Introduction to AI: A brief introduction to Logic

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Outline

Who I am

- Professor of Computer Science, University of Saint-Etienne
- I work at the Hubert Curien lab, team Data Intelligence
- My research domains:
  - Machine Learning/Data Mining (applied to NLP)
  - Deep Learning for fraud and anomaly detection
  - Inductive Logic Programming (ML+Logic)
- How to reach me?
  - by mail: `Francois.Jacquenet@univ-st-etienne.fr`
  - at the Hubert Curien Laboratory (but difficult to enter)
  - during (and after) the lectures

Textbooks and Courses (for this part on Logic)

- *Artificial Intelligence, a modern approach*, Stuart J. Russel and Peter Norvig, 3rd edition, Pearson 2015
- *Introduction to Logic*, 14th edition, Irving M. Copi, Carl Cohen, Kenneth McMahon, Routledge 2010
- *An introduction to formal logic*, Peter Smith, Cambridge University Press, 2003
- Course on Introduction to Logic from Stanford University
- You may find a huge number of courses and videos on Logic on the Internet

Outline of the course

- Reasoning in logic
  - Propositional logic
  - First order logic
- Prolog Programming
- Constraint Logic Programming (maybe)
- Practical sessions in Prolog (and perhaps CLP)
- Reminder: Emilie Morvant teaches AI problems and search algorithms (13h)

Evaluation

- Three exams
  - Logic (33.33%)
  - AI problems and search algos (33.33%)
  - Prolog (33.33%)
- No project this year
- Take care: no resit!

Form of the course

- For some courses I will use slides
- For some courses I will write on the blackboard
- For some courses you will use your own laptop (I will tell you when)

What is Logic?

- "Logic is the study of the methods and principles used to distinguish correct from incorrect reasoning." (Introduction to Logic, I. M. Copi, C. Cohen, K. McMahon)
- Logic is the science of what follows from what.
- Valid and invalid inference
  - Socrates is human. All humans are mortal. Therefore, Socrates is mortal.
  - Socrates is human. All Greeks are human. Therefore, Socrates is Greek.
- Logic investigates what makes the first inference valid and the second invalid.

What is Formal Logic?

- Studies logical properties of formal languages (not English)
  - Logical consequence (what follows from what?)
  - Logical consistency (when do sentences contradict one another?)
- Expressive power (what can be expressed in a given formal language, and how?)
- Formal models (mathematical structures described by formal language)
- Inference and proof systems (how can it be proved that something follows from something else?)

Modeling possible worlds

Suppose we want to model the relationships that exist between members of a chess team.

- The members are Abby, Bess, Cody and Dana
- We can draw an array and fill it with T or F
- An example:

	Abby	Bess	Cody	Dana
Abby	F	F	T	F
Bess	F	F	T	F
Cody	T	T	F	T
Dana	F	F	T	F

Modeling possible worlds

	Abby	Bess	Cody	Dana
Abby	F	F	T	F
Bess	F	F	T	F
Cody	T	T	F	T
Dana	F	F	T	F

What do we model?

- Abby likes Cody but doesn't like the other girls (even herself)
- Bess likes Cody but doesn't like the other girls(even herself)
- Cody likes everyone but herself
- Dana also likes the popular Cody but doesn't like the other girls(even herself)

Modeling possible worlds

Another world could be:

	Abby	Bess	Cody	Dana
Abby	T	F	T	F
Bess	F	T	F	T
Cody	T	F	T	F
Dana	F	T	F	T

In this world, every girl likes exactly two other girls, and every girl is liked by just two girls.

Modeling possible worlds

	Abby	Bess	Cody	Dana
Abby				
Bess				
Cody				
Dana				

- 4 girls
- 16 possible instances of the likes relationship
- 2^{16} (65,536) possible worlds

Modeling possible worlds

- Suppose we do not know the likes and dislikes of the girls
- Suppose we have informants who are willing to tell us about them
- Suppose each informant knows a little about the likes and dislikes of the girls, but no one knows everything.

