Corvinus University of Budapest

Flood Protection –
A Water Level Analysis of Hungarian Rivers

Extreme Value Theory

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Project Description:

Climate change poses a fundamental challenge to critical infrastructure planning, particularly in hydraulic engineering where "stationarity", the assumption that past environmental patterns are reliable predictors of the future, is no longer guaranteed. This report applies Computational Statistics methods to address this challenge, using a case study of Hungary's two principal rivers, the Danube (Duna) and the Tisza.

The core research problem is to assess the stability of extreme hydrological events over the last two decades. Using daily water level records from 2002 to 2024, the project aims to determine whether the statistical probability of rare events, specifically floods and droughts with a 5000-day return period, has undergone a significant regime shift. This requires moving beyond simple descriptive statistics to apply Extreme Value Theory (EVT), utilizing both Block Maxima (GEV) and Peaks-Over-Threshold (GPD) models to extrapolate tail behaviour from limited historical data.

The findings are presented in two parts to address the distinct needs of different stakeholders:

Part I (Policy Brief): A summary for the Home Office of Hungary, translating statistical signals into actionable advice on flood defence resource allocation.

Part II (Technical Report): A detailed statistical analysis for peer review, documenting the modelling assumptions, parameter estimation (MLE), and hypothesis tests used to validate the presence or absence of climate trends.

To: Minister of Interior
From: Data Analytics Team
Subject: Policy Recommendations based on Water Level Analysis (2002-2024)

1. Summary of the Water Level Analysis for the Home Office of Hungary

We compared daily water-level records for 2002-2013 and 2014-2024 on the Tisza (at Szeged) and the Danube (at Esztergom) to assess whether recent climate conditions are associated with changes in extreme high and low water levels that matter for flood defences and low-water management. We summarised annual maxima and minima and modelled rare extremes using standard extreme-value approaches (block maxima/minima and peaks-over-threshold). The aim was not to explain every driver, but to test whether the size of observed shifts is strong enough to be unlikely to reflect random year-to-year variability.

Tisza (Szeged): Flood extremes are clearly lower in 2014-2024 than in 2002-2013. In our two big rivers, floods usually appear either at spring or autumn/early summer. Spring floods are often linked to snowmelt in the mountains of the catchment area (particularly the Eastern Carpathians), but such conditions have become less frequent in recent years. Autumn floods occur due to long rainfalls, the whole catchment area is important, not just the mountains. These events are less predictable; therefore, preparedness and immediate reaction is vital. Typical annual flood peaks fell by more than 0,5 metre (from about 5.9 m to about 5.2 m). Taken together, the data do not support rising flood heights over the past decade at this site. Resource priorities should therefore focus on maintaining reliable flood protection - through maintenance and targeted reinforcement of weak sections - rather than widespread dam heightening. Low-water extremes on the Tisza also show no clear deterioration, with little change in annual minima. Accordingly, the analysis does not justify costly new structural measures for low flows; instead, no-regret actions include improved monitoring, operational drought protocols, and coordinated water use during low-flow periods.

Danube (Esztergom): Changes are smaller and more uncertain. Snowmelt floods are predictable due to the geographic distance from the Alps and also less and less dangerous. Both annual flood peaks and annual minima appear slightly lower in 2014-2024, and threshold-based modelling points in the same direction. However, the evidence is modest and not strong enough to claim a clear trend from these data alone. Autumn floods are still dangerous, heavy rainfalls (as in 2024) can occur. For allocation, the cautious approach is to maintain current flood-protection standards, invest in monitoring, forecasting and systematic inspections. In addition, major dam raising or large structural expansions should be treated as broader policy and cost-benefit decisions, rather than responses compelled by detected increases in extremes. For low flows, the emphasis should be on operational measures rather than new hard infrastructure, unless supported by further studies.

Overall reliability of the report is moderate. Each period is only about 11 years, which limits the power to detect climate-change signals in rivers with high natural variability, and gauge-level changes cannot be cleanly attributed to climate change alone because upstream regulation and human interventions can also affect levels. While the analysis characterises recent extremes, historical water-level data alone cannot reliably anticipate future trends driven by ongoing climate change. The results are therefore most useful for near-term prioritisation of maintenance, monitoring, and targeted upgrades, and should be complemented by broader hydrological and climate analyses for long-term investment decisions.

