香港中文大學(深圳)理工学院
School of Science and Engineering

PHY 1001: Mechanics

Tutorial Session 4

T-05: Feb - 28 - 2024, Wednesday, 19:00~19:50

T-12: Feb - 28 - 2024, Wednesday, 20:00~20:50

Gao Qian

Problem 1

* **Force of a Golf Swing**. A $\mathbf{0.0450 - kg}$ golf ball initially at rest is given a speed of $\mathbf{25.0\ m/s}$ when a club strikes. If the club and ball are in contact for $\mathbf{2.00\ ms = 2.00 \times 10^{-3}\ s}$, what average force acts on the ball? Is the effect of the ball's weight during the time of contact significant? Why or why not?

Solution:

According to the ***impulse-momentum change theorem***, the average force on an object and the object's change in momentum are related by

$$\boldsymbol{F_{av}\Delta t = mv_f - mv_i = m\Delta v} \tag{1}$$

where $\boldsymbol{v_i = 0}$ and $\boldsymbol{v_f = 25\ m/s}$ for the golf ball. Thus

$$\boldsymbol{F_{av} = \frac{m\Delta v}{\Delta t} = \frac{0.0450 * 25.0}{2.00 \times 10^{-3}}\,\mathrm{N} = 563\ \mathrm{N}} \tag{2}$$

The force exerted by the club is much greater than the weight of ball ($w = mg = 0.441$ N), so the effect of the weight of ball during the time of contact is not significant.

QED.

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Problem 2

* **Hockey Puck and Impulse**. A $\mathbf{0.160-kg}$ hockey puck is moving on an icy, frictionless, horizontal surface. At $\boldsymbol{t}=\mathbf{0}$, the puck is moving to the right at $\mathbf{3.00\ m/s}$.

Questions:

(a) Calculate the velocity of the puck (magnitude and direction) after a force of $\mathbf{25.0\ N}$ directed to the right has been applied for $\mathbf{0.050\ s}$.

(b) If, instead, a force of $\mathbf{12.0\ N}$ directed to the left is applied from $\boldsymbol{t}=\mathbf{0}$ to $\boldsymbol{t}=\mathbf{0.050\ s}$, what is the final velocity of the puck?

Problem 2

Question (a):

(a) Calculate the velocity of the puck (magnitude and direction) after a force of $\mathbf{25.0\ N}$ directed to the right has been applied for $\mathbf{0.050\ s}$.

Answer (a):

(1) Based on the ***impulse-momentum change theorem*** $\boldsymbol{J_x = F_x \Delta t = \Delta p_x}$, we know the initial momentum and the impulse so can solve for the final momentum and then the final velocity.

(2) Take the $\boldsymbol{x}-\mathbf{axis}$ to be toward the right, so $\boldsymbol{v_{1x} = +3.00\ \mathrm{m/s}}$. Then compute the impulse

$$\boldsymbol{J_x = F_x \Delta t = (+25\ \mathrm{N})(0.050\ \mathrm{s}) = 1.25\ \mathrm{N \cdot s}} \qquad (1)$$

(3) Final momentum is then $\boldsymbol{p_{2x} = p_{1x} + J_x = 1.73\ \mathrm{N \cdot s}}$ and the final velocity is

$$\boldsymbol{v_{2x} = \frac{p_{2x}}{m} = 10.8\ \mathrm{m/s}} \quad \text{(to the right)} \qquad (2)$$

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Problem 2

Question (b):

(b) If, instead, a force of $\mathbf{12.0\ N}$ directed to the left is applied from $\boldsymbol{t} = \mathbf{0}$ to $\boldsymbol{t} = \mathbf{0.050\ s}$, what is the final velocity of the puck?

Answer (b):

(1) In this case, $\boldsymbol{J_x} = \boldsymbol{F_x}\Delta\boldsymbol{t} = (-\mathbf{12.0\ N})(\mathbf{0.050\ s}) = -\mathbf{0.600\ kg \cdot m/s}$ (negative since force is to left).

(2) Then $\boldsymbol{p_{2x}} = \boldsymbol{J_x} + \boldsymbol{p_{1x}} = (-\mathbf{0.600} + \mathbf{0.480})\mathbf{kg \cdot m/s} = -\mathbf{0.120\ kg \cdot m/s}$. the final velocity is

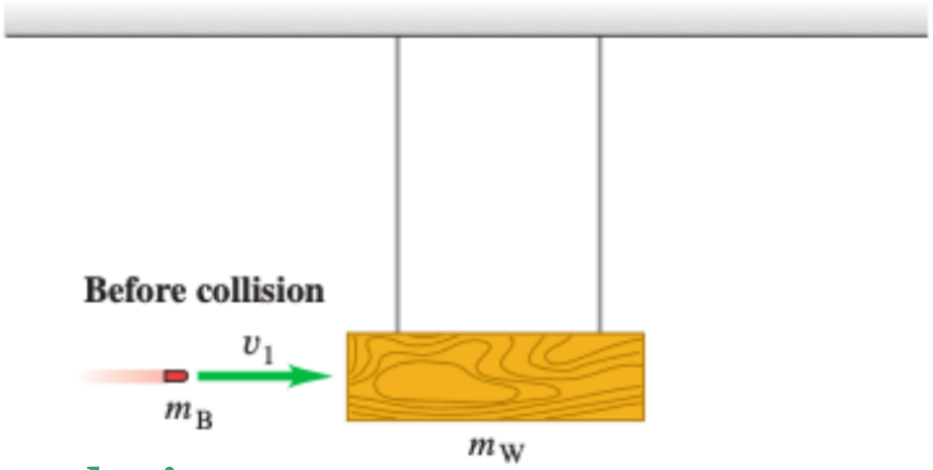
$$\boldsymbol{v_{2x}} = \frac{\boldsymbol{p_{2x}}}{\boldsymbol{m}} = -\mathbf{0.750\ m/s} \quad \text{(to the left)} \qquad (3)$$

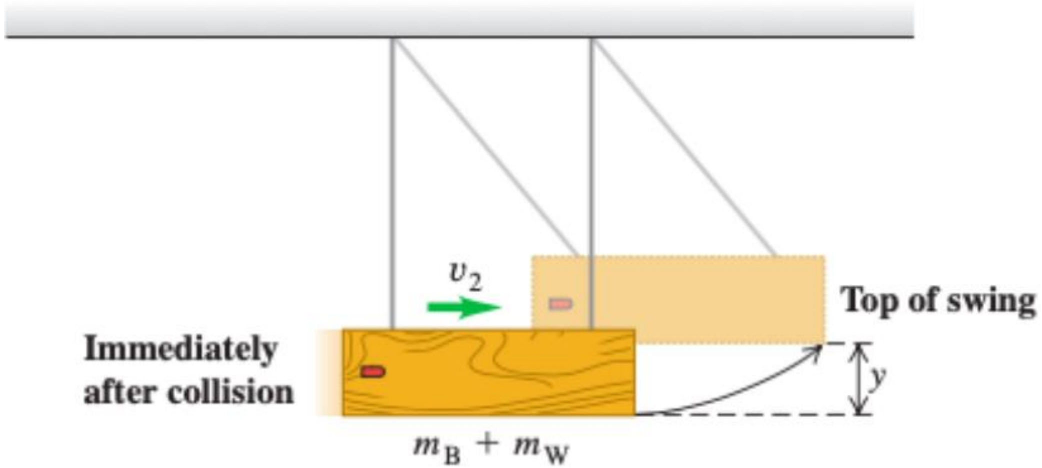
(3) In part (a) the impulse and initial momentum are in the same direction and $\boldsymbol{v_x}$ increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

QED.

