



香港中文大學(深圳)  
The Chinese University of Hong Kong, Shenzhen



Ack: Prof. Jignesh Patel @ CMU  
Prof. Andy Pavlo @CMU

# CSC3170

## 8: B+Tree Indexes

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The Chinese University of Hong Kong, Shenzhen

# Last Lecture

- Hash tables are important data structures that are used all throughout a DBMS.
  - Space Complexity:  $O(n)$
  - Average Time Complexity:  $O(1)$
- Static vs. Dynamic Hashing schemes
- DBMSs use mostly hash tables for their internal data structures.

# This Lecture

- B+Tree Overview
- Design Choices
- Optimizations

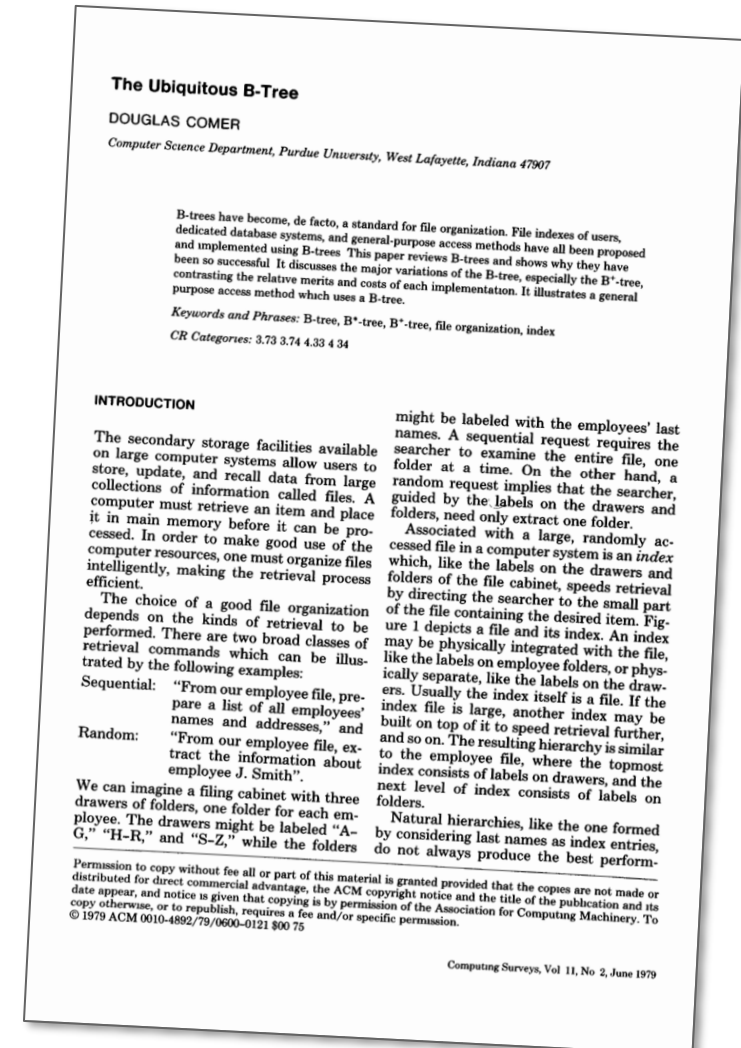
# B+Tree

# B-Tree Family

- There is a specific data structure called a **B-Tree**.
- People also use the term to generally refer to a class of balanced tree data structures:
  - **B-Tree** (1971)
  - **B+Tree** (1973)
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  - **B<sup>link</sup>-Tree** (1981)
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## Efficient Locking for Concurrent Operations on B-Trees

PHILIP L. LEHMAN  
Carnegie-Mellon University  
and  
S. BING YAO  
Purdue University

The B-tree and its variants have been found to be highly useful (both theoretically and in practice) for storing large amounts of information, especially on secondary storage devices. We examine the problem of overcoming the inherent difficulty of concurrent operations on such structures, using a practical storage model. A single additional "link" pointer in each node allows a process to easily recover from tree modifications performed by other concurrent processes. Our solution compares favorably with earlier solutions in that the locking scheme is simpler (no read-locks are used) and only a (small) constant number of nodes are locked by any update process at any given time. An informal correctness proof for our system is given.

Key Words and Phrases: database, data structures, B-tree, index organizations, concurrent algorithms, concurrency controls, locking protocols, correctness, consistency, multiway search trees

CR Categories: 3.73, 3.74, 4.32, 4.33, 4.34, 5.24

### 1. INTRODUCTION

The B-tree [2] and its variants have been widely used in recent years as a data structure for storing large files of information, especially on secondary storage devices [7]. The guaranteed small (average) search, insertion, and deletion time for these structures makes them quite appealing for database applications.

A topic of current interest in database design is the construction of databases that can be manipulated concurrently and correctly by several processes. In this paper, we consider a simple variant of the B-tree (actually of the B\*-tree, proposed by Wedekind [15]) especially well suited for use in a concurrent database system.

Methods for concurrent operations on B\*-trees have been discussed by Bayer and Schkolnick [3] and others [6, 12, 13]. The solution given in the current paper

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This research was supported by the National Science Foundation under Grant MCS76-16604. Authors' present addresses: P. L. Lehman, Department of Computer Science, Carnegie-Mellon University, Pittsburgh, PA 15213; S. B. Yao, Department of Computer Science and College of Business and Management, University of Maryland, College Park, MD 20742.

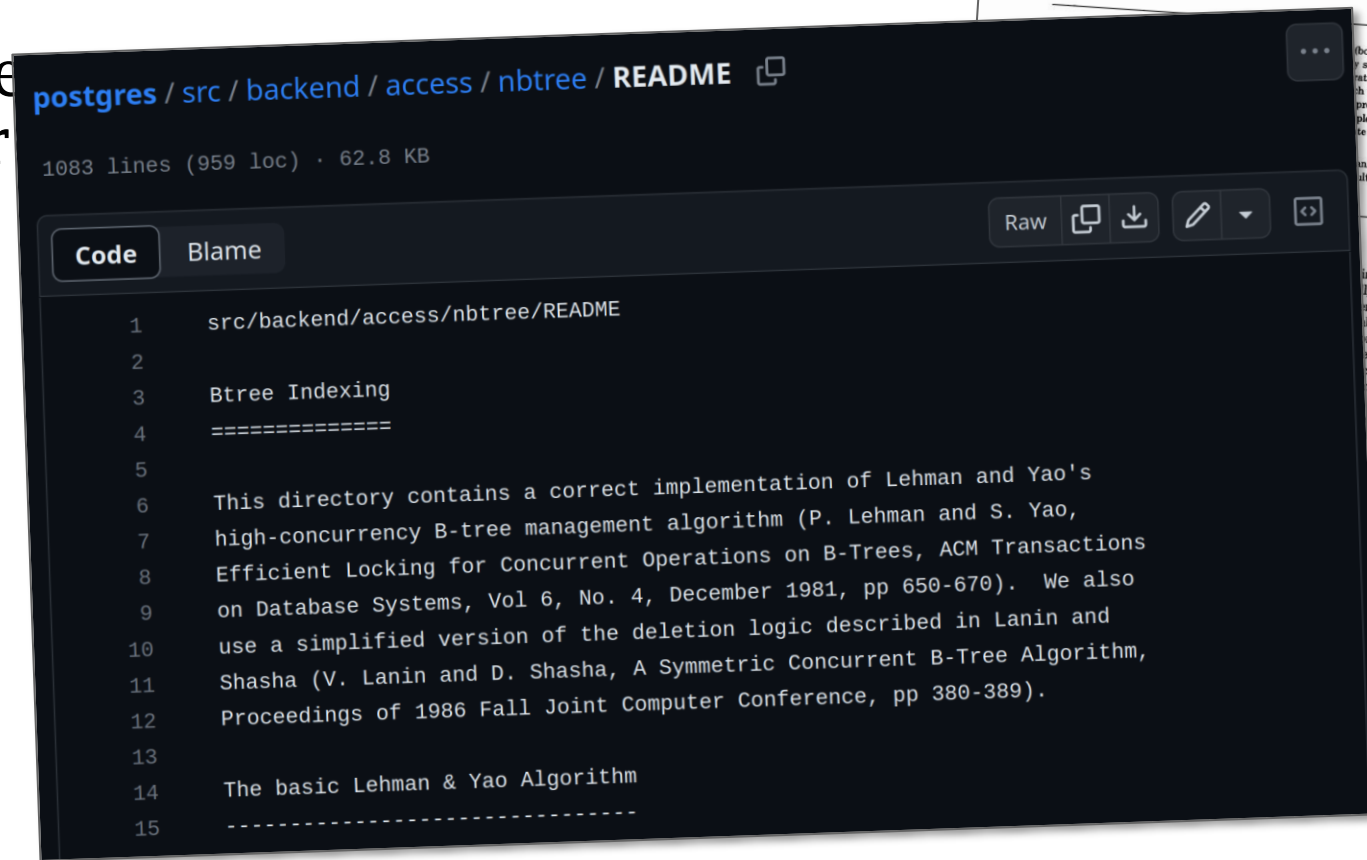
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ACM Transactions on Database Systems, Vol. 6, No. 4, December 1981, Pages 650-670.



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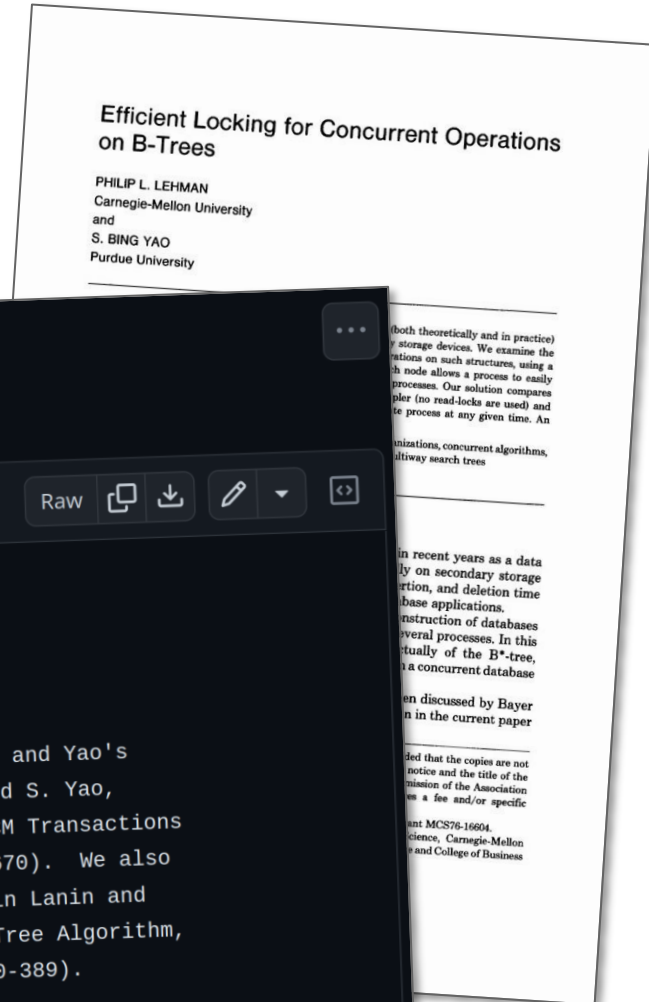


