

STA2001 Tutorial 8

1. 4.1-6. The torque required to remove bolts in a steel plate is rated as very high, high, average, and low, and these occur about 25%, 35%, 20%, and 20% of the time, respectively. Suppose $n = 31$ bolts are rated; what is the probability of rating 9 very high, 10 high, 7 average, and 5 low? Assume independence of the 31 trials.

2. 4.2-7 Let the joint pmf of X and Y be

$$f(x, y) = 1/4$$

where $(x, y) \in S = \{(0, 0), (1, 1), (1, -1), (2, 0)\}$.

(a) Are X and Y independent?

(b) Calculate $\text{cov}(X, Y)$ and ρ .

This exercise also illustrates the fact that dependent random variables can have a correlation coefficient of zero.

3. 4.2-8. A certain raw material is classified as to moisture content X (in percent) and impurity Y (in percent). Let X and Y have the joint pmf given by

| $y \backslash x$ | 1 | 2 | 3 | 4 |
|------------------|------|------|------|------|
| 1 | 0.05 | 0.05 | 0.15 | 0.1 |
| 2 | 0.1 | 0.2 | 0.3 | 0.05 |

- (a) Find the marginal pmfs, the means, and the variances of X and Y , respectively.
- (b) Find the covariance and the correlation coefficient of X and Y .
- (c) If additional heating is needed with high moisture content and additional filtering with high impurity such that the additional cost is given by the function $C = 2X + 10Y^2$ in dollars, find $E(C)$.