Problem 3

* The figure below shows a ballistic pendulum, a simple system for measuring the speed of a bullet. A bullet of mass $\boldsymbol{m_B}$ makes a completely inelastic collision with a block of wood of mass $\boldsymbol{m_W}$, which is suspended like a pendulum. After the impact, the block swings up to a maximum height $\boldsymbol{y}$. In terms of $\boldsymbol{y}$, $\boldsymbol{m_B}$, and $\boldsymbol{m_W}$, what is the initial speed $\boldsymbol{v_1}$ of the bullet?





Analysis:

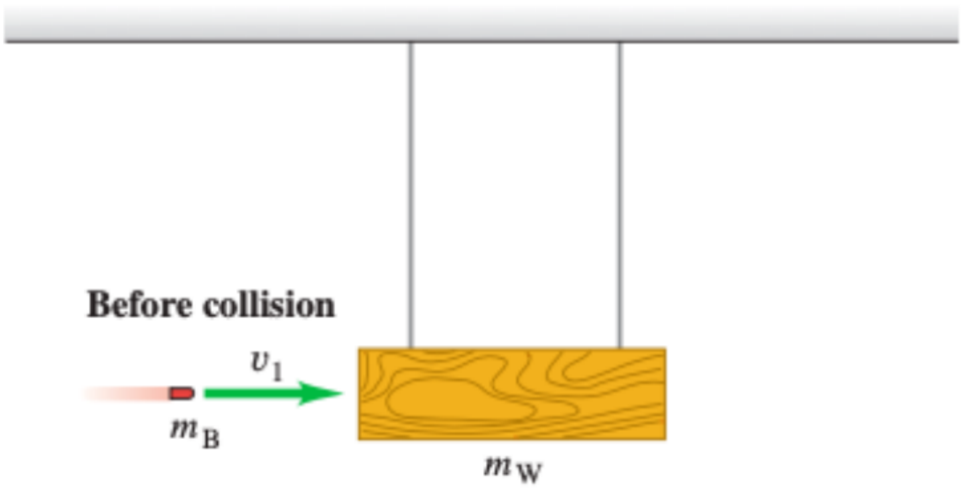
We analyze this event in two stages: (1) the embedding of the bullet in the block; *and* (2) the pendulum swing of the block.

Problem 3

Analysis:

(First stage-the embedding of the bullet in the block)

- During the first stage, the bullet embeds itself in the block so quickly that the block does not move appreciably.
- The supporting strings remain nearly vertical, so negligible external horizontal force acts on the bullet–block system, and the horizontal component of momentum is conserved.
- The collision between bullet–block is inelastic, therefore the mechanical energy is not conserved.

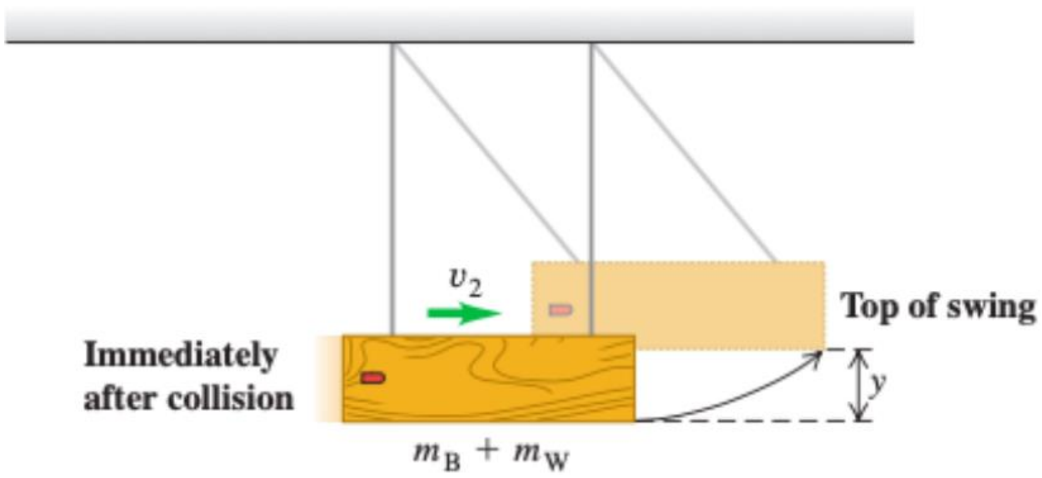


Problem 3

Analysis:

(Second stage-the pendulum swing of the block)

- In the second stage, the block and bullet move together.
- The only forces acting on this system are gravity (a conservative force) and the string tensions (which do no work).
- Thus, as the block swings, mechanical energy is conserved.
- Momentum is not conserved in this process due to the external forces.



Problem 3

Solution:

(1) In the first stage, all velocities are in the $+x$ −direction. Momentum conservation gives

$$m_B v_1 = (m_B + m_W) v_2 \quad (1)$$

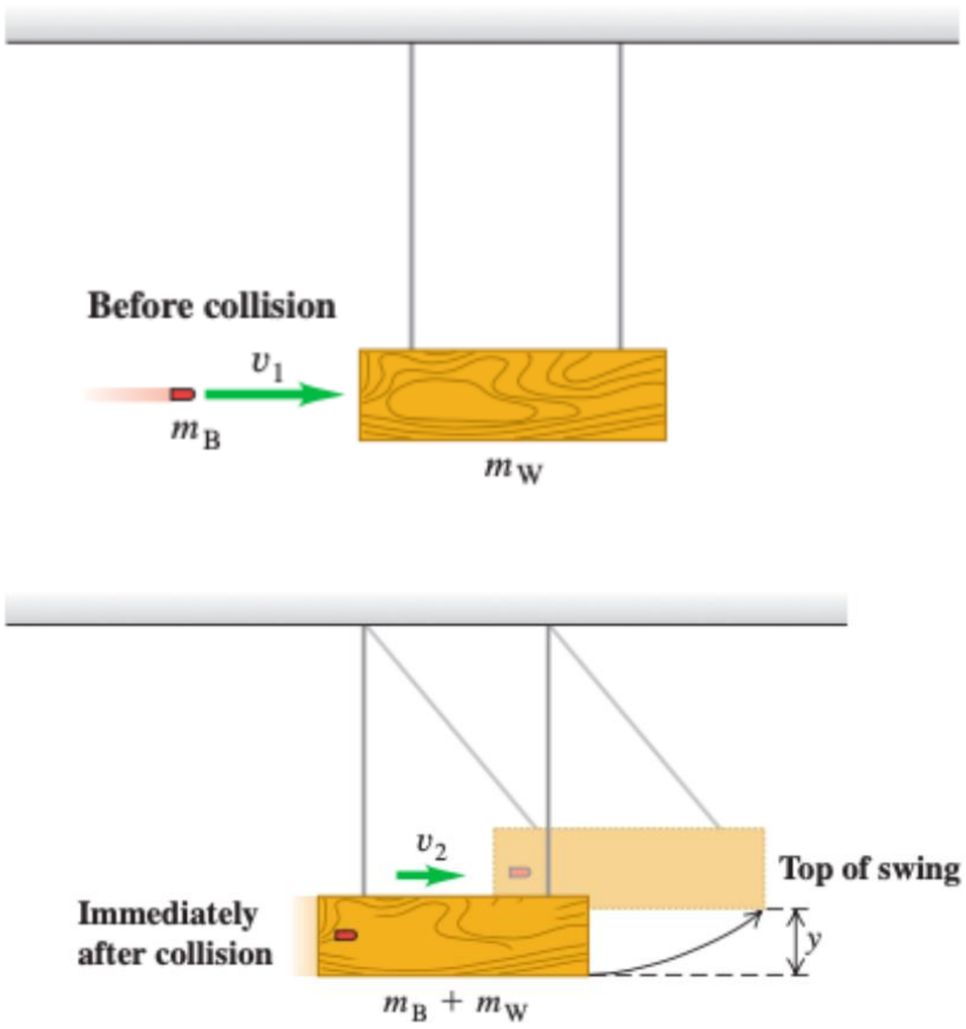
$$v_1 = \frac{(m_B + m_W)}{m_B} v_2 \quad (2)$$