postgres / src / backend / access / nbtree / README

1083 lines (959 loc) · 62.8 KB

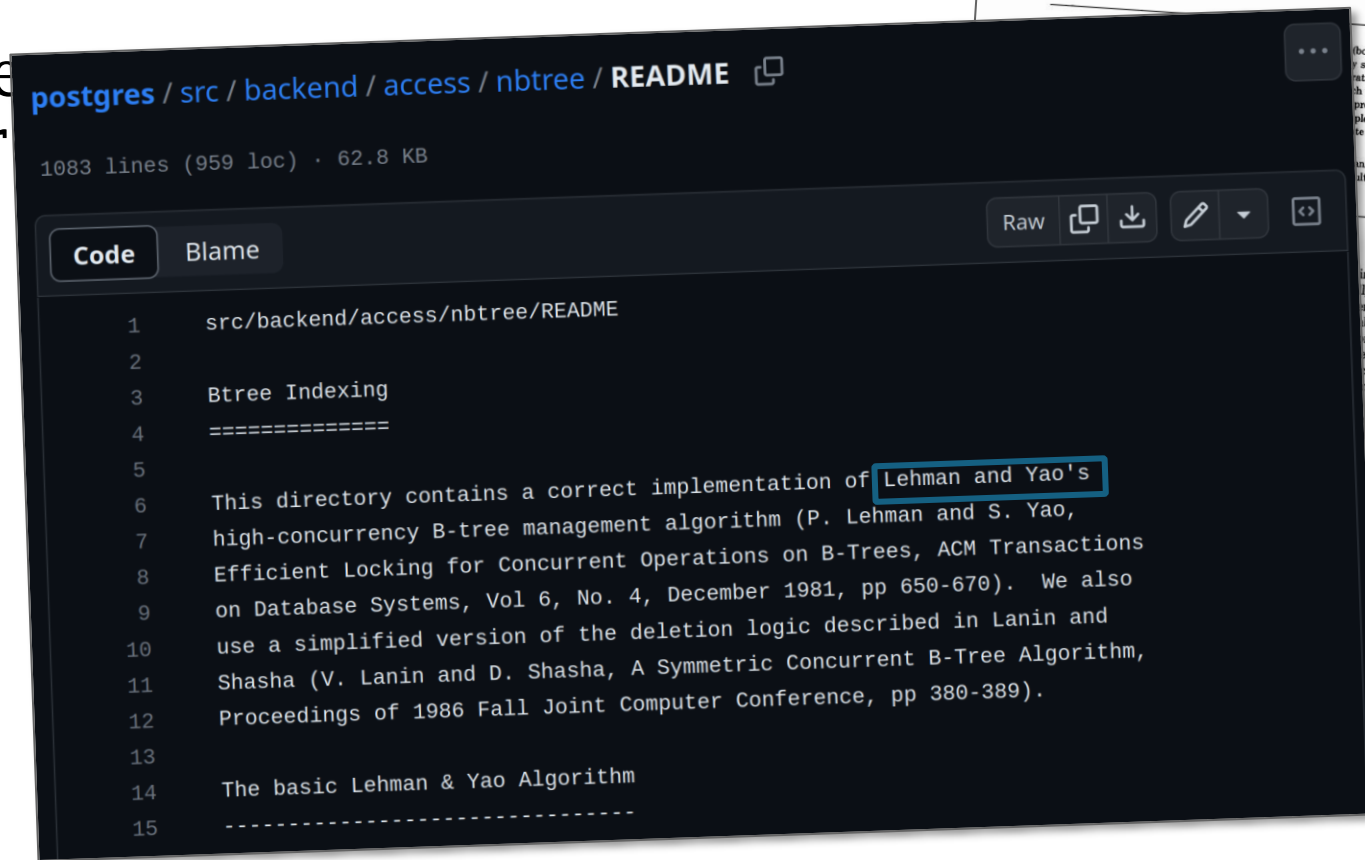
Code Blame

```
1 src/backend/access/nbtree/README
2
3 Btree Indexing
4 =====
5
6 This directory contains a correct implementation of Lehman and Yao's
7 high-concurrency B-tree management algorithm (P. Lehman and S. Yao,
8 Efficient Locking for Concurrent Operations on B-Trees, ACM Transactions
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14 The basic Lehman & Yao Algorithm
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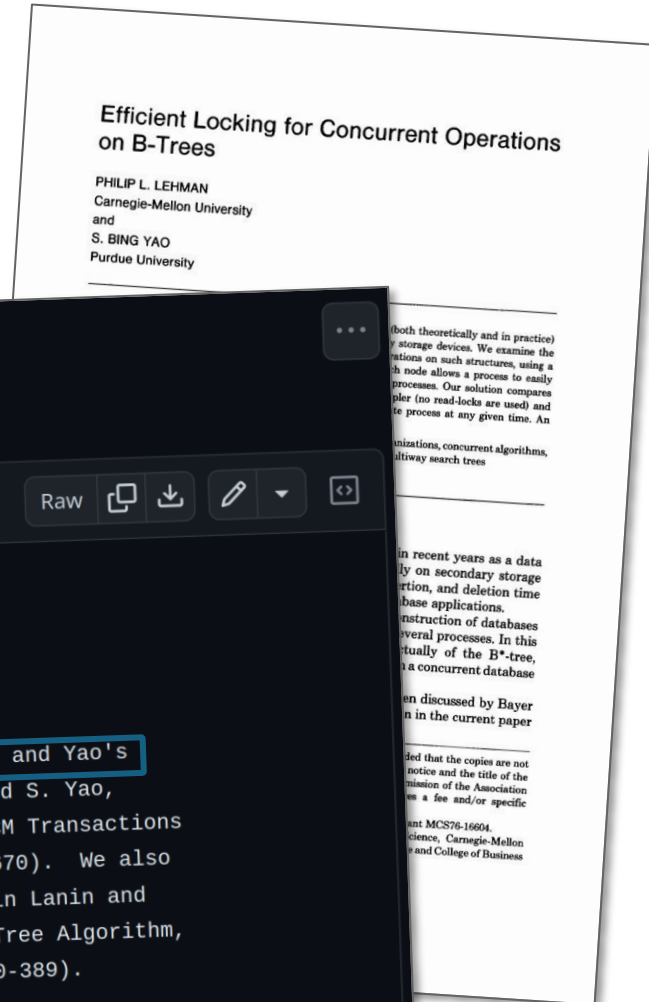
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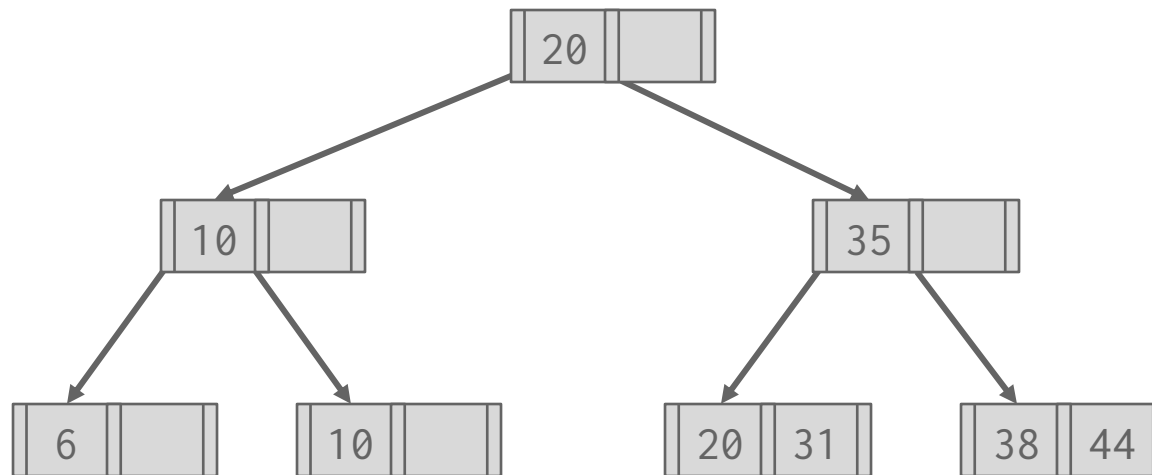
# B+Tree

- A **B+Tree** is a self-balancing, ordered tree data structure that allows searches, sequential access, insertions, and deletions in  $O(\log_m n)$ .
  - Generalization of a binary search tree, since a node can have more than two children.
  - Optimized for systems that read and write large blocks of data.
  - **m** is the fanout of the tree.

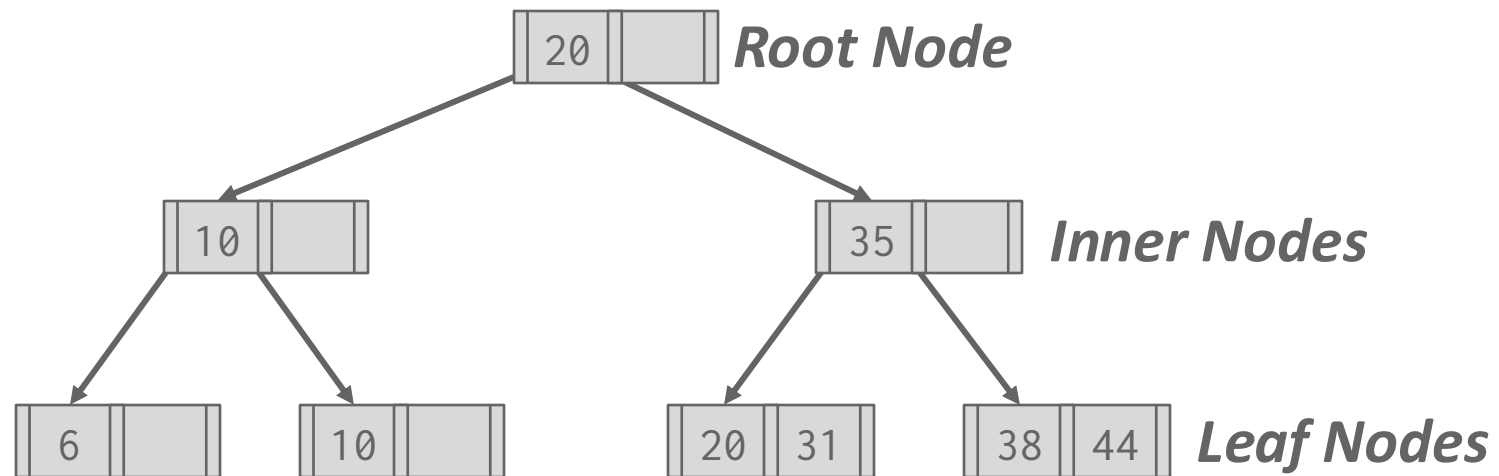
# B+Tree Properties

- A B+Tree is an  $M$ -way search tree with the following properties:
  - It is perfectly balanced (i.e., every leaf node is at the same depth in the tree)
  - Every leaf node is at least half-full  
 $\lceil (M - 1)/2 \rceil \leq \text{\#keys} \leq M - 1$
  - Every non-leaf node other than the root is at least half-full  
 $\lceil M/2 \rceil - 1 \leq \text{\#keys} \leq M - 1$
  - Every inner node with  $k$  keys has  $k+1$  non-null children

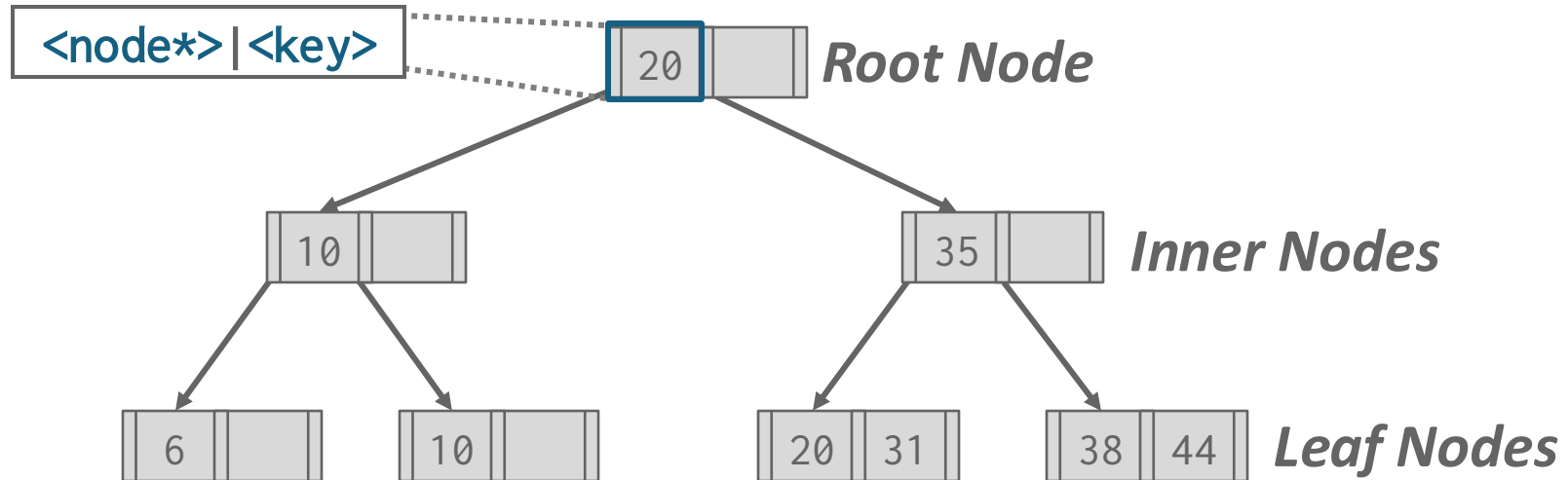
# B+Tree Example



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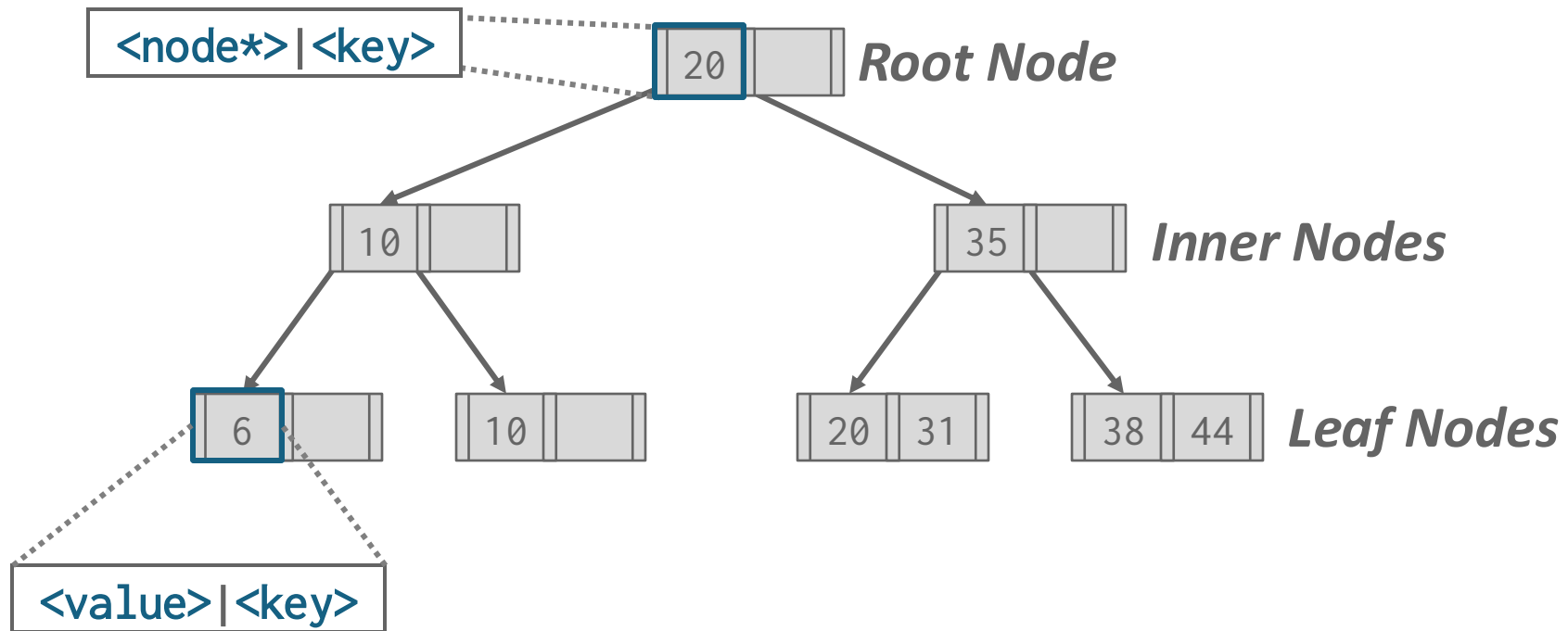


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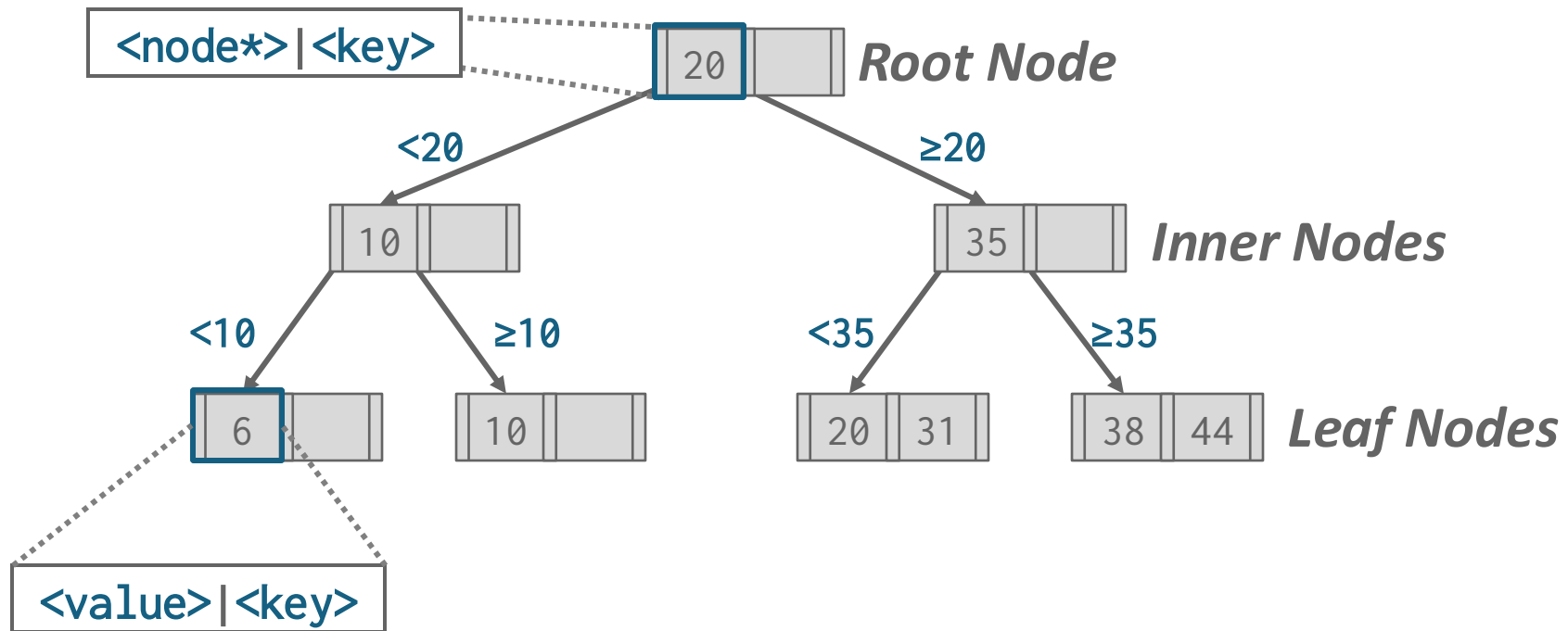




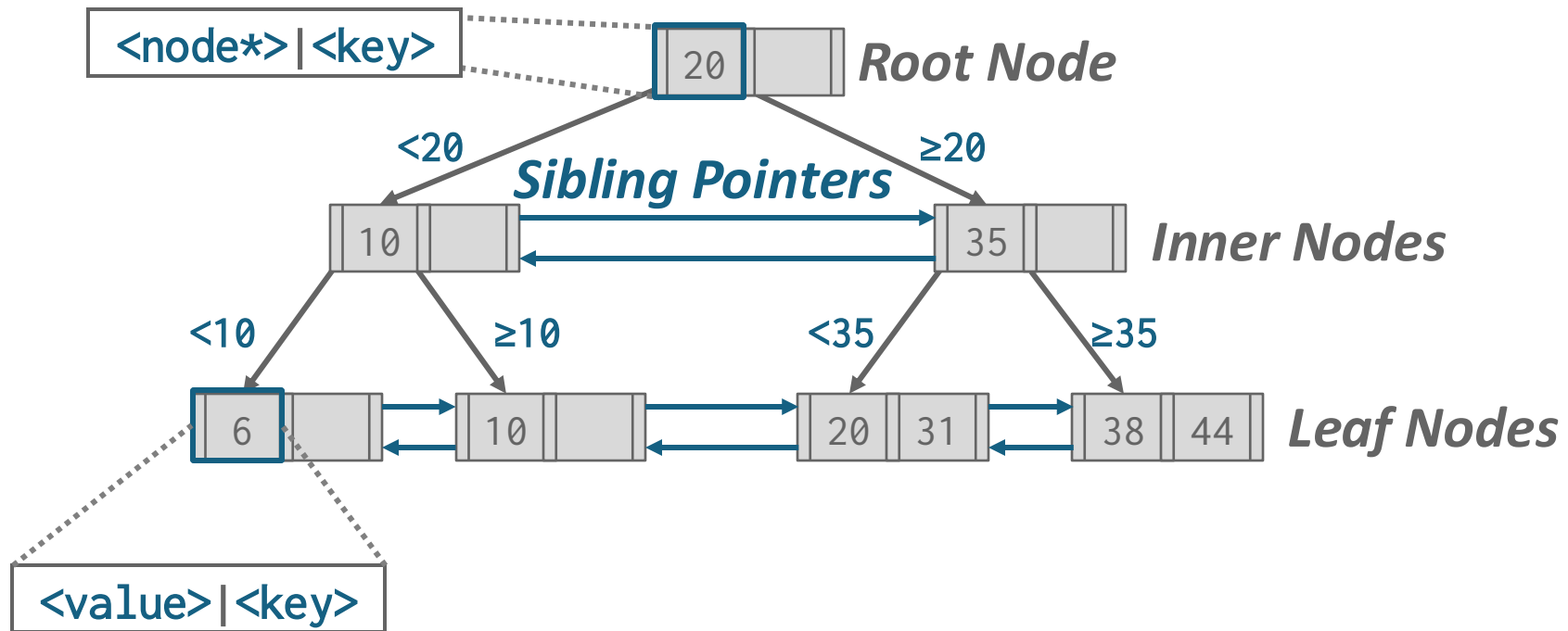
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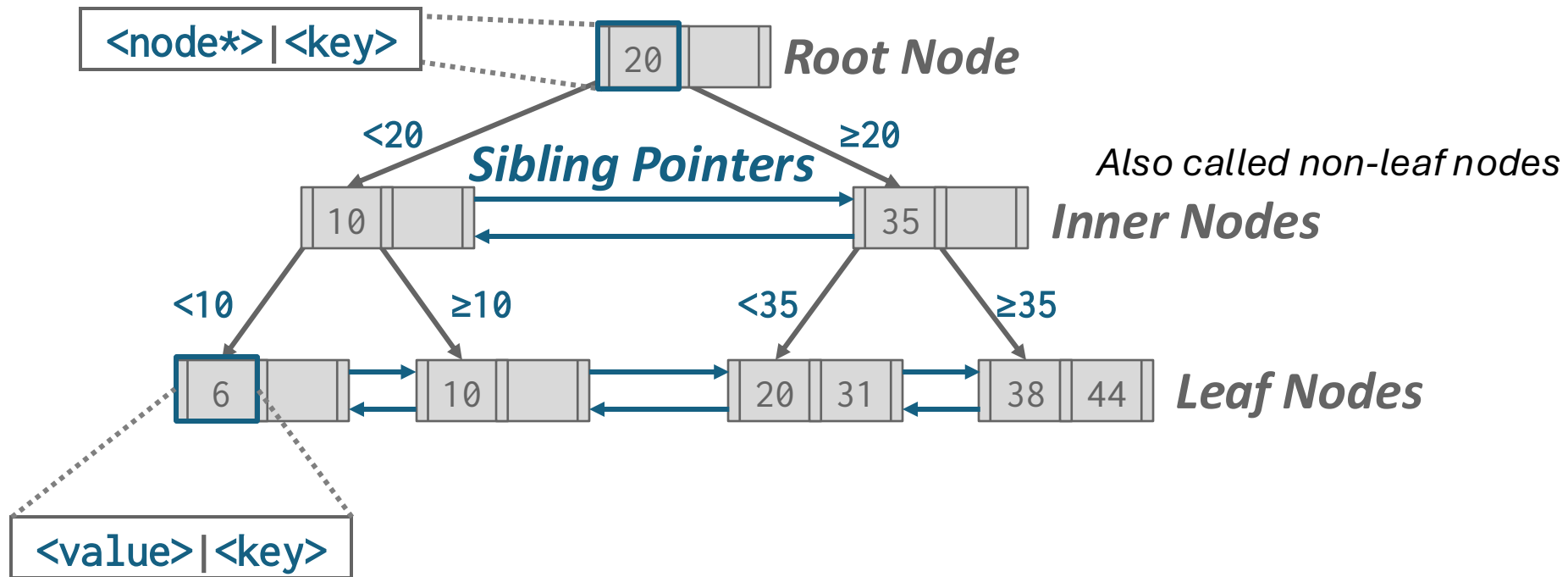
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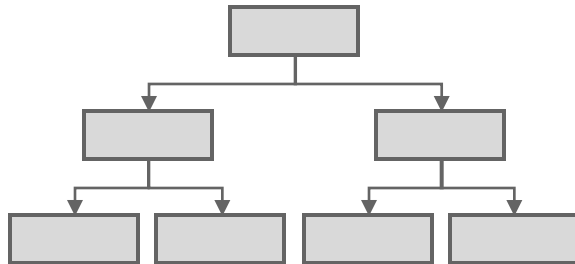
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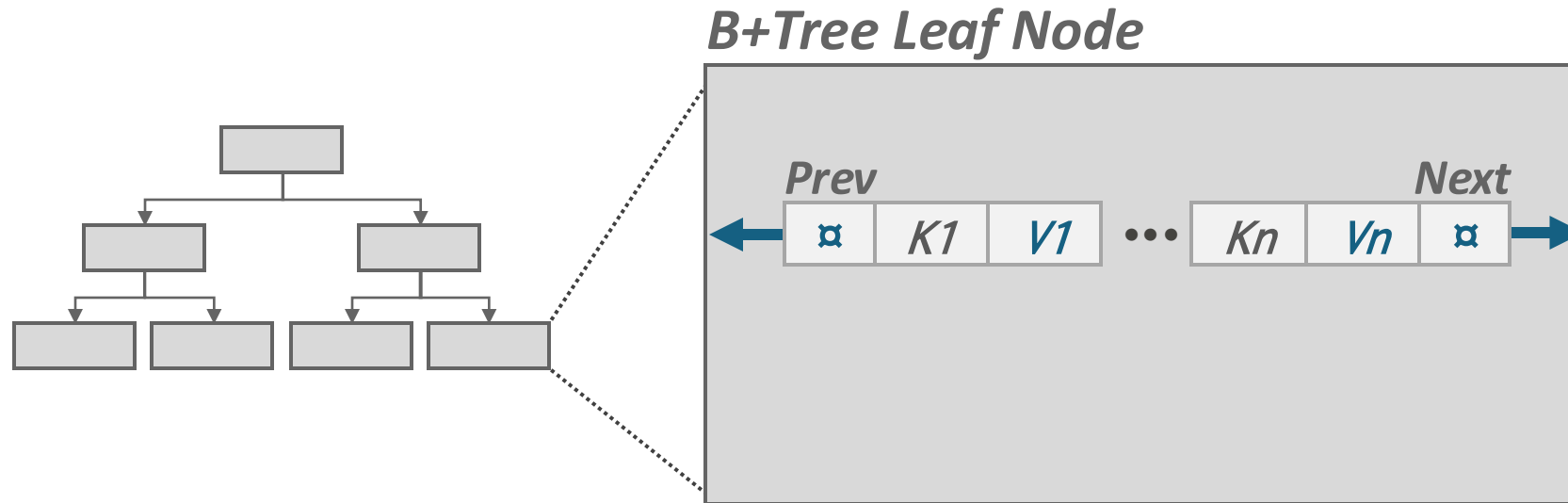
# Nodes

- Every B+Tree node is comprised of an array of key/value pairs.
  - The keys are derived from the attribute(s) that the index is based on.
  - The values will differ based on whether the node is classified as an **inner node** or a **leaf node**.
- The arrays are (usually) kept in sorted key order.
- Store all **NULL** keys at either first or last leaf nodes.

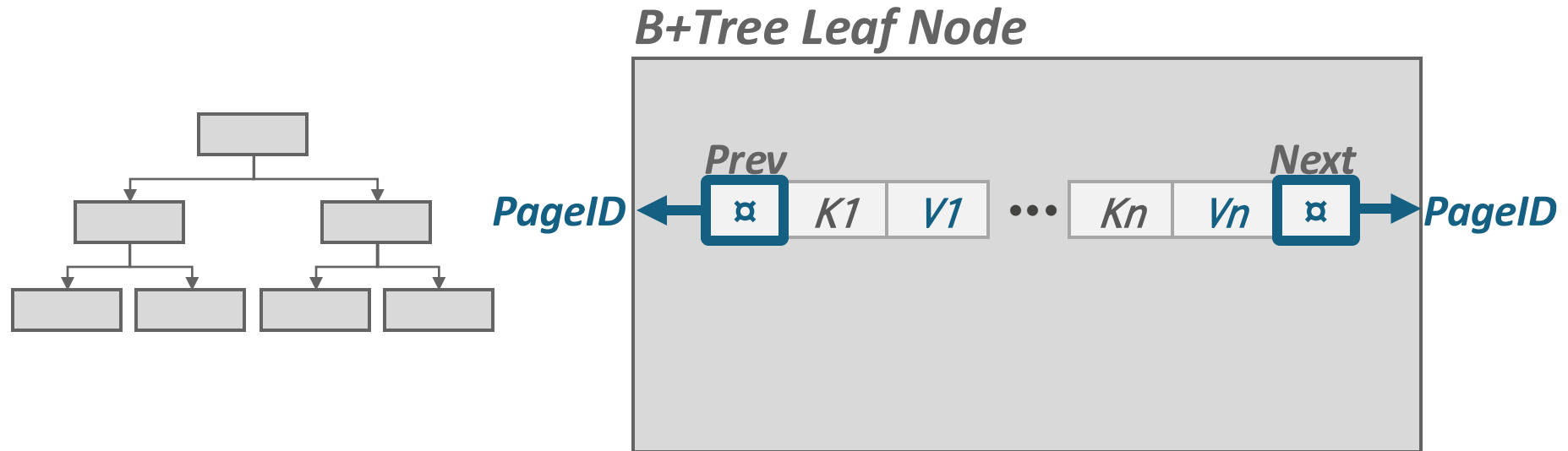
# B+Tree Leaf Nodes



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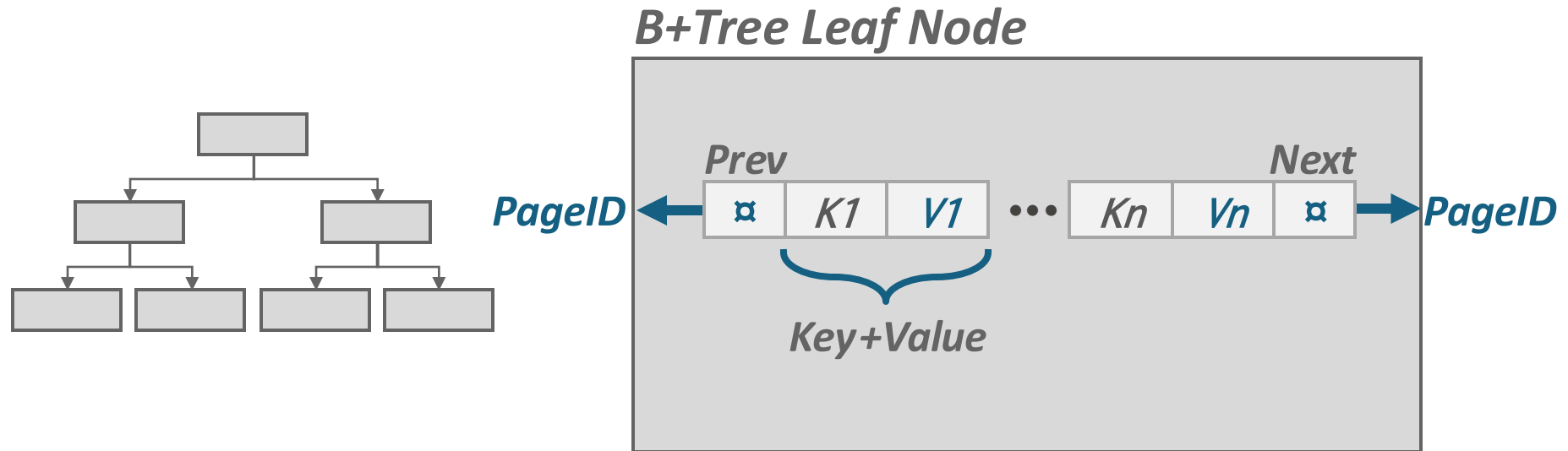


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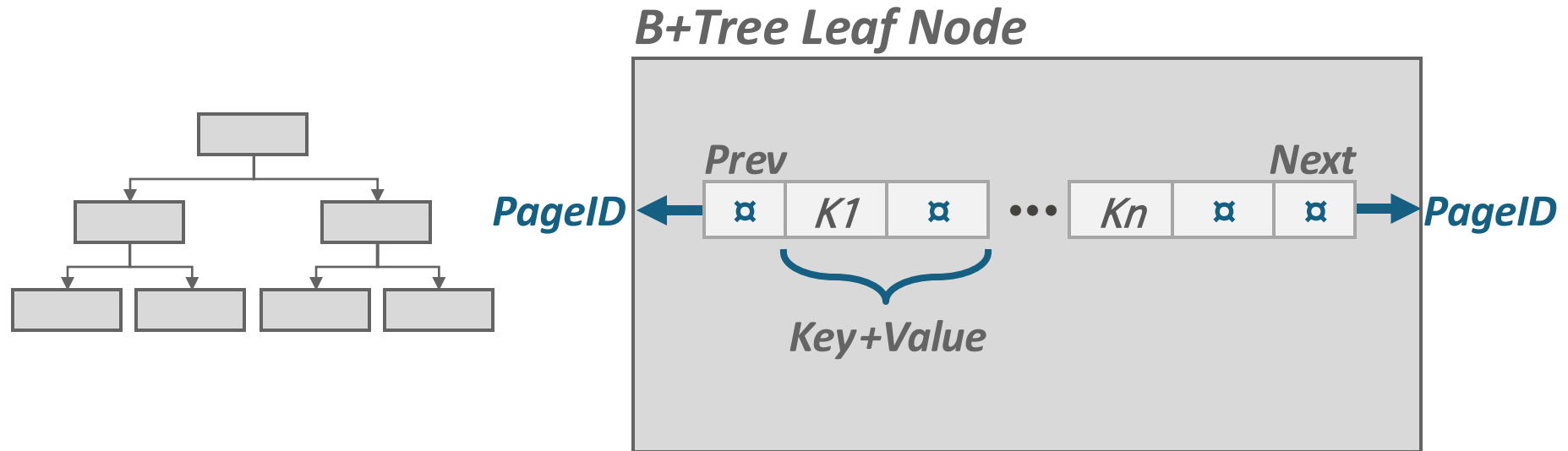




