PHY1001: Mechanics

Show steps in your homework. Correct answers with little or no supporting work will not be given credit. Three-star ∗ ∗ ∗ labels are assigned to the most difficult ones.

date: 2024, March 24th, 23: 59: 00.

1 Homework Problems for Week 7: C11 Rolling, Torque, Angular Momentum

1. ∗ A thin, horizontal rod with length l and mass M placed on a frictionless plane pivots about a vertical axis at one end. A force with constant magnitude F is applied to the other end, causing the rod to rotate in the horizontal plane. The force is maintained perpendicular to the rod and to the axis of rotation. Calculate the magnitude of the angular acceleration of the rod.
   Answers: $\alpha = 3F/(Ml)$.

2. ∗ Neutron stars spin fast!
   Under some circumstances after a supernova explosion, a star can collapse into an extremely dense object made mostly of neutrons and called a neutron star. The density of a neutron star is roughly 10^{14} times as great as that of ordinary solid matter. Suppose we represent the star as a uniform, solid, rigid sphere, both before and after the collapse. The star's initial radius was 7.0×10^5 km (comparable to our sun); its final radius is 16 km. If the original star rotated once in 30 days, find the angular speed of the neutron star.

A supernova remnant with neutron star inside.

   Answers: During the collapse, the angular momentum is roughly conserved. $\omega_{NS} = 4.6 \times 10^3$ rad/s. That is to say that it only takes 1.4×10^{-3} s for the neutron star to rotate once.

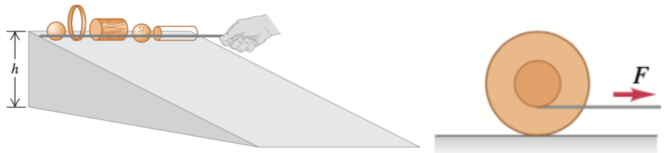
3. ∗ ∗ A Ball Rolling Uphill.
   A ball rolls without slipping up a ramp that slopes upward at an angle β to the horizontal. Treat the ball as a uniform solid sphere.

   (a) Draw the free-body diagram for the ball. Explain why the friction force must be directed uphill.
   Answers: For an uphill rolling (without slipping) ball, its angular velocity and linear velocity must decrease simultaneously $v = \omega R$. Choose the COM as the axis and then find the only non-zero torque is provided by the friction. It has to be uphill to decelerate the angular velocity and keep rotation commensurate with linear motion.

   (b) What is the acceleration of the center of mass of the ball?
   Answers: $a_{cm} = (5/7)g\sin\beta$.

   (c) What minimum coefficient of static friction is needed to prevent slipping?
   Answers: $(2/7)\tan\beta$.

4. ∗ ∗ Race of the rolling bodies
   In a physics demonstration, an instructor "races" various bodies that roll without slipping from rest down an inclined plane (left figure below). What shape should a body have to reach the bottom of the incline first?



   Answers: The one with the smallest $c \equiv I/MR^2$. Solid sphere if all the rigid bodies are uniform.

5. ∗ ∗ A yo-yo is made from two uniform disks, each with mass $m/2$ and radius R, connected by a light axle of radius b. This yo-yo's total moment of inertia about an axis passing through the center can be approximated by $mR^2/2$. A light, thin string is wound several times around the axis, and then the yo-yo is placed upright on a flat table. As shown in the right figure above, the string is pulled with a horizontal force F to the right.

   (a) Suppose the yo-yo is rolling without slipping, find the acceleration of the yo-yo.
   Answers: $a = \frac{2F}{3m}\left(1 - \frac{b}{R}\right)$.

   (b) Given the coefficient of static friction between the yo-yo and the table is μ_s, what is the maximum magnitude of the pulling force F for which the Yo-Yo rolls without slipping?
   Answers: $F \le \frac{3R}{2b+R}\mu_s mg$.

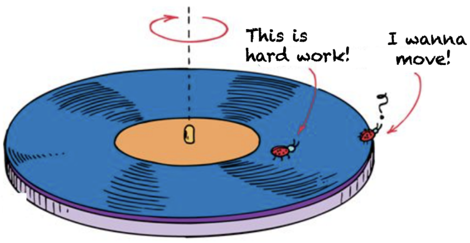
6. ∗ ∗ A spherical ball with mass M and rotational inertia $I_{cm} = (2/5)MR^2$ is given an initial clockwise angular velocity ω_0 and zero linear velocity $v_{cm} = 0$ before it is placed upon a horizontal surface with kinetic friction coefficient μ_k.

