MAT3007 Optimization
Lecture 1 Course Introduction
Optimization Basics

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The Chinese University of Hong Kong, Shenzhen

June 4, 2025

Outline

Outline

1. Course Syllabus
2. Operations Research
3. Optimization
4. Optimization Framework
5. Classifications of Optimization
6. Outcomes of Optimization

Course Information

- Instructor: Dr. Yuang Chen
- Class time: Monday, Wednesday, and Thursday 1:30 - 3:20 pm
- Location: Teaching B 201
- Office hours: Monday, Wednesday, and Thursday 3:20 - 4:20 pm (right after class) or by appointments
- Office: Dao Yuan Building 517
- Email: ychen@cuhk.edu.cn

Teaching Assistants and Tutorials

Teaching Assistants

- Xinyang Feng (Leading TA)
  - Email: 120090445@link.cuhk.edu.cn
  - Office Hour: Tuesday 4-5 pm, Zhi Xin 411
- Kangran Zhao
  - Email: 118010438@link.cuhk.edu.cn
  - Office Hour: Tuesday 2-3 pm, TXB 603
- Guxin Du
  - Email: 224040342@link.cuhk.edu.cn
  - Office Hour: Wednesday 4-5 pm, Zhi Xin 410

Tutorials

- T1: Tuesday and Thursday 6-6:50 pm. Teaching A 101
- T2: Tuesday and Thursday 7-7:50 pm. Teaching A 101
- Tutorials will start on Thursday (June 5)!

About Me

- B.S. in Electrical Engineering from Purdue University, 2015
- M.S. in Electrical Engineering from UCLA, 2016
- Ph.D. in Operations Research from Georgia Tech, 2021
- Assistant Professor at Georgia Tech Shenzhen Institute, 2022-2024
- Assistant Professor (Teaching) at CUHK-SZ from 2025
- Research interests: stochastic optimization, data-driven decision making, infrastructure planning and operation, energy and water systems
- I have been teaching optimization classes since the beginning of my teaching career, but until now, they have all been at the master's level. This is my first time teaching an undergraduate-level optimization course.

Why You Should NOT Take MAT 3007 in the Summer? (1/2)

- **Very fast pace**: The entire course is condensed into 7 weeks (June 3 – July 21) with 21 classes in total, compared to 15 weeks during a regular semester.
- **Limited time to digest**: Optimization is a mathematics course — understanding the concepts requires time for thinking and practice. In a regular semester, you have weekends and breaks to review and digest materials, but the summer schedule leaves little room for this.
- **Heavy prerequisites**: The course builds on prior knowledge in calculus and linear algebra. Due to the accelerated pace, there will be little time for review, and you will be expected to grasp the material quickly on your own.

Why You Should NOT Take MAT 3007 in the Summer? (2/2)

- **Only one instructor**: I will be the only instructor in the summer. In the regular semester, you may have more options to choose an instructor whose teaching style you prefer.
- **Flexible emphasis**: While the syllabus remains the same, I may shift emphasis on certain topics or introduce different perspectives compared to the regular semester.
- **Custom homework and exams**: Although previous homework and exams are useful for reference, I will create my own problems. They may be made upon past questions but are likely to be quite different. However, the previous materials will become valuable during regualr semesters.
- **Competitive grading policy**: I feel I am the nicest instructor in the school, but I must follow university policies. If you do not demonstrate sufficient understanding on exams, you may receive a failing grade.

Why You Should Take MAT 3007 in the Summer?

- **No additional tuition**: Taking summer courses at CUHK-SZ typically does not require extra tuition if you are a full-time student.
- **Short and focused learning**: The intensive 7-week schedule encourages you to stay focused and engaged, often leading to more efficient study habits.
- **Efficient use of summer**: Instead of a long and idle break, you can use the summer to complete a required course and free up your regular semester schedule.
- **Nice instructor**: I am a nice instructor both in person and in grading, you can talk with me more often.

What This Course Will Study?

We study and solve many important optimization problems (linear program, integer program, convex optimization, nonlinear optimization, and unconstrained optimization).

