Probability and Statistics I

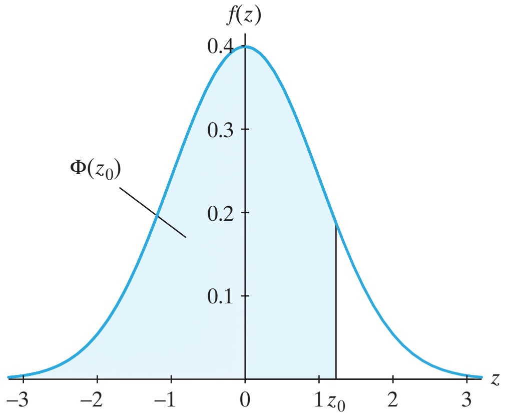
Mid-term Exam
SDS, CUHK(SZ)

March 18, 2023

Name: ____________________ Student ID: ____________________

Answer the multiple choice questions (Section I) in the Answer Card, and answer the regular questions (Section II) in the Answer Book.

Table Va The Standard Normal Distribution Function



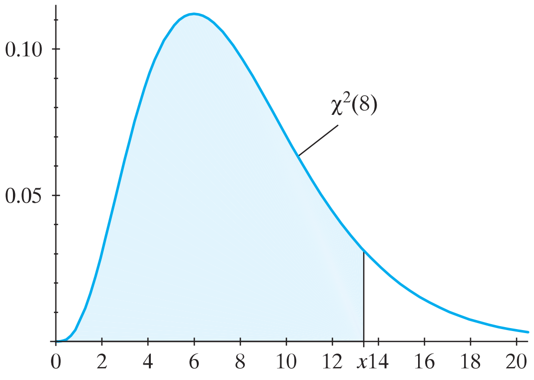
$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

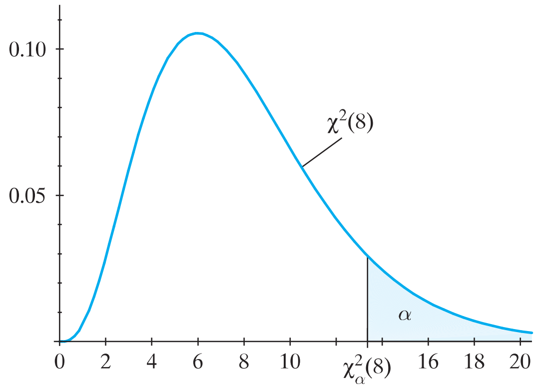
$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291

Table IV The Chi-Square Distribution





$$P(X \leq x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

	$P(X \leq x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E.S.Pearson and H.O.Hartley.

I Multiple Choices (72 points)

- **For each question, choose one and only one out of four given choices (A,B,C and D).**
- **3 points for each correct answer; 0 point for each incorrect or no answer.**

1. Let A, B, and C be three events in the sample space S. Suppose we know that $A \cup B \cup C = S$, $\mathbb{P}(A) = \frac{1}{2}$, $\mathbb{P}(B) = \frac{2}{3}$, $\mathbb{P}(A \cup B) = \frac{5}{6}$.

   (i) $\mathbb{P}(A \cap B) = 1/3$.

   (ii) A and B are independent.

   (iii) $\mathbb{P}\left(C \cap (A \cup B)'\right) = 1/6$.

   (iv) If $\mathbb{P}\left(C \cap (A \cup B)\right) = \frac{5}{12}$, then $\mathbb{P}(C) = 1/2$.

   Determine which of the following four answers is correct where T stands for true and F stands for false.

   A. (T,T,T,F)
   B. (F,T,T,F)
   C. (T,T,F,F)
   D. (T,F,T,F)

2. A box contains three marbles: one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box then replace it in the box and drawing a second marble from the box.

   (i) The sample space has 9 outcomes in total.

   (ii) If, at all times, each marble in the box is equally likely to be selected, the probability for each outcome is equal to each other.

