STA2001 Tutorial 12

1. 5.4-10. Let X equal the outcome when a fair four-sided die that has its faces numbered 0, 1, 2, and 3 is rolled. Let Y equal the outcome when a fair four-sided die that has its faces numbered 0, 4, 8, and 12 is rolled.

   (a) Define the moment-generating function of X.

   (b) Define the moment-generating function of Y.

   (c) Let $W = X + Y$, the sum when the pair of dice is rolled. Find the moment-generating function of W.

   (d) Give the pmf of W; that is, determine $P(W = w), w = 0, 1, \cdots, 15$, from the moment-generating function of W.

2. 5.4-20. The time X in minutes of a visit to a cardiovascular disease specialist by a patient is modeled by a gamma pdf with $\alpha = 1.5$ and $\theta = 10$. Suppose that there is such a patient and have four patients ahead of him/her. Assuming independence, what integral gives the probability that this patient will wait more than 90 minutes?

3. 5.5-10. A consumer buys n light bulbs, each of which has a lifetime that has a mean of 800 hours, a standard deviation of 100 hours, and a normal distribution. A light bulb is replaced by another as soon as it burns out. Assuming independence of the lifetimes, find the smallest n so that the succession of light bulbs produces light for at least $10,000$ hours with a probability of 0.90.

4. 5.6-2. Let $Y = X_1 + X_2 + \cdots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose pdf is $f(x) = (3/2)x^2$, $-1 < x < 1$. Using the pdf of Y, we find that $P(-0.3 \leq Y \leq 1.5) = 0.22788$. Use the central limit theorem to approximate this probability.