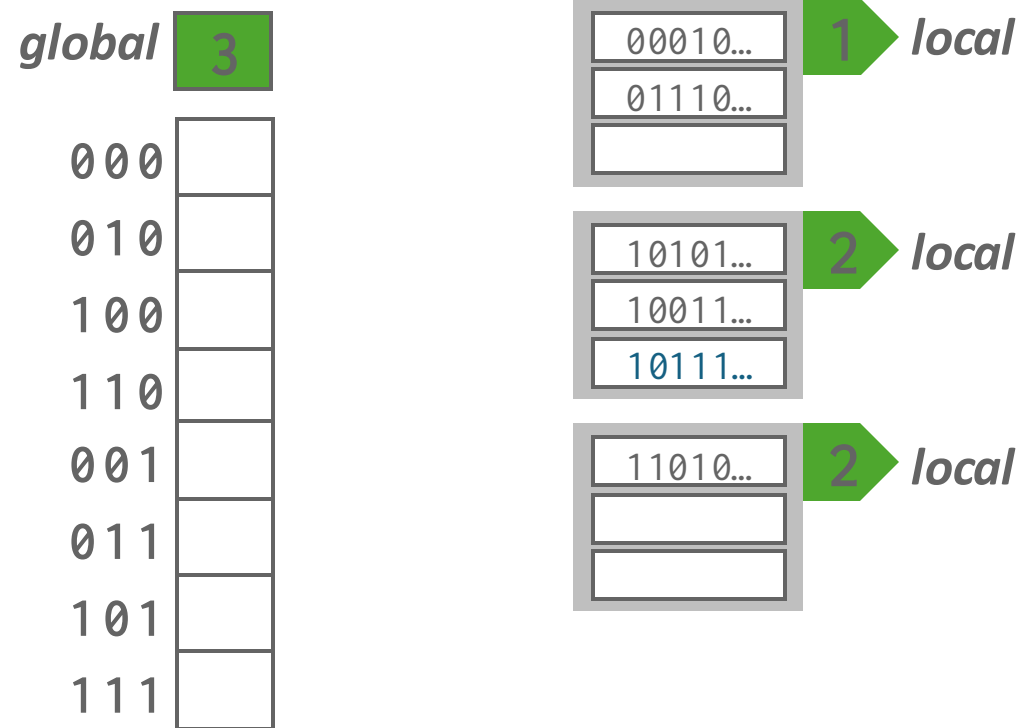


Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Put C
 $hash(C) = 10100...$

Extendible Hashing

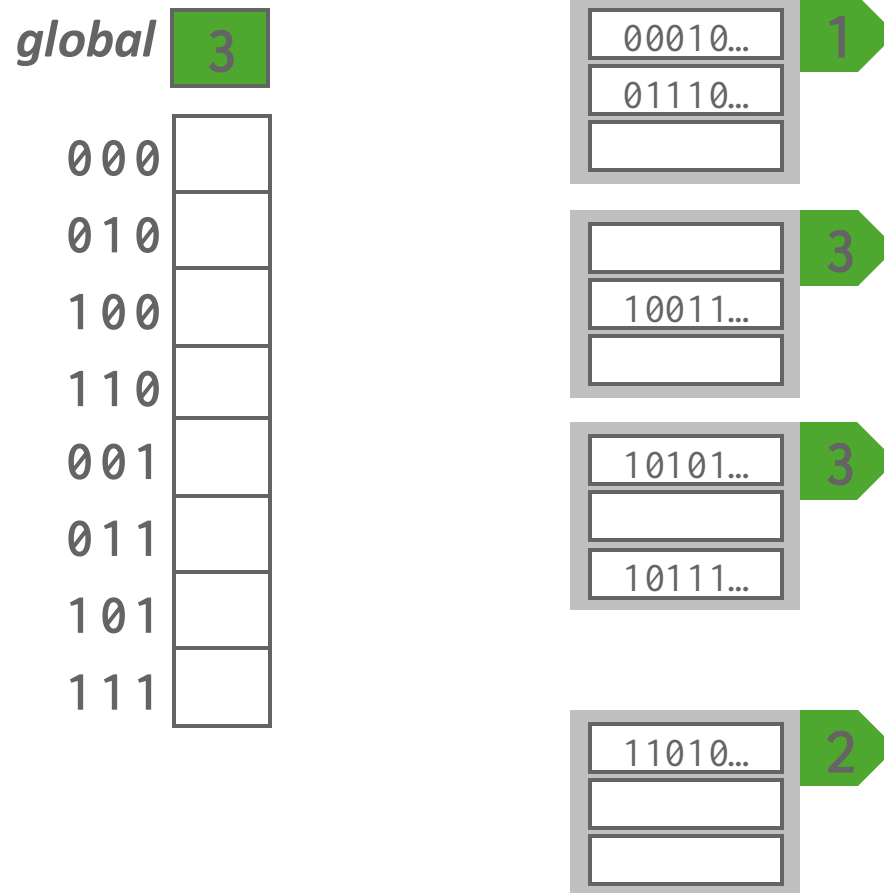


Get A
hash(A) = 01110...

Put B
hash(B) = 10111...

Put C
hash(C) = 10100...

Extendible Hashing

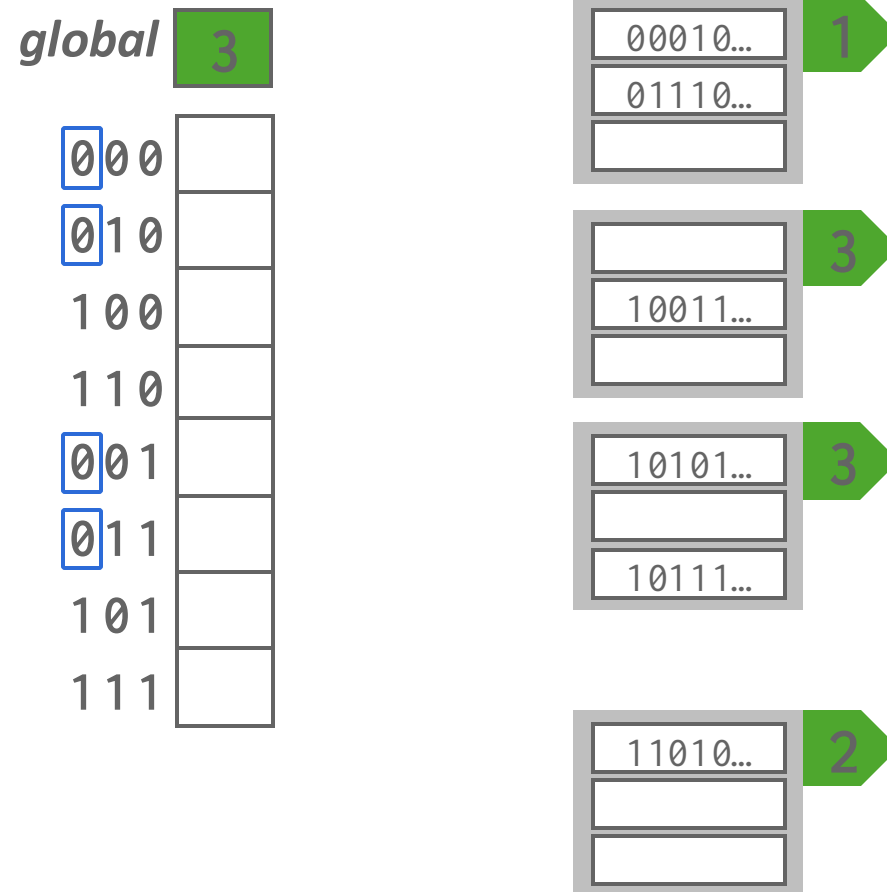


Get A
hash(A) = 01110...

Put B
hash(B) = 10111...

Put C
hash(C) = 10100...

Extendible Hashing

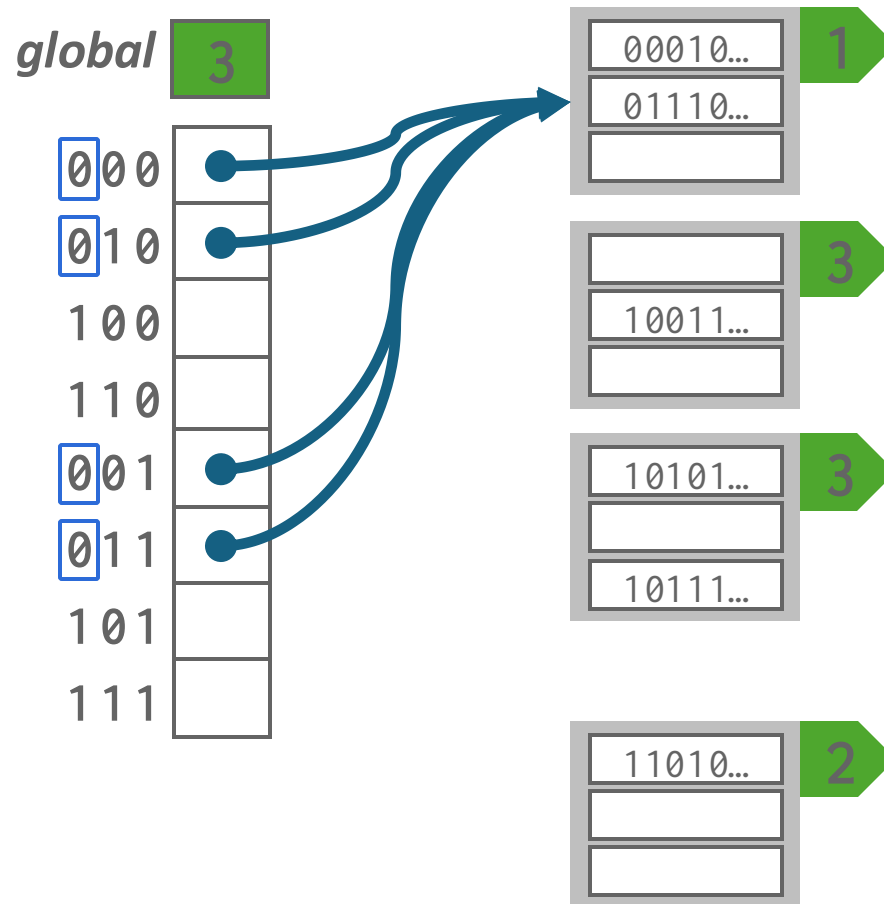


Get A
hash(A) = 01110...

Put B
hash(B) = 10111...

Put C
hash(C) = 10100...

