chapter 12: $F_{net} = \frac{dP}{dt} = 0$ & $\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$

$F_{net,x} = 0$, $F_{net,y} = 0$ and $\tau_{net,z} = 0$

$x_{cog} = x_{com}$ (center of gravity)

elastic modulus: stress = modulus × strain

Tension and compression: $\frac{F}{A} = E \cdot \frac{\Delta L}{L}$ (Young)

横shearing: $\frac{F}{A} = G\frac{\Delta x}{L}$ (shear modulus) $\frac{F}{A} = G\frac{\Delta x}{L}$

Hydraulic stress: $p = B\frac{\Delta V}{V}$ 液压 p: fluid pressure

chapter 11: Rolling smoothly: $V_{com} = \frac{S}{\Delta t} = \frac{ds}{dt} = \frac{d\theta}{dt}R = WR$

$K = \frac{1}{2}I_{com}W^2 + \frac{1}{2}MV_{com}^2$ $a_{com} = \frac{dV_{com}}{dt} = \frac{dW}{dt}R = \alpha R$

$a_{com,x} = -\frac{g\sin\theta}{1 + I_{com}/MR^2}$

Rolling with slipping: $f_k = \mu_k mg = ma_{com}$

$\vec{\tau} = \vec{r} \times \vec{F}$ & $\tau = rF\sin\phi$ torque as a vector

angular momentum: $\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ $\vec{L} = \sum_{i=1}^{n}\vec{l}_i$

$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$ $L = I \cdot W$ conservation: $I_iW_i = I_fW_f$

$\Omega = \frac{d\varphi}{dt} = \frac{Mgr}{IW}$

chapter 17 ($V_{air} = 344$ m/s)

$V = \sqrt{\frac{B}{\rho}}$ → Bulk modulus $= \frac{\Delta P}{-\Delta V/V}$

$= \sqrt{\frac{P\gamma}{\rho}}$ → pressure

$S(x,t) = S_m\cos(kx - wt)$ displacement

$\Delta P(x,t) = \Delta P_m\sin(kx - wt)$ pressure variation

$\Delta P_{max} = \rho \cdot v \cdot w \cdot S_{max}$

sound interference $\phi = 2\pi\frac{\Delta L}{\lambda}$ → 两个点的相位差 / 传播路程之差 $2\pi f$ → $\frac{v}{f}$ 波速 λ波长 $f = \frac{v}{\lambda}$ (频率)

destructive: $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \ldots$

constructive: $\lambda, 2\lambda, 3\lambda$ $\Delta L = con/des \cdot \lambda$

sound intensity: $I = \frac{P}{A} = \frac{1}{2}\rho v w^2 S_m^2 = \frac{P_s}{4\pi r^2}$

$\beta = (10dB)\log\frac{I}{I_0}$ $I_0 = 10^{-12}$ W/m² $= \frac{(\Delta P_{max})^2}{2\rho v}$

standing pipes: two open ends: $f = \frac{v}{\lambda} = \frac{nv}{2L}$ (n = 1, 2, ...)

one closed and one open end: $f = \frac{v}{\lambda} = \frac{nv}{4L}$ (n = 1, 3, 5, ...).

$f_{beat} = |f_1 - f_2|$ Doppler Effect: $f' = f\frac{V \pm V_D}{V \pm V_S}$ $(V > V_S)$

moving wall: $f_{RS} = f_S\left(\frac{V \pm V_0 \pm V_{moving\ wall}}{V \pm V_S}\right)$

$\sin\theta = \frac{Vt}{V_St} = \frac{V}{V_S} = \frac{1}{M}$ → 马赫数

$P_m = \rho v^2 k A_m = BkA_m = \left(\frac{2\pi f}{v}\right)B \cdot A_m = WA_m\frac{B}{V}$.

Chapter 14

$\rho = \frac{m}{V}$, $P = \frac{F}{A}$ $F = \Delta P \cdot A$

$A_1 V_1 = A_2 V_2$.

$P = P_0 + \rho gh$ flowing speed: $V = \sqrt{2gd}$

$F_b = m_f g$ & weight app = Weight − F_b

Volume flow rate: $R_V = A \cdot V$ = a constant. m³/s

mass flow rate: $R_m = \rho A v$ = a constant kg/s

$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2$ kinetic energy density

$P + \frac{1}{2}\rho V^2 + \rho g y$ = constant. speed ↑ pressure ↓

chapter 13. $F = G\frac{m_1m_2}{r^2}$ $G = 6.67 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$ $a_g = G\frac{M}{r^2}$

Apparent weight $= F_G - F_C = G\frac{mM}{R^2} - \frac{mv^2}{R}$ $mg = G\frac{mM}{R^2}$

