$B=-V\frac{dP}{dV}$ $B=\gamma P$ $V_s=\sqrt{\frac{\gamma P}{\rho}}=\sqrt{\frac{\gamma RT}{M}}$ $\Delta E=P\Delta t=\frac{1}{2}\Delta m W^2 A^2$

$f(x-Vt)$ $+x$, $f(x+Vt)$ $-x$ $\frac{1}{k^2}\frac{d^2y}{dx^2}=\frac{1}{w^2}\frac{d^2y}{dt^2}$

$y_m \sin[k(x-Vt)]$

Torque $\vec{\tau}=\vec{r}\times\vec{F}$ N·m 对Rotation而言 $k=\frac{1}{2}Iw^2$ $I=I_{com}+Mh^2$ Rolling = Translation + Rotation

$\tau_{net}=I\alpha$ $a=\alpha R$ (常用联系a与α方向!) $W=\int_{\theta_i}^{\theta_f}\tau d\theta$

$P=\tau w$ 平动与转动 $V_{com}=wR$ (smooth rolling)

力学特征 $a_{com}=\alpha R$, $F_{net}=ma R$, $\tau_{net}=I_{com}\frac{a_{com}}{R}=I\alpha$

旋转启动 $f_s=ma_{com}$, $\tau_{net}=\tau_{app}-f_sR=I_{com}\frac{a_{com}}{R}$

$a_{com}=\frac{\tau_{app}}{mR}\cdot\frac{1}{(1+\frac{I}{mR^2})}$ 力启动 $F_{app}-f_s=ma_{com}$

$\tau_{net}=f_sR=I\frac{a_{com}}{R}$ $a_{com}=\frac{F_{app}}{mR}\cdot\frac{1}{[1+\frac{I}{mR^2}]}$

实例: Bicycle $f_{s后}-f_{s前}=2ma_{com}$ $a_{com}=\frac{\tau_{app}}{2mR}\cdot\frac{1}{[1+\frac{I}{mR^2}]}$

Car: $a'_{com}=\frac{1}{2}a_{com}$ [Bicycle]

Rolling ↓ $a_{com}=-\frac{g\sin\theta}{1+I_{com}/MR^2}$ Rolling ↑ $a_{com}=\frac{g\sin\theta}{1+I_{com}/MR^2}$

能量: $K=\frac{1}{2}I_{com}W^2+\frac{1}{2}MV_{com}^2$

Force启动 $\Delta k=F_{app}\Delta S$ Torque启动 $\Delta k=\tau_{app}\Delta\theta$

Rolling ↑ max height $h=\frac{(W_0R)^2}{2g}[1+\frac{I_{com}}{mR^2}]>h'=\frac{(W_0R)^2}{2g}$ [无f]

Yo-Yo $T-mg=ma_{com}$

$\tau_{net}=TR_0=I_{com}\frac{-a_{com}}{R_0}$ $T=-I_{com}\frac{a_{com}}{R_0^2}$ $a_{com}=\frac{-g}{1+I_{com}/mR_0^2}$

Yo-Yo Sliping $u_kmg=ma_{com}$ $V_{com}=a_{com}t$ $f_kR=u_kmgR=I_{com}\alpha$

smooth → $V_{cm}=WR$ $t=\frac{W_0R}{u_kg[1+\frac{mR^2}{I_{com}}]}$ $[a_{com}\mp\alpha R]$ $W=W_0-\alpha t$

Yo-Yo 的 R!

角动量 $\vec{l}=(\vec{r}\times\vec{v})m$ $\vec{L}=I\vec{w}$ [Symmetric, uniform]

守恒 $L_i=L_f$ $I_iw_i=I_fw_f$ $\vec{\tau}_{net}=\frac{d\vec{L}}{dt}$

$\Omega=\frac{d\varphi}{dt}=\frac{Mgr}{IW}$ 杨氏模量 E $\frac{F}{A}=E\frac{\Delta L}{L}$ (拉伸)

切变模量 G $\frac{F}{A}=G\frac{\Delta x}{L}$ 体积模量 $P=B\frac{\Delta V}{V}$ 开2 $\Delta A=\frac{1}{2}r^2\Delta\theta$

Equilibrium $\begin{cases}F_{net}=0\\ \tau_{net}=0\end{cases}$ 天体物理 分析点放未知力↗减元

$\frac{dA}{dt}=\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{1}{2}r^2W$ $L=mr^2w$ $\frac{L}{2m}=\frac{dA}{dt}$