(2) At the beginning of second stage, the bullet–block system has kinetic energy, which is converted into the potential energy at the end of the second stage. Thus,

$$\frac{m_B + m_W}{2} v_2^2 = (m_B + m_W) g y \rightarrow v_2 = \sqrt{2gy} \quad (3)$$

(3) By substituting this expression for v_2 into the equation above, one gets the **Answer**:

$$v_1 = \frac{(m_B + m_W)}{m_B} \sqrt{2gy} \quad (4)$$



QED.

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Problem 4

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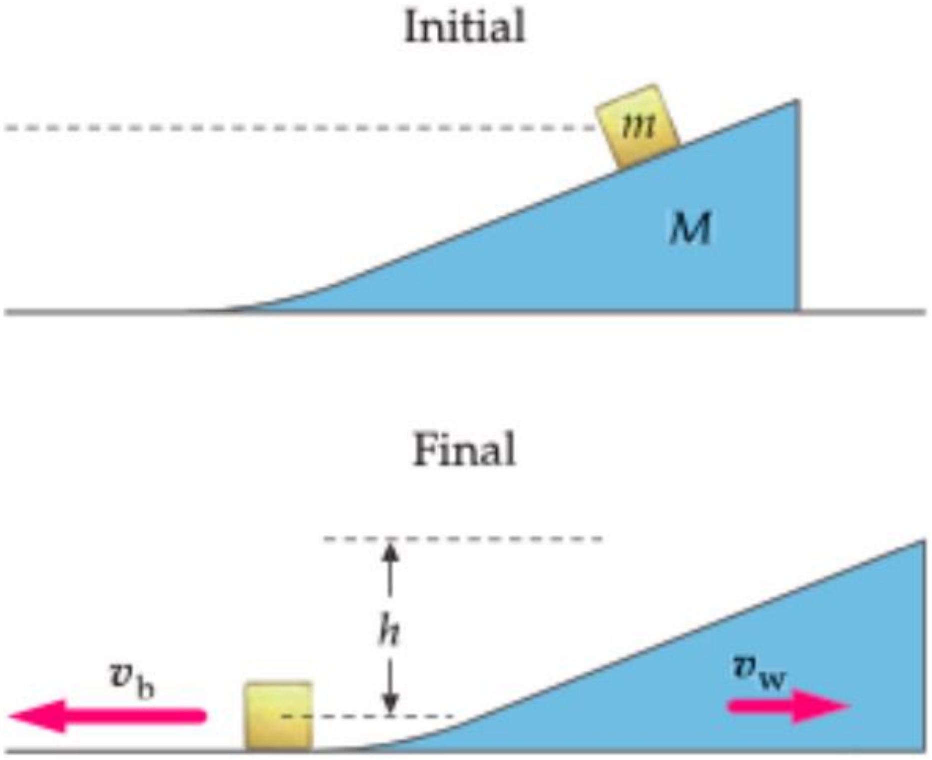
* A wedge of mass $\boldsymbol{M}$ is placed on a frictionless, horizontal surface, and a block of mass $\boldsymbol{m}$ is placed on the wedge, which also has a frictionless surface (see figure below). The block's center of mass moves downward a distance $\boldsymbol{h}$ as the block slides from its initial position to the horizontal floor.

Questions:

(a) What are the speeds of the block and of the wedge as they separate from each other and go their own ways?

(b) Check your calculation plausibility by considering the limiting case when $\boldsymbol{M} \gg \boldsymbol{m}$.



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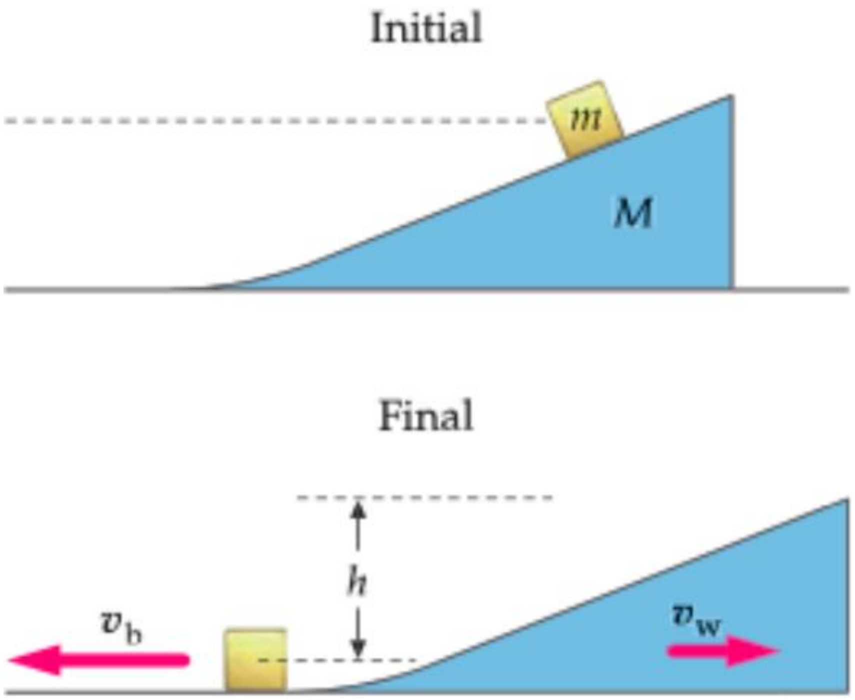
Problem 4

Question (a):

What are the speeds of the block and of the wedge as they separate from each other and go their own ways?

Solution (a):



(1) From the conservation of momentum

$$Mv_w - mv_b = 0 \tag{1}$$

and conservation of energy

$$mgh = \frac{1}{2}mv_b^2 + \frac{1}{2}Mv_w^2 \tag{2}$$

(2) Then solve for v_b and v_w

$$v_b = \sqrt{2gh\frac{M}{m+M}}, \quad v_w = \sqrt{2gh\frac{m^2}{M(m+M)}} \tag{3}$$

Problem 4

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Question (b):