$\frac{dF}{dR} = -2GmMR^{-3}$ $g = a_g - w^2R$ $da_g = -2\frac{GM}{r^3}dr$

Gravity potential energy: $U = -\frac{GMm}{r}$ (two-particle system)

multiple particles: $U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right)\ldots$

potential energy and force: $F = -\frac{dU}{dr} = -G\frac{Mm}{r^2}$ $V = \sqrt{\frac{2GM}{R}}$, $K = \frac{GMm}{2r}$

Kepler ① elliptical orbital, sun at one focus ② $\frac{dA}{dt} = \frac{1}{2}r^2w = \frac{L}{2m}$ = constant ③ $T^2 = \frac{4\pi^2}{GM}r^3$

circular orbit: $E = K + U = -\frac{GMm}{2r} = -K$ elliptical: $E = -\frac{GMm}{2a}$

black hole: $R_S = \frac{2GM}{c^2}$ 求速度 $F = ma$, $a = \frac{v^2}{R}$

escape speed: $\frac{1}{2}mv^2 - \frac{GM \cdot m}{R} = 0 \Rightarrow V = \sqrt{\frac{2GM_E}{R_E}}$

jump to escape: $mgh - \frac{GMm}{R} \geq 0$

Chapter 16.

V ⇒ along x-direction $= f \cdot \lambda = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}$

$W = 2\pi f$, $f = \frac{2\pi}{T}$

wave equation: $\frac{1}{V^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$ $\frac{\partial^2 y(x,t)}{\partial t^2} = V^2\frac{\partial^2 y(x,t)}{\partial x^2}$

$y(x,t) = y_m\sin(kx - wt)$ angular frequency.

displacement. position $K = \frac{2\pi}{\lambda}$ transverse velocity $\frac{\partial y}{\partial t}$

velocity in the wave: $V = \frac{w}{K}$ (wave velocity) constant $= \frac{\lambda}{T}$

particle velocity $u = \frac{\partial y}{\partial t}$ $u_m = wy_m$ → $\frac{1}{2}\sqrt{\mu F}\ w^2A^2$

average power: $P_{avg} = \frac{1}{2}\mu v w^2 y_m^2$ $P(x,t) = -\frac{\partial y}{\partial x}F_T\frac{\partial y}{\partial t}$

wave interference with the same k and w $\phi = 2\pi \times$ wavelength diff.

① same amplitude: $y' = [2y_m\cos(\frac{1}{2}\phi)]\sin(kx - wt + \frac{1}{2}\phi)$

② different amplitude $y'(x,t) = y'_m\sin(kx - wt + \beta)$

③ opposite direction same amplitude: standing wave

$y'(x,t) = 2y_m\sin(kx)\cos(wt)$

nodes: amplitude = 0 $kx = \frac{2\pi}{\lambda}x = n\pi$ $x = n\frac{\lambda}{2}$

Antinodes: $kx = \frac{2\pi}{\lambda}x = (n + \frac{1}{2})\pi$ $x = (n + \frac{1}{2})\frac{\lambda}{2}$

two hard boundaries: $L = n\frac{\lambda}{2}$, n = 1, 2, 3, ..., $f = \frac{v}{\lambda} = n\frac{v}{2L}$

one hard one free: distance: $L = n\frac{\lambda}{4}$, $f = \frac{v}{\lambda} = n\frac{v}{4L}$, n = 1, 3, 5...

two free boundaries: $L = n\frac{\lambda}{2}$, $f = \frac{v}{\lambda} = n\frac{v}{2L}$ n = 1, 2, 3...

chapter 15: $x(t) = x_m\cos(wt + \phi)$ rad

$v(t) = -wx_m\sin(wt + \phi)$ $f = \frac{w}{2\pi}$

$a(t) = -w^2x_m\cos(wt + \phi) = -w^2x(t)$

$m\frac{d^2x}{dt^2} = -(k+k_0)x$ $a_m = w^2x_m$

linear harmonic oscillator: $W = \sqrt{\frac{k}{m}}$, $T = 2\pi\sqrt{\frac{m}{k}}$

$V(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2\cos^2(wt + \phi)$ (spring full extension)

$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}mw^2x_m^2\sin^2(wt + \phi)$ (maximum when moving through mid-point

$t = \frac{1}{2\pi f}\sin^{-1}(\frac{x}{A})$ $= \frac{1}{2}kx_m^2\sin^2(wt + \phi)$

$E = V(t) + K(t) = \frac{1}{2}k(x_m)^2$ → $K = mw^2$.