- Modeling of optimization models
- Structures and properties of optimization models
- Algorithms to solve optimization models

Problem $\rightarrow$ Modeling $\rightarrow$ Structure/Properties $\rightarrow$ Algorithms

Course Pre-requisites

- The class involves many real-world applications, but the class content is a **mathematical class**.
- **You need fundamental knowledge in calculus and linear algebra**. If you are not familiar with calculus (taking derivatives) and linear algebra (equations in matrix form), the class will be difficult for you!

In-class Exercise

Consider you have observations $\{x_1, ..., x_n\}$ that follow Poisson distribution. You derive out the likelihood function is

$$p(x_1, ..., x_n) = \prod_{i=1}^{n} e^{-\mu} \frac{\mu^{x_i}}{x_i!}$$

Now you want to use the maximum (log-)likelihood estimation (MLE) method to find out the best parameter μ, the MLE optimization model is

$$\text{(MLE)} \quad \max_{\mu} \; f(\mu) = \ln \; p(x_1, ..., x_n)$$

Is the function $f(\mu)$ convex in μ? It's equivalent to ask you the sign of the second derivative of $f(\mu)$?

$$f(\mu) = \ln \prod_{i=1}^{n} e^{-\mu} \frac{\mu^{x_i}}{x_i!}$$

$$\boxed{\prod_{i=1}^{n} a_i = a_1 a_2 \cdots a_n}$$

$$= \sum_{i=1}^{n} \ln e^{-\mu} \frac{\mu^{x_i}}{x_i!}$$

$$= \sum_{i=1}^{n} \ln e^{-\mu} + \ln \mu^{x_i} + \ln \frac{1}{x_i!}$$

$$= \sum_{i=1}^{n} \left(-\mu + x_i \ln \mu - \ln x_i! \right)$$

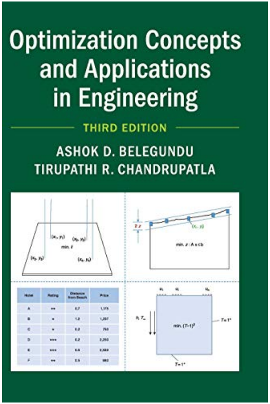
$$f(\mu) = -n\mu + \ln \mu \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \ln x_i!$$

$$f'(\mu) = -n + \frac{\sum_{i=1}^{n} x_i}{\mu}$$

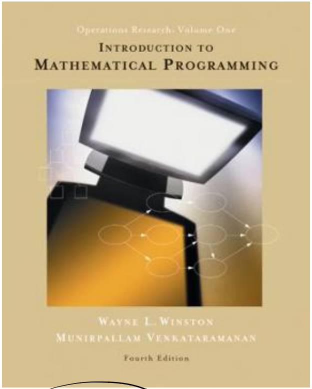
$$f''(\mu) = - \frac{\sum_{i=1}^{n} \underbrace{x_i}_{>0}}{\underbrace{\mu^2}_{>0}} < 0$$

Reference Books (Easy)

Mainly follow the lecture notes. No book is required. However, if you are interested in more details, the following books are recommended.

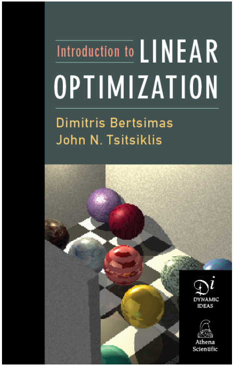


Optimization Concepts and Applications in Engineering by A. Belegundu and R. Tirupathi

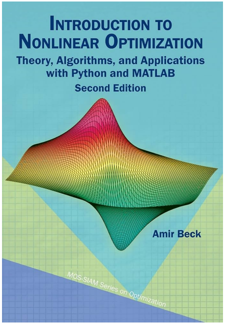


Introduction to Mathematical Programming by W. Winston and M.Venkataramanan

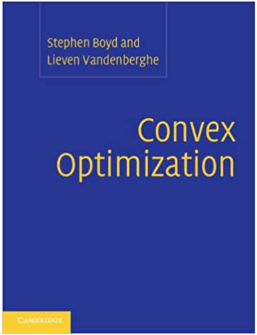
Reference Books



Introduction to Linear Optimization by D. Bertsimas and J. Tsitsiklis



Introduction to Nonlinear Optimization. Theory, Algorithms, and Applications with MATLAB by Amir Beck