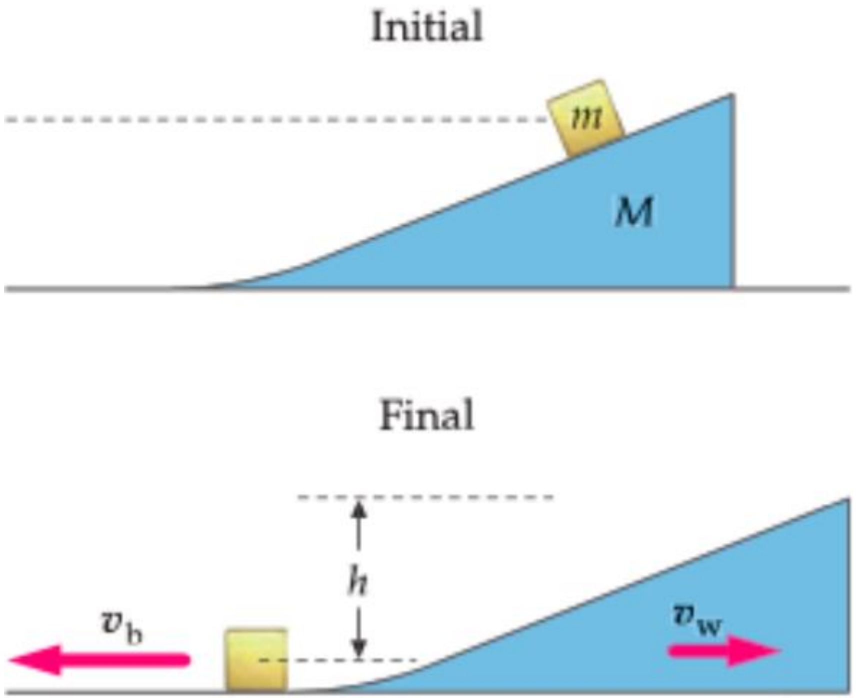
Check your calculation plausibility by considering the limiting case when $\boldsymbol{M} \gg \boldsymbol{m}$.

Solution (b):

In the limit of $\boldsymbol{M} \gg \boldsymbol{m}$, one finds $\boldsymbol{v_b} = \sqrt{\boldsymbol{2gh}}$ and $\boldsymbol{v_w} = \boldsymbol{0}$ as expected.

$$\boldsymbol{mgh} = \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{m}\boldsymbol{v_b}^{\boldsymbol{2}}$$

QED.



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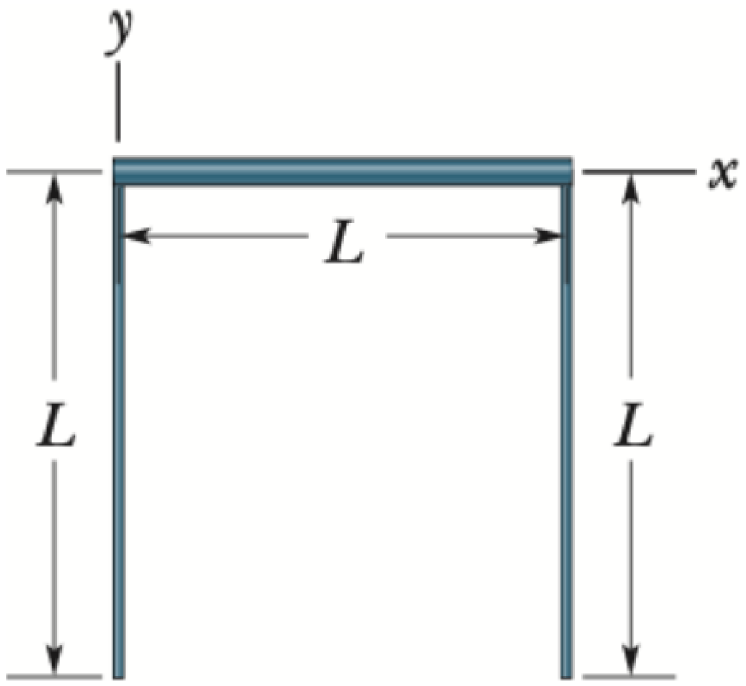
Problem 5

* * (*Halliday, C9_P4*) In the figure below, three uniform thin rods, each of length $\boldsymbol{L} = \mathbf{24\ cm}$, form an inverted **U**. The vertical rods each have a mass of $\boldsymbol{M} = \mathbf{14\ g}$; the horizontal rod has a mass of $\mathbf{3}\boldsymbol{M} = \mathbf{42\ g}$.

Questions:

(a) What is the x coordinate of the system's center of mass?

(b) What is the y coordinate of the system's center of mass?



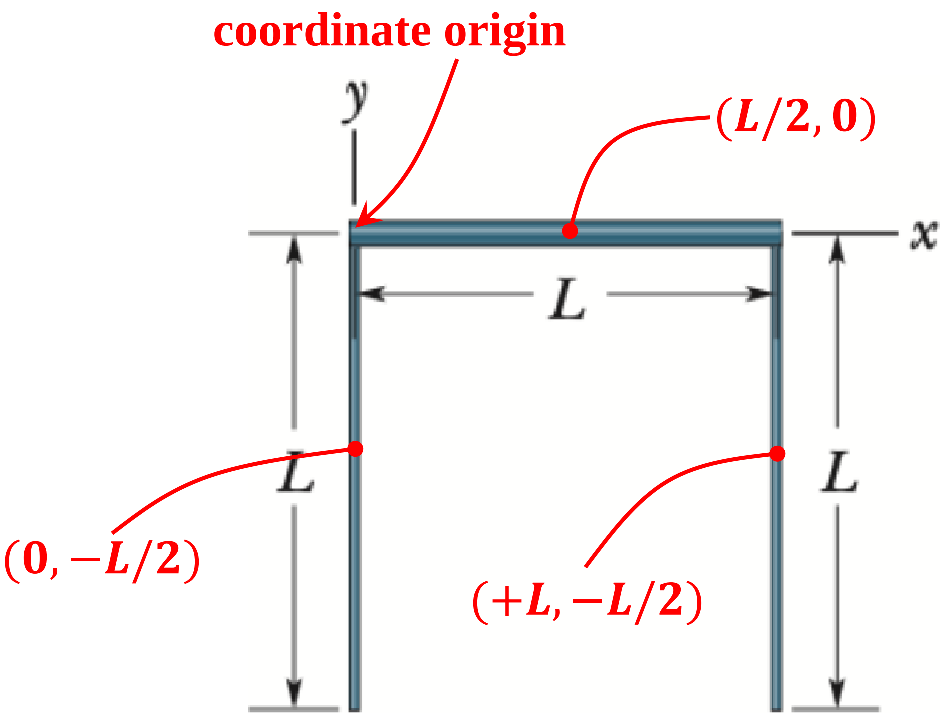
Problem 5

Analysis:

- We will refer to the arrangement as a ‘table’.

- We locate the coordinate origin at the left end of the table-top (as shown in the left figure).

