PHY1001: Mechanics

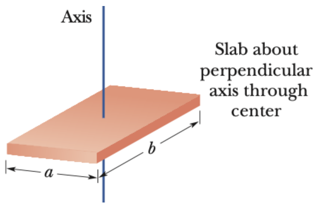
Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star ∗ ∗ ∗ labels are assigned to the most difficult ones.

Due date: 2024, March 17th, 23: 59: 00.

1 Homework Problems for Week 6: C10 and 11

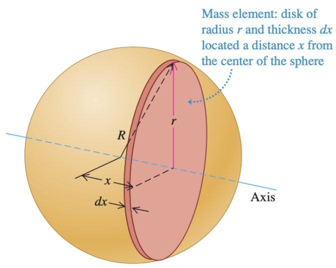
1. ∗ ∗ Find the rotational inertia of a rectangular plate about an axis through its center.

   Answers: $I = \frac{1}{12}M(a^2 + b^2)$



2. ∗ ∗ ∗ Find the rotational inertia of a solid and uniform sphere about an axis through its center.

   Answers: $I = \frac{2}{5}MR^2$



3. ∗ The earth, which is not a uniform sphere, has a moment of inertia of $0.3308MR^2$ about an axis through its north and south poles. It takes the earth 86, 164 s to spin once about this axis.

   (a) Calculate the earth's kinetic energy due to its rotation about this axis.
   Answers: $K_1 = 2.14 \times 10^{29}$ J.

   (b) Compute the earth's kinetic energy due to its orbital motion around the sun.
   Answers: $K_2 = 2.66 \times 10^{33}$ J.

   (c) Explain how the value of the earth's moment of inertia tells us that the mass of the earth is concentrated toward the planet's center.
   Answers: Compare with the moment of inertia of the uniform sphere ($0.400MR^2$), earth's moment of inertia has a smaller coefficient, which means mass is at the center of the earth.

4. ∗ ∗ The moment of inertia of a sphere with uniform density about an axis through its centre is $2/5(MR^2) = 0.400MR^2$. Satellite observations show that the earth's moment of inertia is $0.3308MR^2$. Geophysical data suggest the earth consists of five main regions:

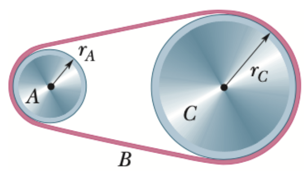
   - the inner core ($r = 0$ to $r = 1220$ km) of average density 12900 kg/m^3,
   - the outer core ($r = 1220$ km to $r = 3480$ km) of average density 10900 kg/m^3,
   - the lower mantle ($r = 3480$ km to $r = 5700$ km) of average density 4900 kg/m^3,
   - the upper mantle ($r = 5700$ km to $r = 6350$ km) of average density 3600 kg/m^3,
   - and the outer crust and oceans ($r = 6350$ km to $r = 6370$ km) of average density 2400 kg/m^3.

   (a) Show that the moment of inertia about a diameter of a uniform spherical shell of inner radius R_1, outer radius R_2, and density ρ is
   $$I = \rho\frac{8\pi}{15}\left(R_2^5 - R_1^5\right).$$
   Hint: Form a shell by superposition of a sphere of density ρ and a smaller sphere of density $-\rho$.

   (b) Compute the mass of the earth by using the above given data. **Answers:** 5.97×10^{24} kg.

   (c) Use the data to calculate the earth's moment of inertia in terms of MR^2. **Answers:** $I = 8.12 \times 10^{37} \mathrm{kg} \cdot m^2 = 0.335MR^2$ with $R = 6370$ km.

5. (Halliday C10-P28)



   In the above figure, wheel A of radius $r_A = 10$ cm is coupled by belt B to wheel C of radius $r_C = 25$cm. The angular speed of wheel A is increased from rest at a constant rate of 2.0 rad/s^2. Find the time needed for wheel C to reach an angular speed of 100 rev/min, assuming the belt does not slip.
   Answers: 13 s.

6. ∗ One force acting on a machine part is $\vec{F} = (-5.00\,N)\hat{i} + (4.00\,N)\hat{j}$. The vector from the origin to the point where the force is applied is $\vec{r} = (-0.450\,m)\hat{i} + (0.150\,m)\hat{j}$.

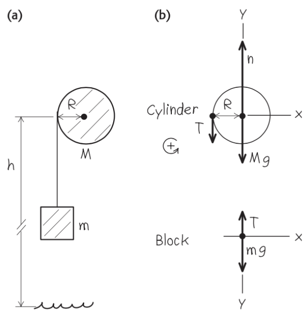
   (a) In a sketch, show $\vec{r}$, $\vec{F}$, and the origin.