Convex Optimization by S. Boyd and L. Vandenberghe

Grading

- Homework: 20% Post on Thursday, due next Sunday
  - 7HWs in the semester, only 5 will be collected.
  - Discuss with other students is encouraged but please write your own answer individually.
  - **Late homework submission will be not be accpeted!**
- Midterm: 40% (tentative schedule: June 26)
- Final 40% (Time TBA)

Topic Sequence: Summer vs Regular Semester

Summer (This Class)

1. Optimization Basics
2. Linear Program (including LP modeling)
3. Integer Program (including IP modeling)
4. Convexity
5. Optimality Conditions
6. Lagrangian Dual and KKT Conditions
7. Algorithms for Unconstrained Optimization

Regular Semester

1. Optimization Basics
2. General Modeling
3. Linear Program
4. Optimality Conditions
5. Convexity
6. Lagrangian Dual and KKT Conditions
7. Algorithms for Unconstrained Optimization
8. Integer Program

WeChat Group

群聊: MAT3007 Optimization
Summer 2025

Outline

What is Operations Research?

- Operations Research: a **quantitative** approach to **decision making** based on the scientific method of problem solving.
- Operations Research is the scientific approach to execute decision making, which consists of:
  - The art of mathematical modeling of complex situations.
  - The science of the development of solution techniques used to solve these models.
  - The ability to effectively communicate the results to the decision maker.

History of Operations Research

- Operation research origins in World War II for military service - planning and scheduling problems for Air Force.
- Urgent need to allocate resources at efficient manner.
- British and US called large number of scientists from discipline were asked to do research on military operation.
- Developed effective method to locate radar (Britain Air Battle).
- Developed a better method to manage convoy and antisubmarine operation(North Atlantic).
- Developed a method to utilize resources efficiently (resource cost reduced one half)
- Invented a branch of operations research called linear programming in 1947 at the pentagon.

Development of Operations Research

- Success of OR in the war spurred interest in outside the military (business, industry and government)
- Two factors played a major for rapid growth of OR
  - Continuous contribution by scientist's to improve the techniques of OR
  - Computer Revolution

Operations Research Models

Deterministic Models

- Linear Programming
- Integer Programming
- Convex Optimization
- Nonlinear Programming
- Combinatorial Optimization

Stochastic Models

- Discrete-Time Markov Chains
- Continuous-Time Markov Chains
- Queuing Theory
- Stochastic Optimization (Markov Decision Process)
- Bayesian Optimization
- Simulation

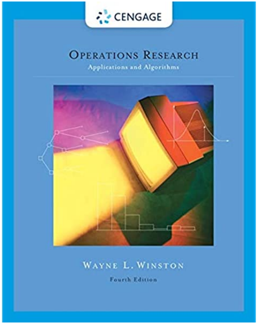
OR
- Optimization
- Stochastic Process
- Simulation

Terminology

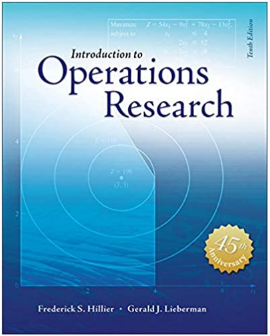
- The British/Europeans refer to “Operational Research”, the Americans to “Operations Research” - but both are often shortened to just ”OR”.
- Another term used for this field is “Management Science” (”MS”). In U.S. OR and MS are combined together to form ”OR/MS” or ”ORMS”.
- Yet other terms sometimes used are “Industrial Engineering” (”IE”) and “Decision Science” (”DS”).
- The Institute for Operations Research and the Management Sciences (INFORMS) is an international society for practitioners in the fields of operations research, management science, and analytics.

Operations Research Textbooks

Operations Research: Applications and Algorithms by Wayne Winston



Introduction to Operations Research by Frederick S Hillier and Gerald J. Lieberman



INFORMS John von Neumann Theory Prize

- Awarded annually to individuals (or occasionally groups) for fundamental and sustained contributions to theory in operations research and management sciences.
- Recognized as the highest honor in the field, often referred to as the "Nobel Prize" of Operations Research.
- 6 out of 50 recipients have also won the Nobel Prize in Economics.
- Three recipients are affiliated with SDS/CUHK-SZ:
  - Yurii Nesterov
  - Yinyu Ye
  - Jim Dai (see: `https://mp.weixin.qq.com/s/SXv3pvYjAFzuF9Y7SNoaqQ`)

Outline

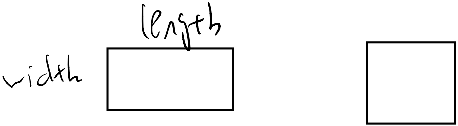
Decision Making

Example: How do I get to school in the morning?