Extendible Hashing

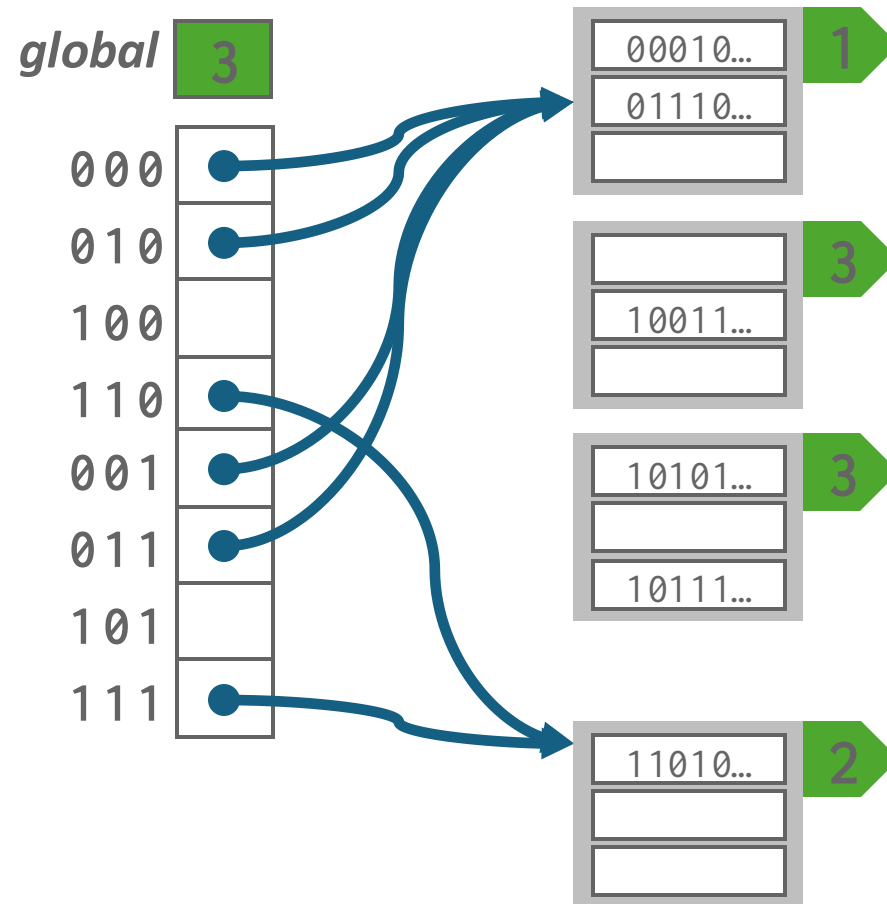


Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Put C
 $hash(C) = 10100...$

Extendible Hashing

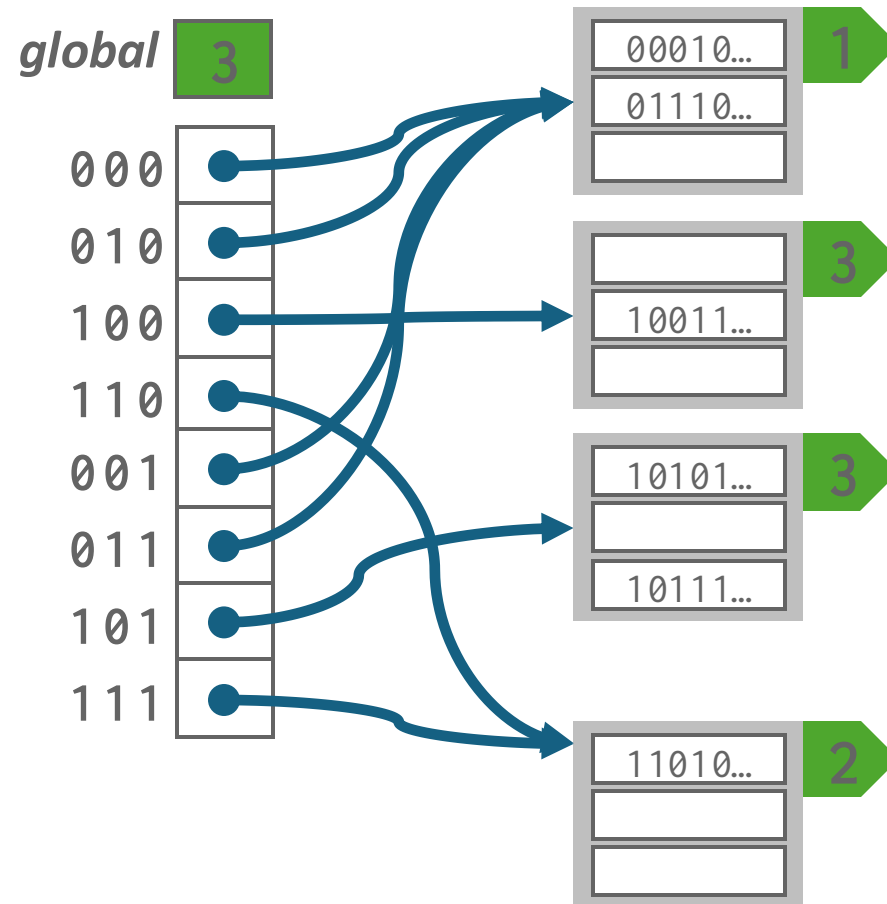


Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Put C
 $hash(C) = 10100...$

Extendible Hashing

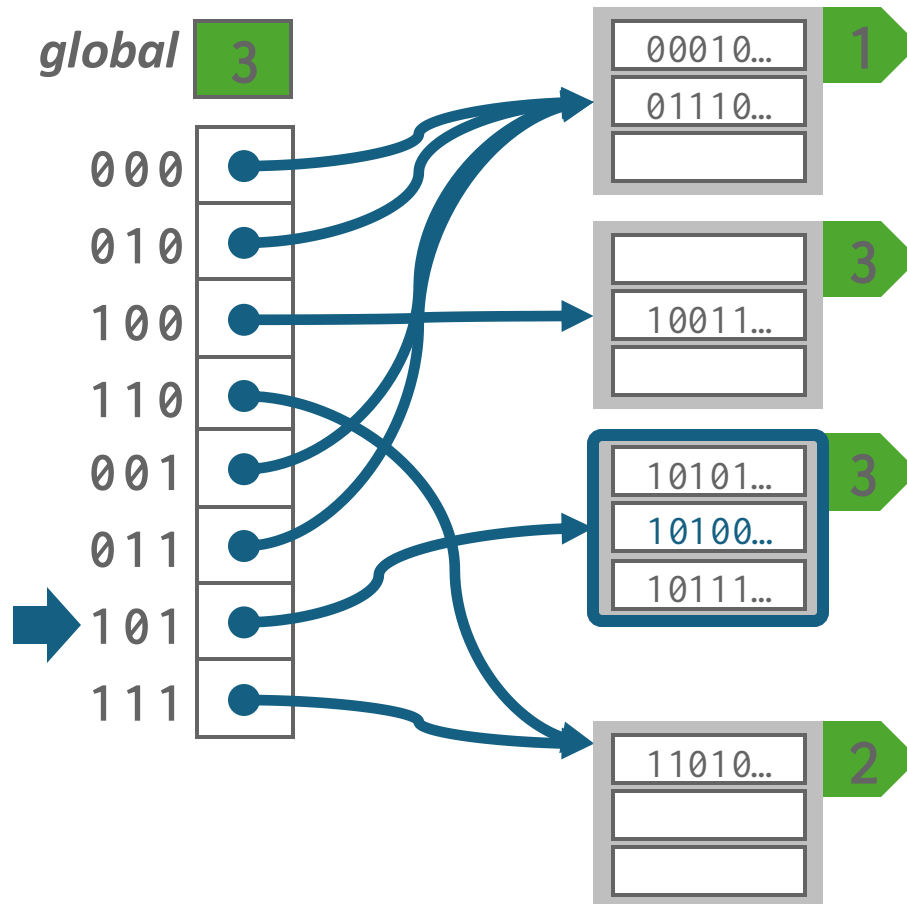


Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Put C
 $hash(C) = 10100...$

Extendible Hashing



Get A
 $hash(A) = 01110...$

Put B
 $hash(B) = 10111...$

Put C
 $hash(C) = 10100...$

Linear Hashing

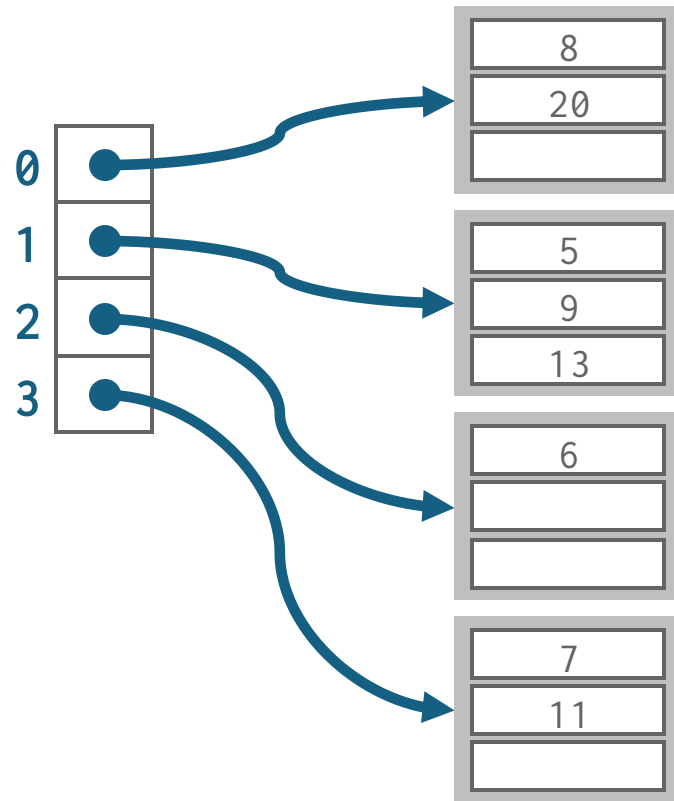
Dynamic Hashing Schemes

Linear Hashing

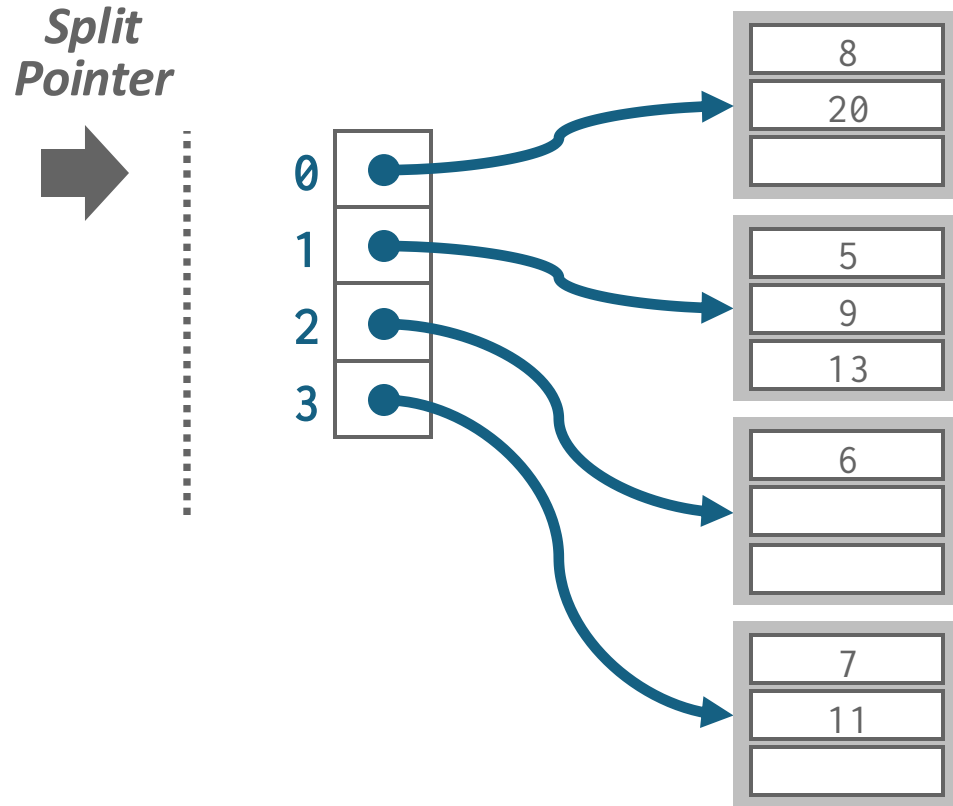
- The hash table maintains a pointer that tracks the next bucket to split.
 - When any bucket overflows, split the bucket at the pointer location.
- Use multiple hashes to find the right bucket for a given key.
- Can use different overflow criterion:
 - Space Utilization
 - Average Length of Overflow Chains



