Indian Institute of Technology Gandhinagar

Control of Inverted Pendulum on a Cart

ME 352 : Mechanical Engineering Lab 2

Lab Report - Experiment 6

Group - 8

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Authors

Pavidhar Jain - 20110136

Priya Gupta - 20110147

Rahul R Pai - 20110155

Raj Krish Dipakkumar - 20110160

Under The Guidance Of

Prof. Jayaprakash

OBJECTIVE

To balance the inverted pendulum by applying a force to the cart that is attached to the pendulum.

ABSTRACT

A feedback control mechanism is needed to sense the position of the inverted pendulum and adjust the force being applied to the cart in real-time in order to maintain its upright position. A PID controller tracks the pendulum's position and adjusts the force being applied to the cart based on the difference between the desired and actual positions. The inverted pendulum can be balanced steadily by adjusting the gains of the P, I, and D components of the PID controller. This problem has numerous practical applications in fields such as robotics, automation, and control engineering.

EXPERIMENTAL DESIGN

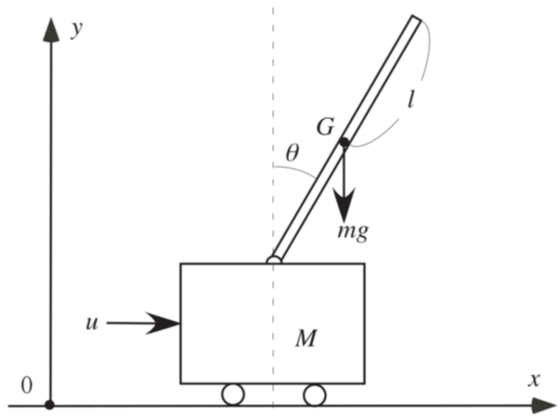

Figure 1 : Schematic of setup

Electrical components used:

- Rotary Encoder
- Arduino UNO
- SMPS
- DC motor
- MOTOR driver MD10C R3

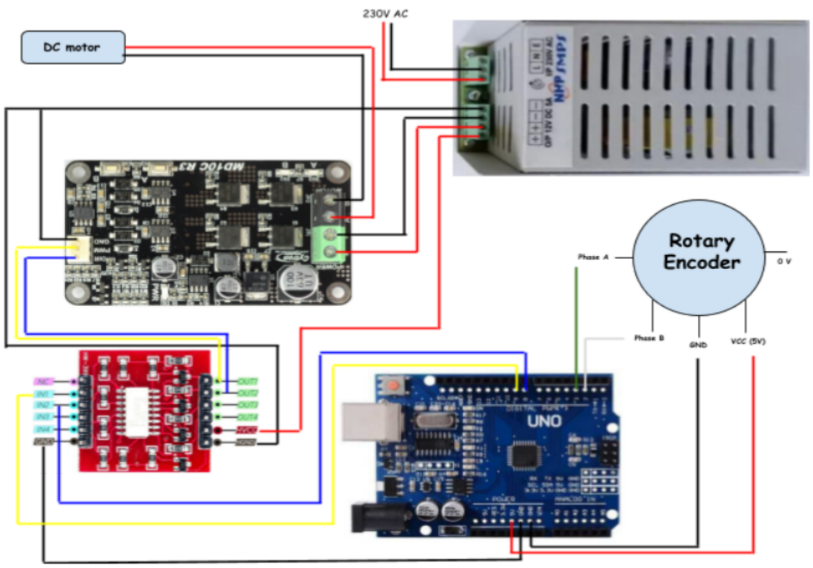

Figure 2 : Circuit diagram

FABRICATION DETAILS

In order to set up our experiment on the inverted pendulum, we utilized a wooden base as the foundation. To properly install the rotatory encoder, we designed, and 3D printed a specialized casing that could be mounted onto the rail with ease. Additionally, we employed laser-cut MDF boxes to elevate the motor and the shaft to the desired height to ensure that the belt was at a consistent vertical level.

To securely attach the various components, we utilized double-sided tape, which helped minimize any unwanted movement during the experiment. The rail was nailed to the base.