$V(x) = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$ $x(V) = \pm\sqrt{A^2 - \frac{m}{k}V^2}$

$x_m = \pm\sqrt{\frac{m}{K}} \cdot V$ $E_0 = E_{f_{eq}}\ \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mV^2$

$a = -\frac{k}{m}x$ $\tau = -k\theta$ (angular form for hooke law) torsion constant

$T = 2\pi\sqrt{\frac{I}{k}}$ (torsion pendulum) $T = 2\pi\sqrt{\frac{L}{g}}$ (simple) no string weight

pendulum: $W = \sqrt{\frac{g}{l}}$ $l = \frac{gT^2}{4\pi^2}$ $g = \frac{4\pi^2 l}{T^2}$

physical ~ (质量分布) . $W = \sqrt{\frac{mgd}{I}}$ → distance of com

damped harmonic motion: $x(t) = x_m e^{-\frac{bt}{2m}}\cos(w't + \phi)$

$W' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ $E(t) = \frac{1}{2}kx_m^2e^{-\frac{bt}{m}}$

IDENTIFY: Apply $\sum\tau_z = 0$ to the post, for various choices of the location of the rotation axis.

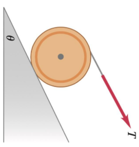
SET UP: When the post is on the verge of slipping, f_s has its largest possible value, $f_s = \mu_s n$.

EXECUTE: **(a)** Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so

$$F\frac{h}{2} = (n - w)h\tan\theta.$$

Taking torques about the upper point (where the rope is attached to the post), $fh = F\frac{h}{2}$. Using $f \leq \mu_s n$

and solving for F, $F \leq 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\ \text{N})\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400$ N.



Show that the tension in the wire must be equal to the friction force, and find this tension.

Answers: $T = \frac{1}{2}Mg\sin\theta$.

Solution: In addition, the direction of the friction is upward parallel to the surface in order to provide opposite torque w.r.t. to the one caused by the tension. The torque equation gives

$$fR - TR = 0 \quad \Rightarrow \quad f = T. \tag{6}$$

From the balancing of the force parallel to the surface of the ramp, one gets

$$T + f - mg\sin\theta = 0, \quad \Rightarrow \quad 2T = mg\sin\theta, \quad \Rightarrow \quad T = \frac{1}{2}mg\sin\theta. \tag{7}$$

IDENTIFY: Use the measurements of the motion of the rock to calculate g_M, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{rad}$ and $T = \frac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward.

$g_M = G\frac{m_M}{R_M^2}$. The radius of Mongo is $R_M = \frac{c}{2\pi} = \frac{2.00 \times 10^8\ \text{m}}{2\pi} = 3.18 \times 10^7$ m. The ship moves in an orbit of radius $r = 3.18 \times 10^7\ \text{m} + 3.00 \times 10^7\ \text{m} = 6.18 \times 10^7$ m.

EXECUTE: **(a)** $v_{0y} = +12.0$ m/s, $v_y = -12.0$ m/s, $a_y = -g_M$ and $t = 6.00$ s. $v_y = v_{0y} + a_y t$ gives

$-g_M = \frac{v_y - v_{0y}}{t} = \frac{-12.0\ \text{m/s} - 12.0\ \text{m/s}}{6.00\ \text{s}}$ and $g_M = 4.00$ m/s².

$m_M = \frac{g_M R_M^2}{G} = \frac{(4.00\ \text{m/s}^2)(3.18 \times 10^7\ \text{m})^2}{6.673 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2} = 6.06 \times 10^{25}$ kg

(b) $F_g = ma_{rad}$ gives $G\frac{m_M m}{r^2} = m\frac{v^2}{r}$ and $v^2 = \frac{Gm_M}{r}$.

$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_M}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_M}} = \frac{2\pi(6.18 \times 10^7\ \text{m})^{3/2}}{\sqrt{(6.673 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(6.06 \times 10^{25}\ \text{kg})}}$

$T = 4.80 \times 10^4\ \text{s} = 13.3$ h

EVALUATE: $R_M = 5.0R_E$ and $m_M = 10.2m_E$, so $g_M = \frac{10.2}{(5.0)^2}g_E = 0.408g_E$, which agrees with the value

3. (a) At depth y the gauge pressure of the water is $p = \rho gy$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\ dy$ and the force it exerts on the dam is $dF = p\ dA = \rho gyW\ dy$. The total force of the water on the dam is

$$F = \int_0^D \rho gyW\ dy = \frac{1}{2}\rho gWD^2 = \frac{1}{2}(1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\ \text{m})(30.0\ \text{m})^2$$
$$= 1.10 \times 10^9\ \text{N}.$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque exerts about O is $D - y$ so the torque it exerts is

$$d\tau = dF(D - y) = \rho gyW(D - y)dy$$

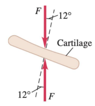
and the total torque of the water is

$$\tau = \int_0^D \rho gyW(D - y)dy = \rho gW\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho gWD^3$$
$$= \frac{1}{6}(1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(250\ \text{m})(30.0\ \text{m})^3 = 1.10 \times 10^{10}\ \text{N} \cdot \text{m}.$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho gWD^3}{\frac{1}{2}\rho gWD^2} = \frac{D}{3} = \frac{30.0\ \text{m}}{3} = 10.0\ \text{m}.$$

