香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

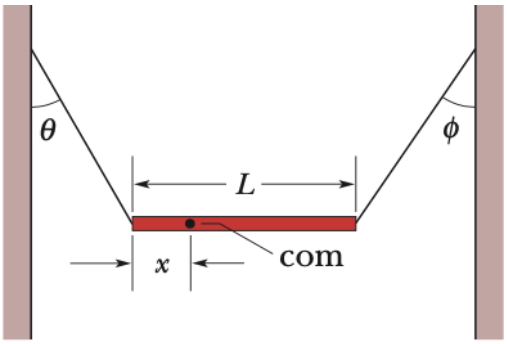
PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star ∗ ∗ ∗ labels are assigned to the most difficult ones.

Due date: 2024, April 7th, 23: 59: 00.

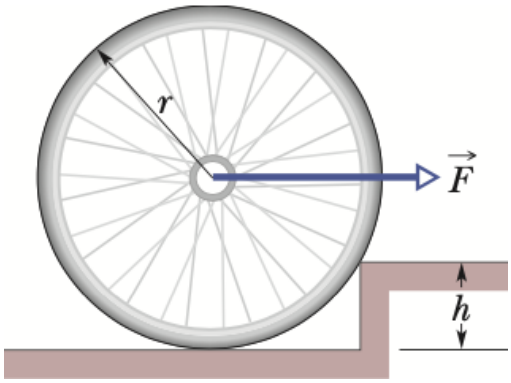
1 Homework Problems for Week 8: Chapter 12 and 13

1. ∗∗ (Halliday C12-P10)
In Figure below, a nonuniform bar is suspended at rest in a horizontal position by two massless cords. One cord makes the angle $\theta = 30.0°$ with the vertical; the other makes the angle $\phi = 60.0°$ with the vertical. If the length L of the bar is 9.50 m, compute the distance x from the left end of the bar to its center of mass.



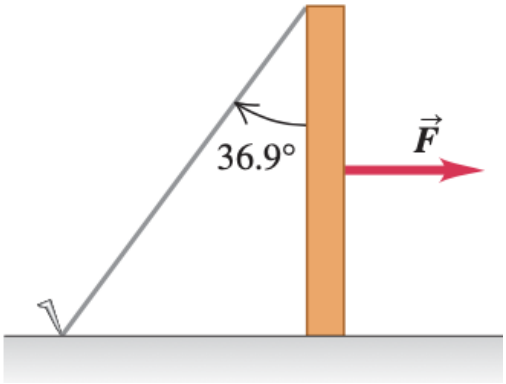
Answers: $x = 2.38$ m.

2. ∗ (Halliday C12-P17)
In Figure below, what magnitude of (constant) force F applied horizontally at the axle of the wheel is necessary to raise the wheel over a step obstacle of height $h = 3.00$ cm? The wheel's radius is $r = 8.00$ cm, and its mass is $m = 0.600$ kg.



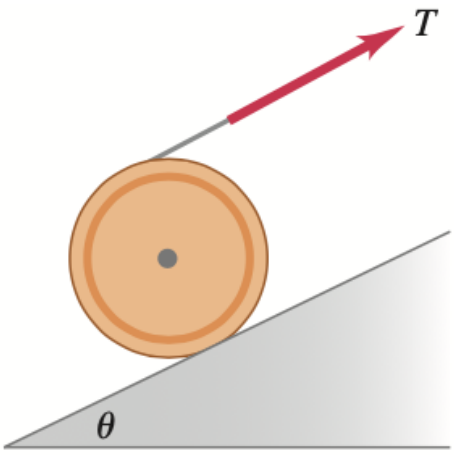
Answers: $F = 7.34$ N.

3. ∗∗ Knocking Over a Post.
One end of a post weighing 400 N and with height h rests on a rough horizontal surface with $\mu_s = 0.30$. The upper end is held by a rope fastened to the surface and making an angle of 36.9° with the post (Fig. below). A horizontal force $\vec{F}$ is exerted on the post as shown. If the force F is applied at the midpoint of the post, what is the largest value it can have without causing the post to slip?



Answers: $F = 400$ N.

4. ∗∗ A uniform solid cylinder of mass M is supported on a ramp that rises at an angle θ above the horizontal by a wire that is wrapped around its rim and pulls on it tangentially parallel to the ramp (Figure below).