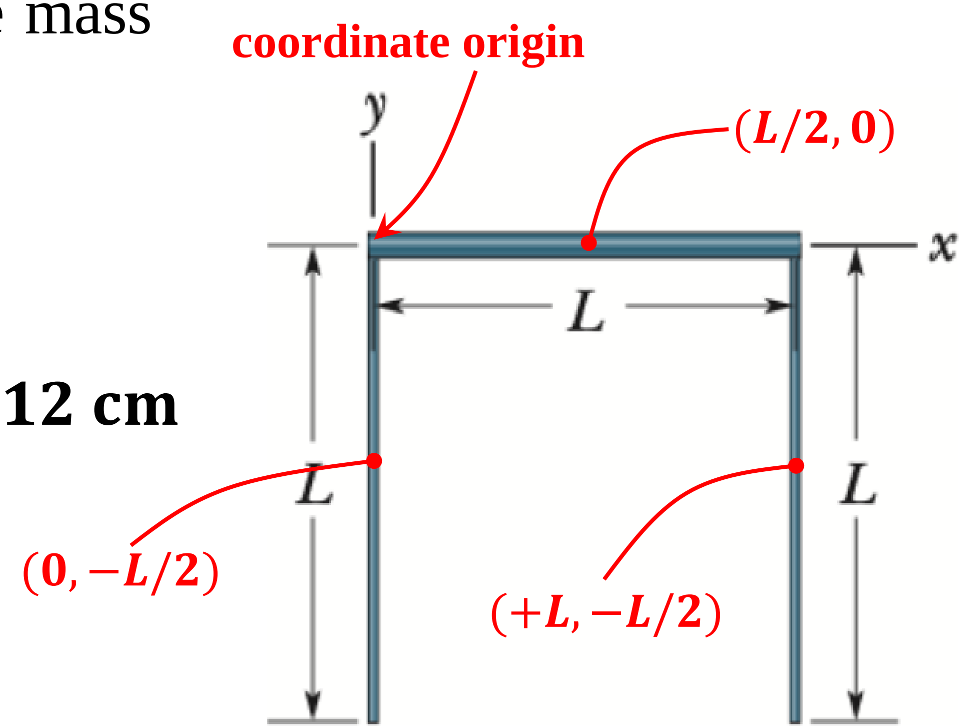
- With $+x$ rightward and $+y$ upward, then
  - the center of mass of the right leg is at $(x, y) = (+L, -L/2)$,
  - the center of mass of the left leg is at $(x, y) = (0, -L/2)$,
  - and the center of mass of the table-top is at $(x, y) = (L/2, 0)$.



Problem 5

Solution (a):

(1) Let $\boldsymbol{M} = \mathbf{14\ g}$ be the mass of the vertical rod, and the mass of the horizontal rod is then $\mathbf{3}\boldsymbol{M} = \mathbf{42\ g}$.



(2) The $\boldsymbol{x}$ coordinate of the (whole table) center of mass is then

$$x_{com} = \frac{M \times (0) + M \times (+L) + 3M \times (+L/2)}{M + M + 3M} = \frac{L}{2} = 12\ \text{cm}$$

Solution (b):

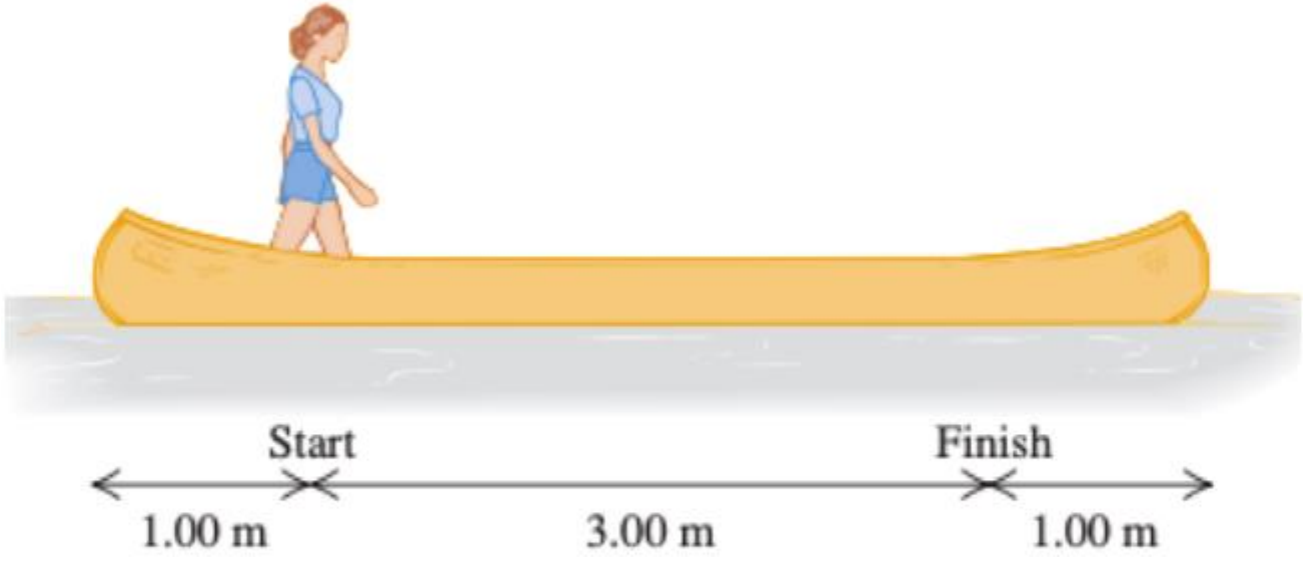
The $\boldsymbol{y}$ coordinate of the (whole table) center of mass is

$$y_{com} = \frac{M \times (-L/2) + M \times (-L/2) + 3M \times (0)}{M + M + 3M} = -\frac{L}{5} = -4.8\ \text{cm}$$

Comment: From the coordinates, we see that the whole table center of mass is a small distance 4.8 cm directly below the middle of the tabletop.

Problem 6

* **Center of Mass**. A **45.0-kg** woman stands up in a **60.0-kg** canoe **5.00 m** long. She walks from a point **1.00 m** from one end to a point **1.00 m** from the other end (Fig. below). If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?



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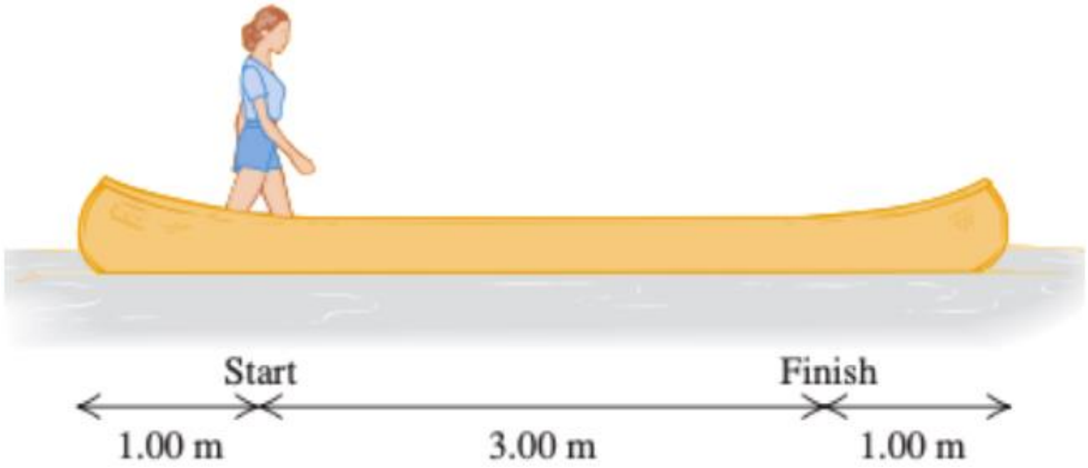
Solution:

(1) There is no net horizontal external force so $\boldsymbol{v_{cm}} = \mathbf{0}$ all the time, and thus the center of mass of the system never moves.

(2) Let **+*x*** be to the right, with the origin at the initial position of the left-hand end of the canoe. Initially, before the woman walks

$$\boldsymbol{x_{cm1}} = \frac{\boldsymbol{m x_{w1} + M x_{c1}}}{\boldsymbol{m + M}} \qquad (\mathbf{1})$$



Where $\boldsymbol{m} = \mathbf{45.0\ kg}$, $\boldsymbol{M} = \mathbf{60.0\ kg}$, $\boldsymbol{x_{w1}} = \mathbf{1.00\ m}$ and $\boldsymbol{x_{c1}} = \mathbf{2.50\ m}$.