Solution: Vigorous downhill hiking produces a shear force on the knee cartilage which could deform the cartilage. The shear strain, or the angle of deformation, can be obtained from the definition as follows

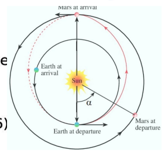


$$\text{Shear Strain:} \quad \phi \equiv \frac{x}{h} = \frac{\text{Shear Stress}}{\text{Shear Modulus}} = \frac{F_{||}/A}{S} = \frac{8 * 110 * 9.8 * \sin(12°)/(10 \times 10^{-4})}{12 \times 10^6} = 0.15\text{rad}.$$

The angle of 0.15rad is 8.6° in degrees.

How long will it take to reach Mars? (Give your answer in years.)

Answers: 0.71 years.

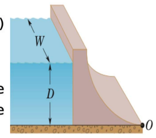


Solution: Let T_E and T be the orbital periods of the earth and the spacecraft in the Hohmann transit orbit (the red orbit in the above figure), respectively. According to Kepler's third law, T^2/a^3 = constant, thus

$$\frac{T^2}{\left(\frac{R_E + R_M}{2}\right)^3} = \frac{T_E^2}{R_E^3}, \quad \text{and} \quad 2a = R_E + E_M \quad \text{(Hohmann Transfer Orbit semi-major)} \quad \Rightarrow \tag{6}$$

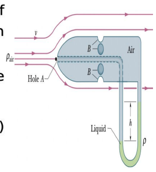
$$\text{or } T = \left(\frac{R_E + R_M}{2R_E}\right)^{3/2} T_E = (3.78/3.00)^{3/2} \times 1\,\text{year} = 1.41\,\text{year}. \tag{7}$$

The flight on the spaceship to Mars takes one half of T, thus the length of the mission is 0.71 years.



To reach Mars from the earth, the launch must be timed so that Mars will be at the right spot when the spacecraft reaches Mars's orbit around the sun. At launch, what must the angle α between a sun–Mars line and a sun–earth line be? **Answers:** 44°.

Solution: To save a lot of fuels in the rocket launch, the spaceship should be launched along the tangent of the earth's orbit and in the same direction of the earth and Mars's rotation. The angular difference α between the earth and Mars at the departure is called "Hohmann angular alignment".



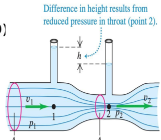
Since the spaceship is required to reach the Mars orbit with Mars at arrival simultaneously, one finds the following relation

$$\frac{T}{2} = \frac{\pi - \alpha}{\omega_M}, \quad \text{with} \quad T_M = \frac{2\pi}{\omega_M} = \left(\frac{R_M}{R_E}\right)^{3/2} T_E = 1.87\text{year}. \tag{8}$$

By plugging in previous result for T, we obtain

$$(\pi - \alpha) = \frac{T}{T_M}\pi = 2.37\text{rad}, \quad \Rightarrow \quad \alpha = 0.77\text{rad} = 44°. \tag{9}$$

Difference in height results from reduced pressure in throat (point 2).



48. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{air}v^2$. However, $\Delta p = p_A - p_B = \rho gh$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho gh = \frac{1}{2}\rho_{air}v^2$, or

$$v = \sqrt{\frac{2\rho gh}{\rho_{air}}}.$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho gh}{\rho_{air}}} = \sqrt{\frac{2(810\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(0.200\,\text{m})}{1.03\,\text{kg/m}^3}} = 55.5\,\text{m/s}.$$

Solution: We assume the fluid is incompressible and steady and has negligible internal friction. Hence we can use Bernoulli's equation and apply this equation to the wide part (point 1) and narrow part (point 2, the throat) of the pipe. Points 1 and 2 have the same vertical coordinate $y_1 = y_2$, so

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2. \tag{3}$$

From the continuity equation, $v_2 = (A_1/A_2)v_1$. Substituting this into the Bernoulli's equation, we get

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2} - 1\right]. \tag{4}$$

Now let us look at the vertical pressure difference. It is important to note that the fluid line is indicating the fluid motion along the horizontal direction while the fluid is stagnant along the vertical direction. For static fluid, one finds $p_1 - p_2 = \rho g h$. Therefore, combining this with the above result and solving for v_1, we get

$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}.$$

down the string. Such a situation can be described by a wave function whose amplitude $A(x)$ depends on x: $y = A(x)\sin(kx - \omega t)$, where $A(x) = A_0 e^{-bx}$. What is the power transported by the wave as a function of x, where $x > 0$? **Answers:** $P = \frac{1}{2}\mu v \omega^2 A_0^2 e^{-2bx}$.