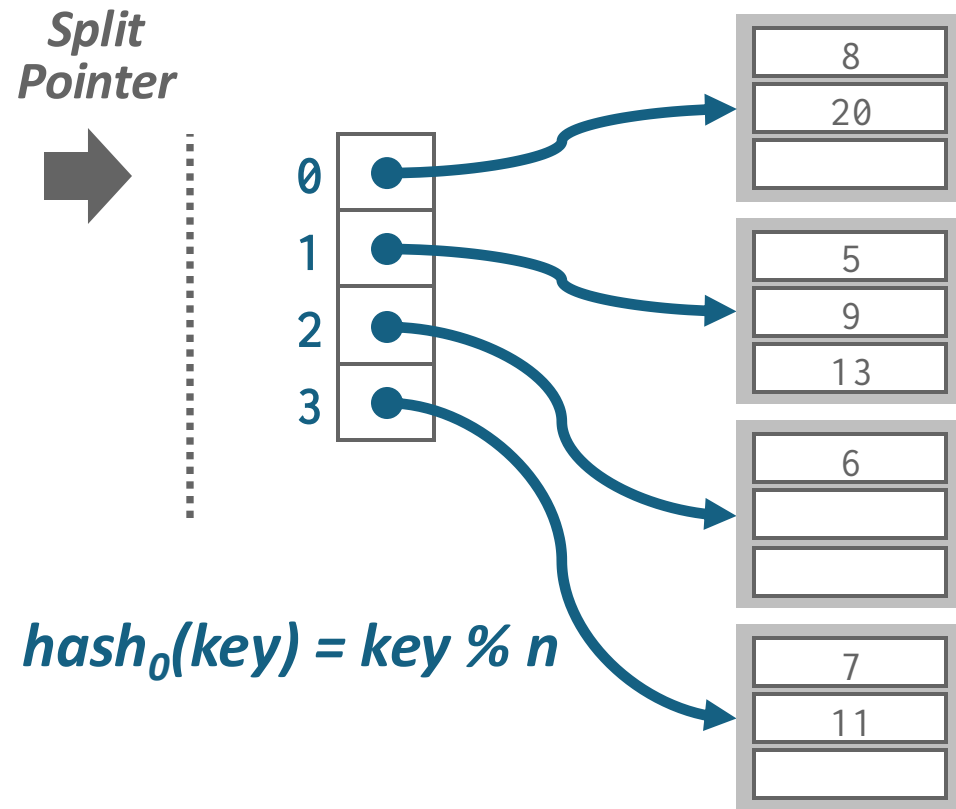
Linear Hashing



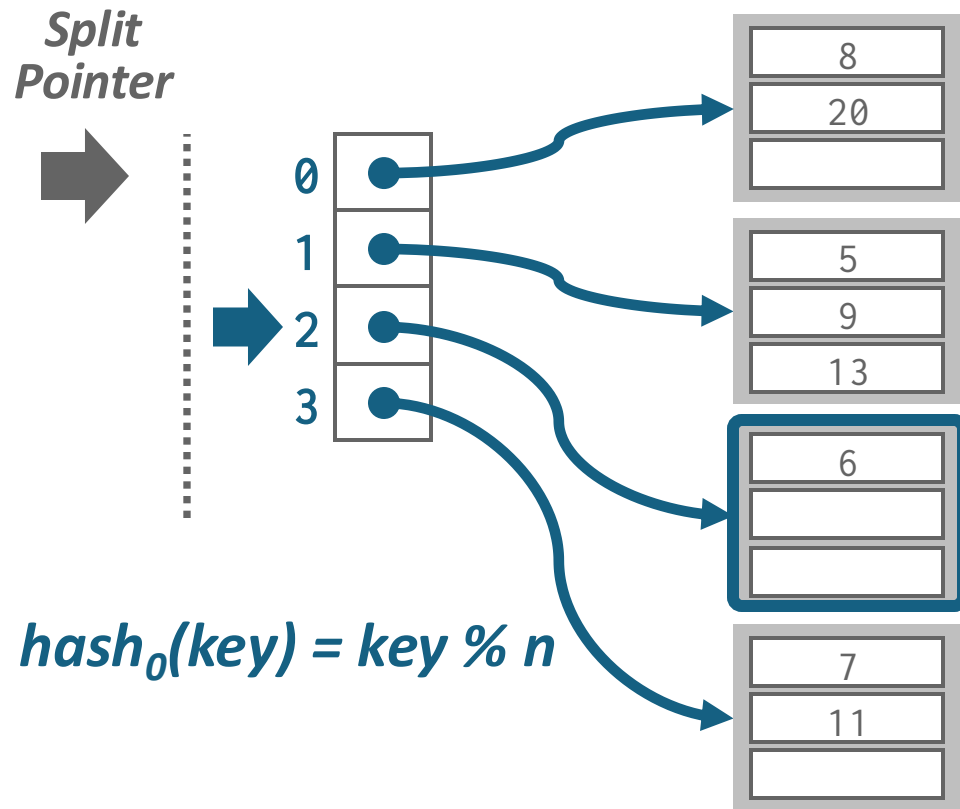
Linear Hashing



Linear Hashing

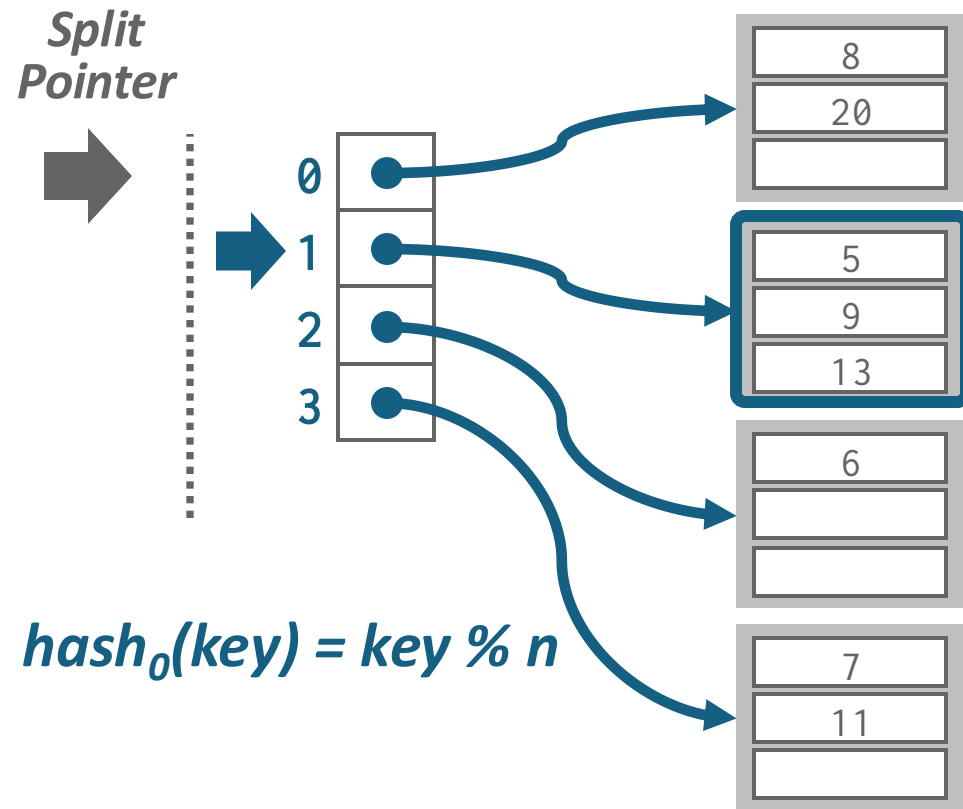


Linear Hashing



Get 6
 $hash_0(6) = 6 \% 4 = 2$

Linear Hashing



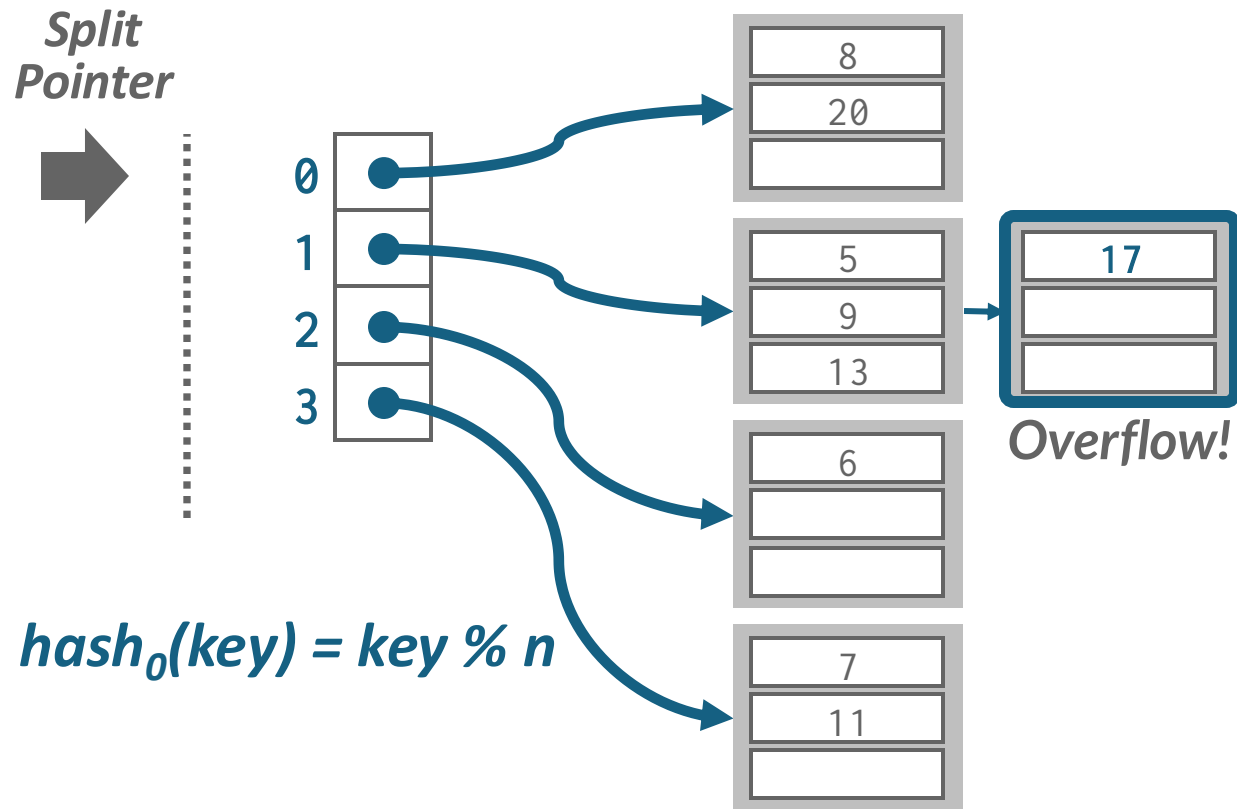
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

Linear Hashing



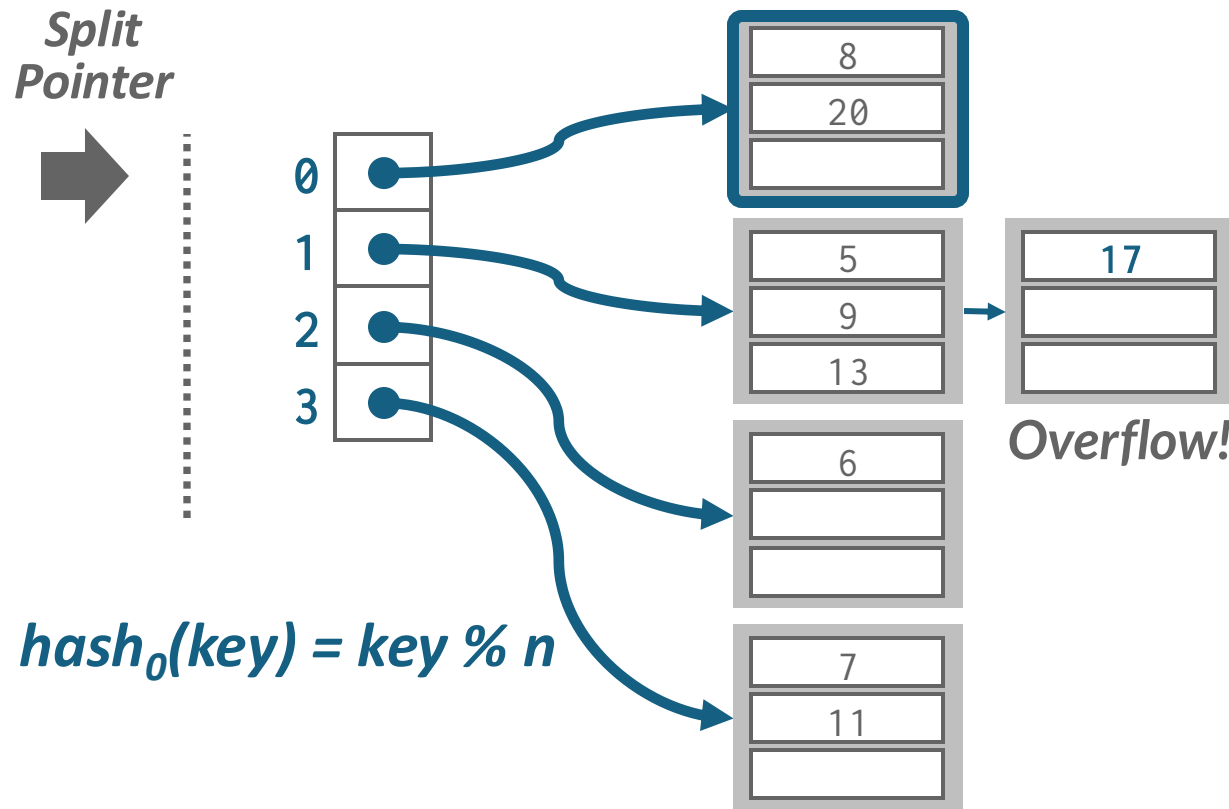
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

Linear Hashing



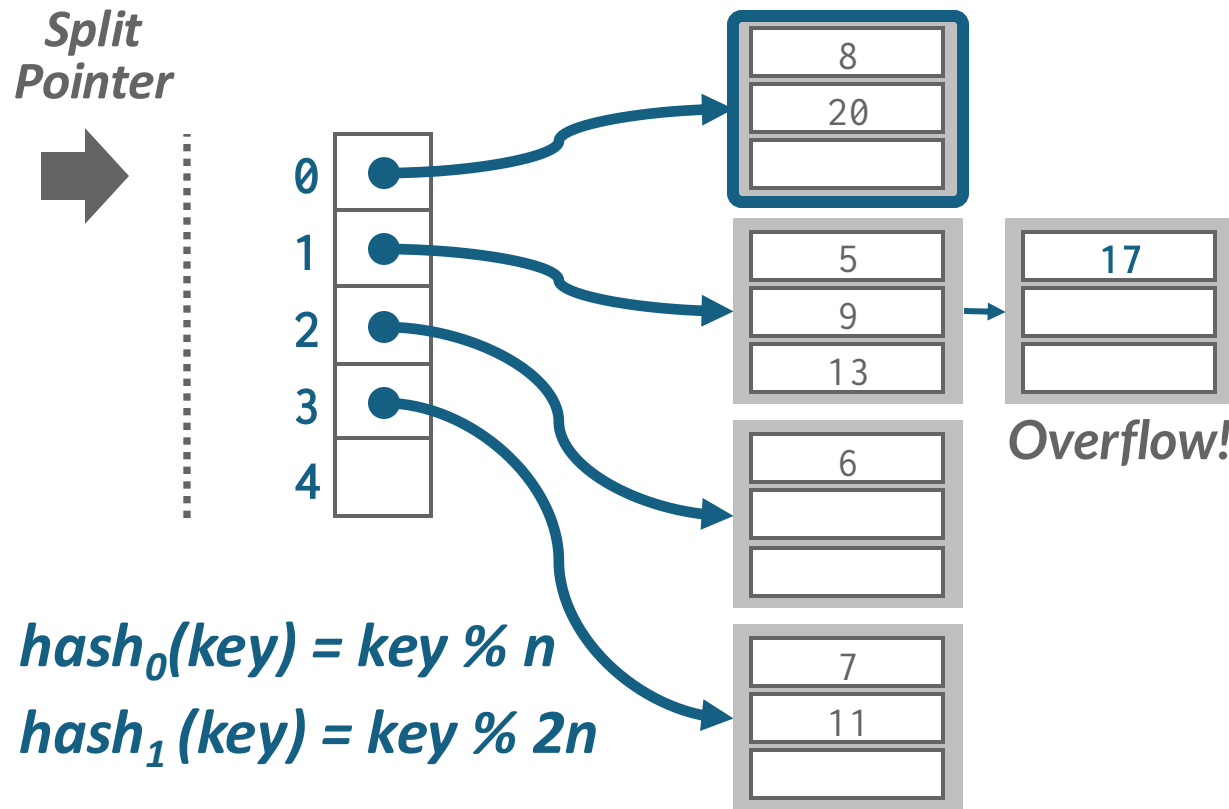
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

