香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

ECE2050 Digital Logic and Systems

Tutorial 4

Yao Junguang

Contact: junguangyao@link.cuhk.edu.cn

Office Hour: 14:00-15:00 Tuesday, Rx902

Boolean Algebra and Logic Simplification

- Outline
  - Intro to laws, rules and theorem about Boolean algebra
  - How to simplify the expressions

Boolean Algebra and Logic Simplification

- Boolean Operations and Expressions
  - Addition -> OR; Multiplication -> AND

Inputs		Output
A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	1

1 = HIGH, 0 = LOW

OR

Inputs		Output
A	*B*	*X*
0	0	0
0	1	0
1	0	0
1	1	1

1 = HIGH, 0 = LOW

AND

Boolean Algebra and Logic Simplification

- Laws of Boolean Algebra

  - Commutative laws

    $A + B = B + A$; $AB = BA$

  - Associative laws

    $A + (B + C) = (A + B) + C$; $A(BC) = (AB)C$

  - Distributive law

    $A(B + C) = AB + AC$

Boolean Algebra and Logic Simplification

- Boolean Operations and Expressions

Number	Axiom	Dual	Name
A1	B = 0 if B ≠ 1	B = 1 if B ≠ 0	Binary Field
A2	$\overline{0} = 1$	$\overline{1} = 0$	NOT
A3	0 • 0 = 0	1 + 1 = 1	AND/OR
A4	1 • 1 = 1	0 + 0 = 0	AND/OR
A5	0 • 1 = 1 • 0 = 0	1 + 0 = 0 + 1 = 1	AND/OR

Dual: Replace: • with +
0 with 1

Boolean Algebra and Logic Simplification

- Rules of Boolean Algebra

TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

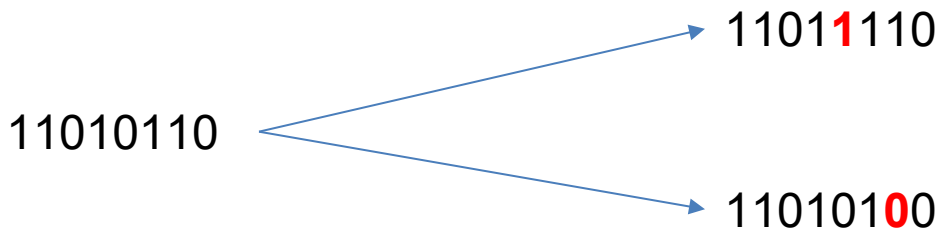
Boolean Algebra and Logic Simplification

- Rules of Boolean Algebra-Extension

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$

Register in Chip

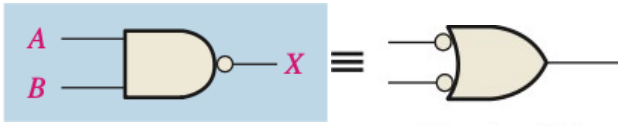
PORTA



Boolean Algebra and Logic Simplification

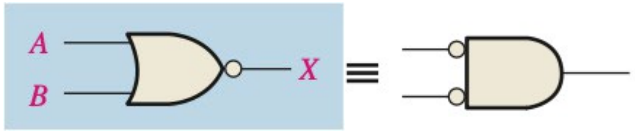
- DeMorgan's Theorems

$$\overline{XY} = \bar{X} + \bar{Y}$$



Negative-OR

$$\overline{X + Y} = \bar{X}\bar{Y}$$



Negative-AND

Boolean Algebra and Logic Simplification

Q1: Using Boolean Algebra to simplify the expression:

$$BCDE + BC(\overline{DE}) + (\overline{BC})DE$$

Boolean Algebra and Logic Simplification

Q1: Using Boolean Algebra to simplify the expression:

$$BCDE + BC(\overline{DE}) + (\overline{BC})DE$$

A1: BC + DE (use rule 6 and 11.)

