PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star ∗ ∗ ∗ labels are assigned to the most difficult ones.

Due date: March 3rd, 2024, 23: 59: 00.

1 Homework Problems for Week 4: Chapter 9 COM and Momentum

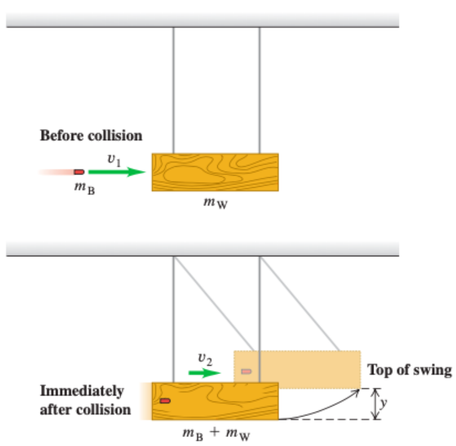
1. ∗ **Force of a Golf Swing.** A 0.0450-kg golf ball initially at rest is given a speed of 25.0 m/s when a club strikes. If the club and ball are in contact for $2.00\text{ms} = 2.00 \times 10^{-3}\text{s}$, what average force acts on the ball? Is the effect of the ball's weight during the time of contact significant? Why or why not?

   Answers: 563N, it is much larger than the ball's weight which is less than 1N.

2. ∗ **Hockey Puck and Impulse.** A 0.160-kg hockey puck is moving on an icy, frictionless, horizontal surface. At $t = 0$, the puck is moving to the right at $3.00m/s$. (a) Calculate the velocity of the puck (magnitude and direction) after a force of 25.0 N directed to the right has been applied for 0.050 s. (b) If, instead, a force of 12.0 N directed to the left is applied from $t = 0$ to $t = 0.050$ s, what is the final velocity of the puck?

   Answers: (a) +10.8 m/s; (b) −0.75 m/s. Choose right as the $+x$ direction.

3. ∗ The figure below shows a ballistic pendulum, a simple system for measuring the speed of a bullet. A bullet of mass m_B makes a completely inelastic collision with a block of wood of mass m_W, which is suspended like a pendulum. After the impact, the block swings up to a maximum height y. In terms of y, m_B, and m_W, what is the initial speed v_1 of the bullet?



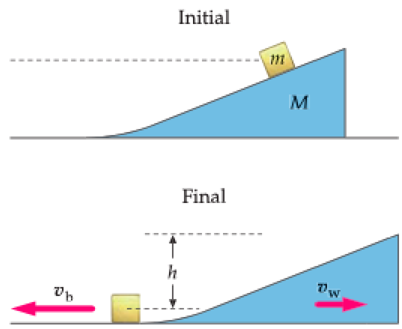
   Answers: The initial speed of the bullet is

$$v_1 = \frac{m_B + m_W}{m_B}\sqrt{2gy}.$$

4. ∗ A wedge of mass M is placed on a frictionless, horizontal surface, and a block of mass m is placed on the wedge, which also has a frictionless surface (see figure below). The block's center of mass moves downward a distance h as the block slides from its initial position to the horizontal floor.

   (a) What are the speeds of the block and of the wedge as they separate from each other and go their own ways?

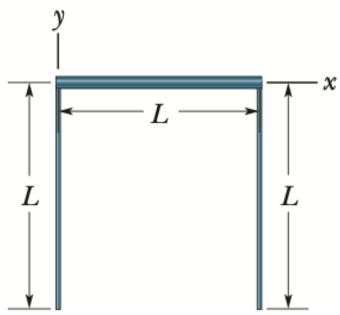
   (b) Check your calculation plausibility by considering the limiting case when $M \gg m$.



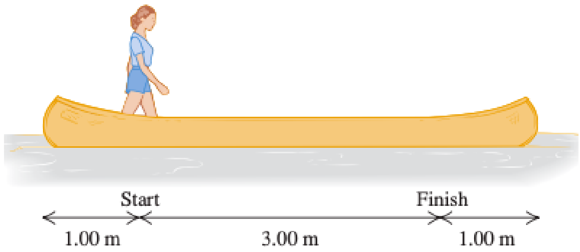
   Answers:

$$v_b = \sqrt{2gh\frac{M}{m+M}}, \quad v_w = \sqrt{2gh\frac{m^2}{M(m+M)}}$$

5. ∗ (Halliday,C9-P4)
   In the figure below, three uniform thin rods, each of length $L = 24$ cm, form an inverted U. The vertical rods each have a mass of $M = 14$ g; the horizontal rod has a mass of $3M = 42$ g. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass?
   Answers: $(x_{com}, y_{com}) = (12, -4.8)$ cm.