Linear Hashing



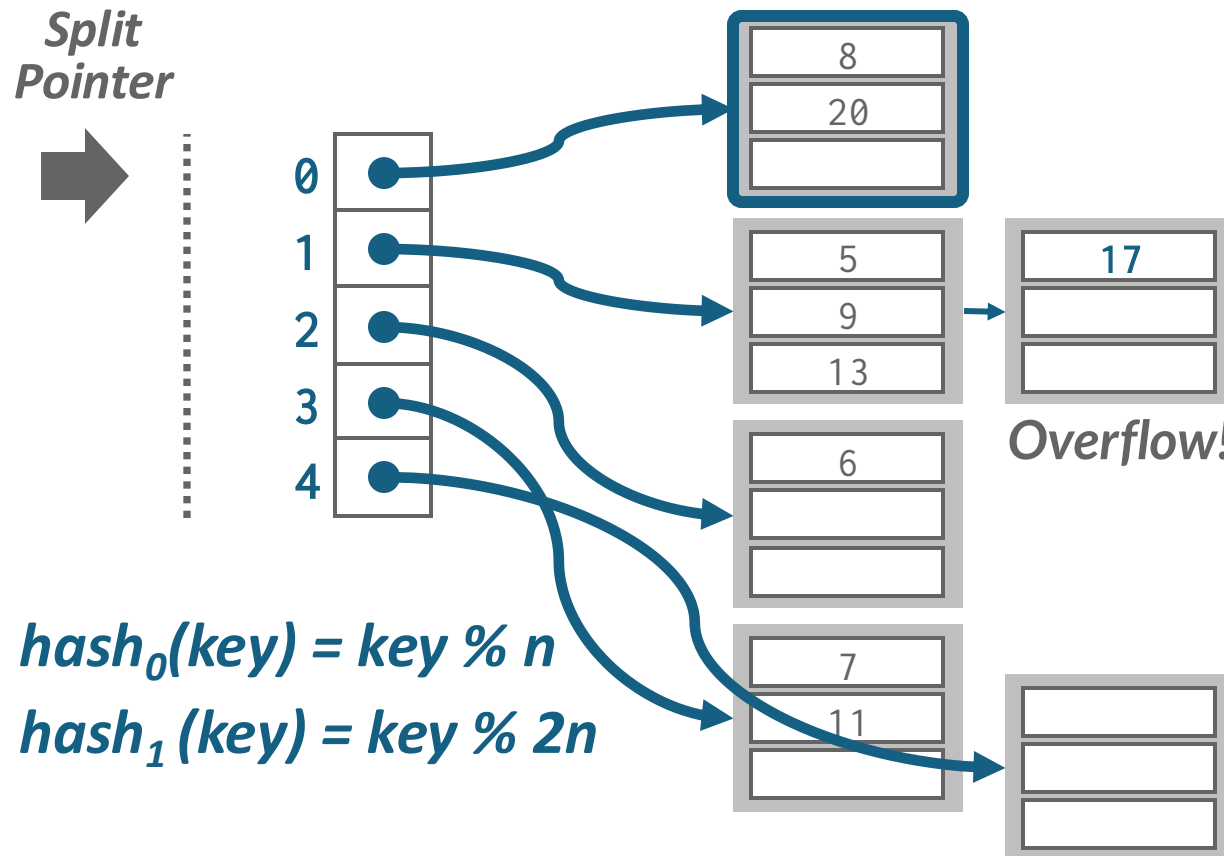
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

Linear Hashing



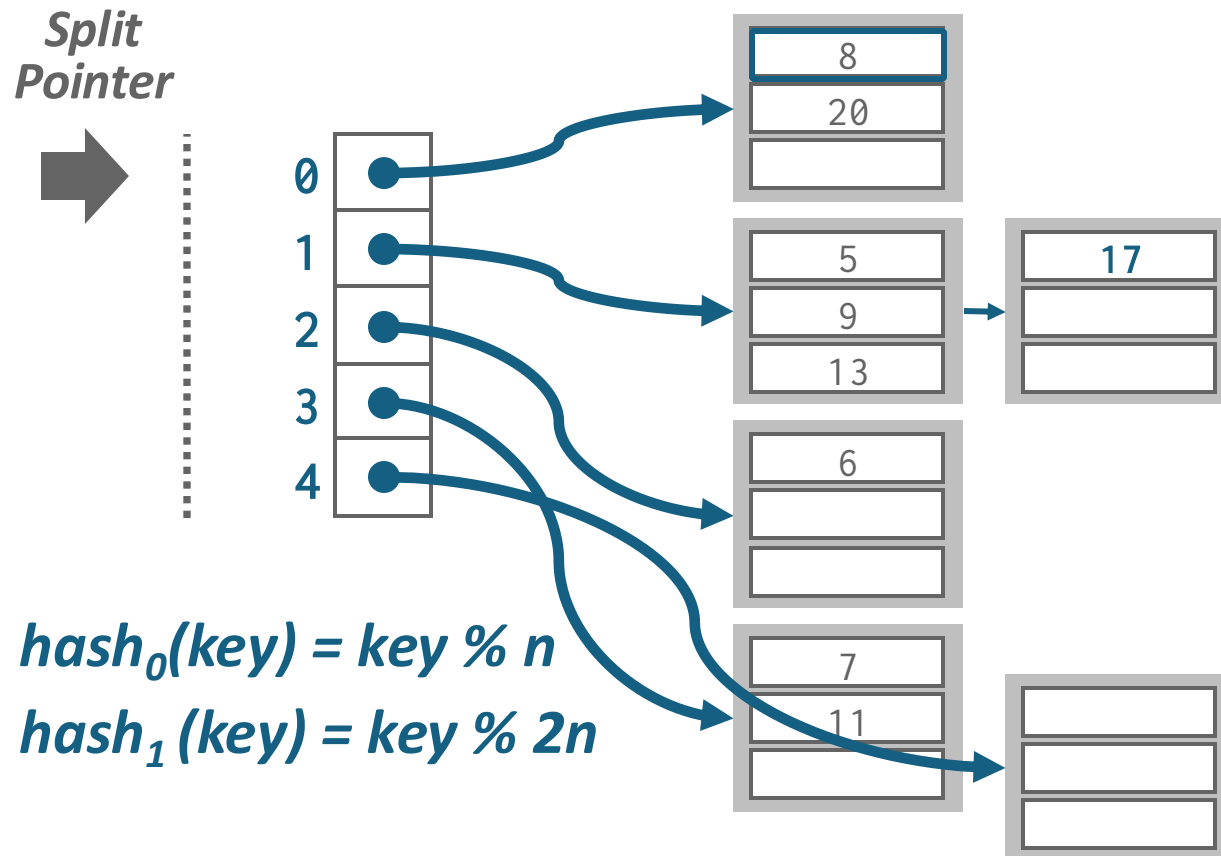
Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

Linear Hashing



Get 6

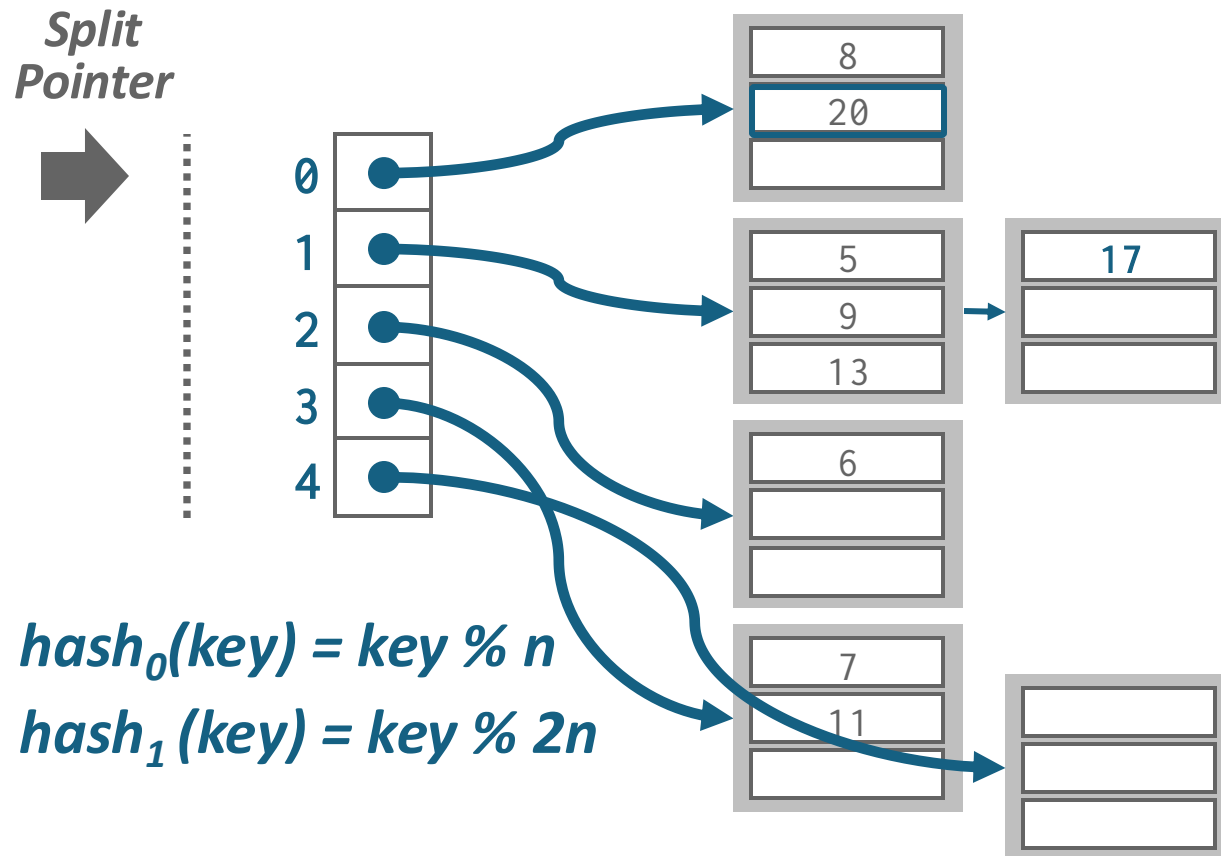
$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

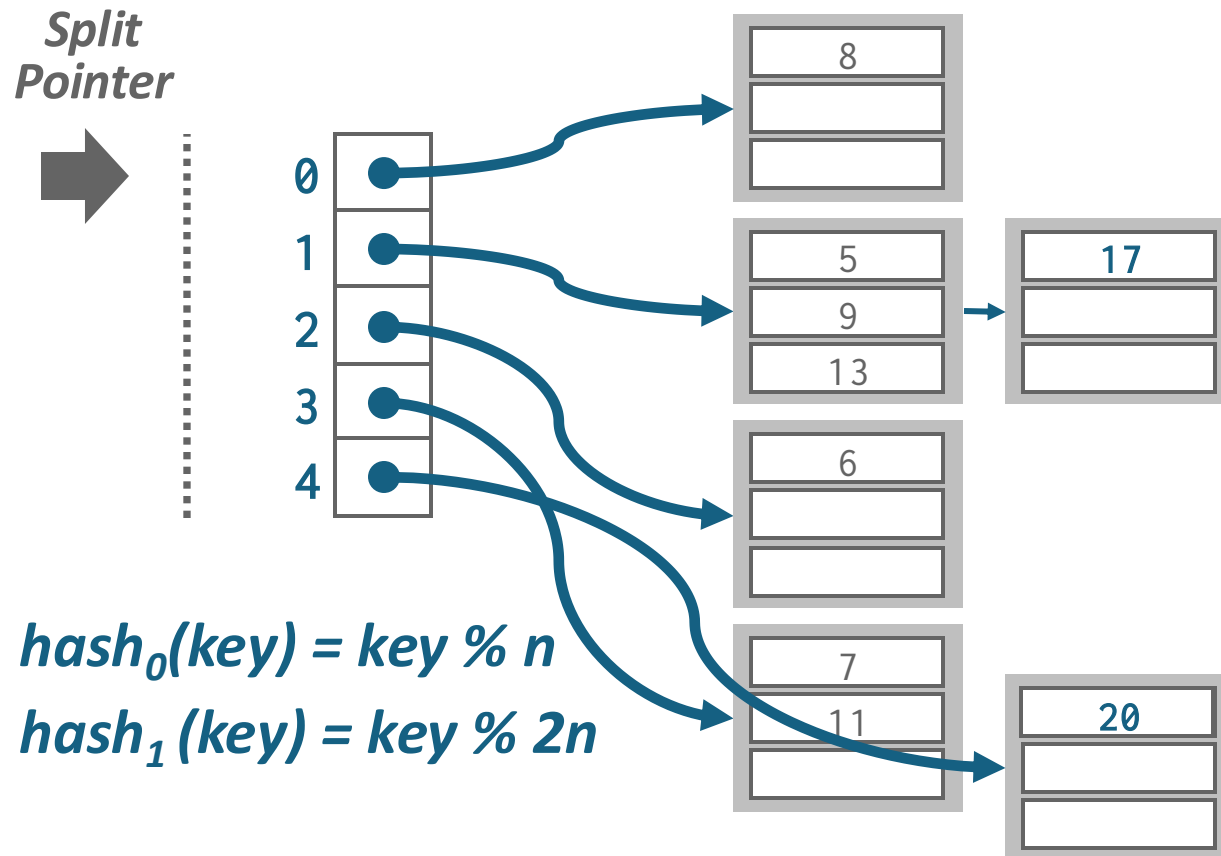
Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

