香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star ∗ ∗ ∗ labels are assigned to the most difficult ones.

Due date: 2024, March 10th, 23: 59: 00.

1 Homework Problems for Week 5: Chapter 10 Rotation

1. ∗ A bicycle wheel has an initial angular velocity of 1.50 rad/s.

   (a) If its angular acceleration is constant and equal to 0.300 rad/s^2, what is its angular velocity at $t = 2.50$ s?
   Answers: 2.25 rad/s.

   (b) Through what angle has the wheel turned between $t = 0$ and $t = 2.50$ s?
   Answers: 4.69 rad.

2. ∗ Rotating Wheel:
   Sketch a wheel lying in the plane of your paper and rotating counterclockwise. Choose a point on the rim and draw a vector $\vec{r}$ from the center of the wheel to that point.

   (a) What is the direction of angular velocity $\vec{\omega}$?
   Answers: Upward, perpendicular to the paper.

   (b) Check that the velocity $\vec{v}$ of the point can be written as $\vec{v} = \vec{\omega} \times \vec{r}$. Remember to show that this is true for both the direction and magnitude.
   Hint: There are two methods you can use. 1. Simply use the right-hand rule to determine the direction of the cross product. 2. Use the mathematical expression of the cross product in terms of unit vectors.

   (c) Check that the radial acceleration of the point is $\vec{a}_{rad} = \vec{\omega} \times \vec{v} = -\omega^2\vec{r}$. Remember to show that this is true for both the direction and magnitude.
   Hint: There are also two methods you can use. 1. Simply use the right-hand rule to determine the direction of the cross product. 2. Apply the following mathematical identity

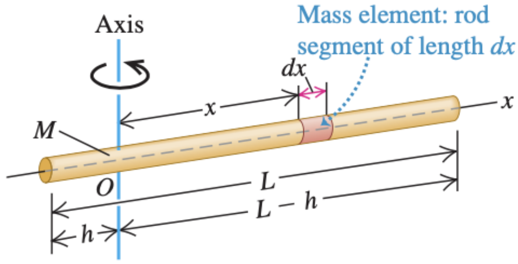
   $$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

   and show that $\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2\vec{r}$.

   (d) Given the mass of the thin-rimmed wheel m, compute the moment of inertia of this wheel.
   Answers: mr^2.

3. ∗ ∗ A Rotating, Uniform Thin Rod:

A thin rod with an axis through O.



The figure above shows a slender uniform rod with mass M and length L. It might be a baton held by a twirler in a marching band (less the rubber end caps).

   (a) Use integration to compute its moment of inertia about an axis through O, at an arbitrary distance h from one end.
   Answers: Moment of inertial:

   $$I = \left[\frac{M}{L}\left(\frac{x^3}{3}\right)\right]_{-h}^{L-h} = \frac{M}{3}(L^2 - 3Lh + 3h^2).$$

   (b) Check the plausibility of the above answer for I. Set $h = 0$ or $h = L/2$, and check whether it agrees with the known answers or not. Use parallel-axis theorem ($I = I_{CM} + Md^2$) to compute the rotational inertia I again.

   (c) Initially the rod is at rest. It is given a constant angular acceleration of magnitude α around the axis through O. Find how much work is done on the rod in a time t.
   Answers: Work equals the increase of kinetic energy

   $$W = \frac{1}{2}I(\alpha t)^2.$$

   (d) At time t, what is the magnitude of the total linear acceleration of the point on the rod farthest from the axis? (Suppose $L - h > h$)
   Hint: Don't forget the acceleration includes the radial and tangential parts.
   Answers: $a = \sqrt{a_\perp^2 + a_\parallel^2}$ with $a_\perp = (\alpha t)^2(L - h)$ and $a_\parallel = \alpha(L - h)$.

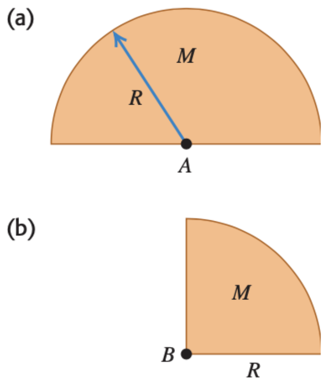
4. ∗ Energy from the Moon?
   Suppose that some time in the future we decide to tap the moon's rotational energy for use on earth. In additional to the usual astronomical data ($M = 7.35 \times 10^{22}$ kg and $R = 1.74 \times 10^6$ m), you may need to know that the moon spins on its axis once every 27.3 days. Assume that the moon is uniform throughout.

