香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

PHY1001 Mechanics

2022-2023 Term 2

Final Examination

May 15, 2023; Time Allowed: 3 Hours

NAME (print) ________________________________

CUHKSZ ID ________________________________

Exam Room No. and Seat No. ________________________________

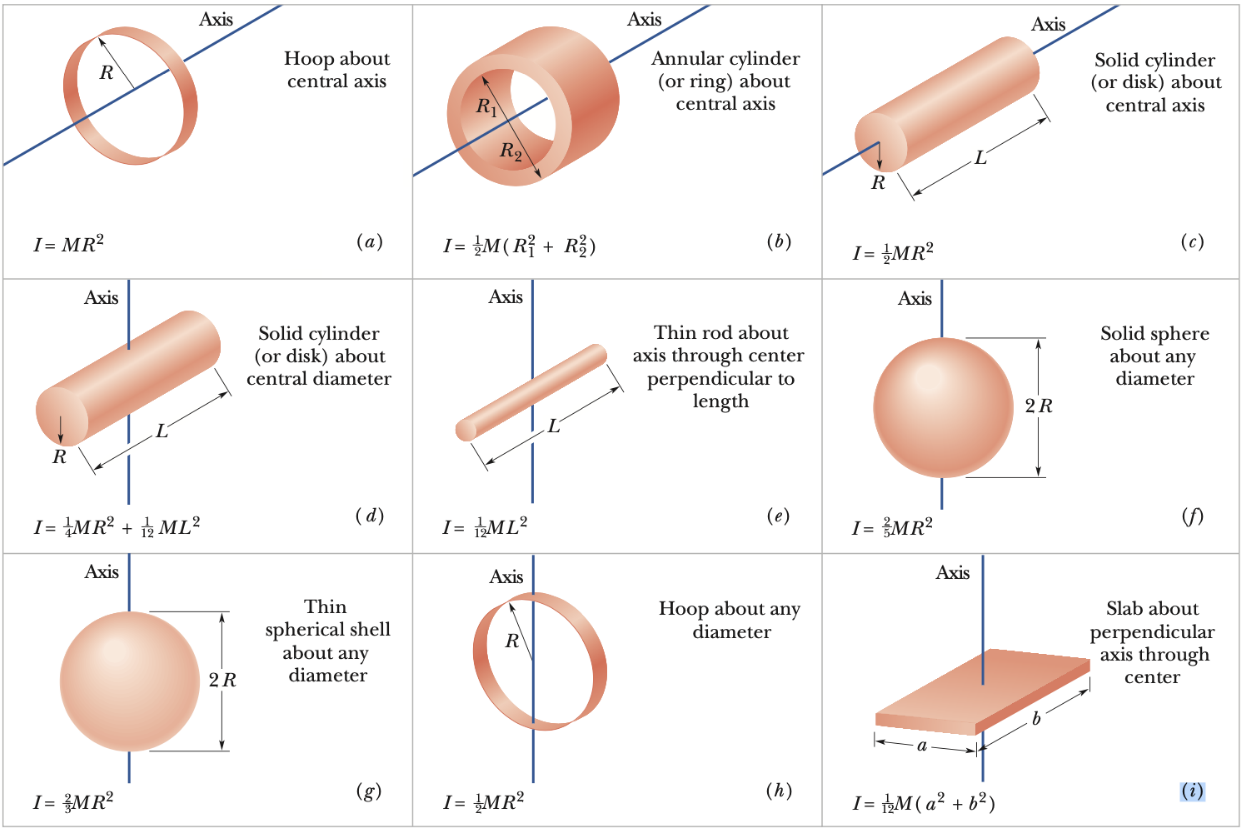
- **Show all your work**. Correct answers with little supporting work will not be given credit.
- Closed Book Exam: One piece of double-sided A4 reference paper, a scientific calculator, and a paper-based dictionary are allowed.
- Unless approved by the instructors, students who arrive more than 30 minutes late will NOT be admitted.
- The total points are 120 points. You need to finish ALL the questions in 3 hours (180 minutes).

