PHY 1001: Mechanics

Tutorial Session 8

T-05: Mar - 27 - 2024, Wednesday, 19:00~19:50

T-12: Mar - 27 - 2024, Wednesday, 20:00~20:50

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Problem 1

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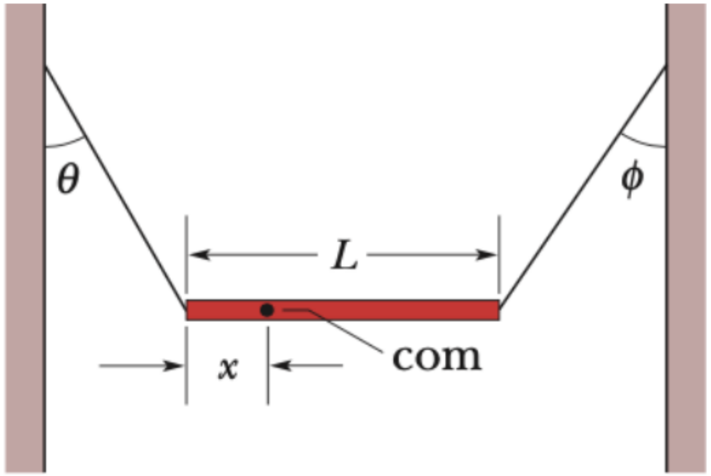
* * (Halliday C12-P10) In Figure below, a nonuniform bar is suspended at rest in a horizontal position by two massless cords. One cord makes the angle $\boldsymbol{\theta = 30.0^\circ}$ with the vertical; the other makes the angle $\boldsymbol{\emptyset = 60.0^\circ}$ with the vertical. If the length $\boldsymbol{L}$ of the bar is $\mathbf{9.50\ m}$, compute the distance $\boldsymbol{x}$ from the left end of the bar to its center of mass.

Solution:

(1) The bar is in equilibrium, so the forces and the torques acting on it each sum to zero.

(2) Let $\boldsymbol{T_l}$ be the tension force of the left-hand cord,
$\boldsymbol{T_r}$ be the tension force of the right-hand cord,
and $\boldsymbol{m}$ be the mass of the bar.



(3) The equations for equilibrium are:

- Vertical force components: $\boldsymbol{T_l \cos\theta + T_r \cos\emptyset - mg = 0}$,
- Horizontal force components: $\boldsymbol{-T_l \sin\theta + T_r \sin\emptyset = 0}$,
- Torque about the left end: $\boldsymbol{-mgx + T_r L \cos\emptyset = 0}$.

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Problem 1

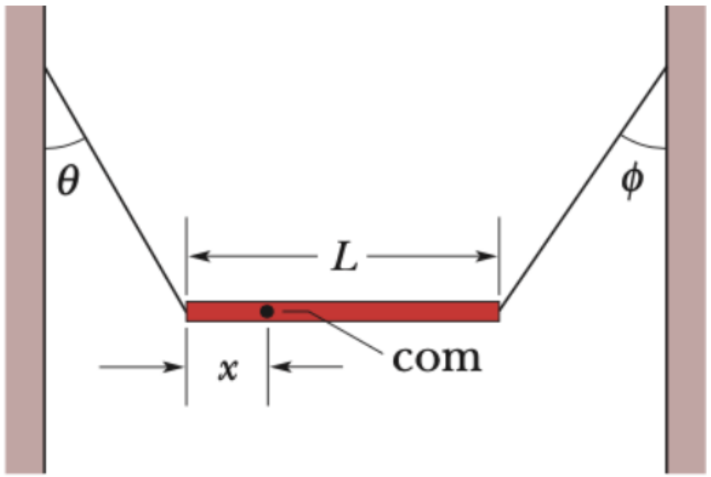
* * (Halliday C12-P10) In Figure below, a nonuniform bar is suspended at rest in a horizontal position by two massless cords. One cord makes the angle $\boldsymbol{\theta = 30.0°}$ with the vertical; the other makes the angle $\boldsymbol{\emptyset = 60.0°}$ with the vertical. If the length $\boldsymbol{L}$ of the bar is $\mathbf{9.50\ m}$, compute the distance $\boldsymbol{x}$ from the left end of the bar to its center of mass.

