香港中文大學(深圳)理工学院
School of Science and Engineering

PHY 1001: Mechanics

Tutorial Session 2

T-05: Jan - 24 - 2024, Wednesday, 19:00~19:50

T-12: Jan - 24 - 2024, Wednesday, 20:00~20:50

Gao Qian

Problem 1

* Two boxes of mass $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ connected by a massless string are being pulled along a horizontal frictionless surface by the tension force in a second string, as shown below.

Question (a):

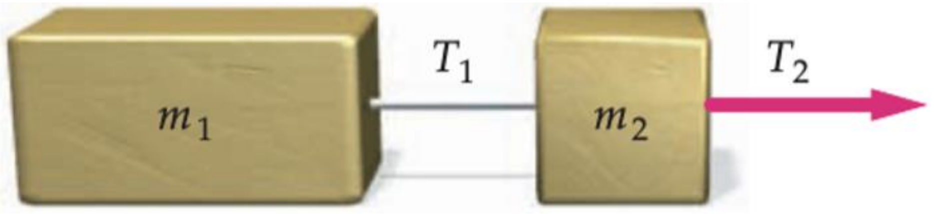
(a) Draw free body diagrams of both boxes separately and show that

$$\boldsymbol{T_1/T_2 = m_1/(m_1+m_2)}$$

Solution (a):

(1) Assuming the same acceleration $\boldsymbol{a}$ for these two bodies, then we have

$$\begin{cases} \boldsymbol{T_1 = m_1 a} & \textbf{(1)} \\ \boldsymbol{T_2 - T_1 = m_2 a} & \textbf{(2)} \end{cases}$$



(2) Combining equations (1) and (2), yields

$$\boldsymbol{T_2 = (m_1+m_2)a \;\rightarrow\; T_1/T_2 = m_1/(m_1+m_2)}$$

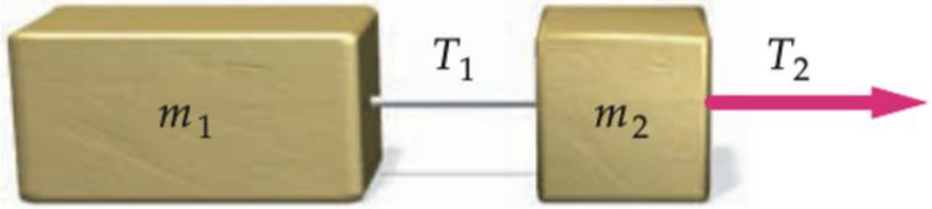
Gao Qian

Problem 1

Question (b):

(b) Is this result plausible? Explain. Does your answer make sense both in the limit that $\boldsymbol{m_2/m_1 \gg 1}$ and in the limit that $\boldsymbol{m_2/m_1 \ll 1}$? Explain.

Solution (b):



(1) Intuitively, the result of (a) is plausible since

$$\boldsymbol{T_1/T_2 = m_1/(m_1+m_2) < 1}$$

(2) In limit case of $\boldsymbol{m_2/m_1 \gg 1}$ (as if $\boldsymbol{m_1}$ does not exist):

$$\boldsymbol{T_1/T_2 = m_1/(m_1+m_2) \to 0} \quad \text{(as if } \boldsymbol{T_1} \text{ does not exist)}$$

(3) In limit case of $\boldsymbol{m_2/m_1 \ll 1}$ (as if $\boldsymbol{m_2}$ does not exist):

$$\boldsymbol{T_1/T_2 = m_1/(m_1+m_2) \to 1} \quad \text{(as if } \boldsymbol{T_1 = T_2}\text{)}$$

QED.

Comments: Taking limits is one of the most common ways to check results in physics.

Gao Qian

Problem 2

* (*Textbook_C5-P8*) A 1.50 kg object is subjected to three forces that give it an acceleration $\vec{a} = -(8.00\,m/s^2)\hat{i} + (6.00\,m/s^2)\hat{j}$. If two of the three forces are

$$\begin{aligned}\vec{F}_1 &= (30.0\,N)\hat{i} + (16.0\,N)\hat{j}\\ \vec{F}_2 &= -(12.0\,N)\hat{i} + (8.00\,N)\hat{j},\end{aligned}$$

find the third force.

Solution:

(1) According to ***Newton's 2nd Law***, we have

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a} = -12\,N\hat{i} + 9\,N\hat{j},$$

(2) Therefore, we have the third force:

$$\begin{aligned}\vec{F}_3 &= -12.0\,N\hat{i} + 9.0\,N\hat{j} - (\vec{F}_1 + \vec{F}_2)\\ &= -(30.0\,N)\hat{i} - (15.0\,N)\hat{j}.\end{aligned}$$

QED.

Problem 3

* (*Textbook_C5-P56*) In **Fig. 5-41*a***, a constant horizontal force $\vec{F}_a$ is applied to block ***A***, which pushes against block ***B*** with a **15.0 N** force directed horizontally to the right. In **Fig. 5-41*b***, the same force $\vec{F}_a$ is applied to block ***B***; now block ***A*** pushes on block ***B*** with a **10.0 N** force directed horizontally to the left. The blocks have a combined mass of **12.0 kg**. What are the magnitudes of **(a)** their acceleration in **Fig. 5-41*a*** and **(*b*)** force $\vec{F}_a$?

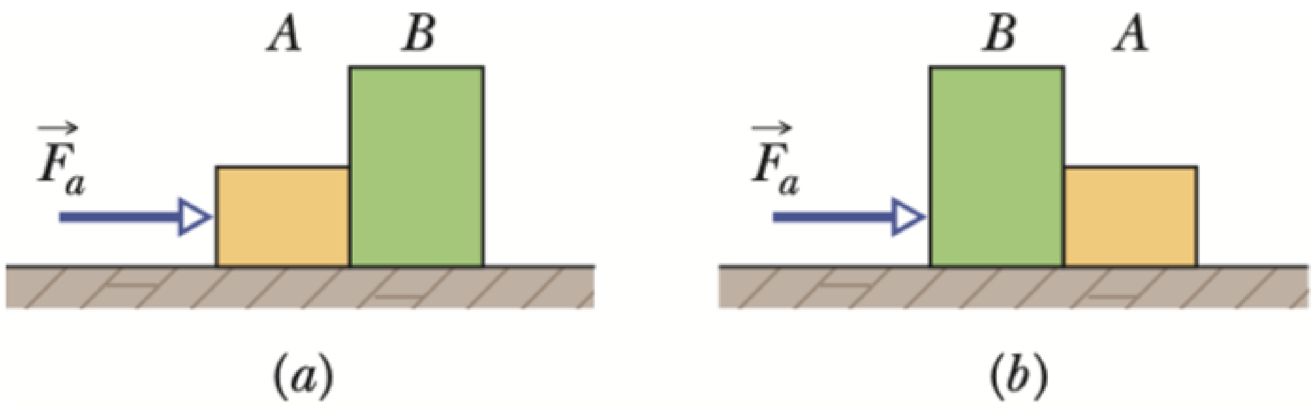
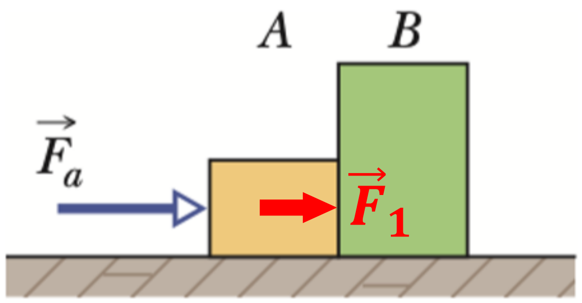


Figure 5-41 Problem 56.

Problem 3

Solution:

(1) 2 situations in (a) and (b) involve the same applied force and the same total mass, so the accelerations must be the same in both figures.