Boolean Algebra and Logic Simplification

- Standard Forms of Boolean Expressions
  - SOP: Sum-of-Products

    Standard SOP Form: Each product term has all variables
  - POS: Product-of-Sums

- SOP form: A+BC
- NOT SOP form: A+B(A+C)
- Standard SOP form: $ABC+AB\overline{C}$

Boolean Algebra and Logic Simplification

- Boolean Expressions and Truth Tables

Q2: Develop the truth table and
the standard POS expression:

$$(A + \overline{B})(A + \overline{B} + \overline{C})(B + C + \overline{D})(\overline{A} + B + \overline{C} + D)$$

Boolean Algebra and Logic Simplification

- Boolean Expressions and Truth Tables

Q2: Develop the truth table and the standard POS expression:

A2:

$$(A + \overline{B})(A + \overline{B} + \overline{C})(B + C + \overline{D})(\overline{A} + B + \overline{C} + D)$$

TABLE P–8

A	*B*	*C*	*D*	*X*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Boolean Algebra and Logic Simplification

- The Karnaugh Map (K-Map)

  - Used to simplify Boolean expressions in a SOP/POS form.

  - Karnaugh maps can be used for expressions with two, three, four, and five variables, and the number of cells in a Karnaugh map indicates the number of possible input variable combinations.

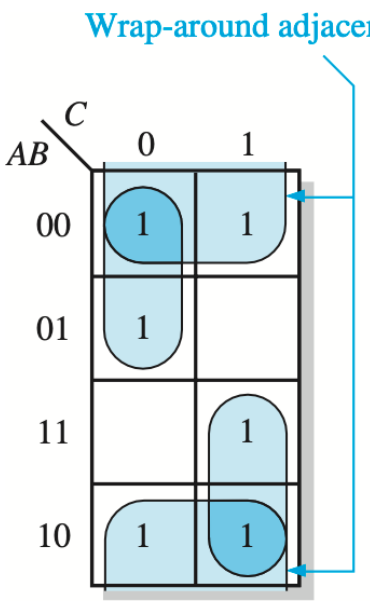
    - *For three variables, the number of cells is $2^3 = 8$. For four variables, the number of cells is $2^4 = 16$.*

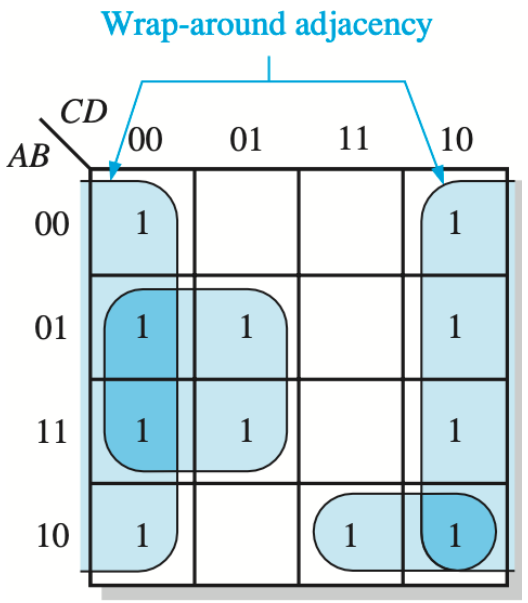
  - Adjacency is defined by a single-variable change between adjacent cells.

Boolean Algebra and Logic Simplification

- Cell Adjacency of The Karnaugh Map (K-Map)

1. Each cell is adjacent to the cells that are immediately next to it on any of its four sides
2. Wrap-around adjacency





Boolean Algebra and Logic Simplification

- SOP Minimization and POS Minimization
  - In SOP, focus on 1's
  - In POS, focus on 0's

- Grouping
  - Goal: Maximize the size of the groups and to minimize the number of groups
  - The number of cells in a group should be powers of 2
  - Overlapping is allowed

- Nonstandard SOP Expression
  - Expand each term

Boolean Algebra and Logic Simplification

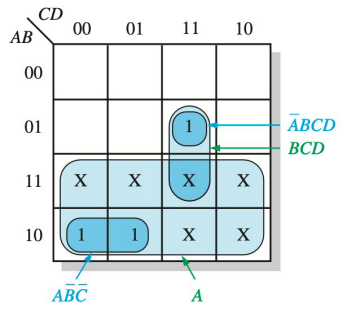
- “Don’t Care” Conditions

  - some states will never occur in an application, they can be treated as “don’t care” terms.