Here is where Logic comes in!

- By writing logical sentences, each informant can express exactly what he or she knows - no more, no less
- For our part, we can use the sentences we have been told to draw conclusions that are logically entailed by those sentences
- And we can use logical proofs to explain our conclusions to others

Logical Sentences

Suppose we get the information:

1. Dana likes Cody.
2. Abby does not like Dana.
3. Dana does not like Abby.
4. Bess likes Cody or Dana.
5. Abby likes everyone that Bess likes.
6. Cody likes everyone who likes her.
7. Nobody likes herself

We can deduce some interesting facts

Logical entailment

Dana likes Cody

	Abby	Bess	Cody	Dana
Abby	?	?	?	?
Bess	?	?	?	?
Cody	?	?	?	?
Dana	?	?	T	?

Each sentence places constraints on the possible world

We can't say anything else about other cells.

Logical entailment

Abby does not like Dana

	Abby	Bess	Cody	Dana
Abby	?	?	?	**F**
Bess	?	?	?	?
Cody	?	?	?	?
Dana	?	?	?	?

We can't say anything else about other cells.

Logical entailment

Dana likes Cody
Abby does not like Dana

	Abby	Bess	Cody	Dana
Abby	?	?	?	F
Bess	?	?	?	?
Cody	?	?	?	?
Dana	?	?	T	?

We can't say anything else about other cells.

Logical entailment

Dana likes Cody
Abby does not like Dana
Dana does not like Abby

	Abby	Bess	Cody	Dana
Abby	?	?	?	F
Bess	?	?	?	?
Cody	?	?	?	?
Dana	F	?	T	?

We can't say anything else about other cells.

Logical entailment

Bess likes Cody or Dana

	A	B	C	D
A	?	?	?	?
B	?	?	T	F
C	?	?	?	?
D	?	?	?	?

	A	B	C	D
A	?	?	?	?
B	?	?	F	T
C	?	?	?	?
D	?	?	?	?

Logical entailment

Dana likes Cody
Abby does not like Dana
Dana does not like Abby
Bess likes Cody or Dana

	A	B	C	D
A	?	?	?	F
B	?	?	T	F
C	?	?	?	?
D	F	?	T	?

	A	B	C	D
A	?	?	?	F
B	?	?	F	T
C	?	?	?	?
D	F	?	T	?

Logical entailment

Dana likes Cody
Abby does not like Dana
Dana does not like Abby
Bess likes Cody or Dana
Abby likes everyone that Bess likes

	A	B	C	D
A	?	?	T	F
B	?	?	T	F
C	?	?	?	?
D	F	?	T	?

	A	B	C	D
A	?	?	F	F/T
B	?	?	F	T
C	?	?	?	?
D	F	?	T	?

Logical entailment

Dana likes Cody
Abby does not like Dana
Dana does not like Abby
Bess likes Cody or Dana
Abby likes everyone that Bess likes
Cody likes everyone who likes her

	A	B	C	D
A	?	?	T	F
B	?	?	T	F
C	T	T	?	T
D	F	?	T	?

Logical entailment

Dana likes Cody
Abby does not like Dana
Dana does not like Abby
Bess likes Cody or Dana
Abby likes everyone that Bess likes
Cody likes everyone who likes her
Nobody likes herself

	A	B	C	D
A	F	?	T	F
B	?	F	T	F
C	T	T	F	T
D	F	?	T	F

Finally: B doesn't like A because otherwise there would be a contradiction with the fact that Abby likes everyone that Bess likes

Logical entailment

Dana likes Cody
Abby does not like Dana
Dana does not like Abby
Bess likes Cody or Dana
Abby likes everyone that Bess likes
Cody likes everyone who likes her
Nobody likes herself

	A	B	C	D
A	F	?	T	F
B	F	F	T	F
C	T	T	F	T
D	F	?	T	F

→ the sentences define 4 possible worlds

Logical entailment

We can observe that some sentences are true in every world that satisfies the 7 given sentences.