#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	TOTAL
10	10	10	10	10	10	10	10	10	10	10	10	120

Summary of Basic Calculus:

$$\frac{d}{dx}x^n = nx^{n-1},$$
$$\frac{d}{dx}e^{ax} = ae^{ax},$$
$$\frac{d}{dx}\ln ax = \frac{1}{x},$$
$$\frac{d}{dx}(uv) = v\frac{d}{dx}u + u\frac{d}{dx}v,$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ \ (n \neq -1),$$
$$\int \frac{dx}{x} = \ln x + C,$$
$$\int e^{ax}dx = \frac{1}{a}e^{ax} + C, \quad C \text{ is a constant.}$$

Table of Moments of Inertia:



Bernoulli's equation

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{Constant}$$

Simple Harmonic Oscillator

Spring Mass System

$$M\frac{d^2x}{dt^2} = -kx,$$

1. A uniform solid ball with mass M and radius R is given an initial clockwise angular velocity ω_0 and zero linear velocity ($v_{com} = 0$) before it is placed on a horizontal surface with kinetic friction coefficient μ_k. **(10 pts)**

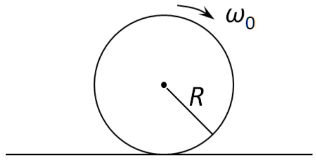


Fig. 1

(a) Which direction is the ball going to move towards? (Hint: Left or Right.) **(2 pts)**

0.5pt The contact point is moving to the left relative to the surface.
0.5pt Thus the friction applied to the ball is to the right &
1pt The ball is moving toward right &

(b) What is the acceleration at the moment that the ball is placed on the surface? **(2 pts)**

Consider the friction f (kinetic).

$f = \mu_k N$ 0.5 pt

$N = Mg$ 0.5 pt

$\Rightarrow a = \frac{f}{M} = \mu_k g$ 1 pt

(c) Show that the corresponding angular acceleration of the ball is $\alpha = 5\mu_k g/(2R)$? **(2 pts)**

Choose C.O.M as the axis

with $f = \mu_k Mg \Rightarrow fR = I\alpha$ 1pt

$I = \frac{2}{5}MR^2$ 1pt

$\Rightarrow \alpha = \frac{5\mu_k g}{2R}$

(d) How long does it take for the ball to roll without slipping? (Hint: The rolling without slipping happens when $\omega R + v = 0$) **(4 pts)**

$$\begin{cases} \omega = -\omega_0 + \alpha t \\ v = at \end{cases}$$
1pt 1pt

$\Rightarrow \omega R + v = 0$ 1pt

$\Rightarrow -\omega_0 R + \frac{5}{2}\mu_k g t + \mu_k g t = 0$

$\Rightarrow t = \frac{2}{7}\frac{\omega_0 R}{\mu_k g}$ 1pt

2. A uniform solid cylinder of mass M is supported on a ramp that rises at angle θ above the horizontal by a wire that is wrapped around its rim and pulls on it tangentially parallel to the ramp as shown in the figure below. The static friction coefficient is $\mu_s = 0.5$. The cylinder is at equilibrium state. **(10 pts)**

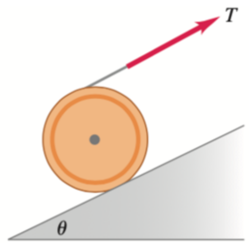


Fig. 2

(a) Show that there must be friction on the surface for the cylinder to stay at equilibrium state. **(2 pts)**

1pt equilibrium state $\Rightarrow$ zero torque and force.

choose COM as axis $fR - TR = 0$

1pt thus there must be friction!

Other correct reasoning can get full 2 pts as well!

(b) From the torque equation, show that the friction f is equal to the tension T ($f = T$). **(2 pts)**

choose COM as axis

1pt $fR - TR = 0$

1pt $\Rightarrow f = T$.

(c) Find the frictional force (in terms of Mg and θ) to keep the cylinder at rest. **(2 pts)**

From $\sum F_i = 0$ along the ramp.

together with $f = T$.

