A Study on Classification Tree and Random Forest Methods

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Contents

1 Introduction

The motivation behind this study is a research paper by *Greg Buchak, Gregor Matvos, Tomasz Piskorski, Amit Seru* on "Fintech, Regulatory Arbitrage and the Rise of Shadow Banks". A part of this research paper is taken for the setup of my study. The aim of this paper is to study the relation between various categorical variables related to loan and borrower characteristics with the type of bank they choose i.e. either traditional bank or shadow banks . The shadow banking system consists of lenders, brokers, and other credit intermediaries who fall outside the realm of traditional regulated banking.

The authors have used logistic regression setup for showing the results. The dependent variable y is an indicator variable which takes the value of *'0'* if the loan is originated by a traditional bank and *'100'* if it is a shadow bank. Moreover, the ethnic and racial differences are seen clearly on county level within markets. I have used a similar setup with 10000 observations using different methods . There is another simulation study wherein I have changed the beta coefficient values and performed the tests to see if there are any drastic changes in the final results.

In this study, the aim is to see whether the same results can be derived using different methods. Thus, I have implemented tree based methods using **classification tree** and **random forest** approaches. I have used these methods because classification trees are more informative in case of categorical data and random forest gives more accuracy than logistic regression as it generates the results from a forest of decision trees. As a result, this study aims at using ML methods to quantify a social situation and improve the results further.

2 Methods

2.1 Classification Tree

A Classification Tree is similar to regression tree, except that it is used to predict qualitative response rather than quantitative. Here, we predict that each observaion belongs to the most commonly occuring class of training observations in the region to which it belongs.

The classification error rate is:

$$E = 1 - \max_k \quad (\hat{p}mk)$$

Gini Index

$$G = \sum_{k=1}^{K} \hat{p}mk(1 - \hat{p}mk)$$

a measure of total variance across kth classes. It is a measure of node-purity, a small value indicates that a node contains predominantly observations from a single class.

Random forest overcomes the problem of correlation between the predictors of the trees by forcing each split to only consider a subset of predictors. Therefore, on average (p - m)/p of the splits will not be considered as strong predictors, and so other predictors will have more of a chance. It can be thought of as a process of de-correlating the trees, thereby making the average of the resulting trees less variable and hence more reliable.

2.2 Random Forest

A Random Forest is a meta-estimator that fits a number of decision trees on various sub-samples of the data-sets and uses averaging to improve the predictive accuracy and control over-fitting. The default criterion is gini-index to measure the quality of split.

Random Forests overcomes the problem of correlation between predictors in the trees by forcing each split to consider only a subset of the predictors. Therefore, on average (p-m)/p of the splits will not even consider the strong predictor, and so other predictors will have more of a chance. It can be though of as a process as de-correlating the trees, thereby making the average of the resulting trees less variable and hence more reliable.

3 Data Generating Process

3.1 Baseline Data Generating Process

My data generating process (dgp) is based on the original dataset used by the authors which deals with loan and borrowers characteristics. It is collected under **Home Mortgagae Disclosure Act (HMDA)**, the year into considertaion is 2015. As the descriptive statistics of the paper was not comprehensive enough, I have taken percentages of various classes in every single categorical variable. These proportions along with the means and standard deviations of numerical variables is consistent with the original data. These all variables are stored in a dictionary called ***"stats_dict"*** which is loaded below. This dgp is a simplified version of the original dgp with **10000** observations.

Moreover, I have created dummy variables to indicate which one a set of categorical variables a data point belongs to. So, dummy variables are created for *applicant's sex*, *race*, *loan purpose* and *loan type*. Further, Hispanic population, African-American population and unemployment rate are county-specific numerical variables. They are taken randomly in proportions based on percentages of each variable from top 25% counties and bottom 25% counties as given by the authors. With this dummy variables along the numerical variables, I create my dependent variable data-frame ***"X_with_dummy"***.

The beta values are taken from the paper [Table 3, Panel B, column (1)] and error term is taken arbitraryly.

Dependent variable $\boldsymbol{y}$ is as follows,

$$
\begin{aligned}
y = & \beta_0 x_{female} + \beta_1 x_{others} + \beta_3 x_{amer-ind} + \beta_4 x_{asian} + \beta_5 x_{black} + \beta_7 x_{race-others} + \beta_9 x_{home} + \\
& \beta_{10} x_{refinance} + \beta_{11} x_{FHA} + \beta_{12} x_{VA} + \beta_{13} x_{FSA} + \beta_{14} x_{income} + \beta_{15} x_{loan-amt} + \beta_{c1} x_{hisponic-popu} + \\
& \beta_{c2} x_{af-amer-popu} + \beta_{c3} x_{unempoyment-rate}
\end{aligned}
$$

Variables of x_males, x_white, x_purchase and x_conventional are the base categories for dummies of applicant's sex, applicant's race, loan-purpose and loan-type.

From the distribution of y, it is defined that y is an indicator variable.

$$y = \begin{cases} y > 30 & 1 : Shadow - Bank \\ y < 30 & 0 : Traditional - Bank \end{cases}$$

3.1.1 Import the libraries and fix the variables

[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeClassifier
from sklearn.tree import plot_tree
from sklearn.model_selection import train_test_split
from sklearn.model_selection import cross_val_score
from sklearn.metrics import confusion_matrix , accuracy_score,␣
 ↪classification_report
from sklearn.metrics import plot_confusion_matrix
from sklearn.ensemble import RandomForestClassifier
```

[2]:
```
np.random.seed(52)
```

[4]:
```
stats_dict = np.load('stats_dict.npy', allow_pickle=True).item()
stats_dict
```

[4]:
```
{'male_percent': 0.6667040818163431,
 'female_percent': 0.2757965985393267,
 'white_percent': 0.7893322348810146,
 'american_indian_percent': 0.0056901030222616614,
 'asian_percent': 0.0562785621428574,
 'black_percent': 0.050144595434575195,
 'home_purchase_percent': 0.5462790104187073,
 'home_improv_percent': 0.030798845213172815,
 'refinance_percent': 0.42292214436811987,
 'conventional_percent': 0.7297290190136025,
 'FHA_percent': 0.17020645750186106,
 'VA_percent': 0.0795952598535091,
 'FSA_percent': 0.020469263631027326,
 'loan_amount_avg': 247.17155322640366,
 'loan_amount_std': 217.16054674622413,
 'applicant_income_avg': 107.13825497859747,
 'applicant_income_std': 129.44754502060457}
```

[5]:
```
male_percent = stats_dict['male_percent']
female_percent = stats_dict['female_percent']

white_percent = stats_dict['white_percent']
american_indian_percent = stats_dict['american_indian_percent']
asian_percent = stats_dict['asian_percent']
```

```
black_percent = stats_dict['black_percent']

home_purchase_percent = stats_dict['home_purchase_percent']
home_improv_percent = stats_dict['home_improv_percent']
refinance_percent = stats_dict['refinance_percent']

conventional_percent = stats_dict['conventional_percent']
FHA_percent = stats_dict['FHA_percent']
VA_percent = stats_dict['VA_percent']
FSA_percent = stats_dict['FSA_percent']

loan_amount_avg = stats_dict['loan_amount_avg']
loan_amount_std = stats_dict['loan_amount_std']

applicant_income_avg = stats_dict['applicant_income_avg']
applicant_income_std = stats_dict['applicant_income_std']
```

[6]:
```
num_values = 10000

num_males = int(num_values*male_percent)
num_females = int(num_values*female_percent)
num_others = num_values - (num_females + num_males)
x_applicant_sex = np.concatenate([np.ones(num_males), 2*np.ones(num_females),␣
 ↪3*np.ones(num_others)])
np.random.shuffle(x_applicant_sex)

num_whites = int(num_values*white_percent)
num_american_indians = int(num_values*american_indian_percent)
num_asians = int(num_values*asian_percent)
num_blacks = int(num_values*black_percent)
num_others = num_values - (num_whites + num_american_indians + num_asians +␣
 ↪num_blacks)
x_applicant_race = np.concatenate([np.ones(num_whites), 2*np.
 ↪ones(num_american_indians),
                                   3*np.ones(num_asians), 4*np.ones(num_blacks),␣
 ↪5*np.ones(num_others)])
np.random.shuffle(x_applicant_race)

num_purchase = int(num_values*home_purchase_percent)
num_home_improv = int(num_values*home_improv_percent)
num_refinance = int(num_values*refinance_percent)
x_loan_purpose = np.concatenate([np.ones(num_purchase), 2*np.
 ↪ones(num_home_improv), 3*np.ones(num_refinance)])
print(x_loan_purpose.shape)
x_loan_purpose = np.concatenate([x_loan_purpose, np.ones(num_values -␣
 ↪len(x_loan_purpose))])
np.random.shuffle(x_loan_purpose)
```

```
num_conventional = int(num_values*conventional_percent)
num_FHA = int(num_values*FHA_percent)
num_VA = int(num_values*VA_percent)
num_FSA = int(num_values*FSA_percent)
x_loan_type = np.concatenate([np.ones(num_conventional), 2*np.ones(num_FHA),␣
 ↪3*np.ones(num_VA), 4*np.ones(num_FSA)])
print(x_loan_type.shape)
x_loan_type = np.concatenate([x_loan_type, np.ones(num_values -␣
 ↪len(x_loan_type))])
np.random.shuffle(x_loan_type)

x_loan_amount = np.random.normal(loan_amount_avg, loan_amount_std, size =␣
 ↪num_values)
x_applicant_income = np.random.normal(applicant_income_avg,␣
 ↪applicant_income_std, size = num_values)
```

```
(9998,)
(9998,)
```

[7]:
```
# Numerical covariates - County variables
x_hisponic_cent = np.concatenate([np.random.normal(2.40, size=5000), np.random.
 ↪normal(8.80, size=5000)])
x_african_american_cent = np.concatenate([np.random.normal(1.06, size=5000), np.
 ↪random.normal(2.83, size=5000)])
x_unemployment_rate = np.concatenate([np.random.normal(6.40, size=5000), np.
 ↪random.normal(7.50, size=5000)])
county_assign = np.arange(num_values)
np.random.shuffle(county_assign)

X = pd.DataFrame({'applicant_sex': x_applicant_sex, 'applicant_race':␣
 ↪x_applicant_race,
                  'loan_purpose': x_loan_purpose, 'loan_type': x_loan_type,
                  'loan_amount_000s': x_loan_amount, 'applicant_income_000s':␣
 ↪x_applicant_income,
                  'hisponic_population': x_hisponic_cent[county_assign],␣
 ↪'african_amer_pop': x_african_american_cent[county_assign],
                  'unemp_rate': x_unemployment_rate[county_assign]})
X
```

[7]:

	applicant_sex	applicant_race	loan_purpose	loan_type	\
0	2.0	1.0	3.0	4.0	
1	1.0	1.0	1.0	2.0	
2	1.0	4.0	1.0	1.0	
3	2.0	5.0	3.0	3.0	
4	2.0	1.0	3.0	1.0	
...	...	...	...	...	

