Report on

Mathematical Modelling and Analysis of PMDC Motor

Submitted by,

Ankit Jain	**(T197043)**
Sanket Kottawar	**(T197197)**
Krushna Garkal	**(T197057)**

Guided by,

Prof. Aniket Gundecha

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This is certify that,

Ankit Jain	**T197043**
Sanket Kottawar	**T197197**
Krushna Garkal	**T197057**

of T.Y. B.Tech. have submitted a Report on,

Mathematical Modelling and Analysis of PMDC Motor

The said work is completed by putting the requirement of hours as per prescribed curriculum during the academic year 2019 – 20. The report is submitted in the partial fulfilment of the requirements for the course **Control Systems** in the Fifth Semester of Degree of Engineering in Electronics and Telecommunication of MIT Academy of Engineering.

Prof. Aniket Gundecha
Project Advisor

Dr. Debashis Adhikari
Dean, SEE

Abstract

In this report, we have performed the analysis and mathematical modelling of PMDC motor using MATLAB software. The motor parameters were used to obtain the transfer function and the state space representation. The modelling was done for both speed and position control. The mathematical modelling includes controllability, observability, step response, ramp response, parabolic response, time domain specification of speed control, root locus plot, bode plot and at last the PID design for the motor system.

Acknowledgement

We would like to express our special thanks of gratitude to the Control System faculty, who gave us the golden opportunity to do this conceptual report of Mathematical modelling of PMDC motor.

We wish to express our profound thanks to our project guide **Prof. Aniket Gundecha** for providing us the technical knowledge and tools required to perform the experiment.

We also want to thank our director **Dr. Mahesh Goudar** for providing us with the basic infrastructure and other facilities.

Ankit Jain
Sanket Kottawar
Krushna Garkal

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Chapter 1

Introduction

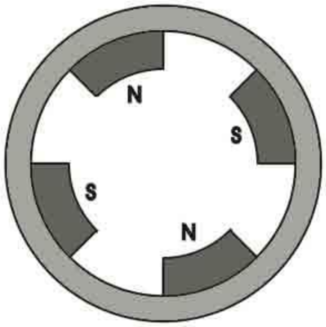
INTRODUCTION TO PMDC:

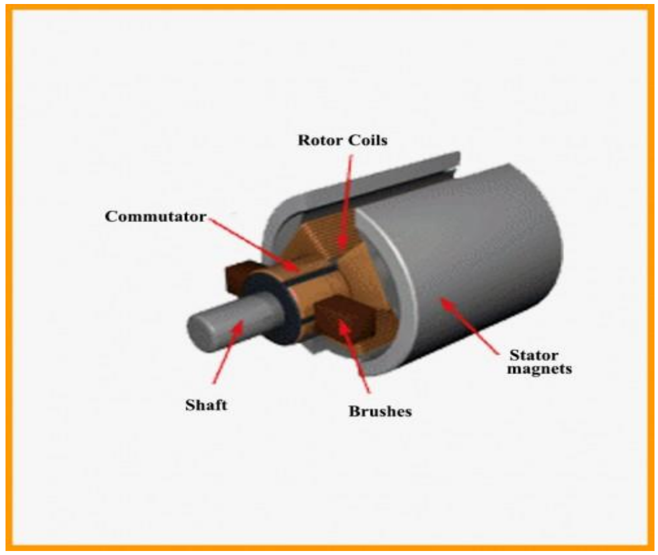
In a DC motor, an armature rotates inside a magnetic field. Basic working principle of DC motor is based on the fact that whenever a current carrying conductor is placed inside a magnetic field, there will be mechanical force experienced by that conductor.

All kinds of DC motors work in this principle only. Hence for constructing a DC motor it is essential to establish a magnetic field. The magnetic field is obviously established by means of magnet. The magnet can by any types i.e. it may be electromagnet or it can be permanent magnet. When permanent magnet is used to create magnetic field in a DC motor, the motor is referred as permanent magnet DC motor or PMDC motor. This battery operates motor is nothing but a permanent magnet DC motor or PMDC motor. These types of motor are essentially simple in construction. These motors are commonly used as starter motor in automobiles, windshield wipers, washer, for blowers used in heaters and air conditioners, to raise and lower windows, it also extensively used in toys.

