PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star $*\,*\,*$ labels are assigned to the most difficult ones.

1 Homework Problems for Week 13: Chapter 17 Wave II

1. $*$ Speed of sound waves

   (a) In a liquid with density $1300 kg/m^3$, longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid.
   Answers: 1.33×10^{10} Pa.

   (b) A metal bar with a length of 1.50 m has density $6400 kg/m^3$. Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this metal?
   Answers: 9.47×10^{10} Pa.

   Solution:

   (a) For waves in general, $v = f\lambda$. For a wave in the liquid, $v = \sqrt{B/\rho}$ with B the bulk modulus and ρ the density. Using these two formulas, one gets

   $$B = v^2\rho = (f\lambda)^2\rho = 1.33 \times 10^{10} Pa. \tag{1}$$

   (b) For wave velocity in general, $v = \Delta L/\Delta t$. For a wave in the metal, $v = \sqrt{Y/\rho}$ with Y the Young's modulus and ρ the density. Using these two formulas, one gets

   $$Y = v^2\rho = (\Delta L/\Delta t)^2\rho = 9.47 \times 10^{10} Pa. \tag{2}$$

2. $*\,*$ Standing Sound Waves and Normal Modes

   (a) Standing sound waves are produced in a pipe that is 1.20 m long. For the fundamental mode (i.e. first harmonic) and first two overtones (i.e., the second and third harmonics), determine the locations along the pipe (measured from the left end) of the displacement nodes and the pressure nodes if the pipe is open at both ends.
   Answers: Location of the displacement nodes (N) measured from the left end: fundamental 0.60 m
   1st overtone 0.30 m, 0.90 m
   2nd overtone 0.20 m, 0.60 m, 1.00 m.
   Answers: Location of the pressure nodes (displacement antinodes (A)) measured from the left end: fundamental 0, 1.20 m
   1st overtone 0, 0.60 m, 1.20 m
   2nd overtone 0, 0.40 m, 0.80 m, 1.20 m.

   (b) What if the pipe is closed at the left end and open at the right end?
   Answers: Location of the displacement nodes (N) measured from the closed end: fundamental 0
   1st overtone 0, 0.80 m
   2nd overtone 0, 0.48 m, 0.96 m.
   Answers: Location of the pressure nodes (displacement antinodes (A)) measured from the closed end: fundamental 1.20 m
   1st overtone 0.40 m, 1.20 m
   2nd overtone 0.24 m, 0.72 m, 1.20 m.

   (c) Singing in the Shower. A pipe closed at both ends can have standing waves inside of it, but you normally don't hear them because little of the sound can get out. But you can hear them if you are inside the pipe, such as someone singing in the shower. Show that the wavelengths of standing waves in a pipe of length L that is closed at both ends are $\lambda_n = 2L/n$ and the frequencies are given by $f_n = nv/(2L)$, where $n = 1, 2, 3, \cdots$. Modeling the shower as a pipe, find the frequency of the fundamental and the first two over-tones for a shower 2.50 m tall. Are these frequencies audible?
   Answers: Derive $\lambda_n = 2L/n$ and $f_n = nv/(2L)$, then find $f_1 = 68.8$ Hz, $f_2 = 138$ Hz, and $f_3 = 206$ Hz. Yes, they are within the range of the audible sound frequency (audible range) from about 20 Hz to $20,000$ Hz.

   Solution:

IDENTIFY and SET UP: An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.
EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.25a. The open ends are displacement antinodes.

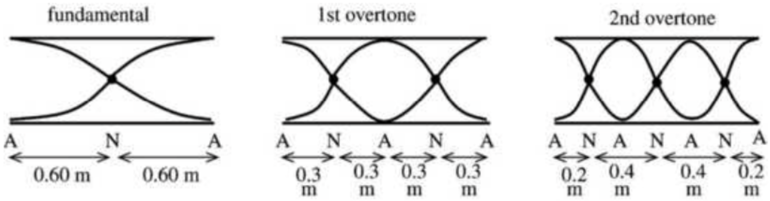


Figure 16.25a

Location of the displacement nodes (N) measured from the left end:
fundamental 0.60 m
1st overtone 0.30 m, 0.90 m
2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:
fundamental 0, 1.20 m
1st overtone 0, 0.60 m, 1.20 m
2nd overtone 0, 0.40 m, 0.80 m, 1.20 m
(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.25b.