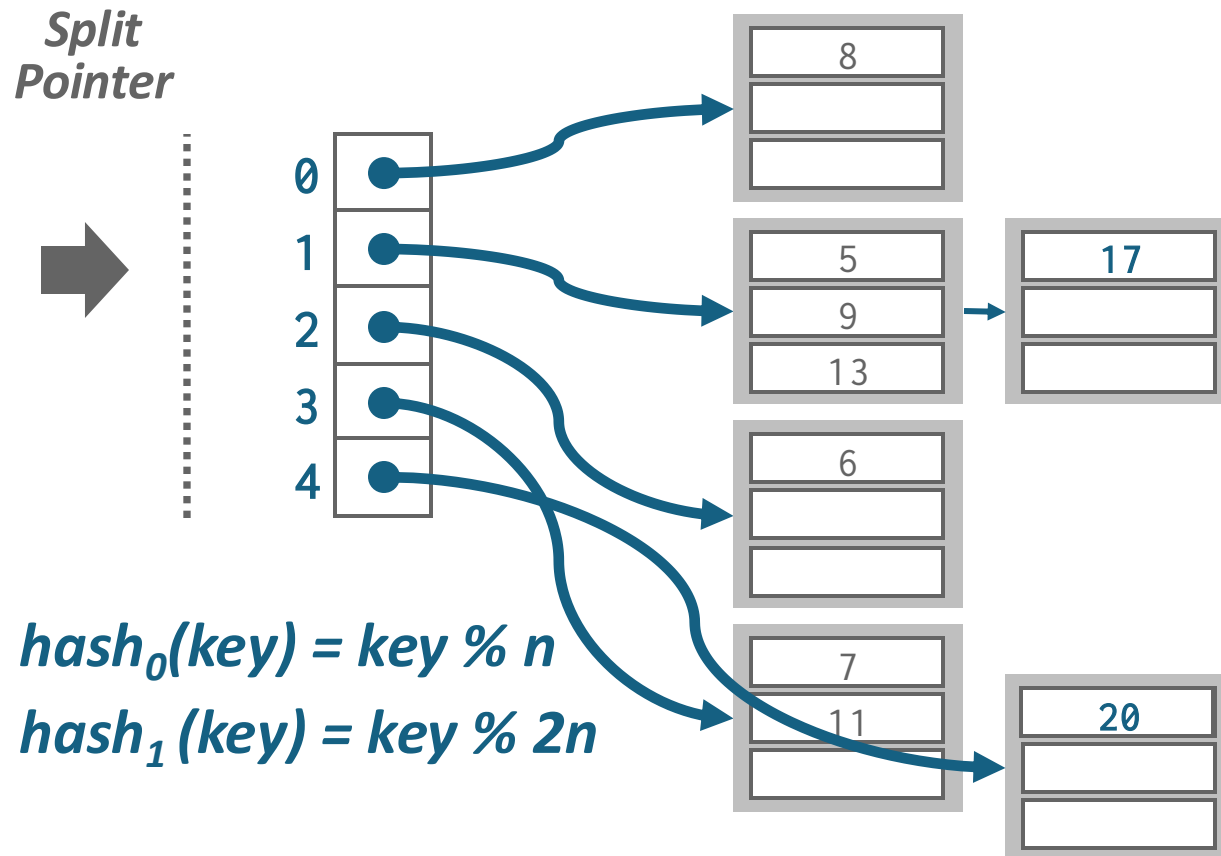
Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

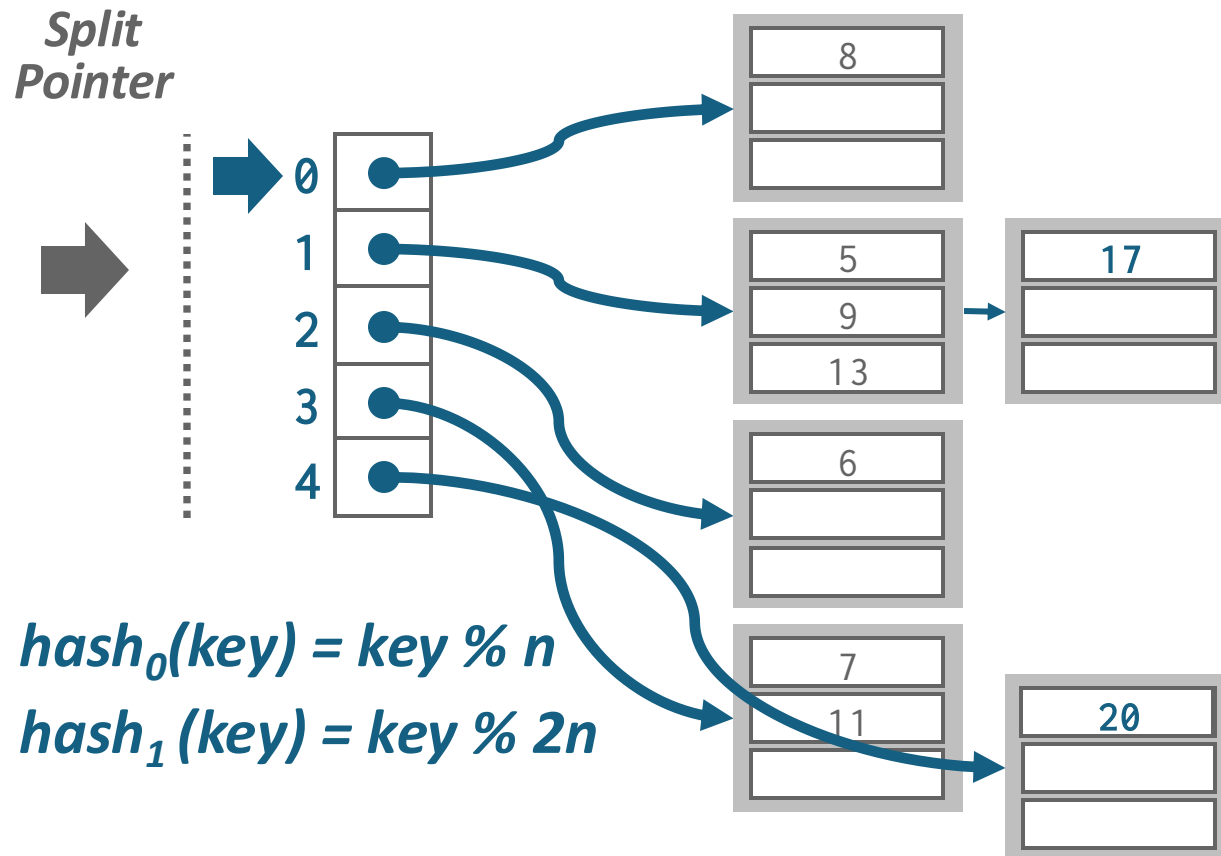
Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

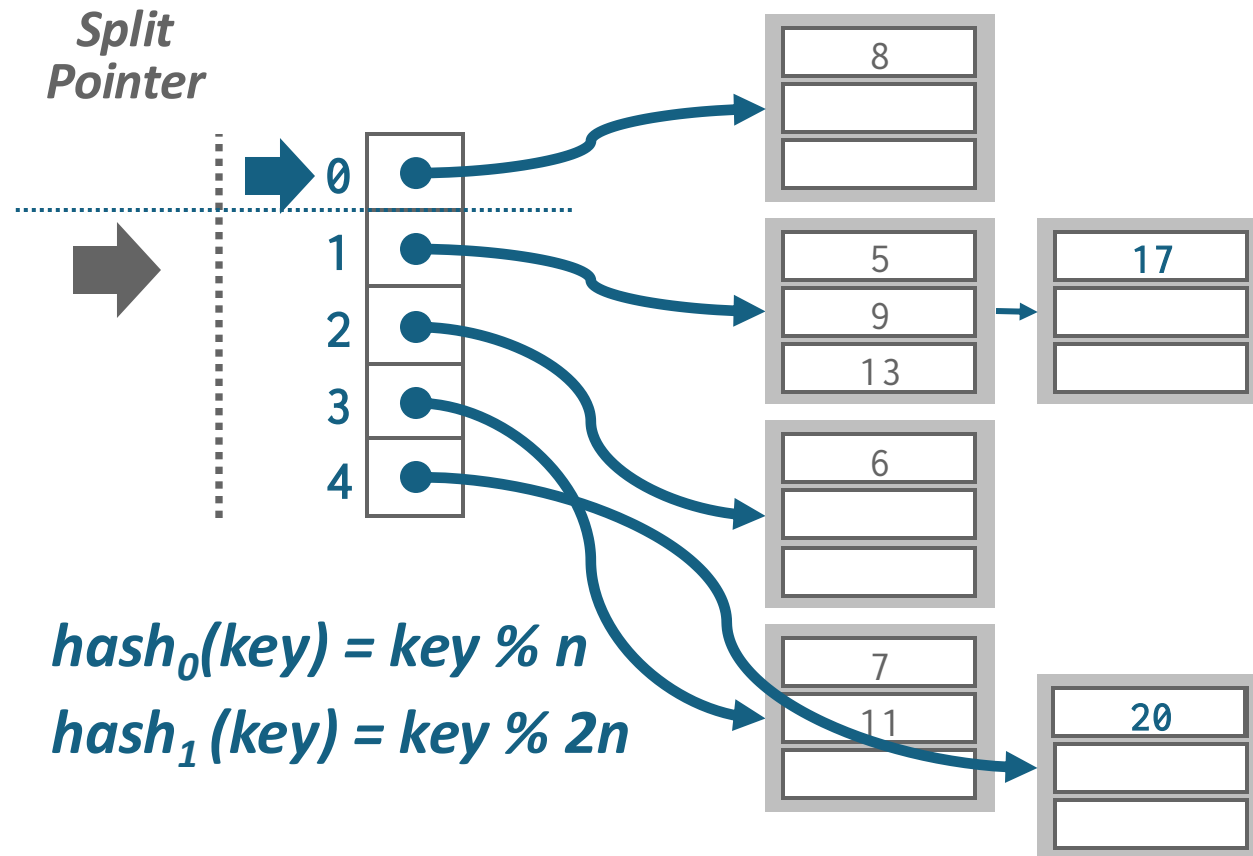
$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Get 20

$$hash_0(20) = 20 \% 4 = 0$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

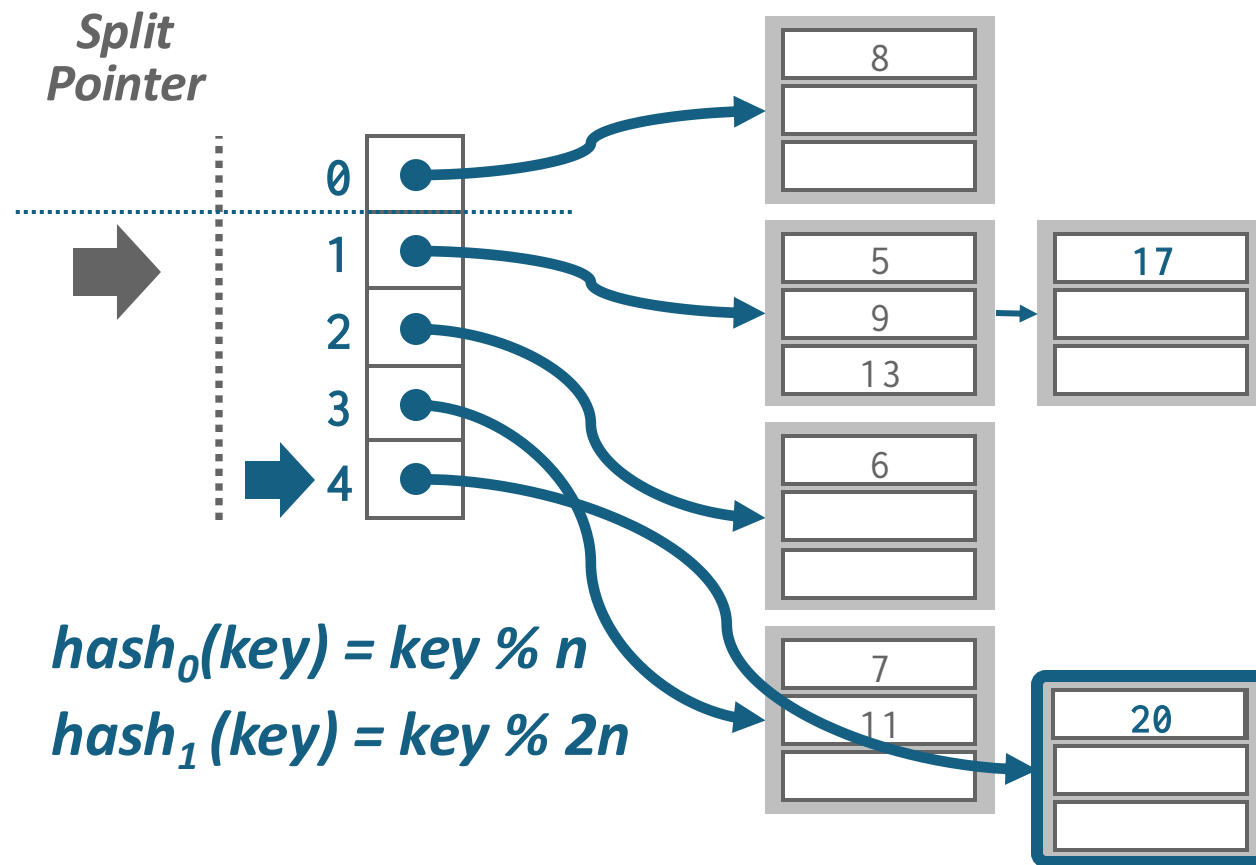
$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Get 20