(a) Which direction is the ball going to move towards? **Answers:** Right.

(b) How long does it take for the ball to roll without slipping? **Answers:** $t = \dfrac{2\omega_0 R}{7\mu_k g}$.

(c) How much energy does it lose during this process? **Answers:** $\Delta K = -MR^2\omega_0^2/7$.

7. ∗∗ A ladybug of mass m lies on the rim of a uniform disk of mass $4m$ and radius R that can rotate freely about its center like a merry-go-round. Initially the ladybug and disk rotate together with an angular velocity of ω_0. Then the ladybug walks halfway to the center of the disk ($R/2$).

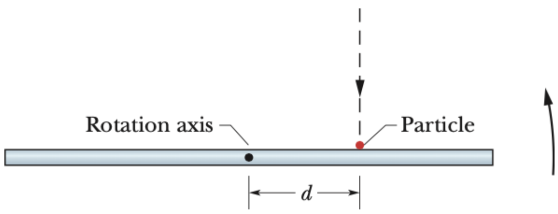


(a) What then is the angular velocity ω of the ladybug–disk system?
Answers: $\omega = 4\omega_0/3$.

(b) What is the ratio K_f/K_0 of the new kinetic energy of the system to its initial kinetic energy?
Answers: $K_f/K_0 = 4/3$.

(c) What accounts for the change in the kinetic energy?
Answers: The ladybug (the centripetal force) does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.
Comments: The exact amount of work is $m\omega_0^2R^2/2$, which comes from integrating along the radial direction. You are not required to find the exact amount of the work.

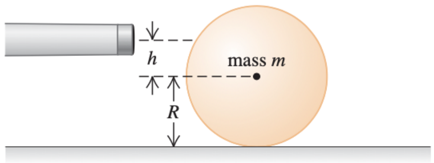
8. ∗ Figure below is an overhead view of a thin uniform rod of length 0.600 m and mass M rotating horizontally at 80.0 rad/s counterclockwise about an axis through its center. A particle of mass $M/3$ and traveling horizontally at speed 40.0 m/s hits the rod and sticks. The particle's path is perpendicular to the rod at the instant of the hit, at a distance d from the rod's center.



(a) At what value of d are rod and particle stationary after the hit?
Answers: 0.180 m

(b) In which direction do rod and particle rotate if d is greater than this value?
Answers: Clockwise.

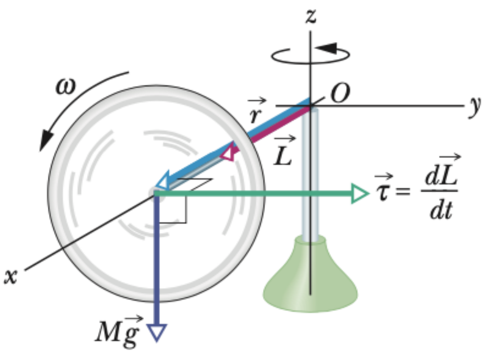
9. ∗∗ Billiard Physics.
A cue ball (a uniform solid sphere of mass m and radius R) is at rest on a level pool table. Using a pool cue, you give the ball a sharp, horizontal hit of magnitude F at a height h above the center of the ball (Figure below). The force of the hit F is much greater than the friction force f that the table surface exerts on the ball. The hit lasts for a very short time Δt.



(a) For what value of h will the ball roll without slipping?
Answers: $h = \dfrac{2R}{5}$.

(b) If you hit the ball dead center $h = 0$ and give it an initial speed v_0, the ball will slide across the table for a while, but eventually it will roll without slipping. What will the speed of its center of mass be then?
Answers: $\dfrac{5}{7}v_0$.

10. ∗ Gyroscope and Precession
Draw a top view of the gyroscope shown below.



(a) Draw labeled arrows on your sketch for $\vec{\omega}$, $\vec{L}$, and $\vec{\tau}$. Draw $d\vec{L}$ produced by $\vec{\tau}$. Draw $\vec{L} + d\vec{L}$. Determine the sense of the precession by examining the direction of $\vec{L}$ and $\vec{L} + d\vec{L}$.

(b) Reverse the direction of the spin angular velocity $\vec{\omega}$ of the rotor (flywheel) and repeat all steps in Part (a).

(c) What happens if you gently add some weight to the end of the flywheel axis farthest from the pivot?
Answers: $d\vec{L} = \vec{\tau}dt$ becomes larger thus it results in fast precession. $\Omega = \tau/I\omega$.