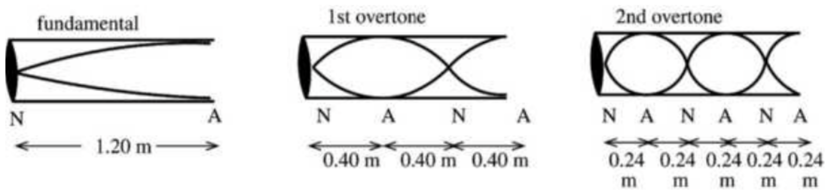


Figure 16.25b

Location of the displacement nodes (N) measured from the closed end:
fundamental 0
1st overtone 0, 0.80 m
2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:
fundamental 1.20 m
1st overtone 0.40 m, 1.20 m
2nd overtone 0.24 m, 0.72 m, 1.20 m
EVALUATE: The node-to-node or antinode-to-antinode distance is $\lambda/2$. For the higher overtones the frequency is higher and the wavelength is smaller.

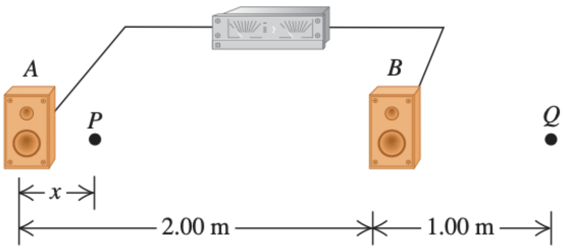
IDENTIFY: There must be a node at each end of the pipe. For the fundamental there are no additional nodes and each successive overtone has one additional node. $v = f\lambda$.

SET UP: $v = 344$ m/s. The node to node distance is $\lambda/2$.

EXECUTE: **(a)** $\dfrac{\lambda_1}{2} = L$ so $\lambda_1 = 2L$. Each successive overtone adds an additional $\lambda/2$ along the pipe, so $n\left(\dfrac{\lambda_n}{2}\right) = L$ and $\lambda_n = \dfrac{2L}{n}$, where $n = 1, 2, 3, \ldots$ $f_n = \dfrac{v}{\lambda_n} = \dfrac{nv}{2L}$.

(b) $f_1 = \dfrac{v}{2L} = \dfrac{344\text{ m/s}}{2(2.50\text{ m})} = 68.8$ Hz. $f_2 = 2f_1 = 138$ Hz. $f_3 = 3f_1 = 206$ Hz. All three of these frequencies are audible.
EVALUATE: A pipe of length *L* closed at both ends has the same standing wave wavelengths, frequencies and nodal patterns as for a string of length *L* that is fixed at both ends.

3. ∗∗ Two loudspeakers, A and B (Figure below), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q. (a) (b)



(a) What is the lowest frequency for which constructive interference occurs at point Q?
Answers: 172 Hz.

(b) What is the lowest frequency for which destructive interference occurs at point Q?
Answers: 86 Hz.

Solution:

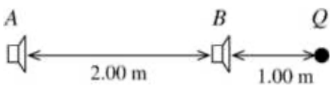


Figure 16.33

(a) IDENTIFY and SET UP: Path difference from points A and B to point Q is $3.00\text{ m} - 1.00\text{ m} = 2.00\text{ m}$, as shown in Figure 16.33. Constructive interference implies path difference $= n\lambda$, $n = 1, 2, 3, \ldots$

EXECUTE: $2.00\text{ m} = n\lambda$ so $\lambda = 2.00\text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{2.00\text{ m}} = \frac{n(344\text{ m/s})}{2.00\text{ m}} = n(172\text{ Hz}), \quad n = 1, 2, 3, \ldots$$

The lowest frequency for which constructive interference occurs is 172 Hz.

