$Y \sim U(0,1)$

$W = a+(b-a)Y$

求 CDF 及 W分布.

$G(w)=P(W\le w)=P(a+(b-a)Y\le w)$

$= P\left(Y \le \frac{w-a}{b-a}\right) = \int_0^{\frac{w-a}{b-a}} dy = \frac{w-a}{b-a}$

$w\le a,\ G(w)=0,\ w\ge b,\ G(w)=1$

$W\sim U(a,b)$.

$f(x)=1/200,\ 0<x<200.$

$x\le n$, profit $x-\frac{1}{2}(n-x)$

$x>n$, profit $n-5(x-n)$.

$$g(x)=\begin{cases} x-\frac{1}{2}(n-x), & 0\le x\le n \\ n-5(x-n), & 200\ge x>n \end{cases}$$

$u=\left[\int_0^n 1.5x-\frac{1}{2}n\,dx+\int_n^{200}-5x+6n\,dx\right]\cdot\frac{1}{200}$

$=\frac{1}{200}(-\frac{13}{4}n^2+1200n-100000)$

$d'=\frac{1}{200}[-\frac{13}{2}n+1200]=0.$

$\int_0^\infty e^{tx}\frac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha}dx \qquad u=(1-\theta t)x/\theta$

$=\frac{1}{\Gamma(\alpha)\theta^\alpha}\int_0^\infty x^{\alpha-1}e^{-x/\theta}e^{tx}dx$

$\int_0^\infty x^{\alpha-1}e^{-x(1-\theta t)/\theta}dx=\int_0^\infty\left(\frac{\theta u}{1-\theta t}\right)^{\alpha-1}e^{-u}\frac{\theta}{1-\theta t}du$

$\Downarrow \frac{1}{\Gamma(\alpha)\theta^\alpha}\cdot\frac{\theta^\alpha}{(1-\theta t)^\alpha}\int_0^\infty u^{\alpha-1}e^{-u}du\to\Gamma(\alpha).$

$=(1-\theta t)^{-\alpha}$

$f(x)=\frac{e^{-x}}{(1+e^{-x})^2} \qquad Y=(1+e^{-X})^{-1}$

$F_X(x)=\int_{-\infty}^x\frac{e^{-w}}{(1+e^{-w})^2}dw=\frac{1}{1+e^{-x}}$

$G(y)=P(1+e^{-X}\ge\frac{1}{y})=P(X\le-\ln(\frac{1}{y}-1))$

$=F_X(-\ln(\frac{1}{y}-1))=y \qquad 0<y<1 \quad Y\sim U(0,1)$

$X\sim N(u,\sigma^2)\quad Y=aX+b.$

$G(y)=P(aX+b\le y)=P(X\le\frac{y-b}{a})=\int_{-\infty}^{\frac{y-b}{a}}\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-u)^2}{2\sigma^2}}dx$

$=\int_{-\infty}^y\frac{1}{a\sigma\sqrt{2\pi}}e^{-(w-b-au)^2/2\sigma^2a^2}dw$

$X=e^Y\quad Y\sim N(10,1)\quad F_Y(y)=\int_{-\infty}^y\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(x-u)^2}{2\sigma^2}}dx$

$F_X(x)=P(e^Y\le x)=P(Y\le\ln x)=F_Y(\ln x).$

$f_X(x)=\frac{1}{x\sqrt{2\pi}}e^{\frac{-(\ln x-10)^2}{2}} \qquad P(10000<X<20000)$

$=P(Y\le\ln 20000)-P(Y\le\ln 10000)=P(Z<-0.10)$

$-P(Z<-0.79)=0.2454 \qquad Z=\frac{Y-u}{\sigma}\sim N(0,1)$

$f(x,y)=\frac{4}{3},\ 0<x<1,\ x^3<y<1$

$P(X>Y)=1-P(X\le Y).$

$=1-\int_0^1dx\int_{x^3}^1\frac{4}{3}dy=\frac{1}{3}$

$2<x<2.5,\ 2<y<2.3.$

$f(x,y)=\frac{20}{3}.$

$P(|X-Y|<0.1)=P(|X-Y|<0.1,\ 2<Y<2.1)$

$+P(|X-Y|<0.1,\ 2.1<Y<2.3)$

$=\int_2^{2.1}dy\int_2^{y+0.1}\frac{20}{3}dx+\int_{2.1}^{2.3}dy\int_{y-0.1}^{y+0.1}dx\cdot\frac{20}{3}$