1pt $\Rightarrow f + T - Mg\sin\theta = 0$

1pt $\Rightarrow f = \frac{1}{2} Mg\sin\theta$

(d) Find the normal force N from the ramp to keep the cylinder at rest. **(2 pts)**

$\perp$ the ramp

$N - Mg\cos\theta = 0 \rightsquigarrow$ 1pt

$\Rightarrow N = Mg\cos\theta \longrightarrow$ 1pt

(e) Find the maximum angle θ to keep the cylinder at rest. **(2 pts)**

1pt $f = \frac{1}{2} Mg\sin\theta \le \mu_s N = \mu_s Mg\cos\theta$

1pt $\Rightarrow \boxed{\tan\theta \le 2\mu_s} = 1.$

this is OK.

$\theta_{max} = 45^\circ$. or $\frac{\pi}{4}$.

3. As shown in the figure below, a point particle of mass m_1 is a distance d from one end of a uniform thin rod with length L and mass M. **(10 pts)**

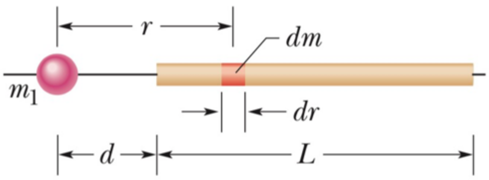


Fig. 3

(a) What is the magnitude of the gravitational force on the particle from the rod? (Hint: As shown above, use $\int \frac{dr}{r^2} = -\frac{1}{r}$ to show that magnitude is $\frac{GMm_1}{d(d+L)}$.) **(5 pts)**

Only compute the amplitude

$$dF = \frac{Gm_1\,dm}{r^2} = \frac{Gm_1M}{L}\frac{dr}{r^2}$$ 2pt

$$F = \frac{Gm_1M}{L}\int_d^{L+d}\frac{dr}{r^2}$$ 2pt

one or two steps can be skipped !!

$$= \frac{Gm_1M}{L}\left(-\frac{1}{r}\right)\Big|_d^{L+d}$$ 1pt

$$= \frac{GMm_1}{d(L+d)}$$

(b) Find the gravitational potential energy U between the particle and the rod. (Take the potential energy to be zero when the rod and particle are infinitely far apart.) **(5 pts)**

$$dU = -\frac{Gm_1\,dm}{r}$$ 1pt

$$= -\frac{Gm_1M}{L}\frac{dr}{r}$$ 1pt

$$U = -\frac{Gm_1M}{L}\int_d^{L+d}\frac{dr}{r}$$ 2pt

$$= -\frac{Gm_1M}{L}\ln\frac{L+d}{d}$$ 1pt

4. A rock with irregular shape is suspended by a light string. When the rock is hanging in the air (suppose the air buoyancy is negligible), the tension in the string is $T_1 = 20.0$ N. When the rock is totally immersed in water (mass density of water is $\rho_w = 1.0 \times 10^3$ kg/m^3), the tension is $T_2 = 10.2$ N. **(10 pts)**

(a) What is the volume of the rock? **(3 pts)**

(1pt) $T_1 = W$

(1pt) $T_2 = W - \rho_w g V$

0.5 pt $V = \frac{(T_1 - T_2)}{g \rho_w}$

0.5 pt $= 1.0 \times 10^{-3}\ m^3$

(b) What is the average mass density of the rock? **(4 pts)**

(2pts) $\rho_R g V = W = T_1$

(1pt) $\rho_R = \frac{T_1}{gV}$

(1pt) $= 2.0 \times 10^3\ kg/m^3$

or $2.04 \times 10^3\ kg/m^3$

(c) When the rock is totally immersed in an unknown liquid, the tension is $T_3 = 7.9$ N. What is the density of the unknown liquid? **(3 pts)**

(1pt) $T_3 + \rho_0 g V - W = 0$

(1pt) $\rho_0 = \frac{W - T_3}{gV}$

(1pt) $= 1.2 \times 10^3\ kg/m^3$

or $1.23 \times 10^3\ kg/m^3$

5. In the figure below, the fresh water behind a reservoir dam has depth $D = 12$ m. A horizontal pipe passes through the dam at depth $d = 6.0$ m. The inner and outer parts of the pipe have cross-section areas of $S_1 = 18.8\text{cm}^2$ and $S_2 = 12.5\text{cm}^2$. A plug secures the pipe opening. **(10 pts)**