- Decision: the way I get to school: I may choose driving, taking the bus, taking a taxi, walking, biking, or using skateboard.
- Objective: minimize time, maximize happiness, minimize cost, minimize social contacts, minimize carbon footprint
- Constraints: budget limitation, time limitation, traffic consideration, route restriction, GHG emission limitation

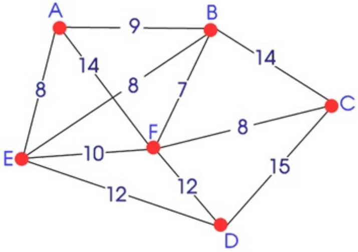
Decision making is an optimization problem.

You have 80 meters of fencing and want to enclose a rectangle yard as large as possible (in area). How should you do it?



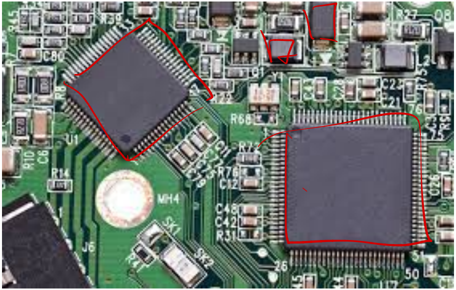
Optimization Example 2: Traveling Salesman Problem

A salesman needs to visit a number of places in a day. How should he schedule his trip so that the total distance is shortest (or the total cost is smallest)?



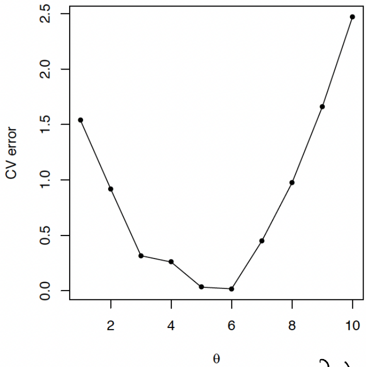
Optimization Example 3: Device Sizing and Locating in Electronic Circuits

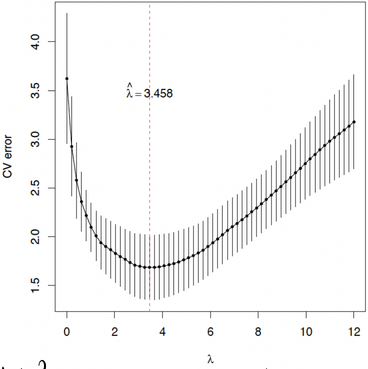
A chip manufacturing company is finding the best device widths, lengths, and placements on an electronic circuit board to minimize power consumption, while adhering to constraints such as manufacturing limitations and timing requirements.



Optimization Example 4: Parameter Tuning

What's the best parameter λ in Lasso regression?

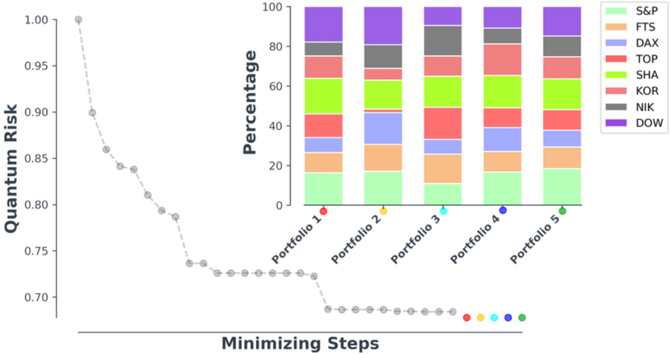




$$\min \|y - \beta X\|_2^2 + \lambda \|\beta\|_1$$

Optimization Example 5: Portfolio Optimization

An investment firm is determining the optimal amounts to allocate across various assets, such as stocks and bonds, to minimize overall risk or return variance, while adhering to constraints including a budget limit, maximum and minimum investment per asset, minimum required return, and regulatory restrictions.



