PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star $* * *$ labels are assigned to the most difficult ones.

Due date: February 25 th, 23: 59: 00, 2024.

1 Homework Problems for Week 3 Chapter 7-8

1. $*$ (Halliday,C7-P8)
   A ice block floating in a river is pushed through a displacement
   $$\vec{d} = (20\,\text{m})\hat{\imath} - (16\,\text{m})\hat{\jmath}$$
   along a straight embankment by rushing water, which exerts a force
   $$\vec{F} = (210\,\text{N})\hat{\imath} - (150\,\text{N})\hat{\jmath}$$
   on the block. How much work does the force do on the block during the displacement?
   Answers: 6.6×10^3 J.

2. $*$ (Halliday,C7-P41)
   Only one force is acting on a 2.8 kg particle-like object whose position is given by
   $$x = (4.0\,\text{m/s})t - (5.0\,\text{m/s}^2)t^2 + (2.0\,\text{m/s}^3)t^3$$
   with x in meters and t in seconds. What is the work done by the force from $t = 0$s to $t = 6.0$ s?
   Answers: 3.6×10^4 J.

3. $*$ A pendulum consists of a bob of mass m attached to a string of length L. The bob is pulled aside so that the string makes an angle θ_0 with the vertical, and is released from rest. As it passes through the lowest point of the arc, find expressions for (a) the speed of the bob, and (b) the tension in the string. Effects due to air resistance are negligible.
   Answers: (a): $v_{\text{bottom}} = \sqrt{2gL(1-\cos\theta_0)}$.
   (b): $T = (3 - 2\cos\theta_0)mg$.

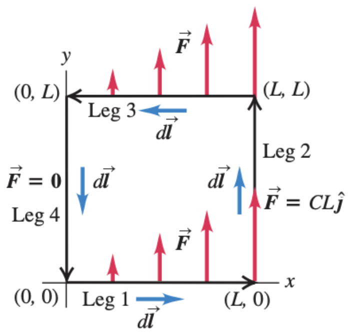
4. $*$ A 1500-kg roller coaster car starts from rest at a height $H = 23.0$ m above the bottom of a 15.0-m-diameter loop. If friction is negligible, determine the downward force of the rails on the car when the upside-down car is at the top of the loop.
   Answers: 1.67×10^4 N.

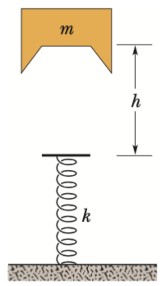
5. $**$ **Conservative or nonconservative?** In a region of space the force on an electron is $\vec{F} = Cx\hat{\jmath}$, where C is a positive constant. The electron moves around a square loop in the xy-plane (see the figure below).

   (a) Calculate the work done on the electron by the force $\vec{F}$ during a counterclockwise trip around the square. **Answers:** CL^2.

   (b) Is this force conservative or nonconservative?

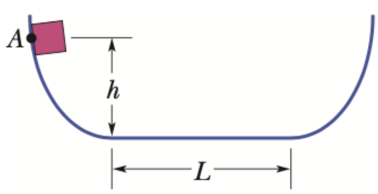
   (c) (Optional bonus part for math and physics enthusiasts!) Compute the curl of the force $\vec{\nabla} \times \vec{F}$. **Comment:** N.B., the curl of a conservative force is zero. **Answers:** $\vec{\nabla} \times \vec{F} = C\hat{k}$.





6. $**$ (Halliday,C8-P24)
   A block of mass $m = 2.0$ kg is dropped from height $h = 0.50$m onto a spring of spring constant $k = 1960$ N/m as shown above. Find the maximum distance the spring is compressed. **Answers:** 0.11 m.

7. $**$ (Halliday,C8-P65)
   A particle can slide along a track with elevated ends and a flat central part, as shown below. The flat part has length $L = 0.40$m. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu_k = 0.20$. The particle is released from rest at point A, which is at height $h = L/2$. How far from the left edge of the flat part does the particle finally stop?



   Answers: 0.20 m.

8. $**$ **Force and the Potential-Energy Function**
   In the region $-a < x < a$ the force on a particle is

represented by the potential-energy function

$$U = -b\left(\frac{1}{a+x} + \frac{1}{a-x}\right),$$

where a and b are positive constants.

(a) Find the force $F_x = -\frac{dU}{dx}$ in $-a < x < a$.

(b) At what value of x is the force zero?
Answers: $x = 0$.

(c) At the location where the force equals zero, is the equilibrium stable or unstable?
Hint: The equilibrium corresponds to the extremum of potential energy,

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} > 0 \quad \Rightarrow \quad \text{minimum},$$
$$\left.\frac{d^2U}{dx^2}\right|_{x=0} < 0 \quad \Rightarrow \quad \text{maximum}.$$

9. ∗∗ **A mathematical derivation of the Circular Motion**: A particle moves in a circle that is centered at the origin and the magnitude of its position vector $\vec{r}$ is constant.