Solution:

The power is defined as

$$P(x,t) = -\frac{\partial y}{\partial x} F_T \frac{\partial y}{\partial t} \quad \text{with} \quad y = A(x)\sin(kx - \omega t) = A_0 e^{-bx}\sin(kx - \omega t).$$

Therefore, by differentiating $y(x,t)$ with x and t

$$\begin{aligned} P(x,t) &= F_T A^2(x) k\omega \cos^2(kx - \omega t) \\ &+ \frac{dA(x)}{dx} F_T A(x)\omega \sin(kx-\omega t)\cos(kx - \omega t). \end{aligned}$$

Now consider the time average of the power P by noting that

$$\frac{1}{T}\int_0^T dt \cos^2(kx - \omega t) = \frac{1}{2}$$
$$\frac{1}{T}\int_0^T dt \sin(kx-\omega t)\cos(kx - \omega t) = 0,$$

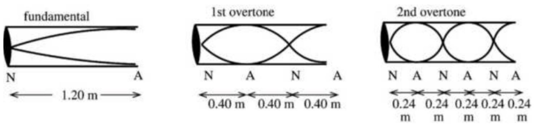
then the time-average power at fixed position is

$P = \frac{1}{2}\mu v \omega^2 A^2(x)$

where μ is the linear density, $\omega = kv$ and $F_T = \mu v^2$.

Then, plug in $A = A_0 e^{-bx}$,

$P = \frac{1}{2}\mu v \omega^2 (A_0 e^{-bx})^2 = \frac{1}{2}\mu v \omega^2 A_0^2 e^{-2bx}$.



Given the coefficient of static friction between the yo-yo and the table is μ_s, what is the maximum magnitude of the pulling force F for which the Yo-Yo rolls without slipping?

From the above result, plus the static friction condition $f \le \mu_s mg$, one finds the maximum magnitude the pulling force is given by

$$F \le \frac{3R}{2b+R}\mu_s mg. \tag{13}$$

If we attach two blocks that have masses m_1 and m_2 to either end of a spring that has a force constant k and set them into oscillation by releasing them from rest with the spring stretched, show that the oscillation frequency is given by $\omega = \sqrt{k/\mu}$, where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the system.

Hint: First consider the separate motions of these two blocks and write their equation of motions as follows

$$m_1\frac{d^2x_1}{dt^2} = -k(x_1 - x_2) \quad \text{and} \quad m_2\frac{d^2x_2}{dt^2} = -k(x_2 - x_1),$$

from which you can obtain the two equations which describe their relative displacement $x_1 - x_2$ (SHM) and their center of mass x_{com} (free motion), respectively.

Solution:

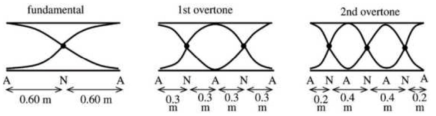
Use Newton's second law and apply it to both blocks separately and obtain

$$m_1\frac{d^2x_1}{dt^2} = -k(x_1 - x_2), \tag{11}$$
$$m_2\frac{d^2x_2}{dt^2} = -k(x_2 - x_1), \tag{12}$$

where x_1 and x_2 are displacements of m_1 and m_2 from their equilibrium positions, respectively. It is interesting to first notice that Eq. (11) + Eq (12) tells us that the center of mass of these two blocks is at rest or moving with constant velocity since the external force is zero.

Let us now compute Eq. (11) / m_1 − Eq (12)/ m_2,

$$\frac{d^2x_1}{dt^2} - \frac{d^2x_2}{dt^2} = -k(1/m_1 + 1/m_2)(x_1 - x_2) \quad \Rightarrow$$
$$\frac{d^2}{dt^2}(x_1 - x_2) = -k\frac{m_1 + m_2}{m_1 m_2}(x_1 - x_2) \quad \Rightarrow$$
$$\mu\frac{d^2x}{dt^2} = -kx \quad \text{with} \quad \mu = \frac{m_1 m_2}{m_1 + m_2},$$



(a) The tension F_T sets the velocity of the mechanical wave of the string $v = \sqrt{F_T/\mu}$, which equals $v = f\lambda$.
At fixed frequency f, the condition for the existence of standing wave $\lambda_n = 2L/n$ $(n = 1, 2, \cdots)$ indicates that only discrete values of v_n, therefore, F_{Tn} can result in standing waves on the string.