(a) Show that there must be friction on the surface for the cylinder to balance this way.
Hint: Suppose that there is no friction and choose COM as the axis for torque, then you always find nonzero torque, which is impossible to balance.

(b) Show that the tension in the wire must be equal to the friction force, and find this tension.
Answers: $T = \frac{1}{2}Mg\sin\theta$.

5. ∗ Stress on a Mountaineer's Rope.
A nylon rope used by mountaineers elongates 1.10

m under the weight of a 65.0-kg climber. If the rope is 45.0 m in length and 7.0 mm in diameter, what is Young's modulus for nylon?
Answers: $Y = 6.77 \times 10^8$ Pa.

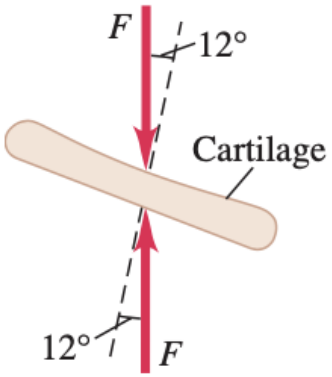
6. ∗∗ Bulk Modulus of an Ideal Gas.
The equation of state (the equation relating pressure, volume, and temperature) for an ideal gas is $pV = nRT$, where n and R are constants.

(a) Show that if the gas is compressed while the temperature T is held constant, the bulk modulus B is equal to the pressure p.

(b) When an ideal gas is compressed without the transfer of any heat into or out of it, the pressure and volume are related by pV^γ = constant, where γ is a constant having different values for different gases. Show that, in this case, the bulk modulus is given by $B = \gamma p$.

Answers:

7. ∗∗ Downhill Hiking.
During vigorous downhill hiking, the force on the knee cartilage (the medial and lateral meniscus) can be up to eight times body weight. Depending on the angle of descent, this force can cause a large shear force on the cartilage and deform it. The cartilage has an area of about 10 cm^2 and a shear modulus of 12 MPa. If the hiker plus his pack have a combined mass of 110 kg (not unreasonable), and if the maximum force at impact is 8 times his body weight (which, of course, includes the weight of his pack) at an angle of 12° with the cartilage (Fig. below), through what angle (in degrees) will his knee cartilage be deformed? (Recall that the bone below the cartilage pushes upward with the same force as the downward force.)
Answers: The target answer is the angle of deformation of the cartilage $\phi \equiv \frac{x}{h} = 0.15$rad, which is also known as the shear strain. It is 8.6° in degrees.



8. ∗∗ Gravity in Three Dimensions.
A point mass m_1 is held in place at the origin, and another point mass m_2 is free to move a distance away at a point P having coordinates x, y, and z. The gravitational potential energy of these masses is found to be $U(r) = -\frac{Gm_1m_2}{r}$, where G is the gravitational constant.

Show by explicit calculation of the partial derivatives ($\vec{F} = -\vec{\nabla}U(r)$) that the components of the force on m_2 due to m_1 are given by

$$F_x = -\frac{Gm_1m_2x}{r^3},$$
$$F_y = -\frac{Gm_1m_2y}{r^3},$$
$$F_z = -\frac{Gm_1m_2z}{r^3},$$

thus $\vec{F} = -\frac{Gm_1m_2}{r^3}\vec{r}$.

9. ∗ Escape speeds
The escape speed is the minimum speed needed for a free, non-propelled object to escape from the gravitational influence of a primary body, thus reaching an infinite distance from it.

(a) Suppose you are launching a spaceship from the earth surface. Estimate the escape speed from earth.
Answers: $v_e = 1.1 \times 10^4$ m/s.

(b) Suppose you are launching a spaceship from the earth orbit. Estimate the escape speed from the solar system.
Answers: $v_s = 4.2 \times 10^4$ m/s.

10. ∗∗ Imagine you are an astronaut who got stuck on the surface of an asteroid. Estimate the order of magnitude of the maximum radius of an asteroid you could escape by jumping.
Hint: First, estimate how high you can jump on earth, 1 m is probably a reasonable estimate of the height. This tells the order of magnitude of how much energy your muscle can generate. Then assume the asteroid is spherical and it has a similar density as the earth, use energy conservation.
Answers: $R \sim 1km - 10km$.