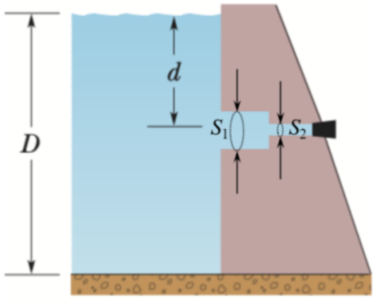


Fig. 4

(a) Find the values of gauge pressure in the horizontal pipe? **(2 pts)**
(Hint: The two diameters of the horizontal pipe are much smaller than d and D. The gauge pressure of water at the cross-section of S_1 approximately equals to that at the cross-section of S_2.) (The gauge pressure is defined as the difference between an absolute pressure (P) and the atmospheric pressure (P_{atm}).)

1pt gauge pressure $\Delta P = \rho g d$

1pt $= 5.9 \times 10^4$ Pa

or 5.88×10^4 Pa.

(b) Find the magnitude of the frictional force between plug and pipe wall. **(2 pts)**

1pt $f - \Delta P S_2 = 0$

1pt $f = \Delta P S_2 = 74N$ (or $73.5N$).

(c) When the plug is removed, what is the water volume that exits the pipe in 1.0 hour? **(3 pts)**

use Bernoulli's equation

$P_0 + \frac{1}{2}\rho v^2 = P_0 + \rho g d \Rightarrow$ 1pt

$v = \sqrt{2gd}$ 1pt

Volume $= vS_2 t = 49\,m^3$ (or $50\,m^3$) 1pt

(d) What is the speed of water flowing through the cross-section of S_1? **(3 pts)**

2pts $vS_2 = V_1 S_1$ Continuity equation

1pt $\Rightarrow V_1 = \frac{vS_2}{S_1} = 7.2\ m/s$

6. As shown in Fig. 5, two stars with masses M_1 and M_2 orbit around their center of mass (COM) at the same angular velocity ω. Both orbits are circular. Let R_1 be the distance between star M_1 and COM, and R_2 be the distance between star M_2 and COM.

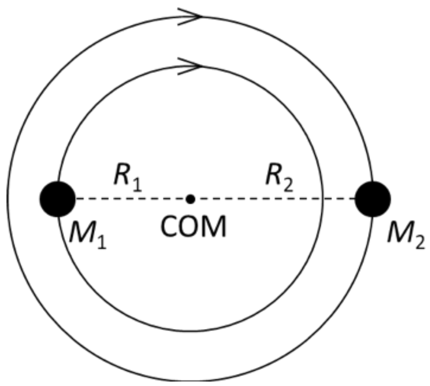


Fig. 5

(a) Find the gravitational force (magnitude and direction) of one star on the other. **(2 pts)**

0.5 $F = \frac{GM_1M_2}{(R_1+R_2)^2}$

0.5 F points from each star to COM

attractive force is OK.

(b) Show that $M_1R_1 = M_2R_2$. **(2 pts)**

1pt $F = M_1\frac{V_1^2}{R_1} = M_2\frac{V_2^2}{R_2}$

or $= M_1\omega^2R_1 = M_2\omega^2R_2$

$\Rightarrow M_1R_1 = M_2R_2$

ω is the same for M_1 and M_2

1pt

(c) Show that $(R1 + R2)^3\omega^2 = G(M_1 + M_2)$. **(2 pts)**

1pt $\frac{GM_1M_2}{(R_1+R_2)^2} = M_1R_1\omega^2 \Rightarrow \frac{GM_2}{(R_1+R_2)^2} = R_1\omega^2$

1pt $\frac{GM_1M_2}{(R_1+R_2)^2} = M_2R_2\omega^2 \Rightarrow \frac{GM_1}{(R_1+R_2)^2} = R_2\omega^2$

1pt $\Rightarrow G(M_1+M_2) = (R_1+R_2)^3\omega^2$

(d) Find angular momentum of star M_1 about COM, including the magnitude and direction. (Hint: is the direction pointing in or out of the exam paper) **(2 pts)**

☆ there are two possible answers!!