```
9995            2.0             1.0           1.0        1.0
9996            1.0             1.0           3.0        2.0
9997            2.0             1.0           3.0        1.0
9998            3.0             1.0           3.0        1.0
9999            2.0             1.0           1.0        1.0

      loan_amount_000s  applicant_income_000s  hisponic_population  \
0           294.317033             247.943429             7.852231
1           423.348033              85.939572             2.447757
2           -44.624067             223.097788             0.782683
3           512.661172             207.261058             2.810321
4            80.391745             402.699774            10.204821
...                ...                    ...                  ...
9995        129.858711             139.927721             0.020925
9996        310.476959            -126.018218             1.504925
9997         10.117781             122.439900             9.385109
9998        148.626139             352.270884             2.099965
9999        -63.866576             -25.187549             1.236570

      african_amer_pop  unemp_rate
0             1.426929    7.478155
1             1.845211    7.045539
2            -0.591626    7.664218
3             0.493730    4.810901
4             2.532785    7.044486
...                ...         ...
9995          1.279827    4.619505
9996          0.704952    6.425754
9997          1.076809    8.736582
9998          1.572109    7.796967
9999         -0.227418    7.706236

[10000 rows x 9 columns]
```

3.2 Dummy varaiables

[8]:
```
X_with_dummy = X.copy()
```

[9]:
```
# Applicant_sex
X_with_dummy.loc[:, 'is_male'] = 0
X_with_dummy.loc[:, 'is_female'] = 0
X_with_dummy.loc[:, 'is_sex_others'] = 0

X_with_dummy.loc[X.applicant_sex == 1, 'is_male'] = 1
X_with_dummy.loc[X.applicant_sex == 2, 'is_female'] = 1
X_with_dummy.loc[X.applicant_sex == 3, 'is_sex_others'] = 1
```

[10]:
```
# Applicant race
X_with_dummy.loc[:, 'is_white'] = 0
X_with_dummy.loc[:, 'is_american_indian'] = 0
X_with_dummy.loc[:, 'is_asian'] = 0
X_with_dummy.loc[:, 'is_black_african'] = 0
X_with_dummy.loc[:, 'is_race_others'] = 0

X_with_dummy.loc[X.applicant_race == 1, 'is_white'] = 1
X_with_dummy.loc[X.applicant_race == 2, 'is_american_indian'] = 1
X_with_dummy.loc[X.applicant_race == 3, 'is_asian'] = 1
X_with_dummy.loc[X.applicant_race == 4, 'is_black_african'] = 1
X_with_dummy.loc[X.applicant_race == 5, 'is_race_others'] = 1
```

[11]:
```
# Purpose
X_with_dummy.loc[:, 'is_purchase'] = 0
X_with_dummy.loc[:, 'is_home'] = 0
X_with_dummy.loc[:, 'is_refinance'] = 0

X_with_dummy.loc[X.loan_purpose == 1, 'is_purchase'] = 1
X_with_dummy.loc[X.loan_purpose == 2, 'is_home'] = 1
X_with_dummy.loc[X.loan_purpose == 3, 'is_refinance'] = 1
```

[12]:
```
# Type
X_with_dummy.loc[:, 'is_conventional'] = 0
X_with_dummy.loc[:, 'is_FHA'] = 0
X_with_dummy.loc[:, 'is_VA'] = 0
X_with_dummy.loc[:, 'is_FSA'] = 0

X_with_dummy.loc[X.loan_type == 1, 'is_conventional'] = 1
X_with_dummy.loc[X.loan_type == 2, 'is_FHA'] = 1
X_with_dummy.loc[X.loan_type == 3, 'is_VA'] = 1
X_with_dummy.loc[X.loan_type == 4, 'is_FSA'] = 1
```

[13]:
```
X_with_dummy
```

[13]:

	applicant_sex	applicant_race	loan_purpose	loan_type	\
0	2.0	1.0	3.0	4.0	
1	1.0	1.0	1.0	2.0	
2	1.0	4.0	1.0	1.0	
3	2.0	5.0	3.0	3.0	
4	2.0	1.0	3.0	1.0	
...	...	...	...	...	
9995	2.0	1.0	1.0	1.0	
9996	1.0	1.0	3.0	2.0	
9997	2.0	1.0	3.0	1.0	
9998	3.0	1.0	3.0	1.0	
9999	2.0	1.0	1.0	1.0	

```
      loan_amount_000s  applicant_income_000s  hisponic_population  \
0           294.317033             247.943429             7.852231
1           423.348033              85.939572             2.447757
2           -44.624067             223.097788             0.782683
3           512.661172             207.261058             2.810321
4            80.391745             402.699774            10.204821
...                ...                    ...                  ...
9995        129.858711             139.927721             0.020925
9996        310.476959            -126.018218             1.504925
9997         10.117781             122.439900             9.385109
9998        148.626139             352.270884             2.099965
9999        -63.866576             -25.187549             1.236570

      african_amer_pop  unemp_rate  is_male  ...  is_asian  is_black_african  \
0             1.426929    7.478155        0  ...         0                 0
1             1.845211    7.045539        1  ...         0                 0
2            -0.591626    7.664218        1  ...         0                 1
3             0.493730    4.810901        0  ...         0                 0
4             2.532785    7.044486        0  ...         0                 0
...                ...         ...      ...  ...       ...               ...
9995          1.279827    4.619505        0  ...         0                 0
9996          0.704952    6.425754        1  ...         0                 0
9997          1.076809    8.736582        0  ...         0                 0
9998          1.572109    7.796967        0  ...         0                 0
9999         -0.227418    7.706236        0  ...         0                 0

      is_race_others  is_purchase  is_home  is_refinance  is_conventional  \
0                  0            0        0             1                0
1                  0            1        0             0                0
2                  0            1        0             0                1
3                  1            0        0             1                0
4                  0            0        0             1                1
...              ...          ...      ...           ...              ...
9995               0            1        0             0                1
9996               0            0        0             1                0
9997               0            0        0             1                1
9998               0            0        0             1                1
9999               0            1        0             0                1

      is_FHA  is_VA  is_FSA
0          0      0       1
1          1      0       0
2          0      0       0
3          0      1       0
4          0      0       0
...      ...    ...     ...
```

```
9995       0       0       0
9996       1       0       0
9997       0       0       0
9998       0       0       0
9999       0       0       0

[10000 rows x 24 columns]
```

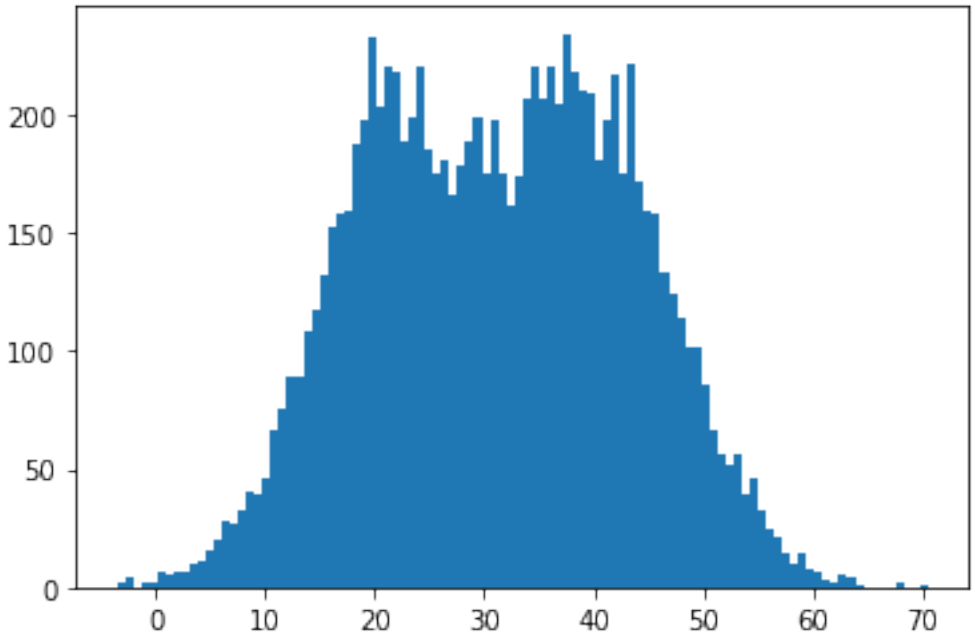
[14]:
```
# Loan and borrower characteristics
betas = [0.163, -4.636, 16.27, -0.227, 3.923, 0.405, 1.363, 7.438, -24.98, -13.
 ↪26, -2.056, 9.418, 2.331, -3.036, -0.00857, 0.00835]

