G H Patel College of Engineering & Technology
(A Constituent College of CVM University)
Academic Year 2022-23, Semester – I
Subject Code: 102000104
Subject Name: CALCULUS
Tutorial-4 Partial Differentiation

Exercise-1: Evaluate $\frac{dw}{dt}$ at the given value of t by using chain Rule

(a) $w = x^2y - y^2, x = sint, y = e^t, t = 0$. Ans: -2
(b) $w = z - sinxy, x = t, y = \ln t, \ z = e^{t-1}, t = 1$. Ans:0
(c) $w = \ln(x^2 + y^2 + z^2)\,,\ x = cost, y = sint, z = 4\sqrt{t}, t = 3$. Ans: $= \frac{16}{49}$

Exercise-2: Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if

(a) $w = xy + yz + zx, x = u + v, y = u - v, z = uv$; at $(u, v) = (1/2, 1)$,
Ans: $\frac{\partial w}{\partial u} = 3,\ \frac{\partial w}{\partial v} = -\frac{3}{2}$.
(b) $w = e^{xyz}, x = 3u + v, y = 3u - v, z = u^2v$; Ans: $\frac{\partial w}{\partial u} = e^{xyz}(3yz + 3xz + 2xyuv),\ \frac{\partial w}{\partial v} = e^{xyz}(yz - xz + xyu^2)$,

Exercise-3: If $u = u(y - z, z - x,\ x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

Exercise-4: If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.

Exercise- 5: If $u = f(x^2 + 2yz,\ y^2 + 2zx)$ then prove that
$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0$.

Exercise-6: Find the value of $\frac{dy}{dx}$ at the given point.

(a) $xe^y + \sin xy + y - log2 = 0$ at (0,log2). Ans: - (2+ln2).
(b) $y^3 + y^2 - 5y - x^2 + 4 = 0$, Ans: $= \frac{-2x}{3y^2+2y-5}$
(c) $\sqrt{xy} = 1 + x^2y$, Ans: $\frac{4(xy)^{3/2}-y}{x-2x^2\sqrt{xy}}$
(d) $x^y = y^x$, Ans: $= \frac{y\,(y-xlogy)}{\mathrm{x}(\mathrm{x}-\mathrm{y}\log x)}$

Exercise- 7: Find the value of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the given point.

(a) $xe^y + ye^z + 2lnx - 2 - 3ln2 = 0$, (1, ln2, ln3), Ans: $\frac{\partial z}{\partial x} = -\frac{4}{3\,ln2}$, $\frac{\partial z}{\partial y} = -\frac{5}{3\,ln2}$.
(b) $x^2 - 3yz^2 + xyz - 2 = 0$, Ans: $\frac{\partial z}{\partial x} = \frac{2x+yz}{6yz-xy}, \frac{\partial z}{\partial y} = -\frac{xz-3z^2}{6yz-xy}$.
(c) $ye^x - 5sin3z = 3z$, Ans: $\frac{\partial z}{\partial x} = \frac{ye^x}{15cos3z+3}, \frac{\partial z}{\partial y} = \frac{e^x}{15cos3z+3}$.

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Tutorial-4 Partial Differentiation

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Subject Name: CALCULUS
Tutorial-5 Partial Differentiation

Exercise-1: Check whether the following functions are homogeneous or not. If yes, find its degree 'n'.

(a) $f(x,y)=\frac{x^3+y^3}{x+y}$,

(b) $f(x,y)=\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}$

(c) $u(x,y)=log\left(\frac{x^7+y^7}{x+y+z}\right)$

(d) $u(x,y)=\text{cosec}^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{x-y}\right)$

(e) $u(x,y,z)=(x^2+y^2+z^2)^{-1/2}$

Exercise-2: Verify Euler's theorem for the function $= sin^{-1}\left(\frac{x}{y}\right)+tan^{-1}\left(\frac{y}{x}\right)$.

Exercise-3: Use Euler's theorem to solve the following problems:

1. If $u=\frac{y^3-x^3}{y^2+x^2}$ find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. Ans:0.
2. If $f(x,y)=x^4y^2\, sin^{-1}\left(\frac{y}{x}\right)$ then find the value of $\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$. Book [4], Ans: $6f(x,y)$.
3. If $u=\log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$.
4. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$; show that (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=sin2u$

   (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2sinucos3u.$;
5. If $u=\sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2cotu$.
6. If $u=\sin^{-1}\left(\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{1}{144}\ tan\, u[tan^2u-11]$,

EXAMPLE -1:

1. If $x = r sin\theta cos\phi, y = r sin\theta sin\phi, z = r cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. Ans: $r^2 \sin\theta$.

2. If $u = \frac{(2x-y)}{2}, v = \frac{y}{2}, w = \frac{z}{3}$, find $J(u, v, w)$. Ans:6

EXAMPLE -2:

Find gradient of a function at the given point.

