Quant Scholarship Program

Report: Stock Process Modeling

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Exercise 2

In exercise 2 we were supposed to test our implementation of Geometric Brownian Motion.

- *Check what happens when the volatility is set to 0. Why?*
  When the volatility is set to 0, the output file contains identical rows, with the same Final Stock Price and Realized Volatility that is equal to zero. This happens, because when we set volatility to 0, we say, that stock prices will not differ from the expected price at all. In the equation for the GBM price, the volatility is a "randomness factor" that is needed to generate different paths from the same starting point. Let's take a look at that equation:

  $$S_{i\frac{t}{n}} = S_{(i-1)\frac{t}{n}} * e^{(r-\frac{\sigma^2}{2})\frac{t}{n}+\sigma\sqrt{\frac{t}{n}}Z_i}$$

  If we now set the volatility to 0, we have:

  $$S_{i\frac{t}{n}} = S_{(i-1)\frac{t}{n}} * e^{r\frac{t}{n}}$$

  In that case, our stock prices in every path will behave the same. Every time then, the Final Stock Price will simply be equal to:

  $$S_t = S_0 * e^{rt}$$

  Let's check my implementation. After running the simulation with the following input parameters:

  $$count = 1500,\ steps = 300,\ price = 100,\ drift = 0.05,\ vol = 0,\ years = 1,\ seed = 5$$

  I got the following output:

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105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
105.127110;0.000000
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105.127110;0.000000
105.127110;0.000000
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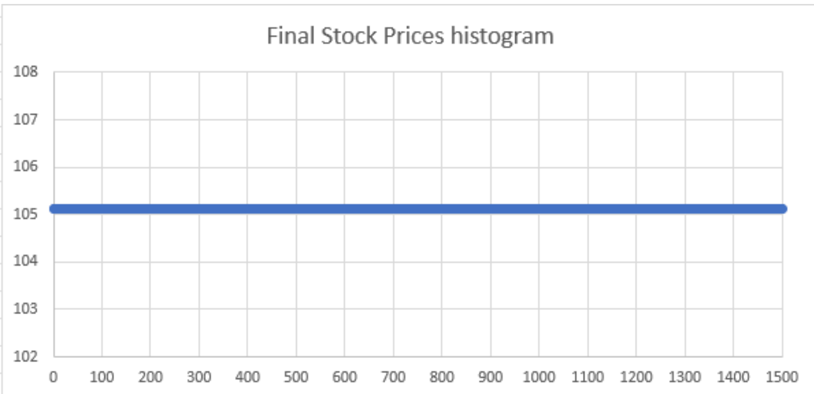

Figure 1: Results of the experiment with the volatility=0.

  It means, that we really got 1500 same values. Let's check if they match the equation:

  $$S_t = S_{\frac{t}{n}} * e^{rt} = 100 * e^{0,05} = 105.12711$$

- *Generate more than 1000 paths of n time steps (where n>=250) starting from the same stock price value, using the same drift and volatility, and ending on the same date (for simplicity: t=1). Then, in a spreadsheet, plot the histogram of the final stock prices and compare these with log normal density.*

  In this exercise I ran the simulation with the following parameters:

$$count = 1500, \ steps = 300, \ price = 100, \ drift = 0.05, \ vol = 0.2, \ years = 1, \ seed = 5$$

Then I analyzed the output data in a spreadsheet. First of all, I used Excel's Data Analysis tool to create a data series for the histogram. This way I got the following table:

Bins	*Midpoints*	*Frequency*
60	55	4
70	65	28
80	75	119
90	85	220
100	95	277
110	105	281
120	115	226
130	125	165
140	135	86
150	145	46
160	155	23
170	165	12
180	175	8
190	185	4
200	195	0

Figure 2: Data prepared for the histogram.

Then I prepared values of the log-normal density using the formula:

$$\frac{1}{x\sqrt{2\pi}\sigma} e^{-\frac{(lnx - lnS_0 - r + \frac{\sigma^2}{2})^2}{2\sigma^2}}$$

The next step was to plot the histogram and the log-normal density curve on one chart. The result was:

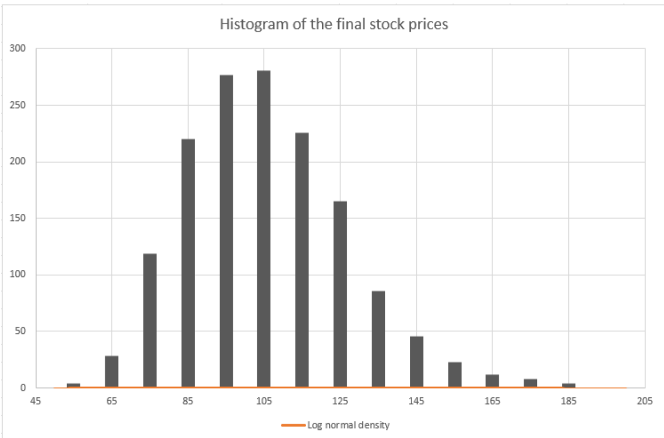

Figure 3: Chart of the histogram and log-normal density curve - first attempt.

