G H Patel College of Engineering & Technology
(A Constituent College of CVM University)
Academic Year 2022-23, Semester – I
Subject Code: 102000104
Subject Name: CALCULUS
Tutorial-4 Partial Differentiation

Exercise-1: Evaluate $\frac{dw}{dt}$ at the given value of $t$ by using chain Rule

(a) $w = x^2y - y^2, x = sint, y = e^t, t = 0.$ Ans: $-2$
(b) $w = z - sinxy, x = t, y = \ln t,\ z = e^{t-1}, t = 1.$ Ans:0
(c) $w = \ln(x^2 + y^2 + z^2),\ x = cost, y = sint, z = 4\sqrt{t}, t = 3.$ Ans: $= \frac{16}{49}$

Exercise-2: Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if

(a) $w = xy + yz + zx, x = u + v, y = u - v, z = uv$; at $(u, v) = (1/2, 1)$,
Ans: $\frac{\partial w}{\partial u} = 3,\ \frac{\partial w}{\partial v} = -\frac{3}{2}$.
(b) $w = e^{xyz}, x = 3u + v, y = 3u - v, z = u^2v$; Ans: $\frac{\partial w}{\partial u} = e^{xyz}(3yz + 3xz + 2xyuv),\ \frac{\partial w}{\partial v} = e^{xyz}(yz - xz + xyu^2)$,

Exercise-3: If $u = u(y - z, z - x,\ x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$

Exercise-4: If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0.$

Exercise- 5: If $u = f(x^2 + 2yz,\ y^2 + 2zx)$ then prove that
$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$

Exercise-6: Find the value of $\frac{dy}{dx}$ at the given point.

(a) $xe^y + \sin xy + y - log2 = 0$ at (0,log2). Ans: - (2+ln2).
(b) $y^3 + y^2 - 5y - x^2 + 4 = 0$, Ans: $= \frac{-2x}{3y^2+2y-5}$
(c) $\sqrt{xy} = 1 + x^2y$, Ans: $\frac{4(xy)^{3/2}-y}{x-2x^2\sqrt{xy}}$
(d) $x^y = y^x$, Ans: $= \frac{y\,(y-xlogy)}{\mathrm{x}(\mathrm{x}-\mathrm{y}\log x)}$

Exercise- 7: Find the value of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the given point.

(a) $xe^y + ye^z + 2lnx - 2 - 3ln2 = 0$, $(1, \ln 2, \ln 3)$, Ans: $\frac{\partial z}{\partial x} = -\frac{4}{3\,ln2}$, $\frac{\partial z}{\partial y} = -\frac{5}{3\,ln2}$.
(b) $x^2 - 3yz^2 + xyz - 2 = 0$, Ans: $\frac{\partial z}{\partial x} = \frac{2x+yz}{6yz-xy}, \frac{\partial z}{\partial y} = -\frac{xz-3z^2}{6yz-xy}$.
(c) $ye^x - 5sin3z = 3z$, Ans: $\frac{\partial z}{\partial x} = \frac{ye^x}{15cos3z+3}, \frac{\partial z}{\partial y} = \frac{e^x}{15cos3z+3}$.

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