$L_1 = M_1V_1R_1$

OK

$= M_1\omega R_1^2 = M_1R_1^2\sqrt{\frac{G(M_1+M_2)}{(R_1+R_2)^3}}$ OK

1pt

1pt

use $M_1 = \frac{M_2R_2}{R_1}$

Another answer

$L_1 = M_1R_1\cdot\sqrt{\frac{GR_1M_2}{(R_1+R_2)}}$ 1pt

OK

(e) How much energy would be required to separate the two stars to infinity? **(2 pts)**

OK

1pt $\Delta E = E_2 - (K+U) = 0 - \frac{1}{2}M_1V_1^2 - \frac{1}{2}M_2V_2^2 + \frac{GM_1M_2}{(R_1+R_2)} = \frac{GM_1M_2}{2(R_1+R_2)}$ OK

1pt $W = \Delta E = \frac{GM_1M_2}{2(R_1+R_2)}$ OK

correct answer gets 2pts

7. Demonstration of Kepler's second law (A planet sweeps out equal areas of the ellipse over equal time intervals.) for a planet that orbits the Sun in an elliptical path. **(10 pts)**

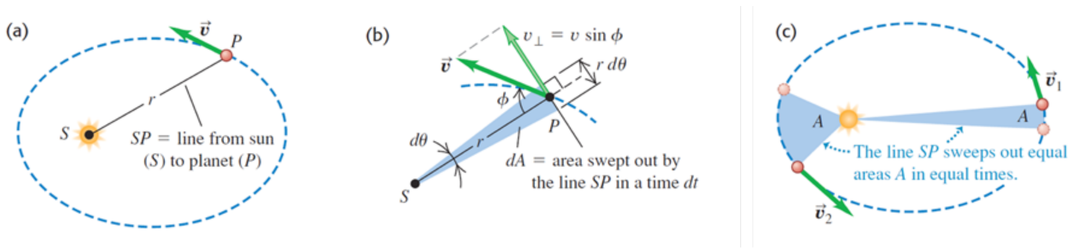


Fig. 6

(a) Write down the gravitational force $\vec{F}_g$ exerted to the planet by the Sun. **(2 pts)**

2pt

$$\vec{F}_g = -\frac{G M_s m}{r^2}\hat{r}$$

$$\text{or} \quad = -\frac{G M_s m}{r^2}\frac{\vec{r}}{r}$$

any similar result is OK

(b) Show that the angular momentum of the planet is conserved. **(3 pts)**

$$\frac{d\vec{L}}{dt} = \vec{\tau} \rightsquigarrow \text{1pt}$$

$$\vec{\tau} = (\vec{r} \times \vec{F}) = 0 \rightsquigarrow \text{1pt}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0 \rightsquigarrow \text{1pt}$$

(c) Show that the area swept out by the line SP per unit time $dA/dt = L/(2m)$. **(3 pts)**

1pt

$$dA = \frac{1}{2} r (v\,dt) \sin\phi$$

$$= \frac{1}{2} |\vec{r} \times \vec{v}|\, dt$$

1pt

$$\vec{L} = \vec{r} \times m\vec{v} \quad \text{and} \quad L = |\vec{r} \times m\vec{v}|$$

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2m} \quad \text{1pt}$$

(d) Demonstrate the Kepler's second law is true, namely, dA/dt is a constant. **(2 pts)**

First method: L angular momentum is conserved from point (b). L = constant 2pt

Second method $\frac{d}{dt}\left(\frac{dA}{dt}\right) = \frac{1}{2m}\frac{d}{dt}(L) = 0 \rightsquigarrow$ 1pt

$$\Rightarrow \frac{dA}{dt} = \text{Constant} \rightsquigarrow \text{1pt}$$

8. The Tale of Two Springs. **(10 pts)**

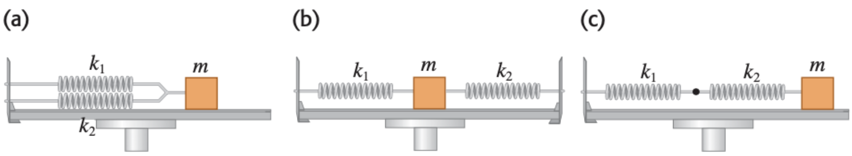


Fig. 7

Two springs with the same unstretched length but different force constants k_1 and k_2 are attached to a block with mass m on a level, frictionless surface.