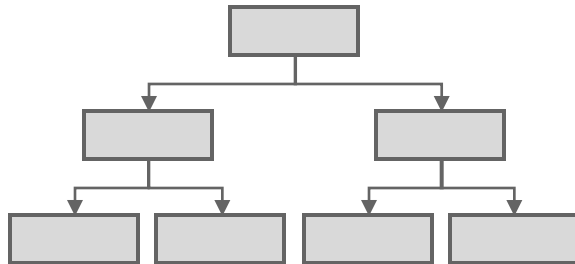
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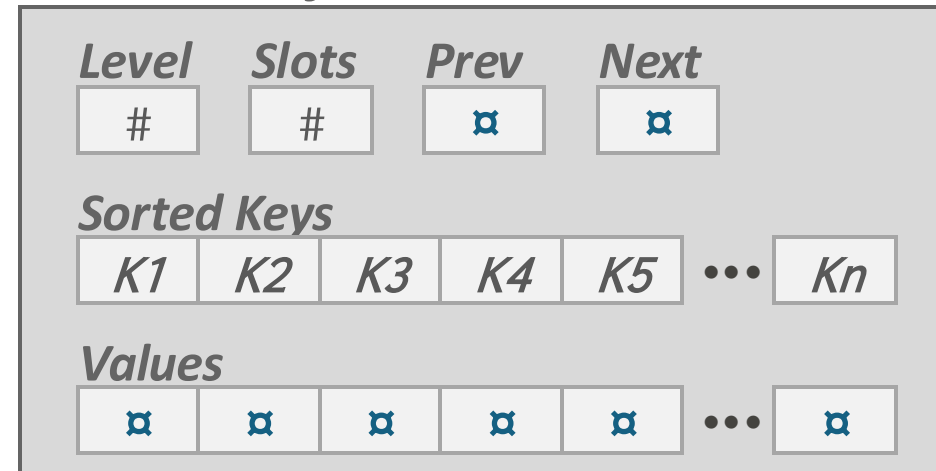
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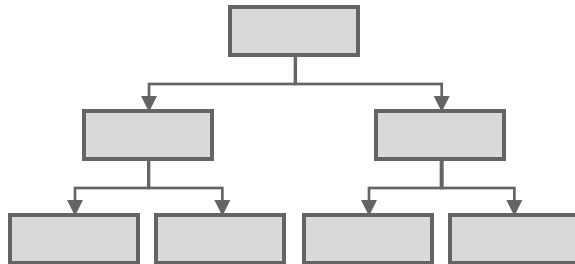
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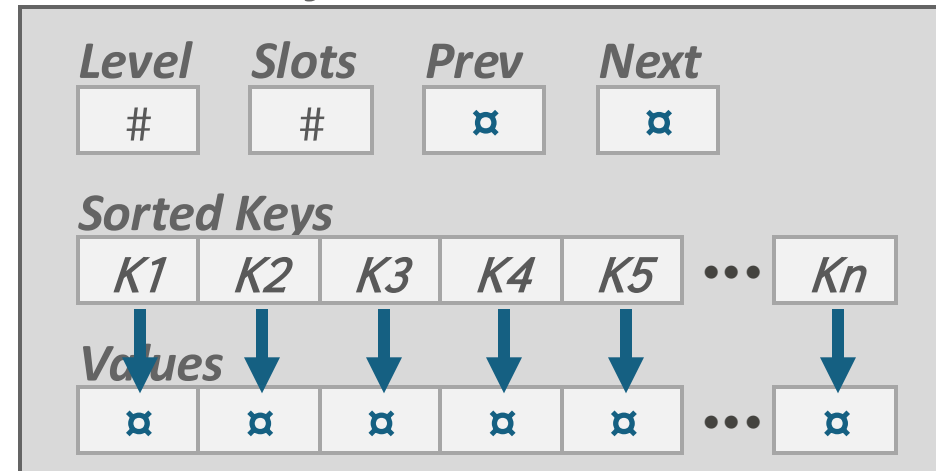
## *B+Tree Leaf Node*



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## *B+Tree Leaf Node*



# Leaf Node Values

- **Approach #1: Record IDs**
  - A pointer to the location of the tuple to which the index entry corresponds.
- **Approach #2: Tuple Data**
  - AKA Index-Organized Storage
  - The leaf nodes store the actual contents of the tuple.
  - Secondary indexes must store the Record ID as their values.

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# B-Tree vs. B+Tree

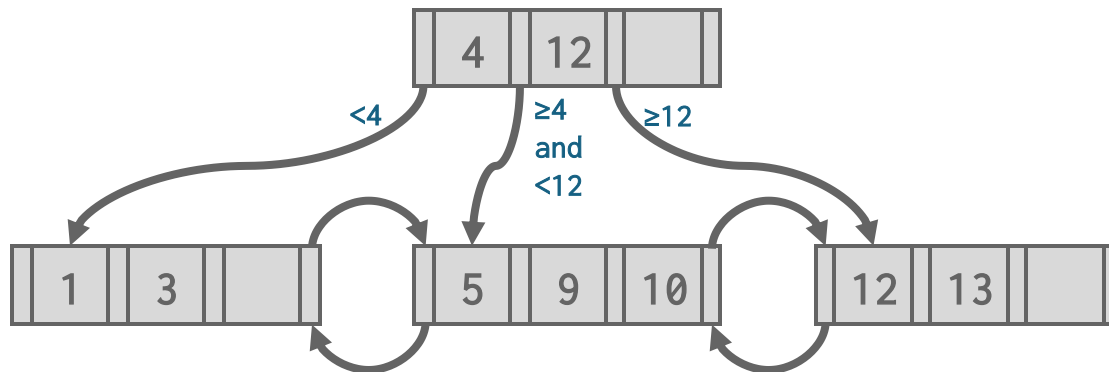
- The original **B-Tree** from 1972 stored keys and values in all nodes in the tree.
  - More space-efficient, since each key only appears once in the tree.
- A **B+Tree** only stores values in leaf nodes. Inner nodes only guide the search process.

# B+Tree: Insert

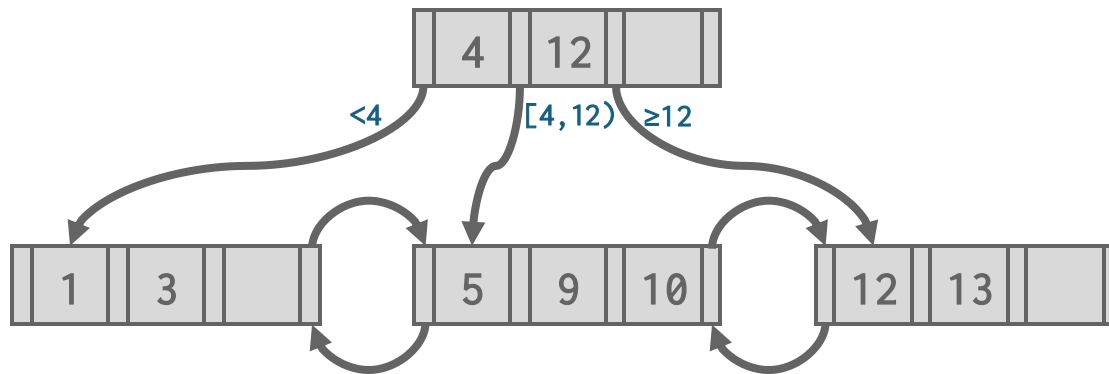
- Find correct leaf node **L**.  
Insert data entry into **L** in sorted order.
- If **L** has enough space, done!
- Otherwise, split **L** keys into **L** and a new node **L2**
  - Redistribute entries evenly, copy up middle key.
  - Insert index entry pointing to **L2** into parent of **L**.
- To split inner node, redistribute entries evenly, but push up middle key.



# B+Tree: Insert

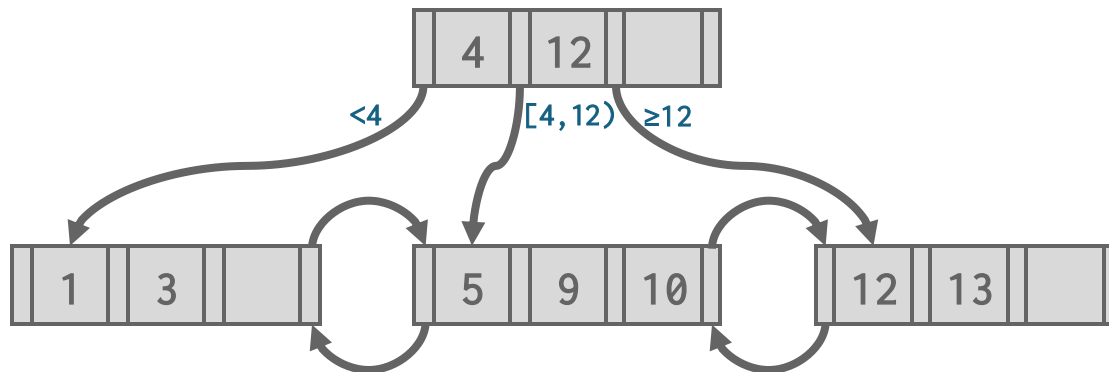


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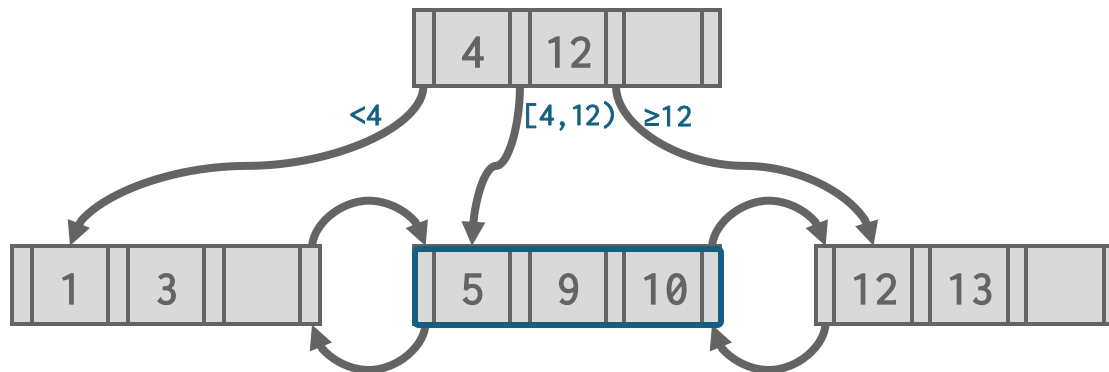
# B+Tree: Insert

Insert 6



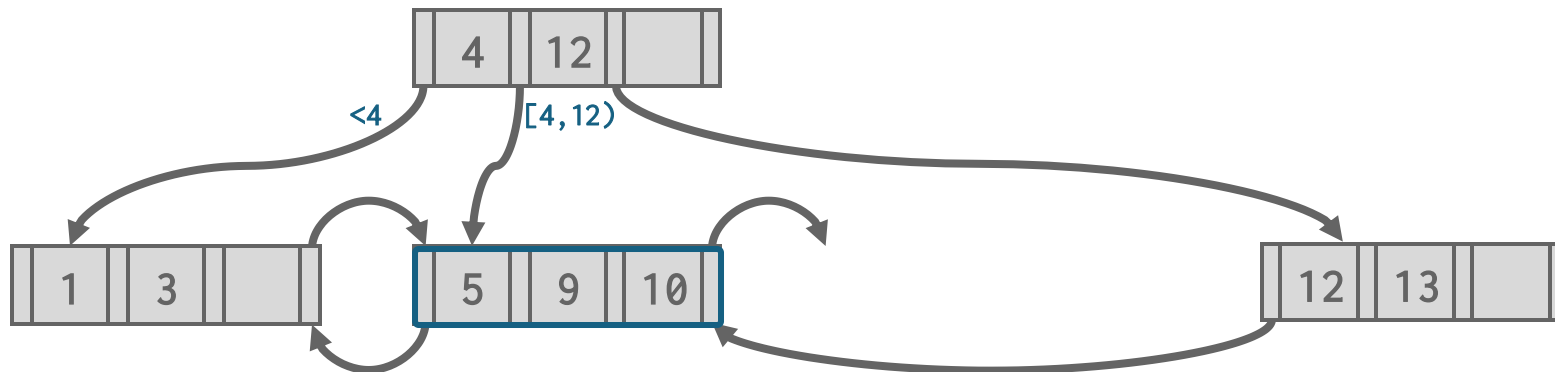
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Insert 6



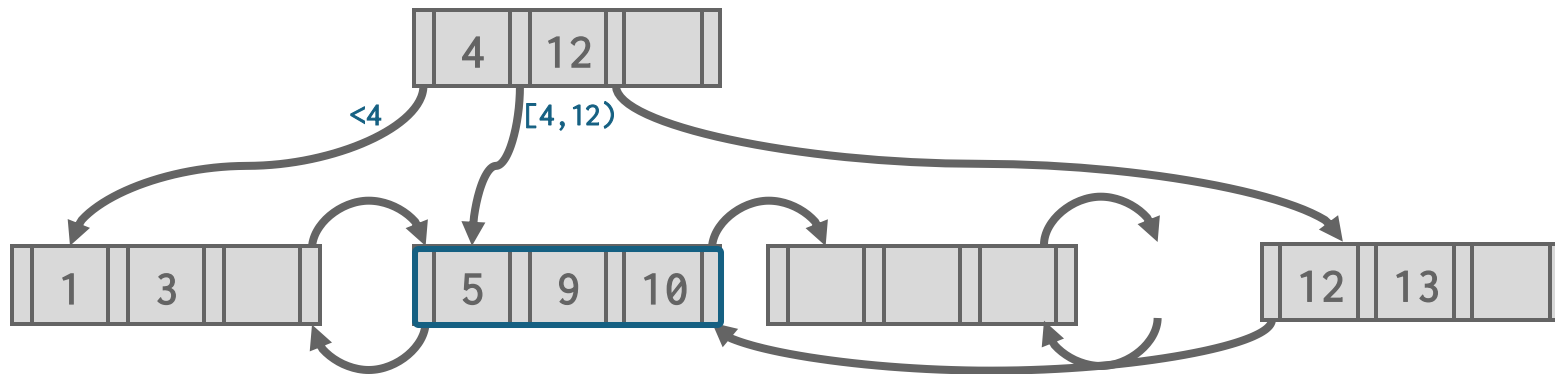
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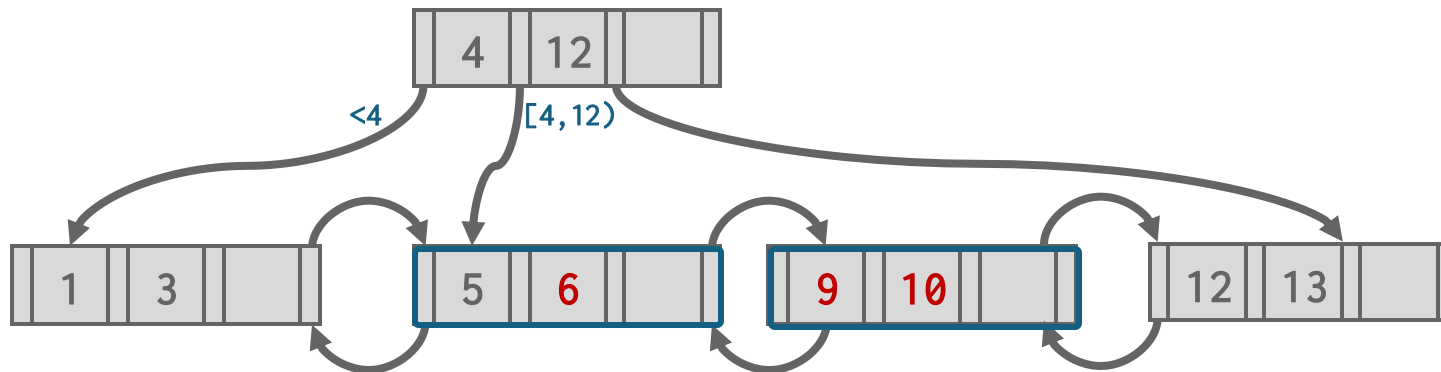
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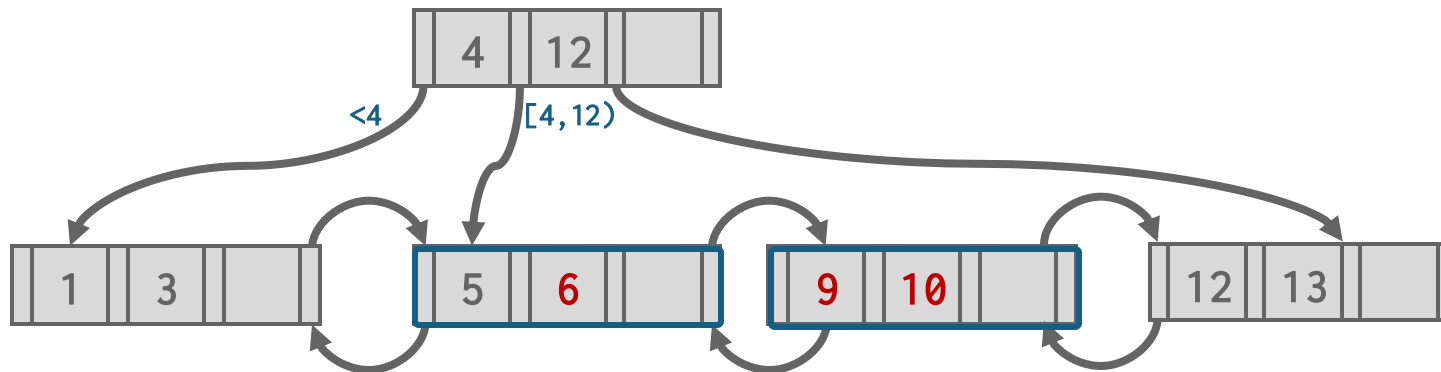
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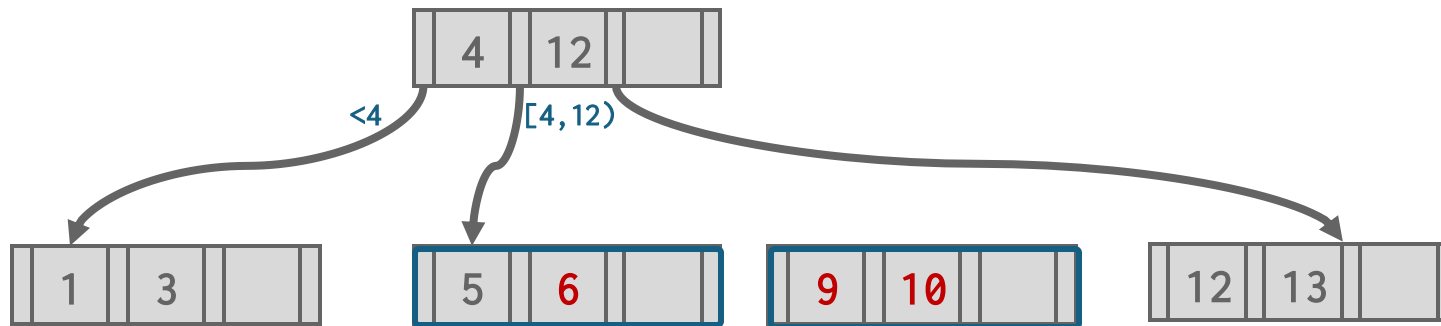
Insert 6





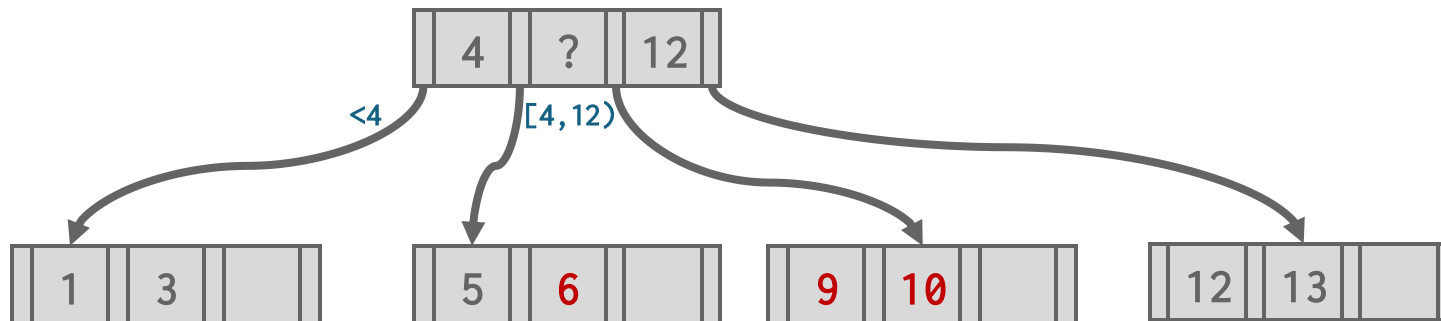
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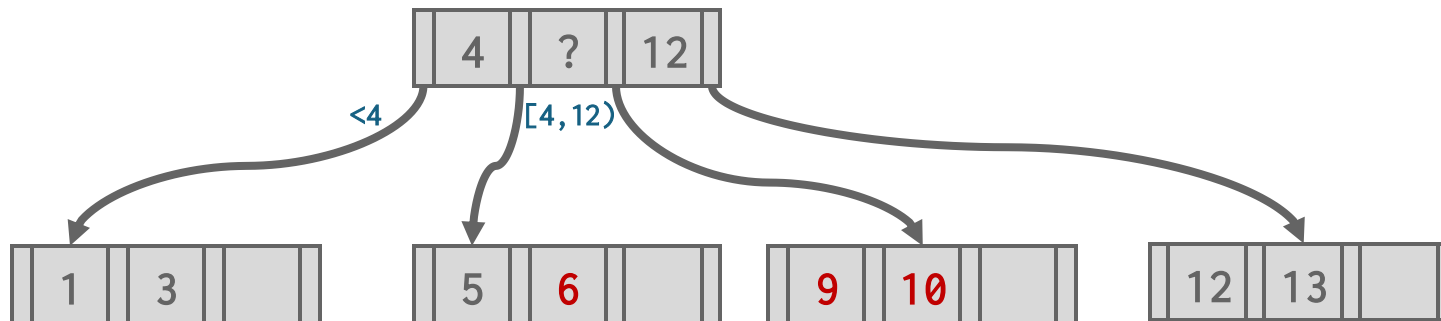
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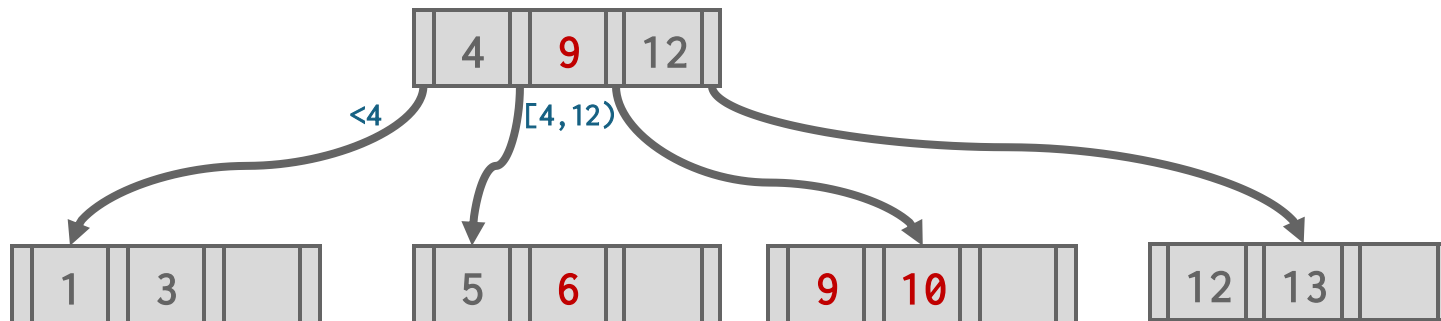
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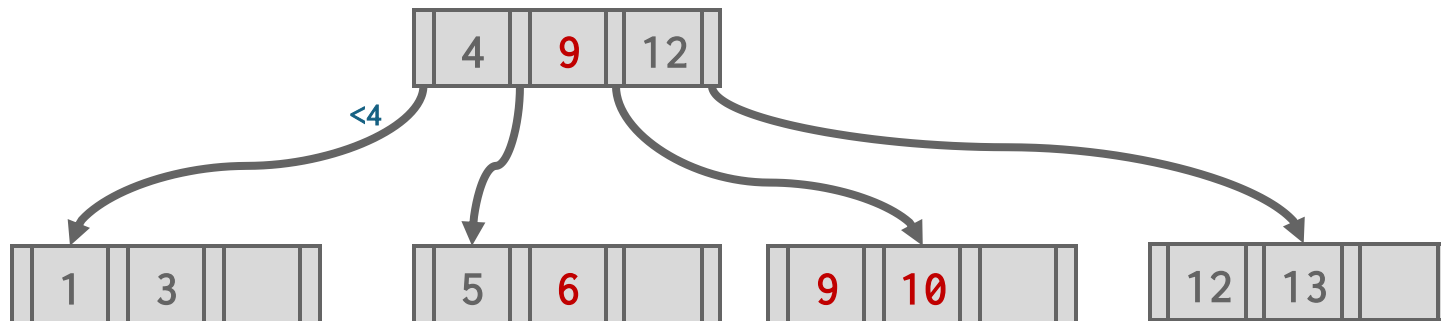
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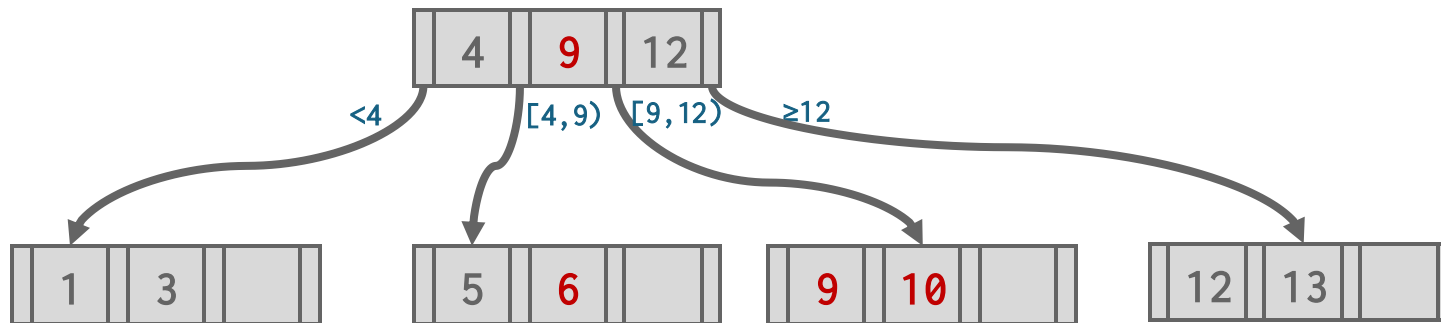
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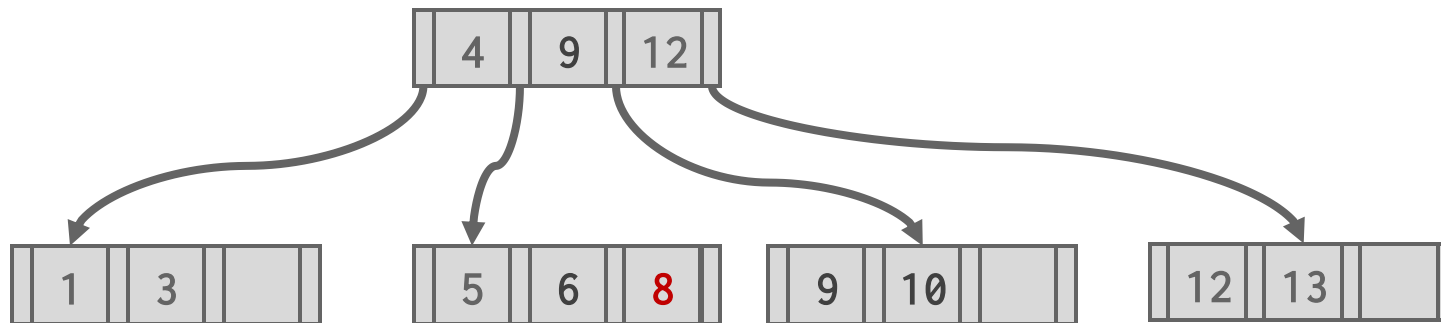
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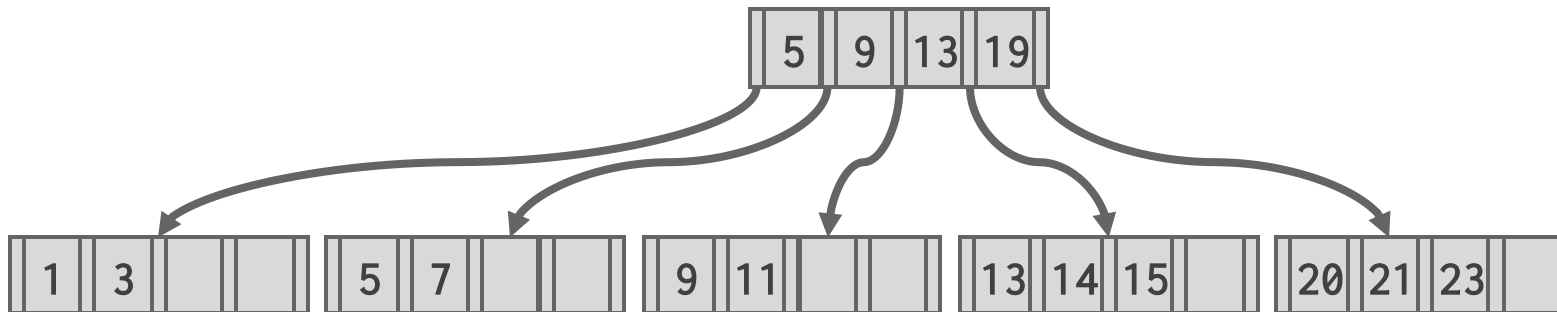