Summary of Optimization Problem

- Components of optimization:
  - Decision
  - Objective
  - Constraints
- Optimization concerns choosing a *decision* (or decisions) to *optimize* certain *objectives* while subject to certain *constraints*
- Optimize could mean *maximize* or *minimize* depending on the problem context.

Why Optimization?

- “Optimization” comes from the same root as “optimal”, which means best. The purpose of optimization is to achieve the “best” to a set of prioritized criteria or constraints.
- When you optimize something, you are “making it best”. When you make a decision, you are optimizing. Every decision-making question is essentially an optimization problem.
- Optimization problems underlie nearly everything we do in real life. A few examples: manufacturing, production, inventory control, transportation, scheduling, network flow, finance, energy system, mechanics, economics, optimal control, marketing, policy making.

Who Uses Optimization?

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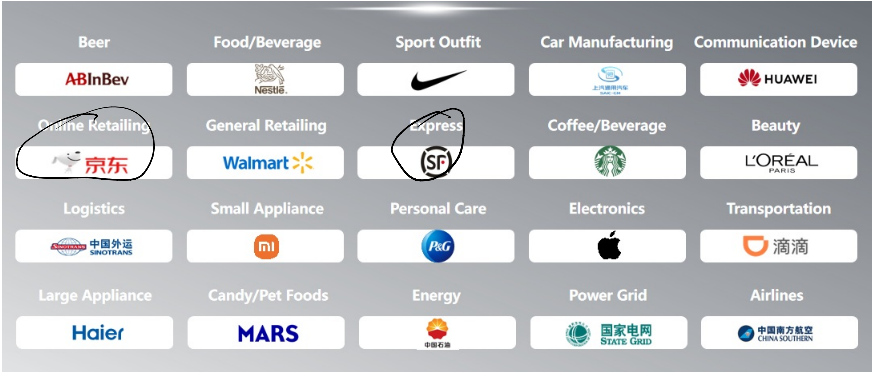


Why Study Optimization?

- Career development: job in data science/machine learning requires basic knowledge in optimization, advanced machine learning scientist (i.e., computer vision) uses a lot of optimization.
- Research development: a lot of (actually almost all) science, engineering, and business/financial research involve different levels of optimization models.
- Until the end, all machine learning models are optimization models. AI research is essentially solving an difficult optimization model.

Program vs. Optimization Problem

- A "program" or "mathematical program" is an optimization problem with a finite number of variables and constraints written out using explicit mathematical (algebraic) expressions.
- The word "program" / "programming" means "plan" / "planning."
- Early applications of optimization arose in planning resource allocations (especially in defense) and gave rise to "programming" to mean optimization (predates computer programming).
- We will use "program" / "programming" and "optimization problem" / "optimization" interchangeably.

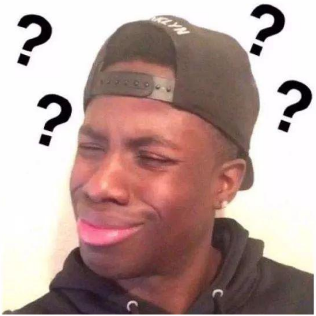
Three well-known optimization solvers

- Xpress: PRESS is worldwide known solver originally developed by Dash Optimization and was acquired by FICO in 2008. It is capable of solving very large optimisation problems especially mixed integer.
- CPLEX: The CPLEX Optimizer was named for the simplex method as implemented in the C programming language at first. Today, it supports other types of mathematical optimization and offers interfaces other than C. CPLEX is actively developed by IBM.
- GuRoBi: Zonghao Gu, Edward Rothberg and Robert Bixby developed GuRoBi in 2008. Bixby was also the founder of CPLEX, while Rothberg and Gu led the CPLEX development team for nearly a decade.

COPT

CVX for Optimization

- CVX is a package for solving convex optimization within MATLAB and Python.
- It provides a user-friendly, high-level interface for formulating optimization problems.
- Easy to prototype and validate small optimization problems.
- Useful for academic research and coursework.
- CVX is mainly used in academic settings; it is rarely used in industry.