(a) Differentiate $\vec{r}\cdot\vec{r} = r^2 =$ constant with respect to time t to show that $\vec{v}\cdot\vec{r} = 0$, therefore $\vec{v} \perp \vec{r}$.
Hint: The differentiation of scalar products also satisfies the Leibniz product rule

$$\frac{d}{dt}r^2 = 2\frac{d\vec{r}}{dt}\cdot\vec{r} = 2\vec{v}\cdot\vec{r} = 0.$$

(b) Differentiate $\vec{v}\cdot\vec{r} = 0$ with respect to time t and show that $\vec{a}\cdot\vec{r} + v^2 = 0$, and therefore the radial acceleration $a_r = -v^2/r$. (This indicates that the radial acceleration's magnitude is v^2/r, and the minus sign means that it is in the opposite direction of $\vec{r}$.)

(c) Differentiate $\vec{v}\cdot\vec{v} = v^2$ with respect to time t and show that

$$\vec{a}\cdot\vec{v} = v\frac{dv}{dt},$$

and thus the tangential acceleration $a_t = dv/dt$.

Comment: Isn't it really nice? We just derived everything about the circular motion.

10. ∗∗∗ **A thrown baseball**
Imagine that you have a baseball in you hand, you throw it straight upwards with an initial velocity $\vec{v}_0$.

(a) Suppose you can neglect the air resistance, find the maximum height $y_{\max}$ that the baseball can reach. **Answers:** $y_{\max} = v_0^2/(2g)$.

(b) Suppose you can neglect the air resistance, what is the velocity of the baseball when it falls back to your hand? Explain this in terms of energy conservation.
Answers: $v_f = v_0$ with direction downward.

(c) Now suppose we can no longer neglect the air resistance, the magnitude of the air resistance is $f = kv^2$ with k a positive constant and v the instantaneous velocity of the baseball. The direction of the air resistance is always in the opposite direction of the velocity of the baseball. Find the maximum height that this baseball can reach.
Answers: Due to the air resistance,

$$y_{\max} = \frac{m}{2k}\ln\left(1 + \frac{kv_0^2}{mg}\right).$$

Consistency check: In the limit of $k \to 0$, one can find

$$\lim_{k\to 0}\frac{m}{2k}\ln\left(1 + \frac{kv_0^2}{mg}\right) = \frac{v_0^2}{2g}$$

Hint: According to Newton's second law, you should be able to obtain the following equation of motion (EOM)

$$m\frac{dv}{dt} = -mg - kv^2 \quad \Rightarrow \quad \frac{dv}{dt} = -g - \frac{k}{m}v^2.$$

In general, for the EOM of the form $\frac{dv}{dt} = f(v)$ with $f(v)$ the function of v, you can multiply $dy = vdt$ on both side of the equation and obtain $dy = \frac{vdv}{f(v)}$. This trick converts dt to dy, then it allows us to solve $y(v)$ directly. Now you can proceed to integrations from here.

$$-\int_0^{y_{\max}} dy = \int_{v_0}^{0} \frac{vdv}{g + \frac{k}{m}v^2} = \frac{m}{2k}\int_{g+\frac{k}{m}v_0^2}^{g} \frac{du}{u}.$$

Of course, you do not have to use the above trick. Other methods involve longer derivations.

(d) With the same air resistance in part (c), remember that the air resistance reverses its direction when the baseball moves downwards, find the velocity of the baseball $\vec{v}_f$ when it falls back to your hand.
Answers: $v_f = v_0/\sqrt{1 + kv_0^2/mg}$.
Consistency check: consider the limit $kv_0^2/mg \gg 1$, do you get the terminal speed?

(e) Is v_f smaller than v_0? Compute the final kinetic energy and qualitatively explain the difference with the initial kinetic energy. (Where does the energy difference go?)

11. ∗∗∗ **Rope wrapped around a pole:** A rope wraps an angle θ around a pole. (Note θ equals $2\pi N$ if one wraps N full revolutions around the pole.) You grab one end and pull with a tension T_0. The other end is attached to a large object, say, a boat. If the coefficient of static friction between the rope and the pole is μ, what is the largest force the rope can exert on the boat, if the rope is not to slip around the pole?
Answers: $T_{\max} = T_0 e^{\mu\theta}$. The exponential behavior here is quite strong after wrapping several revolutions around the pole. This result is why one can usually use strong ropes to tie a boat to a dock as long as the pole can hold.