Inputs				Output
A	*B*	*C*	*D*	*Y*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

(a) Truth table

Don’t cares



(b) Without “don’t cares” $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With “don’t cares” $Y = A + BCD$

FIGURE 4–40 Example of the use of “don’t care” conditions to simplify an expression.

Boolean Algebra and Logic Simplification

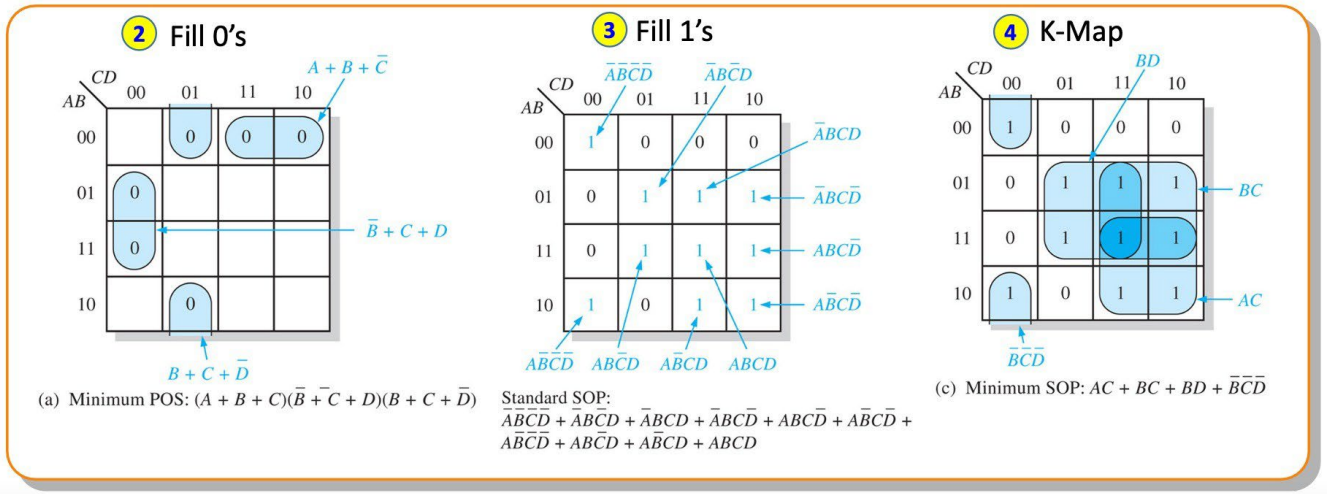
- Converting between POS and SOP with K-Map

Example 4-35 Using a Karnaugh map, convert the following standard POS expression into a min. POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\bar{A}+\bar{B}+C+D)(A+\bar{B}+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+\bar{D})(\bar{A}+B+C+\bar{D})(A+B+\bar{C}+D)$$

1 Find all 0's

1100 0100 0001 0011 1001 0010



(a) Minimum POS: $(A+B+C)(\bar{B}+\bar{C}+D)(B+C+\bar{D})$

Standard SOP:
$\bar{A}\bar{B}\bar{C}\bar{D}+\bar{A}\bar{B}\bar{C}D+\bar{A}\bar{B}CD+\bar{A}B\bar{C}D+\bar{A}BC\bar{D}+A\bar{B}C\bar{D}+$
$A\bar{B}\bar{C}\bar{D}+A\bar{B}\bar{C}D+A\bar{B}CD+ABCD$