(3) After she walks to $\mathbf{1.50\ m}$ to the right of the center of mass of the canoe, the center of mass is

$$\boldsymbol{x_{cm2}} = \frac{\boldsymbol{m x_{w2} + M x_{c2}}}{\boldsymbol{m + M}} \qquad (\mathbf{2})$$

$$\boldsymbol{x_{w2}} = \boldsymbol{x_{c2}} + \mathbf{1.50\ m} \qquad (\mathbf{3})$$

Thus, one finds $\boldsymbol{x_{c2}} = \mathbf{1.21\ m}$ and the canoe moves $\boldsymbol{\Delta x} = \boldsymbol{x_{c2}} - \boldsymbol{x_{c1}} = \mathbf{-1.29\ m}$ to the left.

QED.

Gao Qian

Problem 7

** (*Halliday, C9_P8*) A uniform soda can of mass $\boldsymbol{M = 0.140}$ **kg** is $\boldsymbol{H = 12.0}$ **cm** tall and fully filled with $\boldsymbol{m = 0.354}$ **kg** of soda (Figure shown below). Then small holes are drilled in the top and bottom (with negligible loss of metal) to drain the soda.

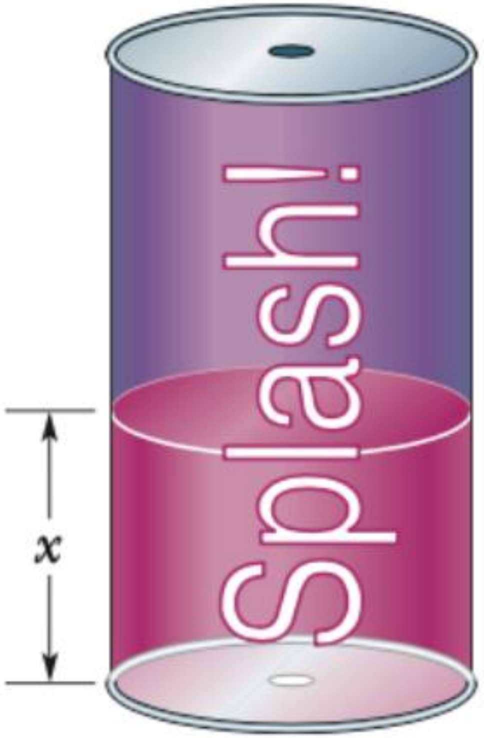
Questions:

(a) What is the height $\boldsymbol{h}$ of the Center of Mass (COM) of the can and contents initially?

(b) What is the height $\boldsymbol{h}$ of the COM of the can and contents after the can loses all the soda?

(c) What happens to $\boldsymbol{h}$ as the soda drains out?

(d) If $\boldsymbol{x}$ is the height of the remaining soda at any given instant, find $\boldsymbol{x}$ when the COM reaches its lowest point.



Problem 7

Question (a):

(a) What is the height $\boldsymbol{h}$ of the Center of Mass (COM) of the can and contents initially?

Solution (a):

(1) Since the can is uniform, its center of mass is at its geometrical center, a distance $\boldsymbol{H/2}$ above its base.

(2) The center of mass of the soda alone is at its geometrical center, a distance $\boldsymbol{H/2}$ above the base of the can, and when the can is full this is $\boldsymbol{H/2}$.

(3) Thus the center of mass of the can and the soda it contains is a distance

$$\boldsymbol{h = \frac{M(H/2) + m(H/2)}{M + m} = \frac{H}{2} = 6.0\ cm} \qquad (1)$$

above the base, on the cylinder axis.

Problem 7

Question (b):

(b) What is the height $\boldsymbol{h}$ of the COM of the can and contents after the can loses all the soda?

Solution (b):

We now consider the can alone. The center of mass is $\boldsymbol{H/2 = 6.0}$ **cm** above the base, on the cylinder axis.

Question (c):

(c) What happens to $\boldsymbol{h}$ as the soda drains out?

Solution (c):

Intuitively, the COM $\boldsymbol{h}$ should decrease from $\boldsymbol{H/2}$ first as $\boldsymbol{x}$ decreases then rise up to $\boldsymbol{H/2}$ again when all the soda is drained. This implies that there must be a minimum in $\boldsymbol{h}$.

Problem 7

Question (d):

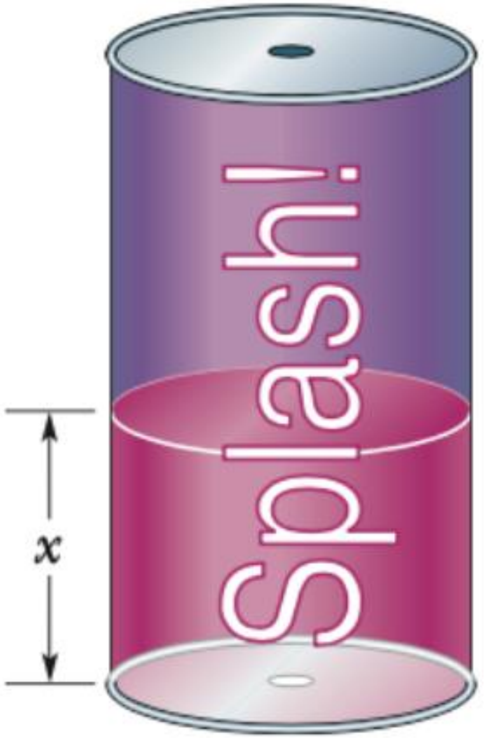
(d) If x is the height of the remaining soda at any given instant, find x when the COM reaches its lowest point.

Solution (d):

(1) When the top surface of the soda is a distance x above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where m is the soda in the can is full ($x = H$).

(2) The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M + m_p} = \frac{MH^2 + mx^2}{2(MH + mx)} \qquad (2)$$



Problem 7

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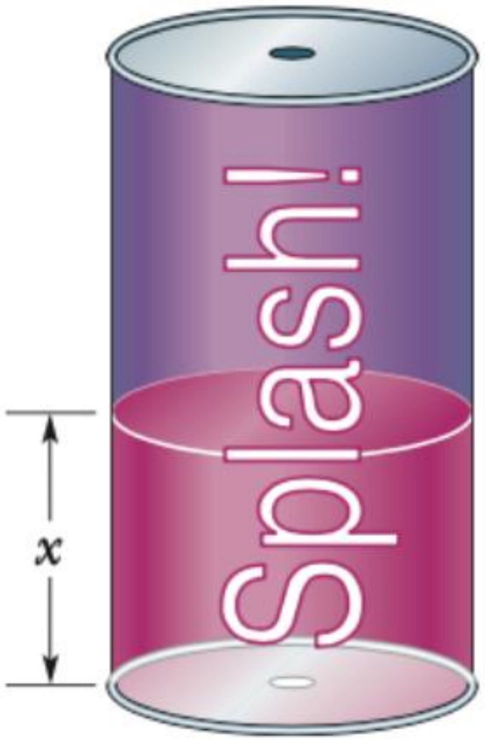
Solution (d):

(3) By setting the derivative of $\boldsymbol{h}$ w.r.t. $\boldsymbol{x}$ equal to $\mathbf{0}$ and solving for $\boldsymbol{x}$, we obtain

$$\frac{dh}{dx} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH + mx)^2} \qquad (3)$$

$$\rightarrow \quad x = \frac{MH}{m}\left[\sqrt{1 + \frac{m}{M}} - 1\right] \qquad (4)$$



(4) In the end, by substituting the above expression found for $\boldsymbol{x}$ into Eq. (2) and some algebraic manipulation, we obtain

$$h = \frac{MH}{m}\left[\sqrt{1 + \frac{m}{M}} - 1\right] = 4.2\ cm \qquad (5)$$

QED.

