

Travelling Salesman Problem (TSP)

- The Traveling Salesman Problem (TSP) is a very well-known integer optimization problem, one of the most intensively studied, most widely applied, and the most difficult integer programs.
- Given a collection of n cities, TSP asks to find a tour of minimum distance that starts from a city and visits each of the other $n - 1$ cities exactly once and comes back to the starting city.

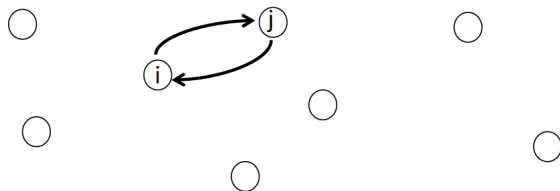
TSP Model

- Let $N = \{1, 2, \dots, n\}$ be the index set of cities.
- Define $x_{ij} = 1$ if the travelling salesman goes from city i directly to city j and 0 otherwise.

$$\begin{aligned} \min \quad & \sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n x_{ij} = 1 \quad \forall i = 1, \dots, n \\ & \sum_{i=1}^n x_{ij} = 1 \quad \forall j = 1, \dots, n \\ & \sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1 \quad \forall S \subset N, |S| \leq n - 1 \\ & x_{ij} \in \{0, 1\} \quad \forall i, j = 1, \dots, n \end{aligned}$$

Subtour Elimination

Subtour elimination constraints: limit the number of selected arcs in each subset of cities S to be no greater than $|S| - 1$



$$S = \{i, j\}$$
$$x_{ij} + x_{ji} \leq 1$$

Subtour Elimination Algorithm for TSP

- There are many (exponential in n) subtour elimination constraints.
- Solve without any subtour elimination constraints.
- Inspect the solution to check if there are subtours.
- Add the subtour elimination constraint corresponding to the found subtour.
- Repeat