ISyE 6669 Deterministic Optimization Homework 3
Due 2:15pm on March 6

1 Logic Constraints

Use appropriate big-M values wherever needed.

(a) x_i for $i = 1, 2, 3$ are continuous nonnegative variables with values in the range [0, 3]. Write a set of constraints to model the requirement that:

$$|2x_1 - x_2 - x_3| \geq 2$$

by introducing an additional binary variable. Justify your formulation.

(b) x_1 and x_2 are integer variables whose values are restricted to be in [0, 10]. Write a set of constraints to model the requirement that: either $x_1 + x_2 \leq 10$ or $2x_1 - x_2 \geq 5$ but not both by introducing an additional binary variable.

2 TSP Subtour Elimination

Consider the TSP with 7 cities. The distances between cities are summarized in the following matrix.

$$d_{ij} = \begin{bmatrix} 0 & 86 & 49 & 57 & 31 & 69 & 50 \\ 86 & 0 & 68 & 79 & 93 & 24 & 5 \\ 49 & 68 & 0 & 16 & 7 & 72 & 67 \\ 57 & 79 & 16 & 0 & 90 & 69 & 1 \\ 31 & 93 & 7 & 90 & 0 & 86 & 59 \\ 69 & 24 & 72 & 69 & 86 & 0 & 81 \\ 50 & 5 & 67 & 1 & 59 & 81 & 0 \end{bmatrix}$$

Code the greedy algorithm discussed in the lecture slide to solve this TSP problem.

3 IP Modeling

An auto manufacturer is considering manufacturing three types of cars: compact, midsize and large. The resources required and profits (per car) for each car type is shown below.

	Compact	Midsize	Large
Steel required (tons)	1.5	3	5
Labor required (hours)	30	25	40
Profit ($)	2000	3000	4000

Currently, 6000 tons of steel and 60,000 hours of labor are available. The following additional restrictions are specified.

1. If the company decides to produce compact cars then it must produce at least 1000 compact cars.
2. If the company decides to produce midsize cars then it must produce at least 800 midsize cars.
3. If the company decides to produce large cars then it can produce at most a total of 1200 compact and midsize cars.

Formulate an integer linear program to maximize the companies profits, while satisfies resource limitations, and the above restrictions. The number of cars of each type produced should be integer valued. If you use big-M's in your model provide good numerical values for these.

4 IP Modeling

Dunwoody baseball little league has to schedule 6 games over a 3 day period. The following table indicates the games and the teams that play in each game.

Game	Team A	Team B	Team C	Team D	Team E
1	x	x			
2	x		x		
3	x				x
4		x		x	
5			x	x	
6			x		x

For example, game 4 is between teams B and D.

The concession revenues for having game i on day t are given as r_{it} for $i = 1, ..., 6$ and $t = 1, 2, 3$. Formulate an integer program to help Dunwoody little league decide which game should be played on which day so as to maximize concession revenues subject to the following restrictions:

- Each game has to be scheduled.
- No team can play more than 2 games in a day.
- Team D cannot play on day 1.
- There has to be at least 2 games assigned on day 3.
- Game 1 can only be assigned on day 1 if game 5 is assigned on day 2 or if game 6 is not assigned on day 3.

5 IP Modeling

Workers at the Execo manufacturing plant work 5 consecutive days a week, e.g. a worker can start work on a Monday and continue till Friday while another can start work on a Wednesday and continue till Sunday etc. Workers who are off on weekends (Saturday and Sunday) are paid \$300 per day (for each of their 5 work days), workers who's schedules involve one weekend day are paid \$320 per day (for each of their 5 work days), and workers who's schedules involve both weekend days are paid \$330 per day (for each of their 5 work days). The goal is to determine the number of workers for each schedule to minimize cost subject to the following restrictions:

- There must be at least 25 workers each day.
- There can be at most 35 workers on at least 3 of the 7 days of the week.
- If the number of workers that start on a Monday exceed 10 then the number of workers available on a Wednesday should be no more than 28.
- The number of workers that start on a Saturday should be less than the number of workers that start on a Monday or a Tuesday.
- A total 40 workers are available.

Formulate the above problem as a mixed-integer linear problem. Use x_i to denote the number (integer) of workers who start work on day i. You may need to introduce additional binary variables. If you use big-M numbers, please provide your best estimates of their values.