   (iii) If, the first draw is without replacement and at all times each marble in the box is equally likely to be selected, then the probability for each outcome in the sample space is equal to each other.

   Determine which of the following four answers is correct where T stands for true and F stands for false.

   A. (T,T,F)
   B. (T,T,T)

C. (T,F,T)
D. (F,T,F)

3. A pair of fair dice is rolled.

   (i) The probability that the second die lands on a higher value than the first does is 5/12.

   (ii) The probability that the second die lands on the same value than the first does is 1/2.

   (iii) The probability that the second die lands on a smaller value than the first does is 1/12.

   Determine which of the following four answers is correct where T stands for true and F stands for false.

   A. (T,T,F)
   B. (T,T,T)
   C. (T,F,F)
   D. (F,T,F)

4. Suppose that we toss 2 fair dice. Define the following three events:

   a The sum of the dice is 6;

   b The first die equals 4;

   c The sum of the dice is 7;

   Determine which of the following four answers is correct.

   A. a and b are independent
   B. b and c are independent
   C. a, b and c are mutually independent
   D. a, b and c are pairwise independent.

5. A coin that, when flipped, comes up heads with probability p is flipped until either heads or tails has occurred twice. Let X be the required number of flips.

   (i) X follows a geometric distribution.

(ii) The range of X is $\{2,3\}$.

(iii) $E(X) = 3 - p^2 - (1-p)^2$.

Determine which of the following four answers is correct where T stands for true and F stands for false.

A. (T,F,T)
B. (T,F,F)
C. (F,T,T)
D. (F,F,F)

6. For some constant c, the random variable X has the probability density function

$$f(x) = \begin{cases} cx^4 & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(i) $c = 5/32$.

(ii) $E(X) = 5/3$.

(iii) Var (X)= 5/63.

Determine which of the following four answers is correct where T stands for true and F stands for false.

A. (T,F,T)
B. (T,F,F)
C. (F,T,T)
D. (T,T,T)

7. Roll two fair dice. The sample space is $\{(x, y)\}$ where $x, y \in (1, 2, 3, 4, 5, 6)$.

(i) Event "the outcome of the first die is 1" is $\{1\}$

(ii) Event "at least one of the two outcomes is 3" is $\{3\}$

(iii) The probability of "the outcome of the first die is strictly larger than that of the second" equals $\frac{5}{12}$

(iv) The probability of "the sum of the two outcomes is an odd number" equals $\frac{1}{2}$

Determine which of the following four answers is correct where T stands for true and F stands for false.

A. (F, T, T, F)
B. (F, F, T, T)
C. (F, T, F, T)
D. (T, F, F, T)

8. Flip two fair coins. The sample space is $\{HH, HT, TH, TT\}$ where H and T denote Head and Tail respectively.

   (i) Events $\{HH\}$ and $\{TT\}$ are independent
   (ii) Events $\{HH, HT\}$ and $\{HT, TT\}$ are independent
   (iii) Events "the first outcome is H" and "the second outcome is T" are mutually exclusive
   (iv) Events "the second outcome is T" and "two outcomes are different" are independent

   Determine which of the following four answers is correct where T stands for true and F stands for false.

   A. (T,F,T,F)
   B. (T,F,F,T)
   C. (F,T,T,F)
   D. (F,T,F,T)

9. You have three coins. The coins are identical except that two of them are fair and the other one is biased, with a probability of Head equal to 4/5. Suppose you pick two coins from the three uniformly at random, and flip, and the outcomes are two Heads. Conditional on that, what is the probability that you picked the biased coin? Redo the calculation if the outcomes are two Tails.

   A. (a) $\frac{16}{21}$ for two Heads, (b) $\frac{4}{9}$ for two Tails
   B. (a) $\frac{6}{7}$ for two Heads, (b) $\frac{4}{9}$ for two Tails
   C. (a) $\frac{16}{21}$ for two Heads, (b) $\frac{5}{9}$ for two Tails
   D. (a) $\frac{6}{7}$ for two Heads, (b) $\frac{5}{9}$ for two Tails

10. Place 15 identical red balls and 12 identical green balls into 10 bins. Each ball is placed into a bin uniformly at random. Compute the

probability of the following events: (a) each bin contains at least one red ball AND one green ball, and (b) each bin contains at least one red ball OR one green ball.