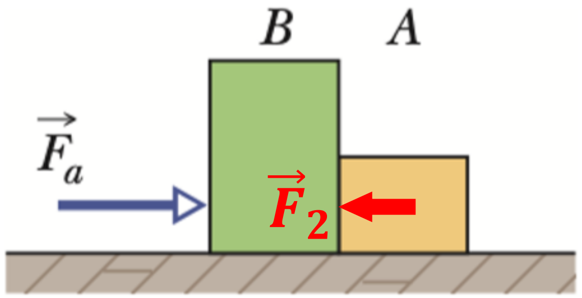
(a)

(2) In **Fig. 5-41*a***, the force $\vec{F}_1$ causing ***B*** to have acceleration ***a*** is **15.0 N**, thus

$$F_1 = m_B a = 15.0\ \mathrm{N} \qquad (1)$$

(3) In **Fig. 5-41*b***, the force $\vec{F}_2$ causing ***A*** to have acceleration ***a*** is **10.0 N**, thus

$$F_2 = m_A a = 10.0\ \mathrm{N} \qquad (2)$$



(b)

(4) Combining equations **(1)** and **(2)**, we have

$$\frac{m_A}{m_B} = \frac{F_2}{F_1} = \frac{2}{3} \qquad (3)$$

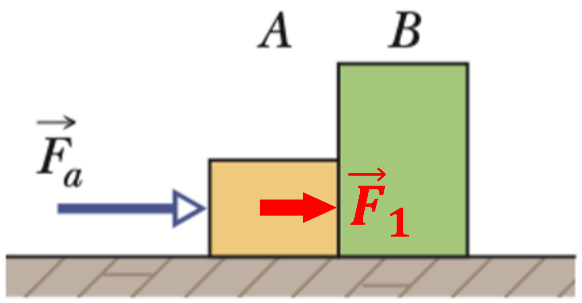
Gao Qian

Problem 3

Solution:

(5) We know that these two blocks have a combined mass of **12.0 kg**, thereby

$$\boldsymbol{m_A + m_B = 12.0\ \mathrm{kg}} \tag{4}$$



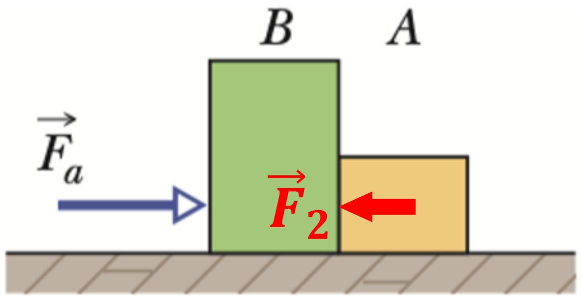
(a)

(6) Combining equations (3) and (4), yields

$$\begin{cases} \boldsymbol{m_A = 4.8\ \mathrm{kg}} \\ \boldsymbol{m_B = 7.2\ \mathrm{kg}} \end{cases} \tag{5}$$

(7) Combining equations **(5)** and **(1)** or **(2)**, yields

$$\boldsymbol{a = \frac{F_1}{m_B} = 2.08\ \mathrm{m/s^2}} \tag{6}$$



(b)

(8) The total force providing the acceleration $\boldsymbol{a}$ of $\boldsymbol{A}$ plus $\boldsymbol{B}$ system is:

$$\boldsymbol{F_a = (m_A + m_B)a = 25\ \mathrm{N}} \tag{7}$$

QED.

Gao Qian

Problem 4

* **Rotating Space Stations**. One problem for humans living in outer space is that they are apparently weightless. One way around this problem is to design a space station that spins about its center at a constant rate. This creates "artificial gravity" at the outside rim of the station.

Question (a): If the diameter of the space station is **800 m**, how many revolutions per minute are needed for the "artificial gravity" acceleration to be $\mathbf{9.8\ m/s^2}$?

Question (b): If the space station is a waiting area for travelers going to Mars, it might be desirable to simulate the acceleration due to gravity on the Martian surface $\mathbf{3.7\ m/s^2}$. How many revolutions per minute are needed in this case?

A spacecraft design using artificial gravity.

Problem 4

Question (a):

(a) If the diameter of the space station is **800 m**, how many revolutions per minute are needed for the “artificial gravity" acceleration to be $\mathbf{9.8\ m/s^2}$?

Solution (a):

(1) Let the Centripetal acceleration $a_{rad} = g$, and we have:

$$a_{rad} = \frac{v^2}{R}$$

v^2 ← *linear velocity*

R ← *rotation radius*

⇒ $v = \sqrt{gR}$

(2) Thus, solving for the rotational period T gives

$$T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R}{g}} = 40.1\ \mathrm{s}$$

(3) So, the number of revolution per minute is

$$60/T = (60\ \mathrm{s})/(40.1\ \mathrm{s}) = 1.5\ \mathrm{rev/min}$$

Gao Qian

Problem 4

香港中文大學(深圳)理工学院
School of Science and Engineering

Question (b):

(b) If the space station is a waiting area for travelers going to Mars, it might be desirable to simulate the acceleration due to gravity on the Martian surface $\mathbf{3.7\ m/s^2}$. How many revolutions per minute are needed in this case?

Solution (b):

By setting Centripetal acceleration $\boldsymbol{a_{rad}} = \boldsymbol{g_m} = \mathbf{3.7\ m/s^2}$, and computing the rotational period $\boldsymbol{T_m}$ following the calculating steps in Question (a), we have

rotational period ⟶ $\boldsymbol{T_m} = \mathbf{65.3\ s}$,

revolutions per minute ⟶ $\mathbf{60}/\boldsymbol{T_m} = \mathbf{0.92\ rev/min}$

Smaller gravitational constant corresponds to a longer period, and hence a lower rotation rate.

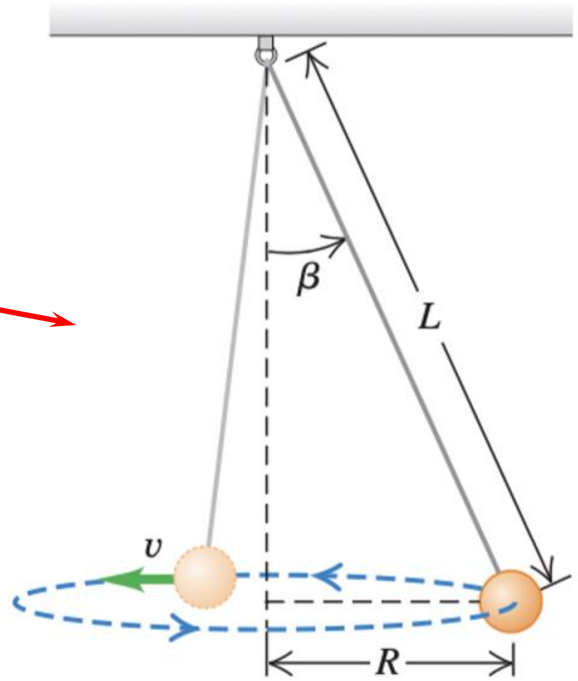
QED.

Gao Qian

Problem 5

* **A conical pendulum**. An inventor designs a pendulum clock using a bob with mass $\boldsymbol{m}$ at the end of a thin wire of length $\boldsymbol{L}$. Instead of swinging back and forth, the bob is to move in a horizontal circle with constant speed $\boldsymbol{v}$, with the wire making a fixed angle $\boldsymbol{\beta}$ with the vertical direction. This is called a conical pendulum because the suspending wire traces out a cone. Find the period $\boldsymbol{T}$.

conical pendulum



Problem 5

香港中文大學(深圳)理工学院
School of Science and Engineering

Solution:

(1) The bob has zero vertical acceleration, thus

$$F\cos\beta - mg = 0 \qquad (1)$$

(2) The horizontal acceleration is toward the center of the circle, and it is the centripetal acceleration, thus

$$F\sin\beta = ma_{rad} = m\frac{v^2}{R} \qquad (2)$$

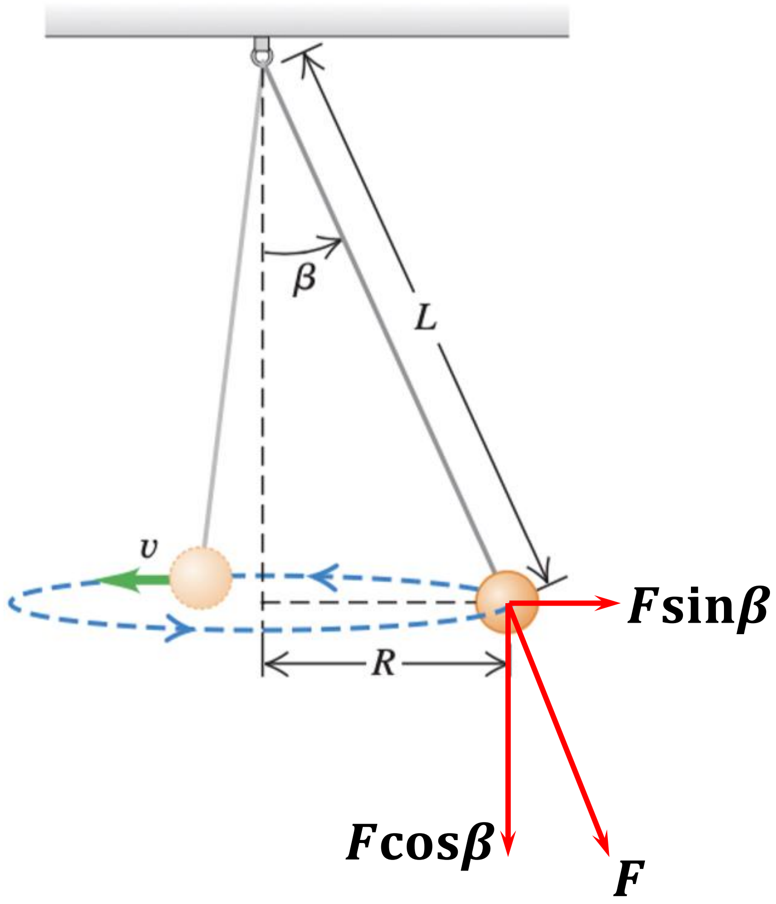
$$\textit{with} \quad R = L\sin\beta$$

(3) Combining Eqs. **(1)** and **(2)**, yields the linear velocity:

$$v = \sqrt{gR\tan\beta} \qquad (3)$$

(4) The rotational period is then be calculated as

$$T = \frac{2\pi R}{v} = \frac{2\pi L\sin\beta}{\sqrt{gL\sin\beta\tan\beta}} = 2\pi\sqrt{\frac{L\cos\beta}{g}} \qquad (4)$$



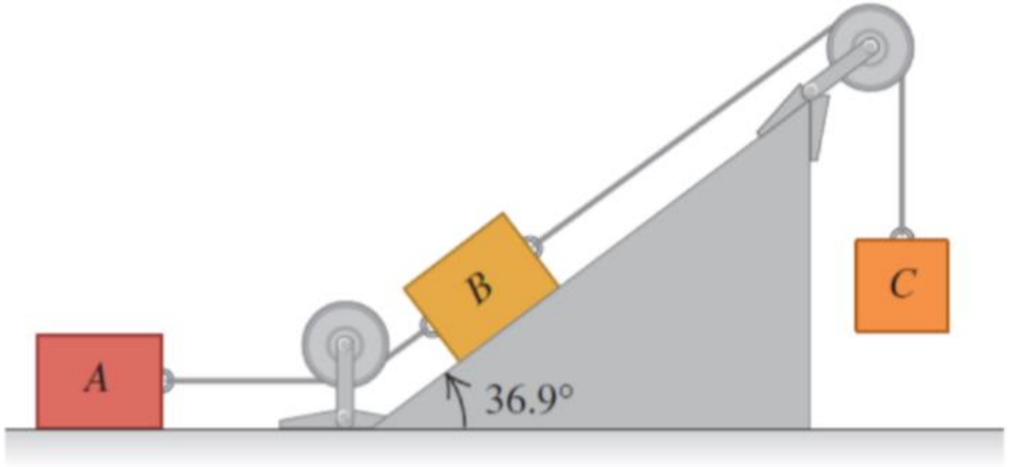
QED.

Problem 6

* * Blocks ***A***, ***B*** and ***C*** are placed as shown below and are connected by ropes of negligible mass. Both ***A*** and ***B*** weigh **25.0 N** each, and the coefficient of kinetic friction between each block and the surface is **0.35**. Block ***C*** descends with constant velocity.

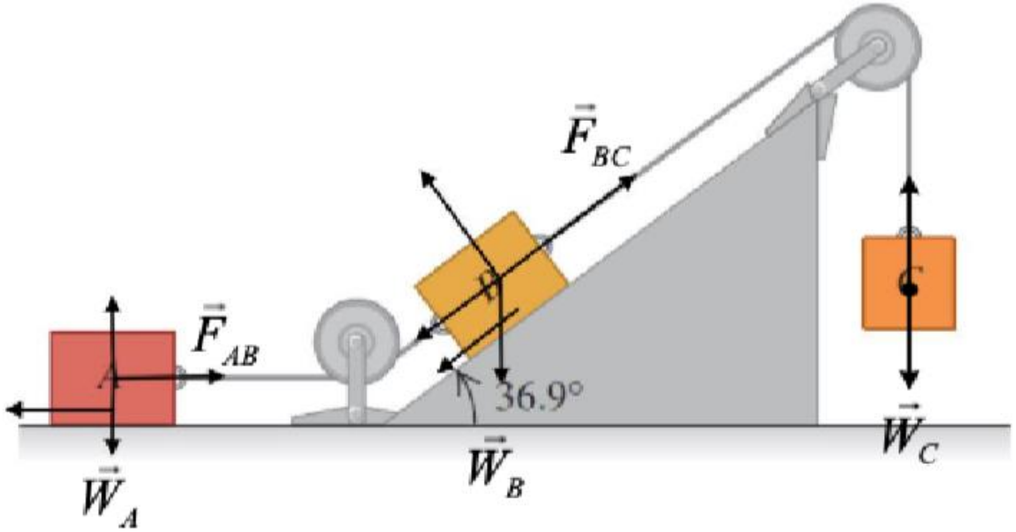
Question (a):

Draw free body diagrams of ***A*** and ***B***.



Solution (a):

The free-body diagram is shown below

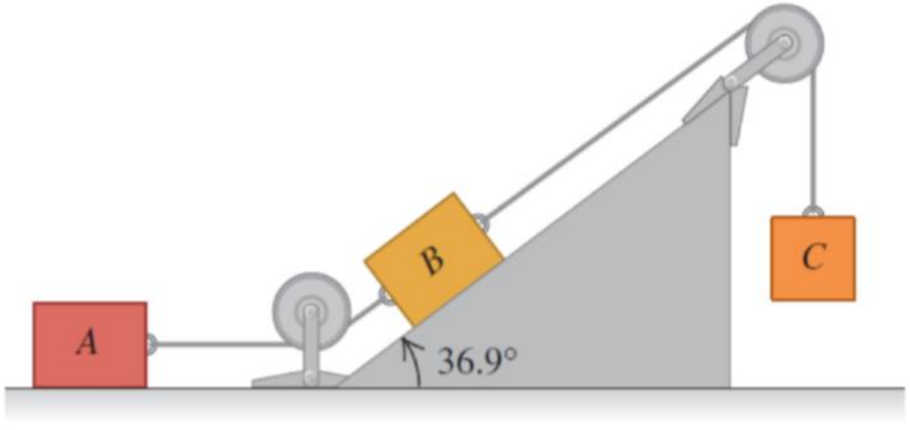


Gao Qian

Problem 6

香港中文大學(深圳)理工学院
School of Science and Engineering

Question (b): Find the magnitude of the tension in the rope connecting blocks $\boldsymbol{A}$ and $\boldsymbol{B}$.