(a) Use Newton's 2nd law, write down the equation of motion for the oscillator in case (a) as shown in the above Fig. 7 (a). Find the corresponding angular frequency ω_a. **(4 pts)**

Suppose the displacement x.

2pt $m\frac{d^2x}{dt^2} = -k_1x - k_2x = -(k_1+k_2)x$

2pt $\Rightarrow \omega_a = \sqrt{\frac{k_1+k_2}{m}}$

(b) Write down the equation of motion for the oscillator in case (b) as shown in the above Fig. 7 (b) and find the corresponding angular frequency ω_b. **(3 pts)**

1pt $m\frac{d^2x}{dt^2} = -(k_1+k_2)x$

2pt $\omega_b = \sqrt{\frac{k_1+k_2}{m}}$

(c) Write down the equation of motion for the oscillator in case (c) as shown in the above Fig. 7 (c) and find the corresponding angular frequency ω_c. **(3 pts)**

Suppose a total displacement $x = x_1 + x_2$

1pt $k_1x_1 = k_2x_2$

$\Rightarrow x_1 = \frac{k_2}{k_1+k_2}x \quad ; \quad x_2 = \frac{k_1}{k_1+k_2}x$

1pt EOM $\Rightarrow m\frac{d^2x}{dt^2} = -k_1x_1 = -\frac{k_1k_2}{k_1+k_2}x$

1pt $\Rightarrow \omega_c = \sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$

9. A certain transverse wave on a rope is described by

$$y_1(x,t) = (6.50\times10^{-3}\mathrm{m})\cos 2\pi\left(\frac{x}{0.280\,\mathrm{m}} - \frac{t}{0.0360\,s}\right)$$

(a) Which direction does the wave propagate (travel)? **(1 pts)**

1pt

+x direction

(b) Determine the wave's amplitude. **(1 pts)**

1pt

$A = 6.50\times10^{-3}\,m$

(c) Find the wavelength λ. **(1 pts)**

1pt

$\lambda = 0.280\ m$

(d) What is the period T? **(1 pts)**

1pt

$T = 0.0360\ S$

(e) What is its speed of propagation (the wave traveling speed)? **(1 pts)**

1pt

$V = \frac{\lambda}{T} = 7.78\ m/s$

(f) The mass per unit length of the rope is $\mu = 0.0500kg/m$. Find the tension F_T. **(1 pts)**

1pt

$V = \sqrt{\frac{F_T}{\mu}} \Rightarrow F_T = V^2\mu = 3.02\ N$

(g) Find the average power $P_{av} = \frac{1}{2}\sqrt{\mu F_T}\omega^2 A^2$ of this wave. **(1 pts)**

1pt

$P_{av} = \frac{1}{2}\sqrt{\mu F_T}\cdot\omega^2A^2$
$= \frac{1}{2}\sqrt{3.02\times0.05}\times\left(\frac{2\pi}{T}\right)^2\cdot A^2 = 0.250\ (W)$

(h) The reflection of the above wave can be described by

$$y_2(x,t) = -(6.50\times10^{-3}\mathrm{m})\cos 2\pi\left(\frac{x}{0.280\,\mathrm{m}} + \frac{t}{0.0360\,s}\right).$$

The superposition of y_1 and y_2 gives rise to a standing wave. Find the form of the standing wave $y(x,t) = y_1(x,t) + y_2(x,t)$ and the location of nodes. **(3 pts)**

1pt

$y = A\cos(kx-\omega t) - A\cos(kx+\omega t)$
$= 2A\sin kx\sin\omega t$

1pt

$= 2\times(6.50\times10^{-3}m)\sin\frac{2\pi x}{0.280m}\sin\frac{2\pi t}{0.0360S}$

locations of nodes $kx = n\pi = \frac{2\pi}{\lambda}x$ 1pt

$\Rightarrow x = \frac{1}{2}n\lambda = (0.140)n\ (m)$

10. Doppler Effects: $f' = \frac{v \pm v_d}{v \pm v_s} f$. **(10 pts)**

A firetruck's siren emits a sinusoidal wave with frequency $f = 170$ Hz. The speed of sound is 340 m/s and the air is still.