(b) Adding an antinode ⇒ increasing n by one ⇒ decreasing λ_n and v_n ⇒ decreasing F_{Tn}

(c) $F_{Tn} = \mu v_n^2 = \mu f^2\lambda_n^2 = 4L^2 f^2\mu/n^2 \propto 1/n^2$.

(d) $4L^2 f^2\mu = 4 \times 1.00^2 \times 80.0^2 \times 0.750 \times 10^{-3} = 19.2$ N
$F_{T1} = 19.2$ N, $F_{T2} = 19.2/2^2 = 4.80$ N and $F_{T3} = 19.2/3^2 = 2.13$ N

Suppose the yo-yo is rolling without slipping, find the acceleration of the yo-yo.

Solution: Suppose the friction is applied to the yo-yo as shown in the above right figure. Choose counterclockwise as the positive direction for rotation and write down the torque equation and force equation as follows

$$\tau = I\alpha = -F(R-b) \quad \text{instantaneous axis with} \quad I = \frac{3}{2}mR^2, \tag{6}$$
$$F - f = ma. \tag{7}$$

Together with the relation $a + R\alpha = 0$ for rolling without slipping (note that α is negative since it is clockwise), one finds the following solutions

$$f = \frac{2b+R}{3R}F, \quad \text{and} \quad a = \frac{2F}{3m}\left(1 - \frac{b}{R}\right). \tag{8}$$

What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?

(b) If the French sub were stationary, the frequency of the reflected wave would be

$$f_r = f_l(v + v_{US})/(v - v_{US}).$$

Since the French sub is moving toward the reflected signal with speed v_F, then

$$f_r' = f_r\left(\frac{v+v_F}{v-v_F}\right) = f_l\frac{(v+v_F)(v+v_{US})}{(v-v_F)(v-v_{US})} = (1.560\times10^3\ \text{Hz})\frac{(5470+48.00)(5470+72.00)}{(5470-48.00)(5470-72.00)}$$
$$= 1.630\times10^3\ \text{Hz}.$$

** A spherical ball with mass M and rotational inertia $I_{cm} = (2/5)MR^2$ is given an initial clockwise angular velocity ω_0 and zero linear velocity $v_{cm} = 0$ before it is placed upon a horizontal surface with kinetic friction coefficient μ_k.

How long does it take for the ball to roll without slipping? **Answers:** $t = \frac{2\omega_0 R}{7\mu_k g}$.

Solution: Choose the COM as the axis and the counterclockwise as the positive direction for the rotation, and write down the equations of motion as follows

$$fR = \frac{2}{5}MR^2\alpha, \quad \Rightarrow \quad \mu_k MgR = \frac{2}{5}MR^2\alpha \quad \Rightarrow \quad \alpha = \frac{5\mu_k g}{2R}, \tag{14}$$
$$f = Ma_{cm}, \quad \Rightarrow \quad a_{cm} = \mu_k g. \tag{15}$$

Since both α and a_{cm} are constants, one can immediately find the angular velocity and linear velocity of the ball after taking into account the initial condition

$$\omega = -\omega_0 + \frac{5\mu_k g}{2R}t, \tag{16}$$
$$v_{cm} = \mu_k g t, \tag{17}$$

where the minus sign in $-\omega_0$ comes from the fact that initial angular velocity is clockwise.

It is important to compute the velocity of the bottom contact point of the ball with the ground as follows

$$v_B = v_{cm} + \omega R = \mu_k g t - \omega_0 R + \frac{5\mu_k g}{2}t. \tag{18}$$

When v_B becomes zero, the ball stops sliding and it starts to roll without slipping, thus the kinetic friction disappears starting from that moment. Setting $v_B = 0$ gives $t = \frac{2\omega_0 R}{7\mu_k g}$.

How much energy does it lose during this process? **Solution:** $\Delta K = -MR^2\omega_0^2/7$.

Solution: At $t = \frac{2\omega_0 R}{7\mu_k g}$, one finds

$$\omega_f = -\frac{2}{7}\omega_0, \quad \text{and} \quad v_f = \frac{2}{7}\omega_0 R. \tag{19}$$

Thus the final kinetic energy is

$$K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}\frac{2}{5}MR^2\omega_f^2 = \frac{2}{35}MR^2\omega_0^2. \tag{20}$$

** Billiard Physics.

A cue ball (a uniform solid sphere of mass m and radius R) is at rest on a level pool table. Using a pool cue, you give the ball a sharp, horizontal hit of magnitude F at a height h above the center of the ball (Figure below). The force of the hit F is much greater than the friction force f that the table surface exerts on the ball. The hit lasts for a very short time Δt.

For what value of h will the ball roll without slipping? **Answers:** $h = \frac{2R}{5}$.