6. ∗ **Center of Mass.** A 45.0-kg woman stands up in a 60.0-kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (Fig. below). If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?
   Answers: 1.29 m to the left.



7. ∗ ∗ (Halliday,C9-P8)
A uniform soda can of mass $M = 0.140$ kg is $H = 12.0$ cm tall and fully filled with $m = 0.354$ kg of soda (Figure shown below). Then small holes are drilled in the top and bottom (with negligible loss of metal) to drain the soda.

(a) What is the height h of the Center of Mass (COM) of the can and contents initially?
Answers: $H/2$.

(b) What is the height h of the COM of the can and contents after the can loses all the soda?
Answers: $H/2$.

(c) What happens to h as the soda drains out?
Answers: Intuitively, the COM h should decrease from $H/2$ first as x decreases then rise up to $H/2$ again when all the soda is drained. This implies that there must be a minimum in h.

(d) If x is the height of the remaining soda at any given instant, draw $h(x)$ vs x figure and find x when the COM reaches its lowest point.
Answers: The lowest point of COM

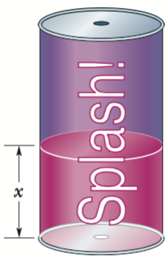
$$h_{\min} = \frac{MH}{m}\left[\sqrt{1+\frac{m}{M}} - 1\right] = 4.2\text{cm}.$$

The corresponding x is the same as $h_{\min}$.

Hint: First, as x decreases find

$$h = \frac{MH^2 + mx^2}{2(MH + mx)}.$$

Then solve for $dh/dx = 0$ and find the corresponding x and h. Note that you will need to go through a few steps of algebraic manipulations.



8. ∗ ∗ (Halliday,C9-P72)
In the two-dimensional collision in Figure below, the projectile particle has mass $m_1 = m$, initial speed $v_{1i} = 3v_0$, and final speed $v_{1f} = \sqrt{5}v_0$. The initially stationary target particle has mass $m_2 = 2m$ and final speed $v_{2f} = v_2$. The projectile is scattered at an angle given by $\tan\theta_1 = 2$.

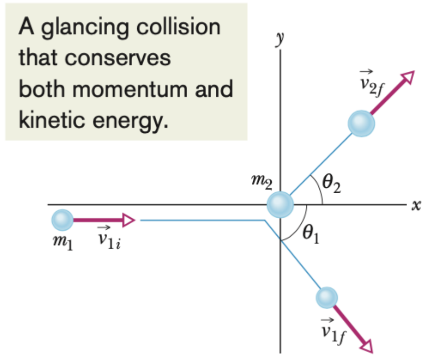


Figure 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

(a) Find angle θ_2.
Answers: $\tan\theta_2 = 1$, $\theta_2 = \pi/4$.

(b) Find v_2 in terms of v_0.
Answers: $v_2 = \sqrt{2}v_0$.

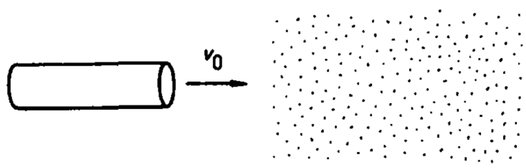
(c) Is the collision elastic?
Answers: Yes, because the kinetic energy is conserved.

9. ∗ ∗ Show that in one-dimensional elastic collision, if the mass and velocity of object 1 are $m1$ and v_{1i}, and if the mass and velocity of object 2 are $m2$ and v_{2i}, then their final velocities v_{1f} and v_{2f} are given by

$$\begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}, \\ v_{2f} &= \frac{m_2 - m_1}{m_1 + m_2}v_{2i} + \frac{2m_1}{m_1 + m_2}v_{1i}. \end{aligned}$$

Check the plausibility of the above answer by considering the limiting case with $m_1 \gg m_2$. Note that in this case the velocity of object 1 is unchanged while the object 2 is like hitting a wall in the reference frame of object 1.

10. ∗ ∗ ∗ Suppose the spacecraft (Enterprise) of mass m_0 and cross-section A is moving with velocity v_0 when it encounters a stationary dust cloud of density ρ at $t = 0$. If the dust sticks to the spacecraft and resistance can be neglected. Solve for the subsequent motion of the spacecraft.



Hint: Momentum is conserved for the spacecraft-dust system. Therefore $d(mv)/dt = 0$, $mv = m_0v_0$, and the mass m increases as Enterprise puts on the dust along its path.

Answers: $v = v_0\sqrt{\dfrac{1}{1 + 2\rho A v_0 t/m_0}}$.