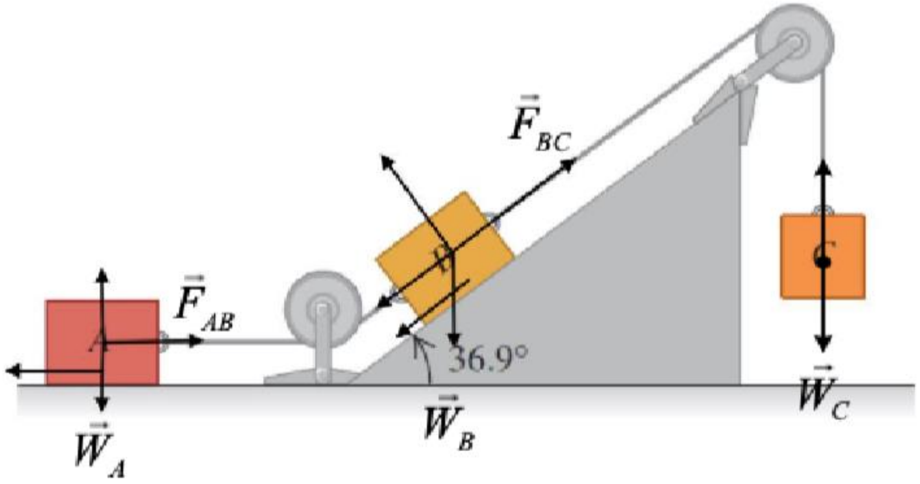


Solution (b):

The magnitude of tension $\boldsymbol{F_{AB}}$ is equal to the kinetic friction of $\boldsymbol{A}$.

$$\boldsymbol{F_{AB} = \mu W_A = 0.35 \times 25.0\ \mathrm{N} = 8.75\ \mathrm{N}}$$

Question (c): What is the weight of block $\boldsymbol{C}$?



Solution (c):

For $\boldsymbol{B}$, we have

$$\boldsymbol{F_{BC} = F_{AB} + W_B \sin(36.9°) + \mu W_B \cos(36.9°)}$$

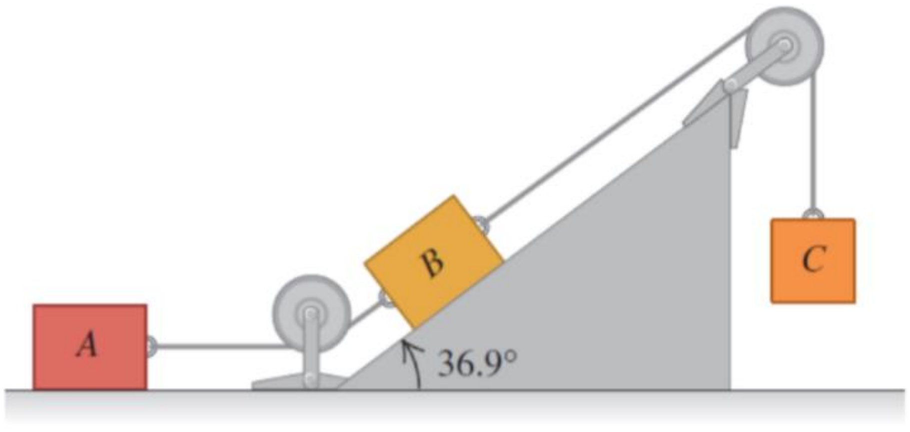
For block $\boldsymbol{C}$, we have $\boldsymbol{F_{BC} = W_C}$. Thus, we derive that

$$\boldsymbol{W_C = 30.75\ \mathrm{N}}$$

Gao Qian

Problem 6

Question (d): If the rope connecting ***A*** and ***B*** is cut, what would be the magnitude of acceleration of ***C***?

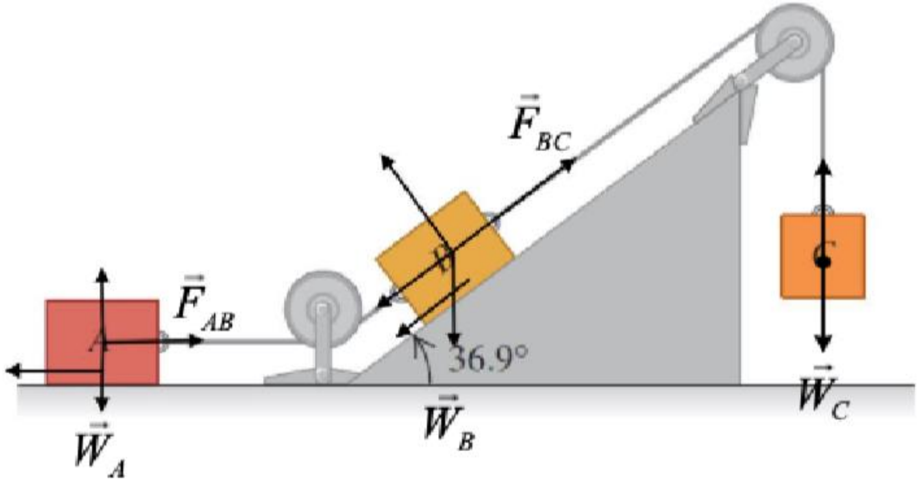


Solution (d):

$$\boldsymbol{a_C = \frac{W_C - (F_{BC} - F_{AB})}{\sum(m_B + m_C)}}$$

$$\boldsymbol{= \frac{W_C - W_B\sin(36.9°) - \mu W_B\cos(36.9°)}{m_B + m_C}}$$

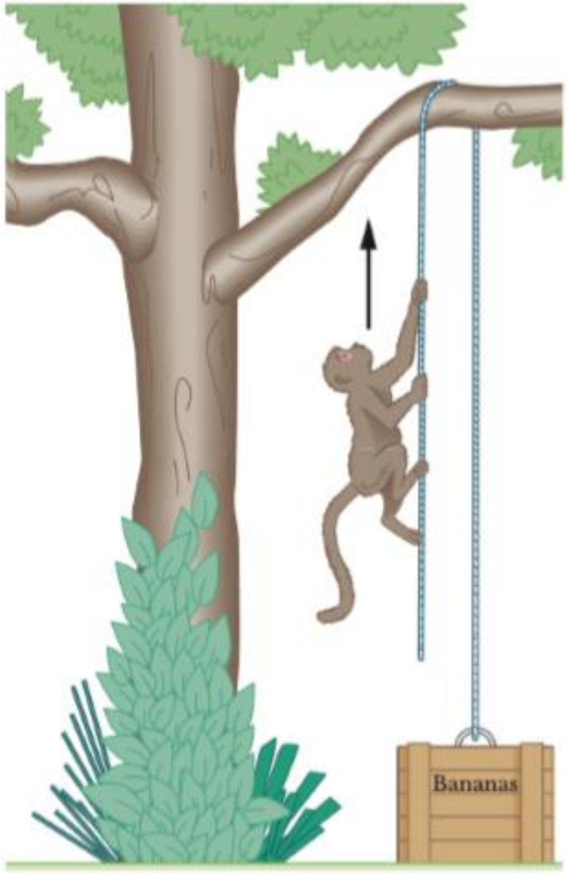
$$\boldsymbol{= \frac{F_{AB}}{m_B + m_C} = 1.54\ \mathrm{m/s^2}}$$



QED.

Problem 7

** (*Textbook, C5_P59*) A **10 kg** monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a **15 kg** package on the ground.



Question (a): What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground?

Solution (a):

(1) If $\boldsymbol{F}$ is the minimum force required to lift the package, then $\boldsymbol{F_N = 0}$ and $\boldsymbol{a_p = 0}$.

(2) Substituting $\boldsymbol{F}$ with $\boldsymbol{m_p g}$ in the equation for the monkey, we can solve for $\boldsymbol{a_m}$ as:

$$\boldsymbol{a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = 4.9\ \mathrm{m/s^2}}$$

Problem 7

Question (b): If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what are the magnitude and direction of the monkey's acceleration and the tension in the rope?