Insert 6

Insert 8



# Insert the Key 17

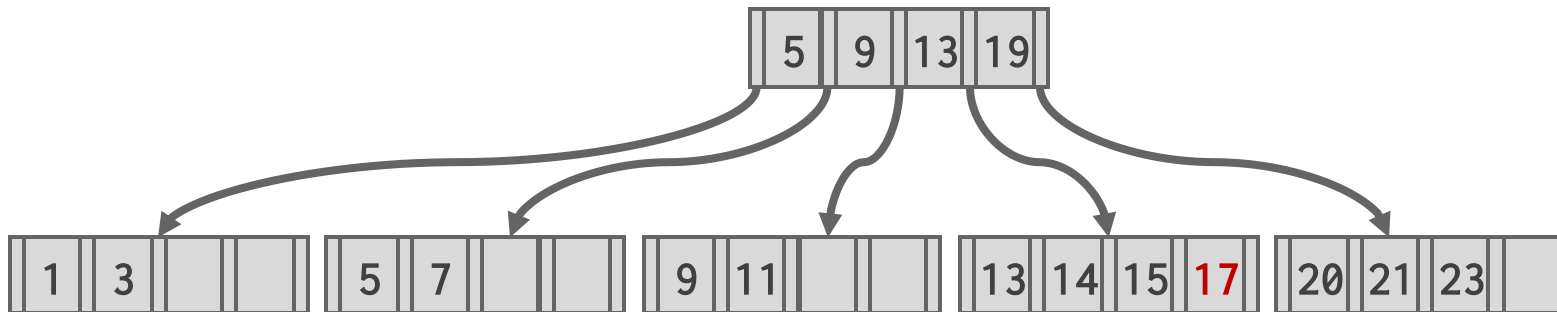
Note: new example/tree.



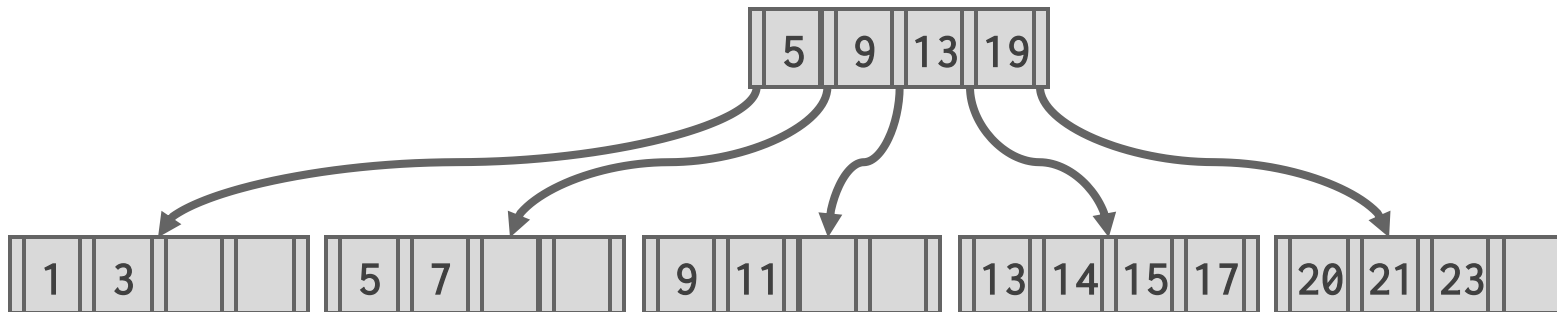


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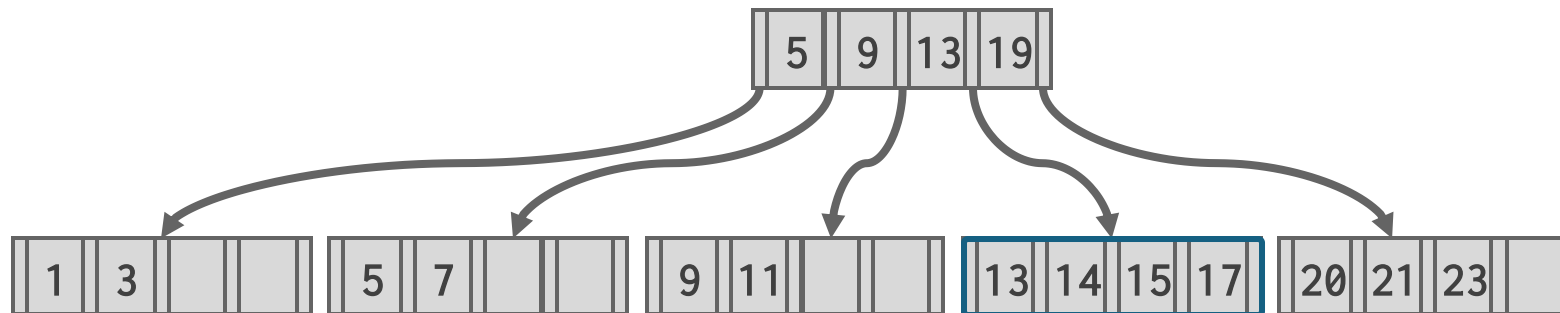
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# Next, Insert the Key 16

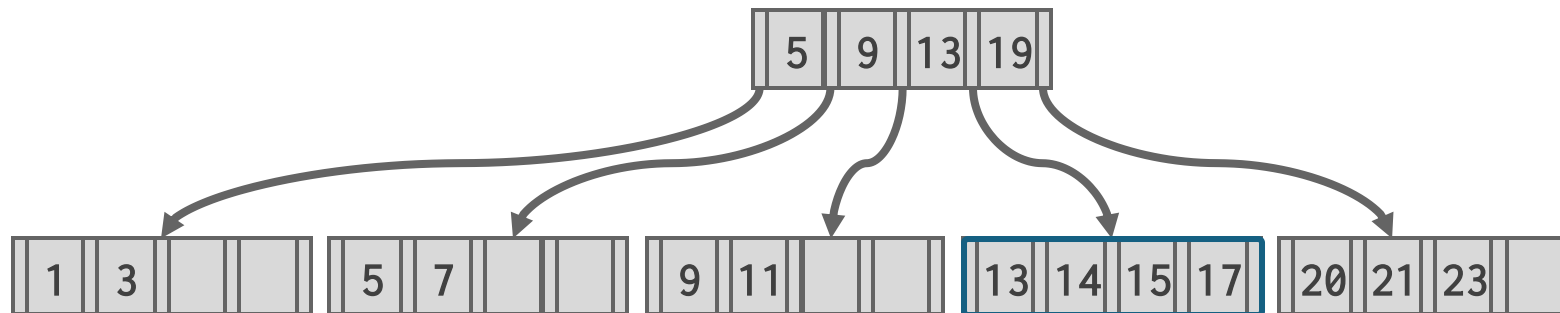


# Next, Insert the Key 16



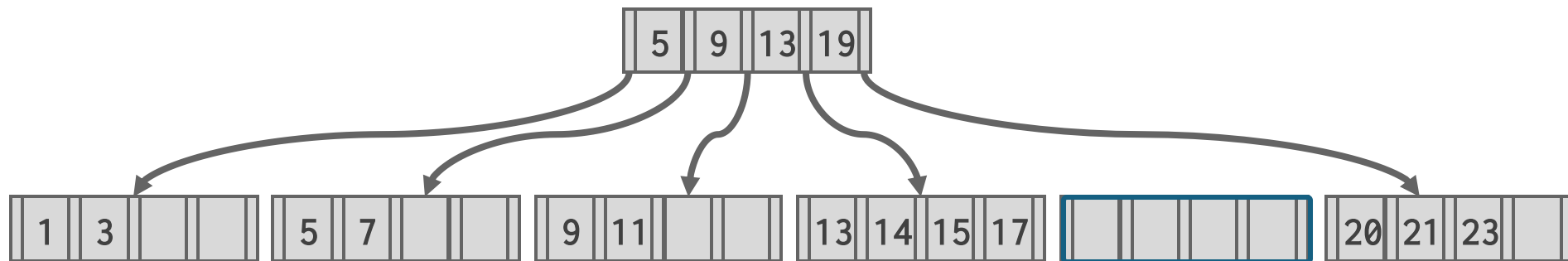
No space in the  
node where the  
new key  
“belongs”.

# Next, Insert the Key 16



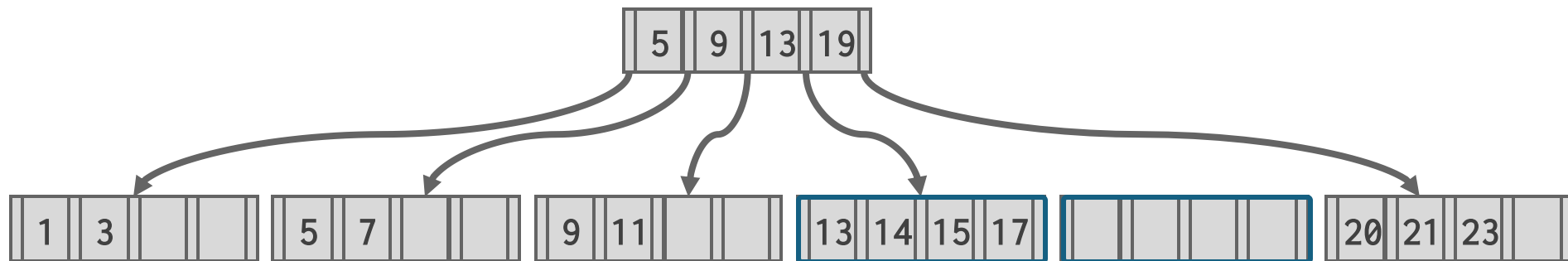
Split the node!  
Copy the middle  
key.  
Push the key up.

# Next, Insert the Key 16



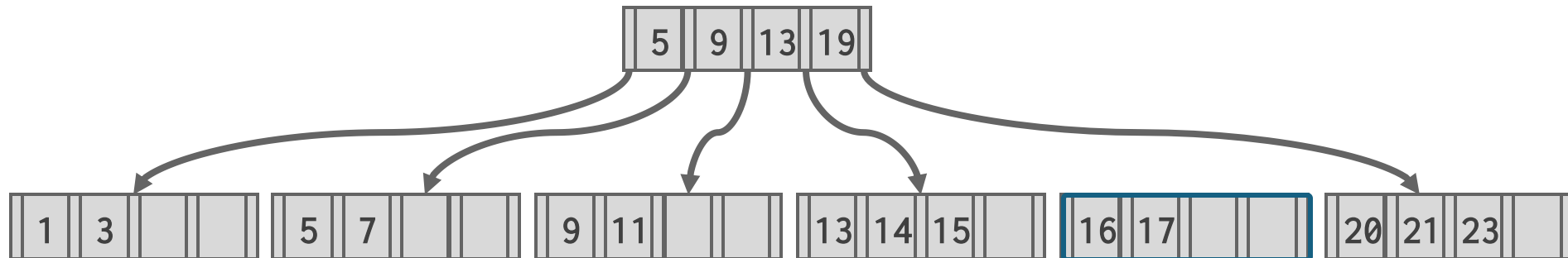
New node.  
Shuffle keys from  
the node that  
triggered the split.

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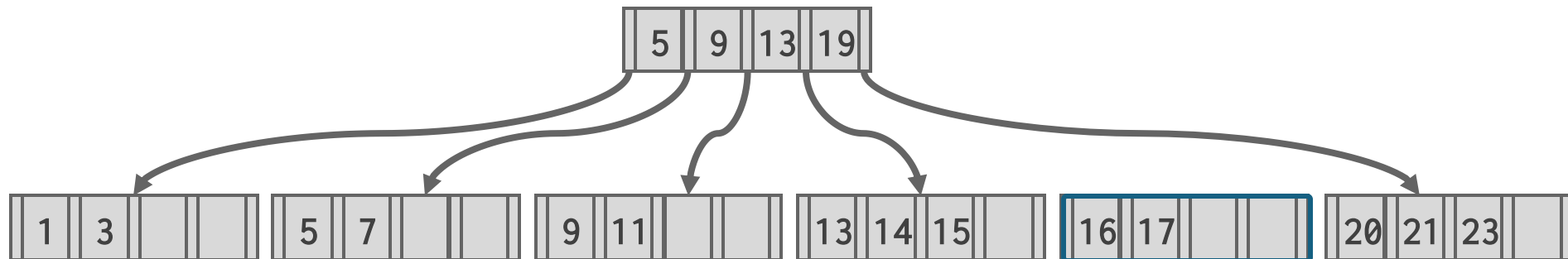


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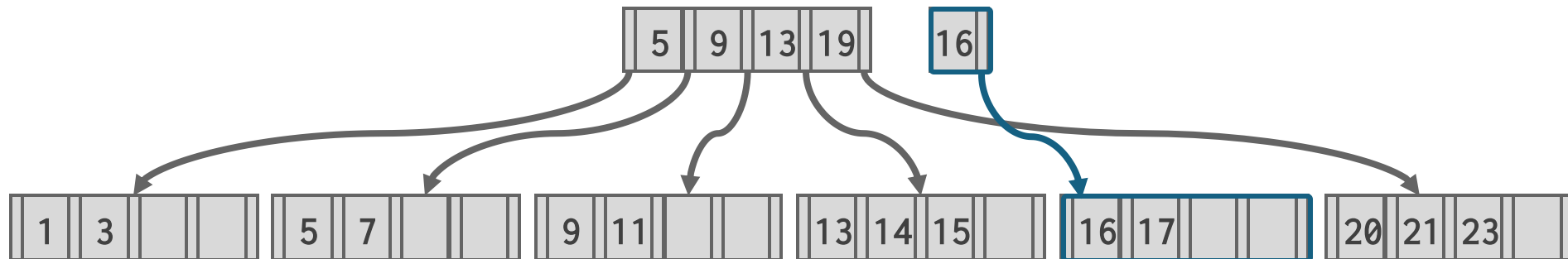
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But, this is an  
“orphan” node. No  
parent node points to  
it.

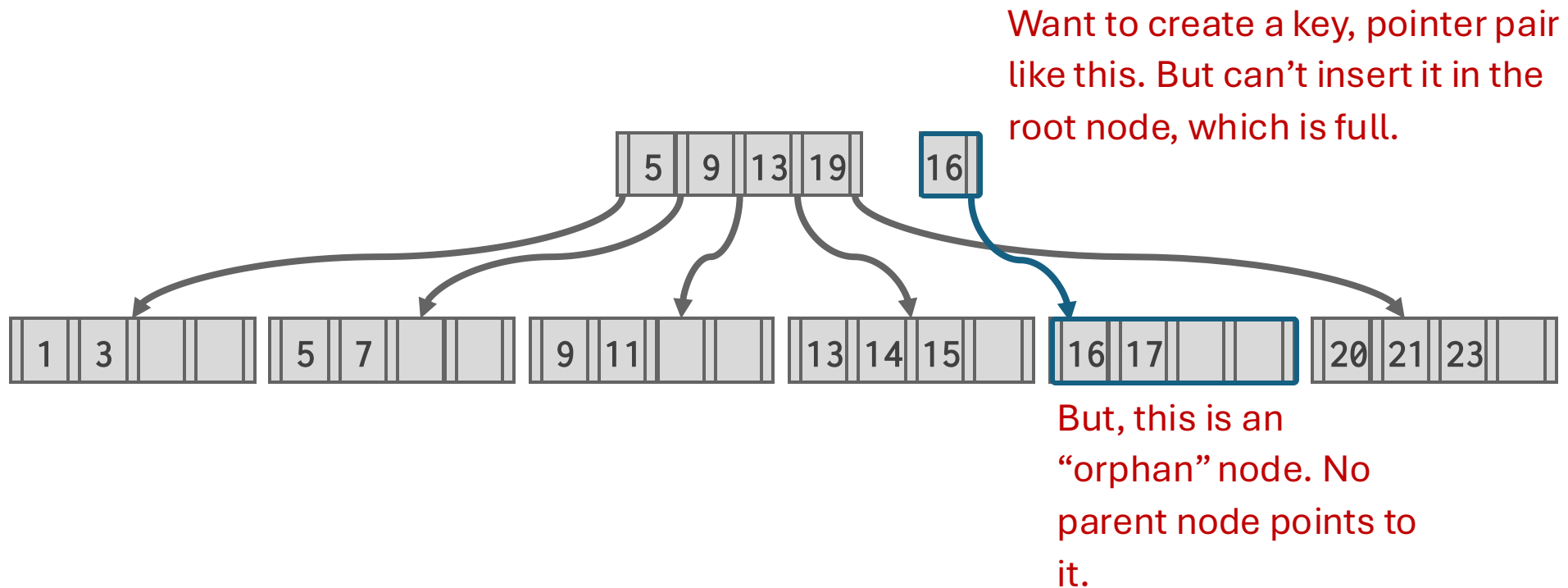


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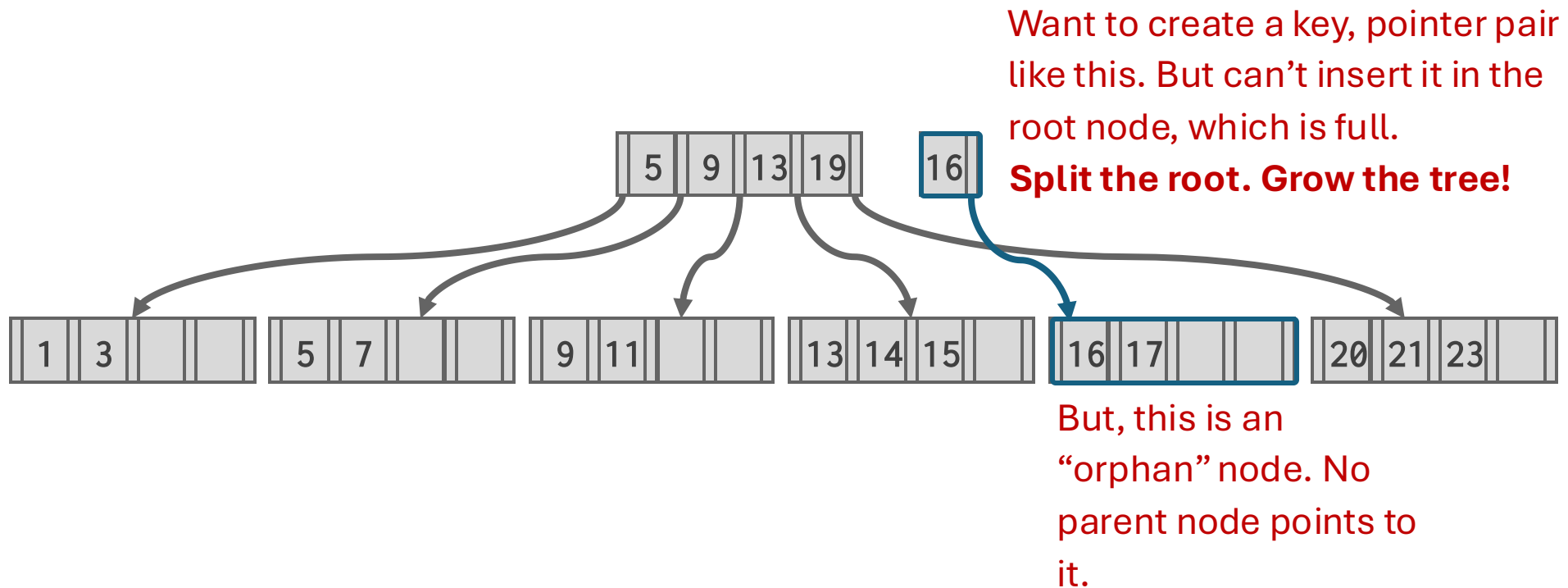


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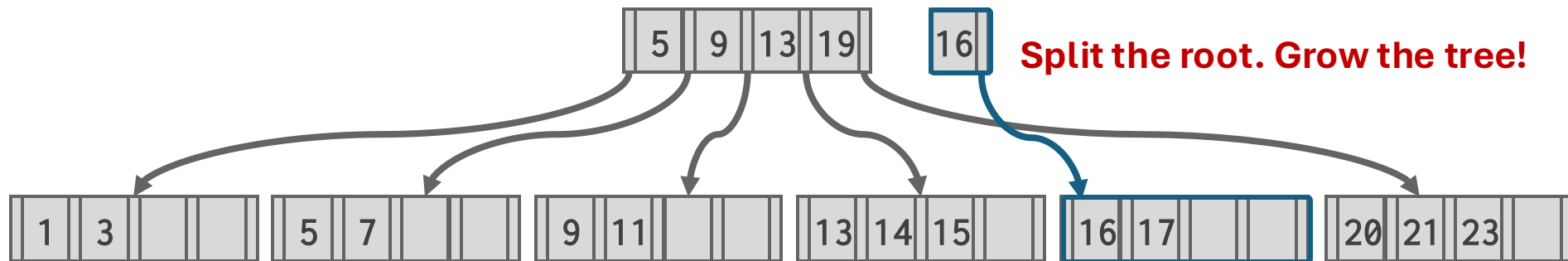
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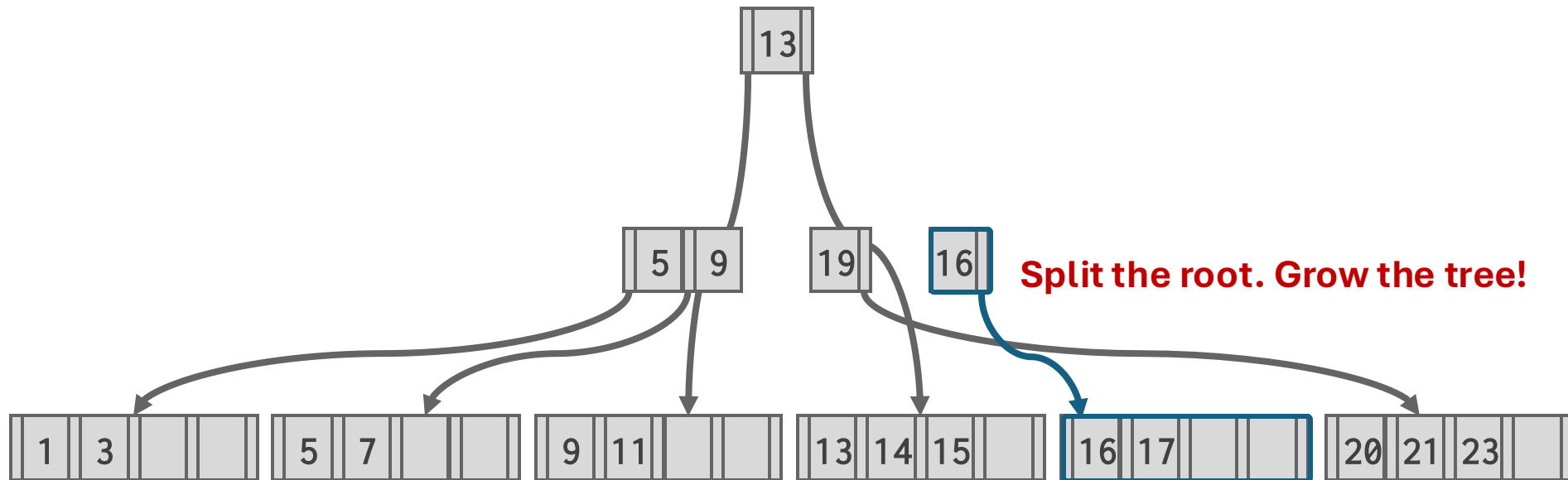
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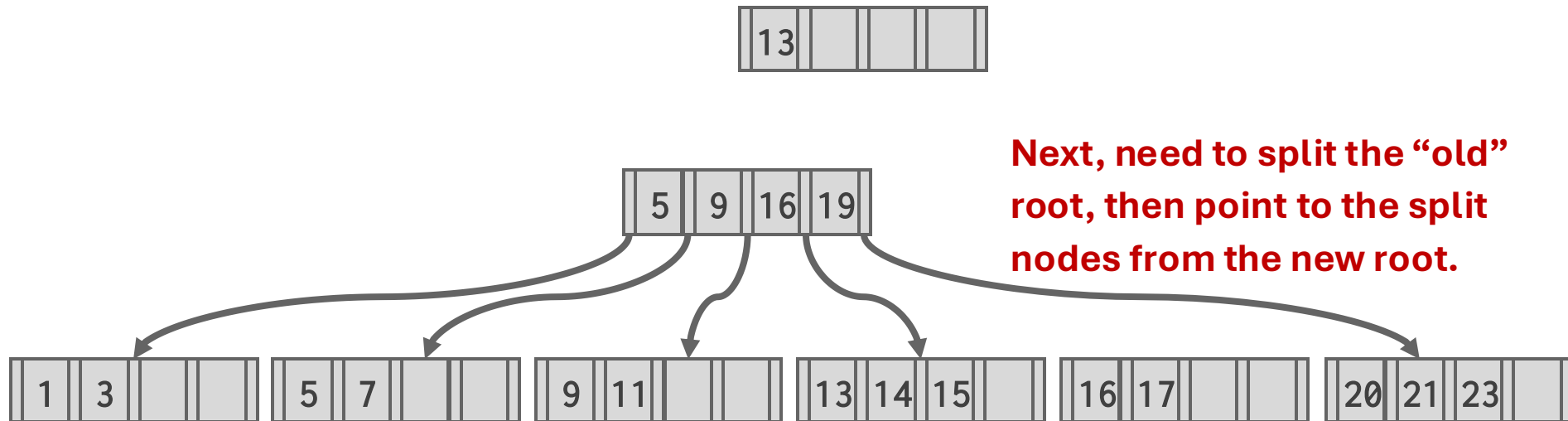
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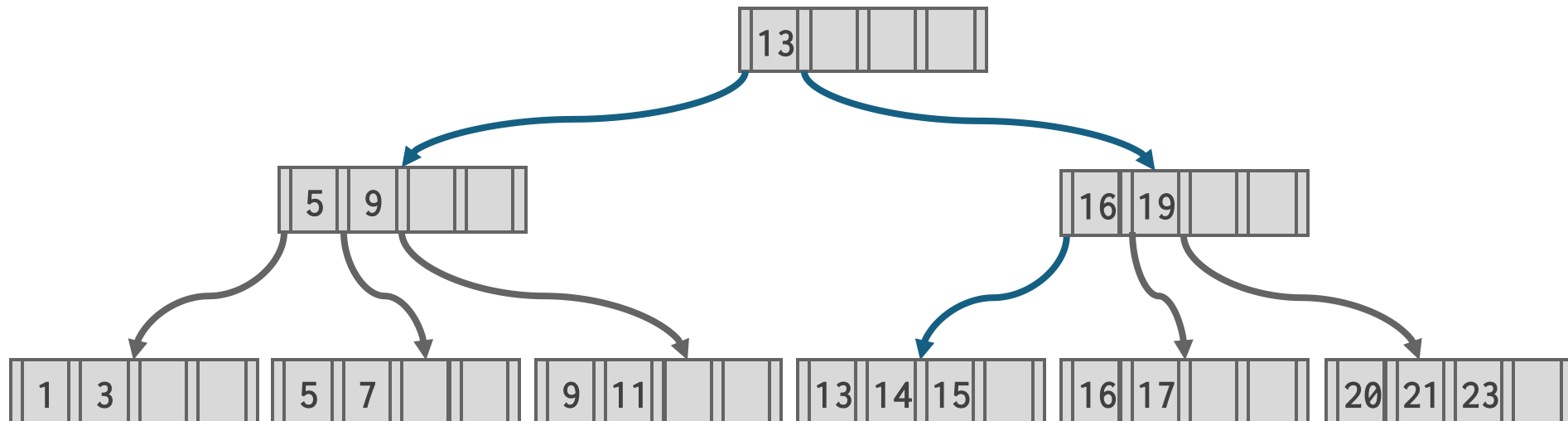
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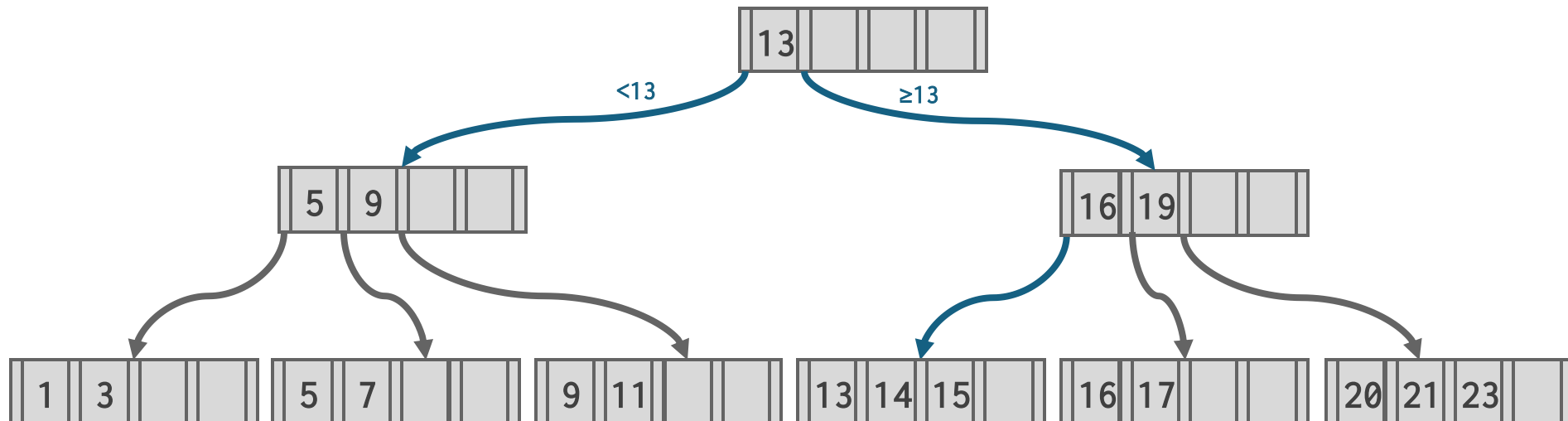
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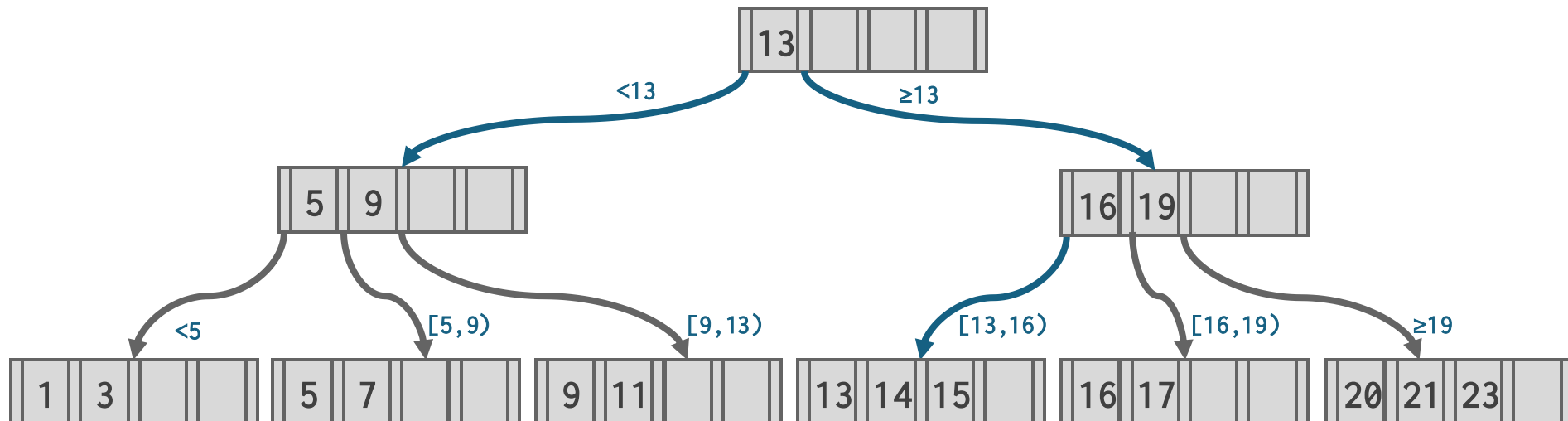


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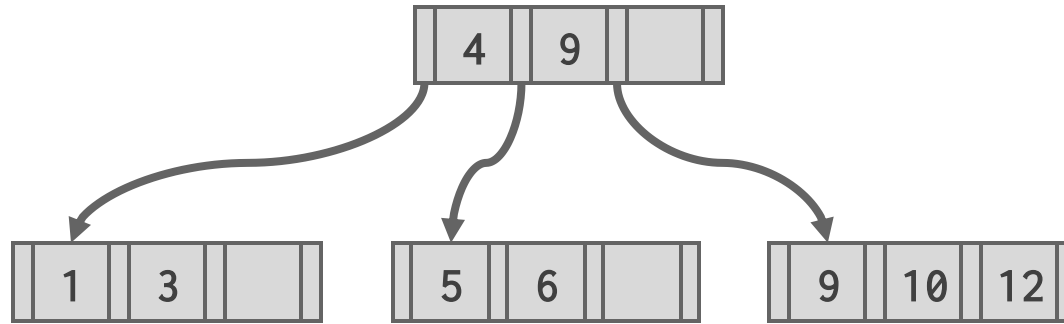
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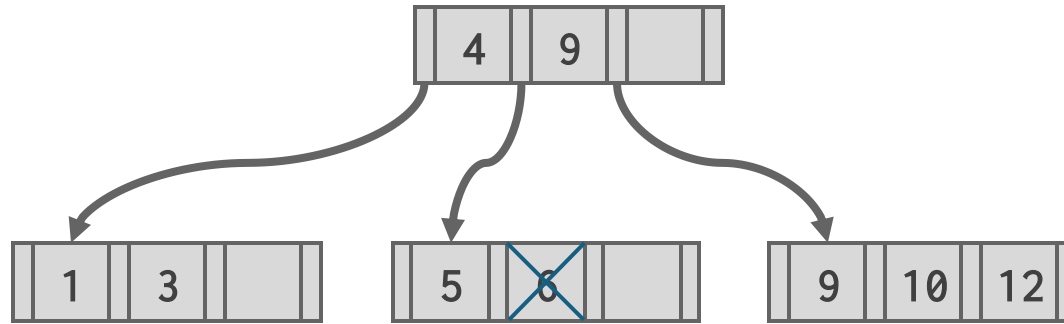
# B+Tree: Delete

- Start at root, find leaf **L** where entry belongs.  
Remove the entry.  
If **L** is at least half-full, done!  
If **L** has only  $M/2-1$  entries,
  - Try to re-distribute, borrowing from sibling (adjacent node with same parent as **L**).
  - If re-distribution fails, merge **L** and sibling.
- If merge occurred, must delete entry (pointing to **L** or sibling) from parent of **L**.

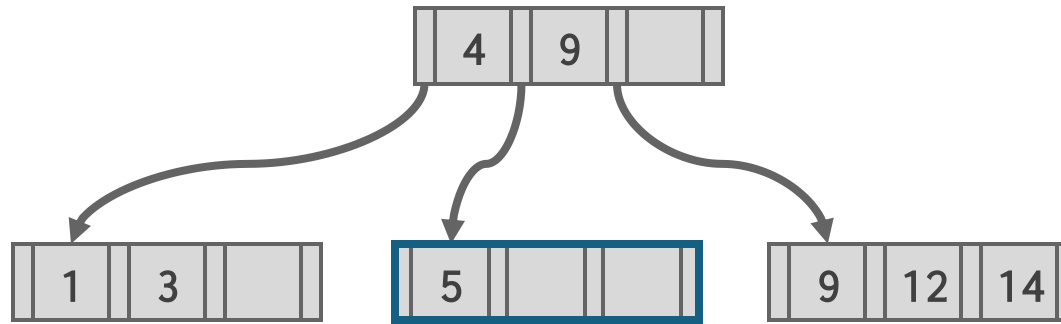
# Delete the Key 6



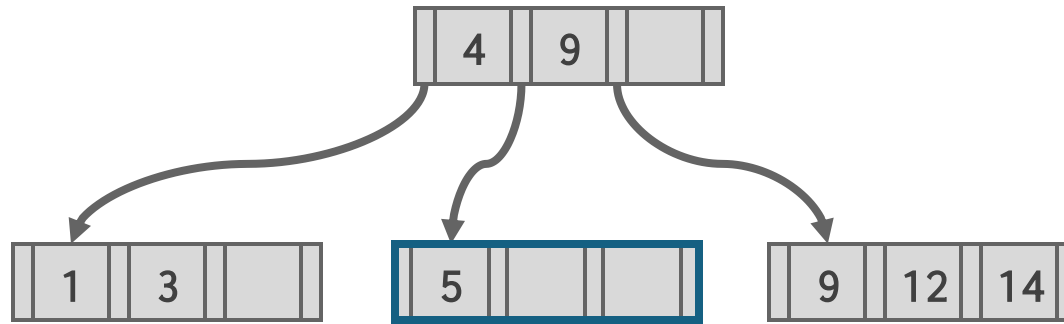
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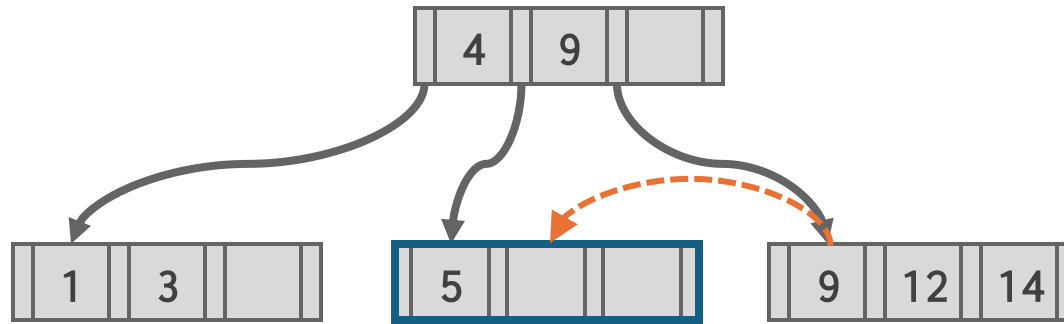


# Delete the Key 6



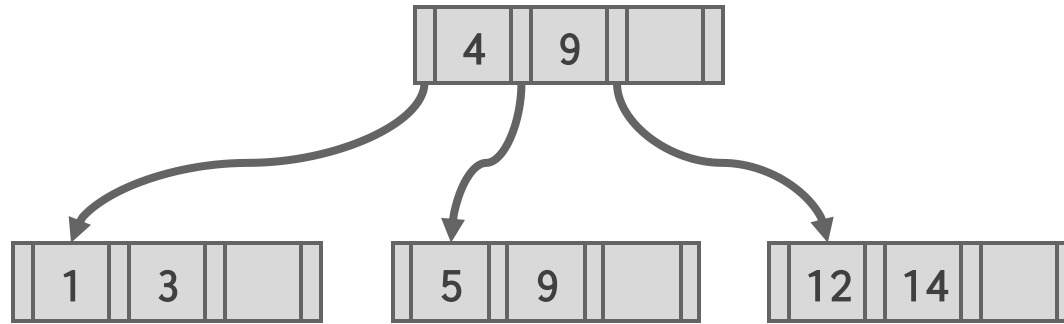
Borrow from a “rich” neighbor.

# Delete the Key 6



Borrow from a “rich” neighbor.

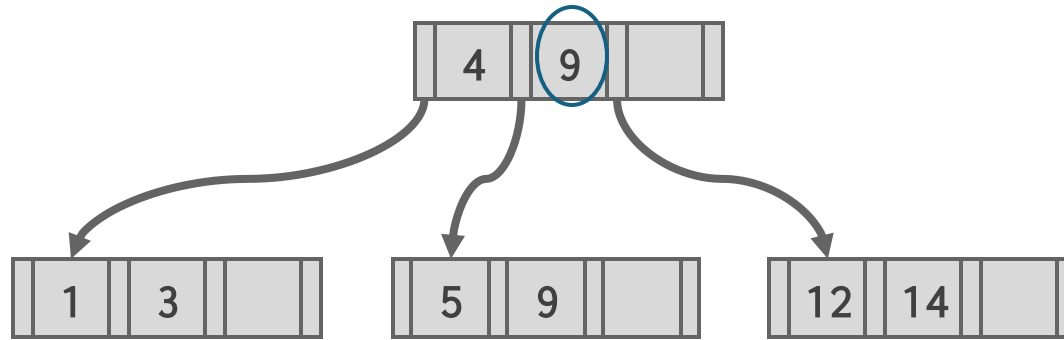