$F=G\frac{m_1m_2}{r^2}$ $\vec{F}=G\frac{m_1m_2}{r^2}\cdot\hat{r}$ 方向Vector

$U=-\frac{GMm}{r}$ $F=-\frac{du}{dr}$ $r\to\infty$ $U\to 0$ 角动量守恒

$GM=gR^2$ $g=a_g-w^2R$ 球壳内 $\frac{d\vec{L}}{dt}=\vec{\tau}=\vec{r}\times\vec{F_g}=0$ $\vec{L}$ is constant

$F=\frac{GMm}{R^3}\cdot r$ 2nd宇速度 $\frac{1}{2}mv^2-\frac{GMm}{R}=0$ 开3

卫星的E $E=K+U=-\frac{GMm}{2r}=-K$ $V=\sqrt{\frac{2GM}{R}}$ 椭圆则 $T^2=\frac{4\pi^2a^3}{GM}$ $E=\frac{-GMm}{2a}$

$K=\frac{GMm}{2r}$ $T^2=\frac{4\pi^2}{GM}r^3$

$U=-\frac{GMm}{r}$

角动量 线动量

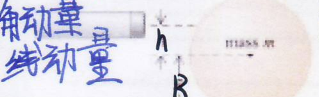
$F\Delta t=mV_{com}=mRW$ $I\alpha=fR$

$F\Delta t h=\frac{2}{5}MR^2W$ $=\frac{2}{5}MR^2\alpha$

$h=\frac{2}{5}R$ $\alpha=\frac{5f}{2MR}$ $F\Delta t=mV_0$ $V_f=\frac{5}{7}V_0$

$W=\alpha t$ $WR=V_0-\frac{f}{M}t$

$V_0=\frac{7f}{2M}t$ $WR=\frac{5f}{2M}t$



$G=6.67\times10^{-11}m^3/kg\cdot s^2$

流体力学 gauge pressure $P_g=P-P_0=\rho gh$

帕斯卡定律 $\frac{F_i}{A_i}=\frac{F_o}{A_o}$ $V_iA_i=V_oA_o$ $F_od_o=F_id_i$

流量 $A_1V_1=A_2V_2$ $R_V=AV$, $R_m=\rho AV$

伯努利 $P_1+\frac{1}{2}\rho V_1^2+\rho gy_1=P_2+\frac{1}{2}\rho V_2^2+\rho gy_2$

y_1, y_2需取参考面

S-H-M $F=-kx=ma=-mw^2x$ $w=\sqrt{\frac{k}{m}}$, $T=2\pi\sqrt{\frac{m}{k}}$

$m\frac{d^2x}{dt^2}+kx=0$ $k=w^2m$

$x(t)=X_m\cos(wt+\varphi)$ $V_m=wX_m$

$v(t)=-wX_m\sin(wt+\varphi)$

$a(t)=-w^2X_m\cos(wt+\varphi)=-w^2x(t)$ $a_m=w^2X_m$

能量 $U=\frac{1}{2}kx^2=\frac{1}{2}kX_m^2\cos^2(wt+\varphi)$ $K=\frac{1}{2}kX_m^2\sin^2(wt+\varphi)$

$E=\frac{1}{2}kX_m^2$ A-SHM $\tau=-k\theta$ 单摆 proof

$T=2\pi\sqrt{\frac{I}{k}}$ $\tau=LF_g\sin\theta=I\alpha$ $\alpha=\frac{mgL}{I}\sin\theta\approx\frac{mgL}{I}\theta$

$\alpha(t)=-w^2\theta(t)$ $T=2\pi\sqrt{\frac{L}{g}}$ $T=2\pi\sqrt{\frac{I}{mgh_{com}}}$

阻尼 $X(t)=X_me^{-\frac{bt}{2m}}\cos(w't+\varphi)$ $w'=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$ $E=\frac{1}{2}kX_m^2e^{-\frac{bt}{m}}$

$P=\frac{dE}{dt}=-bv^2$ $F_d=-bv$

共振 $X(t)=X_m\cos(w_dt+\varphi)$ $w_d=w$

$F(t)=F_d\cos w_dt$ 阻尼 $m\frac{d^2x}{dt^2}=-b\frac{dx}{dt}-kx$

横波 $y=h(x,t)$ $y_m\sin(kx-wt)$ $k=\frac{2\pi}{\lambda}$ $w=2\pi f$ $\frac{1}{v^2}\frac{d^2y}{dt^2}=\frac{d^2y}{dx^2}$!