# County characteristics
betas_1 = [2.0, 2.0, 2.0]

len(betas), len(betas_1)
```

[14]:
```
(16, 3)
```

[15]:
```
eps = np.random.normal(scale = 4,size = num_values)
```

[16]:
```
y = betas[0]*X_with_dummy.is_female + betas[1]*X_with_dummy.is_sex_others + \
    betas[3]*X_with_dummy.is_american_indian + betas[4]*X_with_dummy.is_asian + \
    betas[5]*X_with_dummy.is_black_african + betas[7]*X_with_dummy.
 ↪is_race_others + \
    betas[9]*X_with_dummy.is_home + betas[10]*X_with_dummy.is_refinance + \
    betas[11]*X_with_dummy.is_FHA + betas[12]*X_with_dummy.is_VA +␣
 ↪betas[13]*X_with_dummy.is_FSA + \
    betas[14]*X_with_dummy.applicant_income_000s + betas[15]*X_with_dummy.
 ↪loan_amount_000s + \
    betas_1[0]*X_with_dummy.hisponic_population + betas_1[1]*X_with_dummy.
 ↪african_amer_pop + betas_1[2]*X_with_dummy.unemp_rate + \
    eps
```

[17]:
```
plt.hist(y, bins = 100);
```



[18]:
```
y_class = np.where(y > 30, np.ones(num_values), np.zeros(num_values))
```

[19]:
```
Y = pd.DataFrame({'y': y, 'y_class':y_class})
Y
```

[19]:

	y	y_class
0	31.549360	1.0
1	34.105564	1.0
2	13.214264	0.0
3	25.321486	0.0
4	41.532718	1.0
...	...	...
9995	4.878992	0.0
9996	29.398602	0.0
9997	36.112152	1.0
9998	12.506675	0.0
9999	19.614486	0.0

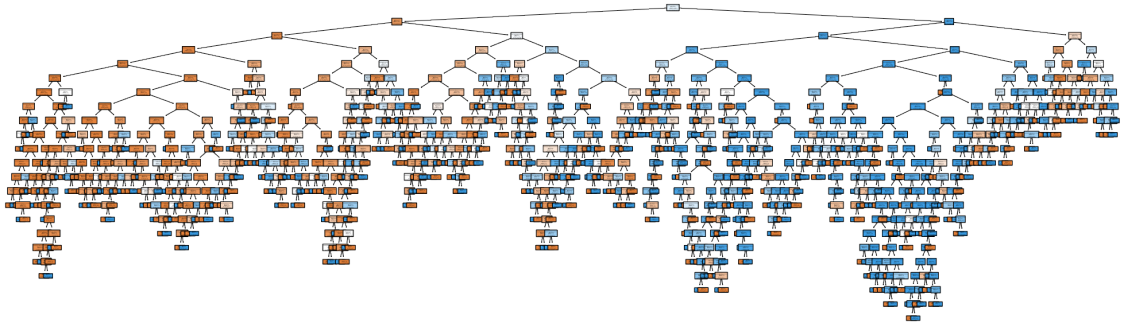
[10000 rows x 2 columns]

3.3 Training and testing data

[20]:
```
## Split the data into training and testing data-sets
X_train, X_test, y_train, y_test = train_test_split(X_with_dummy, Y.y_class,␣
 ↪random_state = 60)
```

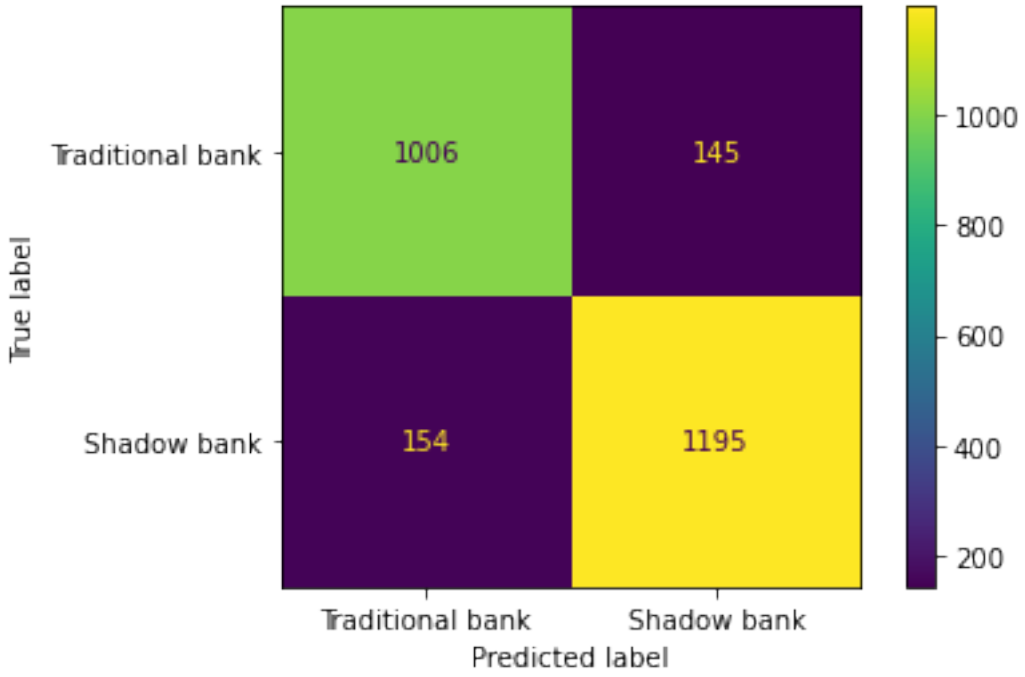
```
#create a decision tree and fit it to training data
clf_dt = DecisionTreeClassifier(random_state = 60)
clf_dt = clf_dt.fit(X_train,y_train)
```

[21]:
```
##plot the long tree
plt.figure(figsize = (25,7.5))
plot_tree(clf_dt,
          filled = True,
          rounded = True,
          class_names = ["Traditional bank","Shadow bank"],
          feature_names = X_with_dummy.columns);
```



[22]:
```
##Plot confusion matrix
plot_confusion_matrix(clf_dt,X_test,y_test,display_labels=["Traditional bank",␣
 ↪"Shadow bank"])
```

[22]:
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x29f9c4a2670>
```



[23]:
```
y_pred = clf_dt.predict(X_test)
accuracy_long_tree = accuracy_score(y_test, y_pred )
accuracy_long_tree
```

[23]: 0.8804

3.3.1 Interpretation

The long tree is very hard to interpret but the confusion matrix tells us that the model has predicted 87.4% of traditional bank borrowers correctly. Similarly, it has correctly predicted 88.5% of shadow bank borrowers. The accuracy of this model is **88.04%**.

3.3.2 Cost complexity pruning

Decision trees are known for the problem of overfitting, thus we need to prune the tree in order to optimize it. Pruning a tree with cost complexity pruning can simplify the whole process of finding a smaller tree that improves the accuracy with the testing data set.

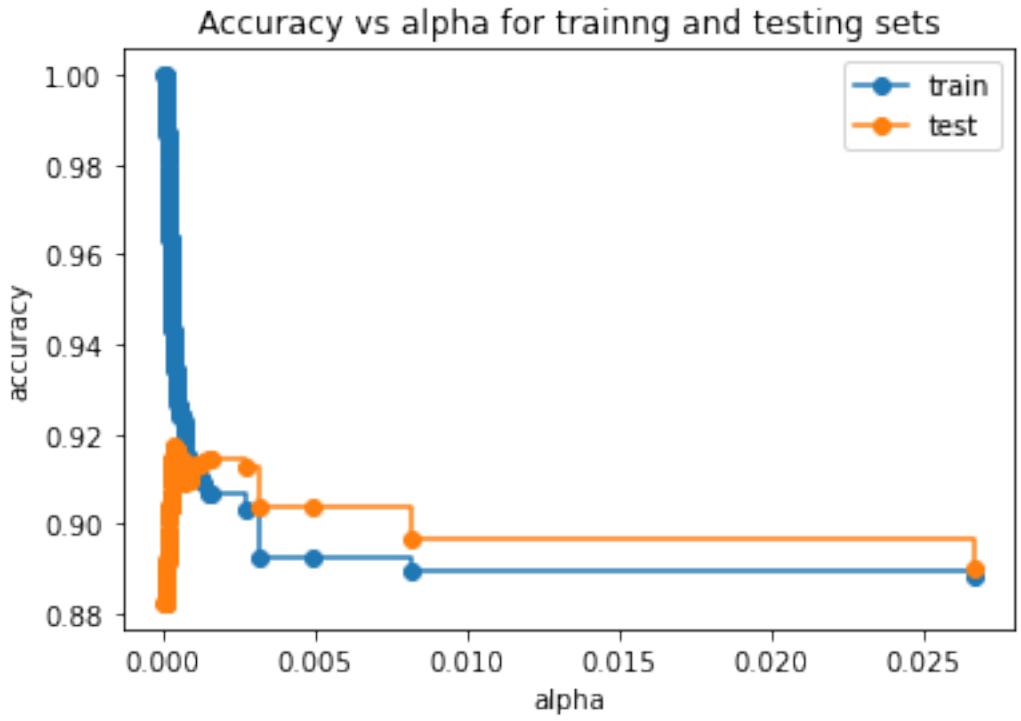
We need to find the value of pruning parameter *"alpha"* which controls the pruning. We find out the value of *"alpha"* by plotting the accuracy of the tree as a function of different values. It is done for both, training and testing datasets.

Here the maximum value of alpha is omitted as it will lead to pruning of all leaves, leaving only a root and not a tree.

[24]:
```
path = clf_dt.cost_complexity_pruning_path(X_train, y_train)
ccp_alphas, impurities = path.ccp_alphas, path.impurities
#print(ccp_alphas, ccp_alphas.shape)
```

[25]:
```
ccp_alphas = ccp_alphas[:-1]

clf_dts = []

for ccp_alpha in ccp_alphas:
    clf_dt = DecisionTreeClassifier(random_state= 0,
                                    ccp_alpha = ccp_alpha)
    clf_dt.fit(X_train, y_train)
    clf_dts.append(clf_dt)
```

[26]:
```
train_scores = [clf_dt.score(X_train,y_train) for clf_dt in clf_dts]
test_scores = [clf_dt.score(X_test,y_test) for clf_dt in clf_dts]

fid, ax = plt.subplots()
ax.set_xlabel("alpha")
ax.set_ylabel("accuracy")
ax.set_title("Accuracy vs alpha for trainng and testing sets")
ax.plot(ccp_alphas,train_scores, marker='o', label="train",␣
 ↪drawstyle="steps-post")
ax.plot(ccp_alphas,test_scores, marker='o', label="test", drawstyle="steps-post")
ax.legend()
plt.show()
```



Here, it can be seen that the accuracy of training data-set is declining from 1.0 which is 100% to 88% and the testing accuracy is increasing till 92% and decline from there. I need to find such a value of alpha which gives me highest testing accuracy. So, I take the argmax of *ccp_alpha* and *test_scores* and use that alpha value further in the analysis.

