MAT 3007 Optimization: Tutorial 4

Guxin DU

The Chinese University of Hong Kong, Shenzhen

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Review: Fundamental LP Theorem

Consider a linear problem **in standard form** and assume that A has full row rank m.

(1) existence of extreme points:
If the feasible set is nonempty, there is a basic feasible solution.
$\Leftrightarrow$ Nonempty polyhedra in standard form have at least one extreme point. Remark: Standard form (especially $x \geq 0$) plays an important role in the existence here!

(2) optimality of extreme points:
If there is an optimal solution, there is an optimal solution that is also a basic feasible solution.
More generally, if feasible, then the optimal cost is either $-\infty$, or finite and can be attained by an extreme point as an optimal solution
Remark: In LP, if optimal cost is **finite**, then it's **attainable**!

Review: Fundamental LP Theorem & Exercise

For each of the following statements, state whether it is true or false. If true, provide a proof, else, provide a counterexample.
Now consider the standard form polyhedron $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$.
Suppose $A \in \mathbb{R}^{m \times n}$ has m linearly independent rows.

(a) if $n = m + 1$, then P has at most two basic feasible solutions.

(b) The set of all optimal solutions is bounded.

(c) At every optimal solution, no more than m variables can be positive.

(d) If there is more than one optimal solution, then there are unaccountably many optimal solutions.

(e) If there are several optimal solutions, then there exist at least two optimal basic feasible solutions.

Exercise 1

For the standard Lp polyhedron $\{x : Ax = b, x \geq 0\}$, the followings are equivalent:

(1) x is an extreme point

(2) x is a basic feasible solution

Exercise 2

Use the simplex method to solve the following problem
(This trivial problem is an illustration of simplex method.)

$$\begin{aligned} \min \quad & 3x_1 + 4x_2 \\ s.t. \quad & x_1 + x_2 \leq 4 \\ & x_2 \leq 5 \\ & x \geq 0 \end{aligned} \tag{1}$$

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