# Delete the Key 6



Borrow from a “rich” neighbor.  
Could borrow from either neighbor.

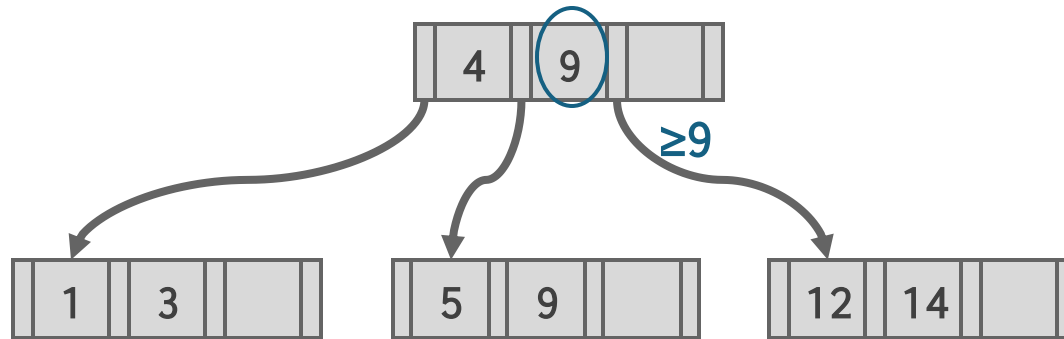


# Delete the Key 6



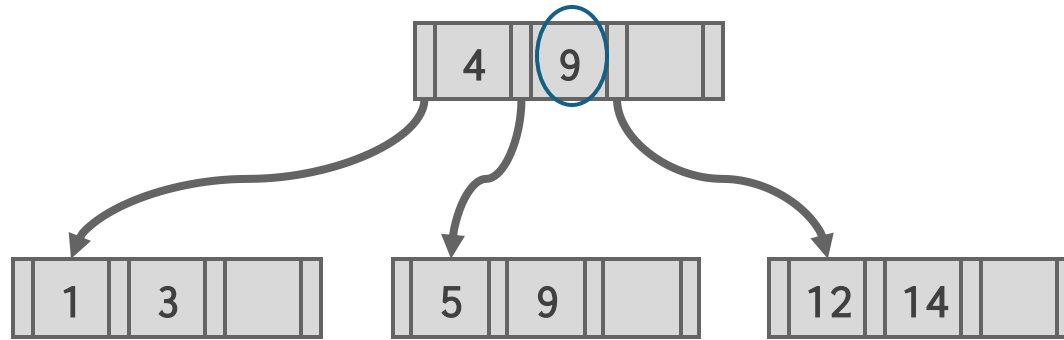
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Could borrow from either neighbor.

# Delete the Key 6



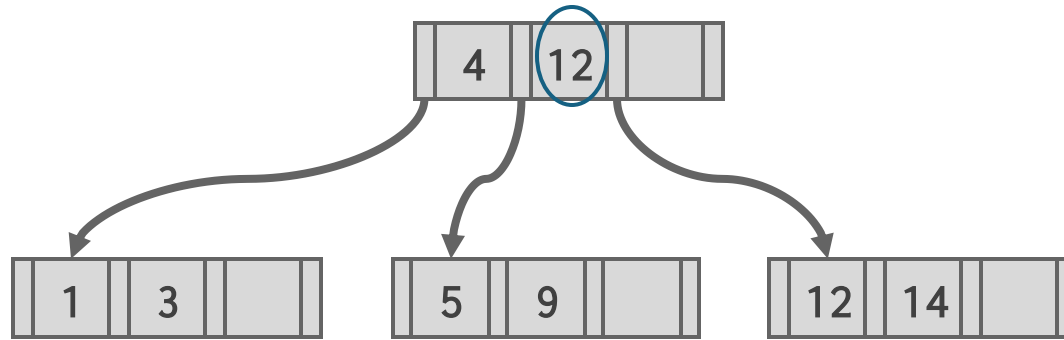
Borrow from a “rich” neighbor.  
Could borrow from either neighbor.

# Delete the Key 6



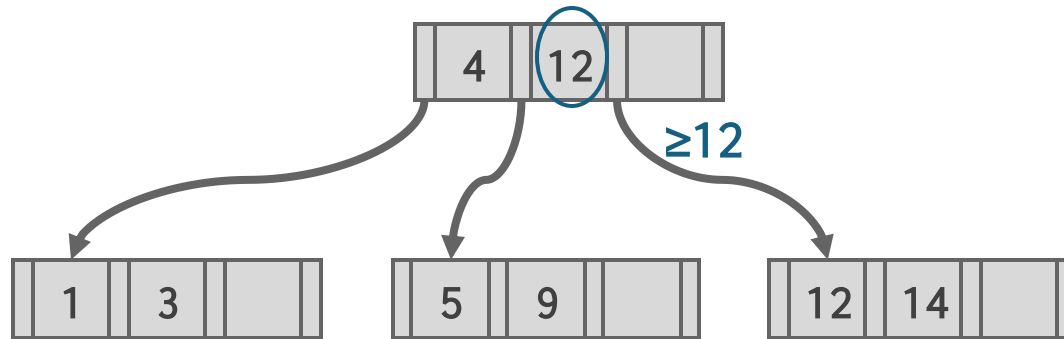
Borrow from a “rich” neighbor.  
Could borrow from either neighbor.

# Delete the Key 6



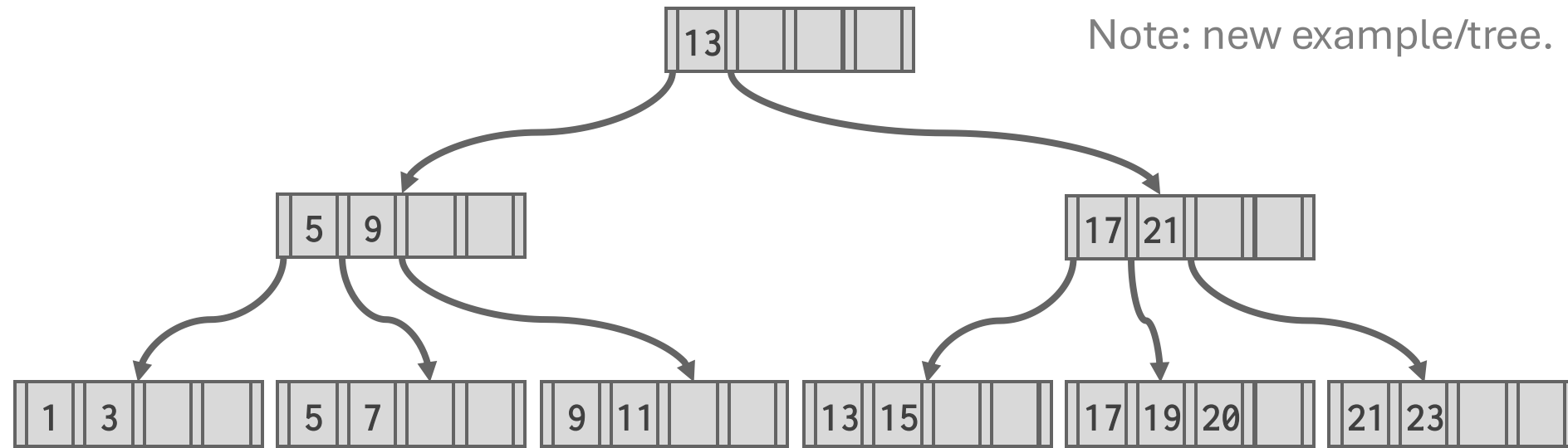
Borrow from a “rich” neighbor.  
Could borrow from either neighbor.

# Delete the Key 6

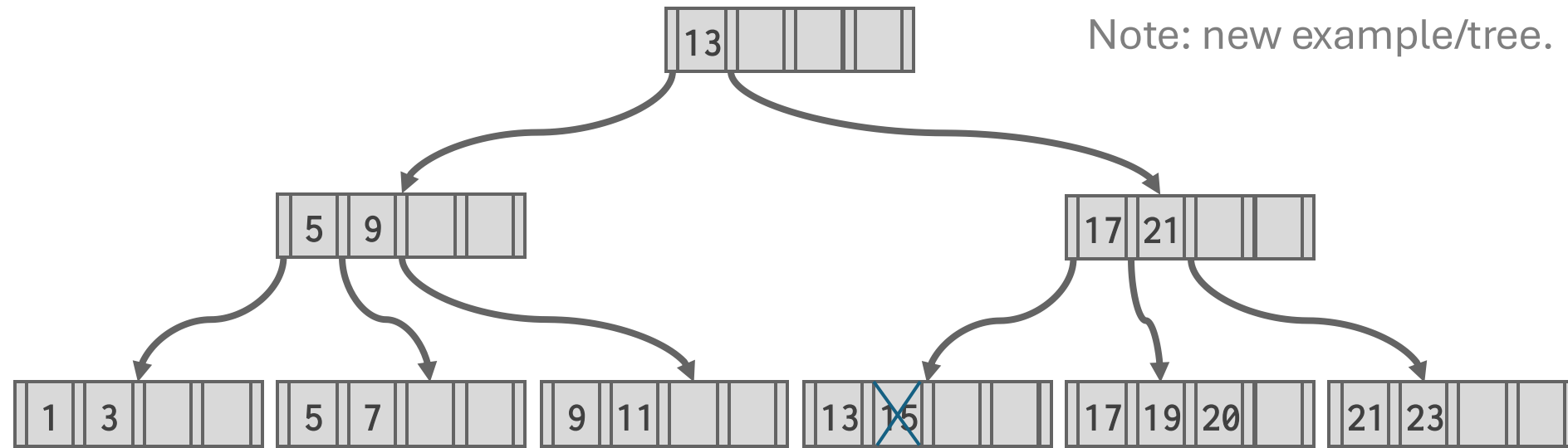


Borrow from a “rich” neighbor.  
Could borrow from either neighbor.

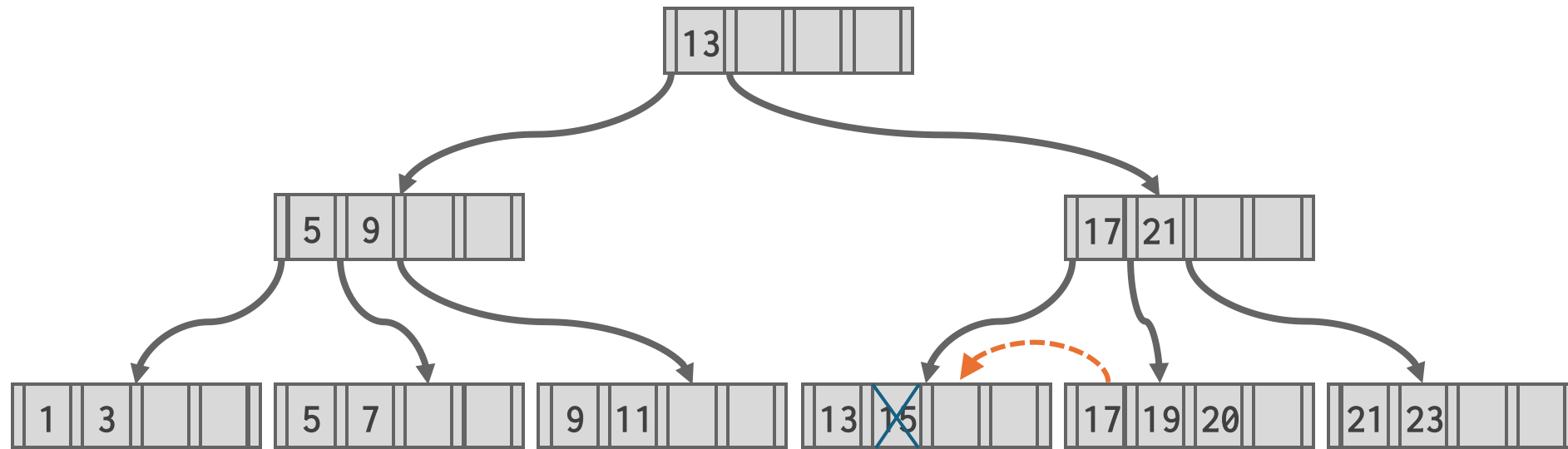
# Delete the Key 15



# Delete the Key 15

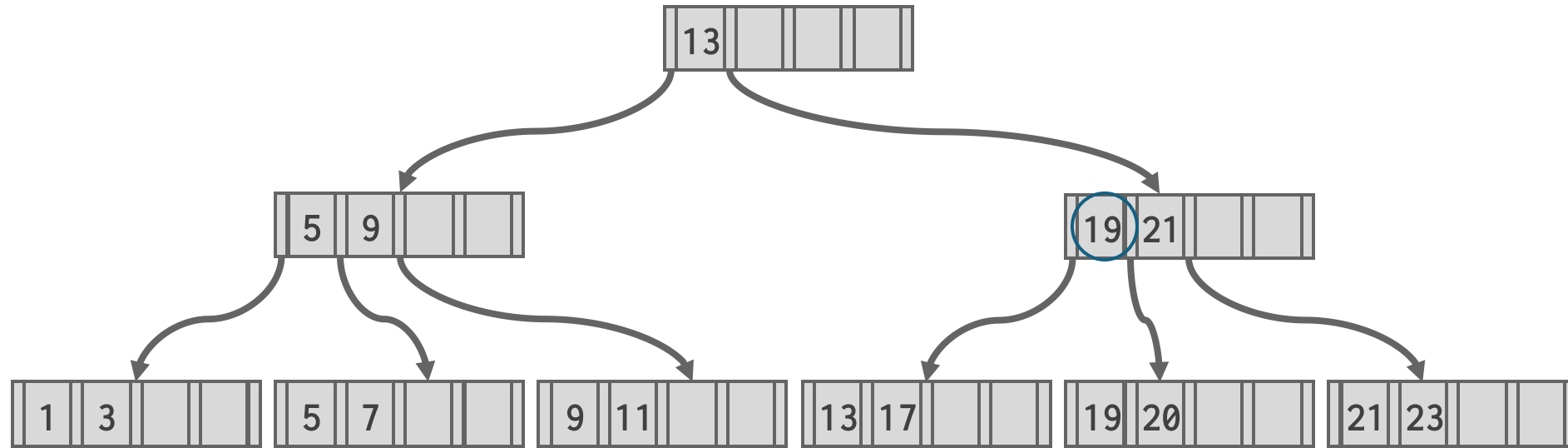


# Delete the Key 15

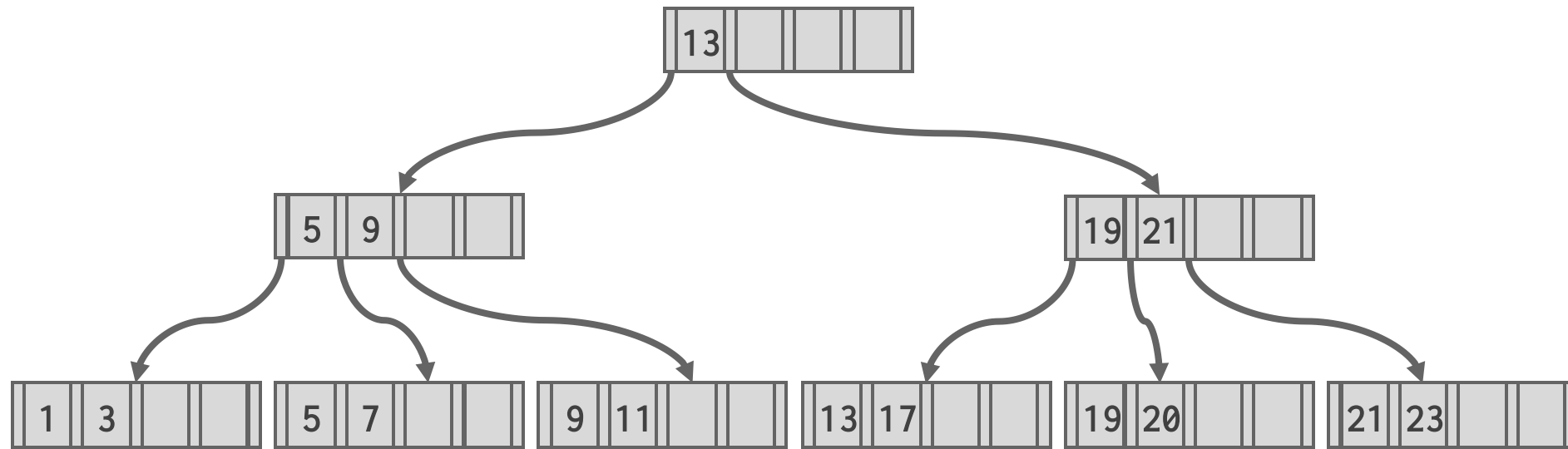




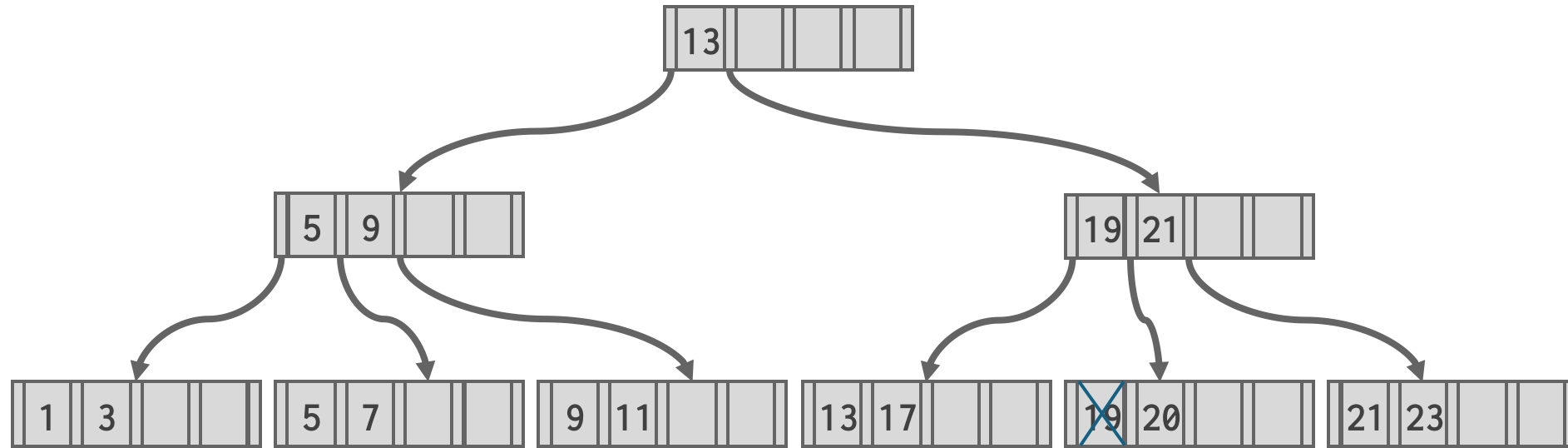
# Delete the Key 15



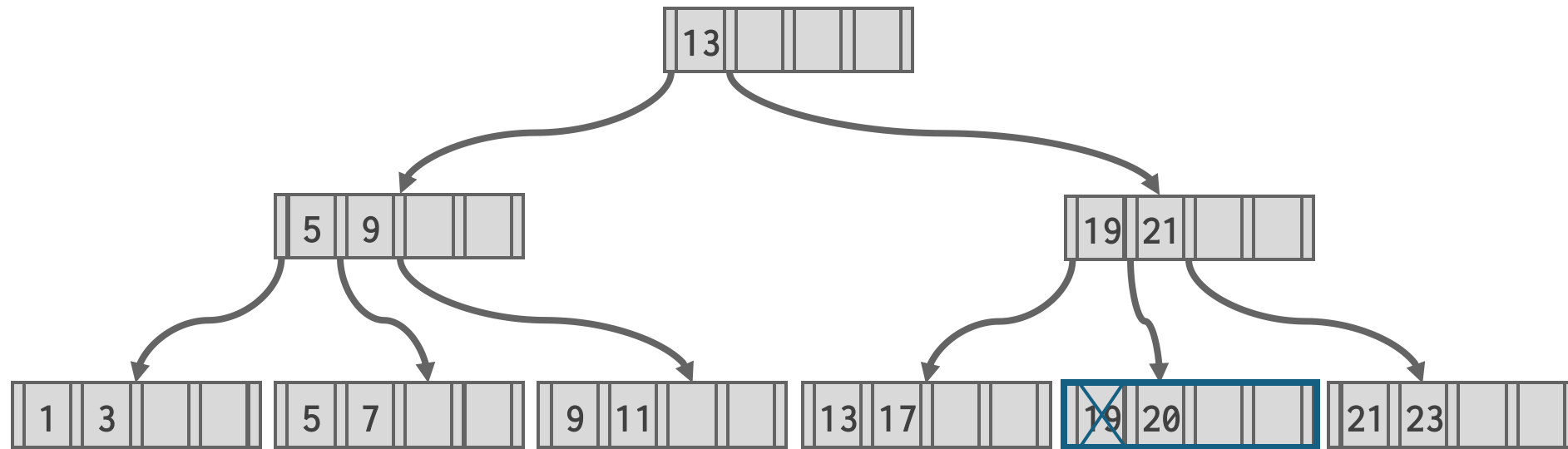
# Delete the Key 19



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# Delete the Key 19

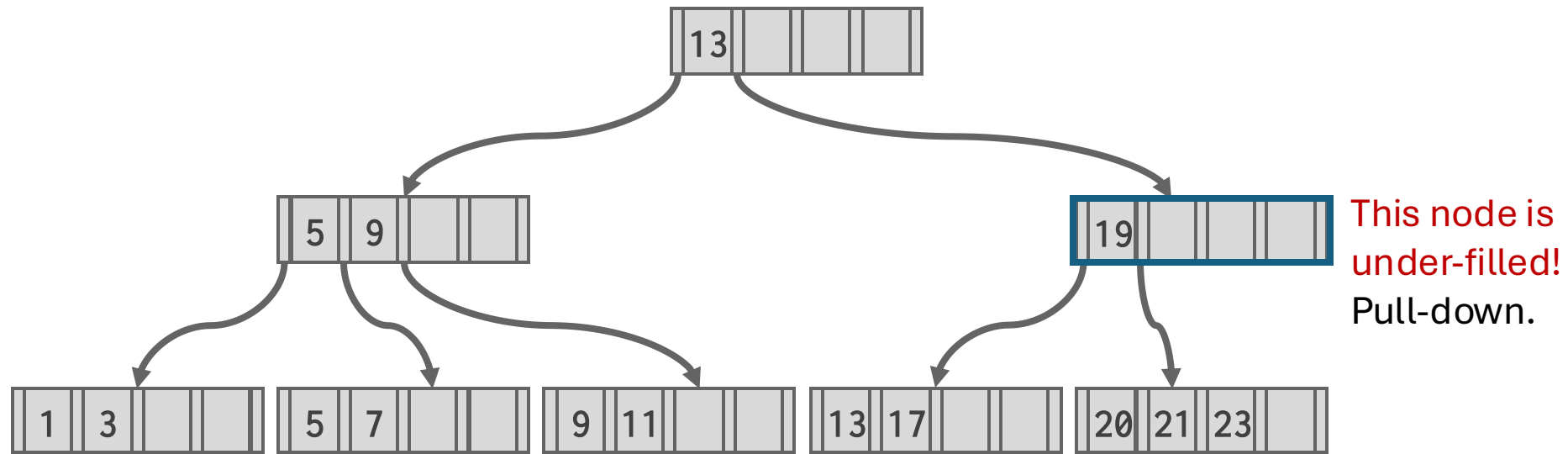


Under-filled.

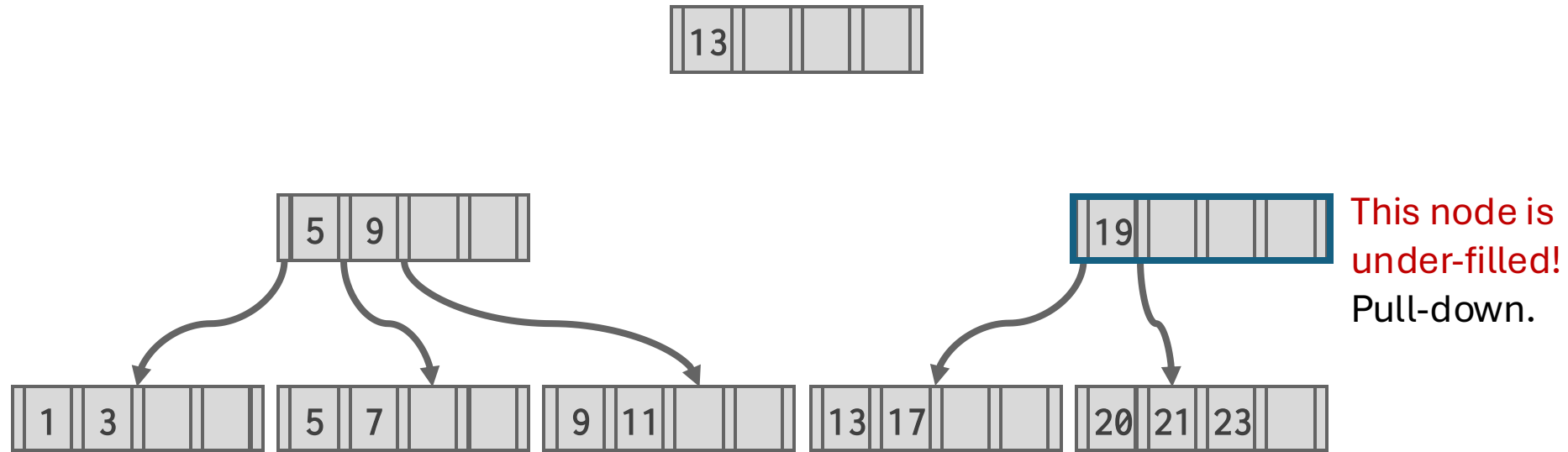
No “rich” neighbors to borrow.

Merge with a sibling

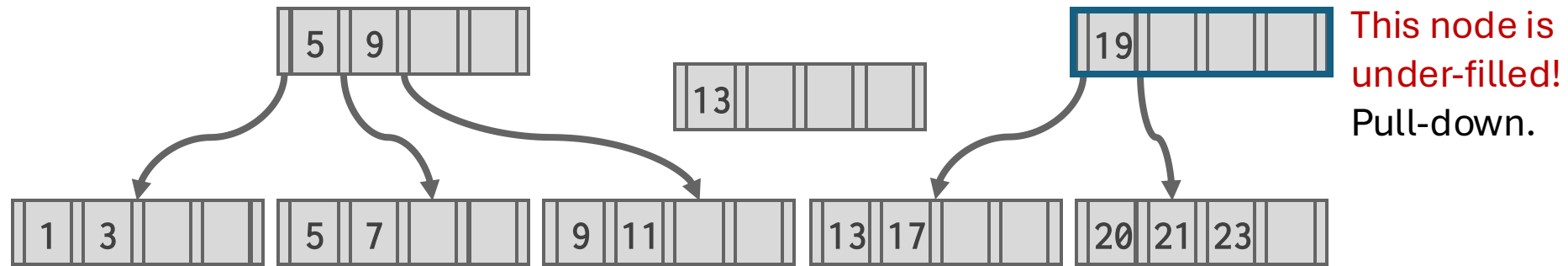
# Delete the Key 19



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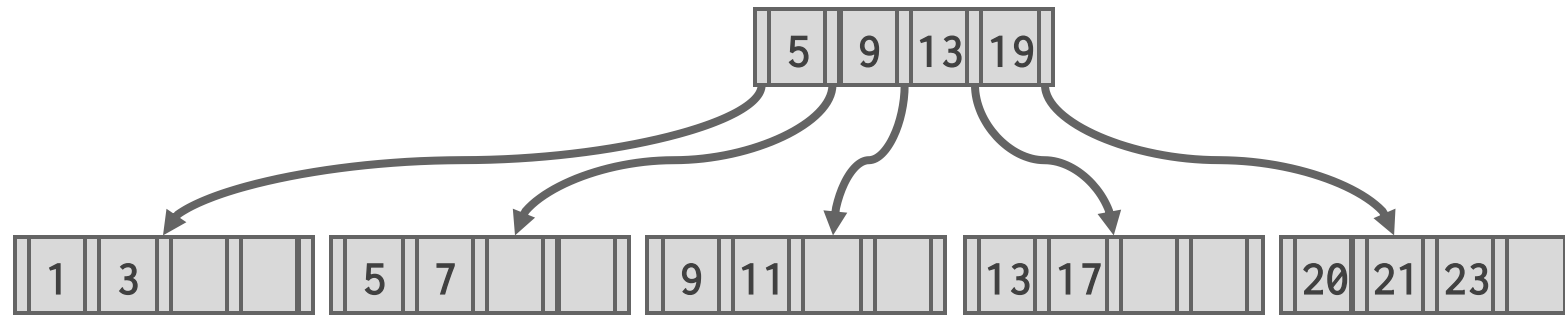


# Delete the Key 19



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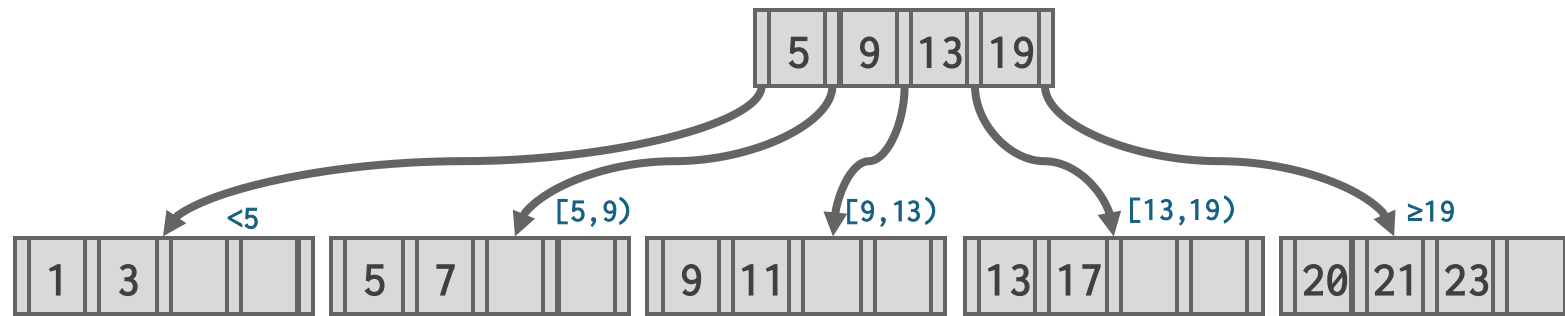
The tree has shrunk in height.





# Delete the Key 19

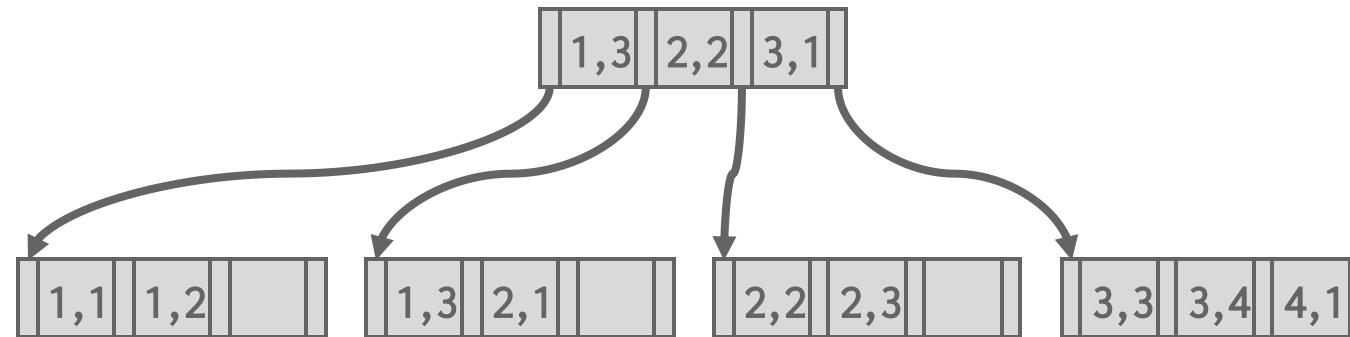
The tree has shrunk in height.



# Composite Index

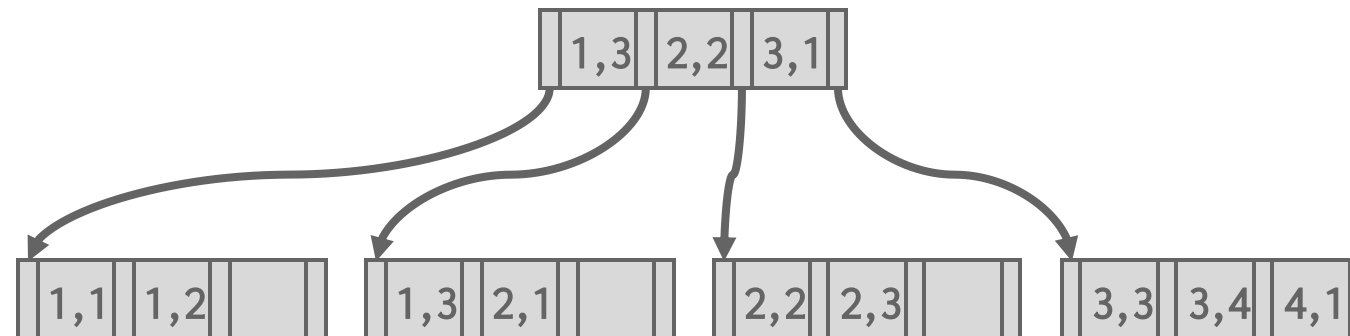
- Composite Index: The key is composed of multiple attributes.
- `CREATE INDEX LFM_name ON artist  
    (last_name, first_name, middle_names NULLS FIRST);`
- Can use a B+Tree index if the query provides a “prefix” of composite key. Example: Index on `<a,b,c>`
  - Supported: `(a=1 AND b=2 AND c=3)`
  - Supported: `(a=1 AND b=2)`
  - NOT (generally) supported: `(b=2), (c=3)`
- For a hash index, we must have all attributes in search key.

# Selection Conditions



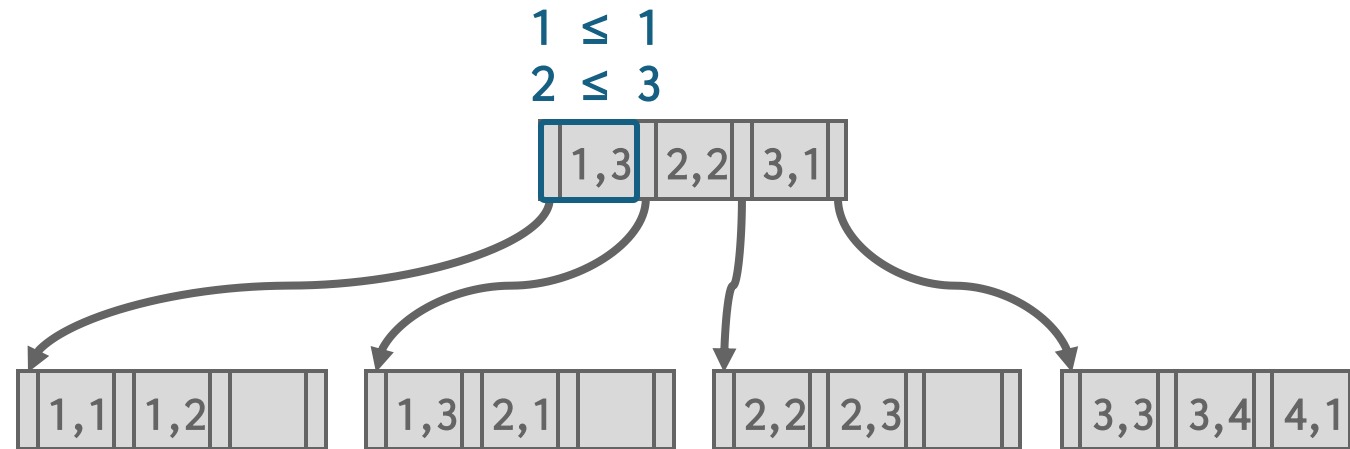
# Selection Conditions

Find Key=(1,2)



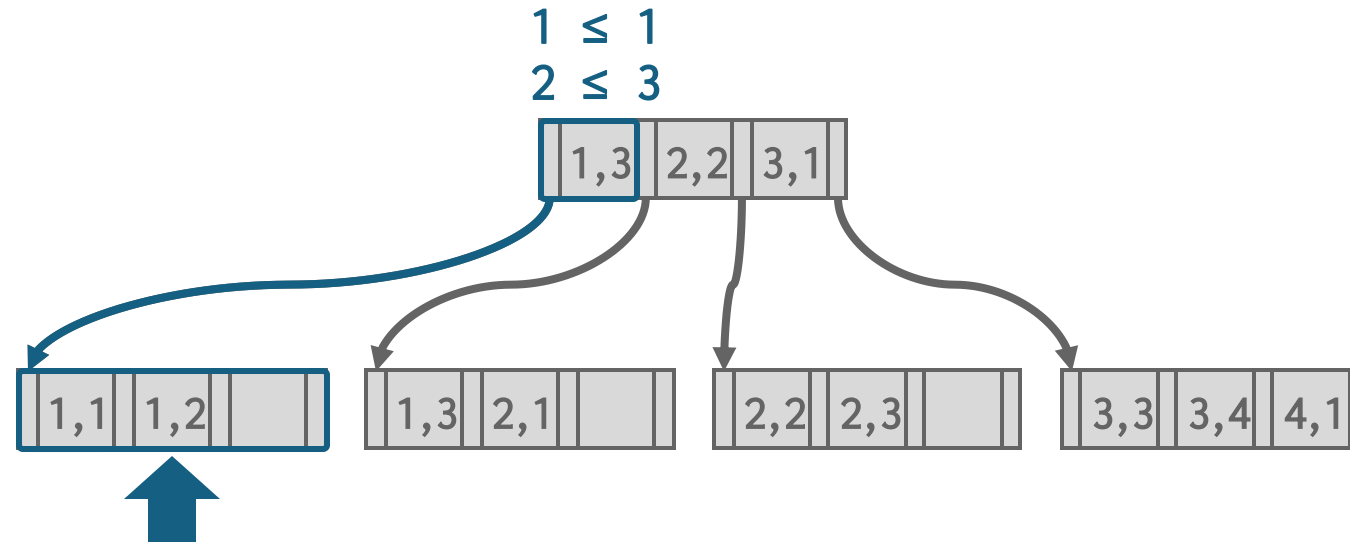
# Selection Conditions

Find Key=(1,2)



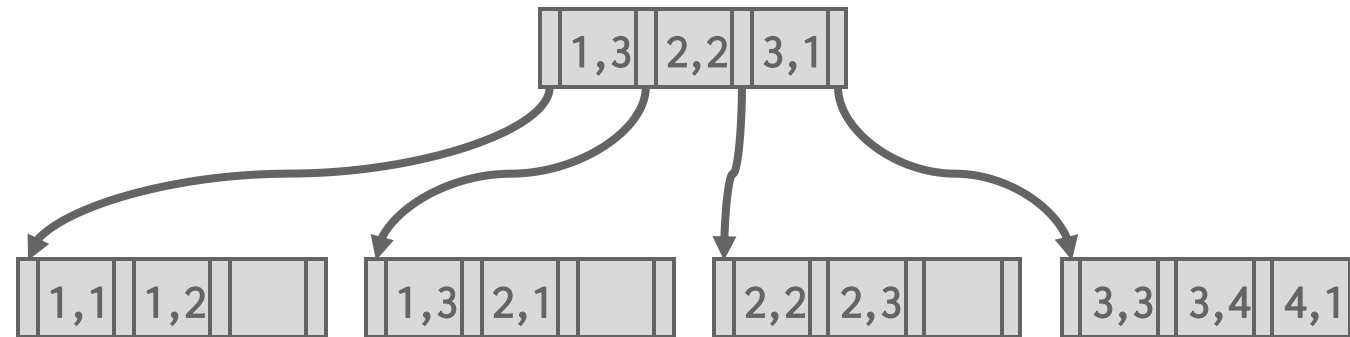
# Selection Conditions

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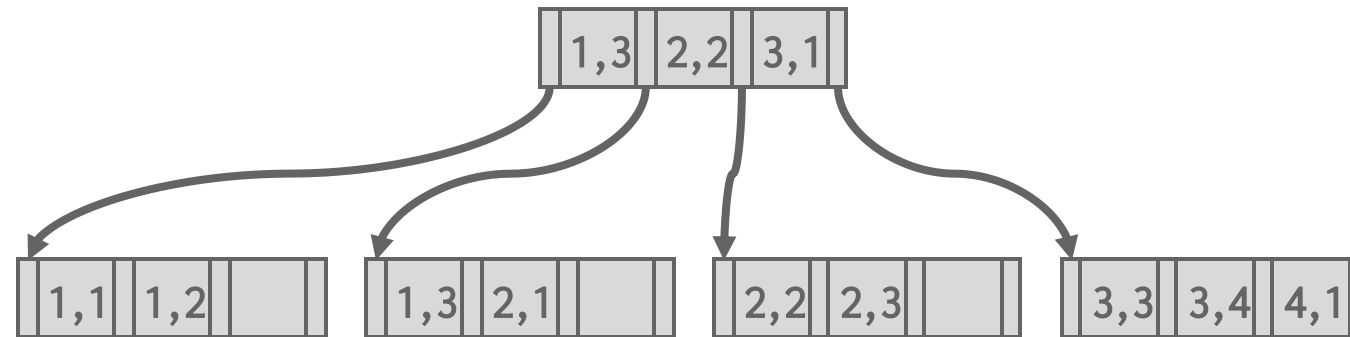
Find Key=(1,2)



# Selection Conditions

Find Key=(1,2)

Find Key=(1,\*)

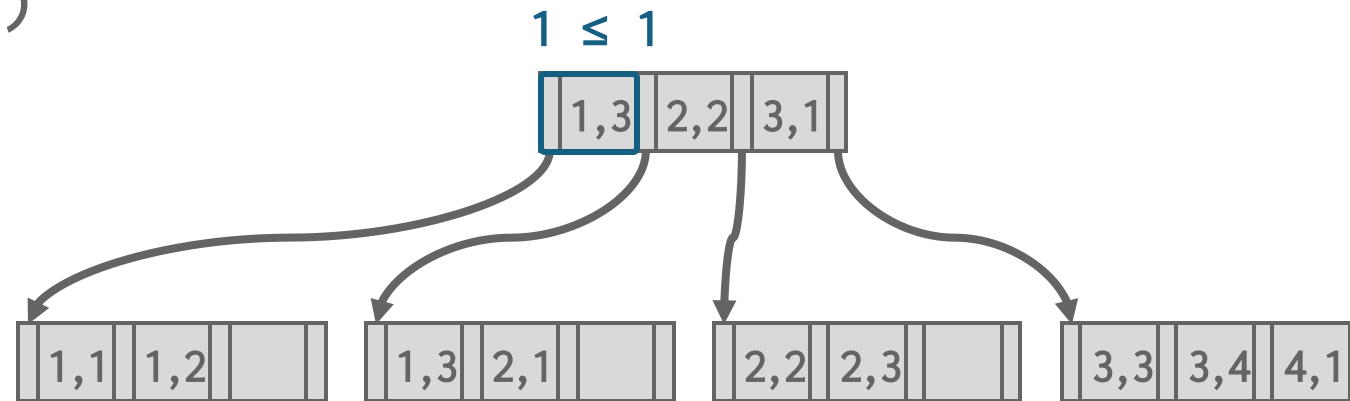




# Selection Conditions

Find Key=(1,2)