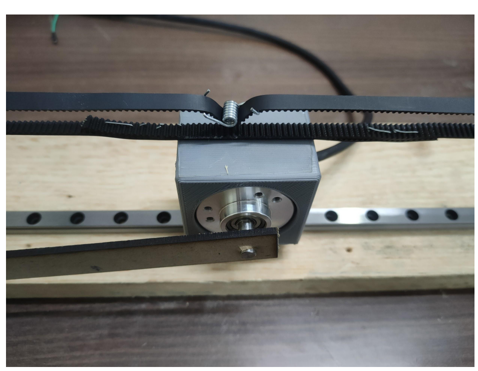

Figure 3: Rotary encoder casing using 3-D printing

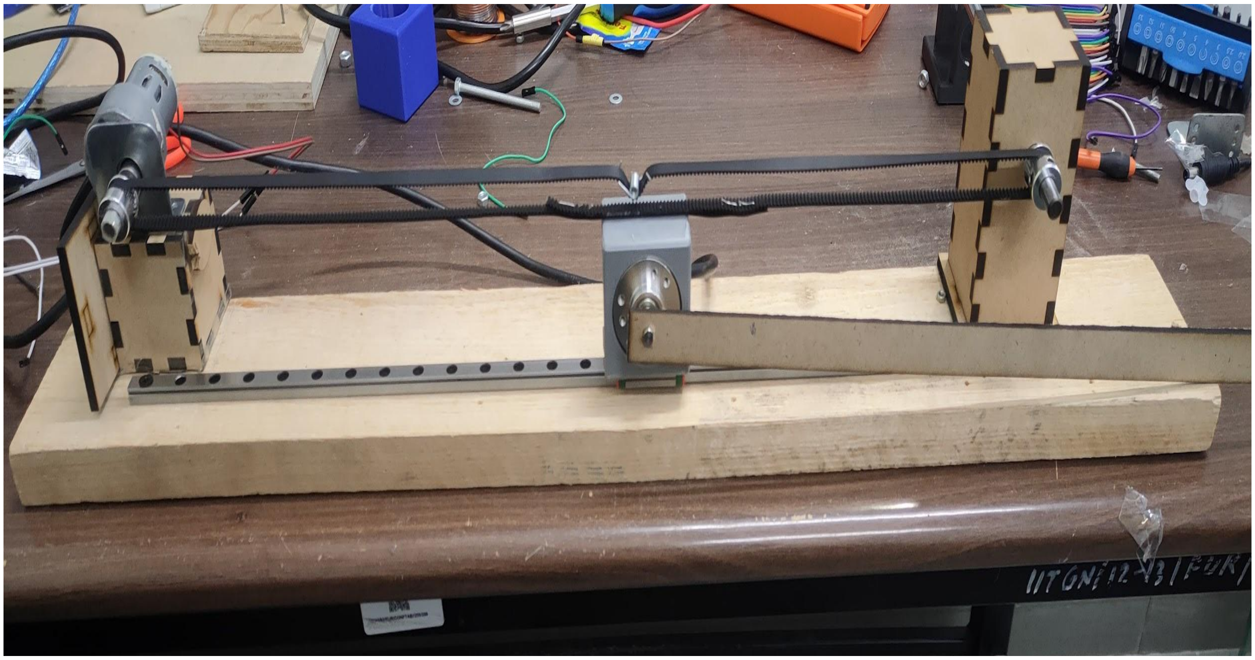

Figure 4 : Final experimental setup

MATHEMATICAL MODELLING

Nomenclature

$m = Mass\ of\ pendulum$

$M = Mass\ of\ cart$

$J = Moment\ of\ inertia\ of\ pendulum$

$H = Horizontal\ force\ exerted\ by\ the\ cart\ on\ pendulum (and\ exerted\ by\ the\ pendulum\ on\ cart)$

$V = Vertical\ \ force\ exerted\ by\ the\ cart\ on\ pendulum (and\ exerted\ by\ the\ pendulum\ on\ cart)$

$g = Acceleration\ due\ to\ gravity$

$u = External\ horizontal\ force\ provided\ to\ the\ cart\ to\ balancethe\ pendulum$

$\theta(t) = Angular\ displacement\ of pendulum (taking\ initial\ position\ as\ vertically\ upwards)$