As the magnetic field strength of a permanent magnet is fixed it cannot be controlled externally, field control of this type of DC motor cannot be possible. Thus, permanent magnet DC motor is used where there is no need of speed control of motor by means of controlling its field. Small fractional and sub fractional KW motors now constructed with permanent magnet.

Construction of Permanent Magnet DC Motor or PMDC Motor:





As it is indicated in name of permanent magnet DC motor, the field poles of this motor are essentially made of permanent magnet. A PMDC motor mainly consists of two parts. A stator and an armature. Here the stator which is a steel cylinder. The magnets are mounted in the inner periphery of this cylinder.

The permanent magnets are mounted in such a way that the N-pole and S-pole of each magnet are alternatively faced towards armature as shown in the figure below. That means, if N-pole of one magnet is faced towards armature then S-pole of very next magnet is faced towards armature. In addition to holding the magnet on its inner periphery, the steel cylindrical stator also serves as low reluctance return path for the magnetic flux. Although field coil is not required in permanent magnet DC motor but still it is sometimes found that they are used along with permanent magnet. This is because if permanent magnets lose their strength, these lost magnetic strengths can be compensated by field excitation through these field coils. Generally, rare earth hard magnetic materials are used for these permanent magnets.

Rotor:

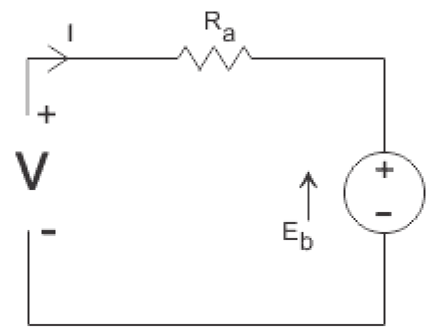
The rotor of PMDC motor is similar to other DC motors. The rotor or armature of permanent magnet DC motor also consists of core, windings and commutator. Armature core is made of number of varnishes insulated, slotted circular lamination of steel sheets.

By fixing these circular steel sheets one by one, a cylindrical shaped slotted armature core is formed. The varnish insulated laminated steel sheets are used to reduce eddy current loss in armature of permanent magnet DC motor. These slots on the outer periphery of the armature core are used for housing armature conductors in them. The armature conductors are connected in a suitable manner which gives rise to armature winding. The end terminals of the winding are connected to the commutator segments placed on the motor shaft. Like other DC motor, carbon or graphite brushes are placed with spring pressure on the commutator segments to supply current to the armature.

Working Principle of Permanent Magnet DC Motor or PMDC Motor

As we said earlier the working principle of PMDC motor is just similar to the general working principle of DC motor. That is when a carrying conductor comes inside a magnetic field, a mechanical force will be experienced by the conductor and the direction of this force is governed by Fleming left hand rule. As in a permanent magnet DC motor, the armature is placed inside the magnetic field of permanent magnet; the armature rotates in the direction of the generated force. Here each conductor of the armature experiences the mechanical force F = B.I.L Newton where, B is the magnetic field strength in Tesla (weber / m2), I is the current in Ampere flowing through that conductor and L is length of the conductor in metre comes under the magnetic field. Each conductor of the armature experiences a force and the compilation of those forces produces a torque, which tends to rotate the armature.

Equivalent Circuit of Permanent Magnet DC Motor or PMDC Motor:



As in PMDC motor the field is produced by permanent magnet, there is no need of drawing field coils in the equivalent circuit of permanent magnet DC motor.

The supply voltage to the armature will have armature resistance drop and rest of the supply voltage is countered by back emf of the motor. Hence voltage equation of the motor is given by,

$$V = IR + E_b$$

Where, I is an armature current and R is armature resistance of the motor.