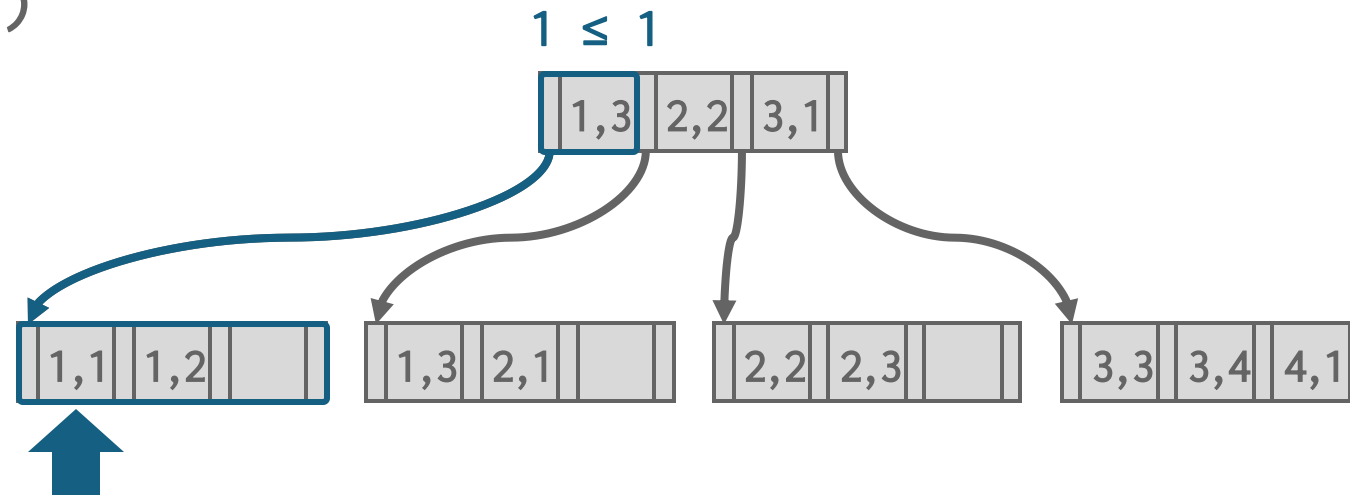
Find Key=(1,\*)



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Find Key=(1,2)

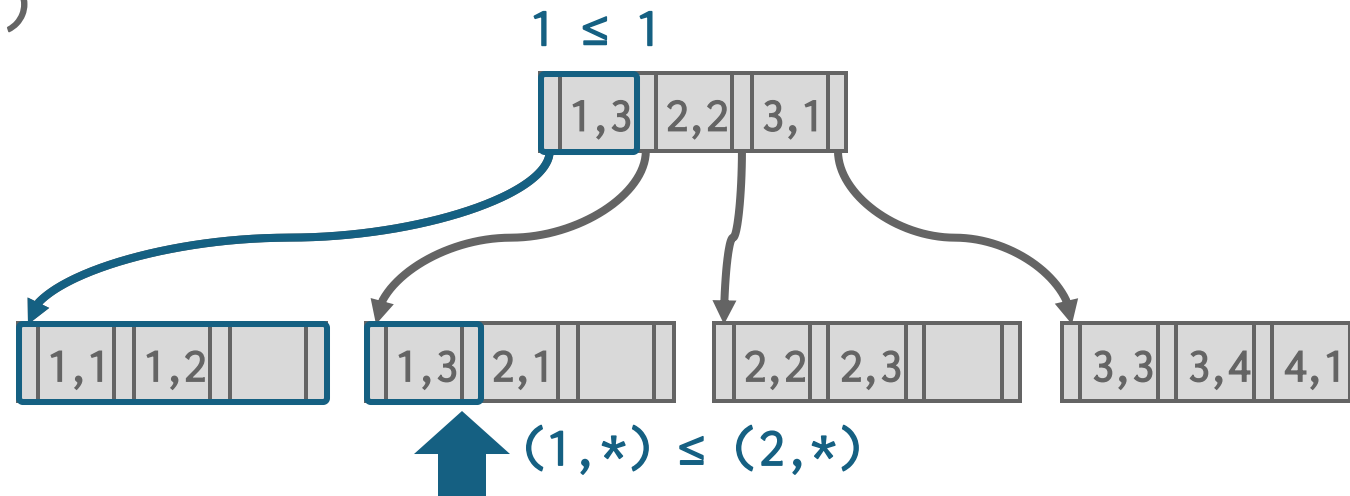
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Find Key=(1,2)

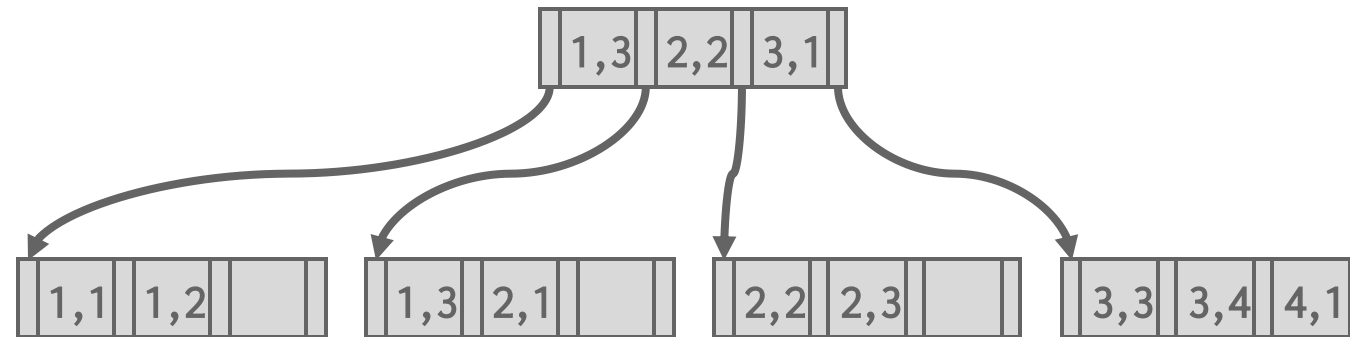
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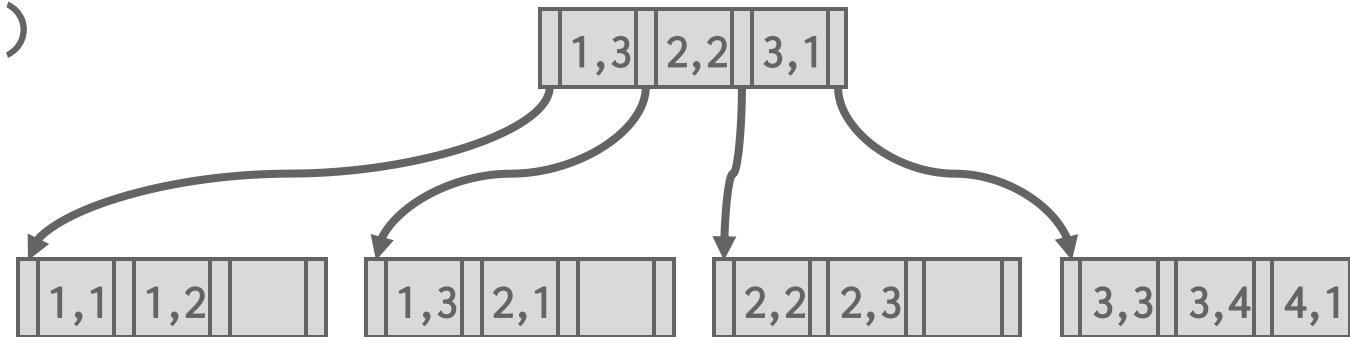


# Selection Conditions

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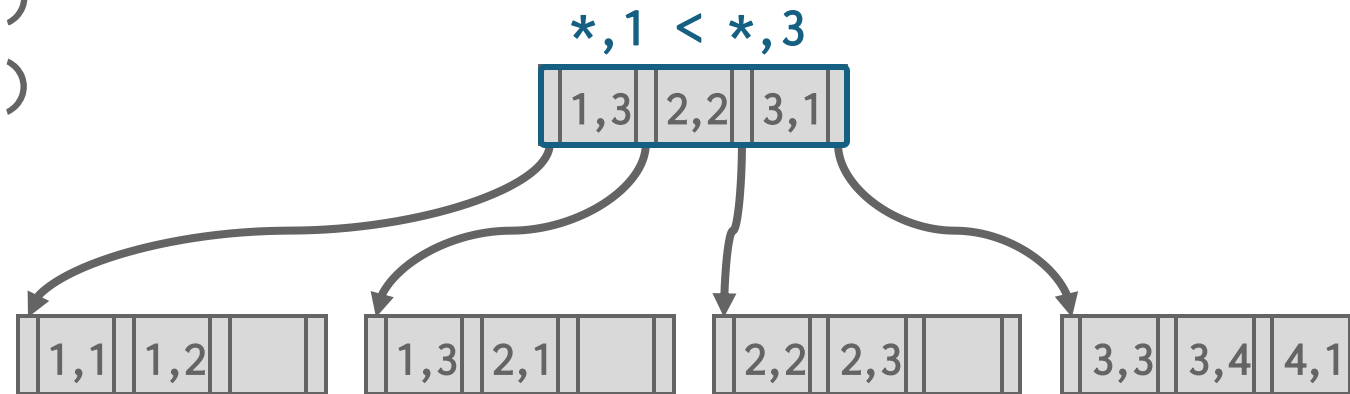


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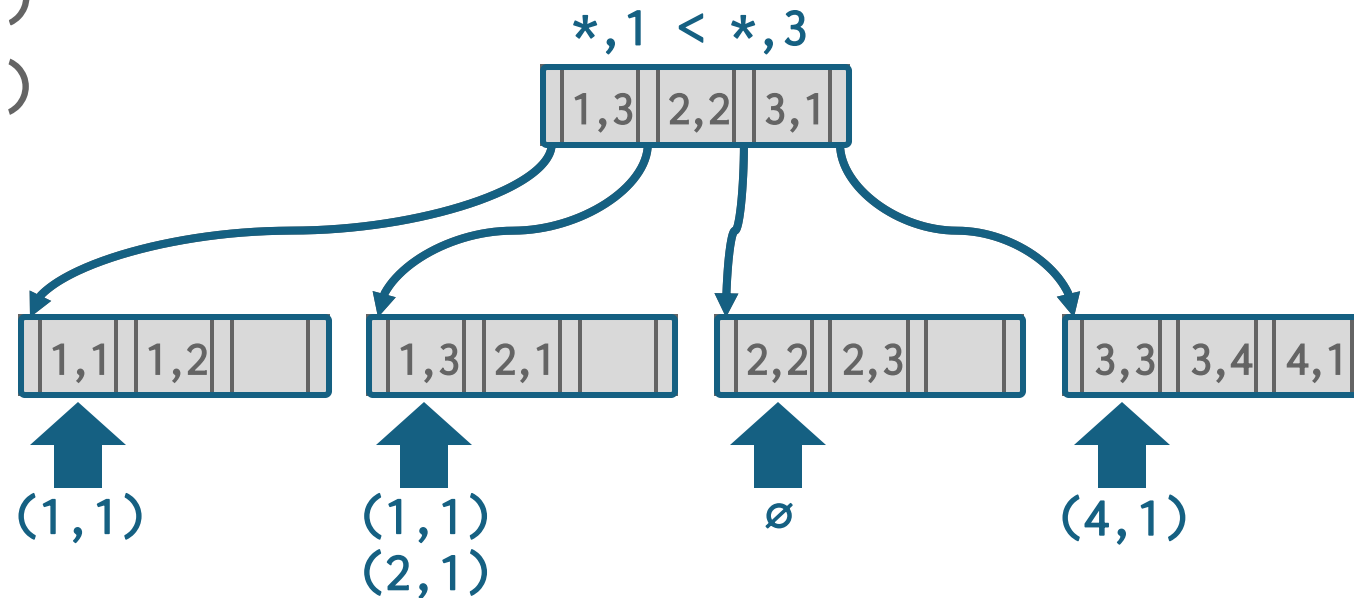


# Selection Conditions

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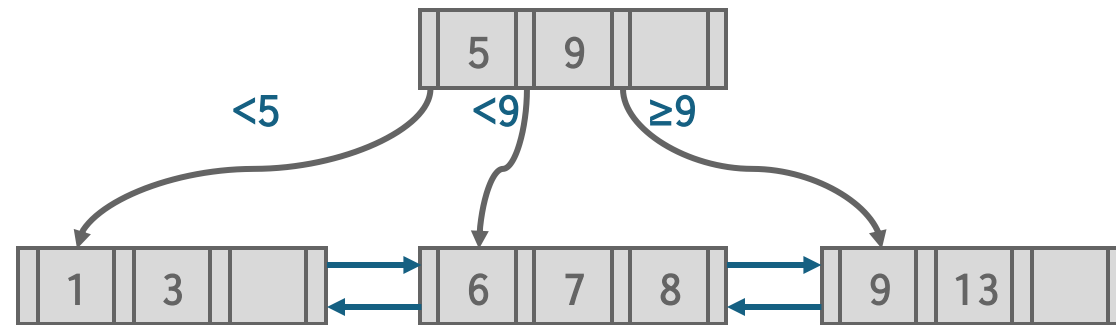


# B+Tree: Duplicate Keys

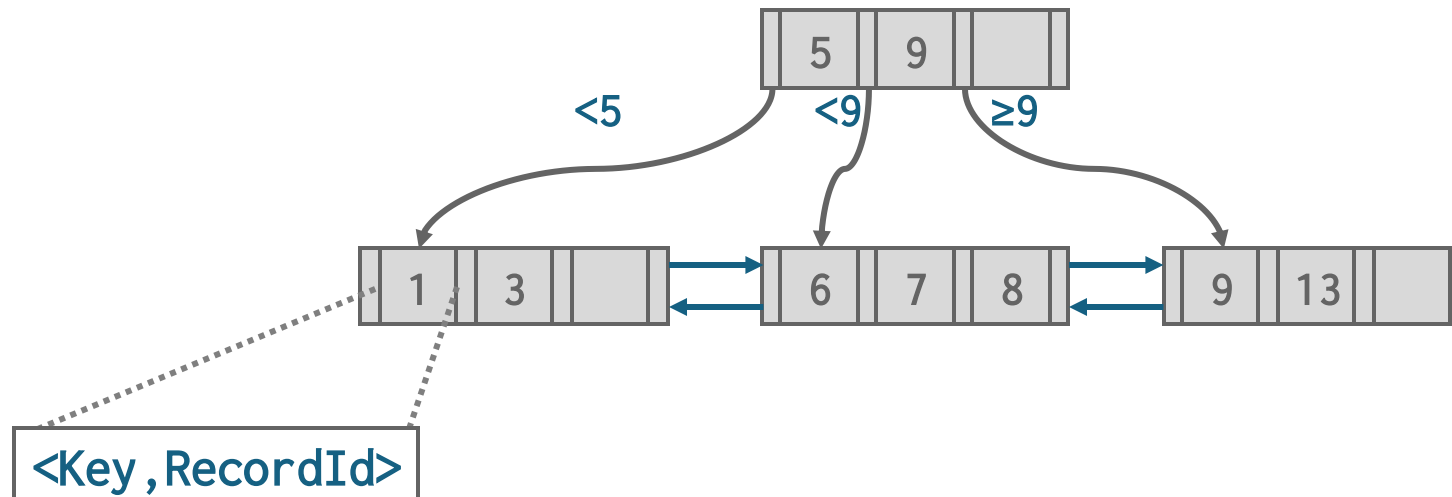
- **Approach #1: Append Record ID**
  - Add the tuple's unique Record ID as part of the key to ensure that all keys are unique.
  - The DBMS can still use partial keys to find tuples.
- **Approach #2: Overflow Leaf Nodes**
  - Allow leaf nodes to spill into overflow nodes that contain the duplicate keys.
  - This is more complex to maintain and modify.



# B+Tree: Append Record ID

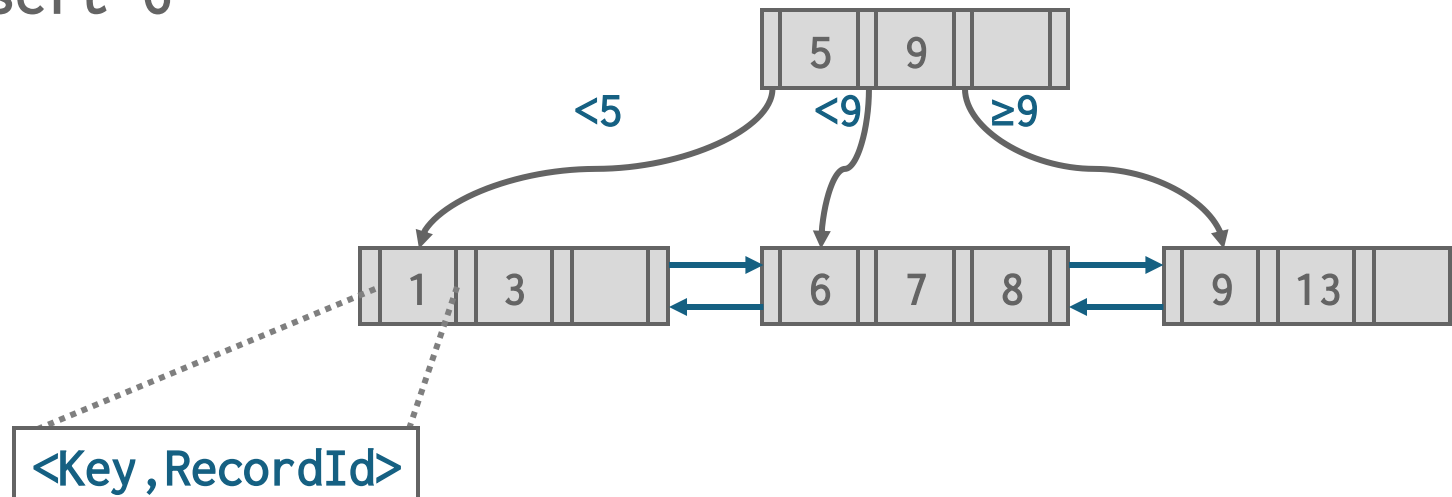


# B+Tree: Append Record ID



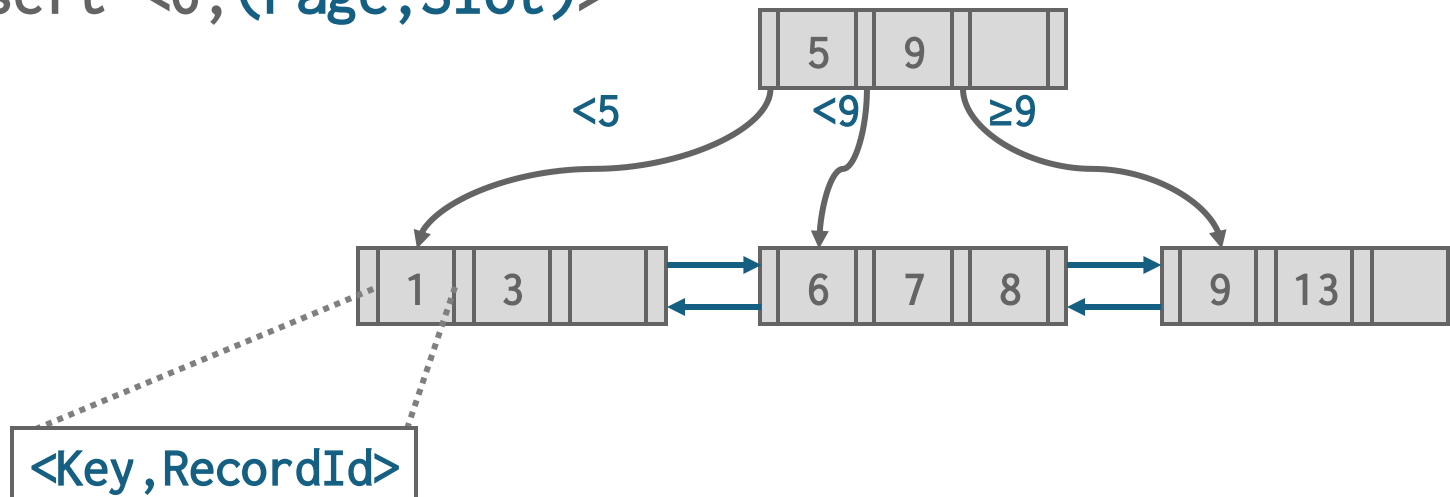
# B+Tree: Append Record ID

Insert 6



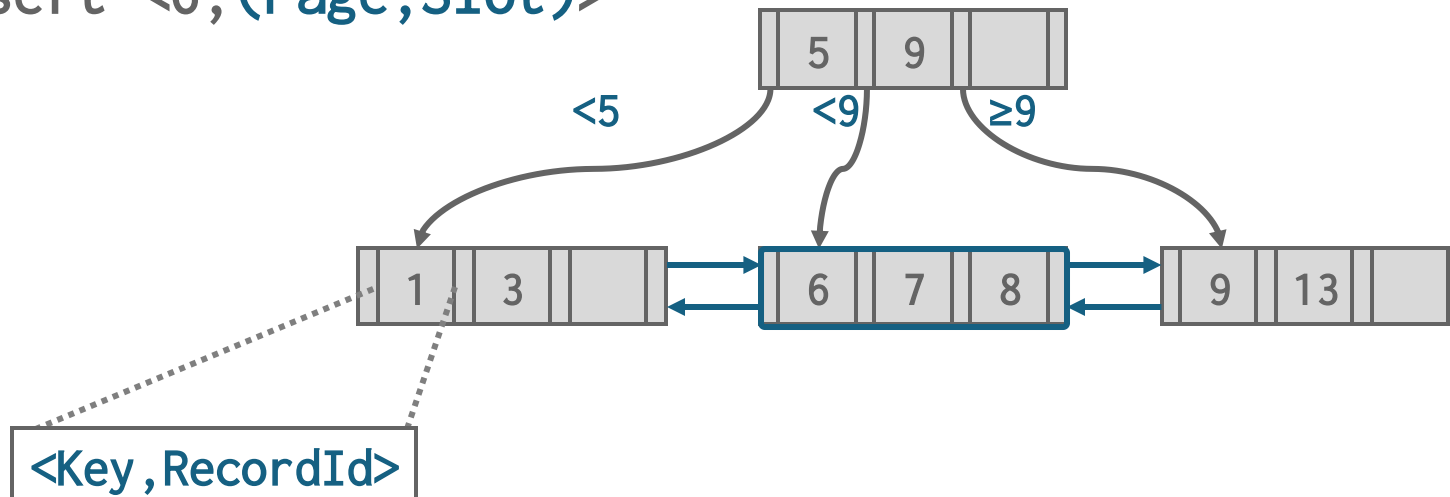
# B+Tree: Append Record ID

Insert  $\langle 6, (\text{Page}, \text{Slot}) \rangle$



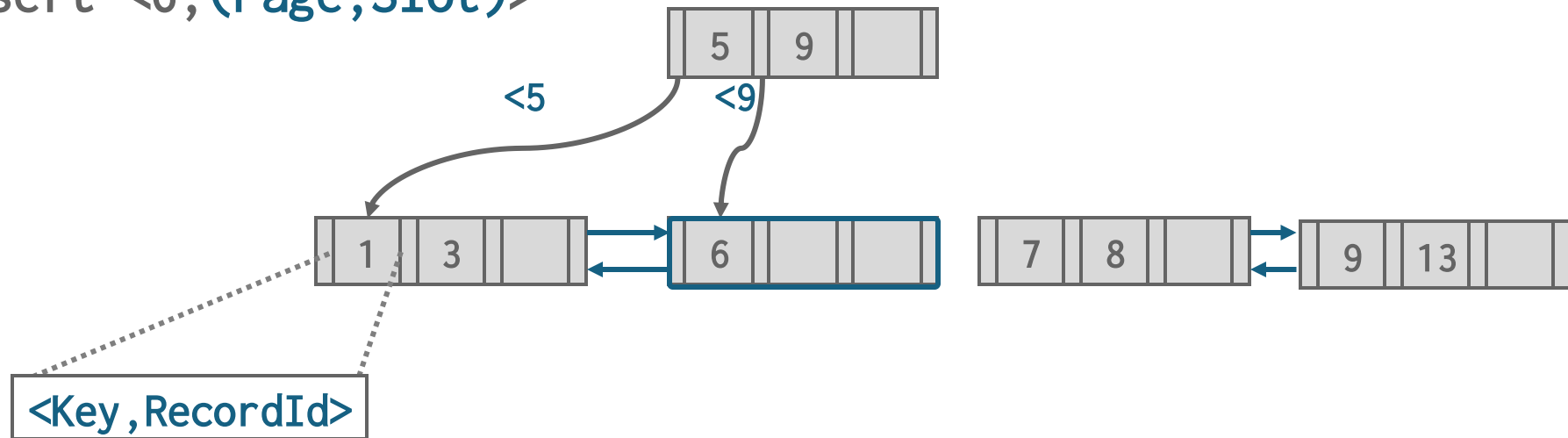
# B+Tree: Append Record ID

Insert  $\langle 6, (\text{Page}, \text{Slot}) \rangle$



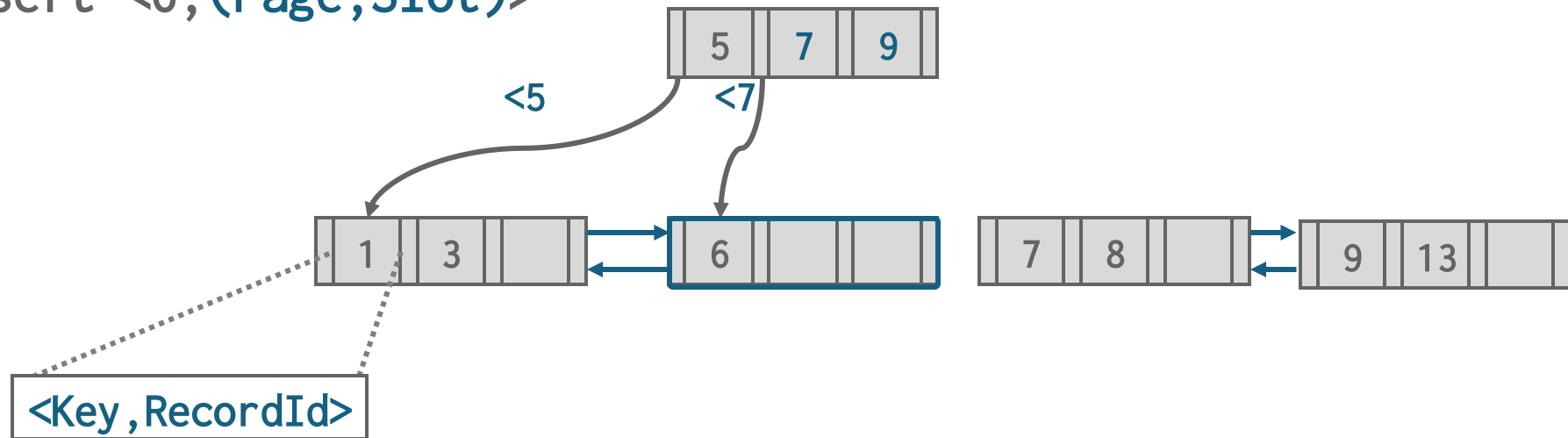
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Insert  $\langle 6, (\text{Page}, \text{Slot}) \rangle$



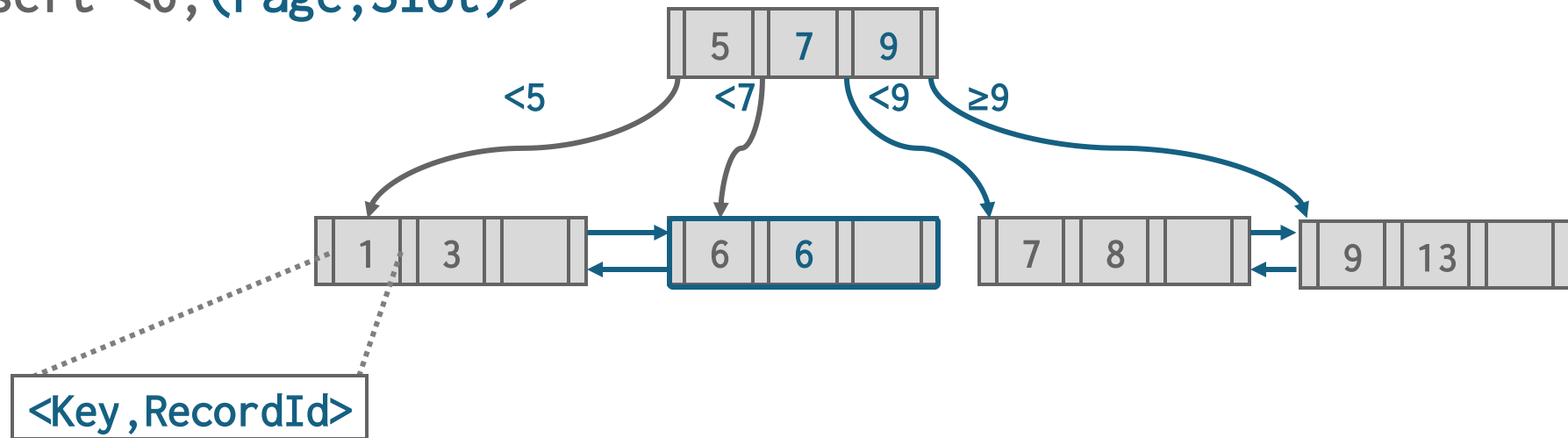
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Insert  $\langle 6, (\text{Page}, \text{Slot}) \rangle$



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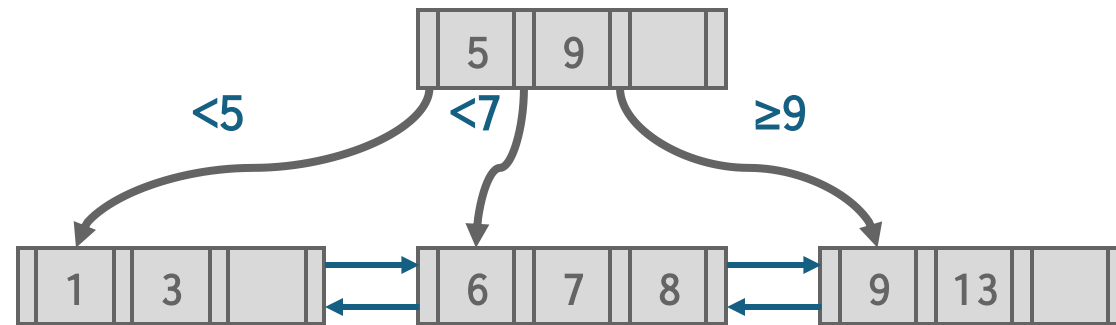
Insert  $\langle 6, (\text{Page}, \text{Slot}) \rangle$





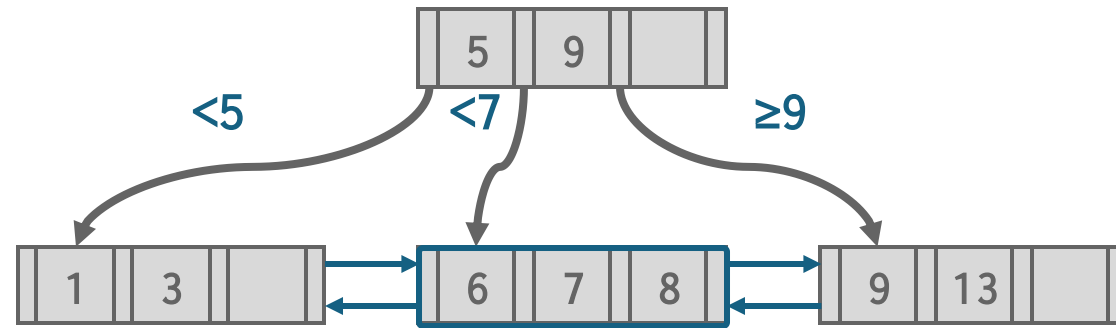
# B+Tree: Overflow Leaf Nodes

Insert 6



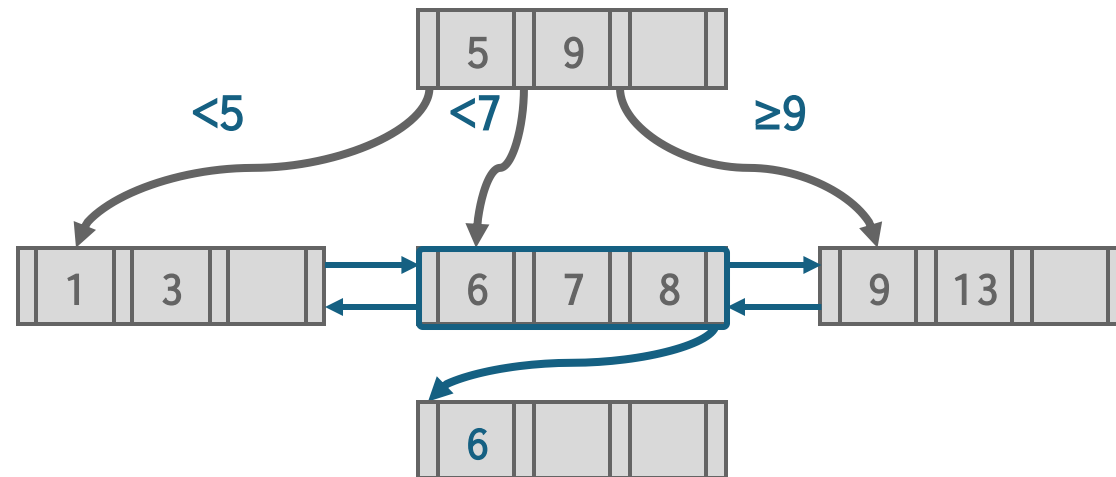
# B+Tree: Overflow Leaf Nodes

Insert 6



# B+Tree: Overflow Leaf Nodes

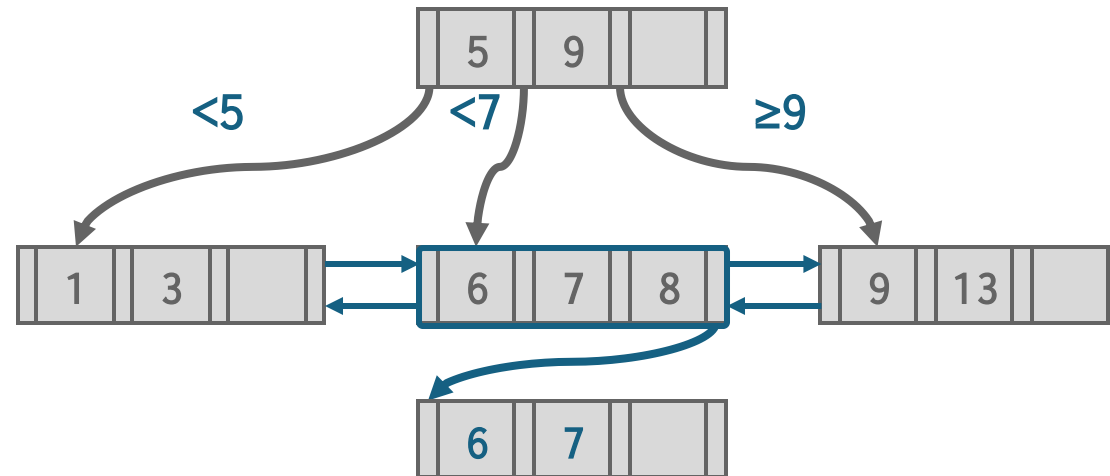
Insert 6



# B+Tree: Overflow Leaf Nodes

Insert 6

Insert 7

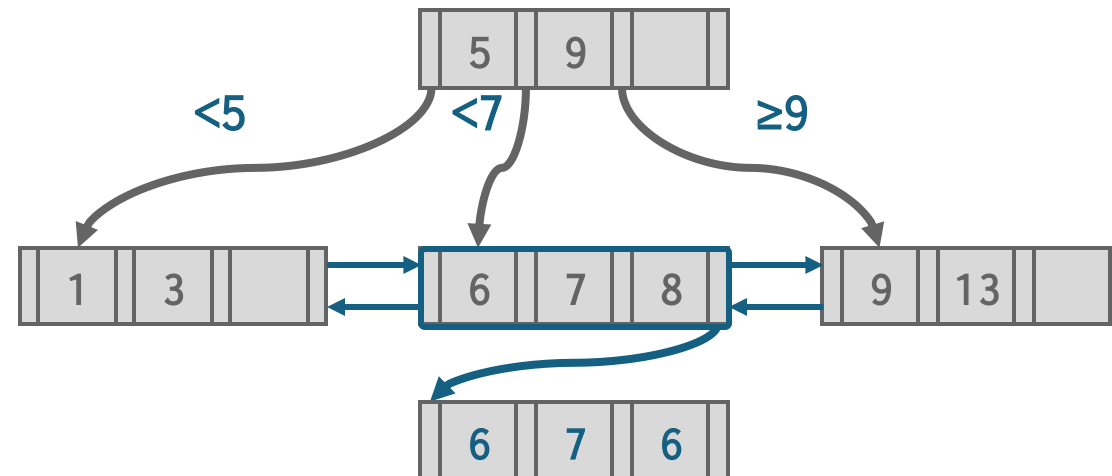


# B+Tree: Overflow Leaf Nodes

Insert 6

Insert 7

Insert 6

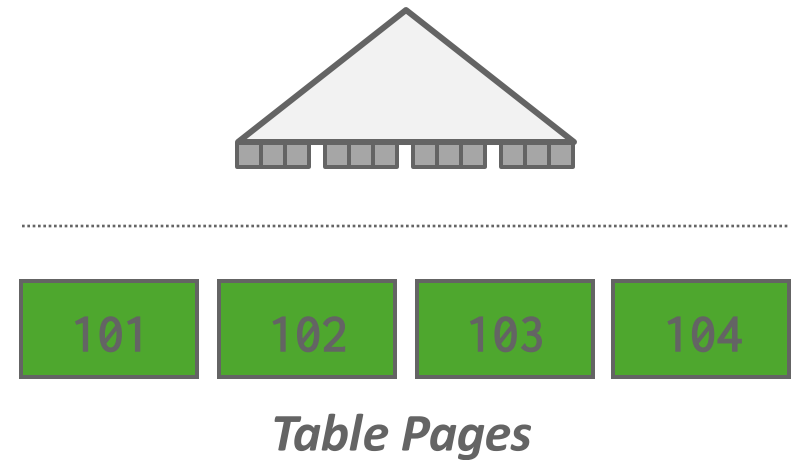


# Clustered Indexes

- The table is stored in the sort order specified by the primary key.
  - Can be either heap- or index-organized storage.
- Some DBMSs always use a clustered index.
  - If a table does not contain a primary key, the DBMS will automatically make a hidden primary key.