$=\frac{11}{30}$

$f(x)=2e^{-2(x-0.2)}.$

$0.2<x<\infty \qquad 0.2<x<\infty$

$f(y|x)=\frac{1}{x+0.1-x+0.1}=5$

$x-0.1<y<x+0.1$

$f(x,y)=f(y|x)\cdot f(x)=10e^{-2(x-0.2)}.$

$E[Y]=\int_{0.2}^\infty dx\int_{x-0.1}^{x+0.1}10ye^{-2(x-0.2)}dy$

$=\int_{0.2}^\infty 5y^2e^{-2(x-0.2)}\Big|_{x-0.1}^{x+0.1}dx$

$=\int_{0.2}^\infty 2xe^{-2(x-0.2)}dx$

$=-\frac{1}{2}(2x+1)e^{-2(x-0.2)}\Big|_{0.2}^\infty=0.7$

$X\sim N(u,\sigma^2)\quad Y=e^X\quad X=\ln Y$

求 $g(y)$ $\quad f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-u}{\sigma})^2}$

$X=\ln y$

$P[e^X\le y]=P[X\le\ln y]$

$=F_X[\ln y]$

$g(y)=\frac{1}{y}\cdot\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{\ln y-u}{\sigma})^2}$

$M(t)=E[e^{tX}].$

$E[Y]=E[e^X]\quad t=1$

$=e^{u+\frac{1}{2}\sigma^2}$

$E[Y^2]=E[e^{2X}]\quad t=2$

$=e^{2u+2\sigma^2}$

$\mathrm{Var}\,Y=e^{2u+2\sigma^2}-e^{2u+\sigma^2}$

$\sigma_X=8100,\ \sigma_Y=10000,\ \sigma(X+Y)=20000.$

$\sigma(X+Y)=\sigma_X+\sigma_Y+2\mathrm{Cov}(X,Y)$

$\mathrm{Cov}(X,Y)=\frac{1900}{2}$

$\mathrm{Var}[X+500+1.08Y]$

$=\sigma_X+2\times1.08\,\sigma_X\sigma_Y+(1.08)^2\sigma_Y$

$=21816.$

$X\sim b(100,0.1)$

Find $P(12\le X\le 14)$

Normal $u=10\quad \sigma^2=9 \qquad 0.2417$

$\Phi\left(\frac{14+0.5-10}{3}\right)-\Phi\left(\frac{12-0.5-10}{3}\right)$

binomial $\sum_{x=12}^{14}\binom{100}{x}0.1^x0.9^{100-x}=0.2244.$

$u=\frac{7}{2},\ \sigma^2=35/12$

max deviation from $u=\frac{5}{2}$

$k\sigma=\frac{5}{2}$

$k=\frac{5}{2}\times\sqrt{\frac{12}{35}}=\sqrt{\frac{15}{7}}$

$P(|X-3.5|<2.5)\ge1-\frac{1}{k^2}=\frac{8}{15}$

$n=15,\ u=80,\ \sigma^2=60$ 有 $\bar{X}$

$P(75<\bar{X}<80)$ lower. $\quad \mathrm{Var}=4$

$P(|\bar{X}-80|<5)=P(|\bar{X}-80|<\frac{5}{2}\cdot2)$

$\ge1-\frac{4}{25}=\frac{21}{25} \qquad k=\frac{5}{2} \quad \sigma_{\bar{X}}=2$

$Y\sim\chi^2(n).\quad W=(Y-n)/\sqrt{2n}\sim N(0,1).$

$Y=\sum_{i=1}^nX_i \qquad X_i\sim\chi^2(1).\ u_Y=n$