However, the result was not satisfying - log-normal density values(*) had to be scaled. To do so, I calculated the scaling factor, by dividing the area under the histogram and the integral of the log-normal density function over the range of the histogram. When I multiplied the values(*) by this factor, the chart I got was:

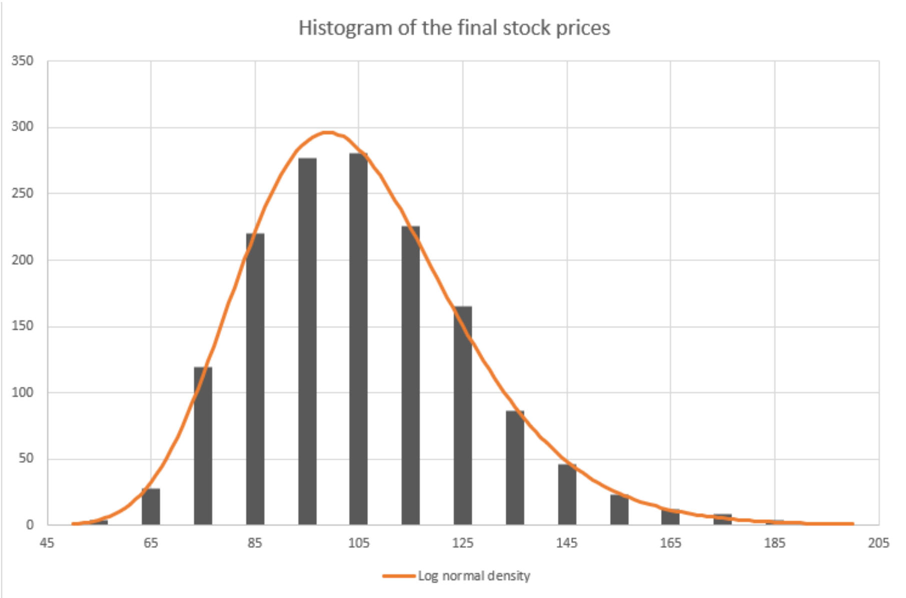

Figure 4: Proper chart.

So the histogram of the final stock prices truly matches the log-normal density curve.

- *For each path calculate the realized volatility, check if the average matches input volatility.*

In this task, I used the built-in function in Excel to calculate the average of Realized Volatility from each row. The result was:

$$averageRealizedVolatility = 0.199819861$$

So this result differs from the input volatility of the simulation by only 0.09%.

Exercise 3

Take some real-life stock daily prices' series, calculate log-returns (in a spreadsheet) and plot the histogram. Plot appropriate normal distribution density function on the same chart (estimate mean and std deviation matching historical returns).

In this exercise, I analyzed last year's daily prices series of HSBC Holdings [HSBC], Cisco Systems [CSCO], and Apple Inc [AAPL] stocks. I downloaded data from the website: https://www.nasdaq.com/market-activity/stocks/*/historical, where * should be replaced by: hsbc, csco, and aapl respectively. Then I calculated log returns in a spreadsheet and plotted the histogram in the same way I did in Excercise 2. Finally, I added the normal distribution density function on those charts, scaled them, and got the following results:

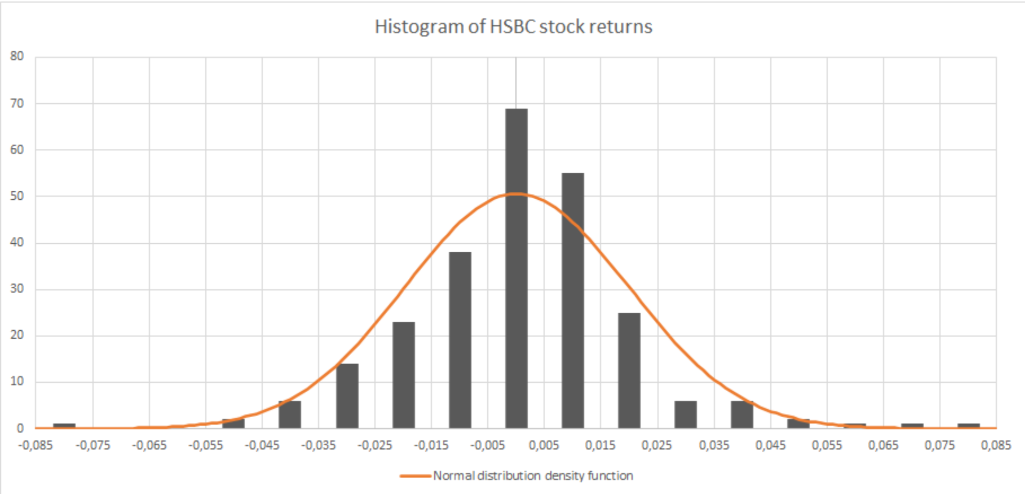

Figure 5: Analysis of HSBC stock returns.

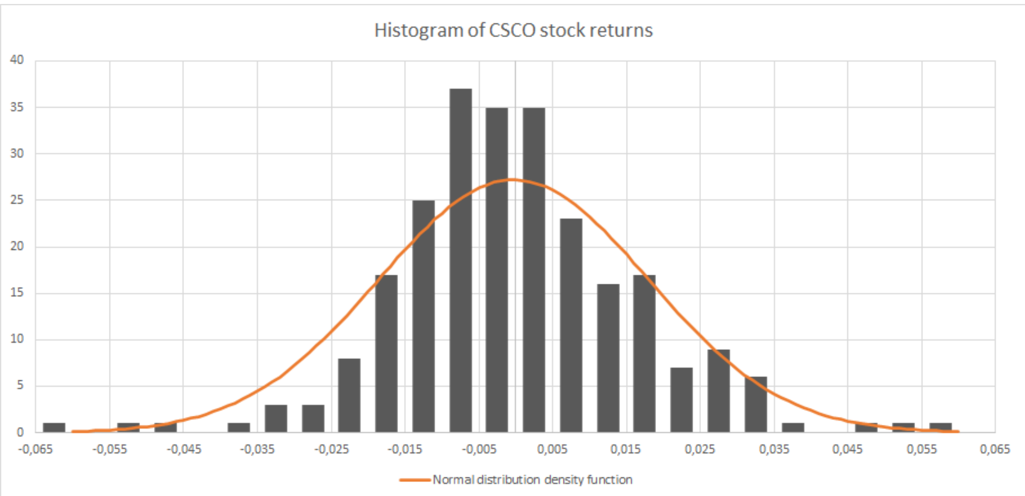

Figure 6: Analysis of CSCO stock returns.

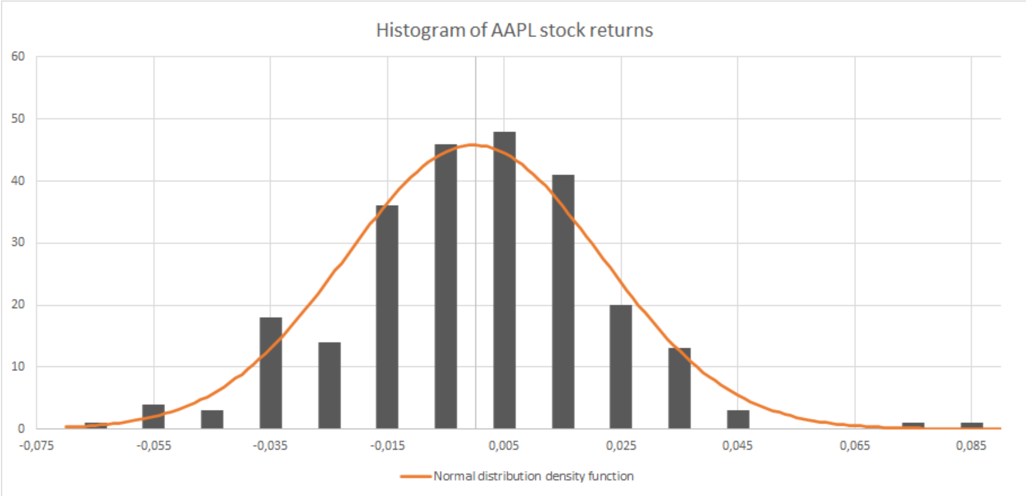

Figure 7: Analysis of AAPL stock returns.

Results presented above show that stock returns may have a distribution similar to the normal distribution, but they differ - especially when it comes to their "tails". Stock returns distributions have in fact fatter "tails", which means that extreme, unexpected events occur more frequently than would be predicted by a normal distribution. However, stock returns are for sure not distributed randomly and in some situations, we might use some of the normal distribution features while analyzing them.