ECE2050 Homework 3

Due: March 8, 2025

Q1 Apply DeMorgan's theorems to each expression

1. $\overline{\overline{(\overline{ABC})(\overline{EFG})} + \overline{(\overline{HIJ})(\overline{KLM})}}$.
2. $\overline{\overline{(\bar{A} + B + C + D)}\,\overline{(A\bar{B}\bar{C}D)}}$.

1. $\overline{\overline{\overline{\overline{ABC}\;\overline{EFG}}} + \overline{\overline{\overline{HIJ}\;\overline{KLM}}}} = (\overline{ABC}\;\overline{EFG})(\overline{HIJ}\;\overline{KLM}) = \bar{A}+\bar{B}+\bar{C}+\bar{E}+\bar{F}+\bar{G}+\bar{H}+\bar{I}+\bar{J}+\bar{K}+\bar{L}+\bar{M}$

2. $\overline{\overline{\bar{A}+B+C+D}\;\overline{A\bar{B}\bar{C}D}} = (\bar{A}+B+C+D)+(A\bar{B}\bar{C}D) = \bar{A}+B+C+D$

Q2 Given the truth table below,

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

1. Write down standard sum-of-product (SOP) form.
2. Write down standard product-of-sum (POS) form.

$$SOP = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D}$$
$$+ A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD$$

$$POS = (A+B+C+\bar{D})(A+B+\bar{C}+D)$$
$$*(A+\bar{B}+C+D)(A+\bar{B}+\bar{C}+\bar{D})$$
$$*(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+\bar{D})$$
$$*(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$$

Q3 A circuit has four inputs and two outputs. The inputs $A_{3:0}$ represent a number from 0 to 15. Output P should be TRUE if the number is not prime (0 and 1 are not prime, but 2, 3, 5, and so on, are prime). Output D should be TRUE if the number is divisible by 4. Give simplified Boolean equations. (Tips: A prime number is a number greater than 1 that has no divisors other than 1 and itself.)

Q3 For numbers from 0 to 15,

Prime numbers: 2, 3, 5, 7, 11, 13

Can be devided by 4: 0, 4, 8, 12

Convert all numbers to binary:

Not Prime: 0000, 0001, 0100,
0110, 1000, 1001, 1010,
1100, 1110, 1111.

Convert to SOP form:

$$P = \bar{A}_3\bar{A}_2\bar{A}_1\bar{A}_0 + \bar{A}_3\bar{A}_2\bar{A}_1 A_0 + \bar{A}_3 A_2\bar{A}_1\bar{A}_0 + A_3\bar{A}_2\bar{A}_1\bar{A}_0$$
$$+ A_3\bar{A}_2\bar{A}_1 A_0 + A_3\bar{A}_2 A_1\bar{A}_0 + A_3 A_2\bar{A}_1\bar{A}_0 + A_3 A_2 A_1\bar{A}_0$$
$$+ A_3 A_2 A_1 A_0$$

Convert 0, 4, 8, 12 to binary:

0000, 0100, 1000, 1100.

$$\therefore D = \bar{A}_1\bar{A}_0$$

Q4 Develop a truth table for each of the SOP expressions:

1. $A\bar{B}\bar{C}D + AC\bar{D} + B\bar{C}D + \bar{A}BC\bar{D}$.
2. $A + B\bar{C} + CD$.

1.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

2.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Q5 Develop a truth table for each of the POS expressions

1. $A(B + \bar{C})(\bar{A} + C)(A + \bar{B} + C)(\bar{A} + B + \bar{C})$.
2. $(X + \bar{Y})(W + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})(W + X + Y + Z)$.

1.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

2.

W	X	Y	Z	Output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Q6 Convert each of the following POS expressions to minimum SOP expressions using a Karnaugh map.

1. $(A+\bar{B}+C+\bar{D})(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$.
2. $A(B+\bar{C})(\bar{A}+C)(A+\bar{B}+C)(\bar{A}+B+\bar{C})$.