Node $x=n\cdot\frac{\lambda}{2}$ Amplitude $=0$

$V=\frac{w}{k}=\lambda f=\frac{\lambda}{T}$ $U=V_y=\frac{dy}{dt}=-wy_m\cos(kx-wt)$ Anti

$a_y=-w^2y_m\sin(kx-wt)$ $u=\frac{m}{L}$ $V=\sqrt{\frac{F}{u}}$ Amplitude max $x=(n+\frac{1}{2})\frac{\lambda}{2}$

K, U均0 $P_{avg}=\frac{1}{2}uvw^2y_m^2$ $\varphi=2\pi$·波长之差 F-F or H-H N-A-N

K, U均max 同K, W, 缺φ叠加 同y_m $L=n\frac{\lambda}{2}$ $n=1,2,3$

$y'=2y_m\cos\frac{1}{2}\varphi$ $\sin(kx-wt+\frac{1}{2}\varphi)$ $f=n\frac{v}{2L}$ $[2y_m\sin kx]\cos wt$

同K, W, 缺φ, 不同 y_m $\sin(kx-wt+\beta)$

正交分解 Calculate y_m, β 驻波 同y_m, K, W, 但传播方向相反

H-F $L=n\frac{\lambda}{4}$ $n=1,3,5$ $f=\frac{nV}{4L}$ 纵波 $V=\sqrt{\frac{B}{\rho}}$ or $\sqrt{\frac{E}{\rho}}$

$S(x,t)=S_m\cos(kx-wt)$ $\Delta P(x,t)=\Delta P_m\sin(kx-wt)$ $\Delta P_m=(V\rho W)S_m$

$\Delta P=BkS_m\sin(kx-wt)$ $I=\frac{P}{4\pi r^2}$ $\frac{I_2}{I_1}=\frac{r_1^2}{r_2^2}$ $\frac{P_{avg}}{A}=\frac{\Delta P_m^2}{2\rho V}=\frac{\Delta P_m^2}{2\sqrt{\rho B}}$

$\Delta P_m=BkS_m=V^2\rho kS_m$ 厚柱式 $I=\frac{P}{A}$ $I=\frac{1}{2}\rho VW^2S_m^2$

$\beta=10dB\log\frac{I}{I_0}$ $I_0=1\times10^{-12}W/m^2$ $I_{beat}=\frac{1}{2}\rho VW^24S_m^2\cos^2(W't)$

叠加 同S_m, K, W, 差φ 驻波 $L=\frac{n}{2}\lambda$

$2S_m\cos\frac{\varphi}{2}\cos(kx-wt+\frac{\varphi}{2})$ 开-开 $f=n\frac{V}{2L}$

$\Delta L=L_2-L_1=n\lambda$ 加强 $\varphi=2\pi\frac{\Delta L}{\lambda}$ 开-闭 $L=\frac{n}{4}\lambda$

$=(n+\frac{1}{2})\lambda$ 减弱 $f_{beat}=|f_1-f_2|$ $n=1,3,5$ $f=\frac{nV}{4L}$

同S_m, K, φ, 缺W $2S_m\cos W't\cos(kx+Wt)$ $W'=\frac{W_1-W_2}{2}$

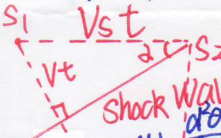
$W_{beat}=2W'$ $f'=f\frac{V\pm V_D}{V\pm V_S}$ $\sin\theta=\frac{Vt}{V_St}=\frac{V}{V_S}$ $W=\frac{W_1+W_2}{2}$

S=0 Vmax, ΔPmax P, K max S=±S_m P, K=0

S D
→ ← $\frac{V+V_D}{V-V_S}$
← → $\frac{V-V_D}{V+V_S}$
→ → $\frac{V-V_D}{V-V_S}$
← ← $\frac{V+V_D}{V+V_S}$

$P(x,t)=-\frac{\partial y}{\partial x}F_T\cdot\frac{\partial y}{\partial t}=F_y(x,t)V_y(x,t)$

$\sin\alpha=\frac{V}{V_S}$ $\tan\alpha=\frac{h}{V_St}\sin\alpha$ $\tan\alpha=\frac{h}{V_ST}$ 则 V_S可求



$W=\sqrt{\frac{I}{L}-\frac{b^2}{4L^2}}$

求T $-mg\sin\theta-Rx\cos\theta=ML\frac{d\theta}{dt}$

$Rx=\theta\cdot L\cdot k$ $-mg\theta-kL\theta=ML\frac{d\theta}{dt^2}$

$\sin\theta\approx\theta$ $d^2\theta/dt^2=-(\frac{g}{L}+\frac{k}{M})\theta$

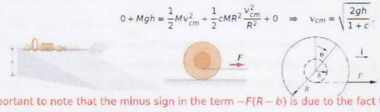
$W=\sqrt{\frac{g}{L}+\frac{k}{M}}$ $T=2\pi/W$

◆◆ Race of the rolling bodies

In a physics demonstration, an instructor "races" various bodies that roll without slipping from rest down an inclined plane (left figure below). What shape should a body have to reach the bottom of the incline first?

