# STA2001 Assignment 3

Due Date: June 27, 2023

(3.1-3). Customers arrive randomly at a bank teller's window. Given that one customer arrived during a particular 10-minute period, let $X$ equal the time within the 10 minutes that the customer arrived. If $X$ is $U(0, 10)$, find
(a) The pdf of $X$.
(b) $P(X \geq 8)$.
(c) $P(2 \leq X < 8)$.
(d) $E(X)$.
(e) $\text{Var}(X)$.

(3.1-5). Let $Y$ have a uniform distribution $U(0, 1)$, and let

$$W = a + (b - a)Y, \quad a < b.$$

(a) Find the cdf of $W$.
Hint: Find $P[a + (b - a)Y \leq w]$.
(b) How is $W$ distributed?

(3.1-6). A grocery store has $n$ watermelons to sell and makes \$1.00 on each sale. Say the number of consumers of these watermelons is a random variable with a distribution that can be approximated by

$$f(x) = \frac{1}{200}, \quad 0 < x < 200,$$

a pdf of the continuous type. If the grocer does not have enough watermelons to sell to all consumers, she figures that she loses \$5.00 in goodwill from each unhappy customer. But if she has surplus watermelons, she loses 50 cents on each extra watermelon. What should $n$ be to maximize profit?
Hint: If $X \leq n$, then her profit is $(1.00)X + (-0.50)(n - X)$; but if $X > n$, her profit is $(1.00)n + (-5.00)(X - n)$. Find the expected value of profit as a function of $n$, and then select $n$ to maximize that function.

(3.2-1). What are the pdf, the mean, and the variance of $X$ if the moment-generating function of $X$ is given by the following?
(a) $M(t) = \frac{1}{1-3t}, t < 1/3$.
(b) $M(t) = \frac{3}{3-t}, t < 3$.

**(3.2-3).** Let $X$ have an exponential distribution with mean $\theta > 0$. Show that

$$P(X > x + y | X > x) = P(X > y)$$

for any $x > 0$.

**(3.2-7).** Find the moment-generating function for the gamma distribution with parameters $\alpha$ and $\theta$.
Hint: In the integral representing $E\left(e^{tX}\right)$, change variables by letting $y = (1 - \theta t)x/\theta$, where $1 - \theta t > 0$.

**(3.2-11).** If $X$ is $\chi^2(17)$, find

(a) $P(X < 7.564)$
(b) $P(X > 27.59)$
(c) $P(6.408 < X < 27.59)$
(d) $\chi^2_{0.95}(17)$
(e) $\chi^2_{0.025}(17)$

**(3.2-22).** Let $X$ have a logistic distribution with pdf

$$f(x) = \frac{e^{-x}}{(1 + e^{-x})^2}, \quad -\infty < x < \infty.$$

Show that

$$Y = \frac{1}{1 + e^{-X}}$$

has a $U(0, 1)$ distribution.

Hint: Find $G(y) = P(Y \leq y) = P\left(\frac{1}{1+e^{-X}} \leq y\right)$, where $0 < y < 1$.

**(3.3-10).** If $X$ is $N\left(\mu, \sigma^2\right)$, show that the distribution of $Y = aX + b$ is $N\left(a\mu + b, a^2\sigma^2\right)$ $a \neq 0$.
Hint: Find the cdf $P(Y \leq y)$ of $Y$, and in the resulting integral, let $w = ax + b$ or, equivalently, $x = (w - b)/a$.

**(3.3-14).** The strength $X$ of a certain material is such that its distribution is found by $X = e^Y$, where $Y$ is $N(10, 1)$. Find the cdf and pdf of $X$, and compute $P(10,000 < X < 20,000)$.

Note: $F(x) = P(X \leq x) = P\left(e^Y \leq x\right) = P(Y \leq \ln x)$ so that the random variable $X$ is said to have a lognormal distribution.