	A	B	C	D
A	F	?	T	F
B	F	F	T	F
C	T	T	F	T
D	F	?	T	F

Example: *Bess likes Cody*

A sentence of this sort is said to be a logical conclusion from the given sentences.

A set of sentences logically entails a conclusion if and only if every world that satisfies the sentences also satisfies the conclusion.

Logical entailment

What other sentences are entailed by the 7 given sentences?
(What other sentences are logical conclusions from the 7 given sentences?)

	A	B	C	D
A	F	?	T	F
B	F	F	T	F
C	T	T	F	T
D	F	?	T	F

Bess does not like Dana
Everybody likes somebody
Everybody is liked by somebody

Logical Proofs

Determining logical entailment by checking all possible worlds is impractical in general $\rightarrow$ logical reasoning

What is it?

- application of reasoning rules to derive logical conclusions and produce logical proofs
- sequences of reasoning steps that leads from premises to conclusions

Logical Proofs

- Aristotle (-384, -322) was the first to recognize that what makes a step of a proof immediately obvious is its form rather than its content
- It doesn't matter whether you are talking about birds or houses or cars, etc
- What matters is the **structure of the facts** with which you are working
- Such patterns are called rules of inference

Logical Proofs

Consider the reasoning steps below:

- **All** Model S cars **are** Teslas
- **All** Teslas **are** American
- **Therefore**, **all** Model S cars **are** American

Consider other reasoning steps:

- **All** Pokemon **are** Animals
- **All** Animals **are** Pretty
- **Therefore**, **all** Pokemon **are** Pretty

Logical Proofs

Those two reasoning steps have the same structure:

- All X are Y
- All Y are Z
- Therefore, all X are Z

This is an inference rule

We may wonder:

- Which inference rules are correct?
- Are there many such inference rules or just a few?

Which inference rules are correct?

Consider the following (faulty) inference rule:

- All X are Y
- Some Y are Z
- Therefore, some X are Z

Now consider an instance of this inference rule:

- **All** Citroëns **are** cars
- **Some** cars **are** made in France
- **Therefore**, **some** Citroëns **are** made in France

The conclusion happens to be **correct**

Which inference rules are correct?

Consider the following (faulty) inference rule:

- All X are Y
- Some Y are Z
- Therefore, some X are Z

Now consider another instance of this inference rule:

- **All** Citroëns **are** cars
- **Some** cars **are** Ferraris
- **Therefore**, **some** Citroëns **are** Ferraris

Here the conclusion happens to be **incorrect**

Which inference rules are correct?

What distinguishes a correct inference rule from one that is incorrect is that it must **ALWAYS** lead to correct conclusions

The conclusions must be correct so long as the premises on which they are based are correct

This is the defining criterion for what we call deduction

Which inference rules are correct?

There are inference rules that are sometimes useful but do not satisfy this strict criterion

Examples are: inductive reasoning, abductive reasoning, reasoning by analogy, and so forth.

Inductive reasoning

Induction is reasoning from the particular to the general

If we see enough cases in which something is true and we never see a case in which it is false, we ***tend*** to conclude that it is always true.

This is the main reasoning principle in Machine Learning

Example:

- I have seen 1,000 black ravens
- I have never seen a raven that is not black
- Therefore, every raven is black

Now try white swans...

Abductive reasoning

Abduction is reasoning from effects to possible causes

Many things can cause an observed result. We often ***tend*** to infer a cause even when our enumeration of possible causes is incomplete.

Example:

- If there is no fuel, the car will not start.
- If there is no spark, the car will not start.
- There is spark.
- The car does not start.
- Therefore, there is no fuel.

What if the car is in a vacuum chamber?...

Reasoning by analogy

Reasoning by analogy is reasoning in which we infer a conclusion based on similarity of two situations

Example:

- The flow in a pipe is proportional to its diameter.
- Wires are like pipes.
- Therefore, the current in a wire is proportional to diameter.

