STA2001 Probability and Statistical Inference I
Tutorial 2

1. Suppose that an experiment is repeated n times. The number of times that an event A actually occurred throughout these n performances is called the *frequency* of A, denoted by $\mathcal{N}(A)$. The ratio $f(A) := \mathcal{N}(A)/n$ is called the relative frequency of event A in these n repetitions of the experiment.

    1. For the sample space S, show $f(S) = 1$.

    2. For two events A and B, if A and B are *mutually exclusive* (i.e., $A \cap B = \emptyset$), prove $f(A \cup B) = f(A) + f(B)$.

    3. For any two events A and B, show that

    $$f(A \cup B) = f(A) + f(B) - f(A \cap B).$$

2. Consider two independent fair coin tosses, in which all four possible outcomes are equally likely. Define the following events

$$H_1 = \{\text{1st toss is a head}\},$$
$$H_2 = \{\text{2nd toss is a head}\},$$
$$D = \{\text{the two tosses have different results}\}.$$

Prove or disprove the following four statements:

(i) The events H_1 and H_2 are dependent.

(ii) Given that the event D has happened, the events H_1 and H_2 are conditionally independent.

(iii) The events H_1, H_2 and D are mutually independent.

(iv) The events H_1, H_2 and D are pairwise independent.

3. A chemist wishes to detect an impurity in a certain compound that she is making. There is a test that detects an impurity with probability 0.90; however, this test indicates that an impurity is there when it is not about 5% of the time. The chemist produces compounds with the impurity about 20% of the time; that is, 80% do not have the impurity. A compound is selected at random from the chemists output. The test indicates that an impurity is present. What is the conditional probability that the compound actually has an impurity?

4. Suppose we have 5 fair coins and 10 unfair coins, which look the same and feel the same. For the fair coins, there is a 50% chance of getting heads and of course 50% chance of getting tails. For the unfair coins, there is a 80% probability of getting heads and 20% tails. Now we randomly pick one coin from all 15 coins and flip it for 6 times. Then we get 4 heads. What is the probability that we have pick a fair coin?