$x(t) = Horizontal\ displacement\ of\ cart$

$\frac{d^2x}{dt^2} = Horizontal\ acceleration\ of\ cart$

$2l = Length\ of\ pendulum$

$L[f(t)] = F(s) = Laplace\ transform\ of\ f(t)$

$COM = Centre\ of\ mass$

Transfer function for the system

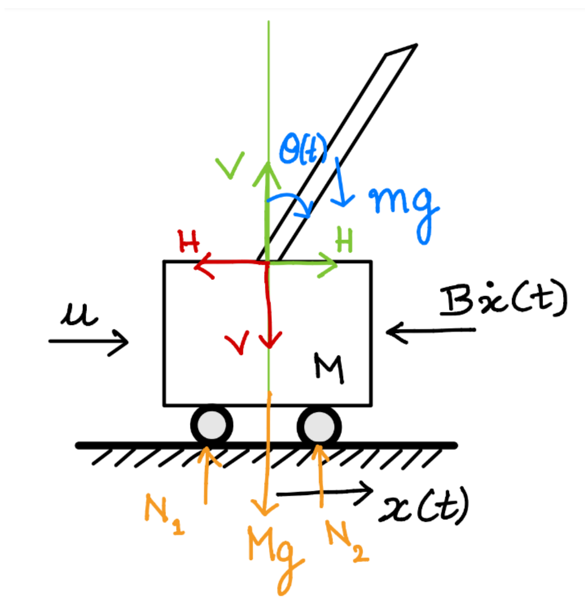

Figure 5: FBD of pendulum and cart

Figure (5) represents the different force which are acting on the system(pendulum and cart). The cart exerts a force on the pendulum and vice versa whose magnitude and direction is unknown. However, we will decompose that force into two components, a horizontal(H) and a vertical(V) component. There is another force acting downwards at the centre of mass(COM) due to its weight(mg).

Considering forces acting in the horizontal direction of the cart:

$$\Sigma F_{x,cart} = M\frac{d^2x}{dt^2}$$

$$\Rightarrow u - H - B\frac{dx}{dt} = M\frac{d^2x}{dt^2} \qquad (1)$$

Considering forces acting in the horizontal direction of the pendulum:

$$\Sigma F_{x,pendulum} = m\frac{d^2}{dt^2}(x + l\, sin\theta)$$

$$\Rightarrow H = m\frac{d^2}{dt^2}(x + l\, sin\theta) \qquad (2)$$

The horzintal displacement of pendulum is due to two components - angular displacement($l\, sin\theta$) of pendulum and horizontal displacement(x) of cart.

Considering forces acting in the vertical direction of the pendulum:

$$\Sigma F_{y,pendulum} = m\frac{d^2}{dt^2}(l - l\, cos\theta)$$

$$\Rightarrow mg - V = m\frac{d^2}{dt^2}(- l\, cos\theta) \qquad (3)$$

Considering torque acting on the pendulum(figure 6):

$$\Sigma T_{pendulum} = J\frac{d^2\theta}{dt^2} \qquad \text{(Taking COM of pendulum as centre of rotation)}$$

$$\Rightarrow V\, sin\theta \times l - H\, cos\theta \times l = J\frac{d^2\theta}{dt^2} \qquad (4)$$

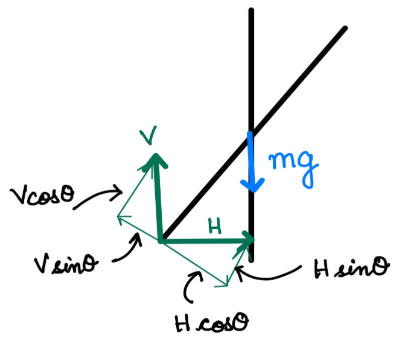

Figure 6: For balancing torque

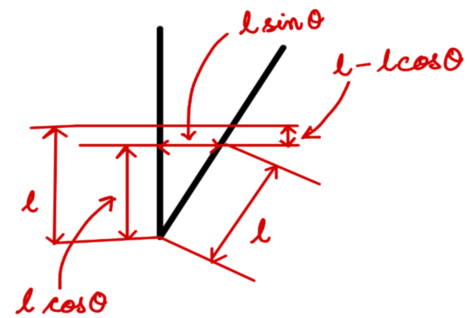

Figure 7: For vertical displacement

We can see that the derived model is non-linear. In order to apply the control and analysis techniques, we have to linearise the equations using the assumption that θ is small. Here, we are assuming that the angular displacement of pendulum with respect to vertical position is very less, i.e., -20° < θ < 20°

If θ is small, then $cos\theta \approx 1$ and $sin\theta \approx \theta$

Now, we will linearise eq. (2), (3), and (4) using the above assumption.