2. Summary of the Water Level Analysis for the Peer Statisticians

2.1 Introduction

The assessment of flood risk and drought severity is critical for civil engineering and national water management. This study aims to quantify changes in the tail behaviour of daily water levels recorded at Esztergom (Danube) and Szeged (Tisza) over a 22-year horizon (2002-2024). The primary estimands are the return levels corresponding to a return period of 5000 days (approximately 13.7 years), which serve as proxies for design dam heights. The analysis specifically addresses the stationarity of these extremes. Given the potential impacts of climate change and catchment management, we test the hypothesis that the statistical distribution of river extremes, specifically the location (μ), scale (σ), and shape (ξ) parameters, has shifted between the early observation period (2002–2013) and the late observation period (2014–2024).

2.2 Data and methodology

The dataset consists of daily water level measurements for both rivers. The daily water level records for both rivers were obtained from the National Water Level Monitoring Service (Országos Vízjelző Szolgálat) via the Hydroinfo database. Prior to analysis, the data underwent quality control procedures. Missing values, which were sparse, were imputed using a local moving average of the nearest four temporal neighbours to preserve short-term autocorrelation structures without artificially inflating variance. To analyse drought risks (minima) within the standard Extreme Value Theory framework, the data were transformed by negating the water levels ($Y_t = -X_t$). This transformation allows theorems derived for maxima to be applied directly to the lower tail of the distribution.

The statistical framework relied on two complementary approaches within Extreme Value Theory: the Block Maxima method and the Peaks-Over-Threshold (POT) method.

Block Maxima and the Generalized Extreme Value (GEV) Distribution

The first approach partitioned the time series into annual blocks, extracting the maximum (or transformed minimum) value for each year. Under the Fisher-Tippett-Gnedenko theorem, the distribution of these block maxima converges asymptotically to the Generalized Extreme Value (GEV) distribution. This method provides a robust, albeit data-sparse, estimation of tail behaviour. The GEV parameters were estimated using Maximum Likelihood Estimation (MLE). To assess changes over time, we fitted non-stationary GEV models where the location and scale parameters were modelled as functions of the categorical time period. Likelihood Ratio (LR) tests were then employed to determine if the inclusion of period-specific parameters significantly improved model fit compared to a stationary null model.

Peaks-Over-Threshold (POT) and the Generalized Pareto Distribution (GPD)

To maximize data efficiency, particularly given the relatively short 11-year observation windows, we also employed the Peaks-Over-Threshold method. This approach utilizes all independent observations exceeding a sufficiently high threshold (u). Under the Pickands-Balkema-de Haan theorem, the excesses over this threshold follow a Generalized Pareto Distribution (GPD). Threshold selection was conducted using Mean Residual Life (MRL) plots and parameter stability plots. For comparative consistency across periods, quantile-based thresholds (95th percentile) were generally adopted. Changes in the GPD parameters between periods were assessed using Wald tests derived from the parameter covariance matrices.

2.3 Statistical Analysis of the Tisza River

The analysis of the Tisza river at Szeged provided strong statistical evidence of a regime shift in flood behaviour. In the period 2002–2013, the river exhibited high volatility and extreme peak levels (see in Appendix, Figure 1). However, in the subsequent period (2014–2024), both the magnitude and the variability of these peaks declined (see in Appendix, Figure 2).

The application of the Likelihood Ratio test to the GEV models confirmed that this change was statistically significant at the 5% level ($p \approx 0.017$). The null hypothesis of a stationary distribution across the full 22-year history was rejected in favor of a model allowing for period-specific location and scale parameters. This finding was corroborated by the POT analysis, where Wald tests indicated a highly significant reduction in the GPD scale parameter ($p < 0.001$) and a shift in the shape parameter toward a lighter tail.

This structural break is clearly reflected in the derived design values. Based on the GEV model, the estimated 5000-day return level for the Tisza dropped from approximately 920 cm in the first decade to approximately 612 cm in the second. The POT model showed an even more dramatic shift, estimating a drop from 1068 cm to 613 cm. This reduction implies that the „safe" dam height required to withstand a 1-in-13.7-year flood event is substantially lower today than it was in the early 2000s. Crucially, the confidence intervals for these return levels do not overlap significantly, reinforcing the conclusion that the underlying data generating process has altered.