A. (a) $\frac{\binom{15}{9}\times\binom{12}{9}}{\binom{25}{9}\times\binom{22}{9}}$, (b) $\frac{\binom{26}{9}}{\binom{36}{9}}$
B. (a) $\frac{\binom{15}{9}\times\binom{12}{9}}{\binom{24}{9}\times\binom{21}{9}}$, (b) $\frac{\binom{26}{9}}{\binom{37}{10}}$
C. (a) $\frac{\binom{14}{9}\times\binom{11}{9}}{\binom{24}{9}\times\binom{21}{9}}$, (b) $\frac{\binom{26}{9}}{\binom{36}{9}}$
D. (a) $\frac{\binom{14}{9}\times\binom{11}{9}}{\binom{24}{9}\times\binom{22}{9}}$, (b) $\frac{\binom{26}{9}}{\binom{37}{10}}$

11. Suppose Y is an exponentially distributed random variable with parameter 1. Compute (a) $P(Y > 5|Y > 3)$ and (b) $P(1 < \sqrt{Y} < 2)$
A. (a) e^{-1}, (b) $e^{-1} - e^{-\sqrt{2}}$
B. (a) e^{-2}, (b) $e^{-1} - e^{-\sqrt{2}}$
C. (a) e^{-2}, (b) $e^{-1} - e^{-4}$
D. (a) e^{-1}, (b) $e^{-1} - e^{-4}$

12. Suppose Z is a standard normal random variable. Find $E[|Z|]$ and $Var(Z^2)$

A. (a) $E[|Z|] = 1$, (b) $Var(Z^2) = 3$
B. (a) $E[|Z|] = \sqrt{\frac{2}{\pi}}$, (b) $Var(Z^2) = 2$
C. (a) $E[|Z|] = 1$, (b) $Var(Z^2) = 3$
D. (a) $E[|Z|] = \sqrt{\frac{2}{\pi}}$, (b) $Var(Z^2) = 2$

13. A man first claimed that in tossing two unfair coins at once, we have only three possible outcomes: "two heads," "one head," and "no heads." Secondly, this man also claimed that each outcome has the same probability 1/3.

(i) {two heads, one head, no heads} are legitimate sample spaces.

(ii) If two unfair coins have different probabilities of heads, then it is possible to make the second claim true.

Determine which of the following four answers is correct where T stands for true and F stands for false.

A. (T,T)
B. (T,F)
C. (F,T)
D. (F,F)

14. Let X be an exponential distribution with mean $\theta = 1/(3^{\frac{1}{3}})$. Find $Var(X^3)$.

A. 2
B. 76
C. 80
D. 84

15. A telephone company employs 5 operators who receive requests independently of one another. The number of the requests received by each operator has a Poisson distribution, and on average 2 requests are received per hour. What is the probability that during a given TWO-hour period, exactly 4 of the 5 operators receive no requests?

A. $5(e^{-8} - e^{-10})$.
B. $4(e^{-16} - e^{-20})$.
C. $5(e^{-10} - e^{-16})$.
D. $5(e^{-16} - e^{-20})$.

16. Let X be a discrete random variable with probability mass function

$$p(x) = \frac{x+1}{\lambda+1}\frac{\lambda^x}{x!}e^{-\lambda}, x = 0, 1, 2 \cdots$$

where $\lambda > 0$. This distribution is a "tilted Poisson distribution" that has its mass pushed to the right and thus has heavier tails than the standard Poisson distribution. Find the mean of X.

A. $\frac{2\lambda}{\lambda+1}$
B. $\frac{\lambda^2+2\lambda}{\lambda+1}$
C. $\frac{2\lambda^2+\lambda}{\lambda+1}$
D. $\frac{\lambda^2+\lambda+1}{\lambda+1}$

17. Customers arrive at a travel agency according to a Poisson process at a rate of 10 per hour. What is the probability that less than 4 customers arrive in 2 hours?