- Other DBMSs cannot use them at all.

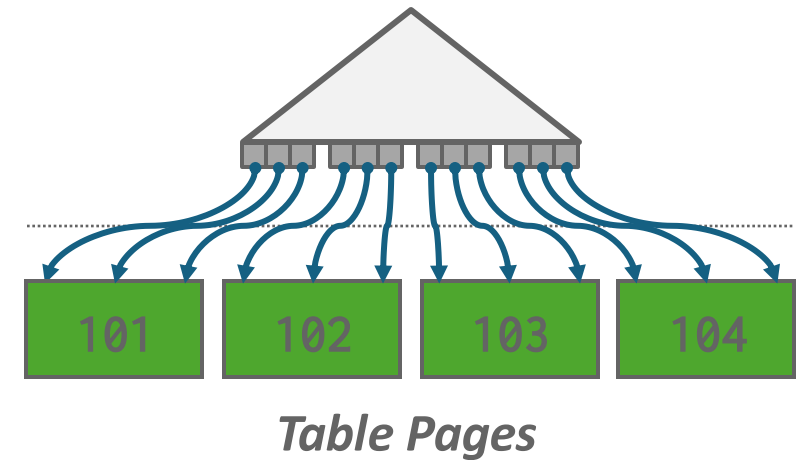
# Clustered B+Tree

- Traverse to the left-most leaf page and then retrieve tuples from all leaf pages.
- This will always be better than sorting data for each query.



# Clustered B+Tree

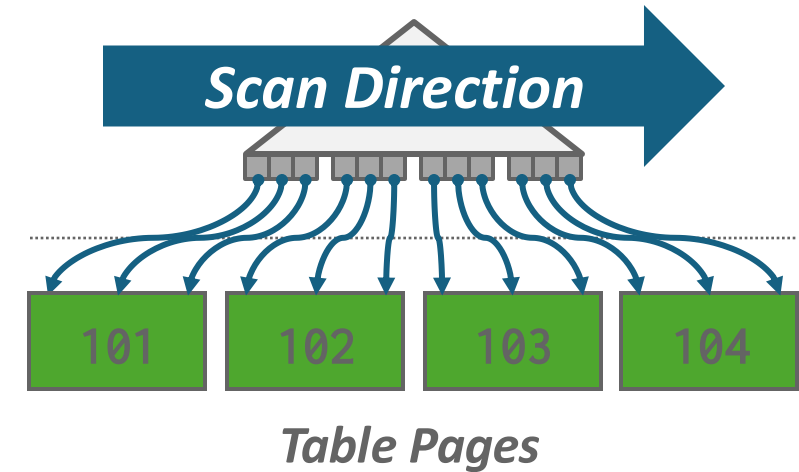
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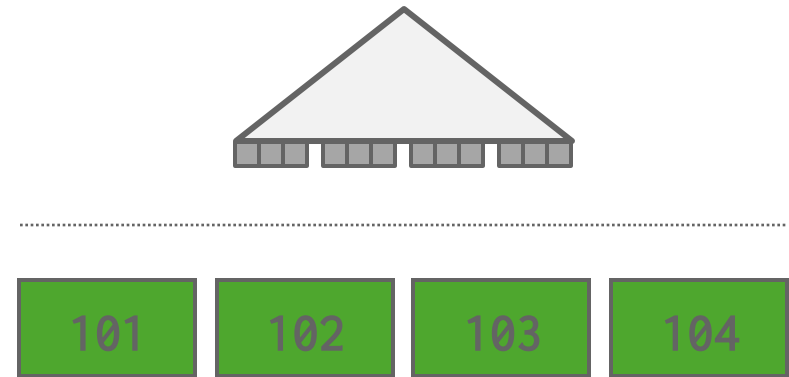
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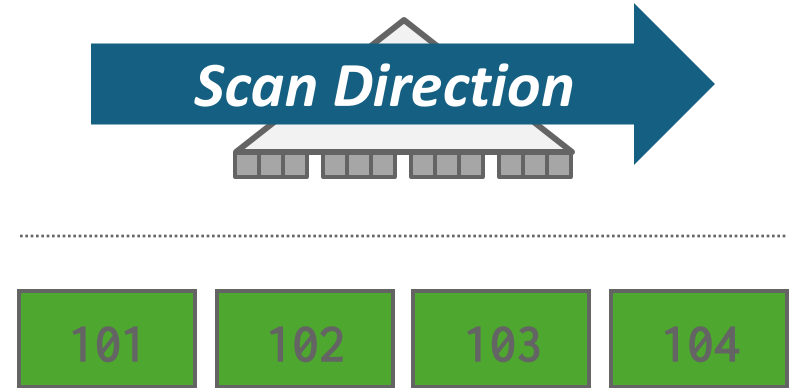
# Index Scan Page Sorting

- Retrieving tuples in the order they appear in a non-clustered index is inefficient due to redundant reads.
- A better approach is to find all the tuples that the query needs and then sort them based on their page ID.
- The DBMS retrieves each page once.



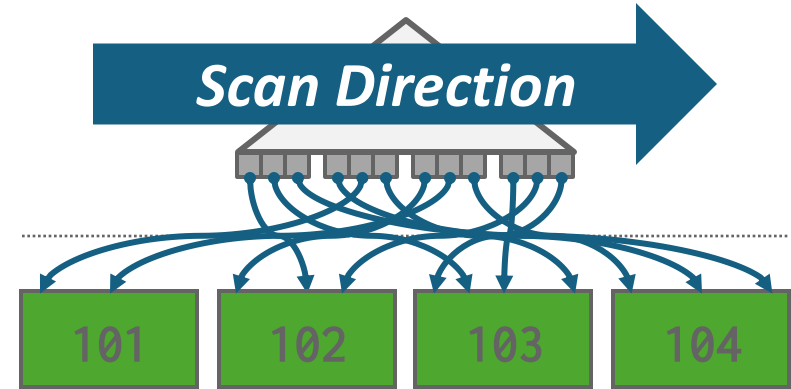
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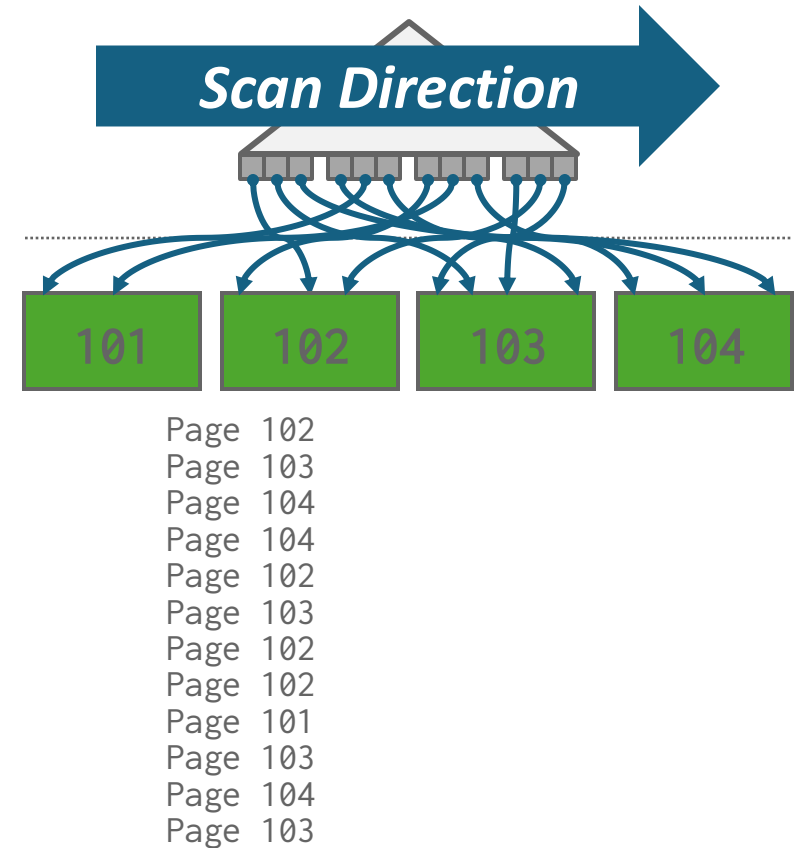
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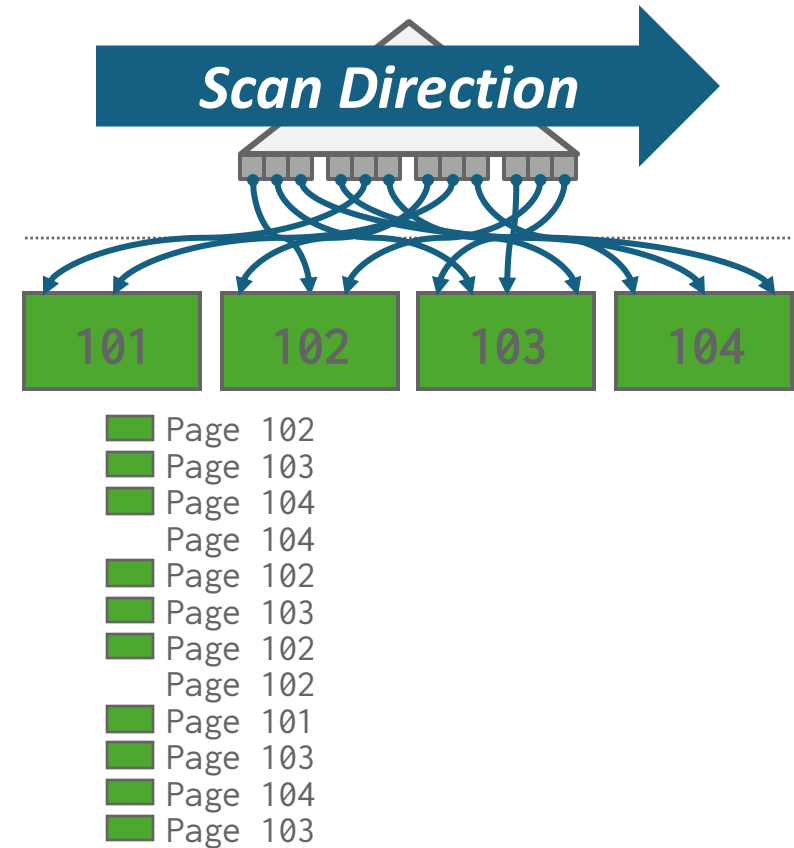
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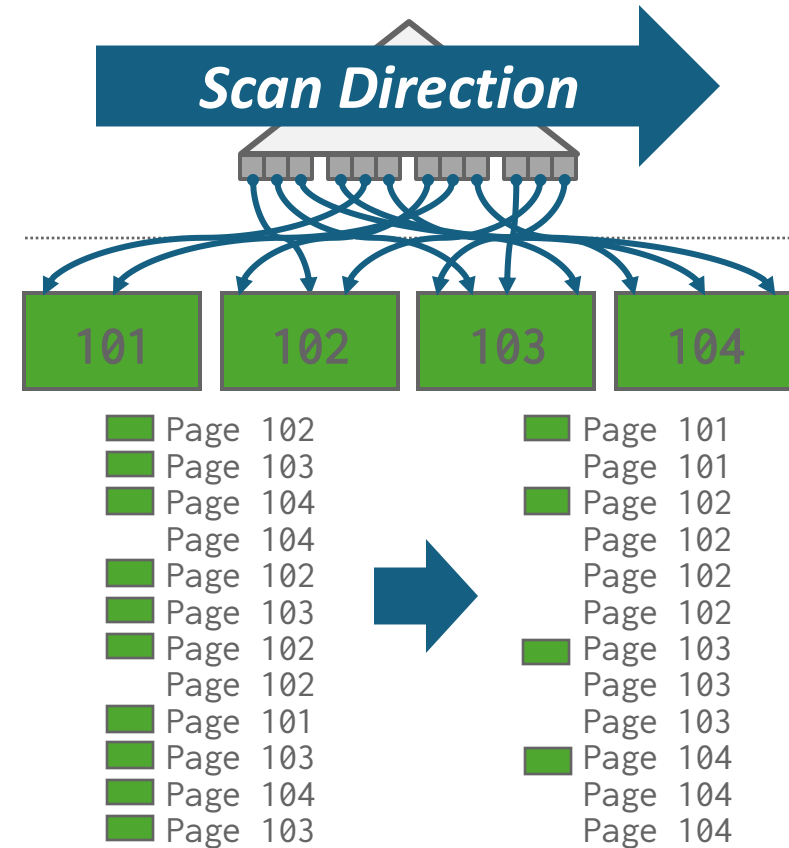
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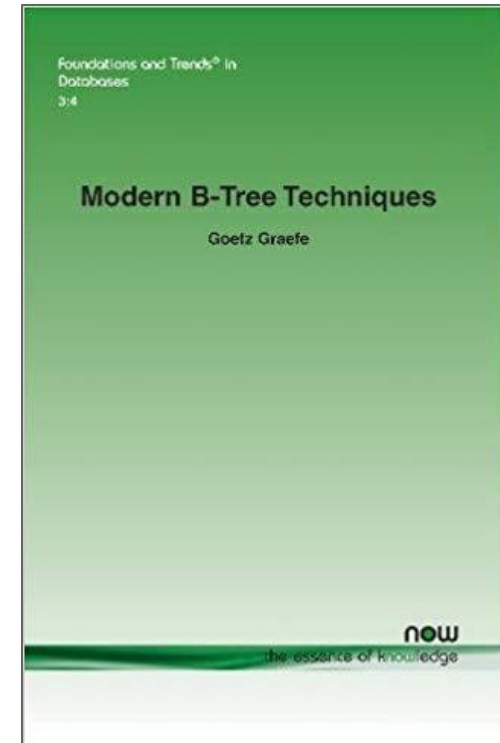
# B+Tree Design Choices

Node Size

Merge Threshold

Variable-Length Keys

Intra-Node Search





# Node Size

- The slower the storage device, the larger the optimal node size for a B+Tree.
  - HDD: ~1MB
  - SSD: ~10KB
  - In-Memory: ~512B
- Optimal sizes can vary depending on the workload
  - Leaf Node Scans vs. Root-to-Leaf Traversals

# Merge Threshold

- Some DBMSs do not always merge nodes when they are half full.
  - Average occupancy rate for B+Tree nodes is 69%.
- Delaying a merge operation may reduce the amount of reorganization.
- It may also be better to just let smaller nodes exist and then periodically rebuild entire tree.
- This is why PostgreSQL calls their B+Tree a “non-balanced” B+Tree ([nbtree](#)).

# Variable-Length Keys

- **Approach #1: Pointers**
  - Store the keys as pointers to the tuple's attribute.
  - Also called [T-Trees](#) (in-memory DBMSs)
- **Approach #2: Variable-Length Nodes**
  - The size of each node in the index can vary.
  - Requires careful memory management.
- **Approach #3: Padding**
  - Always pad the key to be max length of the key type.
- **Approach #4: Key Map / Indirection**
  - Embed an array of pointers that map to the key + value list within the node.

# Intra-Node Search

- **Approach #1: Linear**
  - Scan node keys from beginning to end.
  - Use SIMD to vectorize comparisons.

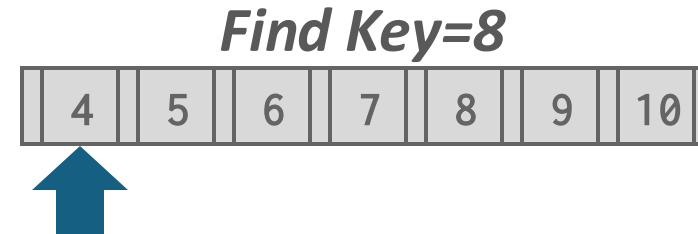
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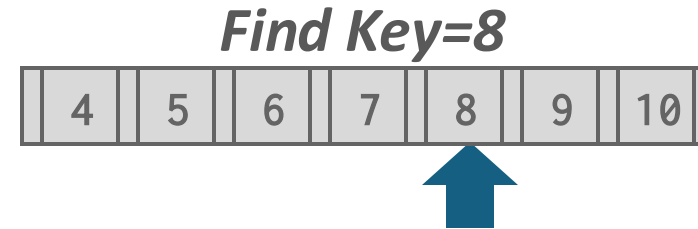
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```
_mm_cmpeq_epi32_mask(a, b)
```