$\sigma^2_Y=2n$

$W=\frac{Y-n}{\sqrt{2n}}=\frac{Y-u_Y}{\sigma_Y}\sim N(0,1).$

$$p(X=x)=\begin{cases}\frac{1}{b-a+1} & x\in\{a\dots b\}\\ 0\end{cases}$$

$u=\sum_{x=a}^b\frac{x}{b-a+1}=\frac{a+b}{2} \qquad Y=X-a+1\quad n=b-a+1$

$\mathrm{Var}\,Y=\mathrm{Var}\,X \quad EY=\sum_{y=1}^n\frac{y}{n}=\frac{n+1}{2} \quad EY^2=\sum_{y=1}^n\frac{y^2}{n}=\frac{(n+1)(2n+1)}{6}$

$\Downarrow EY^2-E^2Y=\frac{n^2-1}{12}=\frac{(b-a+1)^2-1}{12}$

$M_X(t)=\sum_{x=a}^b\frac{e^{tx}}{b-a+1}=\frac{e^{ta}}{b-a+1}[1+e^t\dots e^{(b-a)t}]$

$$=\begin{cases}\frac{e^{at}(1-e^{(b-a+1)t})}{(b-a+1)(1-e^t)}, & t\ne0\\ 1, & t=0\end{cases}$$

$\binom{n}{k}p^k(1-p)^{n-k}\approx\frac{(np)^k}{k!}e^{-np}.$

$X_1\dots X_k$ cts R.V. 有 $f_1(x)\dots f_k(x)$.

$\sum_{i=1}^kc_i=1$. 证 $\sum_{i=1}^kc_if_i(x)$ 是 CTS PDF

$X\to$ PDF如上 求 u,σ^2

$\sum_{i=1}^kc_if_i(x)\ge0 \qquad \int_{-\infty}^\infty\sum_{i=1}^kc_if_i(x)dx=\sum_{i=1}^kc_i\int_{-\infty}^\infty f_i(x)dx=1$

$\int_{-\infty}^\infty xf_i(x)dx=u_i,\ \int_{-\infty}^\infty x^2f_i(x)dx=u_i^2+\sigma_i^2$

$\int_{-\infty}^\infty(x-u_i)^2f_i(x)dx=\sigma_i^2$

$EX=\int_{-\infty}^\infty x\sum_{i=1}^kc_if_i(x)dx=\sum_{i=1}^kc_i\int_{-\infty}^\infty xf_i(x)dx=\sum_{i=1}^kc_iu_i$

$EX^2=\int_{-\infty}^\infty x^2\sum_{i=1}^kc_if_i(x)dx=\sum_{i=1}^kc_i\int_{-\infty}^\infty x^2f_i(x)dx=\sum_{i=1}^kc_i(u_i^2+\sigma_i^2)$

$\mathrm{Var}\,X=\sum_{i=1}^kc_i(u_i^2+\sigma_i^2)-\left(\sum_{i=1}^kc_iu_i\right)^2$

$$X_k=\begin{cases}1, & Y_1+Y_2+Y_3=k\\ -1, & Y_1+Y_2+Y_3\ne k\end{cases} \qquad k=1,2 \qquad Y'=Y_1+Y_2+Y_3$$

$f(-1,-1)=P(Y'=0)+P(Y'=3)=(1-p)^3+p^3$

$f(-1,1)=P(Y'\ne1,\ Y'=2)=3p^2(1-p)$

$f(1,-1)=P(Y'=1,\ Y'\ne2)=3p(1-p)^2$

$f(1,1)=0.$

$$f_{X_2}(x)=\begin{cases}1-3p^2+3p^3, & x_2=-1\\ 3p^2(1-p), & x_2=1\end{cases}$$

$$f_{X_1}(x)=\begin{cases}1-3p+6p^2-3p^3 & x_1=-1\\ 3p(1-p)^2 & x_1=1\end{cases}$$