1.

AB \ CD	00	01	11	10
0 0	0	1	1	1
0 1	1	0	1	1
1 1	1	1	1	1
1 0	1	1	1	0

SOP: $B\bar{D} + \bar{B}D + \bar{A}C + A\bar{C} + AD$

2.

A \ BC	00	01	11	10
0	0	0	0	0
1	0	0	1	0

SOP: ABC

Q7 Prove De Morgan's Theorem for three variables, A, B, and C, using perfect induction.

To prove De Morgan's Theorem for three variables A, B, and C using **perfect induction**, we construct truth tables for all possible combinations of A, B, and C and verify the equalities. Perfect induction involves checking every possible input combination (8 cases for three variables) to confirm the equivalence of both sides of the equations.

First Part: $\overline{A \cdot B \cdot C} = \overline{A} + \overline{B} + \overline{C}$

A	B	C	$A \cdot B \cdot C$	$\overline{A \cdot B \cdot C}$	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A}+\overline{B}+\overline{C}$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

Conclusion:
$\overline{A \cdot B \cdot C}$ is identical to $\overline{A} + \overline{B} + \overline{C}$ in all cases.

Second Part: $\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$

A	B	C	$A+B+C$	$\overline{A+B+C}$	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A} \cdot \overline{B} \cdot \overline{C}$
0	0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	1	0	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0	0

Conclusion:
$\overline{A+B+C}$ is identical to $\overline{A} \cdot \overline{B} \cdot \overline{C}$ in all cases.

Conclusion

By using perfect induction, we have verified both parts of De Morgan's Theorem for three variables A, B, and C across all possible combinations:

1. $\overline{A \cdot B \cdot C} = \overline{A} + \overline{B} + \overline{C}$
2. $\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$

Since the left-hand side equals the right-hand side in every case, De Morgan's Theorem is proven for three variables using perfect induction.

Q8 Find a minimal Boolean equation for the function as shown below.

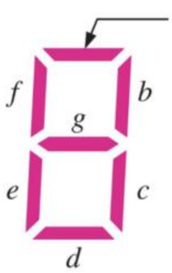
A	B	C	D	Y
0	0	0	0	X
0	0	0	1	X
0	0	1	0	X
0	0	1	1	X
0	1	0	0	0
0	1	0	1	X
0	1	1	0	0
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

AB \ CD	00	01	11	10
00	×	×	×	×
01	0	×	×	0
11	1	1	1	×
10	1	0	1	×

$: AB + A\bar{D} + CD$

Q9 According to the below figure,

1. Draw the true table for c with respect to A, B, C and D;
2. Write down the SOP and POS according to the true table in Q9.1;
3. Simplify the expression by using Karnaugh map.



Segments Inputs							7 Segment Display Output	
a	b	c	d	e	f	g		A B C D
1	1	1	1	1	1	0	0	0 0 0 0
0	1	1	0	0	0	0	1	0 0 0 1
1	1	0	1	1	0	1	2	0 0 1 0
1	1	1	1	0	0	1	3	0 0 1 1
0	1	1	0	0	1	1	4	0 1 0 0
1	0	1	1	0	1	1	5	0 1 0 1
1	0	1	1	1	1	1	6	0 1 1 0
1	1	1	0	0	0	0	7	0 1 1 1
1	1	1	1	1	1	1	8	1 0 0 0
1	1	1	1	0	0	1	9	1 0 0 1

1.

A	B	C	D	c
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	×
1	0	1	1	×
1	1	0	0	×
1	1	0	1	×
1	1	1	0	×
1	1	1	1	×

2. SOP: $\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D}$
$+ A\bar{B}\bar{C}D$

POS: $A + B + \bar{C} + D$

3.

AB \ CD	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	x	x	x	x
10	1	1	x	x

: $B + \bar{C} + D$