mutually exclusive 互斥 $A\cap B=\varnothing$

exhaustive 互补 $A\cup B=S$

Venn图适用 $(A\cap B)\cap C=A\cap C\cap B\cap C$)

$(A\cup B)\cup C=A\cup C\cup B\cup C$ $\quad A\cup C(B\cap C)=$

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ $\quad (A\cup B)\cap(A\cup C)$

$(A\cup B)'=A'\cap B'$ $\quad (A\cap B)'=A'\cup B'$

$\lim_{n\to\infty}\frac{N(A)}{n}=P(A)$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$.

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)$
$-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$.

${}_nP_r=\frac{n!}{(n-r)!}$ Permutation

n物品选r个全排列

ordered 有序, replacement 可重复 (放回).

unordered with size r $\quad {}_nC_r=C_r^n=\frac{n!}{r!(n-r)!}$

Distinguishable permutation

$\frac{n!}{n_1!n_2!\dots n_s}$ $\qquad P(A|B)=\frac{P(A\cap B)}{P(B)}$

$P(A\cap B)=P(A|B)\cdot P(B)=P(B|A)\cdot P(A)$.

A & B Inde

$P(A'|B)=1-P(A|B)$. $\quad A,B'/A',B/A',B'$

$P[A\cap B\cap C]=P(A)\cdot P(B|A)\cdot P(C|A\cap B)$.

Inde. ① $P(A\cap B)=P(A)P(B)$. $\quad A,B,C$ Inde.

② $P(A\cap B\cap C)=P(A)P(B)P(C)$ $\quad A$与$B\cap C$

Pairwise Inde. $\quad A$与$B\cap C'$, A与$B\cup C$

A',B',C' Inde.

全概率.

$P(A)=\sum_{i=1}^{m}P(B_i)P(A|B_i)$. 贝叶斯. $P(B_k|A)=\frac{P(B_k)\cdot P(A|B_k)}{P(A)}$.

Random Variable.

PMF; $f(x)>0$, $S\to(0,1]$

$\sum_{x\in S}f(x)=1$, $P(X\in A)=\sum_{x\in A}f(x)$

cdf $F(x)=P(X\le x)=\sum_{x\le x,\,x\in S}f(x)$

$P(a<X\le b)=F(b)-F(a)$.

Mean. $E[g(x)]=\sum_{x\in S}g(x)f(x)$.

$E[C]=C$ $\quad$ cvg absolutely $\quad M^{(r)}(0)=E[X^r]$

$Var[X]=E[(X-u)^2]$ 注 X可以代 X^2, X^3 etc.

$=E[X^2]-E^2[X]$

$=E[X(X-1)]+E[X]-E^2[X]$ $\quad$ 若rth moment存在 则1…r-1th都存在.

MGF $E[e^{tX}]=\sum_{x\in S}e^{tx}f(x)$ $\quad -h<t<h$.

$M(0)=1$ $\quad M'(0)=u$ $\quad M''(0)=u^2+\sigma^2=E[X^2]$

MGF一样 → probability分布一样 $\quad$ 算$E[X^r]$可用.

Bernoulli [0,1]分布

PMF p^xq^{1-x}, $u=p$

MGF $q+pe^t$, $\sigma^2=pq$

$X\sim b(1,p)$.

Binomial 2项分布

PMF $C_x^n p^x q^{n-x}$ $\quad u=np$

MGF $(q+pe^t)^n$ $\quad \sigma^2=npq$

$X\sim b(n,p)$

CDF $F(x)=P(X\le x)=\sum_{y\in\{y\le x\}}f(y)$

$=\sum_{y=0}^{[x]}C_y^n p^y q^{n-y}$

Geometric 第?次实验1st成功

$X\sim geometric(p)$.

PMF $q^{x-1}p$, $u=\frac{1}{p}$

MGF $\frac{pe^t}{1-qe^t}$ $\quad \sigma^2=\frac{q}{p^2}$

CDF $P(X>k)=(1-p)^k$

$P(X\le k)=1-(1-p)^k$

$t<-\ln(1-p)$.