$E[X_1X_2] = 1-6p+6p^2 \quad p=\frac{1}{2}$ 有 min

$Cov(X_1-X_2, X_2) = Cov(X_1,X_2) - Cov(X_2,X_2)$

$= EX_1X_2 - EX_1EX_2 - Var X_2$

$= -12p^2 - 24p^3 + 144p^4 - 180p^5 + 72p^6$

$Var(X+Y)$ 3 项分布算 Var 变 2 项

$X+Y \sim b(30, p_X+p_Y)$

$= Var X + Var Y - 2Cov(X,Y)$

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

求 u, σ^2, $Y=aX+b$, 求 Y

$M(t) = e^{\frac{1}{2}t^2} \quad u=0$

$M'(t) = te^{\frac{1}{2}t^2} \quad \sigma^2=1$

$M''(t) = e^{\frac{1}{2}t^2} + t^2e^{\frac{1}{2}t^2}$

$E[e^{tY}] = E[e^{taX+tb}]$

$= e^{tb} \cdot e^{\frac{1}{2}(at)^2}$

$= e^{tb+\frac{1}{2}a^2t^2} \quad Y \sim N(b, a)$

$Var = 2\int_0^\infty x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

$z = x^2 \quad dz = 2x\,dx$

$Var = \int_0^\infty \frac{1}{\sqrt{2\pi}} \sqrt{z}\, e^{-\frac{z}{2}} dz$

$y = \frac{1}{2}z \quad dy = \frac{1}{2}dz$

$Var = \frac{1}{\sqrt{2\pi}} \int_0^\infty 2^{\frac{3}{2}} y^{\frac{1}{2}} e^{-y} dy$

$= \frac{2}{\sqrt{\pi}} \int_0^\infty \sqrt{y}\, e^{-y} dy = \frac{2}{\sqrt{\pi}}(-1)\{$

$\sqrt{y}\, e^{-y}|_0^\infty - \frac{1}{2}\int_0^\infty y^{-\frac{1}{2}} e^{-y} dy\}$

$= \frac{2}{\sqrt{\pi}} \frac{1}{2} \int_0^\infty y^{-\frac{1}{2}} e^{-y} dy = \frac{1}{\sqrt{\pi}} \Gamma(\frac{1}{2}) = 1$

$f(x,y) = \frac{1}{8}, \quad 0 \le y \le 4, \quad y \le x \le y+2$

$f_X(x) = \begin{cases} \int_0^x \frac{1}{8} dy = \frac{x}{8}, & 0 \le x \le 2 \\ \int_{x-2}^x \frac{1}{8} dy = \frac{1}{4}, & 2 < x < 4 \\ \int_{x-2}^4 \frac{1}{8} dy = \frac{6-x}{8}, & 4 \le x \le 6 \end{cases}$

$f_Y(y) = \int_y^{y+2} \frac{1}{8} dx = \frac{1}{4}, \quad 0 \le y \le 4$

X_1, X_2 i.i.d $\theta = 2 \quad Z = 2Y_1 + Y_2$

$Y_1 = \min(X_1, X_2) \quad Y_2 = \max(X_1, X_2)$

$F(x) = 1 - e^{-x/2} \quad f(x) = \frac{1}{2}e^{-x/2}$

$G_1(Y) = P(Y_1 \le y) = 1 - P(Y_1 > y)$

$= 1 - P(X_1 > y, X_2 > y) = 1 - [1-F(y)]^2$

$= 1 - e^{-y} \quad G_2(Y) = P(Y_2 \le y)$

$\frac{dG_1(Y)}{dy} = e^{-y} \quad = P(X_1 \le y, X_2 \le y)$

$= (1 - e^{-y/2})^2$

$EY_1 = \int_0^\infty e^{-y} \cdot y\, dy = 1$

$E[Z] = 5$

$\frac{dG_2(Y)}{dy} = e^{-\frac{y}{2}} - e^{-y}$

$EY_2 = \int_0^\infty y e^{-\frac{y}{2}} dy - \int_0^\infty ye^{-y} dy = 4 - 1 = 3$