   (a) How much total energy could we get from the moon's rotation?
   Answers: $K = 3.15 \times 10^{23}$ J.

(b) The world presently uses about 4.00×10^{20} J of energy per year. If in the future the world uses five times as much energy yearly, for how many years would the moon's rotation provide us energy?
Answers: About 158 years. Not seems like quite cost-effective energy source.

5. $*$ A roller in a printing press turns through an angle $\theta(t)$ given by $\theta(t) = \gamma t^2 - \beta t^3$, where $\gamma = 3.20\,\text{rad/s}^2$ and $\beta = 0.500\,\text{rad/s}^3$.
   (a) Calculate the angular velocity of the roller as a function of time.
   Answers: $\omega_z(t) = 2\gamma t - 3\beta t^2$.
   (b) Calculate the angular acceleration of the roller as a function of time.
   Answers: $\alpha_z(t) = 2\gamma - 6\beta t$.
   (c) What is the maximum positive angular velocity, and at what value of t does it occur?
   Answers: The maximum $\omega_z = 6.83$ rad/s and it occurs at $\gamma/3\beta = 2.13$s.

6. $*$ A uniform disk of radius R is cut in half so that the remaining half has mass M (Fig. a).

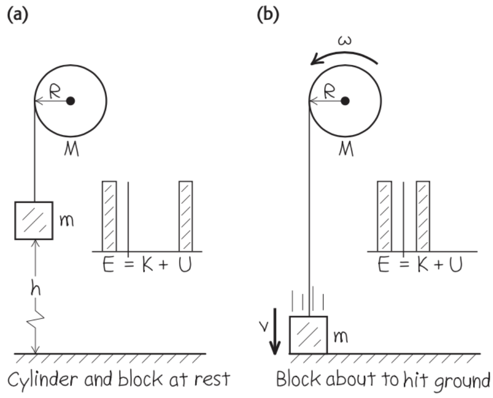


   (a) What is the moment of inertia of this half about an axis perpendicular to its plane through point A?
   (b) Why did your answer in part (a) come out the same as if this were a complete disk of mass M?
   (c) What would be the moment of inertia of a quarter disk of mass M and radius R about an axis perpendicular to its plane passing through point B (Fig. b)?

   Comment: The moment of inertia depends on how the mass of the object is distributed relative to the axis, and this is the same for any segment of a disk.

7. $**$ A thin, uniform rod is bent into a square of side length a. If the total mass is M, find the moment of inertia about an axis through the center and perpendicular to the plane of the square. (**Hint:** Use the parallel-axis theorem.)
   Answers: $Ma^2/3$.

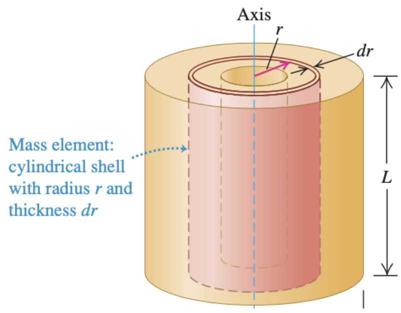
8. $*$ An unwinding cable



   We wrap a light, non-stretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. Find expressions for the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

   Answers: $v = \sqrt{\dfrac{2gh}{1 + I/mR^2}}$ with $I = \dfrac{MR^2}{2}$ and $\omega = v/R$.

9. $**$ Find the rotational inertia of a solid and uniform cylinder about its central axis.
   Answers: $I = \dfrac{1}{2}MR^2$.



10. $***$ A cylinder with radius R and mass M has density that increases linearly with distance r from the cylinder axis, $\rho = \alpha r$, where α is a positive constant.
   (a) Calculate the moment of inertia of the cylinder about a longitudinal axis through its center in terms of M and R.
   Answers: $I = \dfrac{3}{5}MR^2$
   (b) Is your answer greater or smaller than the moment of inertia of a cylinder of the same mass and radius but of uniform density ($I = MR^2/2$)? Explain why this result makes qualitative sense.
   Answers: Greater. (3/5 > 1/2) More mass is distributed in the outer layer.