G H Patel College of Engineering & Technology
(A Constituent College of CVM University)
Academic Year 2022-23, Semester – I
Subject Code: 102000104
Subject Name: CALCULUS
Tutorial-5 Partial Differentiation

Exercise-1: Check whether the following functions are homogeneous or not. If yes, find its degree 'n'.

(a) $f(x,y) = \frac{x^3+y^3}{x+y}$,

(b) $f(x,y) = \frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}$

(c) $u(x,y) = log\left(\frac{x^7+y^7}{x+y+z}\right)$

(d) $u(x,y) = \operatorname{cosec}^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{x-y}\right)$

(e) $u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$

Exercise-2: Verify Euler's theorem for the function $= sin^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y}{x}\right)$.

Exercise-3: Use Euler's theorem to solve the following problems:

1. If $u = \frac{y^3-x^3}{y^2+x^2}$ find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. Ans:0.
2. If $f(x,y) = x^4y^2\, sin^{-1}\left(\frac{y}{x}\right)$ then find the value of $\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. Book [4], Ans: $6f(x,y)$.
3. If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.
4. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$; show that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$

   (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2sinucos3u$.;
5. If $u = \sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2cotu$.
6. If $u = \sin^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{144}\ tan\,u[tan^2u - 11]$,