Hypergeometric 1个选物品

PMF $\frac{C_x^{N_1}C_{n-x}^{N_2}}{C_n^N}$ $\quad u=n\frac{N_1}{N}$

共n个 X好 n-X坏

$\sigma^2=n\frac{N_1}{N}\frac{N_2}{N}\frac{N-n}{N-1}$

Negative Binomial $\quad$ X trials r th success happens

PMF $C_{r-1}^{x-1}p^rq^{x-r}$

$u=\frac{r}{p}$ $\quad \sigma^2=\frac{rq}{p^2}$

MGF $\frac{(pe^t)^r}{(1-qe^t)^r}$

Distinguishable 排列 and number of 无序 samples of size r can be selected out of n物品 with replacement

${}_{n+r-1}C_r=\frac{(n+r-1)!}{r!(n-1)!}$ 选r个

Uniform

从1,2,3,…m随机抽1个数字

PMF $\frac{1}{m}$ $\quad u=\frac{m+1}{2}$

$\sigma^2=\frac{m^2-1}{12}$

Poisson $\quad u=\sigma^2=\lambda$

PMF $\frac{\lambda^xe^{-\lambda}}{x!}$ $\quad$ MGF $e^{-\lambda}\cdot e^{\lambda e^t}=e^{\lambda(e^t-1)}$.

发生次数在unit interval $=\lambda$ $\quad X\sim Poisson(\lambda)$

CTS RV $\quad$ PDF $S\to(0,\infty)$

$f(x)>0$, $x\in S$ $\quad P(X=a)=0$

$\int_S f(x)dx=1$ $\quad$ 不需 PDF CTS

$P(a\le X\le b)=\int_a^b f(x)dx$ or bdd.

CDF $F(x)\triangleq P(X\le x)=\int_{-\infty}^{x}f(t)dt$

nondecreasing

$F'(x)=f(x)$. $\quad P(a\le X\le b)=F(b)-F(a)$

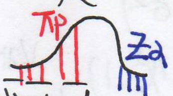
$u=\int_{-\infty}^{\infty}xf(x)dx$ $\quad \sigma^2=\int_{-\infty}^{\infty}(x-u)^2f(x)dx$

MGF $\int_S e^{tx}f(x)dx$

百分位数 $p=\int_{-\infty}^{\pi_p}f(x)dx=F(\pi_p)$.

$P(Z\ge z_\alpha)=\alpha$ $\quad P(Z<z_\alpha)=1-P(Z\ge z_\alpha)$

z_α是$100(1-\alpha)$th位数 $\quad =1-\alpha$



Uniform distribution

PDF $\frac{1}{b-a}$ $\quad$ MGF $\begin{cases}\frac{e^{tb}-e^{ta}}{t(b-a)}, & t\ne0\\ 1, & t=0\end{cases}$

$u=\frac{a+b}{2}$

$\sigma^2=\frac{(b-a)^2}{12}$ $\quad$ CDF $\frac{x-a}{b-a}$ $\quad X\sim U(a,b)$

Exponential Distribution

APP. waiting time until first 发生.

$\theta=\frac{1}{\lambda}$ $\quad \lambda$: avg occurences / unit time

PDF $=\frac{1}{\theta}e^{-\frac{x}{\theta}}$ $\quad$ MGF $=\frac{1}{1-\theta t}$ $\quad t<\frac{1}{\theta}$

$u=\theta$ $\quad \sigma^2=\theta^2$ $\quad$ CDF $1-e^{-\frac{x}{\theta}}$

Gamma Distribution

APP. waiting time until αth 发生

$\theta=\frac{1}{\lambda}$ $\quad \lambda$: avg occurrences / unit time

PDF $\frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-\frac{x}{\theta}}$

关于Gamma函数

MGF $\frac{1}{(1-\theta t)^\alpha}$ $\quad \Gamma(\frac{1}{2})=\sqrt{\pi}$

$\Gamma(t)=\int_0^\infty y^{t-1}e^{-y}dy$

$\Gamma(t)=(t-1)\Gamma(t-1)$

$\Gamma(n)=(n-1)!$ $\quad u=\alpha\theta$, $\sigma^2=\alpha\theta^2$

Chi-square $\quad$ r is integer

$\theta=2$, $\alpha=\frac{r}{2}$

PDF $\frac{1}{\Gamma(\frac{r}{2})2^{\frac{r}{2}}}x^{\frac{r}{2}-1}e^{-\frac{x}{2}}$