By linearising eq. (2), we get:

$$H = m\frac{d^2}{dt^2}(x + l\theta) \tag{5}$$

By linearising eq. (3), we get:

$$mg - V = m\frac{d^2}{dt^2}(-\ l\ ) = 0 \qquad (since\ l\ is\ constant)$$

$$\Rightarrow mg = V \tag{6}$$

By linearising eq. (4), we get:

$$V\theta l - Hl = J\frac{d^2\theta}{dt^2} \tag{7}$$

Eliminating H and V in eq. (7) using eq. (5) and (6), we get:

$$mg\theta l - m\frac{d^2}{dt^2}(x + l\theta)l = J\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \left(J + ml^2\right)\frac{d^2\theta}{dt^2} + ml\frac{d^2x}{dt^2} - mgl\theta = 0 \tag{8}$$

Eliminating H in eq. (1) using eq. (5), we get:

$$u - m\frac{d^2}{dt^2}(x + l\theta) - B\frac{dx}{dt} = M\frac{d^2x}{dt^2}$$

$$\Rightarrow (M + m)\frac{d^2x}{dt^2} + B\frac{dx}{dt} + ml\frac{d^2\theta}{dt^2} = u \tag{9}$$

Obtaining transfer function for the system using Eq. (8) and (9) as these equations describe the linearised model of the pendulum on cart system.

Taking Laplace transform of equation (8), we get:

$$\left(J + ml^2\right)s^2\Theta(s) + ml\,s^2X(s) - mgl\,\Theta(s) = 0$$

Since $L[\theta(t)] = \Theta(s)\ and\ L[x(t)] = X(s)$ (taking initial condition as 0)

$$\Rightarrow \left\{\left(J + ml^2\right)s^2 - mgl\right\}\Theta(s) + ml\,s^2X(s) = 0$$

$$\Rightarrow X(s) = -\frac{\left(J+ml^2\right)s^2 - mgl}{mls^2}\Theta(s) \tag{10}$$

Taking Laplace transform of equation (9), we get:

$$(M + m)s^2X(s) + BsX(s) + ml\,s^2\Theta(s) = U(s) \tag{11}$$

To get transfer function between angular displacement($\Theta(s)$) of pendulum and external force($U(s)$) applied on the cart, we will eliminate X(s) from eq. (11) using eq. (11).

$$-\left((M + m)s + B\right)\left\{\frac{\left(J+ml^2\right)s^2 - mgl}{mls}\right\}\Theta(s) + ml\,s^2\Theta(s) = U(s)$$

$$\Rightarrow \frac{m^2l^2s^3 - (M+m)\left\{\left(J+ml^2\right)s^3 - mgls\right\} + B\left\{\left(J+ml^2\right)s^2 - mgl\right\}}{mls}\Theta(s) = U(s)$$

Assuming there is negligible friction between the surface of the cart and the track, i.e., B = 0

$$\Rightarrow \frac{m^2l^2s^3 - (M+m)\left\{\left(J+ml^2\right)s^3 - mgls\right\}}{mls}\Theta(s) = U(s)$$

$$\Rightarrow \frac{\left[\left(m^2l^2 - (M+m)\left(J+ml^2\right)\right)s^2 + (M+m)mgl\right]}{ml}\Theta(s) = U(s)$$

$$\Rightarrow \frac{\Theta(s)}{U(s)} = \frac{ml}{\left[\left(m^2l^2 - (M+m)\left(J+ml^2\right)\right)s^2 + (M+m)mgl\right]} \tag{12}$$

PID controller

A Proportional-Integral-Derivative(PID) controller is a feedback control mechanism widely used in industrial, engineering, and robotic applications to regulate a system's behavior. It continuously monitors the system's output, compares it to a desired reference or setpoint value, and generates an error signal that adjusts the system's input. A PID controller can be an effective tool to balance an inverted pendulum by continuously measuring the pendulum's angle and using that information to precisely adjust the cart's position in real time.