(a) Find the wavelength of the sound wave if the firetruck is at rest. **(2 pts)**

$$\lambda = vT = v\frac{1}{f}$$ 1pt

$$= \frac{340}{170} m = 2.00 m/s$$ 1pt

(b) What frequency will the stationary listener hear if the firetruck is moving toward him with a speed $v_s = 40$ m/s ? **(2 pts)**

$$f_b = \frac{v}{v - v_s} f$$ 1pt

$$= \frac{340}{300} \cdot \times 170 = 193 \text{ Hz}$$ 1pt

close to 190 is fine.

(c) What frequency will the stationary listener hear if the firetruck is moving away from him with a speed $v_s = 60$ m/s ? **(2 pts)**

$$f_c = \frac{v}{v + v_s} f$$ 1pt

$$= \frac{340 \times 170}{400} = 145 \text{ Hz}$$ 1pt

close to 145 is OK.

(d) Now the firetruck is at rest and the listener is moving toward the firetruck with a speed $v_d = 50$ m/s, what frequency will the moving listener hear? **(2 pts)**

$$f_d = \frac{v + v_d}{v} f$$ 1pt

$$= \frac{390}{340} \times 170 = 195 \text{ Hz}$$ 1pt

(e) Now the firetruck is at rest and the listener is moving away from the firetruck with a speed $v_d = 30$ m/s, what frequency will the moving listener hear? **(2 pts)**

$$f_e = \frac{v - v_d}{v} f$$ 1pt

$$= \frac{340 - 30}{340} \times 170 = 155 \text{ Hz}$$ 1pt

11. A uniform disk with Mass M and radius R mounted on ceiling. Two blocks with mass m_1 and m_2 ($m_2 > m_1$) hang from a massless cord that is wrapped around the rim of the disk as shown in the figure below. **(10 pts)**

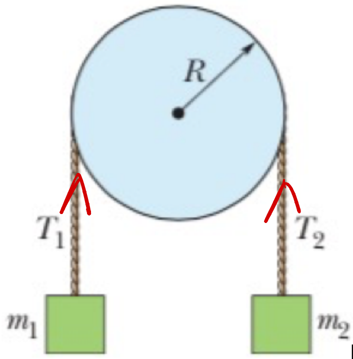


Fig. 8

(a) Let us first set mass of the disk $M = 0$, find the tensions T_1 and T_2 as indicated in the above graph. **(2 pts)**

0.5 $T_1 - m_1 g = m_1 a$

0.5 $m_2 g - T_2 = m_2 a$

$T_1 = T_2$ 0.5

$$\Rightarrow \begin{cases} a = \dfrac{m_2 - m_1}{m_1 + m_2} g \\ T_1 = T_2 = \dfrac{2m_1 m_2}{m_1 + m_2} g \end{cases}$$

0.5

(b) Now assume the disk mass M is nonzero and there is no slipping between the cord and the disk (the disk is rotating as the blocks move), find the tensions T_1 and T_2 as indicated in the above graph. **(4 pts)**

$T_1 - m_1 g = m_1 a$ $\quad$ $m_2 g - T_2 = m_2 a$

1pt $\quad -(T_2 - T_1)R = +\frac{1}{2}MR^2\alpha$

1pt $\quad R\alpha + a = 0$

$\Rightarrow$ 1pt $T_1 = \dfrac{(2m_2 + \frac{1}{2}M)\, m_1 g}{m_1 + m_2 + \frac{1}{2}M}$; $T_2 = \dfrac{(2m_1 + \frac{1}{2}M)\, m_2 g}{m_1 + m_2 + \frac{1}{2}M}$ 1pt

(c) When block m_2 drops a height of h, what is the angular speed of the disk? **(4 pts)**

Two methods:

① use $a = \dfrac{(m_2 - m_1) g}{m_1 + m_2 + \frac{1}{2}M}$ 2pt

$v = \sqrt{2ah}$ 1pt $\quad \omega = \dfrac{v}{R} = \dfrac{1}{R}\sqrt{\dfrac{2(m_2 - m_1)gh}{m_1 + m_2 + \frac{1}{2}M}}$ 1pt

② energy conservation

2pt $(m_2 - m_1) g h = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} \cdot \frac{1}{2} M R^2 \omega^2$

1pt $R\omega = v$

$\Rightarrow \omega = \dfrac{1}{R}\sqrt{\dfrac{2(m_2 - m_1) h g}{m_1 + m_2 + \frac{1}{2}M}}$ 1pt

12. Oscillation in general. **(10 pts)**

(a) For a simple harmonic oscillator, the net force on the body with mass m is given by $F_x = -kx$. What is the potential energy function for this simple harmonic oscillator if we take $U = 0$ at $x = 0$? **(1 pts)**

use work-energy theorem

$$U(x) = -\int_0^x dx\, F_x = \underline{\underline{\frac{1}{2}kx^2}}. \quad \text{——} \quad 1pt$$

(b) Use energy conservation, show that the velocity v_x as a function of x is

$$v_x = \frac{dx}{dt} = \sqrt{\frac{k}{m}(A^2 - x^2)}.$$

where A is the amplitude of the oscillator. **(1 pts)**

$$\underset{\text{potential}}{\frac{1}{2}kA^2 + 0} = \underset{\text{kinetic}}{\frac{1}{2}mv_x^2} + \frac{1}{2}kx^2 \rightsquigarrow 1pt$$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

(c) Separate the variable by writing all factors containing x on one side and all factors containing t on the other side so that each side can be integrated, and use the identity $\int_0^1 \frac{du}{\sqrt{1-u^2}} = \frac{\pi}{2}$ to find the period T of the simple harmonic oscillator. (Hint: One-quarter (1/4) of T is the time for the body to move from $x = 0$ to $x = A$.) **(2 pts)**

Separate variables

$$\int_0^A \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^{T/4} \sqrt{\frac{k}{m}}\, dt \rightsquigarrow 1pt$$

let $u = x/A$

$$\Rightarrow \int_0^1 \frac{du}{\sqrt{1-u^2}} = \frac{\pi}{2} = \sqrt{\frac{k}{m}}\frac{T}{4}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} \rightsquigarrow 1pt$$

(d) For a second oscillator, the net force on the body with mass m is given by $F_x = -cx^3$. What is the potential energy function for this oscillator if we take $U = 0$ at $x = 0$? **(2 pts)**

$$U(x) = -\int_0^x F_x\,dx \quad \rightsquigarrow \text{1pt}$$

$$= \frac{1}{4}cx^4 \quad \rightsquigarrow \text{1pt}$$

(e) Follow part (d), compute the period T_2 for the second oscillator. (Hint: You may find the identity $\int_0^1 \frac{du}{\sqrt{1-u^4}} = 1.31$ useful.) **(3 pts)**

use energy conservation

1pt

$$\frac{1}{4}CA^4 = \frac{1}{4}Cx^4 + \frac{1}{2}mV_x^2$$

$$\Rightarrow \quad V_x = \frac{dx}{dt} = \sqrt{\frac{C}{2m}(A^4 - x^4)}$$

1pt

$$\Rightarrow \int_0^{T_2/4} \sqrt{\frac{C}{2m}}\,dt = \int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \frac{1}{A}\int_0^1 \frac{du}{\sqrt{1-u^4}}$$

1pt

$$\Rightarrow \quad T_2 = 4\sqrt{\frac{2m}{C}} \cdot 1.31\frac{1}{A} = \frac{7.41}{A}\sqrt{\frac{m}{C}}$$

$$\text{or} = \frac{5.24}{A}\sqrt{\frac{2m}{C}}. \quad \text{OK.}$$

(f) According to the result you obtained in part (e), does the period T_2 depend on the amplitude A of the motion? Is this oscillation simple harmonic? **(1 pts)**

Yes, T_2 depends on A.

1pt

No, it is not simple harmonic.