(c) Minimum SOP: $AC+BC+BD+\bar{B}\bar{C}\bar{D}$

Boolean Algebra and Logic Simplification

- K-Map of SOP Minimization

Q3: Minimize the expression with a K-Map:

$$A\overline{B} + A\overline{B}\,\overline{C}D + CD + B\overline{C}D + ABCD$$

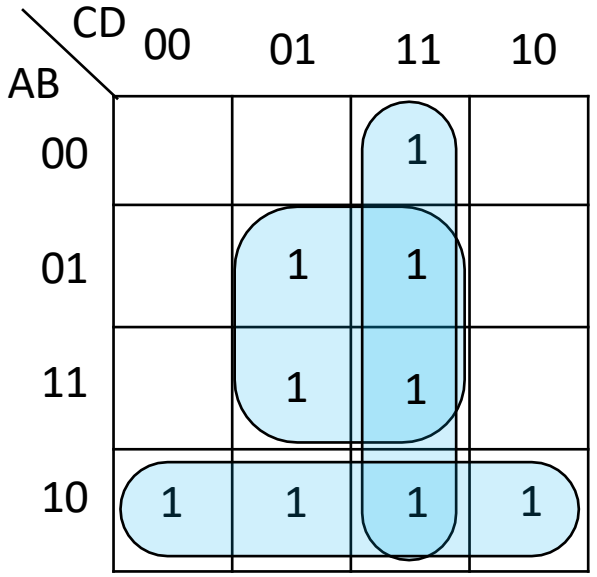
Boolean Algebra and Logic Simplification

- K-Map of SOP Minimization

  Q3: Minimize the expression with a K-Map:

$$A\overline{B} + A\overline{B}\,\overline{C}D + CD + B\overline{C}D + ABCD$$

A3: $A\overline{B}$ +BD+CD



ABCD	*X*
0000	0
0001	0
0010	0
0011	1
0100	0
0101	1
0110	0
0111	1
1000	1
1001	1
1010	1
1011	1
1100	0
1101	1
1110	0
1111	1

(You don't have to draw a truth table if the question does not require)

Boolean Algebra and Logic Simplification

- K-Map of POS Minimization

Q4: Determine the minimum POS expression for the function in Table 4-17:

TABLE 4–17

Inputs				Output
A	*B*	*C*	*D*	*X*
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Boolean Algebra and Logic Simplification

- K-Map of POS Minimization

Q4: Determine the minimum POS expression for the function in Table 4-17:

A4:

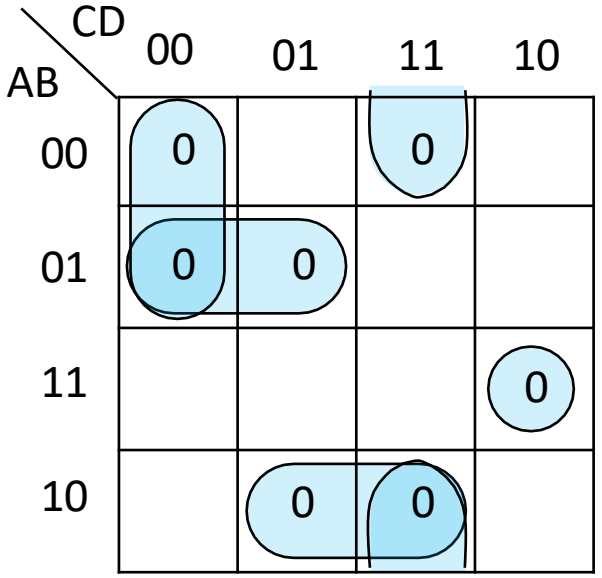


TABLE 4–17

Inputs				Output
A	*B*	*C*	*D*	*X*
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

$$(A + C + D)(A + \bar{B} + C)(\bar{A} + B + \bar{D})(B + \bar{C} + \bar{D})(\bar{A} + \bar{B} + \bar{C} + D)$$

香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

Q&A

香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

Thank You!