A. $\int_2^\infty \frac{10^3}{2} x^2 e^{-10x} dx$
B. $\int_2^\infty \frac{20^3}{2} x^2 e^{-20x} dx$
C. $\int_2^\infty \frac{10^4}{6} x^3 e^{-10x} dx$
D. $\int_2^\infty \frac{20^4}{6} x^3 e^{-20x} dx$

18. Consider the following 4 statements:

(i) If X follows a Gamma distribution with parameter $\alpha = 2$ and $\theta = 1$, then $E[X^4] = 120$
(ii) Consider the Gamma function with a positive integer α, then we could write it as $\Gamma(\alpha) = (\alpha - 1)!$
(iii) If $X \sim N(3, 1)$, then $E[X^3]$=27
(iv) If X follows a Chi-square distribution with 1 degree of freedom, then $P(X \geq 2.706) = 0.9$

A. All statements are true.
B. Only (i), (ii) and (iii) are true.
C. Only (iii) and (iv) are true.
D. Only (i) and (ii) are true.

19. Let $S = \{1, 2, 3, 4\}$ and consider the events $A = \{1, 4\}$, $B = \{2, 4\}$ and $C = \{3, 4\}$.

(i) Suppose $P(\{i\}) = 1/4$, $i = 1, 2, 3, 4$. Are A and B independent?
(ii) Are A, B, and C are pair-wise independent?
(iii) Compute $P(A \cap B \cap C)$ and $P(A)P(B)P(C)$. Are these two quantities equal?
(iv) Suppose $P(\{i\}) = i/10$, $i = 1, 2, 3, 4$. Are A and B independent?

Determine which of the following four answers is correct where T stands for true and F stands for false.

(A) (T, T, T, F)
(B) (T, T, F, F)
(C) (T, F, F, F)
(D) (F, F, F, F)

20. Suppose that A and B are two events such that both $P(A)$ and $P(B)$ are in $(0, 1)$, and consider $P(A|B)$.

(i) Is $P(A|B) = 0$ when A and B are mutually exclusive?
(ii) Is $P(A|B) = P(A)$ when A and B are independent?
(iii) Is $P(A|B) = 1$ if $B \subset A$?
(iv) Is $P(A|B) \leq P(A)$ when $A \subset B$?

Determine which of the following four answers is correct where T stands for true and F stands for false.

(A) (T, T, T, T)
(B) (T, F, F, T)
(C) (T, T, T, F)
(D) (F, F, F, F)

21. Let X_1 and X_2 be independent Bernoulli random variables with success probability p.

(i) Is the probability of $P(X_1 + X_2 = 2|X_1 = 1) = p$?
(ii) Is the probability of $P(X_1 + X_2 = 2|X_1 + X_2 \geq 1) = p/(2 - p)$?
(iii) Alice believes that the two formulas above are correct and claims that families with two children are more likely to have two girls if the first born is a girl compared to families with two children where we only know that at least one of them is a girl. Is Alice correct in her claim under the further assumption that all the births are independent events with the same p, where $p \in (0, 1)$ is the probability of a girl?

Determine which of the following four answers is correct where T stands for true and F stands for false.

(A) (F, F, T)

(B) (T, F, T)

(C) (T, T, F)

(D) (T, T, T)

22. Consider a Bernoulli process with success probability p and define $q = 1 - p$. Let X be the number of Bernoulli trials until the rth success and let N_k be a Binomial random variable with parameters $n = k$ and p.

(i) Is $P(X > k) = P(N_k \leq r - 1)$?

(ii) For $r = 1$ would this yield $P(X > k) = q^k$ for all $k \in \{1, 2, \ldots\}$?

(iii) Is it true that for $r = 1$, $P(\{X > k + l\}|\{X > k\}) = P(X > l)$ for all positive integers l?