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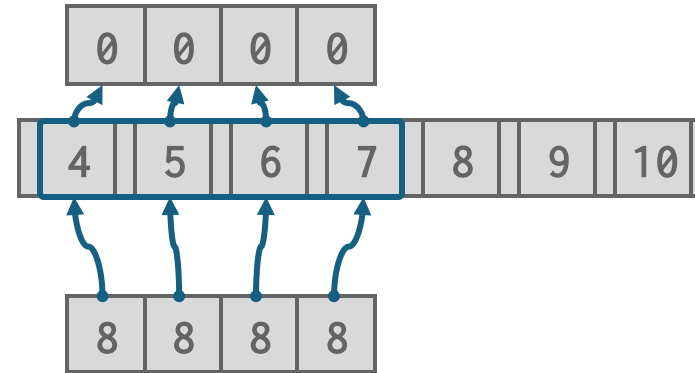
*Find Key=8*



```
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```

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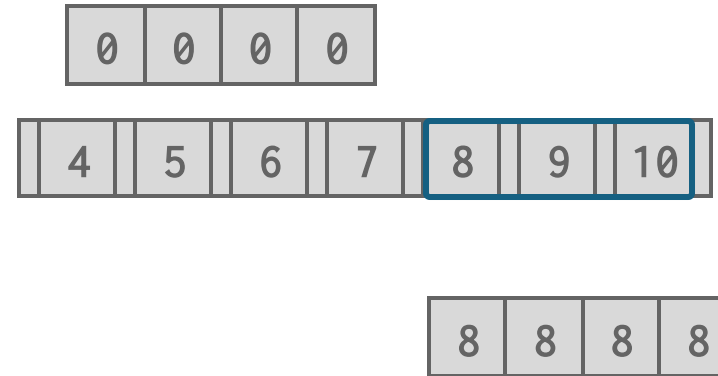
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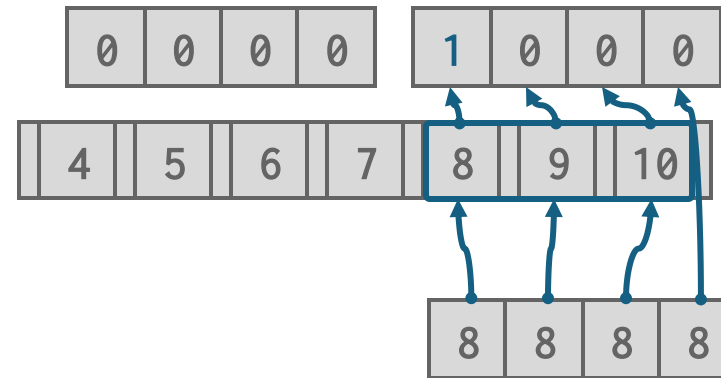
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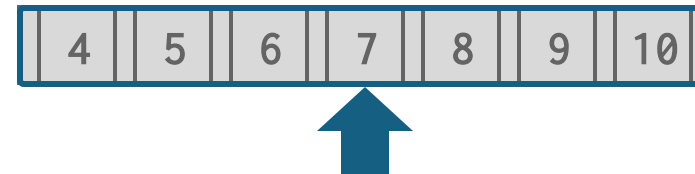
# Intra-Node Search

- **Approach #1: Linear**
  - Scan node keys from beginning to end.
  - Use SIMD to vectorize comparisons.
- **Approach #2: Binary**
  - Jump to middle key, pivot left/right depending on comparison.

4	5	6	7	8	9	10
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# Intra-Node Search

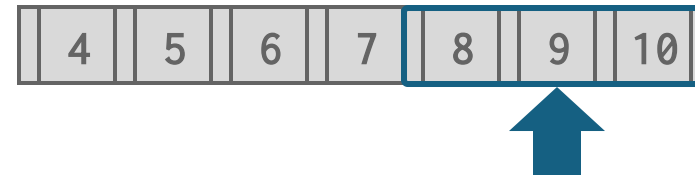
- **Approach #1: Linear**
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  - Use SIMD to vectorize comparisons.
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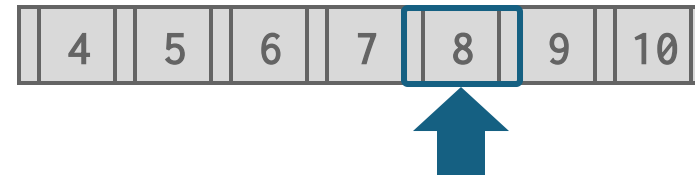
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4	5	6	7	8	9	10
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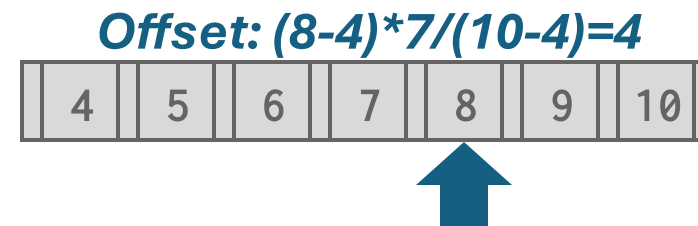
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**Offset:  $(8-4)*7/(10-4)=4$**

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# Optimizations

Prefix Compression, Deduplication, Suffix Truncation, Pointer Swizzling, Bulk Insert, Buffered Updates, Many more...

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- Deduplication
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- Many more...

# Prefix Compression

- Sorted keys in the same leaf node are likely to have the same prefix.
- Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key.
  - Many variations.

robbed	robbing	robot
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<i>Prefix: rob</i>		
bed	bing	ot

# Deduplication

- Non-unique indexes can end up storing multiple copies of the same key in leaf nodes.
- The leaf node can store the key once and then maintain a “posting list” of tuples with that key (similar to what we discussed for hash tables).

K1	V1	K1	V2	K1	V3	K2	V4
----	----	----	----	----	----	----	----

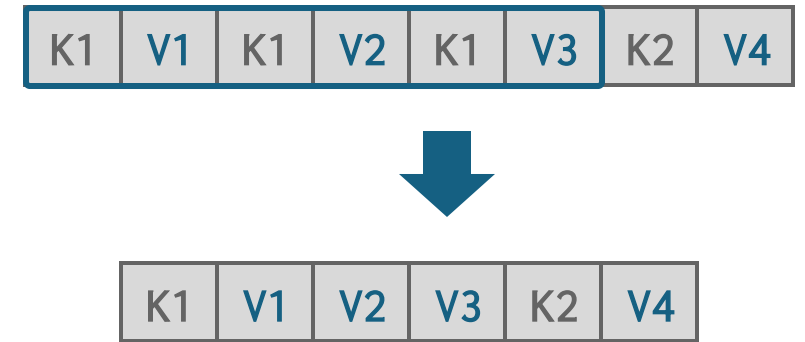
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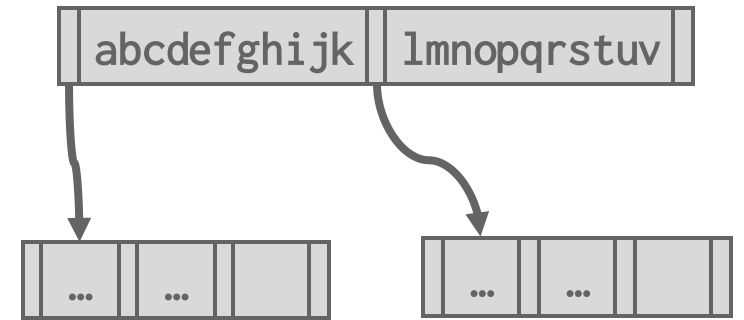
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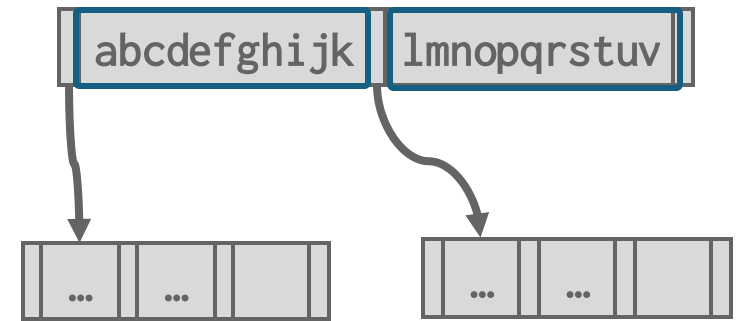
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- The keys in the inner nodes are only used to “direct traffic”.
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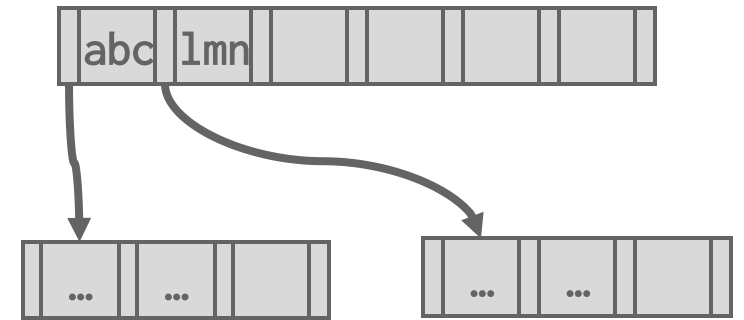
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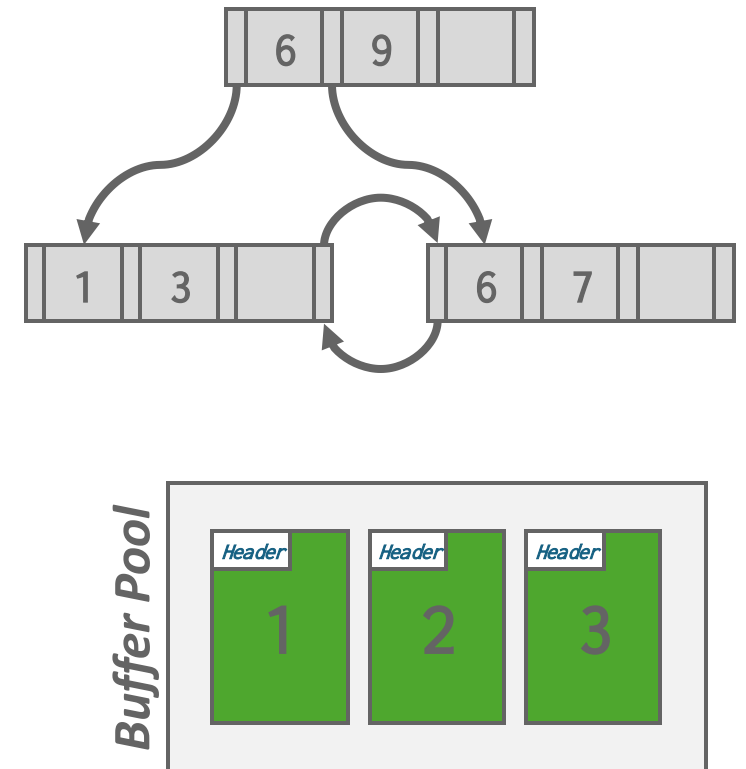
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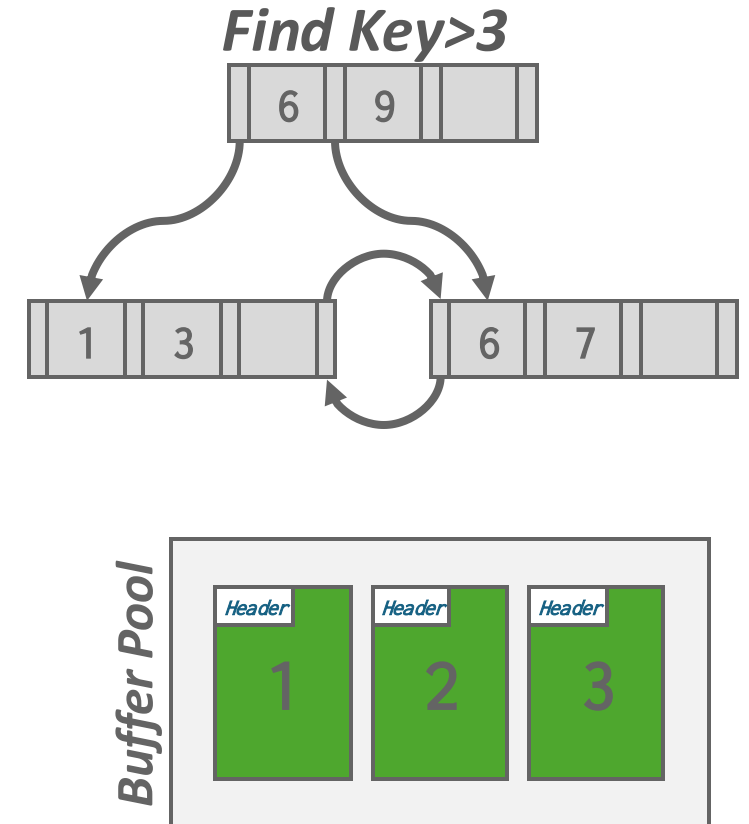
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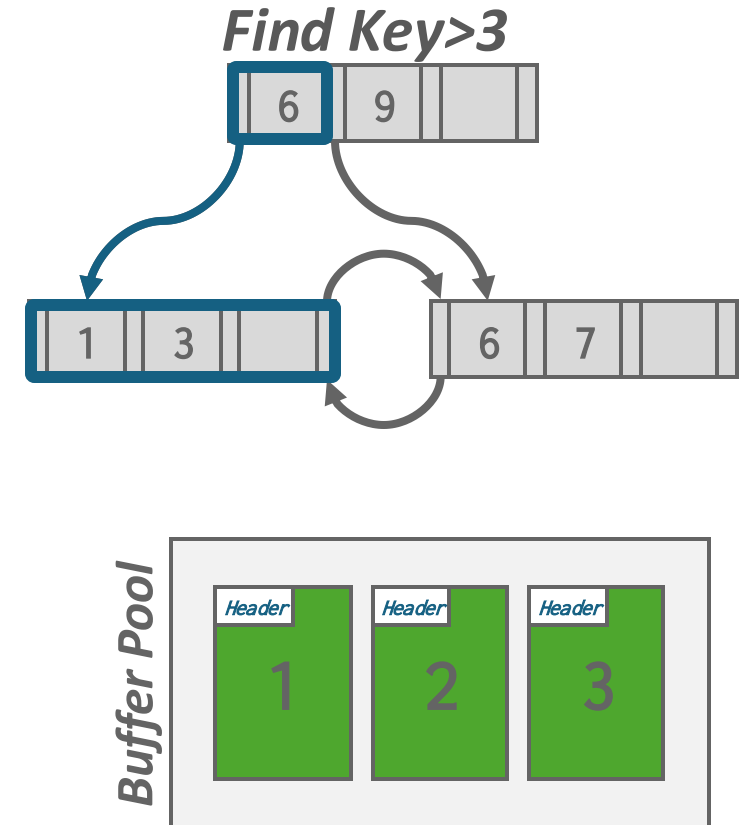
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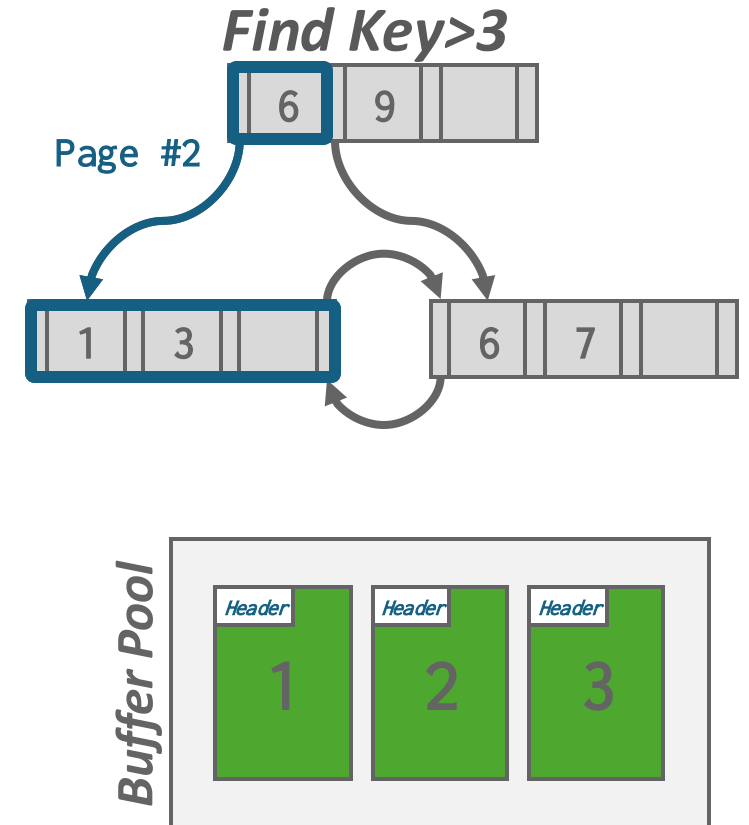
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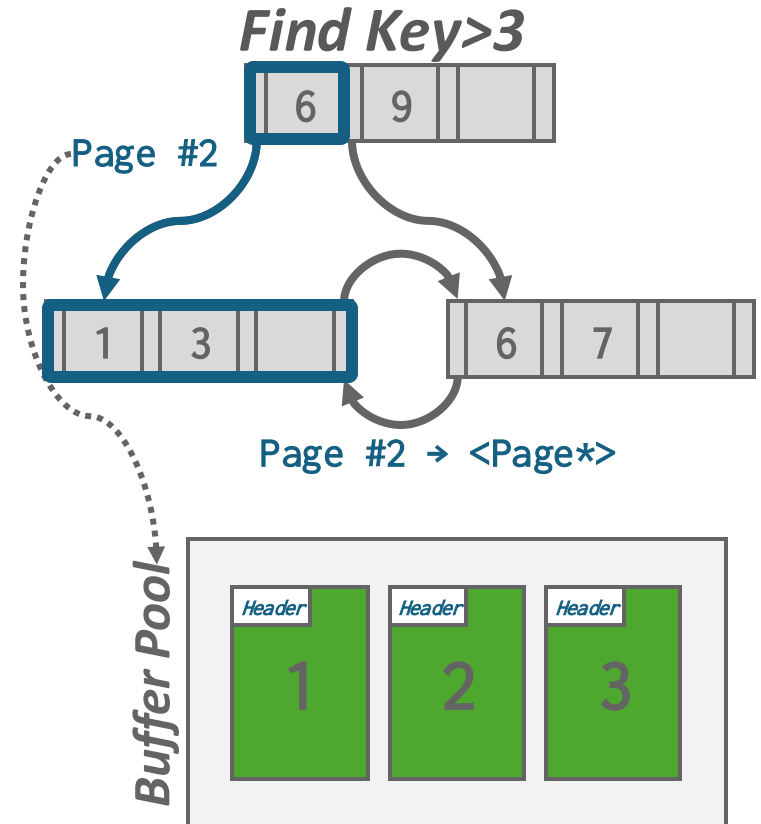
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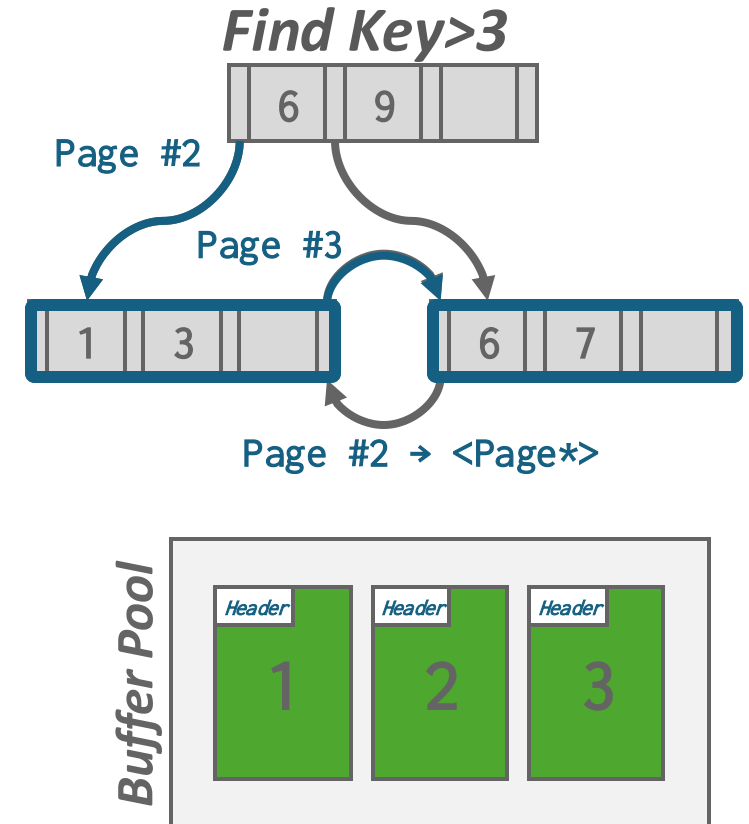
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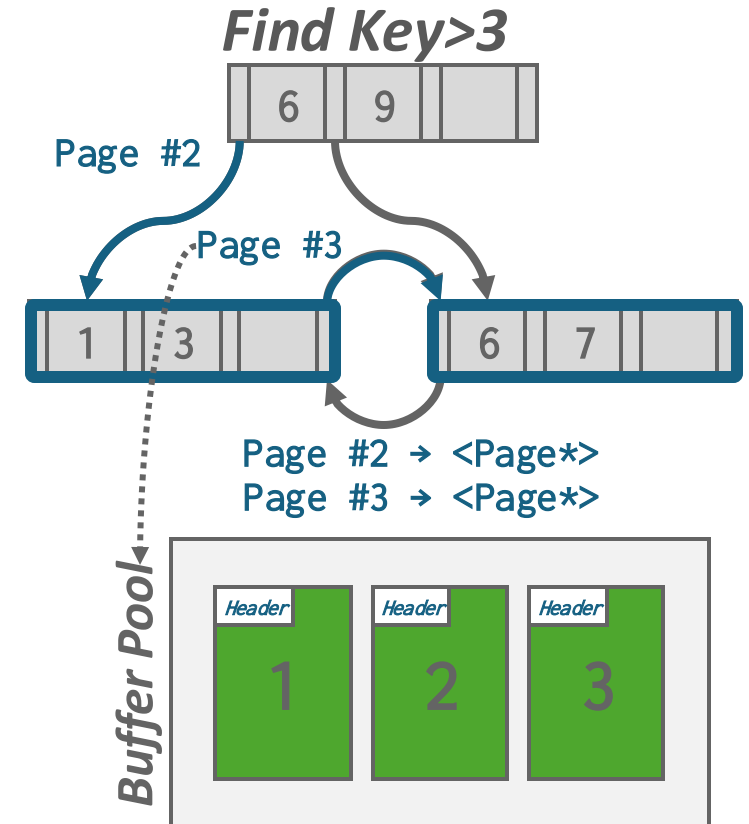
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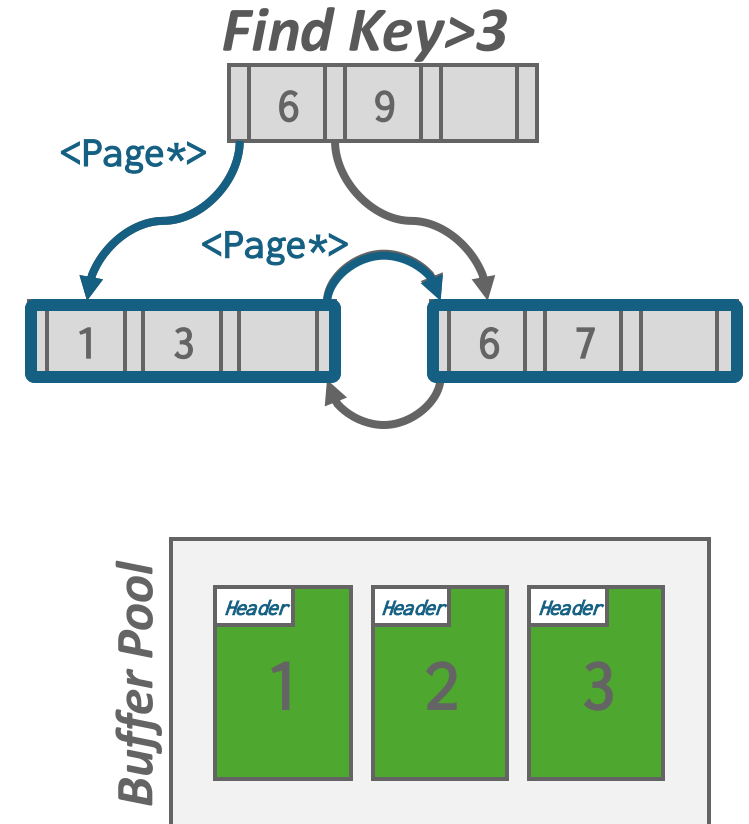
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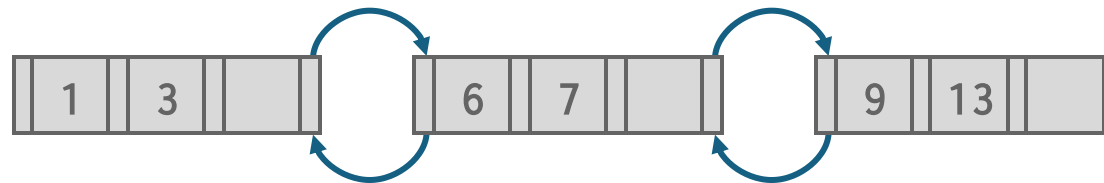
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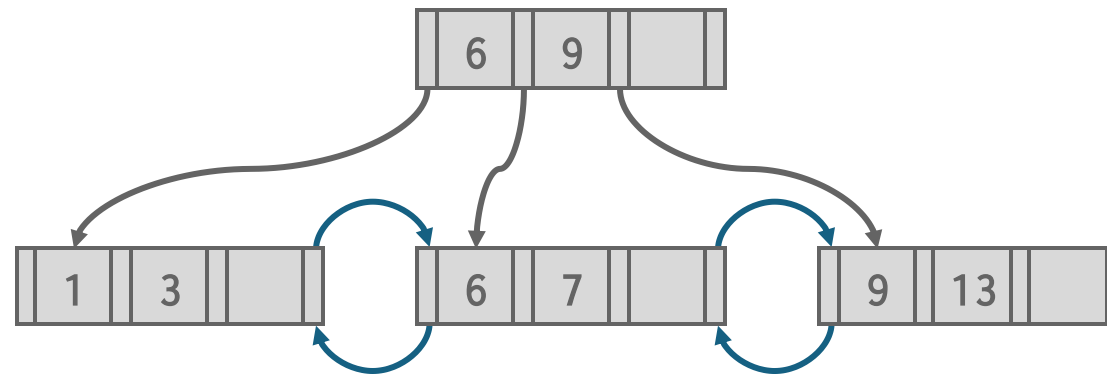


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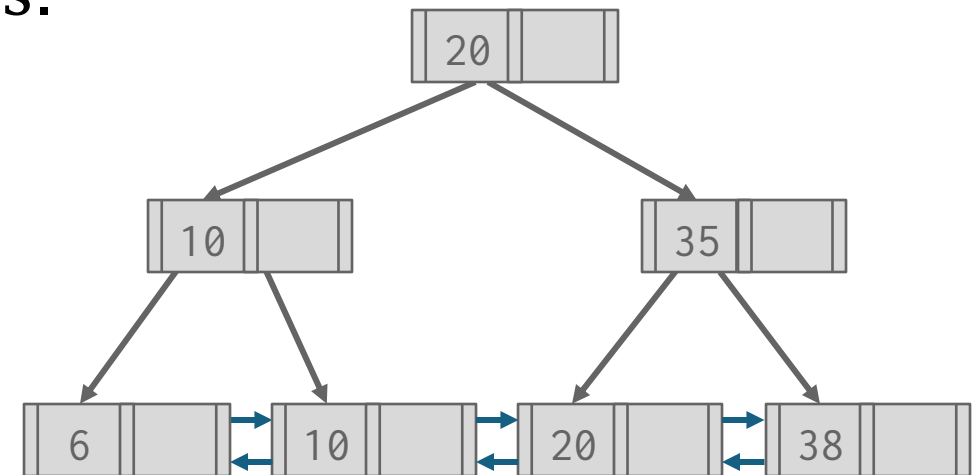


# Observation

- Modifying a B+tree is expensive when the DBMS has to split/merge nodes.
  - Worst case is when DBMS reorganizes the entire tree.
  - The worker that causes a split/merge is responsible for doing the work.
- What if there was a way to delay updates and then apply multiple changes together in a batch?

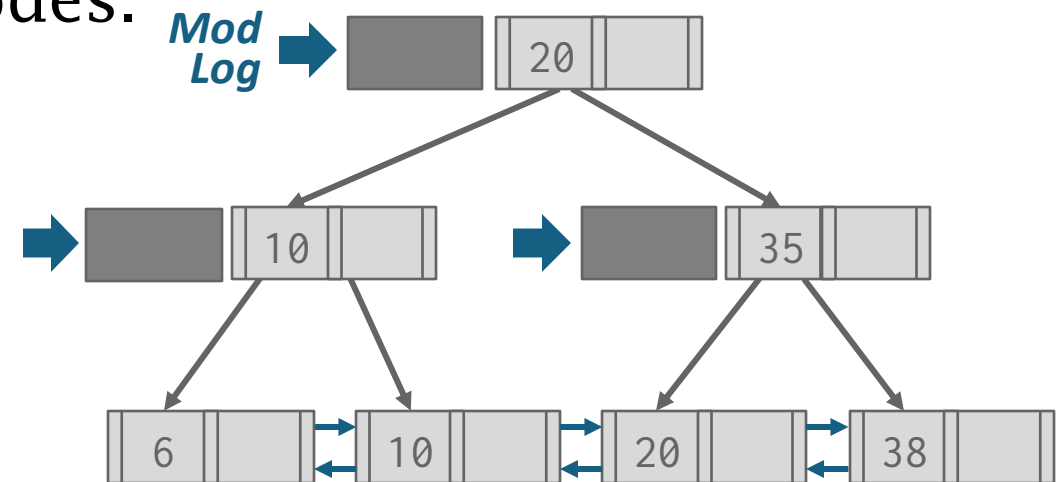
# Write-Optimized B+Tree

- Instead of immediately applying updates, store changes to key/value entries in log buffers at inner nodes.
  - Also known as **B $\epsilon$ -trees**.
- Updates cascade down to lower nodes incrementally when buffers get full.



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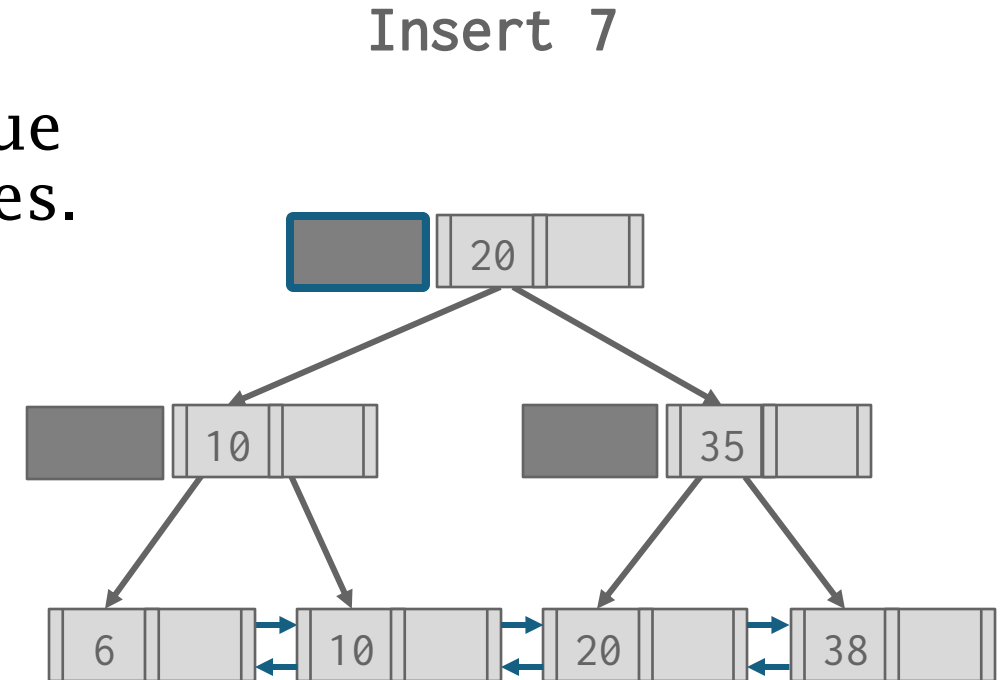
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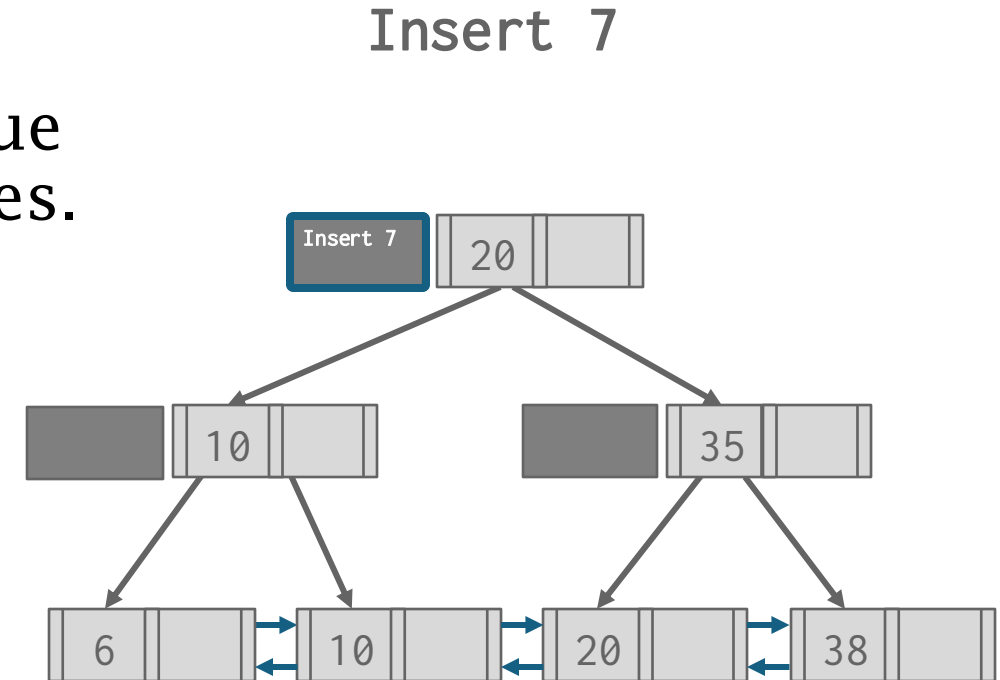
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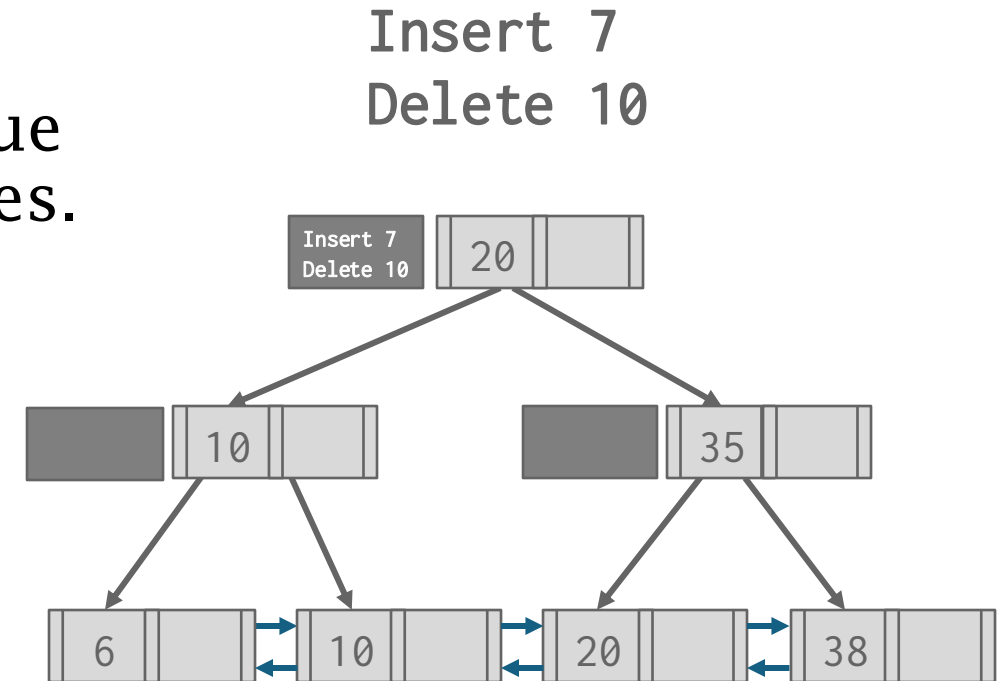
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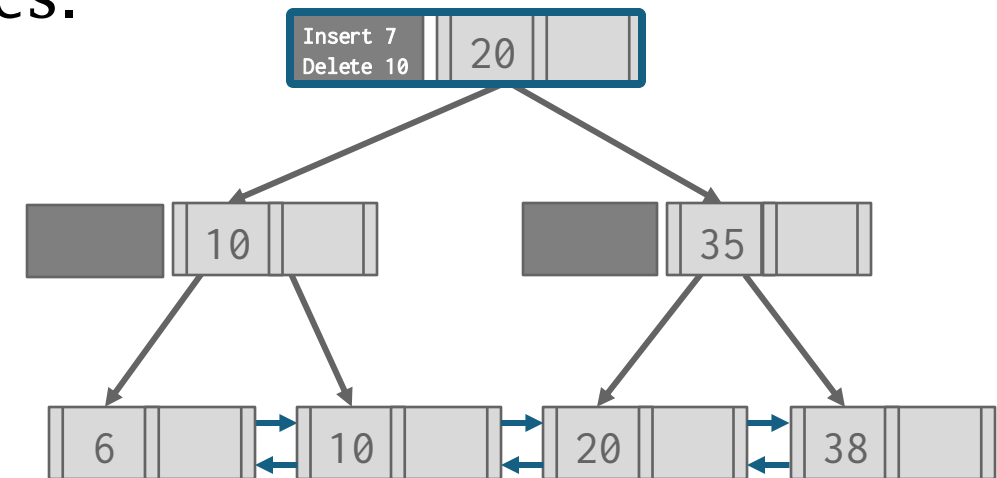
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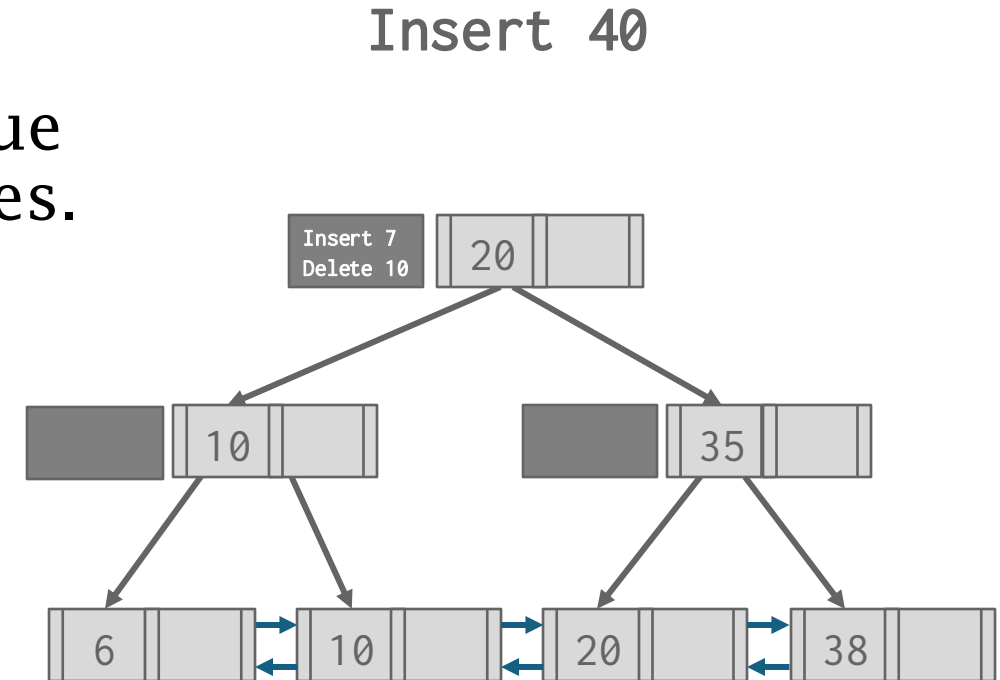
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Find 10



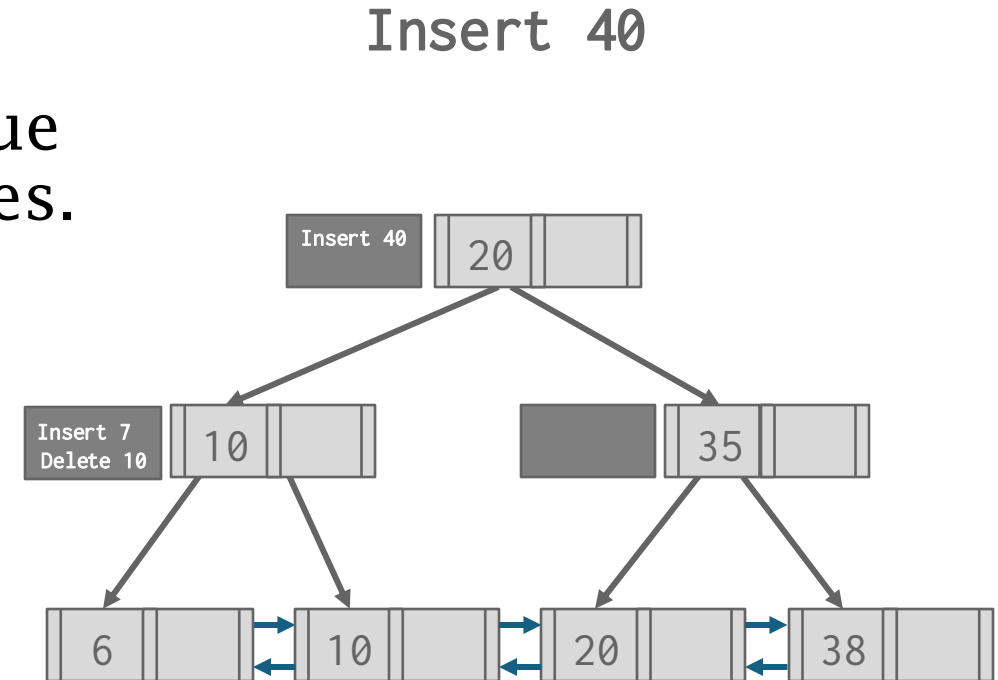
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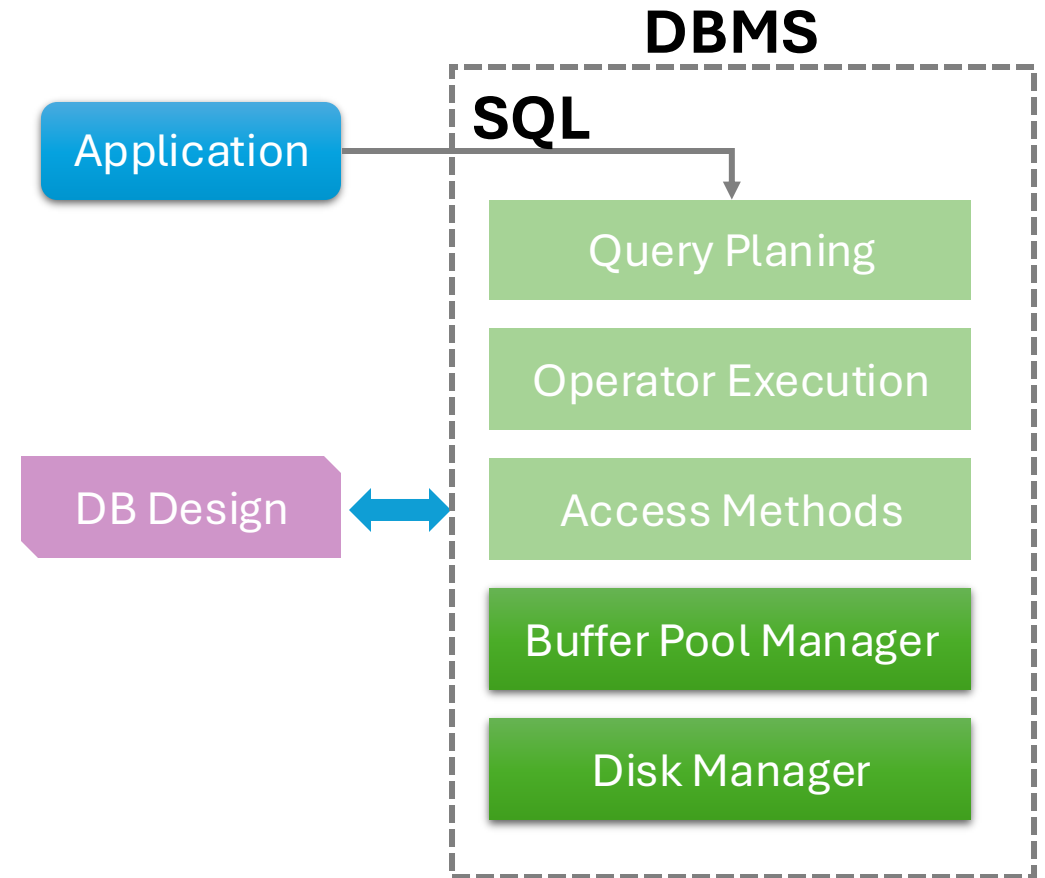
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