G H Patel College of Engineering & Technology
(A Constituent College of CVM University)
Academic Year 2022-23, Semester – I
Subject Code: 102000104
Subject Name: CALCULUS
Tutorial-6 Partial Differentiation

EXAMPLE -1:

1. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. Ans: $r^2\sin\theta$.

2. If $u = \frac{(2x-y)}{2}, v = \frac{y}{2}$, $w = \frac{z}{3}$, find $J(u, v, w)$. Ans:6

EXAMPLE -2:

Find gradient of a function at the given point.

(a) $g(x,y) = \frac{x^2}{2} - \frac{y^2}{2}$, $(\sqrt{2}, 1)$, Ans: $\sqrt{2}\,\hat{\imath} - \hat{\jmath}$.
(b) $f(x,y) = (x^2 + xy)^3$, $(-1,-1)$, Ans: $-36\hat{\imath}-12\hat{\jmath}$.
(c) $\varphi(x,y,z) = \ln(x^2 + y^2 + z^2)$, Ans: $\frac{2(x\hat{\imath}+y\,\hat{\jmath}+z\hat{k})}{x^2+y^2+z^2}$.
(d) $\varphi(x,y,z) = 3x^2y - y^3z^2$, $(1,-2,-1)$, Ans: $-12\hat{\imath} - 9\hat{\jmath} + 16\hat{k}$.

EXAMPLE -3:

Find the derivative of the function at P_0 in the direction of given vector.

(a) $f(x,y) = x^2 \sin 2y$, $P_0(1, \pi/2)$, $\bar{A} = 3\hat{\imath} - 4\hat{\jmath}$, Ans: 8/5.
(b) $f(x,y) = \tan^{-1}(y/x)$, $P_0(-2,2)$, $\bar{v} = -\hat{\imath} - \hat{\jmath}$, Ans: $72/\sqrt{14}$.
(c) $g(x,y,z) = 3e^x\cos(yz)$, $P_0(0,0,0)$, $\bar{A} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, Ans: 2.
(d) $h(x,y,z) = \cos(xy) + e^{yz} + \ln(zx)$, $P_0\left(1, 0, \frac{1}{2}\right)$, $\bar{A} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, Ans: 2
(e) $f(x,y,z) = (x^2 + y^2 + z^2)^{3/2}$, $P_0(1, 1, 2)$, $\bar{A} = 2\hat{\jmath} - \hat{k}$, Ans: $9/2\sqrt{5}$.
(f) $f(x,y,z) = x^2y - yz^3 + z$, $P_0(1,-2,0)$, $\bar{v} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, Ans: - 3.

EXAMPLE -4:

Find equations for the (i) tangent plane and (ii) normal line at the point P_0 on the surface:

(a) $\cos\pi x - x^2y + e^{xz} + yz = 4$, $P_0(0,1,2)$, Ans: 2x+2y+z-4=0, x=2t, y=1+2t, z=2+t.
(b) $x^2 + y^2 - 2xy - x + 3y - z = -4$, $P_0(2,-3,18)$, Ans: (i) $9x - 7y - z = 21$, (ii) $x = 2 + 9t,\ y = -3 - 7t,\ z = 18 - t$.
(c) $z^2 = 4(1 + x^2 + y^2)$, $P_0(2,2,6)$, Ans: (i)$4x + 4y - 3z = -2$, (ii) $x = 2 + 4t$, $y = 2 + 4t,\ z = 6 - 3t$.
(d) $z + 1 = xe^y\cos z$, $P_0\,(1,0,0)$, Ans: (i) $x + y - z = 1$, (ii) $x = 1 + t,\ y = t,\ z = -t$.

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EXAMPLE -5:

Find an equation for the plane that is tangent to the given surface at the given point.

(a) $z = \ln(x^2 + y^2)$, (1,0,0), Ans: 2x - $z = 2$.
(b) $z = \sqrt{y - x}$, (1,2,1), Ans: x – y + 2$z = 1$.
(c) $z = e^{-(x^2+y^2)}$, (0,0,1), Ans: $z = 1$.
(d) z = $4x^3y^2 + 2y$, $(1, -2, 12)$, Ans: $48x - 14y - z = 64$.
(e) $z = e^{3y} sin3x$, $(\pi/6, 0, 1)$, Ans: $3y - z = -1$.

EXAMPLE -6:

Find all the local maxima, local minima and saddle points (if exist) of the below functions.

(a) $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$; Ans: local min at (-3,3). $f(-3,3) = -5$

(b) $f(x, y) = 2xy - x^2 - 2y^2 + 3x + 4$; Ans: local max at $\left(3, \frac{3}{2}\right)$. $f\left(3, \frac{3}{2}\right) = \frac{17}{2}$

(c) $f(x, y) = x^3 - 2xy - y^3 + 6$; Ans: critical points: (0,0), $\left(-\frac{2}{3}, \frac{2}{3}\right)$; saddle point is (0,0), local maximum is $f\left(-\frac{2}{3}, \frac{2}{3}\right) = \frac{170}{27}$

(d) $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$; Ans: critical points: (0,0), $(1, -1)$; saddle point is $(1, -1)$, local minimum is $f(0,0) = 0$

(e) $f(x, y) = 4xy - x^4 - y^4$; Ans: critical points: $(0,0), (1,1), (-1, -1)$; saddle point is (0,0); local min is $f(0,2) = -12$, local max is $f(1,1) = f(-1, -1) = 2$