Flip $n = 8$ fair coins

X_i = 第 i 次出正面 时所需的 i 次数

$Y = \max(X_1 \ldots X_8)$

$P(X_i = k) = (\frac{1}{2})^k$

$P(X_i \le k) = \sum_{n=1}^{k} (\frac{1}{2})^n$

$= 1 - \frac{1}{2^k}$

$P(Y \le y) = P(X_1 \le y) \ldots P(X_8 \le y)$

$= [1 - (\frac{1}{2})^y]^8 \quad P(Y=y) = P(Y \le y)$

$- P(Y \le y-1) = (1-(\frac{1}{2})^y)^8 - (1-(\frac{1}{2})^{y-1})^8$

$Y = X_1 + X_2 + \ldots X_n \quad X_n \sim Poisson(\lambda_n)$

$Y \sim Poisson(\lambda_1 + \ldots \lambda_n)$

$W = X_1 + \ldots X_n$ i.i.d 有 $\alpha = h, \theta$

最后 $M(t) = \frac{1}{(1-\theta t)^h}$ Gamma

$X_1, X_2 \ldots$ geometric $p = \frac{1}{3}$

$Y = X_1 + \ldots X_n$

$Y \sim$ 负二项 $(r = n, p = \frac{1}{3})$

$X_1, X_2 \ldots X_n$ 都 $\alpha = 7, \theta = 5$

$Y = X_1 + \ldots X_n \quad Y \sim Gamma(\alpha = 7n, \theta = 5)$

$Y = X_1 + X_2 \sim \chi^2(r) \quad X_1 \sim \chi^2(r_1)$

$\frac{1}{(1-2t)^{r/2}} = E[e^{tX_1}]E[e^{tX_2}]$

$= \frac{1}{(1-2t)^{r_1/2}} E[e^{tX_2}]$

$\Rightarrow \frac{1}{(1-2t)^{(r-r_1)/2}} \quad X_2 \sim \chi^2(r - r_1)$

$X \sim N(0,1) \quad Y = X^2 \sim \chi^2(1)$

$E[e^{tX^2}] = \int_{-\infty}^\infty e^{tx^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{tx^2 - \frac{x^2}{2}} dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-(x\sqrt{1-2t})^2/2} dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-w^2/2} \frac{1}{\sqrt{1-2t}} dw$

$= \frac{1}{\sqrt{1-2t}} \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw$

$= \frac{1}{\sqrt{1-2t}}$

$f(x) = \frac{x^2}{3}, \quad -1 \le x \le 2, \quad Y = X^2$

求 PDF of Y

$0 \le y \le 1 \quad P(Y < y) = P(-\sqrt{y} < X < \sqrt{y})$

$= F_X(t)|_{-\sqrt{y}}^{\sqrt{y}} \quad f_Y(y) = f_X(\sqrt{y}) \frac{1}{2\sqrt{y}}$

$- f_X(-\sqrt{y}) \frac{-1}{2\sqrt{y}} = \frac{\sqrt{y}}{3}$

$1 < y < 4 \quad f_Y(y) = \frac{\sqrt{y}}{6}$

$EY = \int_0^1 \frac{1}{3} y^{\frac{3}{2}} dy + \int_1^4 \frac{1}{6} y^{\frac{3}{2}} dy = \frac{11}{5}$

$EY^2 = \int_0^1 \frac{1}{3} y^{\frac{5}{2}} dy + \int_1^4 \frac{1}{6} y^{\frac{5}{2}} dy = \frac{43}{7}$

$Var Y = 1922/175$

$F_Y(y) = \begin{cases} \frac{2}{9} y^{\frac{3}{2}}, & 0 < y \le 1 \\ \ldots & 1 < y < 4 \end{cases}$

$F_Y^{-1}(x) = \begin{cases} (\frac{9}{2}x)^{2/3}, & 0 \le x \le \frac{2}{9} \\ (9x-1)^{2/3}, & \frac{2}{9} < x \le 1 \end{cases}$