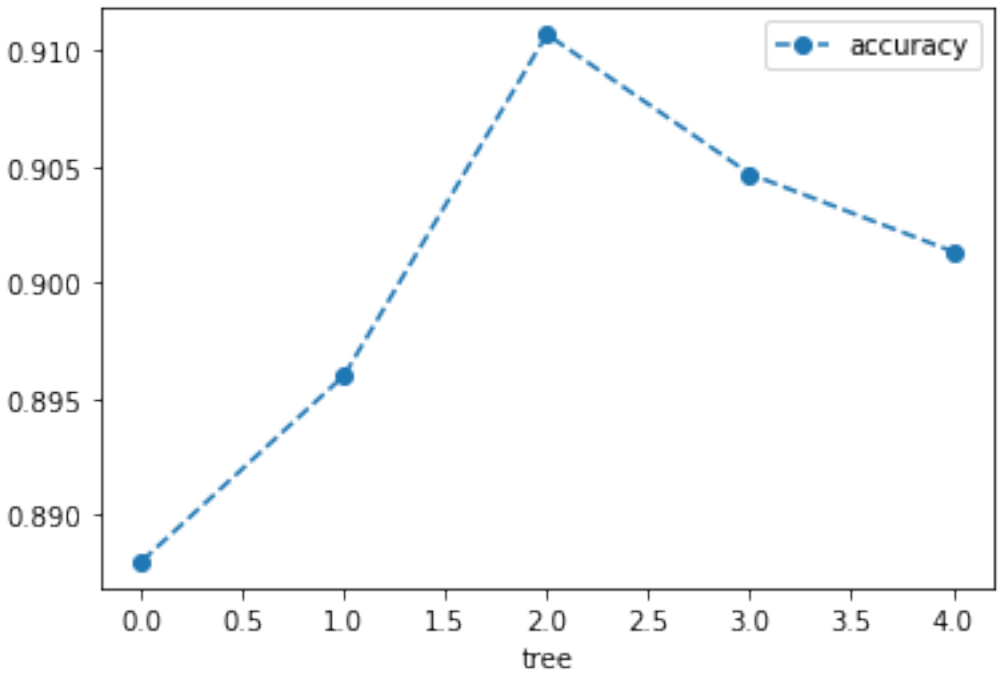
Further, to improve the performance of the model, I use cross-validation with the alpha calculated below. **Cross-Validation** is a resampling method used to improve the model by randomly spliting the sample into training and testing data-sets. It is a popular method because it is simple to understand and generally results in a less biased or less optimistic estimate of the model skill than other methods, such as a simple train/test split.

[27]:
```
ccp_alphas[np.argmax(test_scores)]
```

[27]:
```
0.00037491216655822926
```

[28]:
```
## Cross-validation

clf_dt = DecisionTreeClassifier(random_state = 42, ccp_alpha= 0.
↪00037491216655822926)
scores = cross_val_score(clf_dt, X_train, y_train, cv = 5)

df = pd.DataFrame(data={'tree': range(5), 'accuracy': scores})

df.plot(x = 'tree', y= 'accuracy', marker= 'o', linestyle= '--')
```

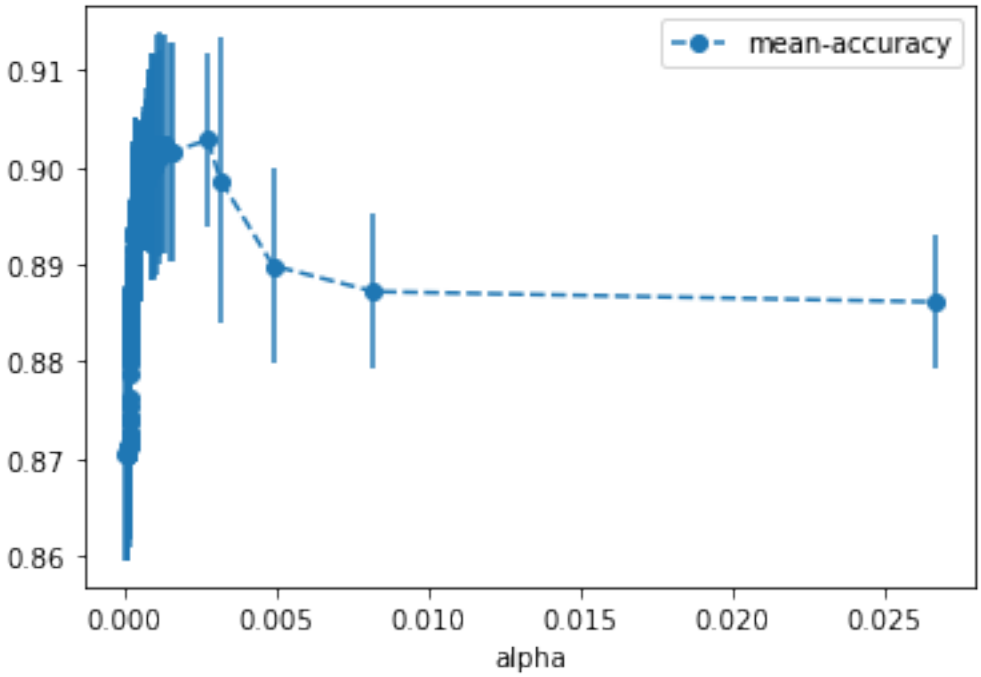
[28]:
```
<AxesSubplot:xlabel='tree'>
```



This graph shows that using different training and testing data with same alpha results in different accuracies which means the pruning parameter is sensitive to datasets. So, instead of picking a single training and testing dataset, lets try to use cross-validation to find the optimal alpha

[29]:
```
## create an array to store the results of each fold during cross validation
alpha_loop_values = []

## for each candidate value for alpha, we will run 5-fold cross validations.
## then we will store all the mean and standard deviation of the scores␣
 ↪(accuracies) for each call
## to cross-val-score in alpha_loop_values
for ccp_alpha in ccp_alphas:
    clf_dt = DecisionTreeClassifier(random_state=0, ccp_alpha=ccp_alpha)
    scores = cross_val_score(clf_dt, X_train, y_train, cv = 4, n_jobs=6)
    alpha_loop_values.append([ccp_alpha, np.mean(scores), np.std(scores)])

## now we draw a graph of mean and standard deviations of the scores
## for each candidate value for alpha
alpha_results = pd.DataFrame(alpha_loop_values,
                             columns = ['alpha','mean-accuracy','std'])

alpha_results.plot(x = 'alpha',
                   y = 'mean-accuracy',
                   yerr = 'std',
                   marker = 'o',
                   linestyle = '--')
```

[29]:
```
<AxesSubplot:xlabel='alpha'>
```



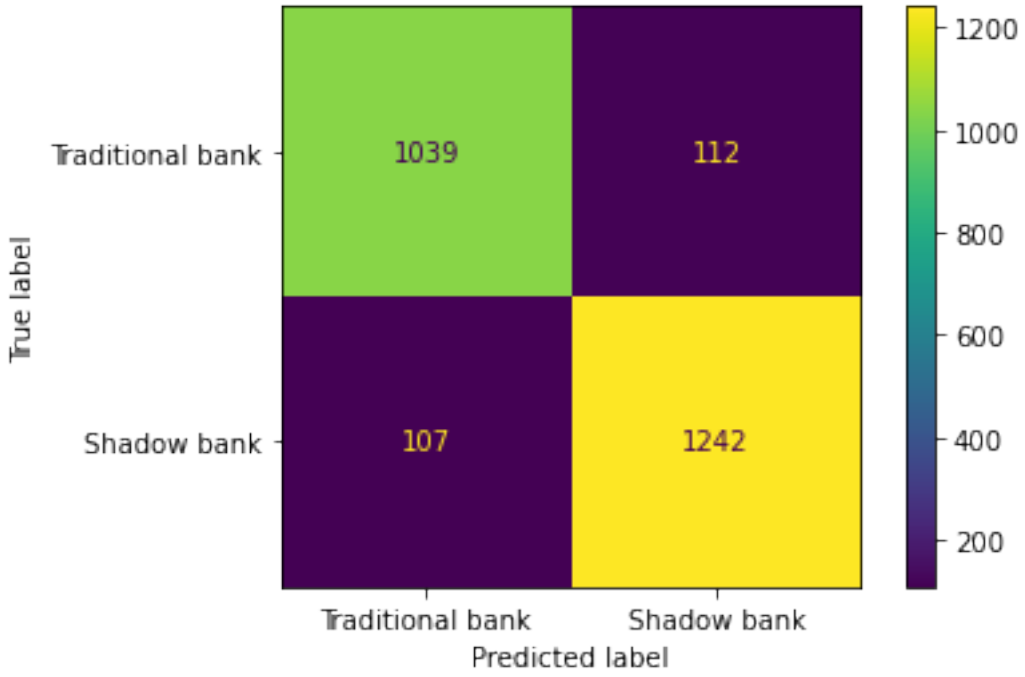
[30]:
```
# To find the ideal value of alpha
alpha_results.loc[alpha_results['mean-accuracy'].idxmax()].alpha
```

[30]:
```
0.002758293022190019
```

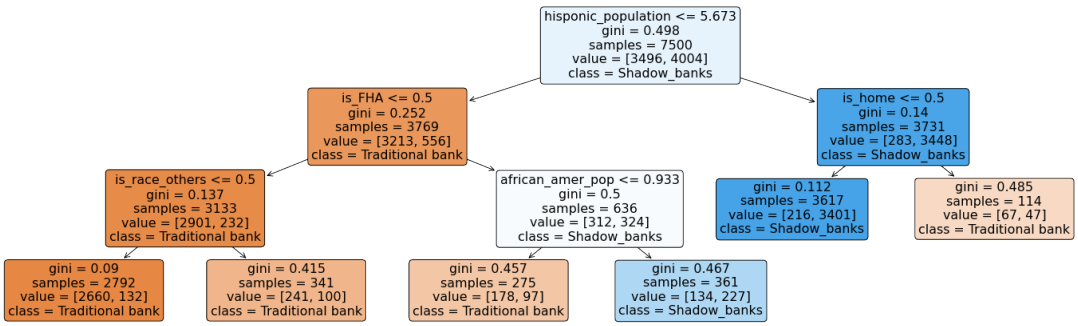
[31]:
```
# We will use the value of ccp_alpha from above to prune the long tree
clf_dt_prunned = DecisionTreeClassifier(random_state = 42,
                                        ccp_alpha = 0.002758293022190019)
clf_dt_prunned = clf_dt_prunned.fit(X_train,y_train)
```

[32]:
```
plot_confusion_matrix(clf_dt_prunned,
                      X_test,
                      y_test,
                      display_labels = ["Traditional bank","Shadow bank"])
```