Regarding the annual minima (drought conditions), both the Likelihood Ratio tests on block maxima and the analysis of threshold excesses failed to find significant changes. The distribution of low-water events on the Tisza appears stable, with no statistically significant trend in either direction.

2.4 Statistical Analysis of the Danube River

The analysis of the Danube river at Esztergom revealed a different statistical profile. Visually and numerically, the data exhibited a decreasing trend in flood peaks similar to the Tisza. The GEV point estimate for the 5000-day return level fell from approximately 783 cm in the early period to 649 cm in the late period, while the POT estimates fell from 904 cm to 758 cm.

However, unlike the Tisza, the statistical significance of this observed decrease is ambiguous. The Likelihood Ratio test comparing the stationary GEV model to the non-stationary alternative yielded a p-value greater than 0.05 ($p \approx 0.13$). Consequently, we cannot reject the null hypothesis that the observed reduction in annual maxima is merely a result of natural stochastic variability. While Wald tests on the POT/GPD models did indicate a significant reduction in the scale of exceedances ($p \approx 0.003$), the shape parameter remained invariant, and the confidence intervals for the return levels in the two periods overlap considerably. This indicates that the uncertainty inherent in extrapolating from short time series is too large to confirm a systematic shift.

Furthermore, seasonality analysis using the POT method indicated significant seasonal dependence in the Danube's exceedances. While incorporating harmonic covariates improved the model fit, it did not alter the fundamental conclusion regarding the lack of robust decadal non-stationarity. Similar to the Tisza, the analysis of minima for the Danube showed no significant changes in drought risk (see in Appendix, Figure 3 and Figure 4).

2.5 Conclusions and Limitations

This study concludes that the two major rivers of Hungary have displayed decoupled behaviours over the last two decades. The Tisza shows a statistically significant structural break toward reduced flood risk, suggesting a fundamental change in catchment hydrology or upstream management. The Danube shows a non-significant decreasing trend that cannot be distinguished from natural variability.

The validity of these conclusions is subject to the limitation of sample size. The target return period of roughly 13.7 years exceeds the length of the individual observation blocks (11 and 12 years). This necessitates extrapolation into the tail of the distribution, introducing uncertainty that is captured in the wide confidence intervals, particularly for the Generalized Extreme Value models. The use of the Peaks-Over-Threshold method mitigated this by increasing the effective sample size, yet the fundamental constraint of the short observation history remains. Based on these results, we advise that any reduction in flood defence specifications be considered only for the Tisza, while the Danube should be treated as maintaining its historical risk profile until further evidence confirms the permanence of the observed trends.

Appendix

For further information please visit our GitHub repository. Here you can find R codes, figures and the data we used for the analysis.