Solution: For rolling without slipping, we can have two ways to do the calculation. First way: choose the instantaneous axis on the ground and let $J = F\Delta t$ be the impulse exerted on the cue ball, thus

$$\text{Angular Momentum}: \quad -J(h+R) = L = I\omega = \frac{7}{5}mR^2\omega, \quad \text{(clockwise AM is negative)} \tag{32}$$
$$\text{Linear Momentum}: \quad J = mv_{cm}, \tag{33}$$
$$\text{No slipping condition}: \quad v_{cm} + \omega R = 0. \tag{34}$$

Solving the above equations yields $h = \frac{2}{5}R$.

Second way, choose COM as the axis, then the equation for the angular momentum becomes $-Jh = I_{cm}\omega = \frac{2}{5}mR^2\omega$ while the rest remains the same. You will the same answer.

If you hit the ball dead center $h = 0$ and give it an initial speed v_0, the ball will slide across the table for a while, but eventually it will roll without slipping. What will the speed of its center of mass be then? **Answers:** $\frac{5}{7}v_0$.

Solution: In this case, it is not obvious where the instantaneous axis is right away. So let choose COM as the axis. Then immediately we find $J * (0) = I_{cm}\omega = 0$, thus no rotation. Simply from the impulse-momentum theorem you find $J = mv_0$.

Now we know that the cue ball starts to slide at first and there must be kinetic friction which is going against the direction of motion. The equations of motion are then

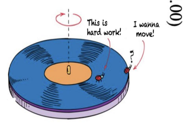
$$-f = ma \quad \Rightarrow \quad a = -f/m \quad \Rightarrow \quad v_{cm} = v_0 - \frac{ft}{m}. \tag{35}$$
$$-fR = I_{cm}\alpha \quad \Rightarrow \quad \alpha = -\frac{5}{2}\frac{f}{mR} \quad \Rightarrow \quad \omega = -\frac{5}{2}\frac{ft}{mR}. \tag{36}$$

We see that v_{cm} is decreasing while the magnitude of ω is increasing. When $v_{cm} + \omega R = 0$, the point that touches the ground will become instantaneously stationary and the ball is going to start rolling without slipping. Let us find that time which is given by

$$v_0 - \frac{ft}{m} + \left(-\frac{5}{2}\frac{ft}{mR}\right)R = 0,$$

namely, $t = \frac{2mv_0}{7f}$. At this moment and the time after, $v_{cm} = \frac{5}{7}v_0$.

** A ladybug of mass m lies on the rim of a uniform disk of mass $4m$ and radius R that can rotate freely about its center like a merry-go-round. Initially the ladybug and disk rotate together with an angular velocity of ω_0. Then the ladybug walks halfway to the center of the disk $(R/2)$.



What then is the angular velocity ω of the ladybug–disk system? **Answers:** $\omega = 4\omega_0/3$.

Solution: The angular momentum is conserved about the center axis in this process, simply because of vanishing torque. The magnitude of the angular momentum of the ladybug-disk system is

$$L = mr^2\omega + I\omega, \tag{22}$$

where $I = \frac{1}{2}4mR^2 = 2mR^2$ for the uniform disk with mass $4m$ and ω is the angular velocity, and r is the distance between the ladybug and the disk center.

Initially, the ladybug is at the rim, thus $r = R$, $\omega = \omega_0$, and the angular momentum is

$$L_i = mR^2\omega_0 + 2mR^2\omega_0. \tag{23}$$

After the ladybug has completed its walk, it reaches to the final location $r = R/2$. The final angular momentum is then

$$L_f = m\frac{R^2}{4}\omega_f + 2mR^2\omega_f. \tag{24}$$

From $L_i = L_f$, we obtain

$$\omega_f = \frac{3mR^2\omega_0}{m\frac{R^2}{4} + 2mR^2} = \frac{4}{3}\omega_0. \tag{25}$$

What is the ratio K_f/K_0 of the new kinetic energy of the system to its initial kinetic energy? **Answers:** $K_f/K_0 = 4/3$.

Solution: The total kinetic energy of the system is

$$K = \frac{1}{2}m(\omega r)^2 + \frac{1}{2}I\omega^2. \tag{26}$$

By comparing with the expression of the angular momentum, we can write the total kinetic energy as $K = \frac{1}{2}L\omega$. Therefore, one can find the ratio of kinetic energy as follows

$$\frac{K_f}{K_0} = \frac{L_f\omega_f}{L_i\omega_0} = \frac{4}{3}. \tag{27}$$

Of course, one can directly compute the ratio $\frac{K_f}{K_0}$ by plugging in the corresponding value of r and ω as in part (a) and obtain

$$\frac{K_f}{K_0} = \frac{\frac{1}{2}(mR^2/4 + I)\omega_f^2}{\frac{1}{2}(mR^2 + I)\omega_0^2} = \frac{(mR^2/4 + 2mR^2)\omega_f^2}{(mR^2 + 2mR^2)\omega_0^2} = \frac{3}{4}\left(\frac{4}{3}\right)^2 = \frac{4}{3}. \tag{28}$$

What is the acceleration of the center of mass of the ball? **Answers:** $a_{cm} = (5/7)g\sin\beta$.

