STA2001 Probability and statistical Inference I
Tutorial 3

1. (2.1-16). (Michigan Mathematics Prize Competition, 1992, Part II) From the set $\{1, 2, 3, \cdots, n\}$, k distinct integers are selected at random and arranged in numerical order (from lowest to highest). Let $P(i, r, k, n)$ denote the probability that integer i is in position r. For example, observe that $P(1, 2, k, n) = 0$, as it is impossible for the number 1 to be in the second position after ordering.
   (a) Compute $P(2, 1, 6, 10)$.
   (b) Find a general formula for $P(i, r, k, n)$.

2. (2.2-3) Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the pmf $f(x) = \frac{5-x}{10}$, $x = 1, 2, 3, 4$. If the patient is to receive 200 dollars from an insurance company for each of the first two days in the hospital and 100 dollars for each day after the first two days, what is the expected payment for the hospitalization?

3. (2.3-9). A warranty is written on a product worth 10,000 dollars so that the buyer is given 8,000 dollars if it fails in the first year, 6,000 dollars if it fails in the second, 4,000 dollars if it fails in the third, 2,000 dollars if it fails in the fourth, and zero after that. The probability that the product fails in the first year is 0.1, and the probability that it fails in any subsequent year, provided that it did not fail prior to that year, is 0.1. What is the expected value of the warranty?

4. Let X be the number of flips of a fair coin to observe different faces (head-tail or tail-head) on consecutive flips. What is the moment generating function of X?