Solution (b):

(1) Now the normal force $\boldsymbol{F_N = 0}$, so we have

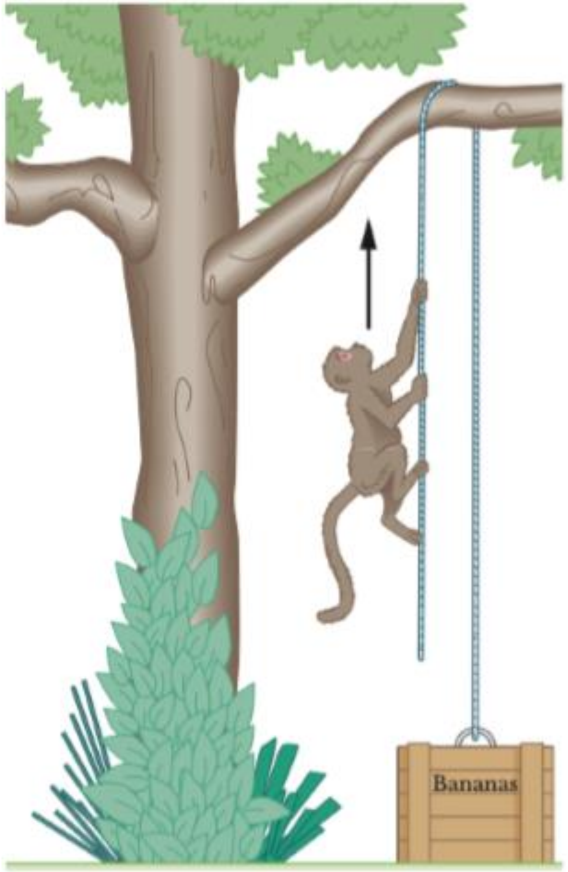
$$\begin{cases} \boldsymbol{F - m_m g = m_m a'_m} \\ \boldsymbol{F - m_p g = m_p a'_p} \end{cases} \rightarrow \boldsymbol{m_m(g + a'_m) = m_p(g + a'_p)}$$

(2) Note that $\boldsymbol{a'_m = -a'_p}$, we derive that

$$\boldsymbol{a'_m = \frac{(m_p - m_m)g}{m_p + m_m} = 1.96\ \mathrm{m/s^2}}$$

upward acceleration

(3) Then, the rope tension: $\boldsymbol{F = m_m(g + a'_m) = 117.6\ \mathrm{N}}$



QED.

Gao Qian

Problem 8

** **Fluid resistance:** Consider a metal ball of mass $\boldsymbol{m}$ falling through a fluid as shown below. For small objects moving at low speeds, the magnitude of the fluid resistance $\boldsymbol{f} = \boldsymbol{kv}$ is approximately proportional to the metal ball's speed, where $\boldsymbol{k}$ is a proportionality constant that depends on the shape and size of the body and the properties of the fluid.

Question (a):

Find the terminal speed of the metal ball.

Question (b):

Find the relationship between velocity $\boldsymbol{v_y}$ and time $\boldsymbol{t}$.

Question (c):

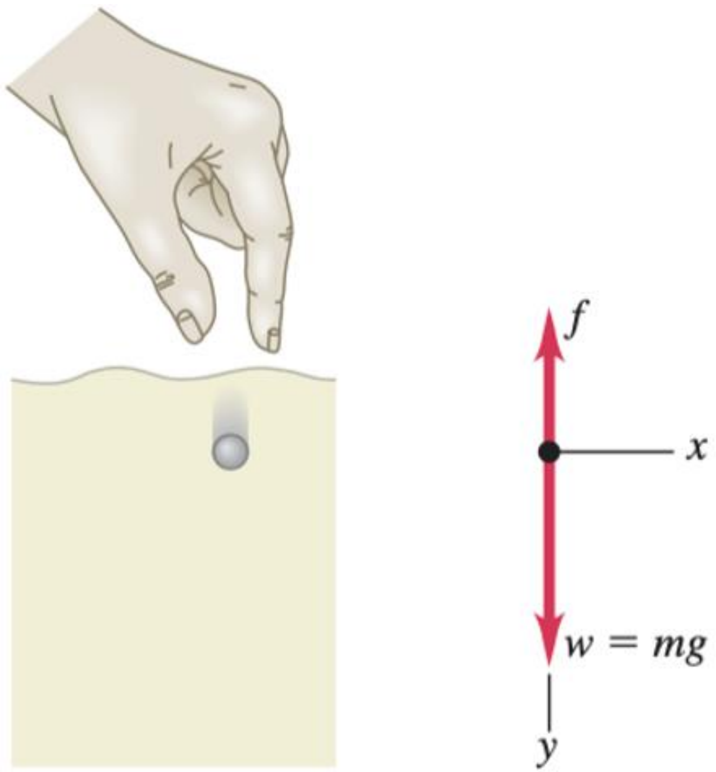
Find the relationship between acceleration $\boldsymbol{a_y}$ and time $\boldsymbol{t}$.

Question (d):

Find relationship between displacement $\boldsymbol{y(t)}$ and time $\boldsymbol{t}$.

Question (e):

Consider the velocity $\boldsymbol{v_y}$ by taking limit $\boldsymbol{t \to 0}$ and $\boldsymbol{t} \to \infty$, explain why you think the results make sense in these two limits.

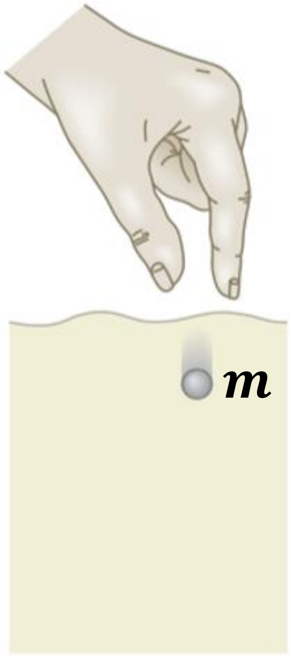


Gao Qian

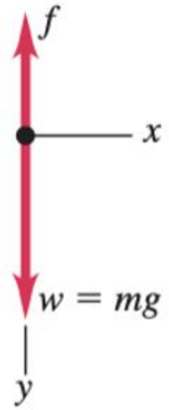
Problem 8

Question (a):

(a) Find the terminal speed of the metal ball.



$$f = kv$$



Solution (a):

(1) When the metal ball starts to fall, $\boldsymbol{v_y = 0}$, the resisting force is zero, and the initial acceleration is $\boldsymbol{g}$.

(2) As the speed increases, the resisting force also increases, until finally it is equal in magnitude to the weight. At this time,

$$\boldsymbol{mg - kv_y = 0}$$

the acceleration becomes zero, and there is no further increase in speed.

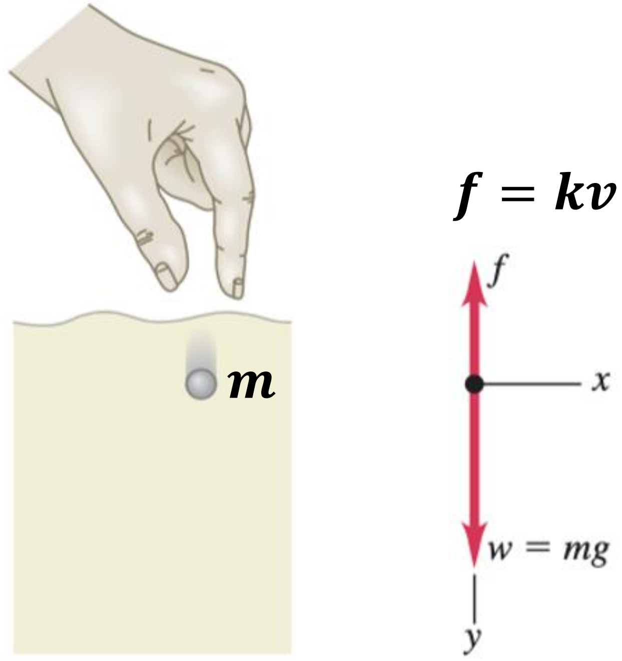
(3) The final speed $\boldsymbol{v_t}$, which is defined as the terminal speed, is then given by

$$\boldsymbol{v_t = \frac{f}{k} = \frac{mg}{k}}$$

Problem 8

Question (b):

(b) Find the relationship between velocity $\boldsymbol{v_y}$ and time $\boldsymbol{t}$.



