STA2001 Tutorial 11

1. 5.3-7 The distributions of incomes in two cities follow the two Pareto-type pdfs

$$f(x) = \frac{2}{x^3}, \quad 1 < x < \infty, \quad \text{and} \quad g(y) = \frac{3}{y^4}, \quad 1 < y < \infty,$$

respectively (Suppose that X and Y are independent). Here one unit represents $\$20,000$. One person with income is selected at random from each city. Let X and Y be their respective incomes. Compute $P(X < Y)$.

2. 5.3-8 Suppose two independent claims are made on two insured homes, where each claim has pdf

$$f(x) = \frac{4}{x^5}, \quad 1 < x < \infty,$$

in which the unit is \$1000. Find the expected value of the larger claim.

Hint: If X_1 and X_2 are the two identical and independent claims and $Y = \max(X_1, X_2)$, then

$$G(y) = P(Y \leq y) = P(X_1 < y)P(X_2 < y) = [P(X \leq y)]^2.$$

Find $g(y) = G'(y)$ and $E(Y)$.

3. 5.3-20. Let X and Y be independent random variables with nonzero variances. Find the correlation coefficient of $W = XY$ and $V = X$ in terms of the means and variances of X and Y.

4. The number of people who enter an elevator on the ground floor, denoted as X, is a Poisson random variable with mean λ. If there are N floors above the ground floor, and if each person is equally likely to get off at any one of the N floors, independently of where the others get off, compute the expected number of stops that the elevator will make in order to discharge all of its passengers.

   - Poisson pmf: $p_X(n) = \frac{e^{-\lambda}\lambda^n}{n!}, n \geq 0$
   - $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$