Robotics

Estimation and Learning
with Dan Lee

Week 2.
Kalman Filter

2.2 System and Measurement Models

Penn Engineering

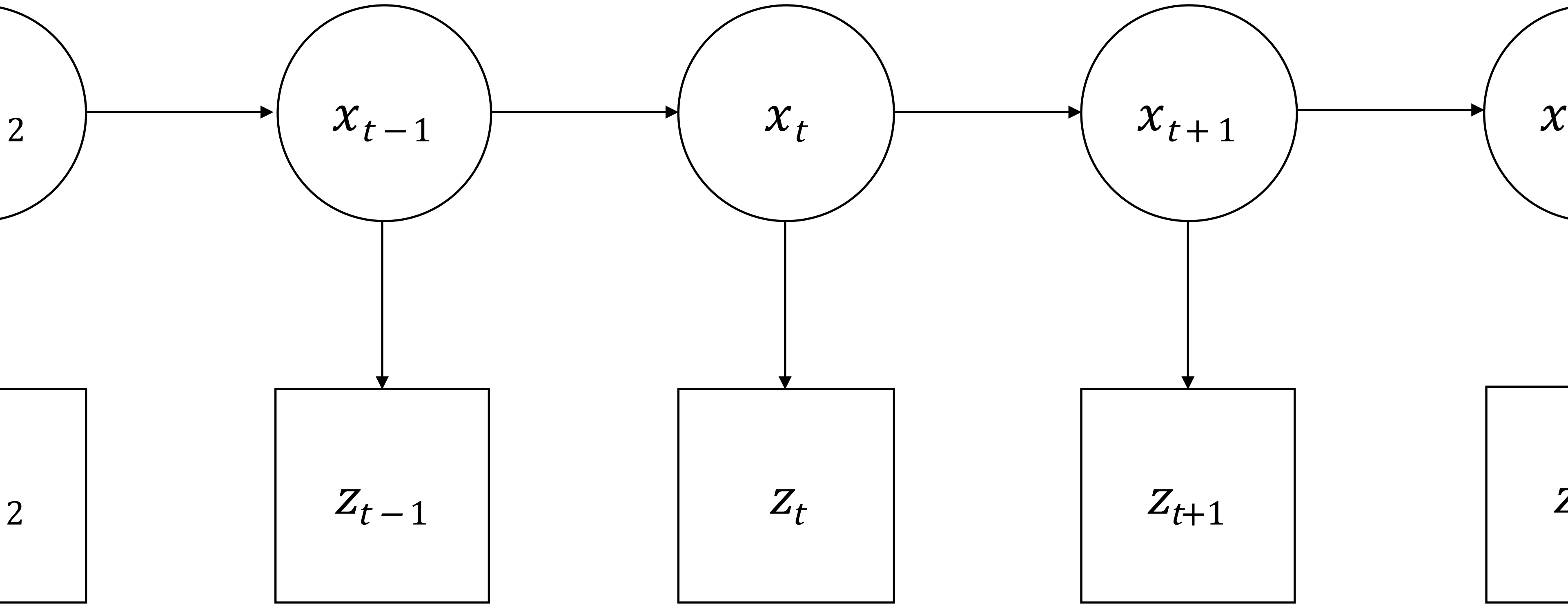
Bayesian Kalman Filtering

- Modeling motion and noise
- Mathematical underpinnings of Kalman filters
- Position tracking example

Linear Modeling

- Discrete Linear dynamical system of motion
  - $x_{t+1} = A\, x_t + B\, u_t$ $\qquad$ $z_t = C\, x_t$
- Simple state vector, x, is position and velocity
  - $x_{t+1} := [\, v \quad {}^{dv}/_{dt} \,]$
- Description of Dynamics
  - $A = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix}$