Solution:

(4) There are three unknown quantities in the above three equations.

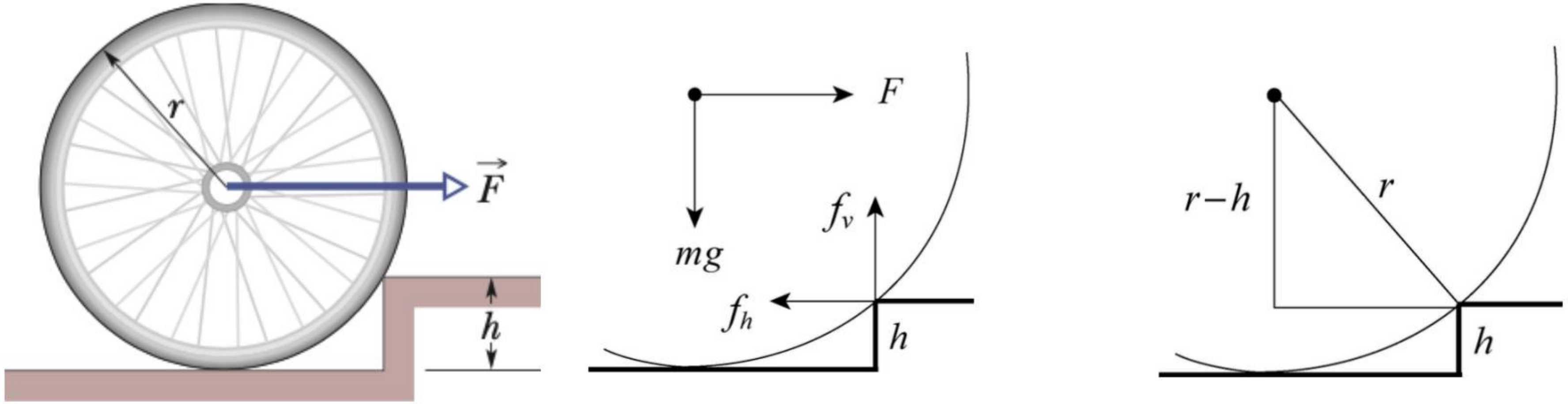
(5) So we eliminate $\boldsymbol{T_l}$ and $\boldsymbol{T_r}$ then solve for $\boldsymbol{x}$ which gives

$$\boldsymbol{x = \frac{\sin\theta\cos\emptyset}{\sin(\theta+\emptyset)}L = 2.38\ m.}$$



Problem 2

* (Halliday C12-P17) In Figure below, what magnitude of (constant) force $\boldsymbol{F}$ applied horizontally at the axle of the wheel is necessary to raise the wheel over a step obstacle of height $\boldsymbol{h = 3.00\ \mathrm{cm}}$? Wheel's radius is $\boldsymbol{r = 8.00\ \mathrm{cm}}$, and its mass is $\boldsymbol{m = 0.600\ \mathrm{kg}}$.



Solution:

(1) At the moment when the wheel is about to leave the lower floor, the floor no longer exerts a force on it. The contact force from the lower floor is zero.

(2) At this critical moment, there are four forces exerted on the wheel as shown above.