MGF $(1-2t)^{-\frac{r}{2}}$

$X\sim\chi^2(r)$ $\quad \sigma^2=2r$

$u=r$

CDF $\int_0^x f(t)dt$

Normal Distribution

PDF $\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-u)^2}{2\sigma^2}}$ $\int_{-\infty}^{\infty} f(x)dx=1$

MGF $e^{ut+\frac{1}{2}\sigma^2t^2}$

$u=u,\ \sigma^2=\sigma^2$ $X\sim N(u,\sigma^2)$

Standard $Y\sim N(0,1)$.

PDF $\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}$ $Y\sim N(u,\sigma^2)$ $X=\frac{Y-u}{\sigma}\sim N(0,1)$

CDF $\int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz$ $X\sim N(u,\sigma^2)$

$\Phi(-y)=1-\Phi(y)$. $(\frac{X-u}{\sigma})^2\sim\chi^2(1)$

$\sum\frac{1}{x!}=e$ 指数分布 $E[X^k]=k!\lambda^{-k}$

Gamma分布 $E[X^k]=\alpha(\alpha+1)\cdots(\alpha+k-1)\theta^k$

No Memory Exponential and Geometric

如 M(t)不是公式里的，观察形式硬求导 CTS RV

或 Discrete RV 找 f(x=1), f(x=2).. etc E[X] $\int_{-\infty}^{\infty}xf(x)dx$

Gamma, Exponential 选取λ要注意

unit interval. PDF $\int_{-\infty}^{\infty}f(x)dx=1$

$X\sim$ Gamma $(u=\frac{3}{2},\ \sigma=\sqrt{\frac{3}{4}})$. $Y=3X$

$M_X(t)=\frac{1}{(1-\frac{1}{2}t)^3}=(\frac{2}{2-t})^3$ $M_Y(t)=E[e^{tY}]$

$=M_X(3t)=(\frac{2}{2-3t})^3$ // $X\sim N(3,1)$ $E[X^3]=2$

$X=Z+3$ $Z\sim N(0,1)$ $E[Z]=0$ $E[Z^3]=0$

$E[Z^2]=1$ $E[X^3]=E[Z^3+9Z^2+27Z+27]=36$

$P[(A\cap B')\cup(A'\cap B)]=P(A)-P(A\cap B)+P(B)-P(A\cap B)$

$M_X(t)=e^tM_X(-t)$ $E(e^{tX})=e^tE(e^{-tX})$

$=E(e^{t(1-X)})$ $t(1-X)=tX$ $X=\frac{1}{2}$

X follows a uniform on [-2,1] $X\sim U[-2,1]$

$Y=X^2$ $F_X(x)=\frac{x+2}{3}$ $-2\le x\le 1$

cdf of Y $(0\le y\le 1)\to -1\le X\le 1$

$F_Y(y)=P(Y\le y)=P(X^2\le y)=P(-\sqrt{y}\le X\le\sqrt{y})$.

$=P(X\le\sqrt{y})-P(X\le-\sqrt{y})$

$=\frac{2\sqrt{y}}{3}$. consider $1<y\le 4$, $-2\le X\le -1$

$P(Y\le y)=P(Y<1)+P(1<Y\le y)$

$=\frac{2}{3}+P(1\le X^2\le y)=\frac{2}{3}+P(-\sqrt{y}\le X\le -1)$

$=\frac{2}{3}+\dots P(X\le-\sqrt{y})=\frac{2}{3}+F_X(-1)-F_X(-\sqrt{y})=\frac{1+\sqrt{y}}{3}$

15 Red 12 Green 放 10 bins

a. each bins at least 1 Red and 1 Green

$\frac{\binom{14}{9}\cdot\binom{11}{9}}{\binom{24}{9}\binom{21}{9}}$ b. at least 1 Red or 1 Green $\frac{\binom{26}{9}}{\binom{36}{9}}$

$Z\sim N(0,1)$

$\int_{-\infty}^{\infty}|z|f(z)dz=2\times\frac{1}{\sqrt{2\pi}}\int_0^{\infty}ze^{-\frac{z^2}{2}}dz$