Solution: The easiest method is to use conservation of energy because all these rigid bodies roll without slipping ($omega=v_{cm}/R$). Each body starts from rest at the top of an incline with height h, so $K_1=0$, $U_1=Mgh$, $U_2=0$, and thus

$0+Mgh=\frac{1}{2}Mv_{cm}^2+\frac{1}{2}cMR^2\frac{v_{cm}^2}{R^2}+0 \Rightarrow v_{cm}=\sqrt{\frac{2gh}{1+c}}$ (5)



It is important to note that the minus sign in the term $-F(R-b)$ is due to the fact that the torque is clockwise

If you decide to write the following equations

$\tau=I\alpha=+F(R-b)$ instantaneous axis with $I=\frac{3}{2}mR^2$,

$F-f=ma$,

You are going get a positive α in this case, thus the pure roll condition becomes $a=R\alpha$.

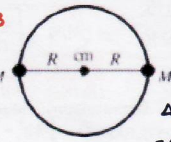
Alternatively, you can write down the torque equation and force equation as follows

$\tau=I_{cm}\alpha=bF-fR$ center of mass axis with $I_{cm}=\frac{1}{2}mR^2$,

$F-f=ma$,

which leads to the same answer with the additional equation $a+R\alpha=0$.

$G(m_1+m_2)=W^2L^3$ $L=R_1+R_2$ $W=\sqrt{\frac{G(m_1+m_2)}{L^3}}$



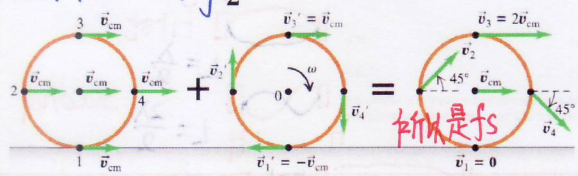
分析所需 Work $k_1+U_1+W=k_2+U_2$ $\Delta E=W_e-k_1-U_1>0$ $=0-2\times\frac{1}{2}MV^2+\frac{GM^2}{2R}=\frac{GM^2}{4R}$

$V=\sqrt{\frac{GM}{4R}}$ $T=4\pi R\sqrt{\frac{R}{GM}}$

M 黑洞 $=\frac{Rs\,c^2}{2G}$ $\frac{1}{2}mV_p^2-\frac{GMsm}{r_p}=\frac{1}{2}mV_a^2-\frac{GMsm}{r_a}$ $mV_pr_p=mV_ar_a$ $r_p=(1-e)a$ $r_a=(1+e)a$ $u=GM$

$V_p=\sqrt{\frac{u}{a}\frac{1+e}{1-e}}$ $V_a=\sqrt{\frac{u}{a}\frac{1-e}{1+e}}$

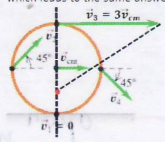
Shear Strain: $\phi\approx\frac{x}{h}=\frac{\text{Shear Stress}}{\text{Shear Modulus}}=\frac{F_{||}/A}{S}=\frac{8*110*9.8*\sin(12°)/(10\times10^{-4})}{12\times10^6}=0.15\text{rad}.$



(3) Use the method in Part (a), we find $K=\frac{1}{2}Mv_{cm}^2+\frac{1}{2}I_{cm}\omega^2=\frac{5}{2}Mv_{cm}^2$

(4) Use the method in Part (b), we find $K=\frac{1}{2}I\omega^2=\frac{5}{8}MR^2\omega^2$

with $I=I_{cm}+\frac{1}{4}MR^2=\frac{5}{4}MR^2$



椭圆轨道 ↑取a

(a) What then is the angular velocity ω of the ladybug-disk system? **Answers:** $\omega=4\omega_0/3$.

Solution: The angular momentum is conserved about the center axis in this process, simply because of vanishing torque. The magnitude of the angular momentum of the ladybug-disk system is

$$L=mr^2\omega+I\omega, \quad (22)$$

where $I=\frac{1}{2}4mR^2=2mR^2$ for the uniform disk with mass 4m and ω is the angular velocity, and r is the distance between the ladybug and the disk center.

Initially, the ladybug is at the rim, thus $r=R$, $\omega=\omega_0$, and the angular momentum is

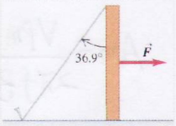
$$L_i=mR^2\omega_0+2mR^2\omega_0. \quad (23)$$

After the ladybug has completed its walk, it reaches to the final location $r=R/2$. The final angular momentum is then

$$L_f=m\frac{R^2}{4}\omega_f+2mR^2\omega_f. \quad (24)$$

From $L_i=L_f$, we obtain

$$\omega_f=\frac{3mR^2\omega_0}{m\frac{R^2}{4}+2mR^2}=\frac{4}{3}\omega_0. \quad (25)$$



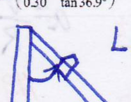
IDENTIFY: Apply $\sum\tau_z=0$ to the post, for various choices of the location of the rotation axis.

