# STA2001 Tutorial 10

1. 4.5-8. Let $X$ and $Y$ have a bivariate normal distribution with parameters $\mu_X = 10$, $\sigma_X^2 = 9$, $\mu_Y = 15$, $\sigma_Y^2 = 16$ and $\rho = 0$. Find

   (a) $P(13.6 < Y < 17.2)$

   (b) $E(Y|x)$

   (c) $\mathrm{Var}(Y|x)$

   (d) $P(13.6 < Y < 17.2|X = 9.1)$

(a). $P(13.6 < Y < 17.2) = P\left(\frac{13.6-\mu_Y}{\sigma_Y} < \frac{Y-\mu_Y}{\sigma_Y} < \frac{17.2-\mu_Y}{\sigma_Y}\right).$

$\underline{Z = \frac{Y-\mu_Y}{\sigma_Y}} \sim N(0,1)$

$P(-0.35 < Z < 0.55).$

$= \Phi(0.55) - \Phi(-0.35).$

$= \Phi(0.55) - (1 - \Phi(0.35)). \approx 0.3456.$

(b). $\underbrace{Y|X=x} \sim N\left(\underbrace{\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x-\mu_X)}, \sigma_Y^2(1-\rho^2)\right).$

$E[Y|X=x] = \quad = 15 + 0 \cdot \frac{4}{3} \cdot (x-10)$

(c). $\mathrm{Var}[Y|X=x] = \sigma_Y^2(1-\rho^2)$

(d). $P(13.6 < Y < 17.2 | X = 9.1).$

$\rho = 0$ means indep.

$\underline{P(13.6 < Y < 17.2).}$ (ref to (a)).

$(X,Y)$. bivariate normal.

2. 5.1-10. Let $X$ has the uniform distribution $U(-1,3)$. Find the pdf of $Y = X^2$.

$p(y) = \frac{d}{dy} P(Y \le y)$. $\quad F(x) = \int_{-1}^{x} \frac{1}{3-(-1)}\,dx = \frac{1}{4}(x+1)$.

$P(Y \le y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y})$.

say: $P(Y \le 1.1) = P(X^2 \le 1.1) = P(\underline{-\sqrt{1.1}} \le X \le \sqrt{1.1}) = P(-1 \le X \le \sqrt{1.1})$.

$-1$. $\quad F(\sqrt{1.1}) - F(-1)$.

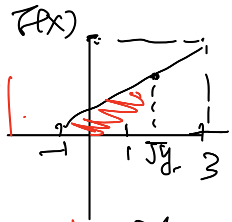
$S_X = [-1, 3]$. $\quad S_Y = [0, 9]$.

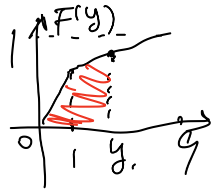
(1). When $Y \in (0,1)$, $-1 < X < 1$:

$P(Y \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) = F(\sqrt{y}) - F(-\sqrt{y})$.

$p(y) = \frac{d}{dy}(P(Y \le y)) = \frac{d}{dy}(F(\sqrt{y}) - F(-\sqrt{y})) = \frac{1}{4\sqrt{y}}$. $y \in (0,1)$.

$\frac{d}{dy}(F(\sqrt{y})) = f(\sqrt{y}) \cdot (\sqrt{y})'$.




(2). When $Y \in [1, 9]$. $1 \le X \le 3$.

$P(Y \le y) = F(\sqrt{y}) - \boxed{F(-\sqrt{y})}$. $\quad -\sqrt{y} \le -1$.

$F(-1)$.

$p(y) = \frac{d}{dy}(P(Y \le y)) = \frac{d}{dy}(F(\sqrt{y}) - F(1)) = \frac{1}{8\sqrt{y}}$. $y \in [1, 9)$.

$$\boxed{p(y) = \begin{cases} \frac{1}{4\sqrt{y}} & y \in (0,1). \\ \frac{1}{8\sqrt{y}} & y \in [1,9) \end{cases}}$$

3. 5.1-14. Let $X$ be $N(0, 1)$. Find the pdf of $Y = |X|$, a distribution that is often called the half-normal.

Hint: Here $y \in S_y = \{y : 0 < y < \infty\}$. Consider the two transformations $x_1 = -y$, $-\infty < x_1 < 0$, and $x_2 = y$, $0 < x_2 < \infty$.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$P(Y \le y) = P(|X| \le y) = P(-y \le X \le y). = F(y) - F(-y)$$
$$= F(y) - (1 - F(y)). = 2F(y) - 1.$$

$$f_Y(y) = \frac{d}{dy} P(Y \le y) = \frac{d}{dy}(F(y) - F(-y)). = \frac{d}{dy}(2F(y) - 1).$$
$$= 2 \frac{d}{dy} F(y). = 2 f_X(y) = \frac{2}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}.$$