Problem 8

** (*Halliday, C9_P72*) In the two-dimensional collision in Figure below, the projectile particle has mass $\boldsymbol{m_1 = m}$, initial speed $\boldsymbol{v_{1i} = 3v_0}$, and final speed $\boldsymbol{v_{1f} = \sqrt{5}v_0}$, The initially stationary target particle has mass $\boldsymbol{m_2 = 2m}$ and final speed $\boldsymbol{v_{2f} = v_2}$. The projectile is scattered at an angle given by $\boldsymbol{tan\,\theta_1 = 2}$.

Questions:

(a) Find angle $\boldsymbol{\theta_2}$.

(b) Find $\boldsymbol{v_2}$ in terms of $\boldsymbol{v_0}$.

(c) Is the collision elastic?

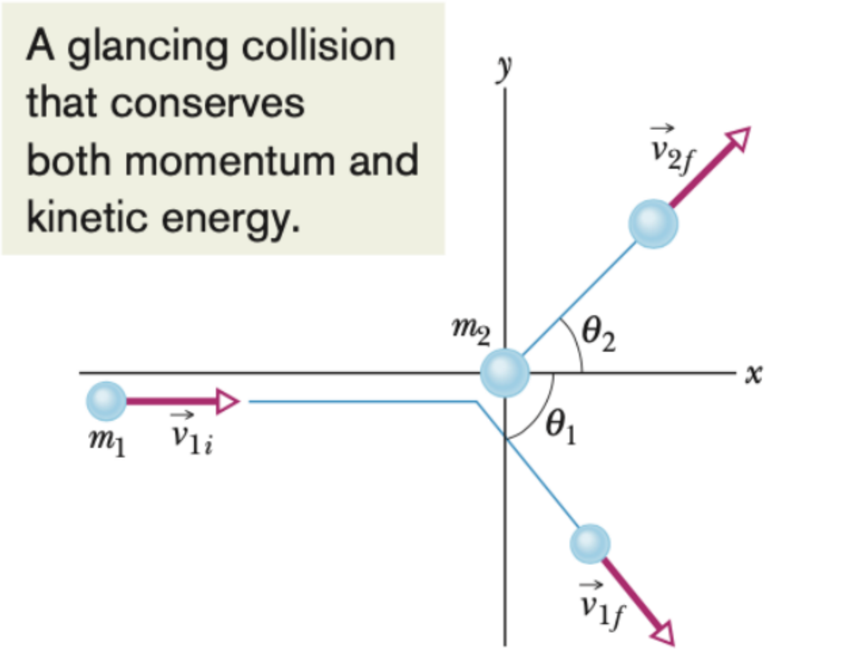


Figure 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

Problem 8

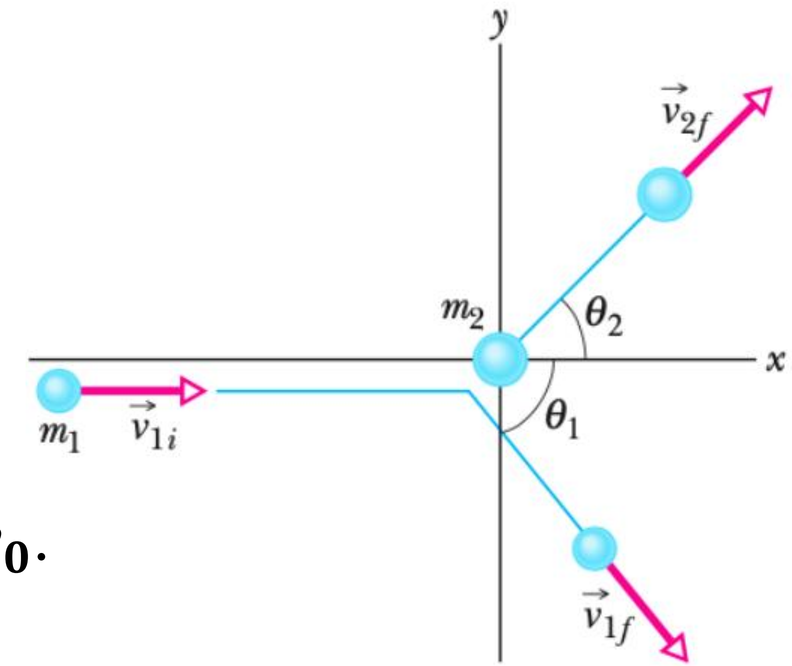
Question (a): Find angle θ_2.

Solution (a):

(1) Momentum conservation along the x and y axes gives

$$3mv_0 = m\sqrt{5}v_0 \cos\theta_1 + 2mv_2 \cos\theta_2 \qquad (1)$$

$$0 = -m\sqrt{5}v_0 \sin\theta_1 + 2mv_2 \sin\theta_2 \qquad (2)$$



(2) Given $\tan\theta_1 = 2$ and acute angle θ_1, trigonometry gives

$$\sin\theta_1 = \frac{2}{\sqrt{5}} \qquad and \qquad \cos\theta_1 = \frac{1}{\sqrt{5}}$$

Combining the above results gives $v_2 \cos\theta_2 = v_2 \sin\theta_2 = v_0$.
Therefore, $\tan\theta_2 = 1$, $\theta_2 = \pi/4$.

Question (b): Find v_2 in terms of v_0.

Solution (b): Since $\theta_2 = \pi/4$, $v_2 = \sqrt{2}v_0$.

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Problem 8

Question (c):

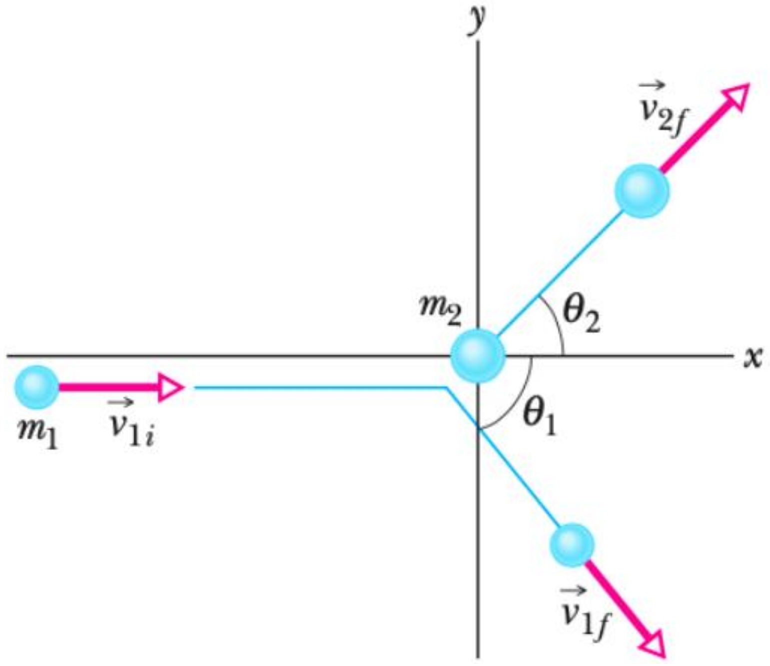
(c) Is the collision elastic?

Solution (c):

(1) Kinetic energy before the collision is $\mathbf{9/2\, m v_0^2}$.

(2) Kinetic energy after the collision is $\mathbf{5/2\, m v_0^2} + \mathbf{4/2 m v_0^2} = \mathbf{9/2 m v_0^2}$.