Solution: From Newton's second law and its rotational analog, one gets

$$ma_{cm} = mg\sin\beta - f_s, \tag{2}$$
$$I\alpha = \frac{2}{5}mR^2\alpha = f_s R. \tag{3}$$

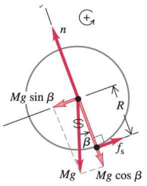
The acceleration and angular acceleration are related by $a_{cm} = R\alpha$. Combining all these equations, one gets $a_{cm} = (5/7)g\sin\beta$.

What minimum coefficient of static friction is needed to prevent slipping? **Answers:** $(2/7)\tan\beta$.

Solution: First, one can also solve for f_s and find $f_s = \frac{2}{7}mg\sin\beta$. In addition, the ball has no motion perpendicular to the ramp, thus

$$n = mg\cos\beta, \quad \Rightarrow \quad \mu_s \ge f_s/n = (2/7)\tan\beta. \tag{4}$$

Comments: $a_{cm} = (5/7)g\sin\beta$ is less than $g\sin\beta$ (in the case of point block without rotation) when friction is present. The ball rolls farther uphill when friction is present, because the friction removes the rotational kinetic energy and converts it to gravitational potential energy. You can think of the rotation as an extra reservoir to store kinetic energy.



Three identical stars of mass M form an equilateral triangle that rotates around the triangle's center as the stars move in a common circle about that center. The triangle has edge length L. What is the speed of the stars?

Answers: $v = \sqrt{GM/L}$.

Solution: Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30 =$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit from the star to the center of mass of these three starts, Newton's second law yields

$$\sqrt{3}\frac{GM^2}{L^2} = \frac{Mv^2}{R}. \tag{4}$$

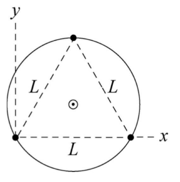
According to geometry, one finds $2R\cos 30 = L$, which implies $R = L/\sqrt{3}$. Substituting the expression for R into the above equation allows us to solve for v and find

$$v = \sqrt{\frac{GM}{L}}. \tag{5}$$

** Using Kepler's third law. We found that for two objects with masses m_1 and m_2 orbiting in a circular orbit at distance R from each other under gravity (about their center of mass), we have

$$\omega^2 R^3 = G(m_1 + m_2),$$

where ω is the angular frequency of the orbital motion. This formula is very useful and let us see some applications.



(a) For communication purposes it is very useful to have satellites that orbit the Earth once a day exactly. This means an antenna on Earth aimed at the satellite remains aimed at it as the Earth rotates. At what radius do such geostationary satellites orbit?

Solution: Geostationary satellites: noting that $T = 1\,day = 24 \times 3600\,s$ and $\omega = \frac{2\pi}{T}$, therefore, after neglecting the mass of the satellite, one gets

$$R = \left(\frac{Gm_E T^2}{(2\pi)^2}\right)^{1/3} = \left(\frac{R_E^2 g T^2}{(2\pi)^2}\right)^{1/3} = 4.22 \times 10^7\,m.$$

(b) The star $S2$ orbits near the center of the Milky Way. We observe that its orbit has a radius of about 930 AU (an AU, or Astronomical Unit, is the radius of Earth's orbit about the Sun and equal to 1.5×10^{11} m. $S2$ has an orbital period of about 15.6 years. Find the mass of the object $S2$ is orbiting, in terms of the mass of the Sun $M_{\odot} = 2.0 \times 10^{30}$ kg. Can you guess what the object is?

Solution: Star S2: Define the mass at the center of the orbit as M and use the Kepler's third law

$$M + m_{s2} = \frac{4\pi^2 R^3}{T^2 G} = 6.59 \times 10^{36}\,kg = 3.31 \times 10^6 M_{\odot}.$$

For a normal star, the mass is roughly around the same order of the solar mass $M_{\odot} = 1.989 \times 10^{30} kg$. Therefore, we can neglect the mass m_{s2} and find $M = 3.31 \times 10^6 M_{\odot}$, which is the mass of a super massive black hole.

We wish to find the effective spring constant for the combination of springs shown in the figure. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{eff} = F/\Delta x$. Since these two springs are the same, the elongation of the spring of each spring is $\Delta x/2$ with internal force $F = k\Delta x/2$. Thus $k_{eff} = F/\Delta x = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. Plugging this into the frequency formula gives

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} = 11.4\,Hz. \tag{5}$$