[32]:
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x29fa22b2910>
```



[33]:
```
plt.figure(figsize = (25,7.5))
plot_tree(clf_dt_prunned,
          filled = True,
          rounded = True,
          class_names = ["Traditional bank","Shadow_banks"],
          feature_names = X_with_dummy.columns);
plt.savefig("pruned_tree.pdf")
```



[34]:
```
y_pred = clf_dt_prunned.predict(X_test)
y_pred
```

[34]:
```
array([0., 1., 0., ..., 1., 0., 1.])
```

[35]:
```
accuracy = accuracy_score(y_test, y_pred )
accuracy
```

[35]:
```
0.9124
```

From the confusion matrix, we can calculate that the traditional bank borrowers are 1039+112 = 1151, 1039 i.e 90.2% borrowers are correctly classified. For the Shadow bank borrowers which are 107+1242 = 1349, 1242 i.e 92.06% are correctly classified, an improvement of around 2% in both the classes.

Pruned-Tree Analysis: * It can be seen the majority class is **shadow bank** which is the class of the first node. The tree is branching from the county-specific Hisponic population variable with the majority class of shadow bank. This supports the authors results under examination. Further branching is happening from loan type and loan purpose variables with one variable inclined to traditional bank and another to shadow bank.

- The gini index at the start of the tree is 0.498 which is high enough to start the branching but such large value of gini also gives weak class majority. It can be seen that as the gini index goes to a smaller number, the color of that node becomes darker. This implies that the specific node contains predominantly observations from a single class.

The **accuracy** of the model is increased from 88.04% to **91.24%** due to pruning of the decision tree

3.4 Random Forest

For random forest, I have used sklearn's ensemble *RandomForestClassifier()*. the dgp is the same as above. I split the data into training and testing data-sets.

Here, I create 120 decision trees with n_estimator set to 120. Further, I predict y with "y_pred" and count the values of 0s and 1s to know the majority class of random forest. Finally I plot the confusion matrix and the accuracy of the model.

[36]:
```
##Split the data into training and testing data-sets
X_train, X_test, y_train, y_test = train_test_split(X_with_dummy, Y.y_class,␣
 ↪random_state = 60)
```

[37]:
```
## Applying Random-Forest classifier with 120 trees with the tarining and␣
 ↪testing data-sets
rf = RandomForestClassifier(n_estimators = 120, random_state = 60, n_jobs= 6,␣
 ↪verbose = 1)
rf.fit(X_train, y_train)
```

```
[Parallel(n_jobs=6)]: Using backend ThreadingBackend with 6 concurrent workers.
[Parallel(n_jobs=6)]: Done  38 tasks      | elapsed:    0.0s
[Parallel(n_jobs=6)]: Done 120 out of 120 | elapsed:    0.1s finished
```

[37]:
```
RandomForestClassifier(n_estimators=120, n_jobs=6, random_state=60, verbose=1)
```

[38]:
```
y_pred = rf.predict(X_test)
y_pred
```

```
[Parallel(n_jobs=6)]: Using backend ThreadingBackend with 6 concurrent workers.
[Parallel(n_jobs=6)]: Done  38 tasks      | elapsed:    0.0s
[Parallel(n_jobs=6)]: Done 120 out of 120 | elapsed:    0.0s finished
```

[38]:
```
array([0., 1., 0., ..., 1., 0., 1.])
```

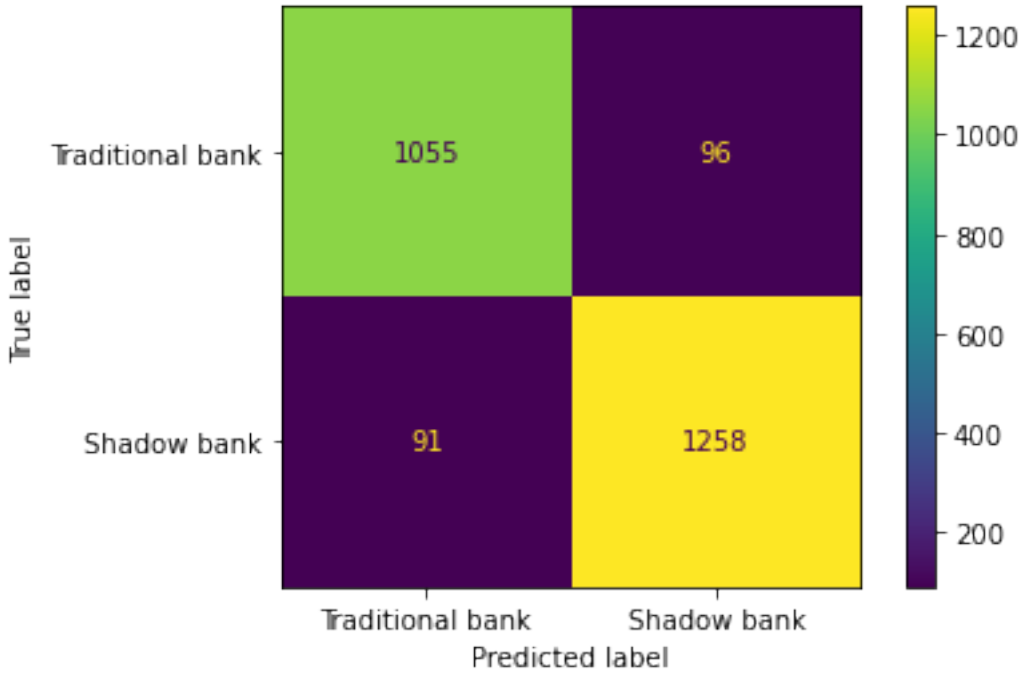
[39]:
```
y_pred.tolist().count(0), y_pred.tolist().count(1)
```

[39]:
```
(1146, 1354)
```

[40]:
```
plot_confusion_matrix(rf,
                      X_test,
                      y_test,
                      display_labels = ["Traditional bank","Shadow bank"])
```

```
[Parallel(n_jobs=6)]: Using backend ThreadingBackend with 6 concurrent workers.
[Parallel(n_jobs=6)]: Done  38 tasks      | elapsed:    0.0s
[Parallel(n_jobs=6)]: Done 120 out of 120 | elapsed:    0.0s finished
```

[40]:
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x29fa1af3070>
```



[41]:
```
print(classification_report(y_test, y_pred))
print( "Accuracy_Score ",accuracy_score(y_test, y_pred))
```

```
              precision    recall  f1-score   support
```

```
         0.0       0.92      0.92      0.92      1151
         1.0       0.93      0.93      0.93      1349

    accuracy                           0.93      2500
   macro avg       0.92      0.92      0.92      2500
weighted avg       0.93      0.93      0.93      2500

Accuracy_Score  0.9252
```

Interpretation:

From the counts of 0s and 1s, it is clear that the majority class is of 1s (1354 as against 1146) which is of **shadow bank**. This is consistent with the classification tree analysis.

The confusion matrix states that the model correctly classifies **91.6%** of traditional bank borrowers and **93.2%** of shadow bank borrowers. The preditions have improved from earlier. In random forest, one can also use ccp_alpha however with the use of ccp_alpha, the results stays unchanged.

The use of random forest has increased the accuracy of the model from 91.2% to **92.5%**.

4 Simulation Study : Changing the beta values

In this simulation study, I have changed the values of beta coefficents to see overall impact on the model. I also wanted to see if the majority class changes if beta values are changed.

So the y variable is the same,

$$\begin{aligned} y_1 =& \beta_0 x_{female} + \beta_1 x_{others} + \beta_3 x_{amer-ind} + \beta_4 x_{asian} + \beta_5 x_{black} + \beta_7 x_{race-others} + \beta_9 x_{home} + \\ & \beta_{10} x_{refinance} + \beta_{11} x_{FHA} + \beta_{12} x_{VA} + \beta_{13} x_{FSA} + \beta_{14} x_{income} + \beta_{15} x_{loan-amt} + \beta_{c1} x_{hisponic-popu} + \\ & \beta_{c2} x_{af-amer-popu} + \beta_{c3} x_{unempoyment-rate} \end{aligned}$$

Variables of x_males, x_white, x_purchase and x_conventional are still the base categories for dummies of applicant's sex, applicant's race, loan-purpose and loan-type.

From the distribution of y, it is defined that y is an indicator variable.

$$y_1 = \begin{cases} y_1 > -5 & 1 : Shadow - Bank \\ y_1 < -5 & 0 : Traditional - Bank \end{cases}$$

[42]:
```
# Numerical covariates - County variables
x_hisponic_cent = np.concatenate([np.random.normal(2.40, size=5000), np.random.
 ↪normal(8.80, size=5000)])
x_african_american_cent = np.concatenate([np.random.normal(1.06, size=5000), np.
 ↪random.normal(2.83, size=5000)])
x_unemployment_rate = np.concatenate([np.random.normal(6.40, size=5000), np.
 ↪random.normal(7.50, size=5000)])
county_assign = np.arange(num_values)
```

```
np.random.shuffle(county_assign)

X = pd.DataFrame({'applicant_sex': x_applicant_sex, 'applicant_race':␣
 ↪x_applicant_race,
                  'loan_purpose': x_loan_purpose, 'loan_type': x_loan_type,
                  'loan_amount_000s': x_loan_amount, 'applicant_income_000s':␣
 ↪x_applicant_income,
                  'hisponic_population': x_hisponic_cent[county_assign],␣
 ↪'african_amer_pop': x_african_american_cent[county_assign],
                  'unemp_rate': x_unemployment_rate[county_assign]})
X
```

[42]:

	applicant_sex	applicant_race	loan_purpose	loan_type	\
0	2.0	1.0	3.0	4.0	
1	1.0	1.0	1.0	2.0	
2	1.0	4.0	1.0	1.0	
3	2.0	5.0	3.0	3.0	
4	2.0	1.0	3.0	1.0	
...	...	...	...	...	
9995	2.0	1.0	1.0	1.0	
9996	1.0	1.0	3.0	2.0	
9997	2.0	1.0	3.0	1.0	
9998	3.0	1.0	3.0	1.0	
9999	2.0	1.0	1.0	1.0	

	loan_amount_000s	applicant_income_000s	hisponic_population	\
0	294.317033	247.943429	9.557256	
1	423.348033	85.939572	9.312414	
2	-44.624067	223.097788	8.154374	
3	512.661172	207.261058	8.044603	
4	80.391745	402.699774	2.944824	
...	...	...	...	
9995	129.858711	139.927721	2.018594	
9996	310.476959	-126.018218	0.570555	
9997	10.117781	122.439900	9.551044	
9998	148.626139	352.270884	8.589867	
9999	-63.866576	-25.187549	7.194338	

	african_amer_pop	unemp_rate
0	3.049126	6.751999
1	2.283303	6.590465
2	1.795839	6.523637
3	5.727903	7.733771
4	0.397821	4.856965
...	...	...
9995	2.081608	7.792276
9996	1.223242	6.373910

```
9997          4.239887    5.885079
9998          2.193260    8.392850
9999          2.771409    8.057605

[10000 rows x 9 columns]
```