Outline

Optimization Generic Formulation

Mathematically, an optimization problem is usually represented as:

Generic Formulation

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in X \end{array}$$

x

- x: Decision (variable) / optimization variable
- $f(\cdot)$: Objective function
- X: Feasible region (constraints)
- $x \in X$: A feasible solution (satisfies all constraints)
- Sometimes, we express the problem using the abstract format: $\min\{f(x) : x \in X\}$.

$$\min_{x \in X} f(x)$$

Mathematical Formulation

An optimization problem can be represented in the following way:

Mathematical Formulation

$\{x : h_i(x) = 0, \forall i,\ g_j(x) \le 0,\ \forall j\}$

$$
\begin{array}{lll}
\text{minimize} & f(x) & \\
\text{subject to} & h_i(x) = 0, & i = 1, 2, ..., N_h \\
 & g_j(x) \le 0, & j = 1, 2, ..., N_g
\end{array}
$$

- Optimization variables: $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$
- Objective function: $f\colon \mathbb{R}^n \to \mathbb{R}$
- Equality constraints functions: $h_i\colon \mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, ..., N_h$
- Inequality constraints functions: $g_j\colon \mathbb{R}^n \to \mathbb{R}$, $j = 1, 2, ..., N_g$
- Feasible solution: a decision that satisfies all constraints
- Feasible region (set): the set of feasible solutions

Optimization Problem

Mathematical Formulation

$f(x^*) \le f(x), \ \forall x \in \mathcal{X}$

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & \underbrace{\begin{aligned} & h_i(x) = 0, \quad i = 1, 2, ..., N_h \\ & g_j(x) \le 0, \quad j = 1, 2, ..., N_g \end{aligned}}_{\mathcal{X}} \end{aligned}$$

Our goal is to find the optimal solution x^* such that the objective function $f(x^*)$ is the smallest among all x vectors that satisfy the constraints. We call $f(x^*)$ the optimal objective function value (optimal value).

Remark

- The problem may be infeasible: you cannot find any vectors that satisfy all constraints.
- The optimal solution can be more than one.
- We don't allow strict inequality constraints. '<' '>' ✗

Minimization vs Maximization

Without loss of generality, it is sufficient to consider a minimization objective since:

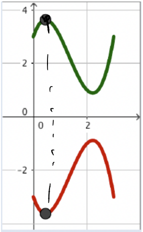
$$\max_x\{f(x) : x \in X\} \equiv -\min_x\{-f(x) : x \in X\}$$

$$\min_x\{f(x) : x \in X\} = -\max_x\{-f(x) : x \in X\}$$

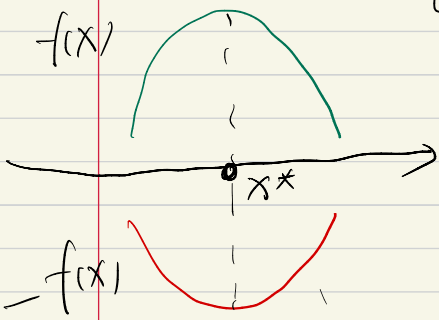
Example:

$$\max\{4 - x^2 + (x-1)^3 : 0 \le x \le 3\}$$

$$\equiv -\min\{-4 + x^2 - (x-1)^3 : 0 \le x \le 3\}$$



Thus, to develop the theory we will only consider minimization problems. When solving problems, we can use the actual min or max objective as needed.



$$\arg\max f(x) = \arg\min. -f(x)$$

$$\arg\min. f(x) = \arg\max. -f(x)$$

Definition of `argmin` and `argmax`

- The **argmin** and **argmax** are operations used to find the optimal solutions where the problem reaches its minimum or maximum.
- **Definition of `argmin`:**

$$\arg\min_{x\in X} f(x) = \{x^* \in X \mid f(x^*) \leq f(x) \;\; \forall x \in X\}$$

- **Definition of `argmax`:**

$$\arg\max_{x\in X} f(x) = \{x^* \in X \mid f(x^*) \geq f(x) \;\; \forall x \in X\}$$

- **Example:**
  - For $f(x) = (x-2)^2$ over $X = \mathbb{R}$:

$$\arg\min_{x\in\mathbb{R}} f(x) = \{2\}.$$

In-class Exercise

Compare the following optimization problems

1. $\max\ f(x)$
2. $\min\ f(x)$
3. $\max\ -f(x)$
4. $\min\ -f(x)$
5. $-\max\ f(x)$
6. $-\min\ f(x)$
7. $-\max\ -f(x)$
8. $-\min\ -f(x)$

- How are the optimal objective values related among these problems?
- How are the optimal solutions related among these problems?

