PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star $* * *$ labels are assigned to the most difficult ones.

Due date: 2024, May 5th, 23: 59 :00 .

1 Homework Problems for Week 12: Chapter 16 Wave I

1. $*$ The wave function for a harmonic wave on a string is $y(x,t) = (1.00mm)\sin(62.8m^{-1}x + 314s^{-1}t)$.

   (a) In what direction does this wave travel, and what is the wave's speed?
   Answers: $-x$ direction and $v = 5.00$ m/s

   (b) Find the wavelength, frequency, and period of this wave.
   Answers: $\lambda = 0.100$ m, $f = 50.0$ Hz, and $T = 0.0200$ s.

   (c) What is the maximum speed of any point on the string?
   Answers: $v_y^{max} = 0.314$ m/s.

2. $*$Does the following wave functions satisfies the wave equation? Show detail derivations.

   (a) $y(x,t) = A\cos(kx + \omega t)$
   Answers: Yes.

   (b) $y(x,t) = A\sin(kx - \omega t)$
   Answers: Yes.

   (c) $y(x,t) = A[\cos(kx) + \cos(\omega t)]$
   Answers: No.

   (d) For the wave of part (b), write the equations for the transverse velocity and transverse acceleration of a particle at point x.
   Answers: For particle at point x

$$v_y = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)\,,$$
$$a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)\,.$$

3. $* *$ These two waves travel along the same string:

$$y_1(x,t) = (4.00mm)\sin(2\pi x - 650\pi t),$$
$$y_2(x,t) = (6.20mm)\sin(2\pi x - 650\pi t + 0.60\pi).$$

   (a) What are the amplitude and the phase angle (relative to wave 1) of the resultant wave?
   Answers: 6.25 mm and 1.23 rad.

   (b) If a third wave of amplitude 5.00 mm is also to be sent along the string in the same direction as the first two waves, what should be its phase angle in order to maximize the amplitude of the new resultant waves?
   Answers: 1.23 rad.

4. $* *$ Speed of Propagation vs. Particle Speed.

   (a) Show that the wave function $y(x,t) = A\cos(kx - \omega t)$ may be written as

$$y(x,t) = A\cos\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

   (b) Use $y(x,t)$ to find an expression for the transverse velocity v_y of a particle in the string on which the wave travels.
   Answers: $v_y = \frac{2\pi A}{\lambda} v\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$.

   (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed v?
   Answers: $v_y^{max} = \frac{2\pi A}{\lambda} v$. When $A = \frac{\lambda}{2\pi}$.

5. $*$ Three pieces of string, each of length L, are joined together end to end, to make a combined string of length $3L$. The first piece of string has mass per unit length μ_1, the second piece has mass per unit length $\mu_2 = 4\mu_1$, and the third piece has mass per unit length $\mu_3 = \mu_1/4$.

   (a) If the combined string is under tension F, how much time does it take a transverse wave to travel the entire length $3L$? Give your answer in terms of L, F, and μ_1.
   Answers: $\frac{7}{2}L\sqrt{\mu_1/F}$

   (b) Does your answer to part (a) depend on the order in which the three pieces are joined together? Explain.
   Answers: NO. v only depends on F and μ.

6. $*$ A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire.

   (a) Calculate the average power carried by the wave.
   Answers: $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = 0.22$ W (keep up to 2 significant figures).

   (b) What happens to the average power if the wave amplitude is halved?
   Answers: $P_{av} = 0.056$ W.

7. $* *$ A commonly used physics experiment that examines resonances of transverse waves on a string is shown in Figure below. A weight is attached to the end of a string draped over a pulley; the other end of