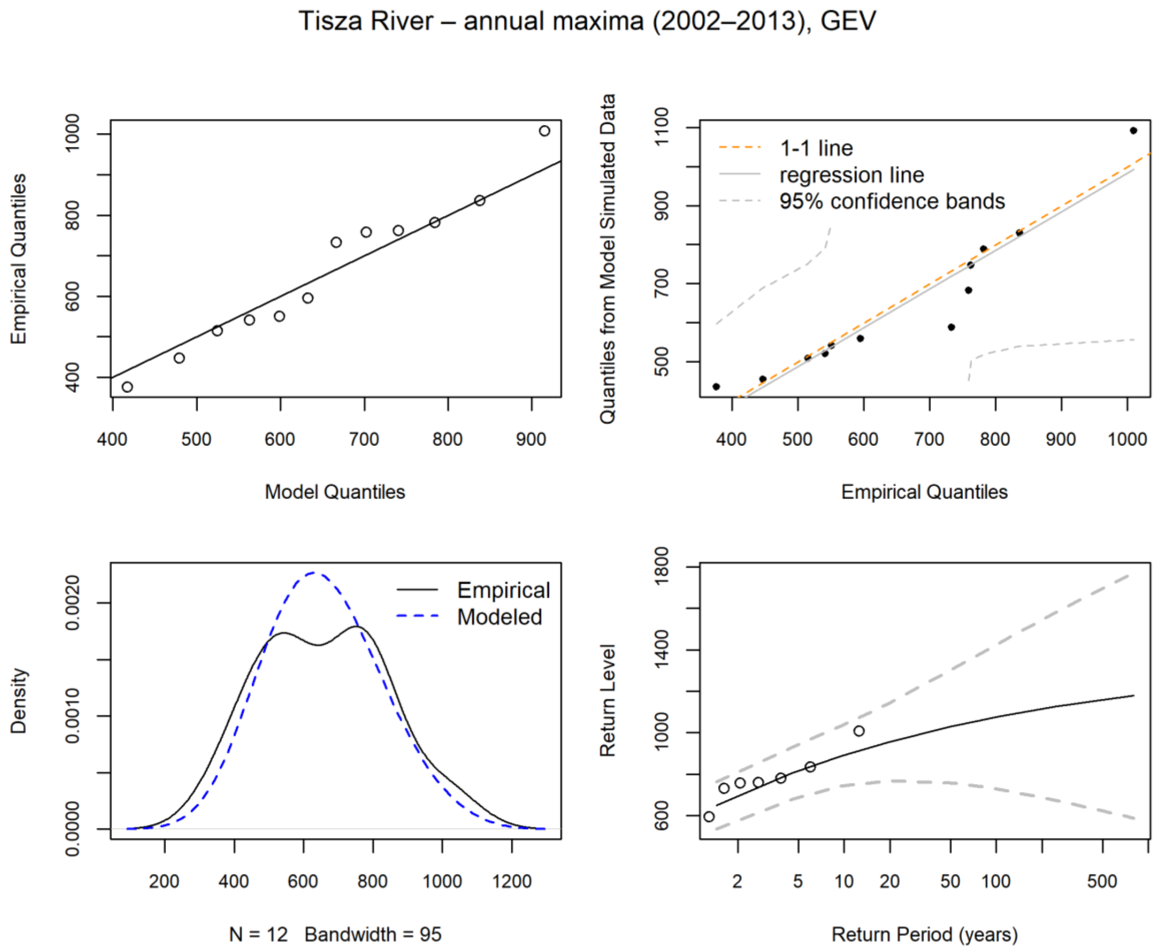

Figure 1: Tisza river – annual maxima (2002-2013), GEV

Source: Own calculations and design based on the analysed data.