The PID controller consists of three control terms: proportional, integral, and derivative.

Proportional term: Computes an output that is proportional to the error signal. The larger the error, the larger the correction signal. It would adjust the cart's position in proportion to the pendulum's angle. This term provides a quick initial response to disturbances but may result in overshooting or instability.

$$P_{out} = K_p e(t)$$

$where, \quad K_p = proportional\ constant = 1000$

$and\ e(t) = angle\ between\ the\ pendulum's\ position\ and\ vertical\ at\ a\ particular\ time$

Integral term: Integrates the error over time, which can help eliminate steady-state errors by gradually adjusting the cart's position over time. It reduces the system's error over time but may also introduce overshoots or oscillations.

$$I_{out} = K_i \int_0^t e(t)dt$$

$where, \quad K_i = integral\ constant = 0.1$

$and, \int_0^t e(t)dt = Integration\ of\ e(t)\ over\ time$

Derivative term: Computes the rate of change of the error signal, which can help predict the future trend and dampen oscillations. It would predict the future trend of the pendulum's angle and help prevent oscillations. It provides a correction signal that is proportional to the rate of change of the error, helping to reduce overshoot and improve stability.

$$D_{out} = K_d \frac{de(t)}{dt}$$

$where, \quad K_d = derivative\ constant = 500$

$and\ \frac{de(t)}{dt} = derivative\ of\ e(t)\ at\ a\ particular\ time$

The three control terms are combined to generate the final output of the controller, which is used to adjust the system's input. The relative weights of the terms are set by tuning the controller's parameters, which depend on the specific application and system.

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt}$$

An optical rotary encoder would be used to measure the pendulum's angle and provide feedback to the controller to implement the PID controller in the pendulum-on-cart system. The controller would compare the pendulum's angle to a desired setpoint(vertical position) and generate an error signal, which would be used to adjust the cart's position. The controller's parameters would be tuned to optimize the system's performance and stability to keep the pendulum in a stable, vertical position.