Solution (b):

(1) First rewrite the Newton's second law as follows:

$$m\frac{dv_y}{dt} = mg - kv_y$$

(2) Then separate the variable $\boldsymbol{v_y}$ from $\boldsymbol{t}$ as: $\dfrac{dv_y}{(mg/k) - v_y} = \dfrac{k}{m}dt$

(3) Integrate both sides (noting that $\boldsymbol{v_y = 0}$ when $\boldsymbol{t = 0}$): $\displaystyle\int_0^{v_y}\frac{dv_y}{(mg/k) - v_y} = \int_0^t \frac{k}{m}dt$

(4) This gives: $ln\dfrac{(mg/k) - v_y}{(mg/k)} = -\dfrac{k}{m}t$ or finally $v_y(t) = \dfrac{mg}{k}\left(1 - e^{-kt/m}\right)$

Problem 8

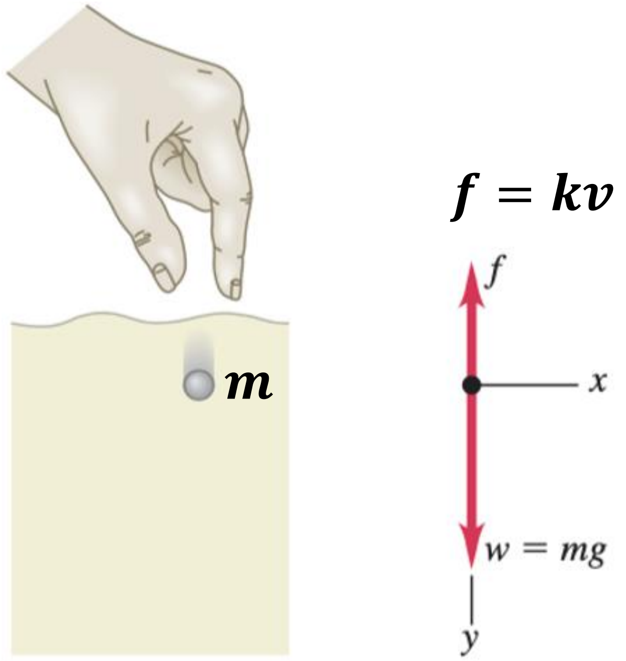
Question (c):

(c) Find the relationship between acceleration $\boldsymbol{a_y}$ and time $\boldsymbol{t}$.

Solution (c):

By definition $\boldsymbol{a_y = \mathrm{d}v_y/\mathrm{d}t}$, one finds:

$$\boldsymbol{a_y = \frac{\mathrm{d}v_y(t)}{\mathrm{d}t} = g e^{-kt/m}}$$



Question (d):

(d) Find the relationship between displacement $\boldsymbol{y(t)}$ and time $\boldsymbol{t}$.

Solution (d):

By definition $\boldsymbol{v_y = \mathrm{d}y/\mathrm{d}t}$, thus:

$$\boldsymbol{y(t) = \int_0^t v_y(t)\,\mathrm{d}t = \frac{mg}{k}t - \frac{m^2 g}{k^2}\left(1 - e^{-kt/m}\right)}$$

Problem 8

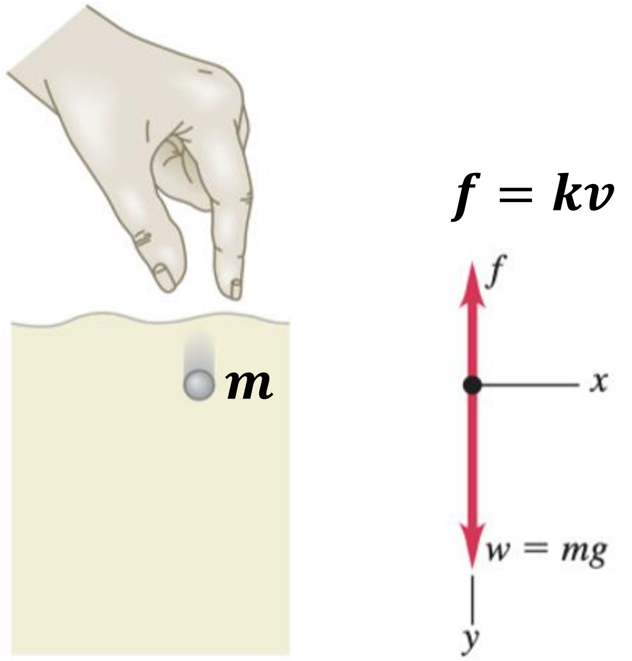
Hint: We use the Taylor expansion of $e^x = 1 + x$ when x is very small

Question (e):

(e) Consider the velocity $\boldsymbol{v_y}$ by taking the limit $\boldsymbol{t \to 0}$ and $\boldsymbol{t \to \infty}$, explain why you think the results make sense in these two limits .

Solution (e):

(1) Originally, we have $\boldsymbol{v_y(t) = \frac{mg}{k}(1 - e^{-kt/m})}$



for $\boldsymbol{t \to 0}$, the Taylor expansion: $\boldsymbol{e^{-kt/m} = 1 - \frac{kt}{m}}$

Then, yield:

$$\boldsymbol{v_y(t) = \frac{mg}{k}\left(1 - 1 + \frac{kt}{m}\right) = gt}$$

This results makes sense since the friction force is very small when $\boldsymbol{v_y}$ is small in the limit $\boldsymbol{t \to 0}$. In this case, the metal ball is accelerating with constant **g**.

Problem 8

Question (e):

(e) Consider the velocity $\boldsymbol{v_y}$ by taking the limit $\boldsymbol{t} \to \boldsymbol{0}$ and $\boldsymbol{t} \to \infty$, explain why you think the results make sense in these two limits .

Answer (e):

(2) Originally, we have

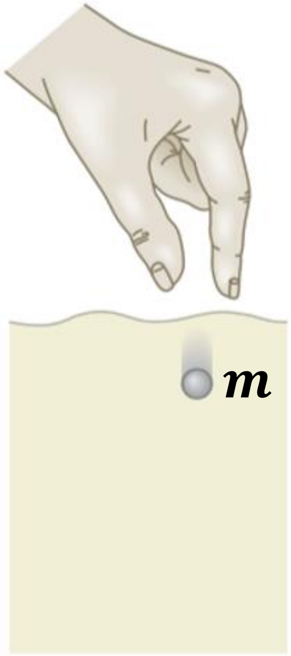
$$\boldsymbol{v_y(t) = \frac{mg}{k}(1 - e^{-kt/m})}$$

for $\boldsymbol{t} \to \infty$, obtaining $\boldsymbol{e^{-kt/m}} \to \boldsymbol{0}$.

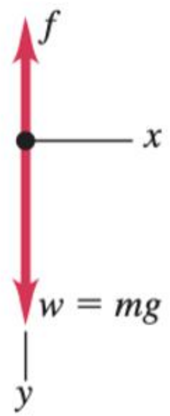
Then, yield:

$$\boldsymbol{v_y(t) = \frac{mg}{k}(1 - 0) = \frac{mg}{k} = v_t}$$

It approaches the terminal velocity as expected.



$\boldsymbol{f = kv}$



QED.

Problem 9

** Suppose you are moving a crate on a level (horizontal) floor with weight $\boldsymbol{w}$ by pulling upward on the rope at an angle of $\boldsymbol{\beta}$ above the horizontal.

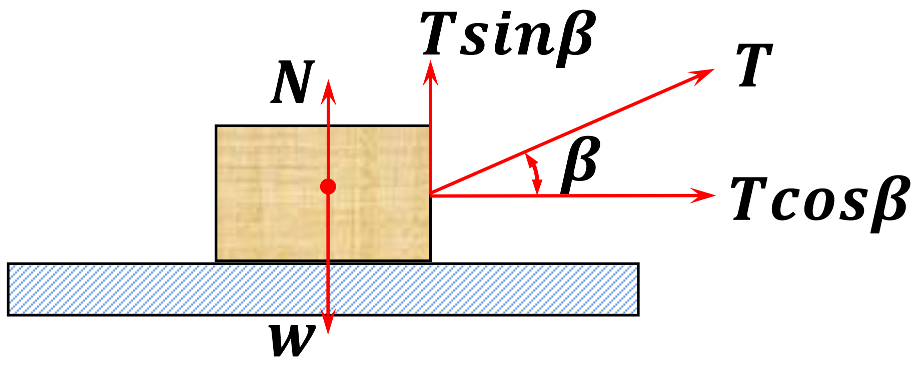
Question (a): Given the coefficient of the kinetic friction $\boldsymbol{\mu}$, how hard must you pull to keep it moving with constant velocity?