(iv) For arbitrary r is it $P(X = k) = P(N_k \leq r - 1) - P(N_{k-1} \leq r - 1)$ correct for all $k \in \{r, r + 1, \ldots, \}$?

Determine which of the following four answers is correct where T stands for true and F stands for false.

(A) (T, T, T, T)

(B) (T, F, F, T)

(C) (T, T, T, F)

(D) (F, F, F, F)

23. Suppose that $X_1, X_2, \ldots, X_n$ are uniform $[0, 1]$ random variables, events associated with X_i, $i = 1, \cdots, n$ are mutually independent, and let $Y_n = \min(X_1, \ldots, X_n)$.

(i) Is $P(Y_n > y) = (1 - y)^n$?

(ii) Is $P(Y_n > z/n) = (1 - z/n)^n$?

(iii) Is $\lim_{n\to\infty} P(nY_n > z) = \exp(-z)$?

(iv) Is $\lim_{n\to\infty} E[nY_n] < 1$?

Determine which of the following four answers is correct where T stands for true and F stands for false.

(A) (T, T, T, T)
(B) (T, T, T, F)
(C) (T, F, F, T)
(D) (F, F, F, F)

24. Let X be a random variable and $\hat{X} = a + bX$ for some constants a and b. Let π_p be the $100p$ percentile of X and $\hat{\pi}_p$ be the $100p$ percentile of $\hat{X}$.

(i) Suppose X is exponential with mean 1. Is $\pi_p = \ln(1-p)$?
(ii) Suppose now that X is normal with mean zero and variance one. Is $\pi_p = \Phi^{-1}(p)$?
(iii) Is $\hat{\pi}_p = a + b\pi_b$ for all reals a and b when $b > 0$?
(iv) Suppose $a = 0$ and $b = \theta$, and X is exponential with mean one, is $\hat{X}$ exponential with mean θ?
(v) Suppose that X is normal, $a = \mu$ and $b = \sigma$, is $\hat{X}$ normal with mean μ and variance σ?

Determine which of the following four answers is correct where T stands for true and F stands for false.

(A) (T, T, T, T, T)
(B) (T, T, T, F, T)
(C) (T, T, T, T, F)
(D) (F, T, F, T, T)

II Regular Questions (28 points)

25. (14 points) The life of a certain type of automobile tire is normally distributed with mean 34,000 miles and standard deviation 4000 miles.
    (a) (3 points) What is the probability that such a tire lasts over 40,000 miles?
    (b) (3 points) What is the probability that it lasts between 30,000 and 35,000 miles?
    (c) (4 points) Given that it has survived 30,000 miles, what is the conditional probability that the tire survives another 10,000 miles?
    (d) (4 points) Suppose the lifetime follows exponential distribution with mean 34,000 miles. Answer the previous question (c) again.

26. (14 points)
    (a) (8 points) Let X be a continuous random variable with its probability density function described by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-0.5(x-\mu)^2/\sigma^2}, x \in (-\infty, \infty). \tag{1}$$

    Prove that

    1. (4 points) its moment generating function $M_X(t)$ is equal to

$$M_X(t) = e^{\mu t + 0.5\sigma^2 t^2}, t \in (-\infty, \infty). \tag{2}$$

    2. (2 points) $\mathrm{E}(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$.
    3. (2 points) for any real constants a, b, $a + bX \sim N(a + b\mu, b^2\sigma^2)$.

    (b) (6 points) Let Y be a continuous random variable whose moment generating function $M_Y(t)$ described by

$$M_Y(t) = e^{0.5t^2}, t \in (-\infty, \infty). \tag{3}$$

    Prove that

    1. (1 points) let $F(y)$ denote the cumulative distribution function of Y, prove that $F(-z_\alpha) = 1 - F(z_\alpha)$, where $\alpha \in (0, 1)$ is a constant, and z_α is the upper 100α percent point.
    2. (5 points) Y^2 has a χ^2 distribution with degrees of freedom 1.