$$hash_0(20) = 20 \% 4 = 0$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

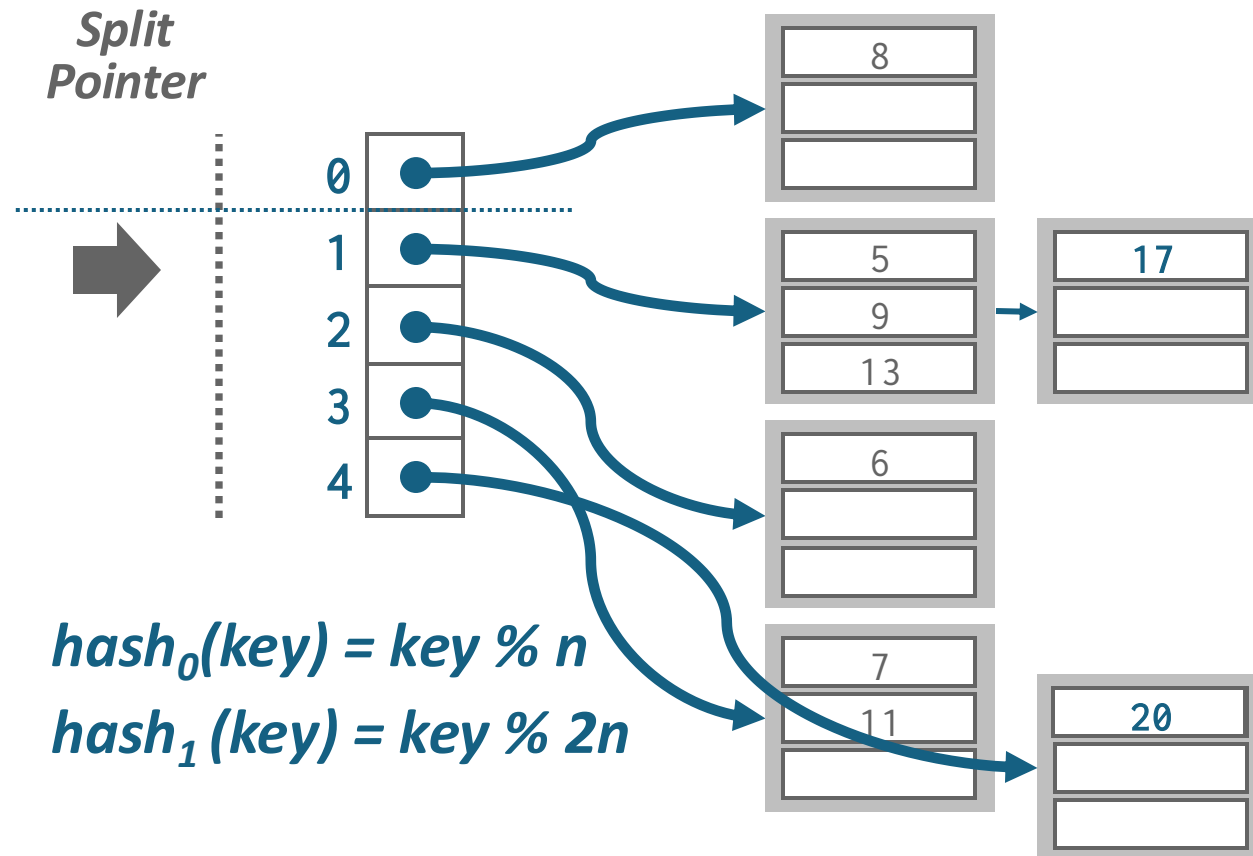
$$hash_1(20) = 20 \% 8 = 4$$

Get 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Get 20

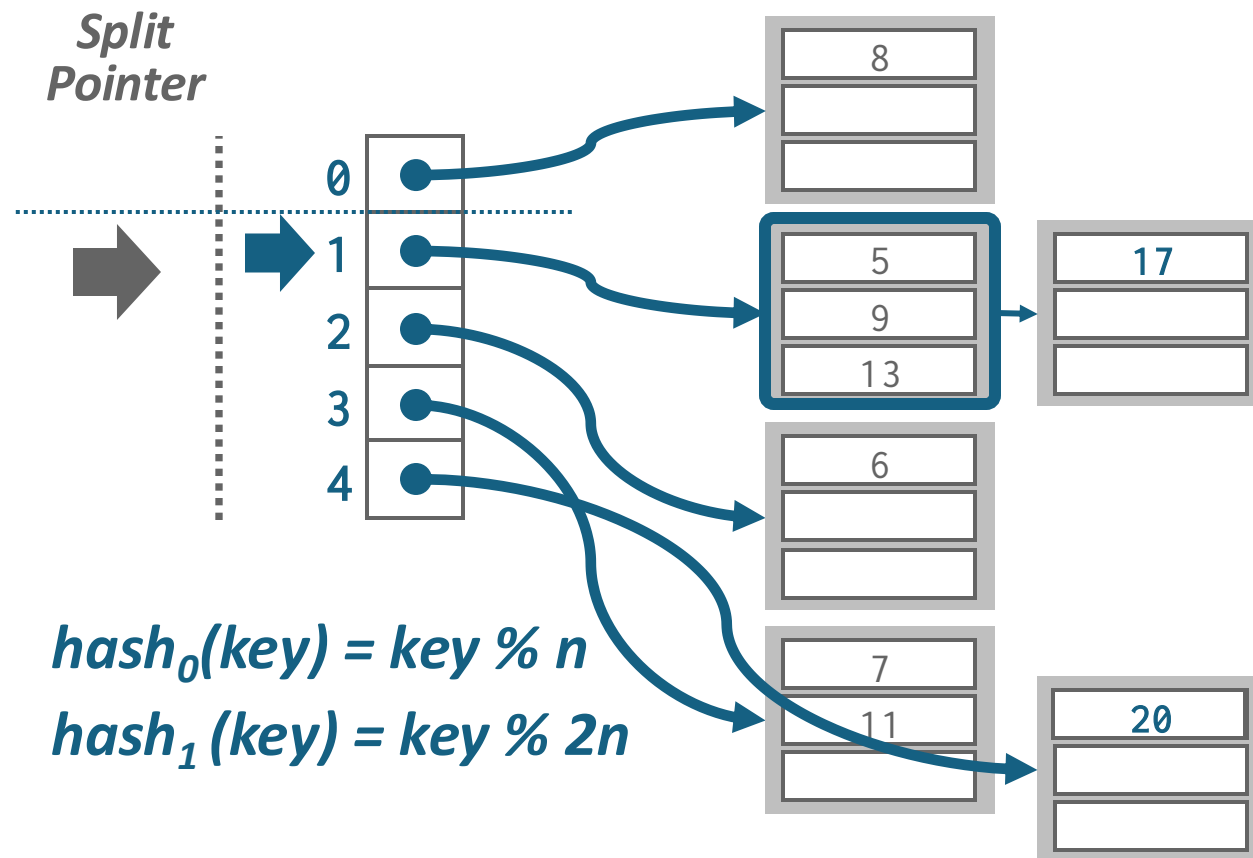
$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Get 9

$$hash_0(9) = 9 \% 4 = 1$$

Linear Hashing



Get 6

$$hash_0(6) = 6 \% 4 = 2$$

Put 17

$$hash_0(17) = 17 \% 4 = 1$$

$$hash_1(8) = 8 \% 8 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Get 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

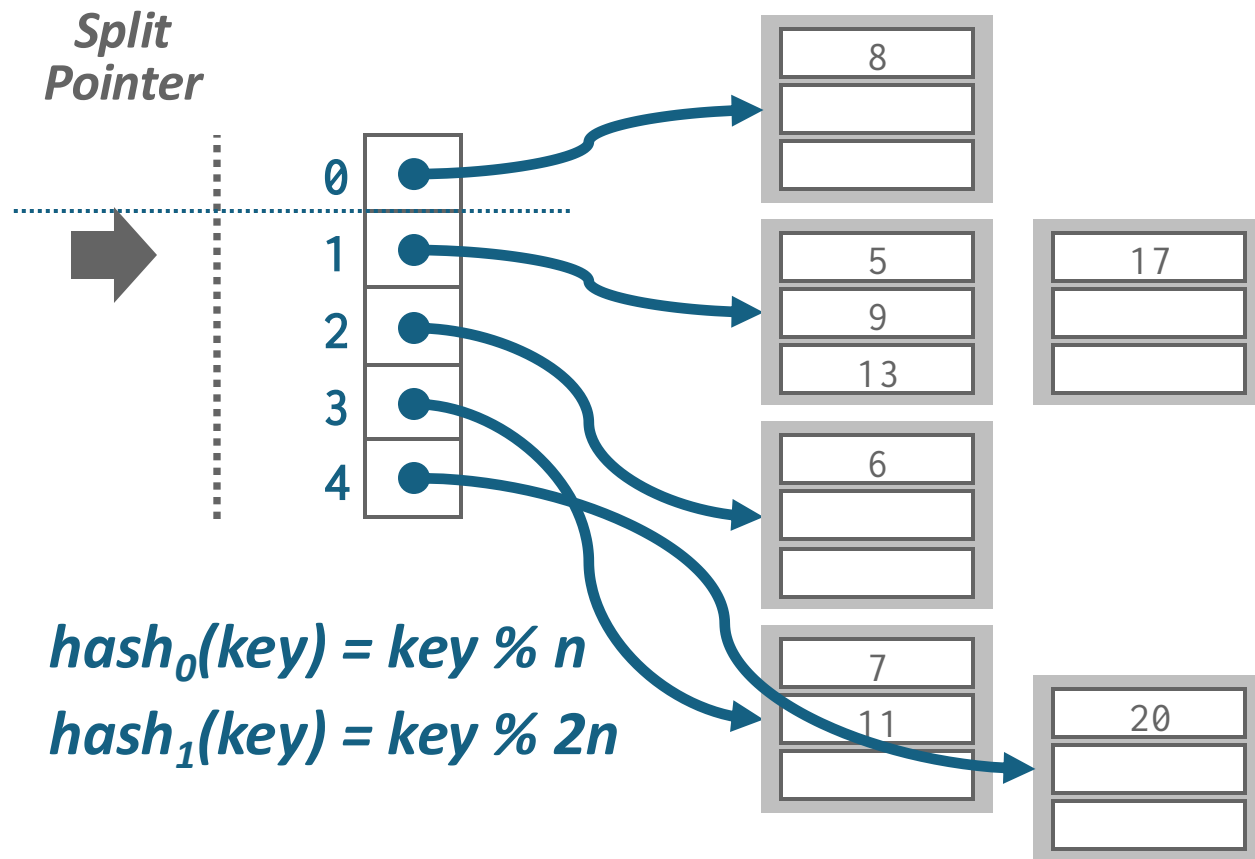
Get 9

$$hash_0(9) = 9 \% 4 = 1$$

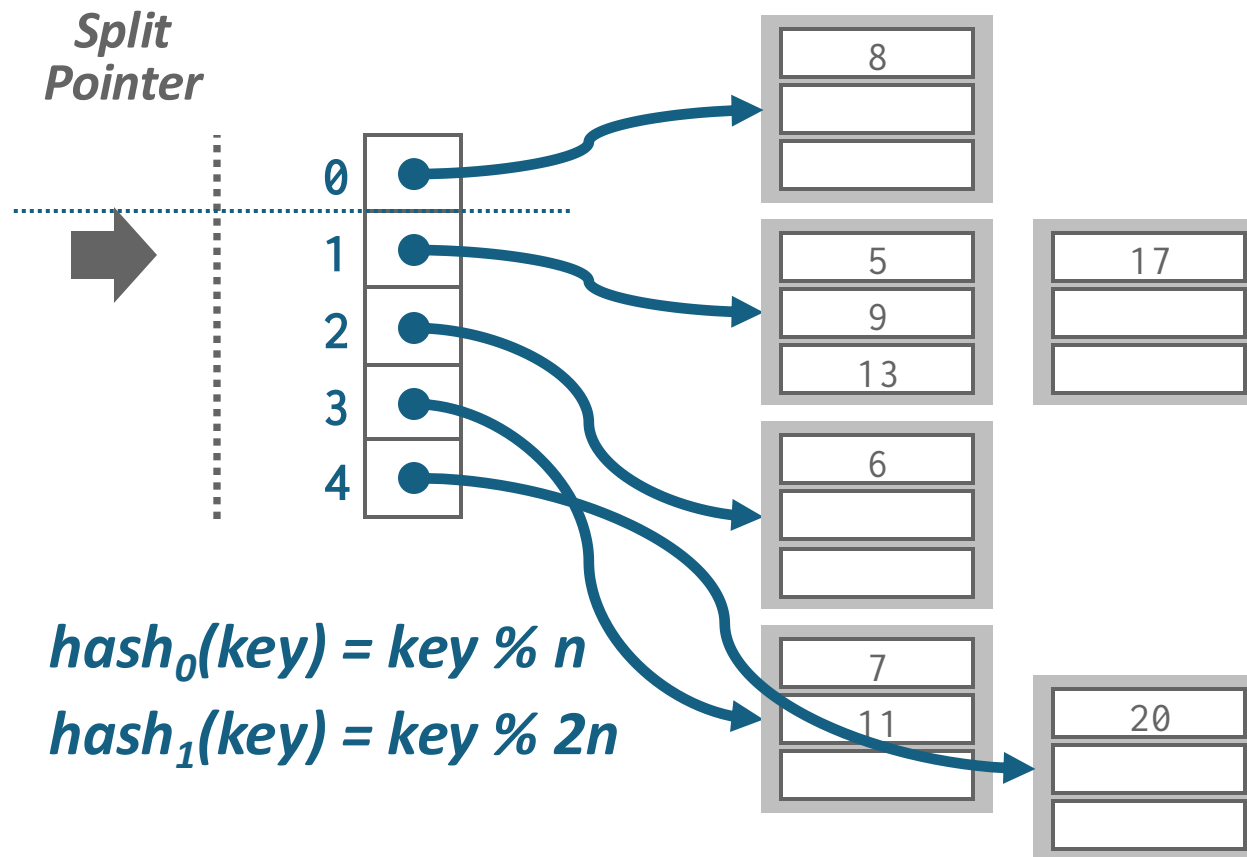
Linear Hashing - Resizing

- Splitting buckets based on the split pointer will eventually get to all overflowed buckets.
 - When the pointer reaches the last slot, remove the first hash function and move pointer back to beginning.
- If the “highest” bucket below the split pointer is empty, the hash table could remove it and move the splinter pointer in reverse direction.