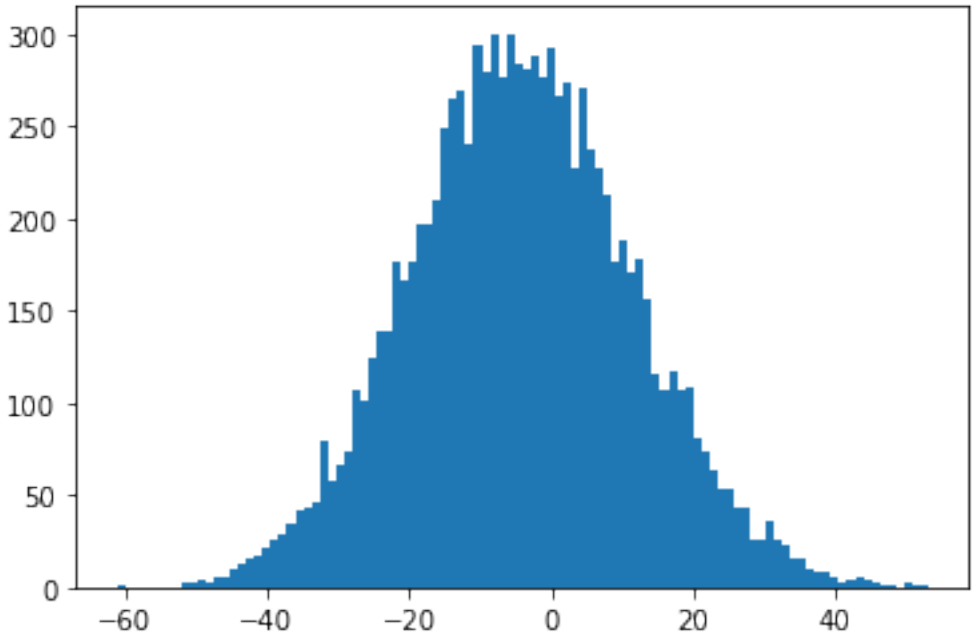
[43]:
```
# Loan and borrower characteristics
betas = [0.005, -2.6, -18.27, -1.5, 15.3, -1.45, 2.6, 15.5, -30.3, -9.8, -4.5, 8.
 ↪6, 2.8, -5.2, -0.01, 0.055]

# County characteristics
betas_1 = [1.0, -1.5, -3.0]

len(betas), len(betas_1)
```

[43]:
```
(16, 3)
```

[44]:
```
eps = np.random.normal(scale = 6,size = num_values)
```

[45]:
```
y1 = betas[0]*X_with_dummy.is_female + betas[1]*X_with_dummy.is_sex_others + \
    betas[3]*X_with_dummy.is_american_indian + betas[4]*X_with_dummy.is_asian + \
    betas[5]*X_with_dummy.is_black_african + betas[7]*X_with_dummy.
 ↪is_race_others + \
    betas[9]*X_with_dummy.is_home + betas[10]*X_with_dummy.is_refinance + \
    betas[11]*X_with_dummy.is_FHA + betas[12]*X_with_dummy.is_VA +␣
 ↪betas[13]*X_with_dummy.is_FSA + \
    betas[14]*X_with_dummy.applicant_income_000s + betas[15]*X_with_dummy.
 ↪loan_amount_000s + \
    betas_1[0]*X_with_dummy.hisponic_population + betas_1[1]*X_with_dummy.
 ↪african_amer_pop + betas_1[2]*X_with_dummy.unemp_rate + \
    eps
```

[46]:
```
plt.hist(y1, bins = 100);
```



[47]:
```
y_class = np.where(y1 > -5, np.ones(num_values), np.zeros(num_values))
```

[48]:
```
Y = pd.DataFrame({'y': y1, 'y_class':y_class})
Y
```

[48]:

	y	y_class
0	-4.044994	1.0
1	4.420076	1.0
2	-34.071280	0.0
3	18.840263	1.0
4	-8.439401	0.0
...	...	...
9995	-5.051181	0.0
9996	13.560076	1.0
9997	-15.686173	0.0
9998	-29.473957	0.0
9999	-27.482584	0.0

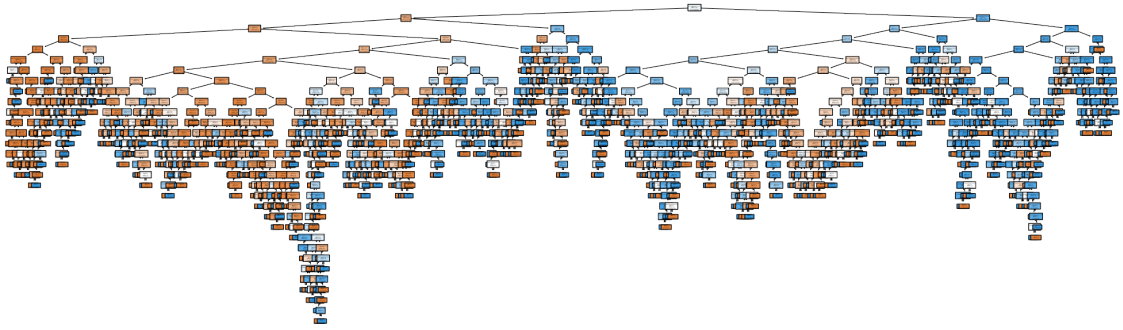
[10000 rows x 2 columns]

4.1 Training and testing datasets

[49]:
```
X_train, X_test, y_train, y_test = train_test_split(X_with_dummy, Y.y_class,␣
↪random_state = 60)
```

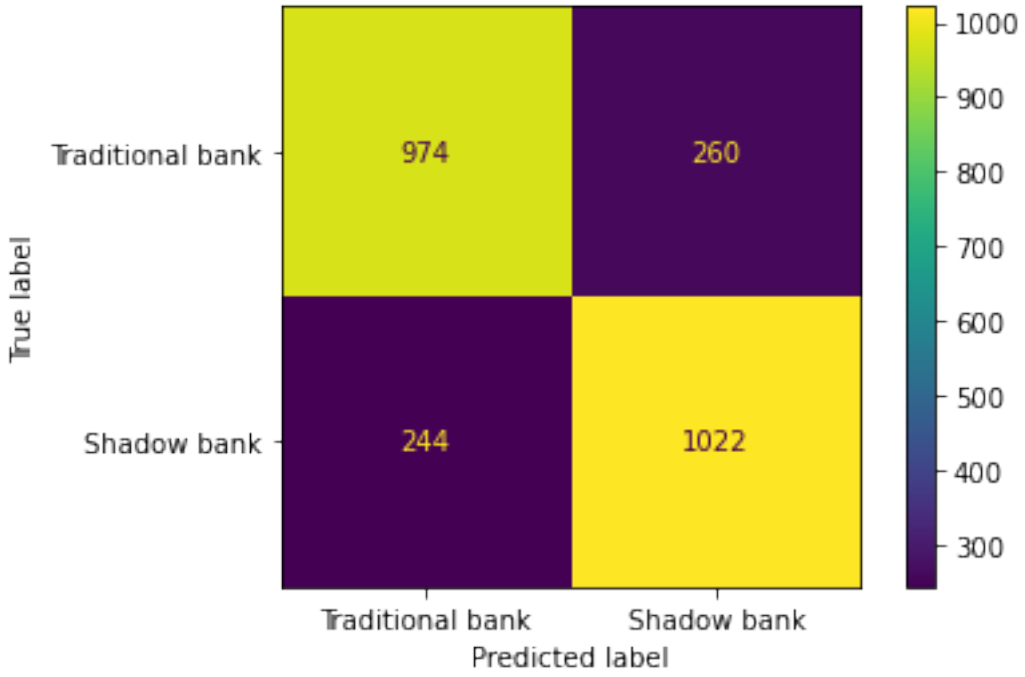
```
#create a decision tree and fit it to training data
clf_dt = DecisionTreeClassifier(random_state = 60)
clf_dt = clf_dt.fit(X_train,y_train)
```

[50]:
```
##plot the long tree
plt.figure(figsize = (25,7.5))
plot_tree(clf_dt,
          filled = True,
          rounded = True,
          class_names = ["Traditional bank","Shadow bank"],
          feature_names = X_with_dummy.columns);
```



[51]:
```
##Plot confusion matrix
plot_confusion_matrix(clf_dt,X_test,y_test,display_labels=["Traditional bank",␣
 ↪"Shadow bank"])
```

[51]: <sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x29fa27c3e80>



[52]:
```
y_pred = clf_dt.predict(X_test)
accuracy_long_tree = accuracy_score(y_test, y_pred )
accuracy_long_tree
```

[52]: 0.7984

Interpretation:

The long tree is very hard to interpret but the confusion matrix tells us that the model has predicted 78.9% of traditional bank borrowers correctly. Similarly, it has correctly predicted 80.7% of shadow bank borrowers. The accuracy of this model is 79.84%

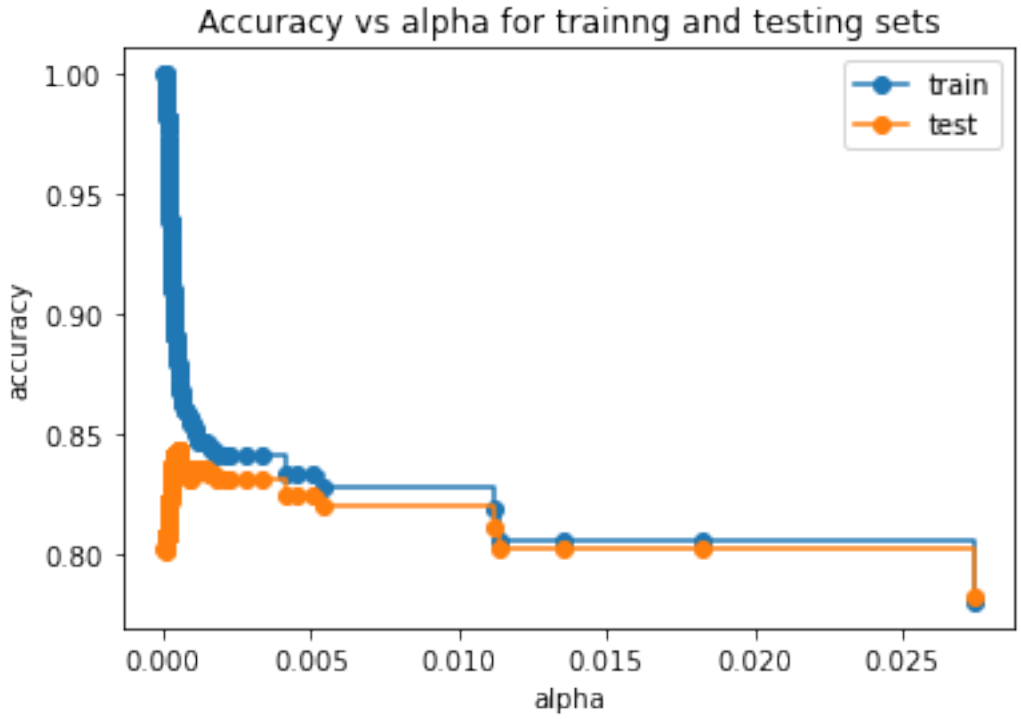
4.2 Cost Complexity Pruning

[53]:
```
path = clf_dt.cost_complexity_pruning_path(X_train, y_train)
ccp_alphas, impurities = path.ccp_alphas, path.impurities
#print(ccp_alphas, ccp_alphas.shape)
```