Solution (a):

(1) From the equilibrium conditions and the equation $\boldsymbol{f = \mu N}$, we have

$$\boldsymbol{T\cos\beta - f = 0} \qquad (1)$$

$$\boldsymbol{T\sin\beta + N - w = 0} \qquad (2)$$



Combing the equations (1) and (2), yields:

$$\boldsymbol{T = \frac{\mu w}{(\cos\beta + \mu\sin\beta)}}$$

Gao Qian

Problem 9

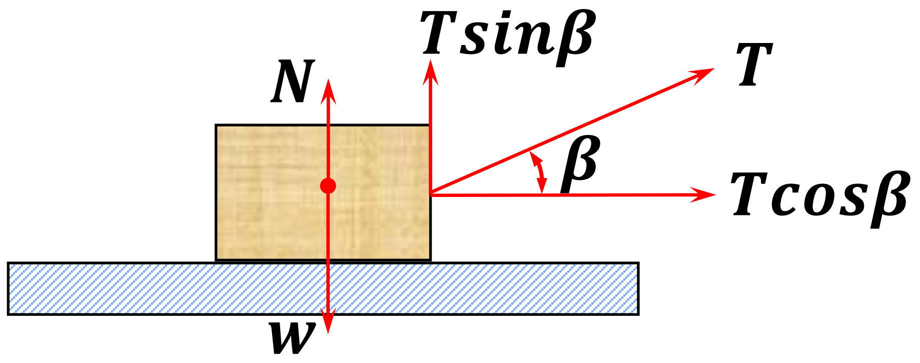
Question (b): Can you find an angle where the required pull is minimum?

Solution (b):

(1) Originally, we have $\boldsymbol{T} = \dfrac{\boldsymbol{\mu w}}{(\boldsymbol{cos\beta} + \boldsymbol{\mu sin\beta})}$

(2) In order to minimize $\boldsymbol{T}$, we can maximize the $\boldsymbol{D(\beta)}$, where

$$\boldsymbol{D(\beta) = cos\beta + \mu sin\beta}$$



(3) Setting $\boldsymbol{D'(\beta) = 0}$ gives

$$\boldsymbol{\frac{dD(\beta)}{d\beta} = -sin\beta + \mu cos\beta \ \rightarrow tan\beta = \mu}$$

(4) In addition, to check the second order derivative, one finds

$$\boldsymbol{\frac{d^2D(\beta)}{d\beta^2} = -cos\beta - \mu sin\beta = -\sqrt{1+\mu^2} < 0}$$

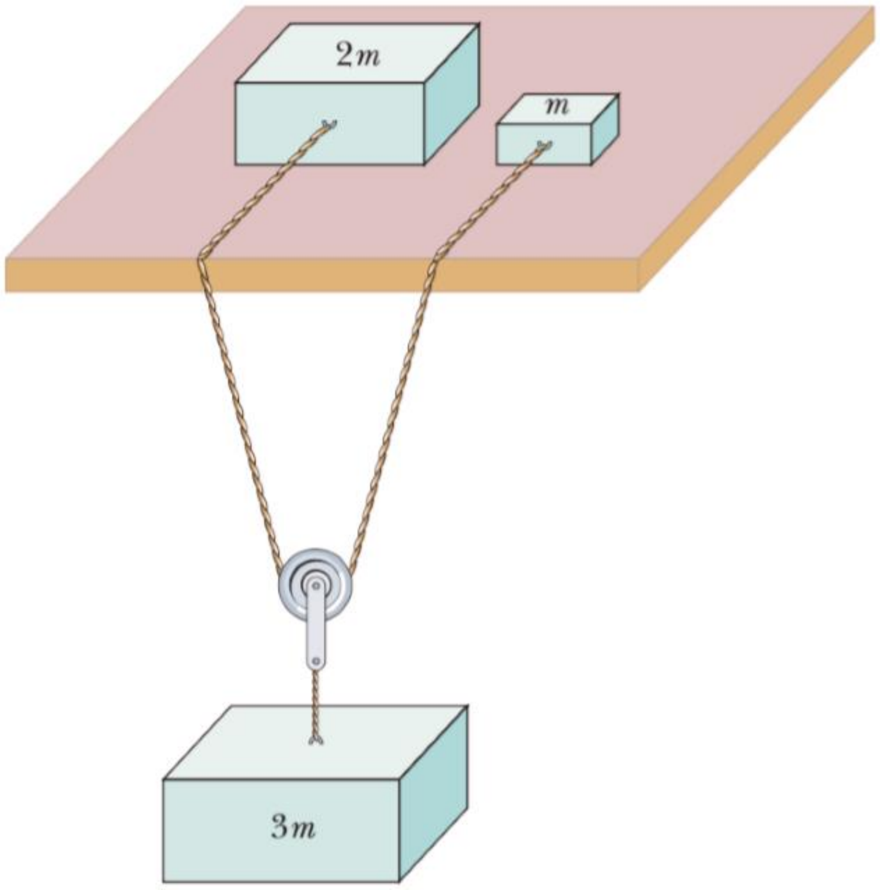
QED.

Gao Qian

Problem 10

香港中文大學(深圳)理工学院
School of Science and Engineering

* * * (*Textbook,C5_P13*) Two particles of masses $\boldsymbol{m}$ and $\mathbf{2}\boldsymbol{m}$ are placed on a smooth horizontal table. A string, which joins these two masses, hangs over the edge supporting a pulley, which suspends a particle of mass $\mathbf{3}\boldsymbol{m}$, as shown below. The pulley has negligible mass. The two parts of the string on the table are parallel and perpendicular to the edge of the table. The hanging parts of the string are vertical. **Find** the acceleration of the particle of mass $\mathbf{3}\boldsymbol{m}$ after releasing it from the equilibrium position.



Hint: $\boldsymbol{a_1} \leftrightarrow \mathbf{2}\boldsymbol{m}$, $\boldsymbol{a_2} \leftrightarrow \boldsymbol{m}$ and $\boldsymbol{a_3} \leftrightarrow \mathbf{3}\boldsymbol{m}$, they have the relation:

$$\boldsymbol{a_3} = (\boldsymbol{a_1} + \boldsymbol{a_2})/2$$

$$\boldsymbol{a_2} - \boldsymbol{a_3} = -(\boldsymbol{a_1} - \boldsymbol{a_3})$$

Problem 10

Solution: (Consider the free-body diagrams for each object separately)

(1) For the mass $\boldsymbol{3m}$, we have

$$3mg - 2T = 3ma_3 \qquad (1)$$

where T is the tension of string.