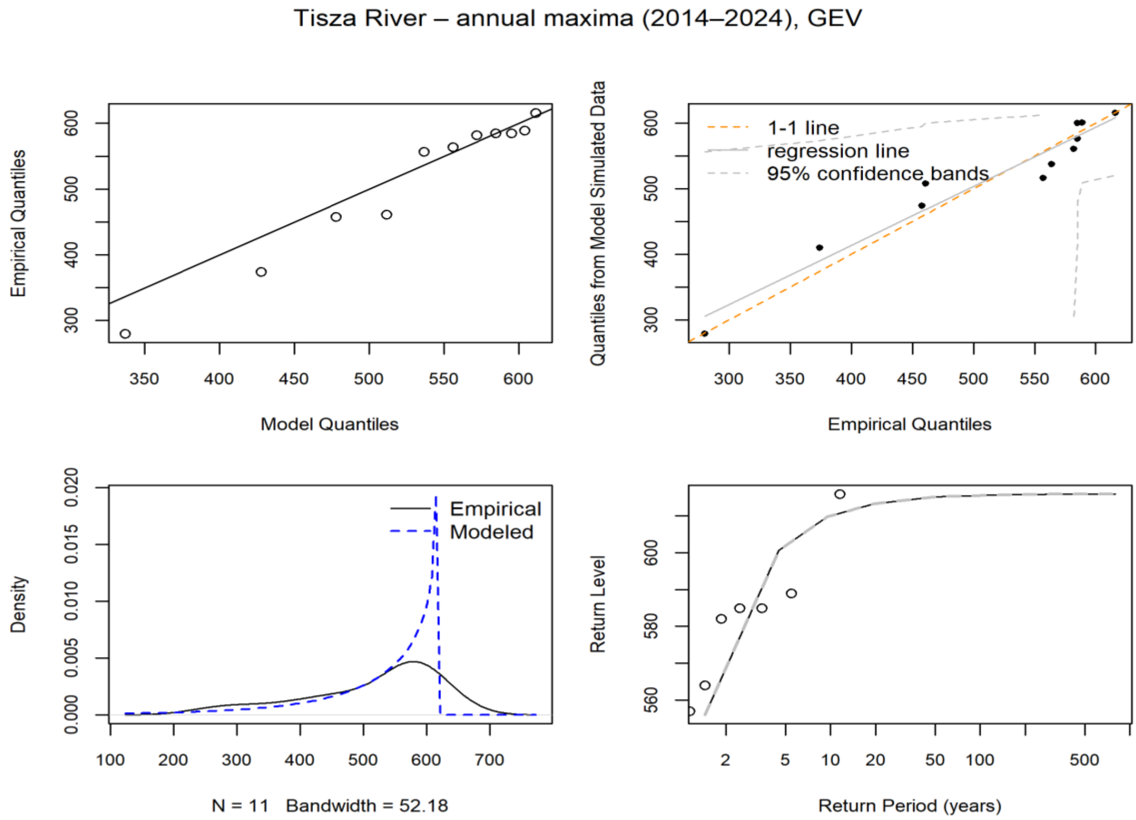

Figure 2: Tisza river – annual maxima (2014-2024), GEV
Source: Own calculations and design based on the analysed data.

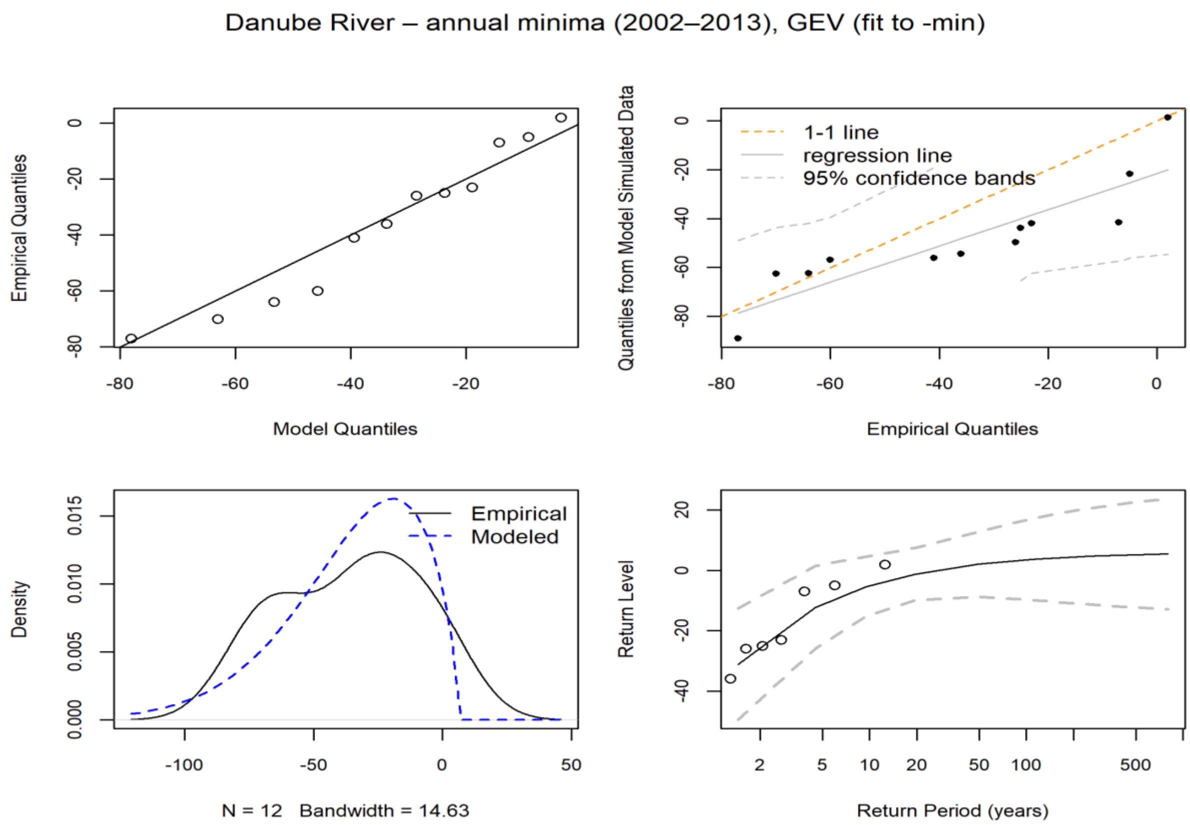

Figure 3: Danube river – annual minima (2002-2013), GEV

Source: Own calculations and design based on the analysed data.