Eb is the back emf and V is the supply voltage.

Advantages of Permanent Magnet DC Motor or PMDC Motor:

PMDC motor have some advantages over other types of DC motors. They are:

1. No need of field excitation arrangement.
2. No input power in consumed for excitation which improve efficiency of DC motor.
3. No field coil hence space for field coil is saved which reduces the overall size of the motor.
4. Cheaper and economical for fractional kW rated applications.

Disadvantages of Permanent Magnet DC Motor or PMDC Motor:

1. In this case, the armature reaction of DC motor cannot be compensated hence the magnetic strength of the field may get weak due to demagnetizing effect armature reaction.
2. There is also a chance of getting the poles permanently demagnetized (partial) due to excessive armature current during starting, reversal and overloading condition of the motor.
3. Another major disadvantage of PMDC motor is that, the field in the air gap is fixed and limited and it cannot be controlled externally. Therefore, very efficient speed control of DC motor in this type of motor is difficult.

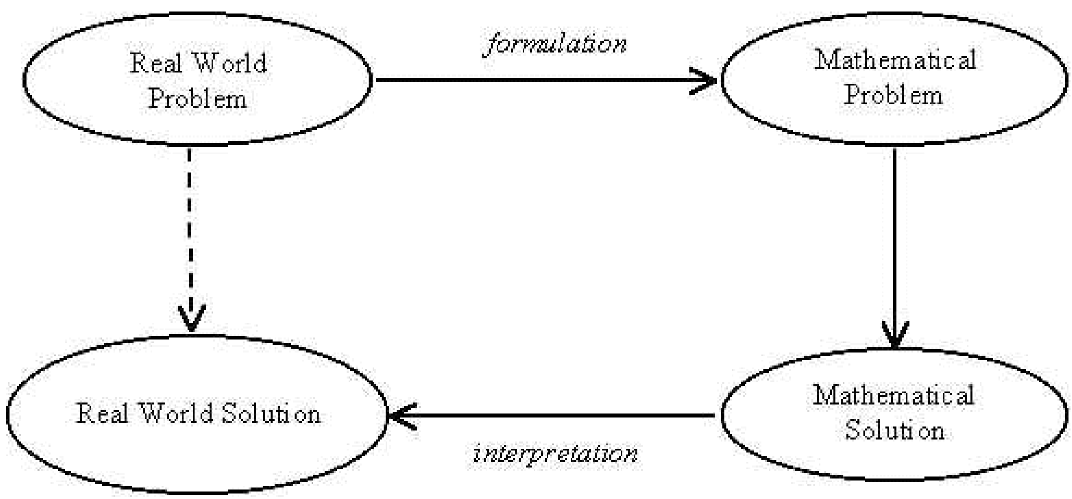
Applications of Permanent Magnet DC Motor or PMDC Motor:

PMDC motor is extensively used where small DC motors are required and also very effective control is not required, such as in automobiles starter, toys, wipers, washers, hot blowers, air conditioners, computer disc drives and in many more.

Chapter 2

Importance of Mathematical Modelling

Mathematical modelling is a process of representing real world problems in mathematical terms in an attempt to find solutions to the problems. A mathematical model can be considered as a simplification or abstraction of a (complex) real world problem or situation into a mathematical form, thereby converting the real-world problem into a mathematical problem. The mathematical problem can then be solved using whatever known techniques to obtain a mathematical solution. This solution is then interpreted and translated into real terms.



In mechanical systems, motion can be of different type i.e., Translational, Rotational or Combination of both. The equations governing such motion in mechanical systems are often directly or indirectly governed by Newton's laws of motion.

The response of dynamic system to an input may be obtained if these differential equations are solved.

Same way electrical systems are governed by Kirchhoff's Law. The mathematical description of the dynamic characteristic of a system is called as mathematical model of a system.