KF: Bayesian filtering



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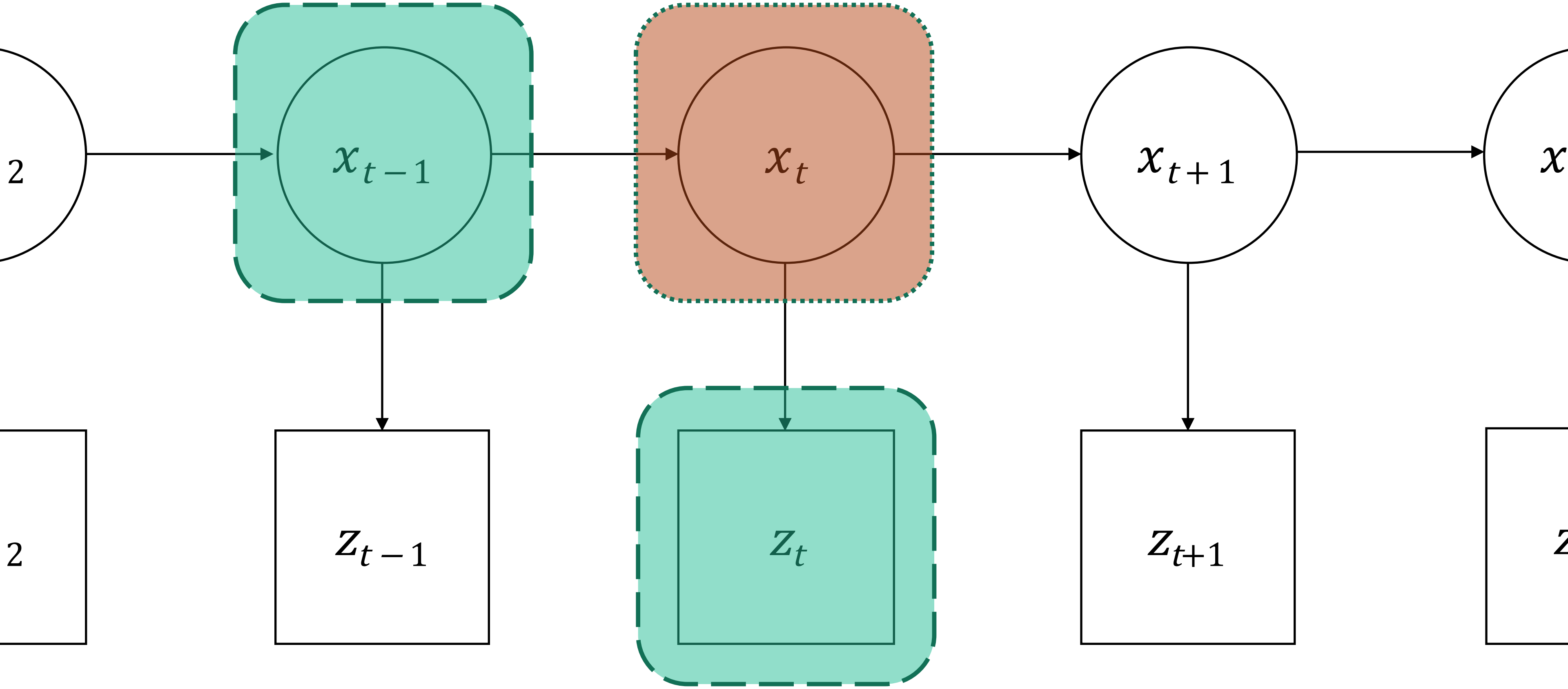

- Find x_t
- Know x_{t-1}
- Know z_t

Bayesian modeling

- Prediction using state dynamics model

$$p(x_{t+1}|x_t)$$

- Inference from noisy measurements

$$p(z_t|x_t)$$

- Model $\mathrm{x_t}$ with a Gaussian (mean and covariance)

$$p(x_t) = \mathcal{N}(x_t, P_t)$$

Bayesian filtering

- Apply linear dynamics

$$p(x_{t+1}|x_t) = Ap(x_t)$$

$$p(z_t|x_t) = Cp(x_t)$$

- Add noise for motion and observations

$$p(x_{t+1}|x_t) = Ap(x_t) + v_m$$

$$p(z_t|x_t) = Cp(x_t) + v_o$$

- Introduce Gaussian model of x_t

$$p(x_{t+1}|x_t) = A\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_m)$$

$$p(z_t|x_t) = C\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_o)$$

Bayesian filtering

- Consolidate expression using special properties

$$p(x_{t+1}|x_t) = A\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_m)$$

$$p(z_t|x_t) = C\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_o)$$

- Apply *linear* transform to Gaussian distributions

$$p(x_{t+1}|x_t) = \mathcal{N}(Ax_t, AP_tA^T) + \mathcal{N}(0, \Sigma_m)$$

$$p(z_t|x_t) = \mathcal{N}(Cx_t, CP_tC^T) + \mathcal{N}(0, \Sigma_o)$$

- Apply summation

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