Problem 2

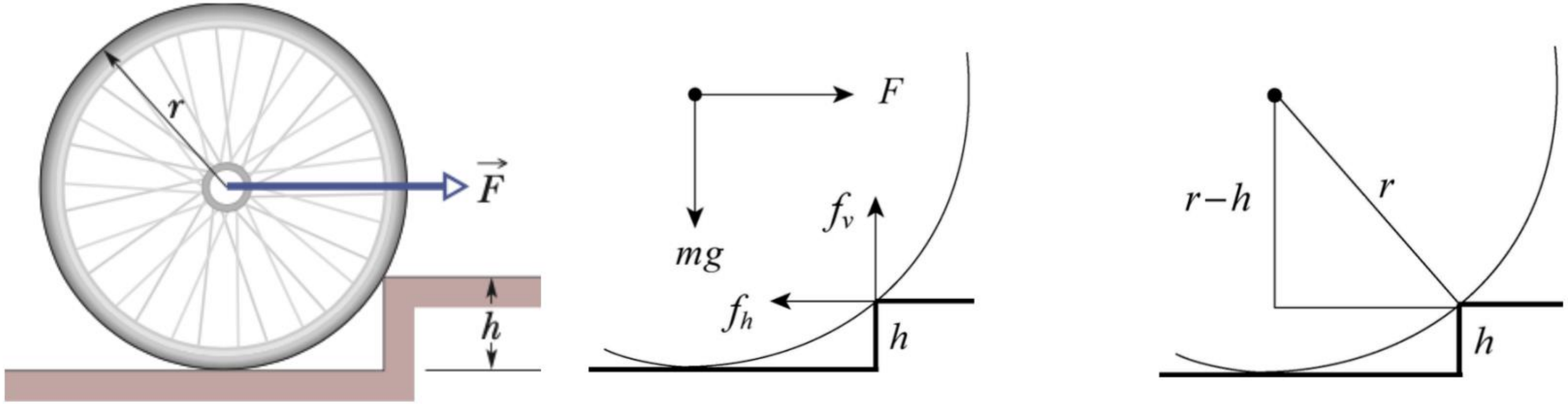
Solution:

(3) If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.

(4) Choose the step corner as the instantaneous rotational center, we can eliminate two unknown forces and direct relate $\boldsymbol{F}$ to $\boldsymbol{mg}$ through the torque equation as follows

$$mg\sqrt{r^2-(r-h)^2}-F(r-h)=0,$$

$$\Rightarrow F=\frac{\sqrt{2rh-h^2}}{r-h}mg=7.34(\mathrm{N}).$$



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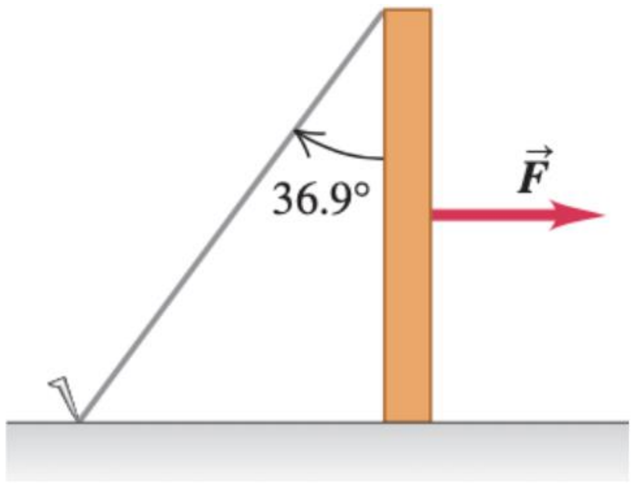
Problem 3

* * **Knocking Over a Post.** One end of a post weighing **400 N** and with height $\boldsymbol{h}$ rests on a rough horizontal surface with $\boldsymbol{\mu_s = 0.30}$. The upper end is held by a rope fastened to the surface and making an angle of $\boldsymbol{36.9°}$ with the post (Fig. below). A horizontal force $\vec{\boldsymbol{F}}$ is exerted on the post as shown. If the force $\boldsymbol{F}$ is applied at the midpoint of the post, what is the largest value it can have without causing the post to slip?

Solution:

Identify: Apply $\sum \boldsymbol{\tau_z} = \boldsymbol{0}$ to the post, for various choices of the location of the rotation axis.

Set Up: When the post is on the verge of slipping, $\boldsymbol{f_s}$ has its largest possible value, $\boldsymbol{f_s = \mu_s n}$.



Problem 3

Execute:

(1) Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $\boldsymbol{h/2}$ and the lever arm of both the weight and the normal force is $\boldsymbol{h\tan\theta}$.