[54]:
```
ccp_alphas = ccp_alphas[:-1]

clf_dts = []

for ccp_alpha in ccp_alphas:
    clf_dt = DecisionTreeClassifier(random_state= 0,
                                    ccp_alpha = ccp_alpha)
```

```
    clf_dt.fit(X_train, y_train)
    clf_dts.append(clf_dt)
```

[55]:
```
train_scores = [clf_dt.score(X_train,y_train) for clf_dt in clf_dts]
test_scores = [clf_dt.score(X_test,y_test) for clf_dt in clf_dts]

fid, ax = plt.subplots()
ax.set_xlabel("alpha")
ax.set_ylabel("accuracy")
ax.set_title("Accuracy vs alpha for trainng and testing sets")
ax.plot(ccp_alphas,train_scores, marker='o', label="train",␣
 ↪drawstyle="steps-post")
ax.plot(ccp_alphas,test_scores, marker='o', label="test", drawstyle="steps-post")
ax.legend()
plt.show()
```



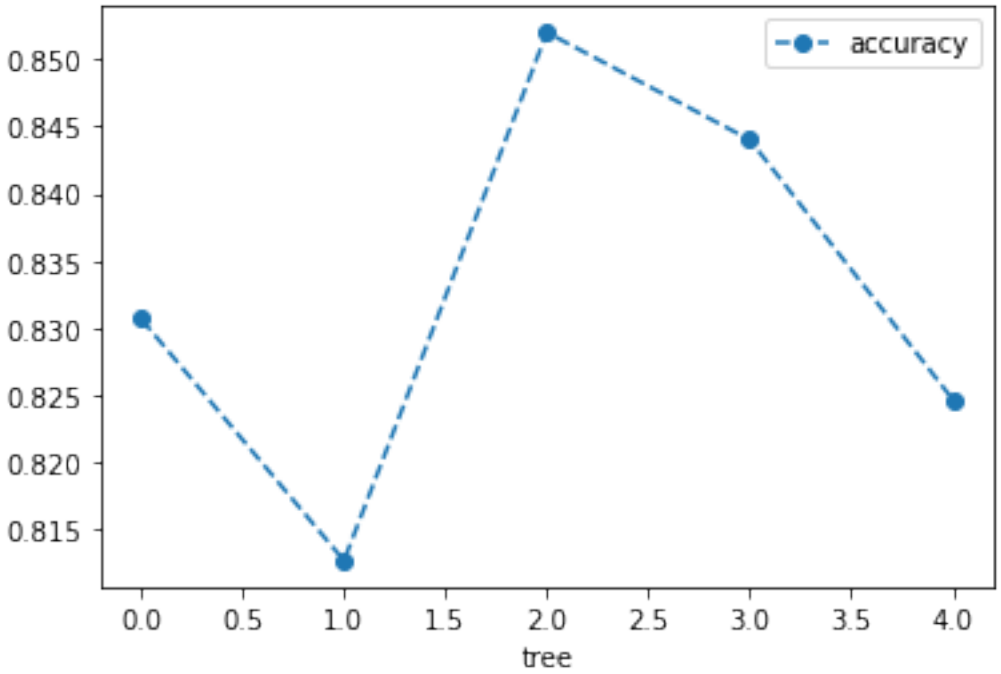
Here, it can be seen that the accuracy of training data-set is declining from 1.0 which is 100% to 75% and the testing accuracy is increasing till 74% and decline from there. I need to find such a value of alpha which gives me highest testing accuracy. So, I take the argmax of ccp_alpha and test_scores and use that alpha value further in the analysis

[56]:
```
ccp_alphas[np.argmax(test_scores)]
```

[56]:
```
0.0005302431670708516
```

[57]:
```
## Cross-validation

clf_dt = DecisionTreeClassifier(random_state = 42, ccp_alpha= 0.
↪0005302431670708516)
scores = cross_val_score(clf_dt, X_train, y_train, cv = 5)

df = pd.DataFrame(data={'tree': range(5), 'accuracy': scores})

df.plot(x = 'tree', y= 'accuracy', marker= 'o', linestyle= '--')
```

[57]:
```
<AxesSubplot:xlabel='tree'>
```

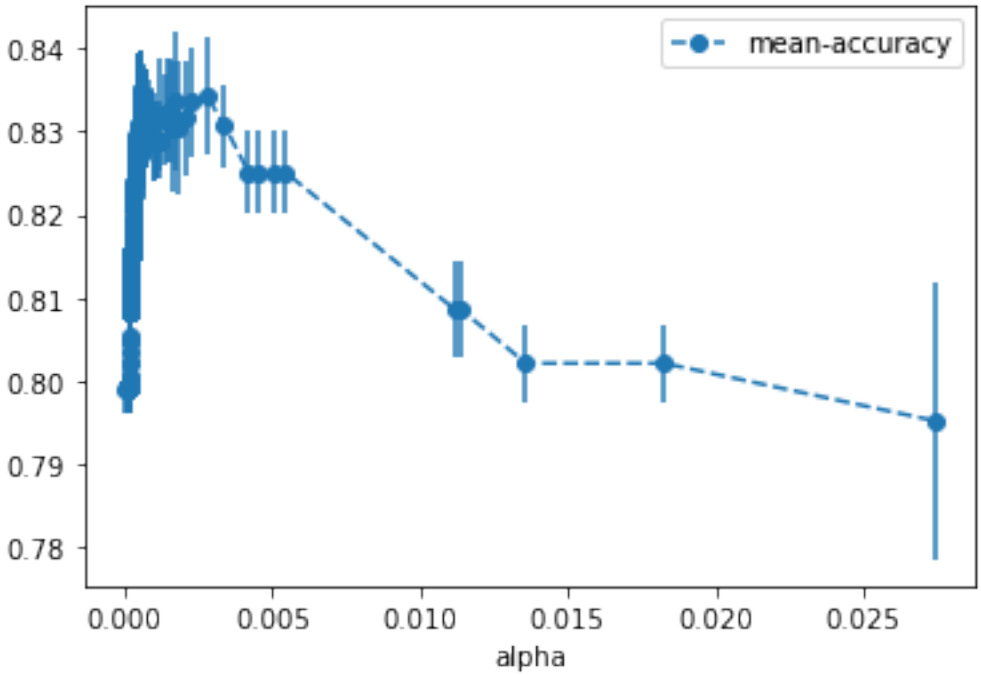


I use cross-validation to find the optimal value of *ccp_alpha*

[58]:
```
## create an array to store the results of each fold during cross validation
alpha_loop_values = []

## for each candidate value for alpha, we will run 5-fold cross validations.
## then we will store all the mean and standard deviation of the scores␣
↪(accuracies) for each call
## to cross-val-score in alpha_loop_values
for ccp_alpha in ccp_alphas:
    clf_dt = DecisionTreeClassifier(random_state=0, ccp_alpha=ccp_alpha)
    scores = cross_val_score(clf_dt, X_train, y_train, cv = 4, n_jobs=6)
    alpha_loop_values.append([ccp_alpha, np.mean(scores), np.std(scores)])
```

```
## now we draw a graph of mean and standard deviations of the scores
## for each candidate value for alpha
alpha_results = pd.DataFrame(alpha_loop_values,
                             columns = ['alpha','mean-accuracy','std'])

alpha_results.plot(x = 'alpha',
                   y = 'mean-accuracy',
                   yerr = 'std',
                   marker = 'o',
                   linestyle = '--')
```

[58]: <AxesSubplot:xlabel='alpha'>



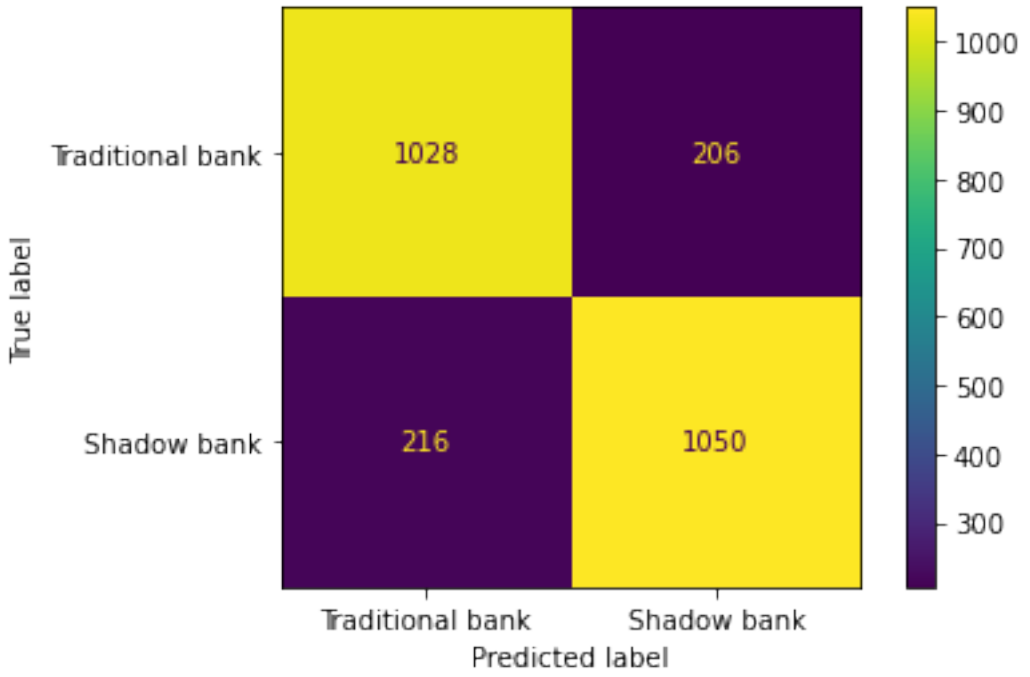
[59]:
```
# To find the ideal value of alpha
alpha_results.loc[alpha_results['mean-accuracy'].idxmax()].alpha
```