(b) IDENTIFY and SET UP: Destructive interference implies path difference $= (n/2)\lambda$, $n = 1, 3, 5, \ldots$

EXECUTE: $2.00\text{ m} = (n/2)\lambda$ so $\lambda = 4.00\text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{4.00\text{ m}} = \frac{n(344\text{ m/s})}{(4.00\text{ m})} = n(86\text{ Hz}), \quad n = 1, 3, 5, \ldots.$$

The lowest frequency for which destructive interference occurs is 86 Hz.

EVALUATE: As the frequency is slowly increased, the intensity at Q will fluctuate, as the interference changes between destructive and constructive.

4. ∗ For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6.0×10^{-5} Pa. (This is a very faint sound and the displacement and pressure amplitudes are very small.) At 20°C for this sound wave (At 20°C, note that the bulk modulus for air is 1.42×10^5 Pa and $v = 344$ m/s.), calculate

(a) the intensity; **Answers:** $4.4 \times 10^{-12}\, W/m^2$

(b) the sound intensity level; (defined as ten times the logarithm in base 10 with $I_0 = 1 \times 10^{-12} W/m^2$) $\beta = (10\, dB) \log_{10} \frac{I}{I_0}$. **Answers:** $6.4\, dB$

(c) the displacement amplitude. **Answers:** 5.8×10^{-11} m.

Solution:

(a) Note that the pressure amplitude p_m and displacement amplitude A_m are related by

$$p_m = \rho v^2 k A_m = BkA_m = \left(\frac{2\pi f}{v}\right) BA_M = \omega A_m B/v. \tag{3}$$

In addition, the intensity

$$I = \frac{1}{2}\rho v\omega^2 A_m^2 = \frac{1}{2}\rho v\left(\frac{p_m v}{B}\right)^2 = \frac{1}{2}\rho v\frac{p_m^2}{B^2}\frac{B}{\rho} = \frac{vp_m^2}{2B} = \frac{344\text{m/s}(6.0\times 10^{-5})^2\text{Pa}^2}{2\times 1.42\times 10^5 Pa} = 4.4\times 10^{12}\text{W/m}^2. \tag{4}$$

(b) $\beta = (10\,dB)\log_{10}\frac{I}{I_0} = (10\,dB)\log_{10}4.4 = 6.4\,dB.$

(c) From above relation, one gets $A_m = \frac{vp_m}{2\pi f B} = 5.8\times 10^{-11}$ m, which is very small size in length.

5. ∗ A police car's siren emits a sinusoidal wave with frequency $f_s = 300$ Hz. The speed of sound is 340 m/s and the air is still.

   (a) Find the wavelength of the waves if the siren is at rest.
   Answers: 1.13 m.

   (b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.
   Answers: 1.03 m (in front) and 1.23 m (behind).

   Solution:

   (a) In this part there is no Doppler effect because neither source nor listener is moving with respect to the air, thus the formula $v = \lambda f$ gives the wavelength. When the source is at rest,

$$\lambda = \frac{v}{f_s} = 1.13m. \tag{5}$$

   (b) The wavelength is shorter in front of the siren and longer behind it according to the Doppler effect. In front of the siren and behind the siren, we find

$$\lambda_{\text{in front}} = \frac{v - v_s}{f_s} = 1.03m/s, \quad \lambda_{\text{behind}} = \frac{v + v_s}{f_s} = 1.23m/s. \tag{6}$$

6. ∗ Dune
On the planet Arrakis (also known as Dune and featured in the Dune series of novels by Frank Herbert), a male ornithoid is flying toward his friend at 25.0 m/s while singing at a frequency of 1200 Hz. If the stationary female hears a tone of 1240 Hz, what is the speed of sound in the atmosphere of Arrakis? **Answers:** 780 m/s.
Solution:

IDENTIFY: Apply the Doppler shift equation $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$.

SET UP: The positive direction is from listener to source. $f_S = 1200$ Hz. $f_L = 1240$ Hz.