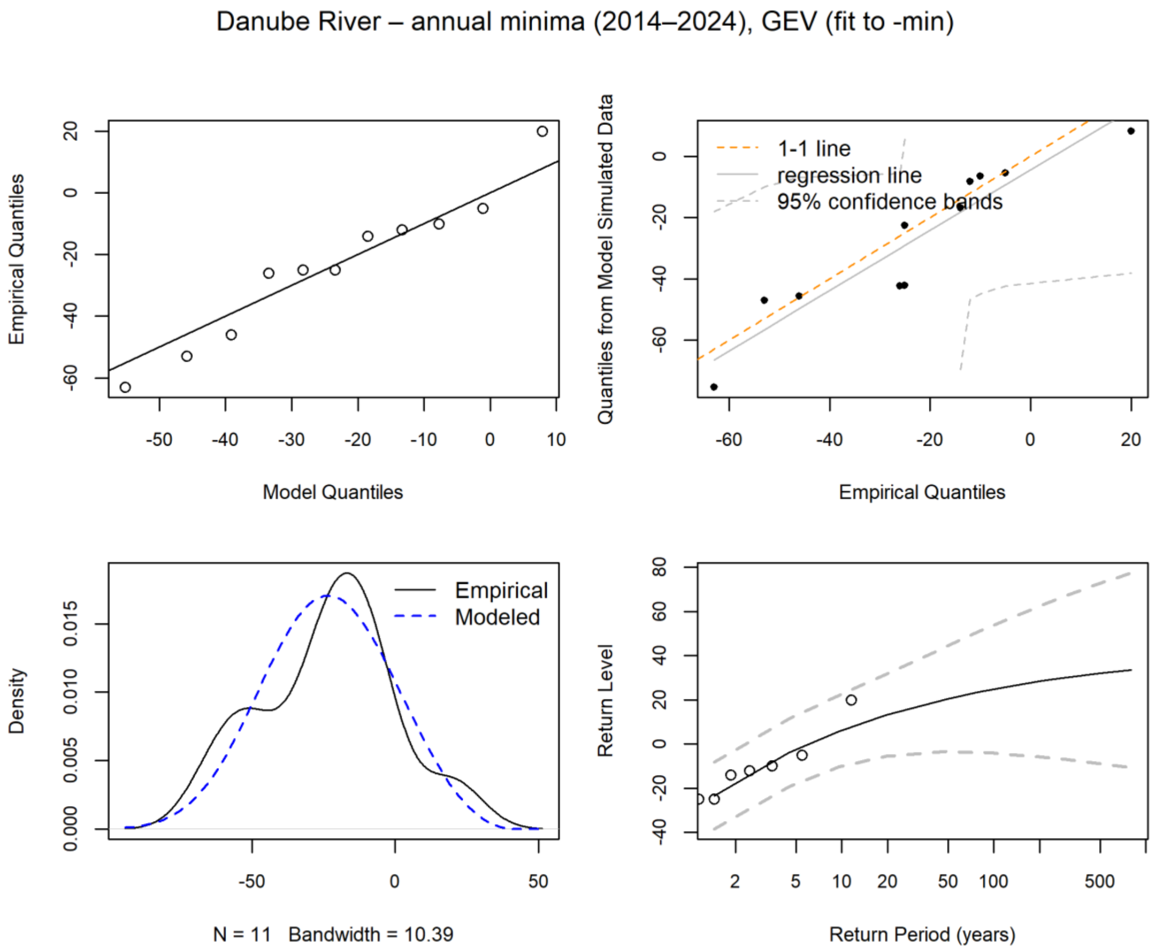

Figure 4: Danube river – annual minima (2014-2024), GEV
Source: Own calculations and design based on the analysed data.

Declaration of use of Large Language Models (LLMs)

During the preparation of this study, we used the ChatGPT and Gemini websites, while writing, and editing the program codes in order to more efficiently detect and resolve any errors that may arise.

After using the tool, we reviewed and edited the received content, and we take full responsibility for what is described in the study.