But:

- A light bulb is hot when turned on and cold when turned off.
- The sun is like a bulb.
- Therefore, if the sun is not shinning, it is turned off and then it is cold.

Why studying deduction?

Of all types of reasoning, deduction is the only one that guarantees its conclusions in all cases, it produces only those conclusions that are logically entailed by one's premises

$\rightarrow$ we will concentrate entirely on deduction during this course

Formalization

Natural language is not a good candidate for reasoning

- NL sentences can be too complex
- NL sentences can be ambigous

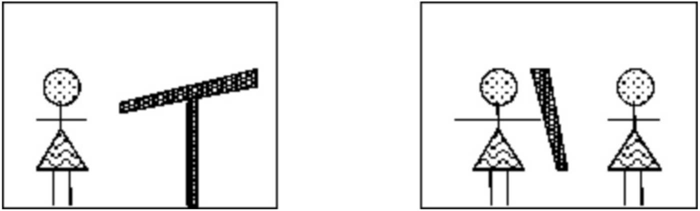
Formalization

Example of complexity (University of Michigan lease agreement):
The University may terminate this lease when the Lessee, having made application and executed this lease in advance of enrollment, is not eligible to enroll or fails to enroll in the University or leaves the University at any time prior to the expiration of this lease, or for violation of any provisions of this lease, or for violation of any University regulation relative to resident Halls, or for health reasons, by providing the student with written notice of this termination 30 days prior to the effective date of termination, unless life, limb, or property would be jeopardized, the Lessee engages in the sales of purchase of controlled substances in violation of federal, state or local law, or the Lessee is no longer enrolled as a student, or the Lessee engages in the use or possession of firearms, explosives, inflammable liquids, fireworks, or other dangerous weapons within the building, or turns in a false alarm, in which cases a maximum of 24 hours notice would be sufficient.

Formalization

Example of ambiguity

There's a girl in the room with a telescope



Formalization

Problem with reasoning in natural language

- Champagne is better than beer
- Beer is better than soda
- Therefore, champagne is better than soda
- $\rightarrow$ OK

- Bad food is better than nothing
- Nothing is better than good food
- Therefore, bad food is better than good food
- $\rightarrow$ NOT OK

$\Longrightarrow$ Use a formal language for encoding information

Formalization

In fact there are similarities between formal logic and, for example, algebra

Suppose we want to solve the following problem: *John is three times as old as Mary. John's age and Mary's age add up to twelve. How old are John and Mary?*

First step: Express the information in the form of equations. If we let x represent the age of Xavier and y represent the age of Yolanda, we can capture the essential information of the problem as:

$$x - 3y = 0$$
$$x + y = 12$$

Formalization

Second step: Manipulate the expressions to solve the problem

First we subtract the second equation from the first.

$$\begin{array}{r} x - 3y = 0 \\ x + y = 12 \\ \hline -4y = -12 \end{array}$$

Next, we divide each side of the resulting equation by -4 to get a value for y

Then substituting back into one of the preceding equations, we get a value for x

We get $x = 9$ and $y = 3$

Formalization

Now consider another problem: *If Mary loves Pat, then Mary loves Quincy. If it is Monday and raining, then Mary loves Pat or Quincy. If it is Monday and raining, does Mary love Quincy?*
As in the algebra problem, the first step is formalization

- Let p represent the possibility that Mary loves Pat
- let q represent the possibility that Mary loves Quincy
- let m represent the possibility that it is Monday
- let r represent the possibility that it is raining.

We can then represent the information of this problem with the following logical sentences:

- The first says that p implies q ($p \Rightarrow q$)
- The second says that m and r implies p or q ($m \wedge r \Rightarrow p \vee q$)

Formalization

As with Algebra, Formal Logic defines certain operations that we can use to manipulate expressions.