(2) Thus, we have

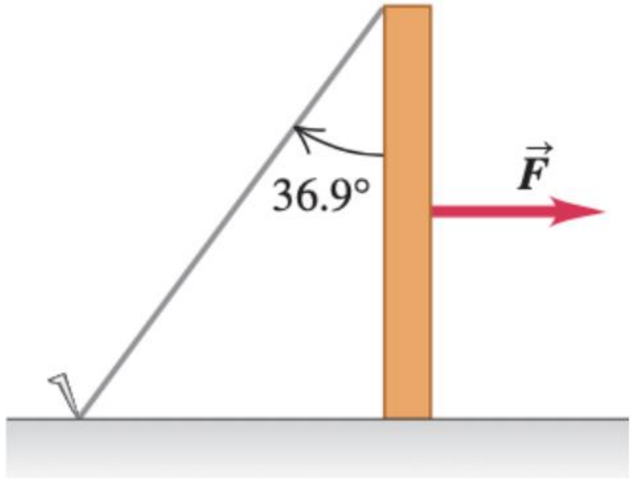
$$F\frac{h}{2} = (n - w)h\tan\theta\,.$$

(3) Taking torques about the upper point (where the rope is attached to the post),

$$fh = F\frac{h}{2}$$

(4) Using $f \le \mu_s n$ and solving for F,

$$F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\text{ N})\left(\frac{1}{0.300} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\text{ N}$$



Problem 4

* * A uniform solid cylinder of mass $\boldsymbol{M}$ is supported on a ramp that rises at an angle $\boldsymbol{\theta}$ above the horizontal by a wire that is wrapped around its rim and pulls on it tangentially parallel to the ramp (Figure below).

Question (a):

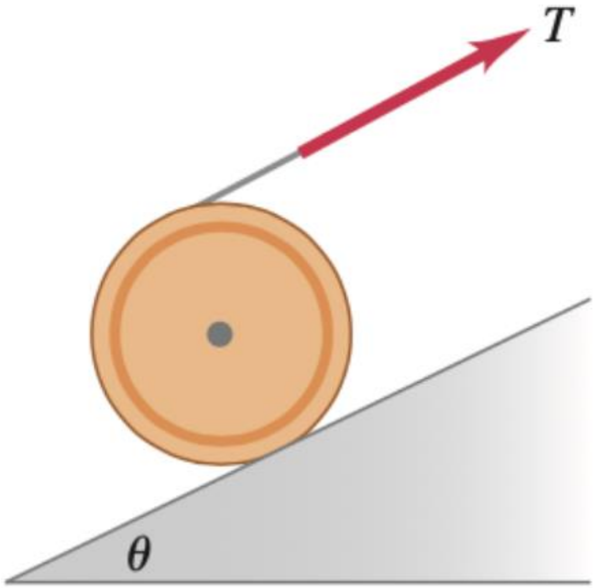
Show that there must be friction on the surface for the cylinder to balance this way.

Solution (a):

(a) Suppose that there is no friction and choose COM as the axis for torque, then you always find nonzero torque, which is impossible to balance the cylinder.

(b) Note that the gravitational force and normal force do not contribute to the torque since their arm lengths are zero.

(c) Therefore, there must be friction.



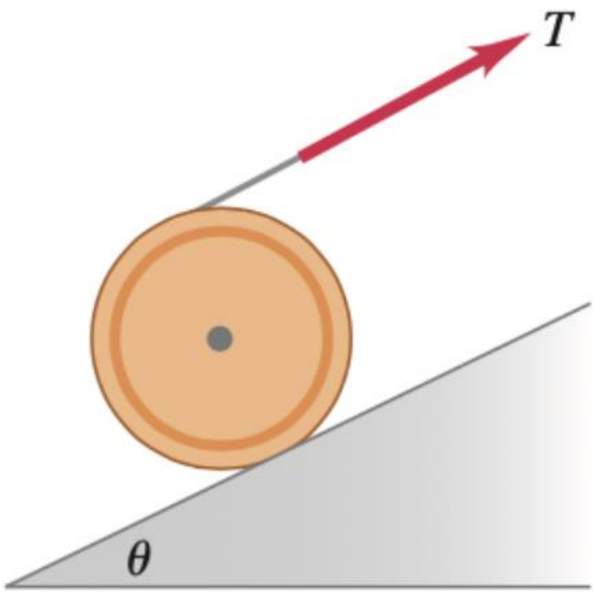
Problem 4

Question (b):

Show that the tension in the wire must be equal to the friction force, and find this tension.