(2) For the mass $\boldsymbol{2m}$, we have

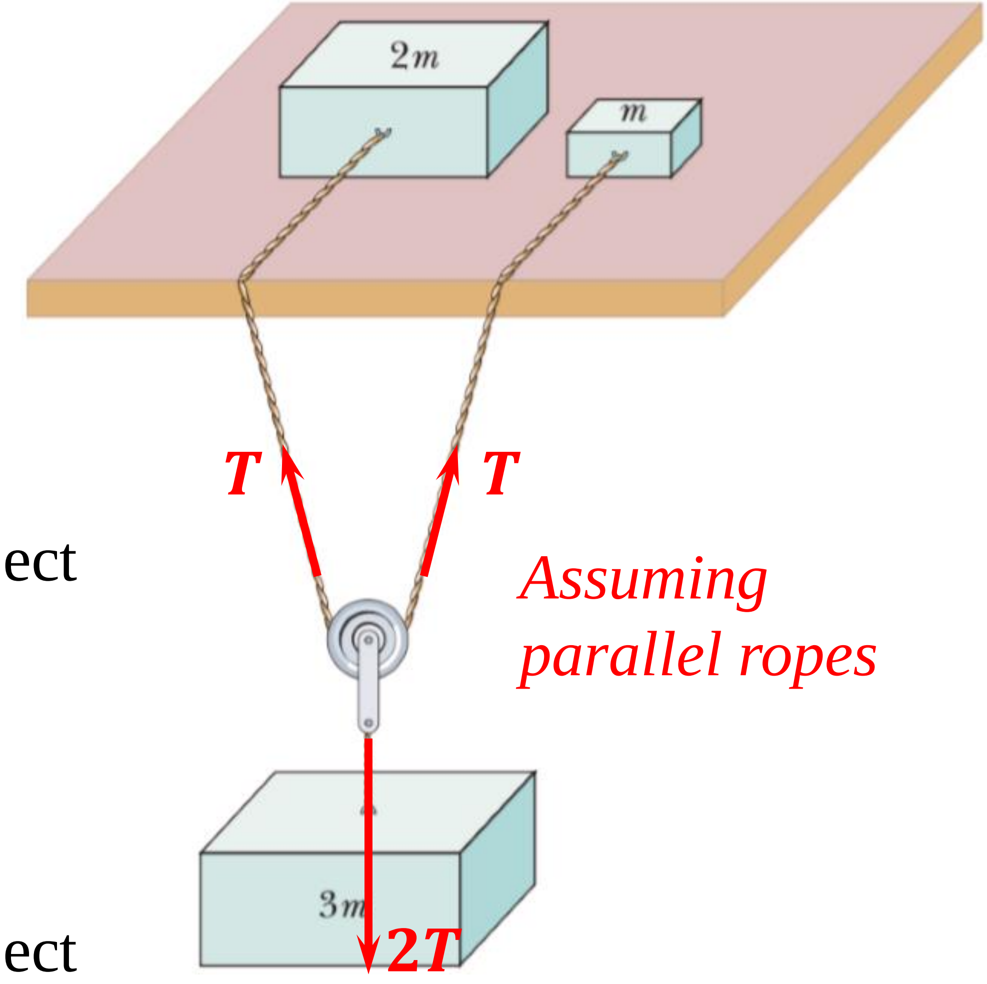
$$T = 2ma_1 \qquad (2)$$

where a_1 is defined as the acceleration of the object with mass of $\boldsymbol{2m}$.

(3) For the mass $\boldsymbol{m}$, we have

$$T = ma_2 \qquad (3)$$

where a_2 is defined as the acceleration of the object with mass of $\boldsymbol{m}$.



Gao Qian

Problem 10

Solution:

(4) Combining equations (1), (2) and (3), we can obtain:

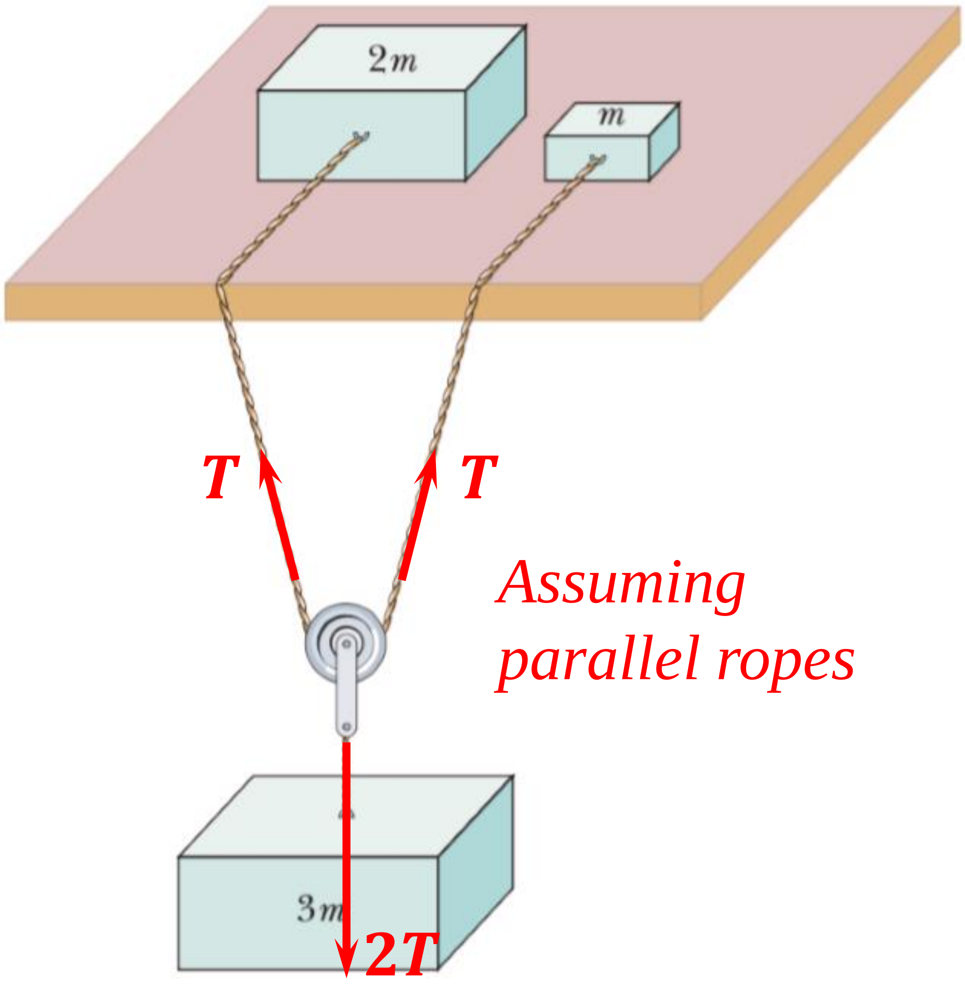
$$\begin{cases} a_2 = 2a \\ a_1 = a \end{cases} \qquad (4)$$

and further

$$a_3 = \frac{a_1 + a_2}{2} = \frac{3a}{2} \qquad (5)$$

(5) According to the geometric constraint of the pulley system, putting equations **(2)~(5)** into **(1)** gives

$$\begin{cases} a = \dfrac{6}{17} g \\ a_3 = \dfrac{9}{17} g = 5.2\ m/s^2 \end{cases} \qquad (6)$$



QED.

Problem 11

* * Suppose we know the force $f(v)$ as function of v

Question (a): The net force on a body moving along the $\boldsymbol{x}$-axis equals $-\boldsymbol{C}\boldsymbol{v}^2$. Use Newton's second law written as $\boldsymbol{F} = \boldsymbol{m}\boldsymbol{d}\boldsymbol{v}/\boldsymbol{d}\boldsymbol{t}$ and two integrations to **show** that $\boldsymbol{x} - \boldsymbol{x}_0 = \frac{\boldsymbol{m}}{\boldsymbol{C}}\ln\frac{\boldsymbol{v}_0}{\boldsymbol{v}}$, with $\boldsymbol{x}_0$ and $\boldsymbol{v}_0$ the initial position and velocity, respectively.

Solution (a):

(1) Start from the equation of motion $-\boldsymbol{C}\boldsymbol{v}^2 = \boldsymbol{m}\boldsymbol{d}\boldsymbol{v}/\boldsymbol{d}\boldsymbol{t}$, and separate variables before integration as follows

$$-\frac{C}{m}dt = \frac{dv}{v^2} \Longrightarrow -\frac{C}{m}t = -\frac{1}{v} + \frac{1}{v_0}$$

(2) which can be cast into

$$v = \frac{dx}{dt} = \frac{v_0}{1 + Cv_0t/m}$$

Problem 11

(3) Integrate the above formula w.r.t. time again, one gets

$$x - x_0 = \frac{m}{C}\ln\left(1 + \frac{Cv_0t}{m}\right) = \frac{m}{C}\ln\frac{v_0}{v}$$

Question (b): Use the chain rule to show that Newton's second law can be written as $F = mvdv/dx$. **Derive** the same expression as in part (a) using this form of the second law and one integration.

Solution (b):

(1) By the chain rule, $\dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} = \dfrac{dv}{dx}v$

(2) and using the expression for the net force $-Cv^2$, one finds the Second law becomes $-Cv^2 = mvdv/dx$.

(3) Rewrite this expression: $dx = -\dfrac{m}{C}\dfrac{dv}{v} \xrightarrow{\text{integrate on its both sides}} x - x_0 = \dfrac{m}{C}\ln\dfrac{v_0}{v}$

QED.

Gao Qian