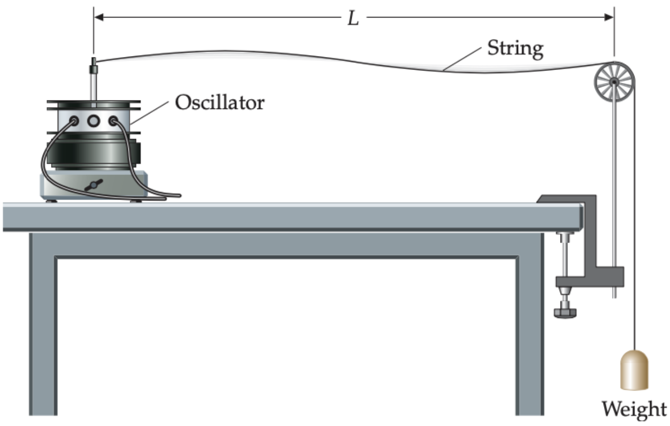
the string is attached to a mechanical oscillator that moves up and down at a frequency f that remains fixed throughout the demonstration. The length L between the oscillator and the pulley is fixed, and the tension is equal to the gravitational force on the weight. For certain values of the tension, the string resonates. Assume the string does not stretch or shrink as the tension is varied. The amplitude of the motion at oscillator is **small enough** for that point to be considered a node. You are in charge of setting up this apparatus for a lecture demonstration.

(a) Explain why only certain discrete values of the tension result in standing waves on the string.
Answers: For fixed f, only discrete values of λ and v can result in standing waves. This leads to discrete values of F_T since $v = \sqrt{F_T/\mu}$.

(b) Do you need to increase or decrease the tension to produce a standing wave with an additional antinode? Explain.
Answers: Decrease F.

(c) Prove your reasoning in Part (b) by showing that the values for the tension F_{Tn} for the n—th standing-wave mode are given by $F_{Tn} = 4L^2 f^2 \mu/n^2$, and thus the F_{Tn} is inversely proportional to n^2.

(d) For your particular setup to fit onto the lecture table, you chose $L = 1.00$ m, $f = 80.0$ Hz, and $\mu = 0.750$ g/m. Calculate how much tension is needed to produce each of the first three modes (standing waves) of the string.
Answers: $F_{T1} = 19.2$ N, $F_{T2} = 19.2/2^2 = 4.80$ N and $F_{T3} = 19.2/3^2 = 2.13$ N.



8. ∗∗ On a real string with the linear mass density μ, some of the energy of a wave dissipates as the wave travels down the string. Such a situation can be described by a wave function whose amplitude $A(x)$ depends on x: $y = A(x)\sin(kx - \omega t)$, where $A(x) = A_0 e^{-bx}$. What is the power transported by the wave as a function of x, where $x > 0$?
Answers: $P = \frac{1}{2}\mu v \omega^2 A_0^2 e^{-2bx}$.

9. ∗∗ (Halliday C16-P59)
A string oscillates according to the equation

$$y_s = (0.80\,cm)\sin\left[\left(\frac{\pi}{3}\,cm^{-1}\right)x\right]\cos\left[(40\pi s^{-1})t\right].$$

(a) What are the amplitude of the two waves (identical except for direction of travel) whose superposition gives this oscillation?
Answers: 0.40 cm.

(b) What are the speed of these two waves?
Answers: 1.2 m/s

(c) What is the distance between nodes?
Answers: 3.0 cm.

(d) What is the transverse speed of a particle of the string at the position $x = 2.1$ cm when $t = 0.50$ s?
Answers: $v_y = \partial y/\partial t = 0$ at $x = 2.1$ cm when $t = 0.50$.

10. ∗∗∗ Waves of Arbitrary Shape.

(a) Explain why any wave described by a function of the form $y(x,t) = f(x - vt)$ moves in the $+x$-direction with speed v.

(b) Show that $y(x,t) = f(x - vt)$ satisfies the wave equation, no matter what the functional form of f.
Hint: Write $y(x,t) = f(u)$, where $u = x - vt$. Then to take partial derivatives of $y(x,t)$ use the chain rule

$$\frac{\partial y(x,t)}{\partial t} = \left(\frac{df}{du}\right)(-v), \quad \text{and} \quad \frac{\partial y(x,t)}{\partial x} = \left(\frac{df}{du}\right)$$

(c) A wave pulse is described by the function $y(x,t) = De^{-(Bx - Ct)^2}$, where B, C, and D are all positive constants. What is the speed of this wave?
Answers: According to the wave equation in part b), $v = C/B$.