Solution (b):

(1) In addition, the direction of the friction is upward parallel to the surface in order to provide opposite torque *w.r.t.* to the one caused by the tension.



(2) The torque equation gives

$$fR - TR = 0 \quad \Longrightarrow \quad f = T.$$

(3) From the balancing of the force parallel to the surface of the ramp, one gets

$$T + f - mg\sin\theta = 0, \quad \Longrightarrow \quad 2T = mg\sin\theta,$$

$$\Longrightarrow \quad T = \frac{1}{2}mg\sin\theta.$$

Problem 5

* **Stress on a Mountaineer's Rope.**

A nylon rope used by mountaineers elongates **1.10 m** under the weight of a **65.0-kg** climber. If the rope is **45.0 m** in length and **7.0 mm** in diameter, what is Young's modulus for nylon?

Solution:

(1) Young's modulus is defined as

$$Y = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{65.0 * 9.80/\pi/\left(3.5 \times 10^{-3}\right)^2}{1.10/45.0}(\mathrm{Pa}) = 6.77 \times 10^{8}(\mathrm{Pa}).$$

(2) Note that in the above calculations, you do not have to keep track of the units.

(3) As long as you the numbers that you put in are all in *SI* units, the resultant unit will automatically be the correct unit for ***Y*** in *SI* unit.

(4) In this case, it is **Pa**.

Problem 6

* * **Bulk Modulus of an Ideal Gas.**

The equation of state (the equation relating pressure, volume, and temperature) for an ideal gas is $pV = nRT$, where n and R are constants.

Question (a): Show that if the gas is compressed while the temperature T is held constant, the bulk modulus B is equal to the pressure p.

Solution (a):

(1) The bulk modulus B is defined as

$$B = -\frac{\Delta p}{\Delta V/V} = -V\frac{dp}{dV}.$$

(2) From the equation of motion $pV = nRT$ together with constant T, one gets $d(pV) = 0 = Vdp + pdV$, which is $dp/dV = -p/V$.

(3) Therefore, $B = p$ for constant T.

Problem 6

Question (b): When an ideal gas is compressed without the transfer of any heat into or out of it, the pressure and volume are related by $\boldsymbol{pV^{\gamma} = \text{constant}}$, where $\boldsymbol{\gamma}$ is a constant having different values for different gases. Show that, in this case, the bulk modulus is given by $\boldsymbol{B = \gamma p}$.

Solution (b):

(1) For the adiabatic process (no heat transfer), similarly one gets $\boldsymbol{d(pV^{\gamma}) = 0}$, which implies $\boldsymbol{V^{\gamma}dp + \gamma p V^{\gamma-1} dV = 0}$ and $\boldsymbol{dp/dV = -\gamma p/V}$.

(2) Plugging it into the definition of the bulk modulus $\boldsymbol{B}$

$$\boldsymbol{B = -\frac{\Delta p}{\Delta V/V} = -V\frac{dp}{dV} = \gamma p}.$$

(3) Usually $\boldsymbol{\gamma > 1}$ and it means that ideal gas is harder to compress without heat transfer than the gas with constant temperature.

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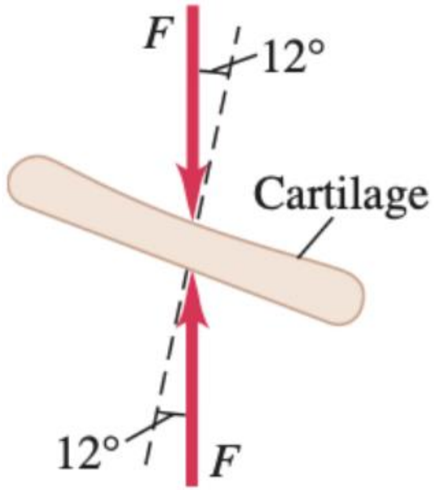
Problem 7