(3) Since the kinetic energy of the system is the same before and after the collision, we conclude that the **<u>collision is elastic</u>**.



QED.

Problem 9

** Show that in one-dimensional elastic collision, if the mass and velocity of object **1** are $\boldsymbol{m_1}$ and $\boldsymbol{v_{1i}}$, and if the mass and velocity of object **2** are $\boldsymbol{m_2}$ and $\boldsymbol{v_{2i}}$, then their final velocities $\boldsymbol{v_{1f}}$ and $\boldsymbol{v_{2f}}$ are given by

$$\boldsymbol{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}}$$

$$\boldsymbol{v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i}}$$

Check the plausibility of the above answer by considering the limiting case with $\boldsymbol{m_1 \gg m_2}$. Note that in this case the velocity of object **1** is unchanged while the object **2** is like hitting a wall in the reference frame of object **1**.

Problem 9

Solution: 1D elastic collision.

(1) From conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\rightarrow\ m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \qquad (1)$$

(allow v's to take positive and negative values to capture its vector feature.)

(2) According to conservation of energy

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \ \rightarrow$$

$$\frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}m_2 v_{2f}^2 - \frac{1}{2}m_2 v_{2i}^2 \ \rightarrow$$

$$m_1(v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2(v_{2i} + v_{2f})(v_{2f} - v_{2i}) \qquad (2)$$

Problem 9

Solution: **1D elastic collision.**

(3) Divide Eq. (2) by Eq. (1) ⇒

$$v_{1i} + v_{1f} = v_{2i} + v_{2f} \qquad (3)$$

Comments: The above equation essentially indicates that the relative velocity changes sign before and after the collision, namely,

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f}) \qquad (4)$$

Solve the final velocities from Eq. (1) and Eq. (3) and find

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i}$$

QED.

Problem 10

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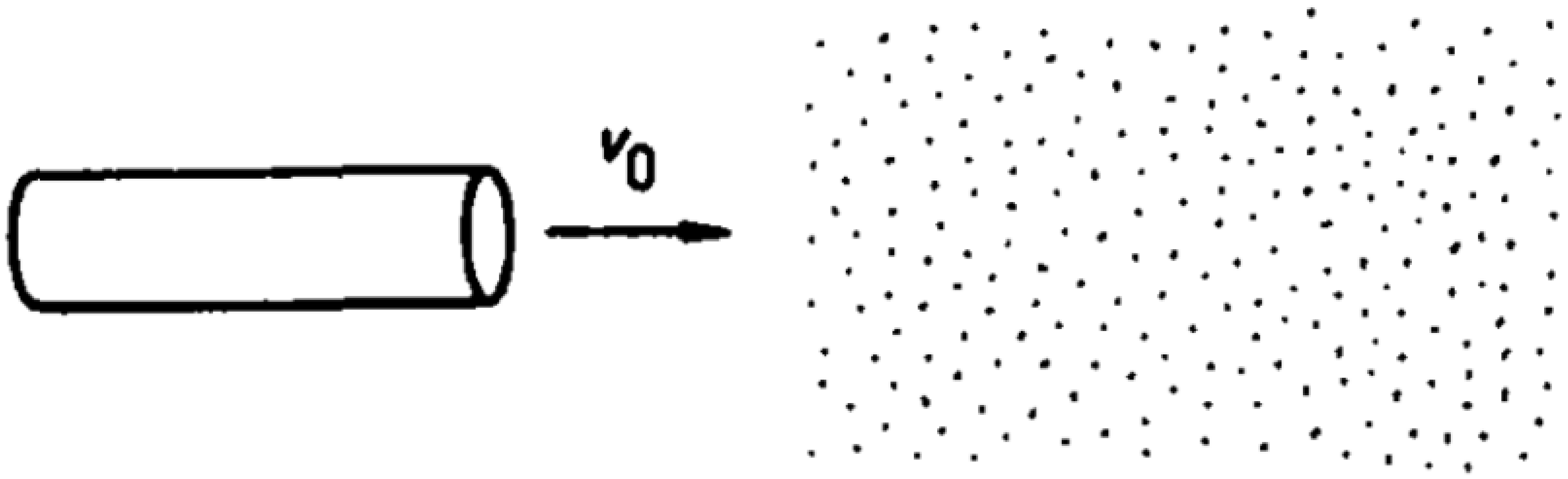
* * * Suppose the spacecraft (Enterprise) of mass $\boldsymbol{m_0}$ and cross-section $\boldsymbol{A}$ is moving with velocity $\boldsymbol{v_0}$ when it encounters a stationary dust cloud of density $\boldsymbol{\rho}$ at $\boldsymbol{t=0}$. If the dust sticks to the spacecraft and resistance can be neglected. Solve for the subsequent motion of the spacecraft.



Solution:

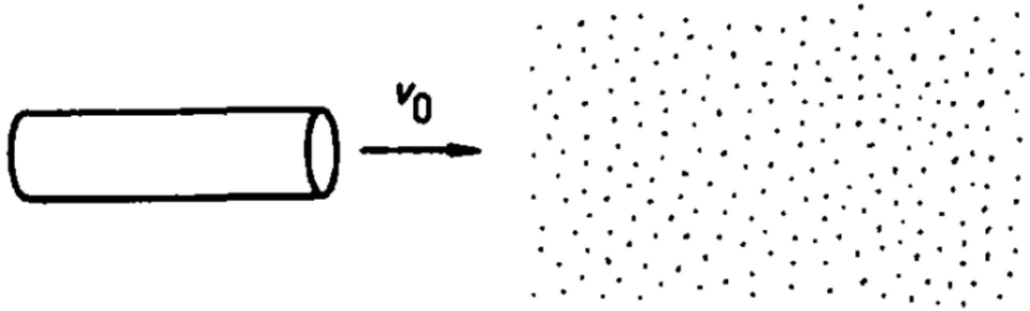
(1) Suppose the dust gives no drag resistance to the spacecraft (Enterprise), then the spacecraft-dust system is isolated. ***Newton's second law*** gives

$$\frac{d(mv)}{dt} = 0 \qquad \text{or} \qquad m\frac{dv}{dt} + v\frac{dm}{dt} = 0 \tag{1}$$

which implies $mv = m_0 v_0$ or $m = \dfrac{m_0 v_0}{v}$.

(2) As the spacecraft picks up the dust along its path, its mass increases at the rate

$$\frac{dm}{dt} = \rho A v \tag{2}$$



Problem 10

Solution:

(3) Combining the above two equations yields

$$m\frac{dv}{dt}+\rho Av^2=0 \quad plus \quad m=\frac{m_0v_0}{v} \quad \rightarrow \tag{3}$$

$$\frac{m_0v_0}{v}\frac{dv}{dt}+\rho Av^2=0 \quad \rightarrow \tag{4}$$

$$separate\ variables \quad \frac{dv}{v^3}=-\frac{\rho A}{m_0v_0}dt \tag{5}$$

(4) Integrating on both sides gives

$$\int_{v_0}^{v}\frac{dv}{v^3}=-\frac{\rho A}{m_0v_0}\int_0^t dt \quad \rightarrow \tag{6}$$

$$-\frac{1}{2}\left(\frac{1}{v^2}-\frac{1}{v_0^2}\right)=--\frac{\rho A}{m_0v_0}t \quad \rightarrow \quad v=v_0\sqrt{\frac{1}{1+2\rho Av_0t/m_0}} \tag{7}$$

QED.

Gao Qian