SET UP: When the post is on the verge of slipping, f_s has its largest possible value, $f_s=\mu_sn$.

EXECUTE: (a) Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so $F\frac{h}{2}=(n-w)h\tan\theta$.

Taking torques about the upper point (where the rope is attached to the post), $fh=F\frac{h}{2}$. Using $f\le\mu_sn$ and solving for F, $F\le 2w\left(\frac{1}{\mu_s}-\frac{1}{\tan\theta}\right)^{-1}=2(400\text{ N})\left(\frac{1}{0.30}-\frac{1}{\tan 36.9°}\right)^{-1}=400\text{ N}.$

Roll without slipping contact pt's velocity = 0 → 逆打木块



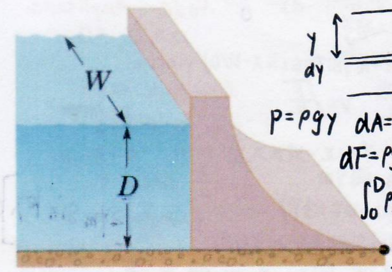
$L_i=mVL=L_f=I\omega$ 垂直的! No Sliping $V_{com}+WR=0$

$P=\frac{dF_L}{dA}$ $B=-\frac{\Delta P}{...}$

... only fs

$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ $\quad\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ $\quad\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$



y, dy

$p=\rho g y \quad dA=W\,dy$

$dF=\rho g y W dy$

$\int_0^D \rho g y W dy$

$\frac{1}{2}\rho g W D^2$

$d\tau$

$D=\rho g y W(D-y)dy$

$\int_0^D \rho g y W(D-y)dy$

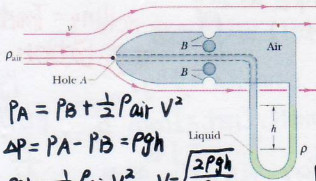
$=\rho g W[\frac{1}{2}D^3-\frac{1}{3}D^3]$

$p_1+\frac{1}{2}\rho v_1^2+\rho g h_1$

$=p_2+\frac{1}{2}\rho v_2^2+\rho g h_2$

$p_2=p_0+\frac{1}{2}\rho(v_1^2-v_2^2)$

$p_g=p_2-p_0$



$p_A=p_B+\frac{1}{2}\rho_{air}v^2$

$\Delta p=p_A-p_B=\rho g h$

$\rho g h=\frac{1}{2}\rho_{air}v^2 \quad v=\sqrt{\frac{2\rho g h}{\rho_{air}}}$

$p_1+\rho g y_1+\frac{1}{2}\rho v_1^2$

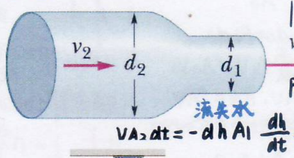
$=p_2+\rho g y_2+\frac{1}{2}\rho v_2^2$

$v_1A_1=v_2A_2$

$p_1-p_2=\frac{1}{2}\rho v_1^2[\frac{A_1^2}{A_2^2}-1]$

$=\rho g h$

$v_1=\sqrt{\frac{2gh}{(\frac{A_1}{A_2})^2-1}}$



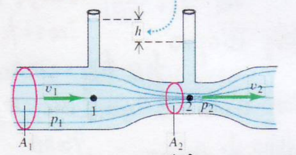
$p_0+\rho g h=p_0+\frac{1}{2}\rho v^2$

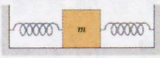
$v=\sqrt{2gh}$

$v A_2 dt=-dh A_1 \quad \frac{dh}{dt}=-\frac{A_2}{A_1}v$

$\frac{dh}{\sqrt{h}}=-\frac{A_2}{A_1}\sqrt{2g}\,dt$

H, A_1, A_2



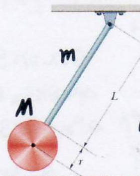


$$m\frac{d^2x}{dt^2}=-k_1x-k_2x.$$

Substituting $x=x_m\cos(\omega t+\phi)$ and simplifying, we find

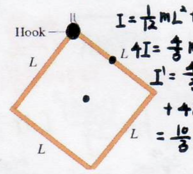
$$\omega^2=\frac{k_1+k_2}{m} \quad\text{and}\quad f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}.$$

$$f_1=\frac{1}{2\pi}\sqrt{\frac{k_1}{m}}=30\,Hz \quad\text{and}\quad f_2=\frac{1}{2\pi}\sqrt{\frac{k_2}{m}}=50\,Hz,\quad f=\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}=\sqrt{f_1^2+f_2^2}=58\,Hz.$$