** **Downhill Hiking.**

During vigorous downhill hiking, the force on the knee cartilage (the medial and lateral meniscus) can be up to eight times body weight. Depending on the angle of descent, this force can cause a large shear force on the cartilage and deform it. The cartilage has an area of about **10 cm^2** and a shear modulus of **12 MPa**. If the hiker plus his pack have a combined mass of **110 kg** (not unreasonable), and if the maximum force at impact is **8** times his body weight (which, of course, includes the weight of his pack) at an angle of **12°** with the cartilage (Fig. below), through what angle (in degrees) will his knee cartilage be deformed? (Recall that the bone below the cartilage pushes upward with the same force as the downward force.)



Problem 7

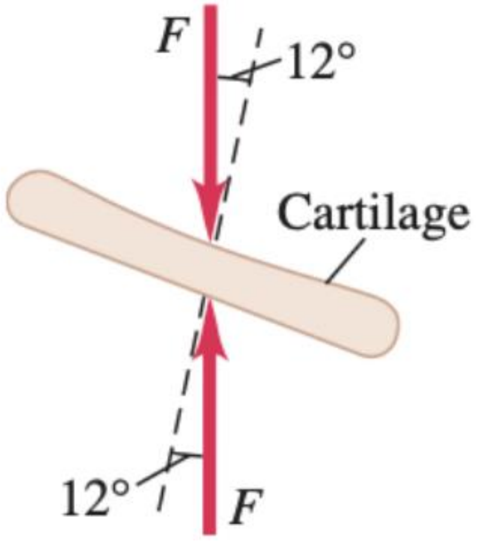
Solution:

(1) Vigorous downhill hiking produces a shear force on the knee cartilage which could deform the cartilage.

(2) The **shear strain**, or the angle of deformation, can be obtained from the definition as follows

$$\emptyset \equiv \frac{x}{h} = \frac{\text{Shear Stress}}{\text{Shear Modulus}} = \frac{F_{\parallel}/A}{S}$$

$$= \frac{8 * 110 * 9.8 * \sin(12°)\,/\left(10 \times 10^{-4}\right)}{12 \times 10^{6}} = 0.15\ \text{rad}.$$



(3) The angle of **0.15 rad** is **8.6°** in degrees.

Problem 8

* * **Gravity in Three Dimensions.**

A point mass $\boldsymbol{m_1}$ is held in place at the origin, and another point mass $\boldsymbol{m_2}$ is free to move a distance away at a point $\boldsymbol{P}$ having coordinates $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$. The gravitational potential energy of these masses is found to be $\boldsymbol{U(r) = -\frac{Gm_1m_2}{r}}$, where $\boldsymbol{G}$ is the gravitational constant.

Show by explicit calculation of the partial derivatives ($\boldsymbol{F = -\vec{\nabla}U(r)}$) that the components of the force on $\boldsymbol{m_2}$ due to $\boldsymbol{m_1}$ are given by

$$F_x = -\frac{Gm_1m_2x}{r^3},$$

$$F_y = -\frac{Gm_1m_2y}{r^3},$$

$$F_z = -\frac{Gm_1m_2z}{r^3},$$

thus $\vec{F} = -\frac{Gm_1m_2}{r^3}\vec{r}.$

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Problem 8

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Solution:

Writing out the definition of the gradient of the potential with $\boldsymbol{r} = \sqrt{\boldsymbol{x^2 + y^2 + z^2}}$ gives:

$$\vec{F} = -\vec{\nabla}U(r) = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) \equiv F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k},$$

$$F_x = -\frac{\partial U}{\partial x} = -\frac{Gm_1m_2}{r^2}\frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x} = -\frac{Gm_1m_2}{r^2}\frac{x}{r}$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{Gm_1m_2}{r^2}\frac{\partial\sqrt{x^2+y^2+z^2}}{\partial y} = -\frac{Gm_1m_2}{r^2}\frac{y}{r}$$

$$F_z = -\frac{\partial U}{\partial z} = -\frac{Gm_1m_2}{r^2}\frac{\partial\sqrt{x^2+y^2+z^2}}{\partial z} = -\frac{Gm_1m_2}{r^2}\frac{z}{r},$$

where the chain rule has been used repeatedly. In the end, one can summarize the components of $\boldsymbol{F}$ into the vector form:

$$\vec{F} = -\frac{Gm_1m_2}{r^3}\vec{r}.$$

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Problem 9

* **Escape speeds:**
  The escape speed is the minimum speed needed for a free, non-propelled object to escape from the gravitational influence of a primary body, thus reaching an infinite distance from it.