EXECUTE: $v_L = 0$. $v_S = -25.0$ m/s. $f_L = \left(\frac{v}{v + v_S}\right) f_S$ gives

$$v = \frac{v_S f_L}{f_S - f_L} = \frac{(-25\text{ m/s})(1240\text{ Hz})}{1200\text{ Hz} - 1240\text{ Hz}} = 780\text{ m/s}.$$

EVALUATE: $f_L > f_S$ since the source is approaching the listener.

7. ∗∗ Moving Source vs. Moving Listener.

   (a) A sound source producing 1.00-kHz waves moves toward a stationary listener at one-half the speed of sound. What frequency will the listener hear?
   Answers: 2000 Hz.

   (b) Suppose instead that the source is stationary and the listener moves toward the source at one-half the speed of sound. What frequency does the listener hear? How does your answer compare to that in part (a)? Explain on physical grounds why the two answers differ.
   Answers: 1500 Hz.
   Comments: It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

   Solution:

IDENTIFY: Apply $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S$.

SET UP: $f_S = 1000$ Hz. The positive direction is from the listener to the source. $v = 344$ m/s.

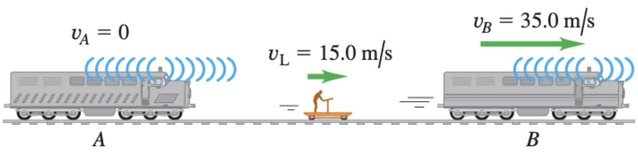
EXECUTE: **(a)** $v_S = -(344 \text{ m/s})/2 = -172$ m/s, $v_L = 0$.

$$f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S = \left(\frac{344 \text{ m/s}}{344 \text{ m/s} - 172 \text{ m/s}}\right)(1000 \text{ Hz}) = 2000 \text{ Hz}$$

(b) $v_S = 0$, $v_L = +172$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} + 172 \text{ m/s}}{344 \text{ m/s}}\right)(1000 \text{ Hz}) = 1500 \text{ Hz}$

EVALUATE: The answer in (b) is much less than the answer in (a). It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

8. ∗∗ Two train whistles, A and B, each have a frequency of 392 Hz. A is stationary and B is moving toward the right (away from A) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s (Figure below). No wind is blowing.



(a) What is the frequency from A as heard by the listener?
Answers: 375 Hz.

(b) What is the frequency from B as heard by the listener?
Answers: 371 Hz.

(c) What is the beat frequency detected by the listener?
Answers: 4 Hz.

Solution:

IDENTIFY: Apply the Doppler shift equation $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S$.

SET UP: The positive direction is from listener to source. $f_S = 392$ Hz.

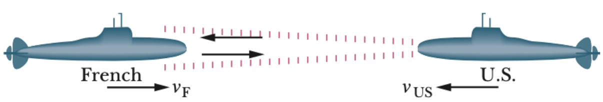
(a) $v_S = 0$. $v_L = -15.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} - 15.0 \text{ m/s}}{344 \text{ m/s}}\right)(392 \text{ Hz}) = 375 \text{ Hz}$

(b) $v_S = +35.0$ m/s. $v_L = +15.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} + 35.0 \text{ m/s}}\right)(392 \text{ Hz}) = 371 \text{ Hz}$

(c) $f_{\text{beat}} = f_1 - f_2 = 4$ Hz

EVALUATE: The distance between whistle A and the listener is increasing, and for whistle A $f_L < f_S$. The distance between whistle B and the listener is also increasing, and for whistle B $f_L < f_S$.

9. ∗∗ (Halliday C17-P31) As shown in figure below, a French submarine and a U.S. submarine move toward each other during maneuvers in motionless water in the North Atlantic. The French sub moves at the speed $v_F = 48.00$ km/h, and the U.S. sub at $v_{US} = 72.00$ km/h. The French sub sends out a sonar signal (sound wave in water) at 1.560×10^3 Hz. Sonar waves travel at 5470 km/h.