(a) $g(x,y) = \frac{x^2}{2} - \frac{y^2}{2}$, $(\sqrt{2}, 1)$, Ans: $\sqrt{2}\,\hat{\imath} - \hat{\jmath}$.
(b) $f(x,y) = (x^2 + xy)^3$, $(-1,-1)$, Ans: $-36\hat{\imath}-12\hat{\jmath}$.
(c) $\varphi(x,y,z) = \ln(x^2+y^2+z^2)$, Ans: $\frac{2(x\hat{\imath}+y\,\hat{\jmath}+z\hat{k})}{x^2+y^2+z^2}$.
(d) $\varphi(x,y,z) = 3x^2y - y^3z^2$, $(1,-2,-1)$, Ans: $-12\hat{\imath} - 9\hat{\jmath} + 16\hat{k}$.

EXAMPLE -3:

Find the derivative of the function at P_0 in the direction of given vector.

(a) $f(x,y) = x^2 sin2y$, $P_0(1, \pi/2)$, $\bar{A} = 3\hat{\imath} - 4\hat{\jmath}$, Ans: 8/5.
(b) $f(x,y) = tan^{-1}(y/x)$, $P_0(-2,2)$, $\bar{v} = -\hat{\imath} - \hat{\jmath}$, Ans: $72/\sqrt{14}$.
(c) $g(x,y,z) = 3e^x\cos(yz)$, $P_0(0,0,0)$, $\bar{A} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, Ans: 2.
(d) $h(x,y,z) = \cos(xy) + e^{yz} + \ln(zx)$, $P_0\left(1, 0, \frac{1}{2}\right)$, $\bar{A} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, Ans: 2
(e) $f(x,y,z) = (x^2+y^2+z^2)^{3/2}$, $P_0(1, 1, 2)$, $\bar{A} = 2\hat{\jmath} - \hat{k}$, Ans: $9/2\sqrt{5}$.
(f) $f(x,y,z) = x^2y - yz^3 + z$, $P_0(1,-2,0)$, $\bar{v} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, Ans: - 3.

EXAMPLE -4:

Find equations for the (i) tangent plane and (ii) normal line at the point P_0 on the surface:

(a) $cos\pi x - x^2y + e^{xz} + yz = 4$, $P_0(0,1,2)$, Ans: 2x+2y+z-4=0, x=2t, y=1+2t, z=2+t.
(b) $x^2 + y^2 - 2xy - x + 3y - z = -4$, $P_0(2,-3,18)$, Ans: (i) $9x-7y - z = 21$, (ii) $x = 2 + 9t$, $y = -3 - 7t$, $z = 18 - t$.
(c) $z^2 = 4(1 + x^2 + y^2)$, $P_0(2,2,6)$, Ans: (i)$4x + 4y - 3z = -2$, (ii) $x = 2 + 4t$, $y = 2 + 4t$, $z = 6 - 3t$.
(d) $z + 1 = xe^y\cos z$, $P_0(1,0,0)$, Ans: (i) $x + y - z = 1$, (ii) $x = 1 + t$, $y = t$, $z = -t$.

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EXAMPLE -5:

Find an equation for the plane that is tangent to the given surface at the given point.

(a) $z = \ln(x^2 + y^2)$, (1,0,0), Ans: 2x - $z = 2$.
(b) $z = \sqrt{y - x}$, (1,2,1), Ans: x – y + 2$z = 1$.
(c) $z = e^{-(x^2+y^2)}$, (0,0,1), Ans: $z = 1$.
(d) z = $4x^3y^2 + 2y$, $(1, -2,\ 12)$, Ans: $48x - 14y - z = 64$.
(e) $z = e^{3y} sin3x$, $(\pi/6, 0, 1)$, Ans: $3y - z = -1$.

EXAMPLE -6:

Find all the local maxima, local minima and saddle points (if exist) of the below functions.

(a) $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$; Ans: local min at (-3,3). $f(-3,3) = -5$

(b) $f(x, y) = 2xy - x^2 - 2y^2 + 3x + 4$; Ans: local max at $\left(3, \frac{3}{2}\right)$. $f\left(3, \frac{3}{2}\right) = \frac{17}{2}$

(c) $f(x, y) = x^3 - 2xy - y^3 + 6$; Ans: critical points: (0,0), $\left(-\frac{2}{3}, \frac{2}{3}\right)$; saddle point is (0,0), local maximum is $f\left(-\frac{2}{3}, \frac{2}{3}\right) = \frac{170}{27}$

(d) $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$; Ans: critical points: (0,0), $(1, -1)$; saddle point is $(1, -1)$, local minimum is $f(0,0) = 0$

(e) $f(x, y) = 4xy - x^4 - y^4$; Ans: critical points: (0,0), (1,1), $(-1, -1)$; saddle point is (0,0); local min is $f(0,2) = -12$, local max is $f(1,1) = f(-1, -1) = 2$