Question (a): Suppose you are launching a spaceship from the earth surface. Estimate the escape speed from earth.

Solution (a):

(1) To escape from earth gravitational pull, minimum velocity must satisfy the condition

$$\frac{1}{2}mv_e^2 - \frac{GM_E m}{R_E} = 0,$$

which means that the total energy is large enough for the spaceship to go to infinity relative to the earth. Thus, $v_e = \sqrt{2GM_E/R_E} = \sqrt{2gR_E} = 1.1 \times 10^4\ \mathrm{m/s}$.

(2) In the last step, we have used the fact $g = GM_E/R_E^2$ on the surface of the earth.

Problem 9

* **Escape speeds:**

The escape speed is the minimum speed needed for a free, non-propelled object to escape from the gravitational influence of a primary body, thus reaching an infinite distance from it.

Question (b): Suppose you are launching a spaceship from the earth orbit. Estimate the escape speed from the solar system.

Solution (b):

To escape from the solar system, the minimum velocity must satisfy the condition

$$\frac{1}{2}mv_s^2 - \frac{GM_S m}{R_S} = 0,$$

where R_S is the radius of the earth orbit around the Sun. Using the data $M_S = 2.0 \times 10^3$ **kg** and $R_S = 1.5 \times 10^{11}$ **m**, then get

$$v_s = \sqrt{2GM_S/R_S} = 4.2 \times 10^4 \text{ m/s}.$$

Problem 10

* * Imagine you are an astronaut who got stuck on the surface of an asteroid. Estimate the order of magnitude of the maximum radius of an asteroid you could escape by jumping.

Hint:

(a) First, estimate how high you can jump on earth, **1 m** is probably a reasonable estimate of the height. This tells the order of magnitude of how much energy your muscle can generate.

(b) Then assume the asteroid is spherical and it has a similar density as the earth, use energy conservation.

Solution:

(1) Generally speaking, you roughly change the height of your center of mass by one meter (two meters for professional athletes.) when you jump on earth.

(2) In the process, the work done is approximately $\boldsymbol{mgh}$ with $\boldsymbol{h = 1\ \mathrm{m}}$ and $\boldsymbol{g}$ is the gravitational acceleration on the earth surface.

Problem 10

Solution:

(3) It is reasonable to suppose that when one jumps on an asteroid one would consume the same amount of energy as on the earth.

(4) Then to escape from the asteroid of mass $\boldsymbol{M}$ and radius $\boldsymbol{R}$ by jumping, the critical requirement is

$$mgh - \frac{GMm}{R} \geq 0, \quad \Rightarrow \quad gh \geq \frac{GM}{R}.$$

(5) Assuming that the density of the asteroid is the same as that of the earth, one gets

$$\frac{M}{M_E} = \frac{R^3}{R_E^3}, \quad \Rightarrow \quad M = \frac{R^3}{R_E^3} M_E,$$

where $\boldsymbol{M_E}$ and $\boldsymbol{R_E}$ are the mass and radius of the earth.

Gao Qian

Problem 10

Solution:

(6) Therefore, combining the above two equations gives

$$gh \geq \frac{GM_E}{R_E^3} R^2$$

$$\Rightarrow R \leq \sqrt{hR_E} = \sqrt{(1\,m)(6400 \times 10^3\,m)} \simeq 3 \times 10^3\,m = 3\,km,$$

where $\boldsymbol{g = GM_E/R_E^2}$ and $\boldsymbol{R_E = 6400}$ **km** have been used in the above derivations.

(7) Since this is just an order of magnitude estimate, any number ranging from **1 km** to **10 km** is acceptable.

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