[59]: 0.002795318756171502

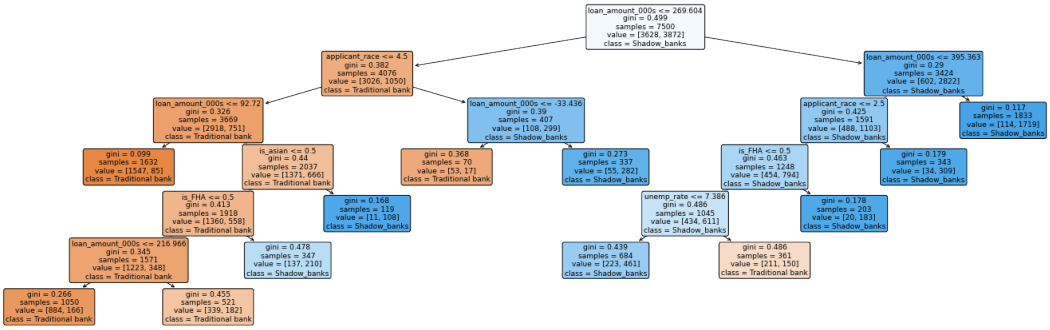
[60]:
```
# We will use the value of ccp_alpha from above to prune the long tree
clf_dt_prunned = DecisionTreeClassifier(random_state = 42,
                                        ccp_alpha = 0.002795318756171502)
clf_dt_prunned = clf_dt_prunned.fit(X_train,y_train)
```

[61]:
```
plot_confusion_matrix(clf_dt_prunned,
                      X_test,
                      y_test,
                      display_labels = ["Traditional bank","Shadow bank"])
```

[61]: <sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x29fa27eeca0>



[62]:
```
plt.figure(figsize = (25,7.5))
plot_tree(clf_dt_prunned,
          filled = True,
          rounded = True,
          class_names = ["Traditional bank","Shadow_banks"],
          feature_names = X_with_dummy.columns);
plt.savefig("pruned_tree.pdf")
```



[63]:
```
y_pred = clf_dt_prunned.predict(X_test)
y_pred
```

[63]:
```
array([0., 0., 0., ..., 1., 0., 1.])
```

[64]:
```
accuracy = accuracy_score(y_test, y_pred )
accuracy
```

[64]:
```
0.8312
```

From the confusion matrix, we can calculate that the traditional bank borrowers are 1028+206 = 1234, 1039 i.e **83.3%** borrowers are correctly classified. For the Shadow bank borrowers which are 216+1050=1266, 1082 i.e **82.9%** are correctly classified, an improvement of around 4% in both the classes.

Pruned-Tree Analysis:

It can be seen the majority class is **shadow bank** which is the class of the first node. The tree is branching from **loan amount_000s** variable with the majority class of shadow bank. This supports the authors results under examination. Further branching is happening from loan type and racial groups variables with one variable inclined to traditional bank and another to shadow bank.

The gini index at the start of the tree is 0.492 which is high enough to start the branching but such large value of gini also gives weak class majority. It can be seen that as the gini index goes to a smaller number, the color of that node becomes darker. This implies that the specific node contains predominantly observations from a single class.

The accuracy of the model is increased from 79.84% to **82.9%** due to pruning of the decision tree.

The results from both pruned trees are similar, however, the accuracy has a considerable change. This is due to the beta values being changed. In mostly of the cases, the beta value is lowered which lowers the significance of the variables.

4.3 Random forest

[65]:
```
##Split the data into training and testing data-sets
X_train, X_test, y_train, y_test = train_test_split(X_with_dummy, Y.y_class,␣
 ↪random_state = 60)
```

[66]:
```
## Applying Random-Forest classifier with 120 trees with the tarining and␣
 ↪testing data-sets
rf = RandomForestClassifier(n_estimators = 120, random_state = 60, n_jobs= 6,␣
 ↪verbose = 1, ccp_alpha=0.00095)
rf.fit(X_train, y_train)
```

```
[Parallel(n_jobs=6)]: Using backend ThreadingBackend with 6 concurrent workers.
[Parallel(n_jobs=6)]: Done  38 tasks      | elapsed:    0.0s
[Parallel(n_jobs=6)]: Done 120 out of 120 | elapsed:    0.2s finished
```

[66]:
```
RandomForestClassifier(ccp_alpha=0.00095, n_estimators=120, n_jobs=6,
                       random_state=60, verbose=1)
```

[67]:
```
y_pred = rf.predict(X_test)
y_pred
```

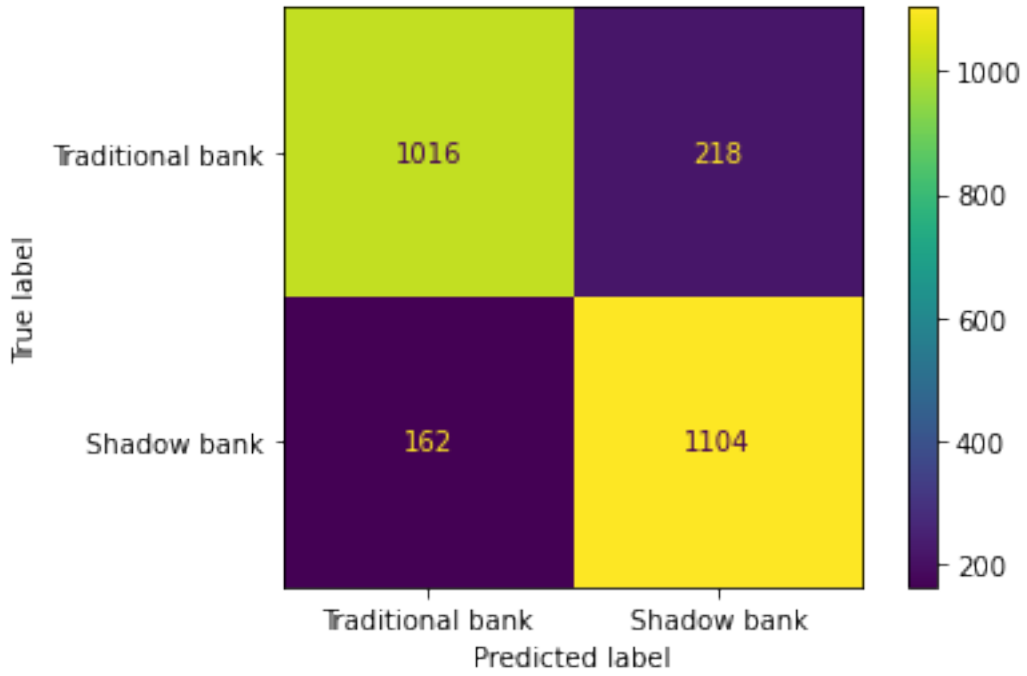
```
[Parallel(n_jobs=6)]: Using backend ThreadingBackend with 6 concurrent workers.
[Parallel(n_jobs=6)]: Done  38 tasks      | elapsed:    0.0s
[Parallel(n_jobs=6)]: Done 120 out of 120 | elapsed:    0.0s finished
```

[67]:
```
array([0., 0., 0., ..., 0., 0., 1.])
```

[68]:
```
plot_confusion_matrix(rf,
                      X_test,
                      y_test,
                      display_labels = ["Traditional bank","Shadow bank"])
```

```
[Parallel(n_jobs=6)]: Using backend ThreadingBackend with 6 concurrent workers.
[Parallel(n_jobs=6)]: Done  38 tasks      | elapsed:    0.0s
[Parallel(n_jobs=6)]: Done 120 out of 120 | elapsed:    0.0s finished
```

[68]:
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x29fa296a850>
```



[69]:
```
print(classification_report(y_test, y_pred))
print( "Accuracy_Score: ",accuracy_score(y_test, y_pred))
```

```
              precision    recall  f1-score   support

         0.0       0.86      0.82      0.84      1234
         1.0       0.84      0.87      0.85      1266

    accuracy                           0.85      2500
   macro avg       0.85      0.85      0.85      2500
weighted avg       0.85      0.85      0.85      2500

Accuracy_Score:  0.848
```

Interpretation:

The confusion matrix states that the model correctly classifies **82.3%** of traditional bank borrowers and **87.2%** of shadow bank borrowers. The preditions have improved from earlier. In this random forest, I have used *ccp_alpha* which was derived during pruning of the classification tree .

The confusion matrix also states that the majority class is **Shadow banks** with 1104 borrowers as against 1016 traditional bank borrowers.

The use of random forest has increased the accuracy of the model from **82.9%** to **84.8%**.

5 Conclusion

I would like quote the authors that the residential mortgage market has changed dramatically in the years following the financial crisis and the Great Recession. The rise of shadow banking can be seen by my study as well as the research paper. People prefer to get loan from non-traditional banks due their technological advancement (fintech banks) and the regulatory arbitrage from the traditional commercial banks. People from certain racial and ethnic groups prefer shadow banking due to the element of anonymity which is diffcult to achieve in commercial banks.

The aim of this study was to generate output which is quantitatively accurate and descriptive while keeping its social factors and importance intact.Both the methods of classification tree and random forest have given accurate results supporting the authors arguments. However, it is open to develop more precise results using different approaches and variables.

I would like to conclude by cautioning against an interpretation only based on graphs and numbers. These results definitely shows the rise of shadow banks, however, we need to also focus on the fact that shadow banks needs more regulations to avoid any upcoming banking and mortgage crisis. The primary function of such banks to loan origination, but they also have to focus on maintaining and monitoring on the asset side to retain their stability. We could find a balance between both kinds of banking systems to have more financial inclusion and greater fiscal stability.

6 References

1. **Buchak G., Matvos G., Piskorski T., Seru A**. *Fintech, regulatory arbitrage, and the rise of shadow banks*, Journal of Financial Economics, Volume 130, Issue 3,2018,Pages 453-483, ISSN 0304-405X

2. **James, G., Hastie, T. J., Tibshirani, R., Witten, D., James, G.** (2021). Tree-Based Methods . *In An introduction to statistical learning: With applications in R* (2nd ed.). essay, Springer.