Fundamental content before midterm- A brief review

MAT2040 Linear Algebra

Brief Review- Linear system

1. Three equation operations. Equivalent linear system.

2. Augmented matrix. Three elementary row operations. Row-echelon form by using Gaussian Elimination. Reduced row-echelon form by using Gauss-Jordan Elimination.

3. How to determine the three type of solutions.

4. How to determine independent variable, dependent variable.

5. Write solution in parametric vector form.

6. Write solutions in using Span for $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$

7. The relations for the solutions of $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$

8. How to solve the linear system $A\mathbf{x} = \mathbf{b}$ by using the LU decomposition of A.

Brief Review- Matrix

1. Matrix operation: matrix addition, scalar multiplication

2. Matrix algebra: matrix multiplication, block matrix multiplication.

3. Matrix inverse, matrix transpose, the interaction between inverse and transpose. Special matrices, symmetric matrix, anti-symmetric matrix.

5. Elementary matrices and its inverse, LU and LDU decomposition for matrix with good property, fast algorithm to compute the LU decomposition. Permutation matrix and its property.

6. Row equivalent matrices, method to find A^{-1} and $A^{-1}B$.

7. Equivalent conditions for an invertible matrix (7-8 important conditions).

8. Nonsingular matrices product has nonsingular terms, one-sided inverse verification is sufficient.

Brief Review- Vectors

1. Euclidean Vector Space $\mathbb{R}^n$.
2. Linearly independent, Linearly dependent.
3. Span, Spanning set.
4. The solution of the linear system $A\mathbf{x} = \mathbf{b}$ in parametric vector form.

Brief Review- Vector Space

1. $\mathbb{R}^n, P_n, \mathbb{R}^{m\times n}, C[a,b]$ are vector spaces.

2. Definition for Subspaces. Three conditions for verifying a subspace.

3. Basis and dimension for vector space.

4. Definition of the coordinate of a vector with respect to a basis.

5. Transition matrix for two bases in a vector space. Coordinate change between two different bases.

Theorem (**Transition Matrix between two bases**) Let $\mathcal{U} = \{\mathbf{u}_1, \cdots, \mathbf{u}_n\}$ and $\mathcal{V} = \{\mathbf{v}_1, \cdots, \mathbf{v}_n\}$ be two bases of vector space V. Then

$$[\mathbf{x}]_{\mathcal{V}} = A[\mathbf{x}]_{\mathcal{U}}$$

where the jth column of A is $[\mathbf{u}_j]_{\mathcal{V}}$.

Remark: $\dim(\{\mathbf{0}\}) = 0$, since $\{\mathbf{0}\}$ has no basis vector.

Brief Review- Matrix Vector Space

1. Null space of a matrix A. Using the solution of $A\mathbf{x} = \mathbf{0}$ in parametric vector form to find a basis for Null(A), the dimension of Null(A)=the number of non-pivot columns of A. The relation of the solution set for the linear system $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$.

2. Column space of a matrix A. Pivot columns of A form a basis for Col(A). Let $A \in \mathbb{R}^{m\times n}$. $\dim(Col(A)) + \dim(Null(A)) = n$.

3. Row space of a matrix A. The transpose of nonzero rows in reduced row echelon-form of A form a basis for Row(A).

4. Rank of A. rank(A)=$\dim(Col(A)) = \dim(Row(A))$. Rank-Nullity theorem.

5. Three important properties of row operations: (1). Preserve the linear dependence relation between column vectors, (2). Do not preserve the column space. (3). Preserve the row space.