$Y_1 = \min. f(x)$

$Y_2 = \min. -f(x)$

$Y_3 = \max. f(x)$

$Y_4 = \max. -f(x)$

$Y_5 = -\min. f(x)$

$Y_6 = -\min. -f(x)$

$Y_7 = -\max. f(x)$

$Y_8 = -\max. -f(x)$

$X_1 = \arg\min. f(x)$

$X_2 = \arg\min. -f(x)$

$X_3 = \arg\max. f(x)$

$X_4 = \arg\max. -f(x)$

$X_5 = -\arg\min. f(x)$

$X_6 = -\arg\min. -f(x)$

$X_7 = -\arg\max. f(x)$

$X_8 = -\arg\max. -f(x)$

$Y_1 = -Y_5 = Y_8 = -Y_4$ $\qquad X_1 = X_4 = -X_5 = -X_8$

$Y_2 = -Y_6 = -Y_3 = Y_7$ $\qquad X_2 = X_3 = -X_6 = -X_7$

Outline

Classifications

$x \in \mathbb{R}^n$

$$\begin{aligned} \text{minimize}_x \quad & f(x) \\ \text{subject to} \quad & g_i(x) \le 0, \quad \forall i = 1, \dots, s \\ & h_j(x) = 0, \quad \forall j = 1, \dots, t \end{aligned}$$

- Unconstrained optimization: If $s = t = 0$. Otherwise constrained optimization.
- Linear optimization (LP): Constraints and objective function are linear in the decision variables.
- Nonlinear optimization (NLP): Either some of the constraints or the objective function is nonlinear.
- Convex optimization: if $f(x)$ and all $g_i(x)$ are convex, and $h_j(x)$ are linear (i.e., the feasible set is convex).
- Integer/Discrete optimization (IP): Some of the decision variables have to be integers or discrete.
- Other classifications: continuous, quadratic, mixed integer, binary, etc

Classifications

- Constrained vs Unconstrained
- Linear vs Nonlinear
- Continuous vs Discrete

$x \in \mathbb{R}^n$ $\quad$ $x \in \mathbb{Z}$

By default, when we talk about an optimization problem, we assume it is continuous, unless we explicitly say that it is *discrete*

LP is easy.

Classifications of Optimization

min. D

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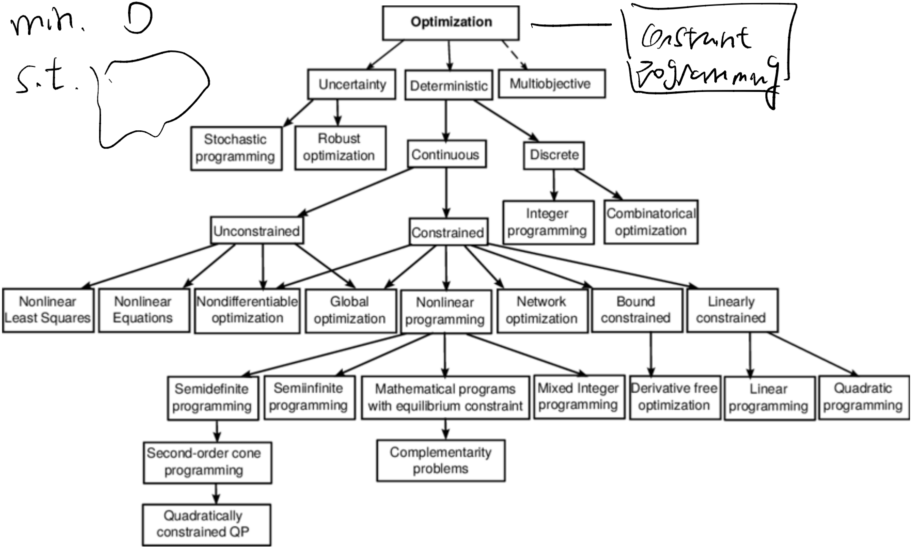

Constraint programming

Remarks

- Sometimes, an NLP can be equivalently transformed to an LP.
- Sometimes, an IP can be equivalently transformed to a continuous optimization problem.