(a) What is the signal's frequency as detected by the U.S. sub?
Answers: 1.595×10^3 Hz.

(b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?
Answers: 1.630×10^3 Hz.

Solution:

31. (a) The frequency as detected by the U.S. sub is

$$f_1' = f_1\left(\frac{v + v_{\mathrm{US}}}{v - v_{\mathrm{F}}}\right) = (1.560\times10^3\ \mathrm{Hz})\left(\frac{5470\ \mathrm{km/h} + 72.00\ \mathrm{km/h}}{5470\ \mathrm{km/h} - 48.00\ \mathrm{km/h}}\right) = 1.595\times10^3\ \mathrm{Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be

$$f_r = f_1(v+v_{\mathrm{US}})/(v - v_{\mathrm{US}}).$$

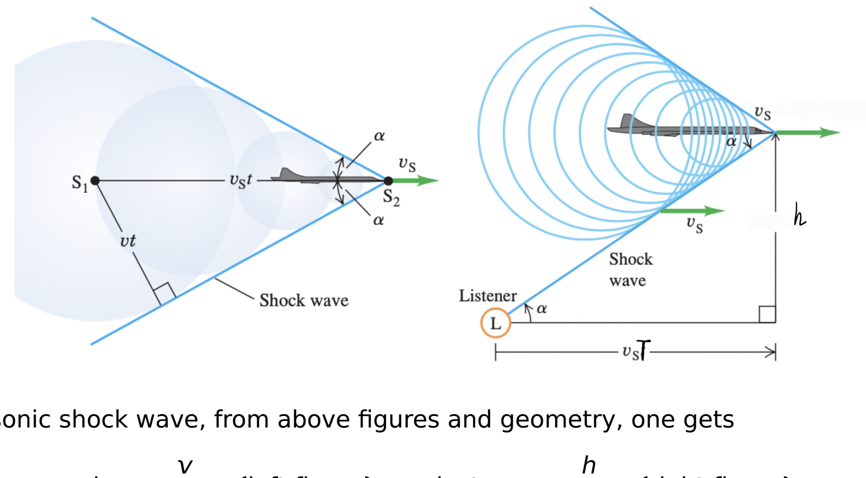
Since the French sub is moving toward the reflected signal with speed v_{F}, then

$$f_r' = f_r\left(\frac{v + v_{\mathrm{F}}}{v - v_{\mathrm{F}}}\right) = f_1\frac{(v+v_{\mathrm{F}})(v+v_{\mathrm{US}})}{(v-v_{\mathrm{F}})(v-v_{\mathrm{US}})} = (1.560\times10^3\ \mathrm{Hz})\frac{(5470+48.00)(5470+72.00)}{(5470-48.00)(5470-72.00)}$$
$$= 1.630\times10^3\ \mathrm{Hz}.$$

10. ∗∗ Supersonic Shock Waves.
On a clear day you see a jet plane flying overhead. From the apparent size of the plane, you determine that it is flying at a constant altitude h. You hear the sonic boom at time T after the plane passes directly overhead. Show that if the speed of sound v is the same at all altitudes, the speed of the plane v_s is

$$v_s = \frac{hv}{\sqrt{h^2 - v^2T^2}}.$$

Solution:



In the case of supersonic shock wave, from above figures and geometry, one gets

$$\sin\alpha = \frac{v}{v_s}, \quad \text{(left figure)} \quad \text{and} \quad \tan\alpha = \frac{h}{v_sT} \quad \text{(right figure)}. \tag{7}$$

Then use the trigonometric identity which relates $\sin\alpha$ to $\tan\alpha$, one finds

$$\tan\alpha = \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{v}{\sqrt{v_s^2 - v^2}} = \frac{h}{v_sT}. \tag{8}$$

In the end, one can solve for v_s and find it is given by $v_s = \dfrac{hv}{\sqrt{h^2 - v^2T^2}}$.

For a given h, the faster the speed v_s of the plane, the greater is the delay time T. The maximum delay time is h/v as expected when $v_s \to \infty$.