$I=\frac{1}{2}Mr^2+M(r+L)^2+\frac{1}{12}mL^2+m(\frac{1}{2}L)^2$

$d_{com}=\frac{M\cdot(L+r)+m\cdot\frac{L}{2}}{m+M}$ $\quad T=2\pi\sqrt{\frac{I}{(m+M)g\,d_{com}}}$



$I=\frac{1}{12}mL^2+m(\frac{L}{2})^2$

$4I=\frac{4}{3}mL^2$

$I'=\frac{4}{3}mL^2$

$+4m(\frac{L}{\sqrt{2}})^2$

$=\frac{10}{3}mL^2$

$f=\frac{1}{2\pi}\sqrt{\frac{Mg\,d_{com}}{I'}}$

Choose COM of the rod as the axis.

Angular momentum conservation ——— 2pt

$mv\frac{L}{2}=I_{cm}\omega$ ①

Momentum Conservation ——— 2pt

$mv=MV_{cm}$ ②

Kinetic energy conservation ——— 2pt

$\frac{1}{2}mv^2=\frac{1}{2}MV_{cm}^2+\frac{1}{2}I_{cm}\omega^2$ ③ with $I_{cm}=\frac{1}{12}ML^2$

①+②+③ ⇒ $M=4m$ ——— 2pts

$$U=-\frac{GMm}{L}\ln\frac{\sqrt{\frac{L^2}{4}+a^2}+\frac{L}{2}}{\sqrt{\frac{L^2}{4}+a^2}-\frac{L}{2}}\qquad\text{(See integral identities on Page 1.)}$$

Solution: Potential energy.

$U=\int dU=-\int\frac{G\,dM\,m}{r}=-\frac{GMm}{L}\int_{-L/2}^{L/2}\frac{dx}{r}$ ——— 1pt

$=-\frac{GMm}{L}\int_{-L/2}^{L/2}\frac{dx}{\sqrt{x^2+a^2}}$ ——— 2pt

use $\int dx\frac{1}{\sqrt{x^2+a^2}}=\frac{1}{2}\ln\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}+C.$ ——— 1pt

$=-\frac{GMm}{L}\ln\frac{\sqrt{\frac{L^2}{4}+a^2}+\frac{L}{2}}{\sqrt{\frac{L^2}{4}+a^2}-\frac{L}{2}}$ ——— 1pt

(b) The sound from the horn reflects from the cliff back to the engineer on the train. What i the frequency of the reflected sound? **(2 pts)**

The cliff is at rest. The reflection simply means the cliff takes the sound wave, reverses its direction, then emits it towards the tra

Frequency is the same as $f'=693\,(Hz)$ — 2pt

(c) What is the frequency f'' that the engineer on the train hears? **(4 pts)**

For the engineer on the train, he is now the new (moving) listener.

$f''=\frac{v+u}{v}f'$ (set $V_D=u$, $V_S=0$) — 2pt

$=\frac{v+u}{v-u}f=736\,(Hz)$ — 2pt

(a) Find the wavelength of the sound wave if the firetruck is at rest.

$\lambda=vT=v\frac{1}{f}$ → 1pt

$=\frac{340}{170}\,m=2.00\,m/s$ → 1pt

$$F=\frac{GMm}{a\sqrt{\frac{L^2}{4}+a^2}}\qquad\text{(See integral identities on Page 1.)}$$

We consider an infinitesimal element dx, with a coordinate x in the rod, the y componen of the gravitational force between this element and the mass m is

$$dF_y=\frac{G\frac{M}{L}dx\cdot m}{x^2+a^2}\frac{a}{\sqrt{x^2+a^2}} \qquad 2\text{ pts}$$

Considering the symmetry of the rod, the total gravitational force will be along the y direction, and

$$F_y=2\int_0^{L/2}\frac{GMma}{L}\frac{1}{(x^2+a^2)^{3/2}}dx \qquad 2\text{ pts}$$

$$=\frac{GMm}{a\sqrt{a^2+L^2/4}} \qquad 1\text{ pt}$$

(c) Write down the equation of motion for the oscillator in case (c) as shown in the above Fig. 7 (c) and find the corresponding angular frequency ω_c. **(3 pts)**

Suppose a total displacement $X=x_1+x_2$

1pt $k_1x_1=k_2x_2$

⇒ $x_1=\frac{k_2}{k_1+k_2}X$; $x_2=\frac{k_1}{k_1+k_2}X$

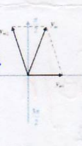
2pt → EOM ⇒ $m\frac{d^2x}{dt^2}=-k_1x_1=-\frac{k_1k_2}{k_1+k_2}X$

1pt → ⇒ $\omega_c=\sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$

a) According to phasor below,

阻尼

$m\frac{d^2x}{dt^2}=-b\frac{dx}{dt}-kx$



we have

$y_m=(y_{m1}+y_{m2}\cos(0.6\pi),\ y_{m2}\sin(0.6\pi))=(2.084\text{mm},\ 5.897\text{mm})$.