   (b) Use the right-hand rule to determine the direction of the torque.
   Answers: $-z$ direction

(c) Calculate the vector torque for an axis at the origin produced by this force. Verify that the direction of the torque is the same as you obtained in part (b).
Answers: $\tau = (-1.05\,\mathrm{N}\cdot\mathrm{m})\hat{k}$.

7. $*$ The flywheel of an engine has moment of inertia 2.50 kg $\cdot$ m^2 about its rotation axis. What constant torque is required to bring it up to an angular speed of 400 rev/min in 8.00 s, starting from rest?
Answers: $\tau_z = 13.1\,\mathrm{N}\cdot\mathrm{m}$.

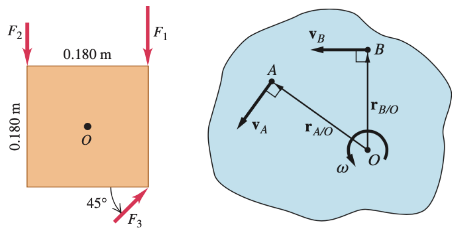
8. $*$ An unwinding cable (see also Week 5 Problem 8)



We wrap a light, non-stretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping.

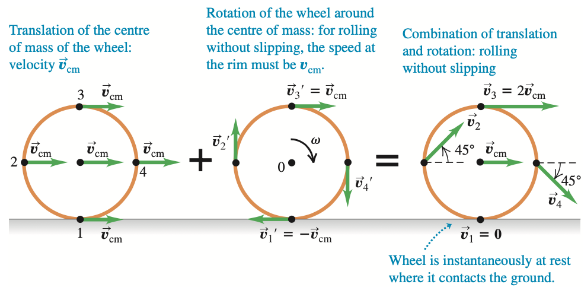
Question: what are the acceleration of the falling block and the tension in the cable?

9. $*$ A square metal plate 0.180 m on each side is pivoted about an axis through point O at its center and perpendicular to the plate (Fig. below). Calculate the net torque about this axis due to the three forces shown in the figure if the magnitudes of the forces are F_1 = 18.0 N, F_2 = 26.0N,and F_3 = 14.0 N.The plate and all forces are in the plane of the page.
Answers: 2.50 N·m



10. $**$ Instantaneous Axis: The instantaneous axis (point), the point O as shown in the above figure, of a rigid body undergoing planar motion is defined to be the point that has zero velocity at the instant under consideration. This point may be either in a body or outside the body (in the "body extended"). It is often useful to the instantaneous axis to compute the velocity and kinetic energy of a rigid body.

Consider the following wheel which is rolling without slipping.



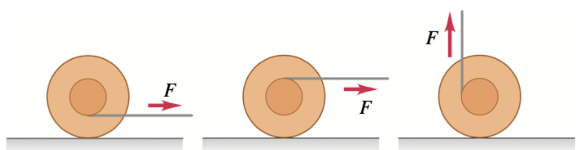
(a) Think of the motion as the combination of the translation of the center of mass (COM) and the rotation about the axis through COM, and compute the kinetic energy of the wheel.
Answers: $K = MR^2\omega^2$.

(b) Think of the motion as rotating about the instantaneous axis that passes through the point of contact with the ground, and compute the kinetic energy of the wheel.
Answers: $K = MR^2\omega^2$.

(c) Imagining that you now increase the magnitude of ω to $2v_{cm}/R$ and the wheel is slipping. Where is the instantaneous axis now? Repeat the calculation for part (a) and (b).
Answers: It is located at $1/2R$ above the ground, $K = (5/8)MR^2\omega^2 = (5/2)Mv_{cm}^2$.

(d) Can you use the parallel-axis theorem to show that you will always get consistent answers for kinetic energy using these two points of view?
Hint: There is only pure rotation about the instantaneous axis with the moment of inertia $I = I_{cm} + Md^2$ according to the parallel-axis theorem, where d is the distance between the instantaneous axis and the center of mass. In addition, convince yourself that the COM is rotating about instantaneous axis with speed $v_{cm} = \omega d$.

11. $*$ Figure below shows three identical yo-yos initially at rest on a horizontal surface. For each yo-yo, the string is pulled in the direction shown. In each case, there is sufficient friction for the yo-yo to roll without slipping. Draw the free-body diagram for each yo-yo. In what direction will each yo-yo rotate? Explain your answers.



Hint: You do not have to do any calculation for this problem. You only need to find the "magical axis" in the free-body diagram and use the axis to compute the torque about it. And - voilà - all these three yo-yos should rotate clockwise.