$Y \sim b(n, \frac{1}{2}), \quad P(|Y/n - 0.5| < 0.08)$

$u_Y = 0.5n, \quad \sigma_Y^2 = 0.25n$

$P(|Y - 0.5n| < 0.08n) = P(|Y - u| < 0.08n)$

$= P(|Y - u| < \frac{4\sqrt{n}}{25} \cdot 0.5\sqrt{n}) \ge 1 - \frac{625}{16n}$

$Y \sim Poisson(\lambda = 3n)$ show

$W = (Y - 3n)/\sqrt{3n} \sim N(0,1)$

$u_Y = n \cdot 3 = 3n, \quad \sigma_Y^2 = n \cdot \sigma_{X_i}^2 = n \cdot 3 = 3n$

$W = \frac{Y - 3n}{\sqrt{3n}} = \frac{Y - u_Y}{\sigma_Y} \sim N(0,1)$

$X \sim N(1,1). \quad Z_n = 2 + \frac{1}{n}X$

$\lim_{n\to\infty} P(|Z_n - 2| \ge \varepsilon) = 0$

$EY_n = \frac{1}{n} \quad Var Y_n = \frac{1}{n^2}$

$|Y_n| = |Y_n - EY_n + EY_n|$

$\le |Y_n - EY_n| + |EY_n| = |Y_n - \frac{1}{n}| + \frac{1}{n}$

$0 \le P(|Z_n - 2| \ge \varepsilon) = P(|Y_n| \ge \varepsilon)$

$\le P(|Y_n - \frac{1}{n}| + \frac{1}{n} \ge \varepsilon)$

$= P(|Y_n - \frac{1}{n}| \ge \varepsilon - \frac{1}{n})$

$0 \le P(|Z_n - 2| \ge \varepsilon) \le \frac{1}{(n\varepsilon - 1)^2}$

RHS $\to 0$, ↓ goes $\to 0$

$X \sim U[-2, 1], \quad Y = X^2$

$F_X(x) = \frac{x+2}{3}, \quad -2 \le x \le 1$

$0 \le y \le 1 \Rightarrow -1 \le x \le 1$

$F_Y(y) = P(X \le \sqrt{y}) - P(X \le -\sqrt{y})$

$= 2\sqrt{y}/3$

$1 < y \le 4, \quad -2 \le x < -1$

$P(Y \le y) = P(Y < 1) + P(1 < Y \le 4)$

$= \frac{2}{3} + P(1 \le X^2 \le y) = \frac{2}{3} +$

$P(-\sqrt{y} \le X \le -1) = \frac{2}{3} + F_X|_{-\sqrt{y}}^{-1}$

$= \frac{1 + \sqrt{y}}{3}$

$= E[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i] = n\frac{1}{2}$

$P(|Y - EY| \ge k) \le \frac{Var Y}{k^2}$ 求 $P(|Y - \frac{n}{2}| < \varepsilon n)$.

$P(|Y - \frac{n}{2}| < \varepsilon n) \ge 1 - \frac{1}{4\varepsilon^2 n}$

$\varepsilon \ge 4\varepsilon^2 n$ 取 $n \ge \frac{1}{4\varepsilon^3}$

$X \sim U(-1, 3) \quad Y = X^2$ 求 PDF

$Y = X^2 \in [0, 9] \quad 0 < y < 1 \quad x = -\sqrt{y}$

$g(y) = \begin{cases} \frac{1}{4\sqrt{y}} & 0 < y < 1 \\ \frac{1}{8\sqrt{y}} & 1 \le y < 9 \end{cases}$

$nCr = \frac{n!}{r!(n-r)!}$

n 物品 size = r

$f(x) = \lambda e^{-\lambda x}, \quad Y = \max(a, X)$

$P(Y > y) = P(a > y) + P(X > y$

$\cap a \le y), \quad P(X > y) = e^{-\lambda y}$

$P(X \le y) = 1 - e^{-\lambda y}$

$P(Y > y) = \begin{cases} e^{-\lambda y} & y \ge a \\ 1, & y < a \end{cases}$

X, X' i.i.d.

$\min(X, X')$ 是 exp d.