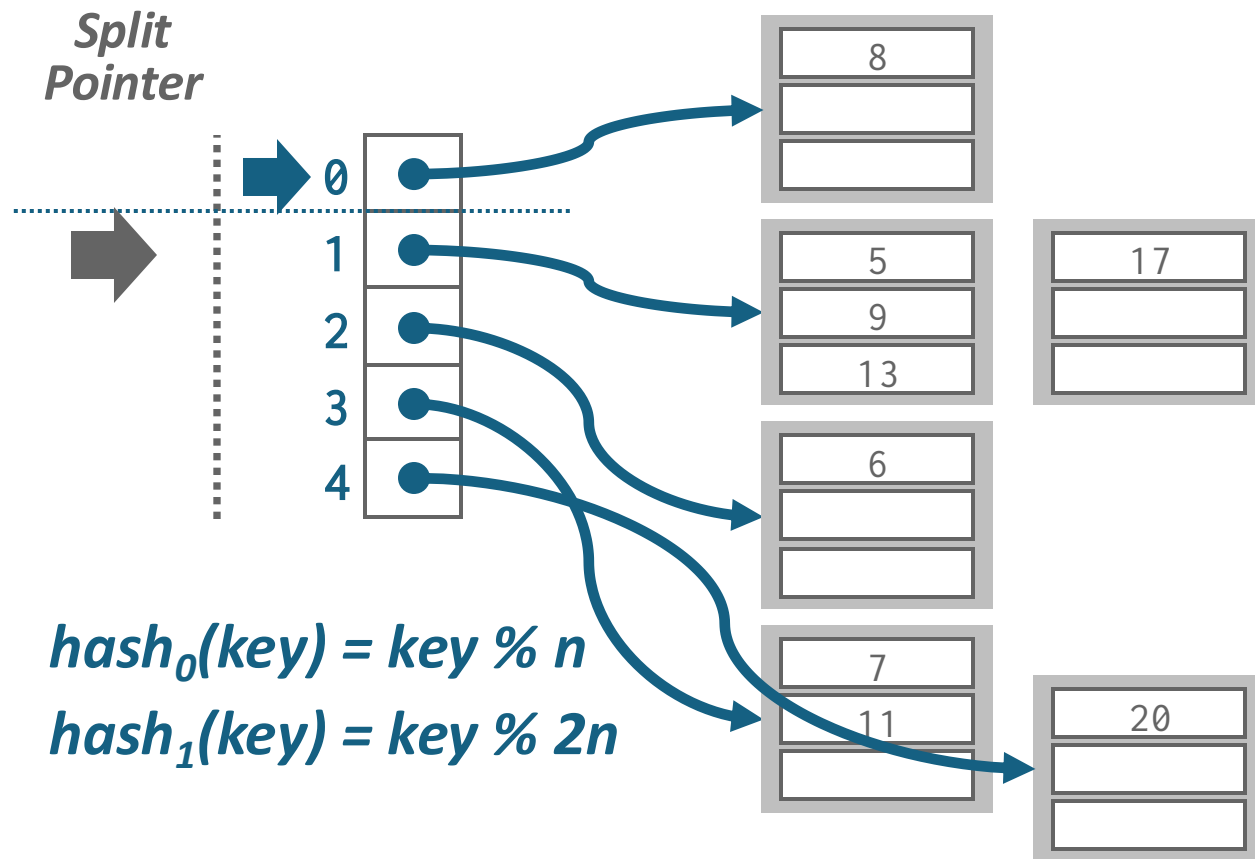
Linear Hashing - Deletes



Linear Hashing - Deletes

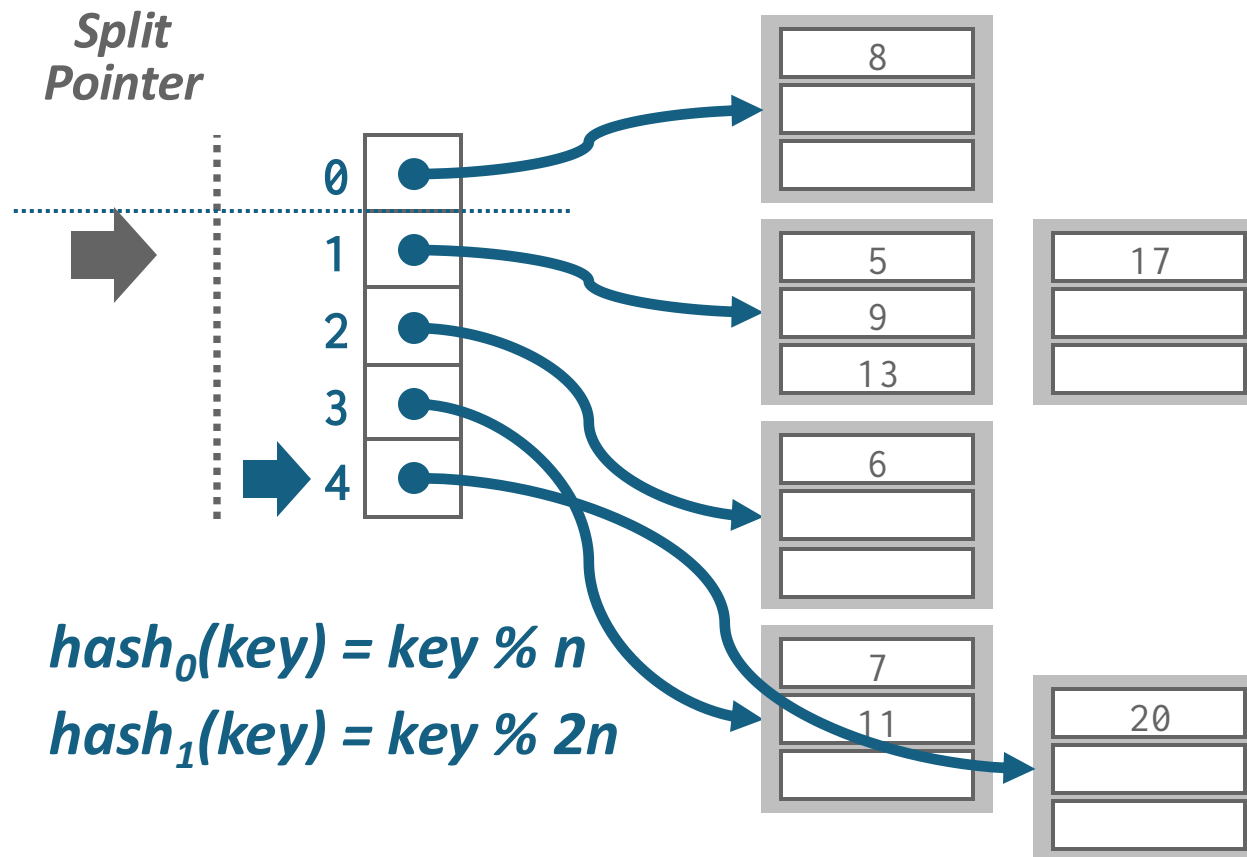


Linear Hashing - Deletes



Delete 20
 $hash_0(20) = 20 \% 4 = 0$

Linear Hashing - Deletes

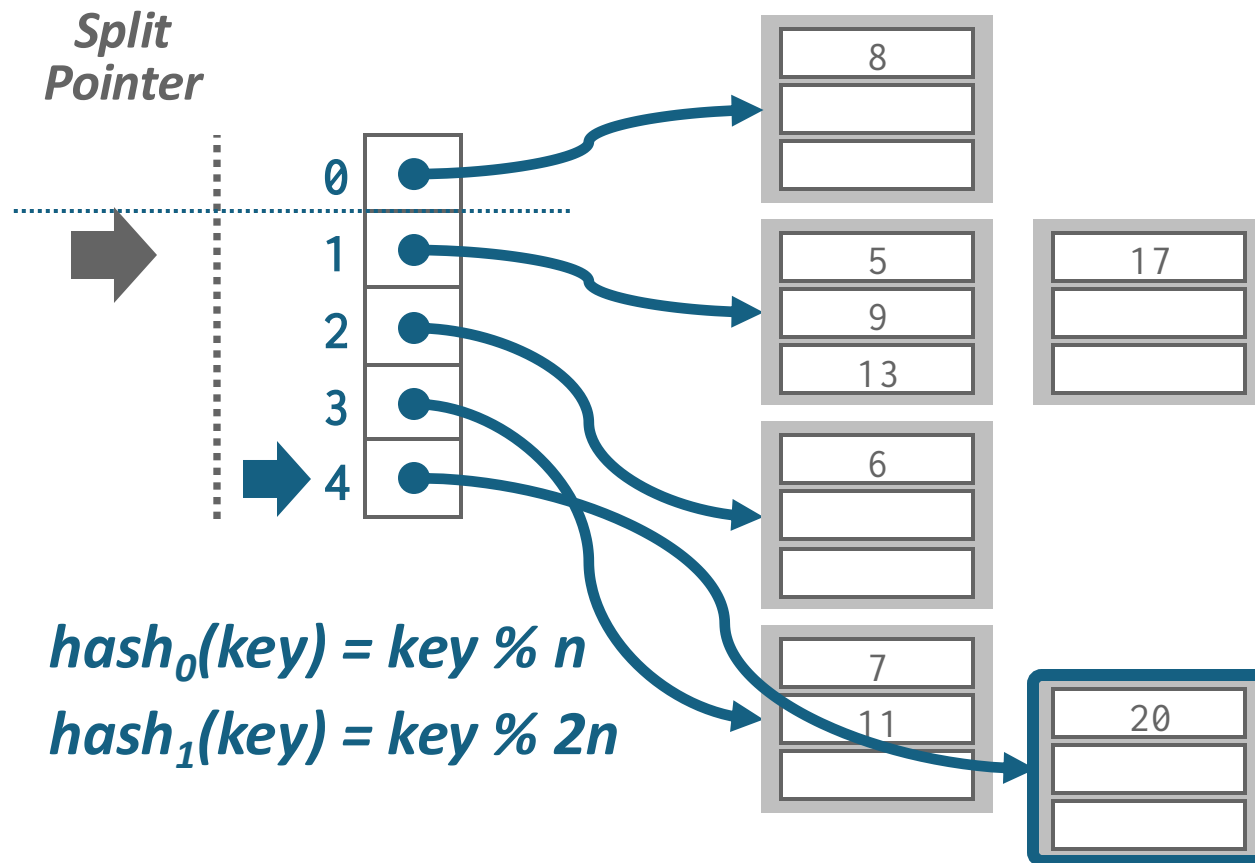


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

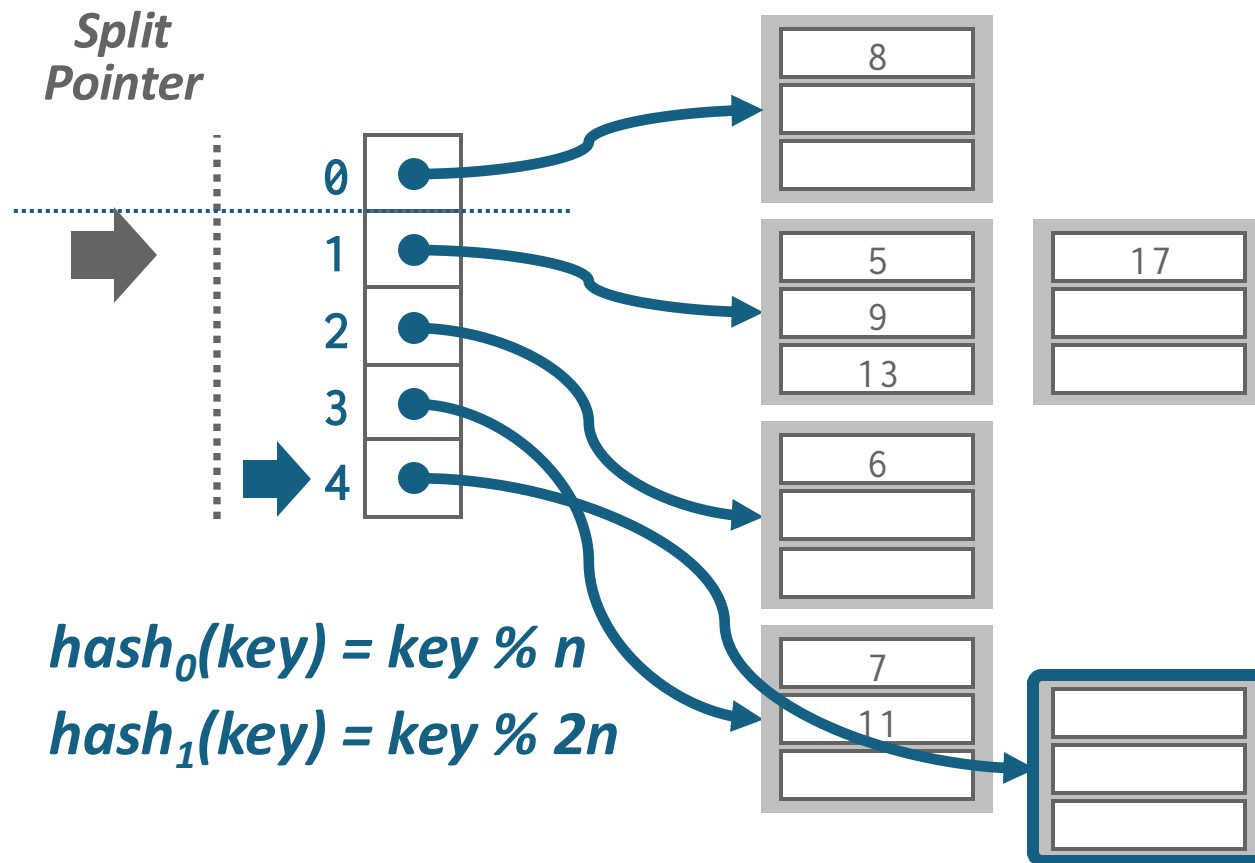


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

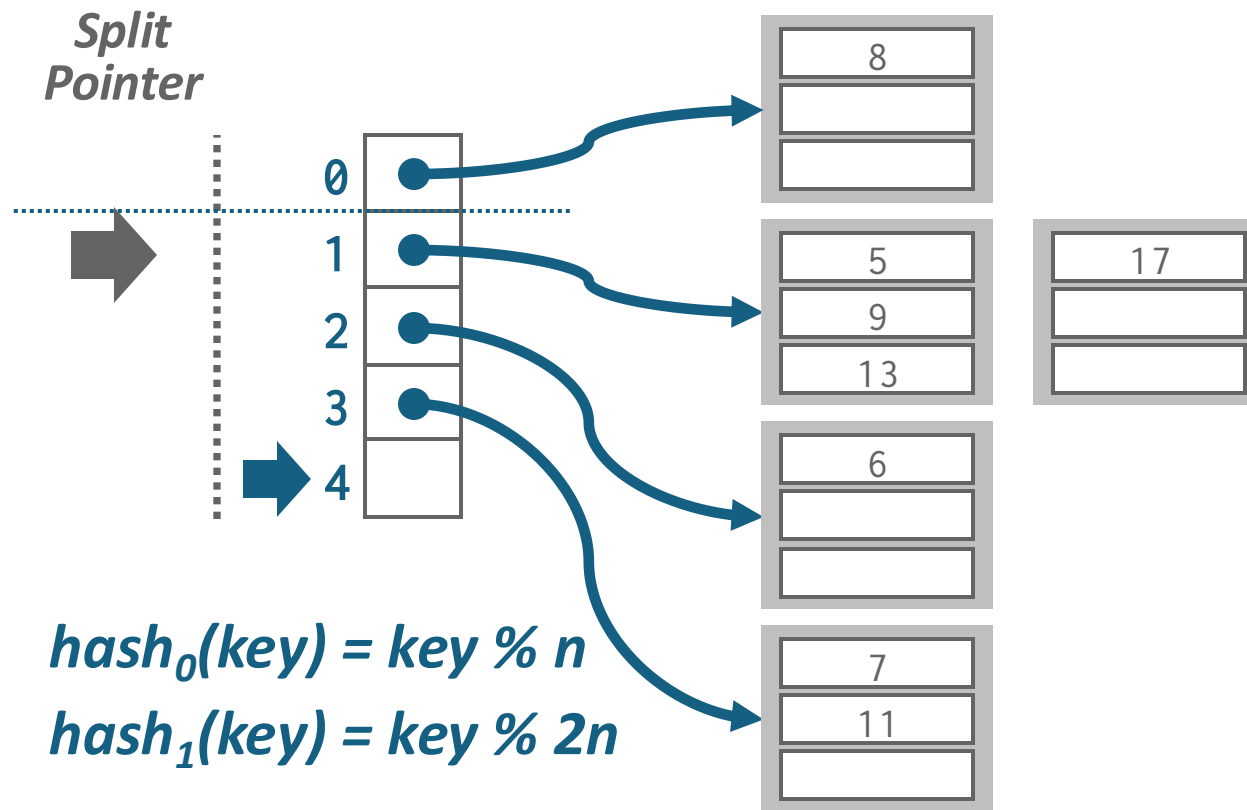


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

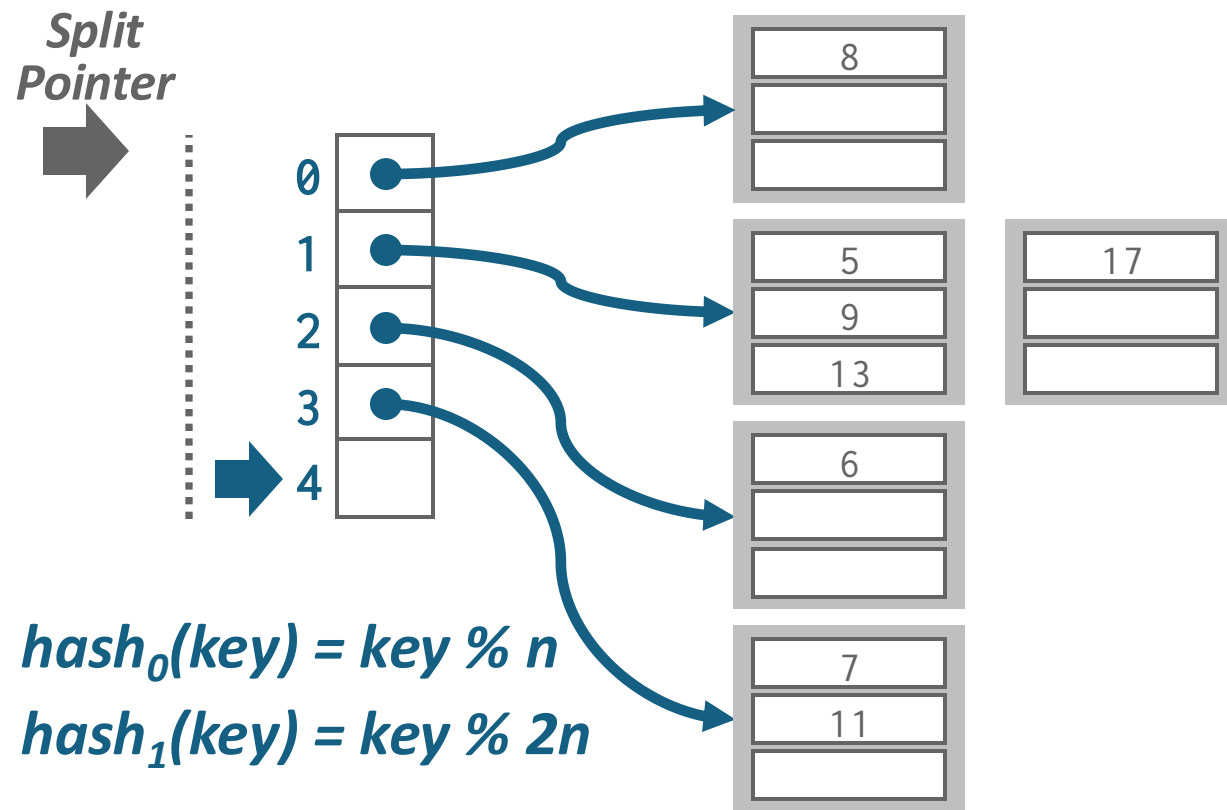


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

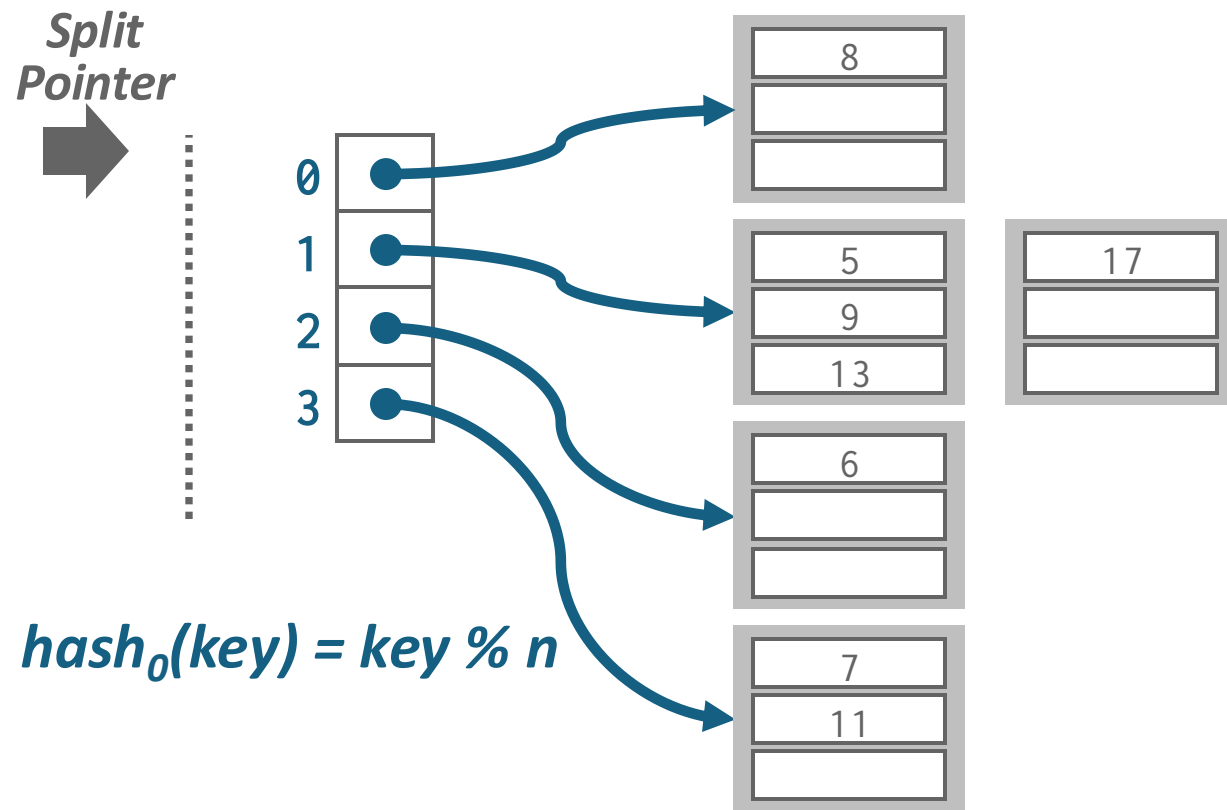


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes

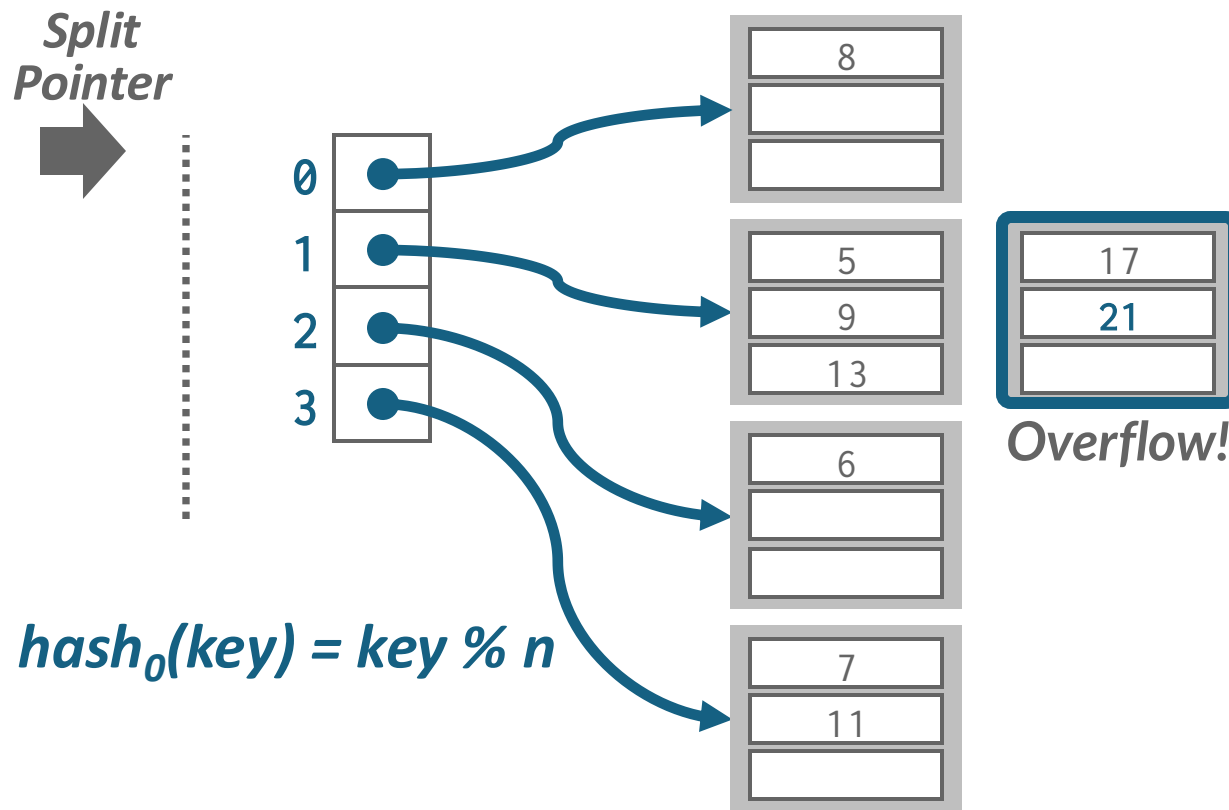


Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Linear Hashing - Deletes



Delete 20

$$hash_0(20) = 20 \% 4 = 0$$

$$hash_1(20) = 20 \% 8 = 4$$

Put 21

$$hash_0(21) = 21 \% 4 = 1$$

Conclusion

- Fast data structures that support $O(1)$ look-ups that are used all throughout DBMS internals.
 - Trade-off between speed and flexibility.
- Hashing schemes get used for both in-memory and on-disk.
 - Linear Probing, Cuckoo Hashing, Chained Hashing: Generally used for in-memory hash tables; e.g., in hash join and aggregate operators.
 - Extendible Hashing, Linear Hashing: for disk-based hash indexing.

Next Lecture

- **B+Trees**
 - aka “The Greatest Data Structure of All Time”