Thus, the amplitude is 6.25mm and the phase angle is 1.23rad.

b) Obviously, the third wave should have the same phase with old resultant wave, i.e., 1.23rad.

(d) Find angular momentum of star M_1 about COM, including the magnitude and direction. **(2 pts)**

(Hint: is the direction pointing in or out of the exam paper)

$L_1=M_1V_1R_1$ OK

$=M_1\omega R_1^2=M_1R_1^2\sqrt{\frac{G(M_1+M_2)}{(R_1+R_2)^3}}$ 1pt 1pt

we are 2 possible answers! OK

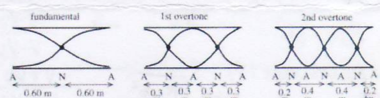
Another answer $L_1=M_1R_1\frac{\sqrt{GR_1M_2}}{(R_1+R_2)}$ 1pt use $M_1=\frac{M_2R_2}{R_1}$ OK

(e) How much energy would be required to separate the two stars to infinity? OK **(2 pts)**

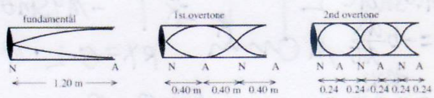
1pt $\Delta E=E_2-(K+U)=0-\frac{1}{2}M_1V_1^2-\frac{1}{2}M_2V_2^2+\frac{GM_1M_2}{(R_1+R_2)}=\frac{GM_1M_2}{2(R_1+R_2)}$ OK

1pt $W=\Delta E=\frac{GM_1M_2}{2(R_1+R_2)}$ OK correct answer gets 2pts

EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.25a. The open ends are displacement antinodes.



(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.25b.



5. A police car's siren emits a sinusoidal wave with frequency $f_s=300$ Hz. The speed of sound is 340 m/s and the air is still.

(a) Find the wavelength of the waves if the siren is at rest.

(b) Find the wavelengths of the waves in front of and behind the siren if it is moving at 30 m/s.

(a) In this part there is no Doppler effect because neither source nor listener is moving with respect to the air, thus the formula $v=\lambda f$ gives the wavelength. When the source is at rest,

$$\lambda=\frac{v}{f_s}=1.13\text{m}.$$

(b) The wavelength is shorter in front of the siren and longer behind it according to the Doppler effect. In front of the siren and behind the siren, we find

$$\lambda_{\text{in front}}=\frac{v-v_s}{f_s}=1.03\text{m/s},\quad \lambda_{\text{behind}}=\frac{v+v_s}{f_s}=1.23\text{m/s}.$$

S向D动 $\qquad$ S远离D.

(a) What is the signal's frequency as detected by the U.S. sub

(b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub

31. (a) The frequency as detected by the U.S. sub is

$$f'=f_1\left(\frac{v+v_{US}}{v-v_F}\right)=(1.560\times10^3\text{ Hz})\left(\frac{5470\text{ km/h}+72.00\text{ km/h}}{5470\text{ km/h}-48.00\text{ km/h}}\right)=1.595\times10^3\text{ Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be $f_r=f_1(v+v_{US})/(v-v_{US})$.

Since the French sub is moving toward the reflected signal with speed v_F, then

$$f_r'=f_r\left(\frac{v+v_F}{v}\right)=f_1\frac{(v+v_F)(v+v_{US})}{(v-v_F)(v-v_{US})}=(1.560\times10^3\text{ Hz})\frac{(5470+48.00)(5470+72.00)}{(5470-48.00)(5470-72.00)}$$

$=1.630\times10^3$ Hz.

(a) If we attach two blocks that have masses m_1 and m_2 to either end of a spring that has a force constant k and set them into oscillation by releasing them from rest with the spring stretched, show that the oscillation frequency is given by $\omega=\sqrt{k/\mu}$, where $\mu=m_1m_2/(m_1+m_2)$ is the reduced mass of the system.

Hint: First consider the separate motions of these two blocks and write their equation of motions as follows

$$m_1\frac{d^2x_1}{dt^2}=-k(x_1-x_2)\quad\text{and}\quad m_2\frac{d^2x_2}{dt^2}=-k(x_2-x_1),$$

from which you can obtain the two equations which describe their relative displacement x_1-x_2 (SHM) and their center of mass x_{com} (free motion), respectively.