For example, consider a variant of what is called Propositional Resolution (the expressions above the line are the premises of the rule, and the expression below is the conclusion)

$$\frac{\begin{array}{c} p_1 \wedge \cdots \wedge p_k \Rightarrow q_1 \vee \cdots \vee q_l \\ r_1 \wedge \cdots \wedge r_m \Rightarrow s_1 \vee \cdots \vee s_n \end{array}}{p_1 \wedge \cdots \wedge p_k \wedge r_1 \wedge \cdots \wedge r_m \Rightarrow q_1 \vee \cdots \vee q_l \vee s_1 \vee \cdots \vee s_n}$$

Formalization

Applying this rule to our problem we get:

$$\frac{\begin{array}{c}(p \Rightarrow q) \\ (m \wedge r \Rightarrow p \vee q)\end{array}}{m \wedge r \wedge p \Rightarrow q \vee p \vee q}$$

Which may be simplified to: $m \wedge r \Rightarrow q \vee q$

and then to: $m \wedge r \Rightarrow q$

Which means: *If it is Monday and it is raining, then Mary loves Quincy*

Formalization

Conclusion:

This example showcases the formal language for encoding logical information

As with algebra, we use symbols to represent relevant aspects of the world in question, and we use operators to connect these symbols in order to express information about the things those symbols represent

Resolution has the property of being complete for an important class of logic problems (it is the only operation necessary to solve any problem in the class)

What is Logic Good For? (Philosophy)

- (Informal) logic originates in philosophy (Aristotle)
- Valid inference cornerstone of philosophical research
- Logic itself a subdiscipline of philosophy
- Formal tools of logic useful to make intuitive philosophical notions precise
  - Possibility and necessity
  - Time
  - Moral obligation and permissibility
  - Belief and knowledge

What is Logic Good For? (Mathematics)

- Formal logic developed in the quest for foundations of mathematics (19th C.)
- Logical systems provide precise foundational framework for mathematics
    - Axiomatic systems (e.g, geometry)
    - Algebraic structures (e.g., groups)
    - Set theory (e.g, Zermelo-Fraenkel with Choice)
- Formal methods make mathematics more precise
    - Formal language can make mathematical claims more precise
    - Formal structures can point to alternatives, unveil gaps in proofs
    - Formal proof systems make proofs rigorous
    - Formal proofs make mechanical proof checking and proof search possible
- Logical tools can be applied to mathematical problems

What is Logic Good For? (Computer Science)

- Combinational logic circuits
- Database query languages
- Logic programming
- Knowledge representation
- Automated reasoning
- Formal specification and verification (of programs, of hardware designs)
- Theoretical computer science (theory of computational complexity, semantics of programming languages)

Exercise 1

Consider the state of the Chess Team World depicted below.

	Abby	Bess	Cody	Dana
Abby	F	T	T	F
Bess	F	F	T	F
Cody	T	T	F	T
Dana	F	T	T	F

For each of the following sentences, say whether or not it is true in this state of the world.

1. Abby likes Dana.
2. Dana does not like Abby.
3. Abby likes Cody or Dana.
4. Abby likes someone who likes her.
5. Somebody likes everybody.

Exercise 2

Build a table of likes and dislikes for the Chess Team World that makes *all* of the following sentences true (there is more than one such table)

1. Dana likes Cody.
2. Abby does not like Dana.
3. Bess likes Cody or Dana.
4. Abby likes everyone whom Bess likes.
5. Cody likes everyone who likes her.
6. Nobody likes herself.

Exercise 3

Consider a set of Chess Team World premises that are true in the four states of Sorority World shown in Figure bellow.

	A	B	C	D
A	F	?	T	F
B	F	F	T	F
C	T	T	F	T
D	F	?	T	F

For each of the following sentences, say whether or not it is logically entailed by these premises.

1. Abby likes Bess or Bess likes Abby.
2. Somebody likes herself.
3. Everybody likes somebody.

Exercise 4

Say whether or not the following inference rules are logically correct.

1. All X are Z. All Y are Z. Therefore, some X are Y.
2. Some X are Y. All Y are Z. Therefore, some X are Z.
3. All X are Y. Some Y are Z. Therefore, some X are Z.