Linear optimization is the most well-studied and the easiest optimization problem.

- Nonlinear optimization and integer optimization could be significantly harder than LP.
- Therefore, in many cases, people strive to find LP formulations for problems.

In the first half of the semester, we will focus on linear optimization, then we will discuss integer and nonlinear (convex) optimization in the second half of the semester.

Outline

Feasible Solutions and Infeasible problem

Mathematical formulation

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \mathcal{X} \end{aligned}$$

- Any $x \in \mathcal{X}$ is a feasible solution of the optimization problem.
- Feasible solution $=$ A solution that satisfies all the constraints.
- If $\mathcal{X} = \phi$; then no feasible solutions exist, and the problem is said to be infeasible.
- The problem $\min\{3x + 2y : x + y \leq 1, x \geq 2, y \geq 2\}$ is infeasible.

Unbounded Problem

Mathematical formulation

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \mathcal{X} \end{aligned}$$

- The optimization problem is unbounded, if there are feasible solutions with arbitrarily small objective values (for minimization problem).
- Formally, the problem is unbounded if there exists a sequence of feasible solutions $\{x^i\} \in \mathcal{X}$ such that $\lim_{i\to\infty} f(x^i) = -\infty$.
- An unbounded problem must be feasible.
- The problem $\min\{x : x \leq 1\}$ is unbounded.

Optimal Solution Exists

Mathematical formulation

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \mathcal{X} \end{aligned}$$

- A feasible solution x^* is an **optimal solution** of the optimization problem if

$$f(x^*) \leq f(x) \quad \forall x \in \mathcal{X}$$

- The objective value corresponding to an optimal solution (if it exists) is called the **optimal (objective function) value** of the optimization problem.
- The problem $\min\{x : x \geq 1\}$ has one unique solution. The problem $\min\{x : x \geq 1, y \leq 2\}$ has infinite number of optimal solutions.

Remark:

An optimization problem can have

<u>0 - ∞</u> optimal solutions

and <u>0-1</u> optimal obj fun values.

Optimal Solution Cannot be Achieved/Attained

- $\min\{e^x : x \in \mathbb{R}\}$
- $\min\{\frac{1}{x} : x \geq 0\}$

Four Outcomes of Optimization Problem

Mathematical Formulation

$$\begin{array}{rl} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathcal{X} \end{array}$$

1. Infeasible: $\mathcal{X} = \phi$
2. Unbounded: $\exists \{x^i\} \in \mathcal{X}$, s.t. $f(x^i) \to -\infty$
3. Feasible and bounded but the minimizer is not achieved (attained)
4. An optimal solution x^* exists

Existence of Optimal Solutions: Weierstrass Theorem

Definition

A function f is **continuous** if for all convergent sequences $\{x^i\} \subseteq dom(f) : \lim_{i\to\infty} x^i = x^0$ such that $\lim_{i\to\infty} f(x^i) = f(x^0)$.
A set $\mathcal{X}$ is **closed** if for all convergent sequences $\{x^i\} \subseteq \mathcal{X}$ such that $\lim_{i\to\infty} x^i = x^0 \in \mathcal{X}$.
A set $\mathcal{X}$ is **bounded** if $\exists M > 0, ||x|| \leq M, \forall x \in \mathcal{X}$.

Weierstrass Theorem

For an optimization problem, if the objective function $f : \mathbb{R}^n \to \mathbb{R}$ is continuous, and the feasible region $\mathcal{X} \in \mathbb{R}^n$ is nonempty, closed, bounded, then the problem has an optimal solution.

True or False Exercise

- The problem: $\max\{f(x) : x \in [a, b]\}$ has an optimal solution.
- Any optimization problem whose feasible region is unbounded cannot have an optimal solution.
- For an optimization problem, if the objective function $f : \mathbb{R}^n \to \mathbb{R}$ is not continuous, and the feasible region $\mathcal{X} \in \mathbb{R}^n$ is nonempty, open, unbounded, then the problem has no optimal solution.

Weierstrass Theorem: Sufficient But Not Necessary