Use Newton's second law and apply it to both blocks separately and obtain

m_1 m_2

$$m_1\frac{d^2x_1}{dt^2}=-k(x_1-x_2),\qquad(11)$$

$$m_2\frac{d^2x_2}{dt^2}=-k(x_2-x_1),\qquad(12)$$

here x_1 and x_2 are displacements of m_1 and m_2 from their equilibrium positions, respectively. It is interesting first notice that Eq. (11) + Eq (12) tells us that the center of mass of these two blocks is at rest or moving ith constant velocity since the external force is zero.

et us now compute Eq. (11) / m_1 – Eq (12)/ m_2.

$$\frac{d^2x_1}{dt^2}-\frac{d^2x_2}{dt^2}=-k(1/m_1+1/m_2)(x_1-x_2)\ \Rightarrow$$

$$\frac{d^2}{dt^2}(x_1-x_2)=-k\frac{m_1+m_2}{m_1m_2}(x_1-x_2)\ \Rightarrow$$

$$\mu\frac{d^2x}{dt^2}=-kx\quad\text{with}\quad\mu=\frac{m_1m_2}{m_1+m_2},$$

$W=\sqrt{\frac{k}{u}}$

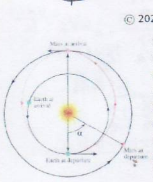
H H First $L=\frac{\lambda}{2}$

Second $L=\frac{2\lambda}{2}$

H H F n=1 First

H N A N A F n=3 Third $L=\frac{n\lambda}{4}$

Suppose you are working out a flight plan to travel from earth to Mars in an elliptical orbit with its perihelion at earth and its aphelion at Mars. Of course the Sun is located at the focus near earth as shown in the above figure. This orbit is known as the Hohmann transfer orbit, which provides one of the most efficient way to send a spacecraft to Mars. Assume that the orbits of earth and Mars are circular with radii $R_E=1.50\times10^{11}$ m and $R_M=2.28\times10^{11}$ m, respectively. Neglect the gravitational effects of the planets on your spaceship.



(a) How long will it take to reach Mars?

$$\frac{T^2}{\left(\frac{R_E+R_M}{2}\right)^3}=\frac{T_E^2}{R_E^3},\quad\text{and}\quad 2a=R_E+E_M\quad\text{(Hohmann Transfer Orbit semi-major)}\ \Rightarrow$$

$$\text{or } T=\left(\frac{R_E+R_M}{2R_E}\right)^{3/2}T_E=(3.78/3.00)^{3/2}\times1\,\text{year}=1.41\,\text{year}.$$

To reach Mars from the earth, the launch must be timed so that Mars will be at the right spot when the spacecraft reaches Mars's orbit around the sun. At launch, what must the angle α between a sun–Mars line and a sun–earth line be?

Solution: To save a lot of fuels in the rocket launch, the spaceship should be launched along the tangent of the earth's orbit and in the same direction of the earth and Mars's rotation.

Since the spaceship is required to reach the Mars orbit with Mars at arrival simultaneously, one finds the following relation

$$\frac{T}{2}=\frac{\pi-\alpha}{\omega_M},\quad\text{with}\quad T_M=\frac{2\pi}{\omega_M}=\left(\frac{R_M}{R_E}\right)^{3/2}T_E=1.87\text{year}.$$

$$(\pi-\alpha)=\frac{T}{T_M}\pi=2.37\,\text{rad},\quad\Rightarrow\quad\alpha=0.77\,\text{rad}=44°.$$

open open n=1 First

A N A

$\frac{1}{2}\rho A^2$

$I=\frac{1}{2}\rho v\omega^2A_m^2=\frac{1}{2}\rho v\cdot\left(\frac{P_mV}{B}\right)^2$

n=2

$F=2\tau\sin\theta\sim2\tau\theta=\frac{VP_m^2}{2B}$ $\quad A_m=\frac{VP_m}{2\pi fB}$

$=\tau\frac{\Delta l}{R}$

A N A N A

End Open First

Third

$\vec{\tau}=\frac{d\vec{L}}{dt}$ $\quad\tau=Mgr$ $\quad d\phi=\frac{dL}{L}=\frac{Mgr\,dt}{I\omega}$

$L=I\omega$ $\quad d\vec{L}=\vec{\tau}dt=Mgr\,dt$ $\quad\Omega=d\phi/dt=\frac{Mgr}{I\omega}$

$a=\frac{v^2}{R}$ $\quad\frac{T\Delta l}{R}=u\Delta l\cdot\frac{v^2}{R}$ $\quad V=\sqrt{\frac{T}{u}}$