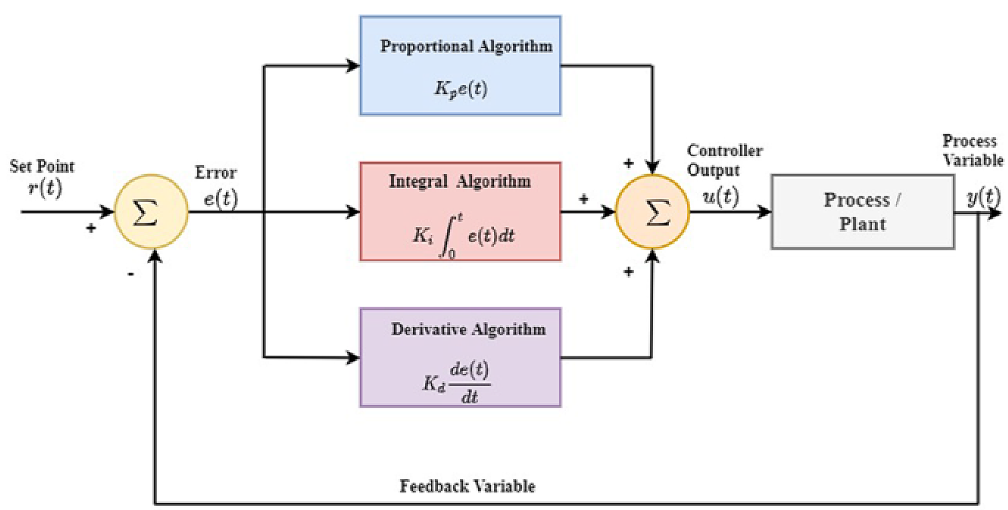

Figure 8 : PID Controller

CODE

```
#define ENCA 3
#define ENCB 2
#define PWM 9
#define IN1 8
#include "ArrbotMonitor.h"
volatile int encoderCount = 0;
volatile int encoderState = 0;
volatile int lastEncoderState = 0;
long prevT = 0;
float eprev = 0;
float eintegral = 0;
void setup() {
Serial.begin(9600);
pinMode(9,OUTPUT);
pinMode(8,OUTPUT);
pinMode(ENCA,INPUT_PULLUP);
pinMode(ENCB,INPUT_PULLUP);
attachInterrupt(digitalPinToInterrupt(ENCA), readEncoder, CHANGE);
attachInterrupt(digitalPinToInterrupt(ENCB), readEncoder, CHANGE);
Serial.println("target pos");
}
void loop() {
// set target position
int target = 0;
// PID constants
float kp = 1000;
float kd = 500;
float ki = 0.1;
// time difference
```

```
long currT = micros();
float deltaT = ((float) (currT - prevT))/( 1.0e6 );
prevT = currT;
// error
int e = (int)(encoderCount/10)-target;
// derivative
float dedt = (e-eprev)/(deltaT);
// integral
eintegral = eintegral + e*deltaT;
// control signal
if(abs(e)>1){
float u = kp*e + kd*dedt + ki*eintegral;
// motor power
float pwr = fabs(u);
if( pwr > 255 ){
pwr = 255;
}
// motor direction
int dir = 1;
if(u<0){
dir = -1;
}
// store previous error
eprev = e;
// signal the motor
setMotor(dir,pwr,PWM,IN1);
}
Serial.print(target);
Serial.print(" ");
Serial.print((int)encoderCount);
```

```
Serial.print(" ");
Serial.println();
}
void setMotor(int dir, int pwmVal, int pwm, int in1){
analogWrite(pwm,pwmVal);
if(dir == 1){
digitalWrite(in1,HIGH);
}
else if(dir == -1){
digitalWrite(in1,LOW);
}
else{
digitalWrite(in1,LOW);
}
}
void readEncoder(){
encoderState = digitalRead(ENCA) << 1 | digitalRead(ENCB);
if (encoderState != lastEncoderState) {
if ((lastEncoderState == 0b00 && encoderState == 0b01) || (lastEncoderState ==
0b01
&&
encoderState == 0b11) || (lastEncoderState == 0b11 && encoderState == 0b10) ||
(lastEncoderState == 0b10 && encoderState == 0b00)) {
encoderCount++;
} else {
encoderCount--;
}
lastEncoderState = encoderState;
}
}
```

RESULTS

- The system was able to balance the stick as long as the stick's deviation from the mean position was small.
- For large angles of deviation, the system was losing control.
- The final PID gains obtained are:

  $K_p = 1000$

  $K_d = 500$

  $K_i = 0.1$

REASONS FOR MISMATCHING

- The initial position had to be set manually at the beginning of the experiment, which could have led to inaccuracies due to possible human error.
- The friction approximation in the guided rails may have been inaccurate, potentially causing discrepancies.
- The PID gains were determined through a trial-and-error method, whereas using a more advanced program to calculate the gains could have eliminated inaccuracies.
- The motor used to move the pendulum had limited torque or speed, which can make it difficult to achieve the desired control resulting in imbalances at higher inclination angles.
- The dynamics of the inverted pendulum are nonlinear, which can make it difficult to design a control algorithm that works well across the entire range of operating conditions.
- There may be a delay between when the control algorithm calculates the necessary control input and when it is applied to the system, leading to errors.

SCOPE OF IMPROVEMENT

- Model-based control: Using a more accurate model of the inverted pendulum system can lead to better control performance. This can be achieved through system identification techniques or using more advanced modeling techniques such as state-space modeling.
- Advanced control techniques: There are several advanced control techniques, such as Model Predictive Control (MPC) or Nonlinear Model Predictive Control (NMPC), that can be used to improve the control performance of the inverted pendulum.
- Sensor fusion: Combining data from multiple sensors such as accelerometers, gyroscopes, and encoders can improve the accuracy of the measurements and reduce the impact of sensor noise.
- Nonlinear control: Using nonlinear control techniques such as sliding mode control or feedback linearization can improve the control performance of the system, especially in cases where the system dynamics are highly nonlinear.

Overall, incorporating these advanced techniques can help improve the control performance of the inverted pendulum system and make it more robust to uncertainties and disturbances.

REFERENCES

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- https://ctms.engin.umich.edu/CTMS/index.php?example=InvertedPendulum&section=SystemModeling
- https://deustotech.github.io/DyCon-Blog/tutorial/dp00/P0003

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