$P(\min(X, X') > X)$

$= P(X > x, X' > x)$

$= e^{-\lambda x} \cdot e^{-\lambda x} = e^{-2\lambda x}$

$P(|\bar{X} - \lambda| \ge \frac{\lambda}{2}) \le \frac{Var(\bar{X})}{(\frac{\lambda}{2})^2}$

平方 $= \frac{Var(X)}{n(\frac{\lambda}{2})^2} = \frac{2}{n\lambda^2} \quad X_n \xrightarrow{d} X$

$/X \equiv X$

prove $X_n \xrightarrow{P} X$

$\lim_{n\to\infty} P(|X_n - X| \le \varepsilon) = \lim_{n\to\infty} P(X - \varepsilon \le X_n \le X + \varepsilon)$

$= \lim_{n\to\infty} F_{X_n}(X + \varepsilon) - F_{X_n}(X - \varepsilon) = F_X(X + \varepsilon) - F_X(X - \varepsilon)$

$= 1 \quad (X, Y)$ 双正态, $Z = X + Y$

$u_Z = \sigma X + \sigma Y, \quad \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y$

$Cov(X, Z) = Cov(X, X) + Cov(X, Y)$.

$= \sigma_X^2 + \rho\sigma_X\sigma_Y \quad \sigma_Z = \sqrt{\sigma_Z^2}$

$\rho_{XZ} = \frac{Cov(X, Z)}{\sigma_X \sigma_Z}$ ρ 相关系数 $X_1 \sim \lambda_1$

$Y = X_1 + X_2 \quad Y \sim \lambda_1 + \lambda_2 \quad X_2 \sim \lambda_2$

$\rho(X_1, Y) = \frac{Cov(X_1, Y)}{\sqrt{Var X_1 \cdot Var Y}} \to Cov(X_1, X_1) + Cov(X_1, Y)$

$Var Y = Var X_1 + Var X_2 = \lambda_1 + \lambda_2$

$Cov(X_1, Y) = \lambda_1 + \rho\sigma_{X_1}\sigma_{X_2} = \lambda_1 \quad \sqrt{\frac{\lambda_1}{\lambda_1 + \lambda_2}}$

inde $\Rightarrow \rho = 0 \quad \rho(X_1, Y) = \frac{\lambda_1}{\sqrt{(\lambda_1 + \lambda_2)\lambda_1}}$

$P(X_1 = k_1, X_2 = k_2 | Y = m)$

$= \frac{P(X_1 = k_1)P(X_2 = k_2)}{P(Y = m)} = \frac{\lambda_1^{k_1}\lambda_2^{k_2} e^{-(\lambda_1 + \lambda_2)}}{k_1!\, k_2!}$

$\frac{m!}{(\lambda_1 + \lambda_2)^m e^{-(\lambda_1 + \lambda_2)}}$ 长度为 n 的

二元序列 $X_1 \ldots X_n$

$Y = \sum_{i=1}^{n} X_i \quad X_i = 1. \; EY$

$Var Y = n(E[X_i^2] - E[X_i]^2)$

$= n \cdot (\frac{1}{2} - \frac{1}{4}) = \frac{n}{4}$

$P(|Y - \frac{n}{2}| \ge \varepsilon n)$

$n \to \infty \quad \le \frac{Var Y}{(\varepsilon n)^2}$

$1 - \frac{1}{4\varepsilon^2 n} \ge 1 - \varepsilon$

$f_X = \frac{1}{4} \quad -1 \le x \le 3$

$dx/dy = \frac{1}{-2\sqrt{y}} \quad x = \sqrt{y} \quad dx/dy = \frac{1}{2\sqrt{y}}$

$P(|X - u| < \varepsilon) \ge 1 - \frac{\sigma^2}{\varepsilon^2}$

$g(y) = \frac{1}{\Gamma(\alpha)\theta^\alpha} \cdot \frac{(\ln y)^{\alpha - 1}}{y^{1 + \frac{1}{\theta}}} \quad u = (1-\theta)^{-\alpha}$

$\sigma^2 = (1 - 2\theta)^{-\alpha} - (1 - \theta)^{-2\alpha}$