$=\frac{2}{\sqrt{2\pi}}\int_0^{\infty}e^{-w}dw=\sqrt{\frac{2}{\pi}}$

$Var(Z^2)$ $V=Z^2$ $M(t)=\int_{-\infty}^{\infty}e^{tz^2}f(z)dz$

$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{tz^2}e^{-\frac{z^2}{2}}dz$

$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-(\frac{1}{2}-t)z^2}dz$

$=\frac{1}{\sqrt{1-2t}\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}w^2}dw$

$=\frac{1}{\sqrt{1-2t}}$ $M'(0)=1$

$E[V]=1,\ E[V^2]=M''(0)=3$

$Var(V)=2$ 法2

$Var(Z^2)=Var(Z^4)-Var^2(Z^2)$

$\frac{1}{1+\theta t}=\frac{\lambda}{\lambda-t}$ $E[e^{tX}]$

$=\frac{1}{1-t/\lambda}=\sum_0^{\infty}(\frac{t}{\lambda})^k$

$E[e^{tX}]=\sum_{k=0}^{\infty}\frac{E[X^k]t^k}{k!}$

$E[X^k]=k!\lambda^{-k}$

$\sum x\frac{\lambda^x}{x!}e^{-\lambda}=\lambda$

$\sum x^2\frac{\lambda^x}{x!}e^{-\lambda}=\lambda^2+\lambda$

有时先用Gamma/Exponential Poisson 算单次事件P，再2项分布. $Var[X]=E[X^2]-E[X]^2$

给了PDF可以 $\int xf(x)dx$ 求E[X] $\int x^2f(x)dx$ 求 $E[X^2]$ 区分泊松分布与过程所取入

几局几胜→考虑前n-1场胜了几场

什么时候得一套东西 $X\sim geometric(p)$.

36口香糖 共10个口味 每个口味有0~36个口香糖

组合 $C^{r+n-1}_{r,(n-1)}=\frac{45!}{36!9!}$

$P(A\cup C|B)=P(A|B)+P(C|B)-P(A\cap C|B)$.

Poisson process {考虑 Poisson分布 / 考虑 伽马, 指

有可能先用正态求P 再2项. PMF $\sum_0^{\infty}f(x)=1$

$P(X=x)=p(x)=\begin{cases}\frac{1}{b-a+1} & x\in\{a..b\}\\ 0, & otherwise\end{cases}$

$E[X]=\sum_{x=a}^{b}\frac{x}{b-a+1}=\frac{a+b}{2}$

$Y=X-a+1$ $n=b-a+1$

$Var\,Y=Var\,X$ 求 Var Y

$EY=\sum_{x=1}^{n}\frac{y}{n}=\frac{n+1}{2}$

$EY^2=\sum_{x=1}^{n}\frac{y^2}{n}=\frac{(n+1)(2n+1)}{6}$

$\binom{n}{k}p^k(1-p)^{n-k}\approx\frac{(np)^k}{k!}e^{-np}$

求CDF 例 $W=a+(b-a)Y$

$P[W\le w]$

$=P[a+(b-a)Y\le w]$

$=P[Y\le\frac{w-a}{b-a}]$

$P(\frac{1}{1+e^{-X}}\le y)=P(X\le -\ln(\frac{1}{y}-1))$

公理 $P(A)\ge 0,\ P(S)=1$

$A\subseteq B$ $P(A)\le P(B)$

roll 3个3⇒A, roll了个3或5⇒B, 3在5之前roll ⇒C

$P(C)=P(A|B)$

抽板法 $E[e^{tX}]=\sum_{x\in S}e^{tx}f(x)$

$(A\cap B')\cup(A'\cap B)$

$=A-A\cap B+B-A\cap B$

$1+2+\dots n^2=\frac{n(n+1)(2n+1)}{6}$

$1+8+27+\dots n^3=[\frac{n(n+1)}{2}]^2$

$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$

$\frac{1}{1-x}=\sum_0^{\infty}x^n$

$\frac{1}{1+x}=\sum_0^{\infty}(-1)^nx^n$

$\ln(1+x)=\sum_1^{\infty}\frac{(-1)^{n-1}x^n}{n}$