MIDTERM EXAM

MAT 3007
Nov 2020

INSTRUCTIONS

a) Write ALL your answers in this exam paper.

b) One piece of note is allowed. No computer or calculator is allowed.

c) The exam time is 10:00am - 11:30am.

d) There are 6 questions and 100 points in total. Except the true or false questions, write down the reasonings for your answers.

In taking this examination, I acknowledge and accept the instructions.

NAME (signed) ______________________________

NAME (printed) ______________________________

For grading use. Don't write in this part

	1 (18pts)	2 (28pts)	3 (20pts)	4 (15pts)	5 (9pts)	6 (10pts)	Total
Points							

Problem 1: True/False (18pts)

State whether each of the following statements is True or False. For each part, only your answer, which should be one of True or False, will be graded. Explanations will not be read.

(a) For linear optimization problems, if the primal problem has a feasible solution, then the dual problem must also have a feasible solution.

(b) Some linear optimization problems may have exactly two solutions.

(c) Adding a constraint to a linear programming problem increases the size of the feasible region

(d) Consider a standard LP with n variables and m constraints. Suppose it has a finite optimal solution. Then the optimal solution returned by the simplex method must have no more than m strictly positive entries.

(e) In portfolio problems, we want to minimize the number of stocks chosen, so we prefer to use the interior method rather than the simplex method.

(f) In a standard linear optimization problem, if we remove the nonnegative constraint of a variable, then the optimal value (suppose exist for both problems) must not increase.

Problem 2: Simplex Method (28pts)

Consider the following linear optimization problem:

$$\begin{array}{rrrrcl} \text{maximize} & x_1 & +x_2 & +x_3 & & \\ \text{s.t.} & 2x_1 & -2x_2 & +x_3 & = & 4 \\ & 3x_1 & & -x_3 & = & 1 \\ & x_1 \text{ free}, & x_2 \leq 0, & x_3 \geq 0 & & \end{array}$$

a) Convert the problem into the standard form. (5pts)

b) Solve it using two-phase simplex method. Please clearly write down each step. Please also clearly write down your final solution to the original problem. (15pts)

c) Write down the dual problem (of the original problem, not the standard form). What is the optimal solution to the dual problem? (8pts)

Problem 3: Sensitivity Analysis (20pts)

General Mills produces three types of cereal products, A, B, and C. The ingredient requirements for each product, the availability of each ingredient and the selling prices of each product are given in the following table:

	Product A	Product B	Product C	Availability
Wheat	2	2	1	140
Corn	1	2	3	120
Oat	1	1	2	160
Price	\$4	\$5	\$7	

Let x_1, x_2 and x_3 denote the amount of product A, B, C to produce, respectively. Then a linear program for this problem can be written as:

$$
\begin{array}{llrrrcl}
(\mathbf{P1}) & \text{maximize} & 4x_1 & +5x_2 & +7x_3 & & \\
& \text{s.t.} & 2x_1 & +2x_2 & +x_3 & \leq & 140 \\
& & x_1 & +2x_2 & +3x_3 & \leq & 120 \\
& & x_1 & +x_2 & +2x_3 & \leq & 160 \\
& & x_1, & x_2, & x_3 & \geq & 0
\end{array}
$$

After using simplex method on the standard form, the final tableau is as follows:

B							
B	0	1	0	1	2	0	380
1	1	0.8	0	0.6	-0.2	0	60
3	0	0.4	1	-0.2	0.4	0	20
6	0	-0.6	0	-0.2	-0.6	1	60

Please answer the following questions (for part a, b and c, please clearly state the range of coefficient as your final answer):

a) In what range can the price of product B vary without changing the optimal basis? (5pts)

b) In what range can the price of product C vary without changing the optimal basis? (5pts)

c) In what range can the availability of wheat vary without changing the optimal basis? (5pts)

d) Suppose there is an additional type of product (product D), which requires 3 units of wheat, corn and oat respectively. What is the minimum selling price to make it worth producing? (5pts)

Problem 4: Complementarity Conditions (15pts)

In a production planning problem, a firm can produce products 1, 2, 3 using two types of resources. Each of the product has a certain profit. The linear optimization formulation and its solutions are shown below. The formulation and solution were 100% correct and 100% accurate, but the printer smudged some of the values. The pound sign # in the following tables represents a number that has been smudged. Find those missing values (only the numbers in the box will be graded, no explanation needed).

$$\begin{array}{rc} \text{maximize} & \#x_1 + 7x_2 + 9x_3 \\ \text{subject to} & x_1 + x_2 + x_3 \le 10 \\ & 30x_1 + 40x_2 + \#x_3 \le 500 \\ & x_1, x_2, x_3 \ge 0 \end{array}$$

Solution:

x_1^*	x_2^*	x_3^*	Objective value z^*
#	8	#	#

Dual variable solution (y_{slack}'s are the slack variables for the dual problem):

y_1^*	y_2^*	$y_{slack-1}^*$	$y_{slack-2}^*$	$y_{slack-3}^*$
5.4	0.04	1.6	0	0

Your answer (A_{23} is the missing constraint entry, c_1 is the missing objective coefficient):

x_1^*	x_3^*	A_{23}	c_1	z^*

Problem 5: LP Formulation (9pts)

Consider an investment company that is interested in investing in n assets. The current prices of those assets are $p_1, ..., p_n$. After analysis, the company forecasts that there are m future scenarios. In scenario i, the price of asset j will be q_{ij}. Also, there is a current budget C for the company. We assume that no negative amount of investment is allowed (i.e., no short selling is allowed).

Write an LP formulation such that the worst-case market value among all the scenarios is maximized.

Problem 6: Unit eigenvectors of stochastic matrices (10pts)

We say that an $n \times n$ matrix $\mathbf{P}$, with entries p_{ij}, is stochastic if all of its entries are nonnegative and

$$\sum_{j=1}^{n} p_{ij} = 1, \ \ \forall i,$$

that is, the sum of the entries of each row is equal to 1.

Use duality to show that if $\mathbf{P}$ is a stochastic matrix, then the system of equations

$$x^T \mathbf{P} = x^T, \ \ x \geq 0$$

has a nonzero solution.