Significance of mathematical modelling:

1. Control systems is the arrangement of physical elements and that physical elements are analysed to make governing equations
2. Mathematical modelling helps in easy analysis of control systems.
3. As mathematical model is in Laplace domain it is easy to analyse big systems also.

Chapter 3

Application of Mathematical Modelling in Real life

Application 1: Drug Dosage

In prescribing drug dosage, physicians know that residual build-up depends on the time interval between administration of drug doses. They use sub-models for decay rate, assimilation rate and drug concentration with repeated equal doses for prescribing a safe and effective dosage of drug concentration and dose schedule.

Application 2: Time of Death

At the beginning of a murder investigation, a forensic pathologist will go out into the field to examine the scene and then uses Newton's Law of Cooling to approximately determine a victim's time of death.

Application 3: Reactor Risk Assessment

A process operator uses ODEs to analyse the transient response of a reactor in a manufacturing plant (i.e., petrochemical, semiconductor, pharmaceutical) to ensure that it is safe for continuous production without interruption.

Chapter 4

Mathematical Model for Speed control and Position control

- Transfer function representation
  Code

```
syms s
Ra=input('enter the value of terminal resistance : ');
Kt=input('enter the value of torque constant : ');
Kb=input('enter the value of back emf constant : ');
B=input('enter the value of viscocity friction : ');
J=input('enter the value of MI of rotor : ');

Tm=((Ra*J)/(Ra*B+Kb*Kt));
Km=Kt/(Ra*B+Kb*Kt);
num=Km
den=[Tm 1]
sys=tf(num,den)  %velocity tf
num1=Km
den1=[Tm 1 0]
sys1=tf(num1,den1) %Position tf
```

Output

enter the value of terminal resistance : 8

enter the value of torque constant : 0.5

enter the value of back emf constant : 0.5

enter the value of viscocity friction : 0.02

enter the value of MI of rotor : 0.03

num =

1.2195

```
den =

    0.5854    1.0000


sys =

      1.22
  ------------
  0.5854 s + 1

Continuous-time transfer function.


num1 =

    1.2195


den1 =

    0.5854    1.0000         0


sys1 =

       1.22
  --------------
  0.5854 s^2 + s

Continuous-time transfer function.
```

- State space representation
  Code

```
clc;clear all; close all;
% Transfer function to state space model
syms s;
num = input('Enter the num = ');
den = input('Enter the den = ');
num1 = sym2poly(num);
den1 = sym2poly(den);
[A B C D] = tf2ss(num1,den1)
```

Output-

Enter the num = 1.22*s^0

Enter the den = 0.5854*s+1

A =

-1.7082

B =

1

C =

2.0840

D =

0

For position transfer function

Output –

Enter the num = 1.22*s^0

Enter the den = 0.5854*s^2+s

A =

```
   -1.7082         0
    1.0000         0


B =

     1
     0


C =

         0    2.0840


D =

     0
```

- Controllability and Observability check Code

```
clc; clear all; close all;
A = input ('Enter the Matrix A : ');
B = input ('Enter the Matrix B : ');
C = input ('Enter the Matrix C : ');

if B==0
    disp('Matrix B=0')
else
    Co = ctrb(A,B)
    if length(A)==rank(Co)
       disp('System is controllable')
    else
       disp('System is not controllable')
    end
end
```

```
if C==0
    disp ('Matrix C=0')
else
    Ob = obsv(A,C)
    if length(A)==rank(Ob)
        disp('System is observable')
    else
        disp('System is not observable')
    end
end
```

For Velocity transfer function
Enter the Matrix A : [-1.7082]
Enter the Matrix B : [1]
Enter the Matrix C : [2.0840]

Co =

1

System is controllable

Ob =

2.0840

System is observable

For Position transfer function
Enter the Matrix A : [-1.7082 0;1 0]
Enter the Matrix B : [1;0]
Enter the Matrix C : [0 2.0840]

Co =

1.0000 -1.7082

0   1.0000

System is controllable

Ob =

0   2.0840
2.0840   0

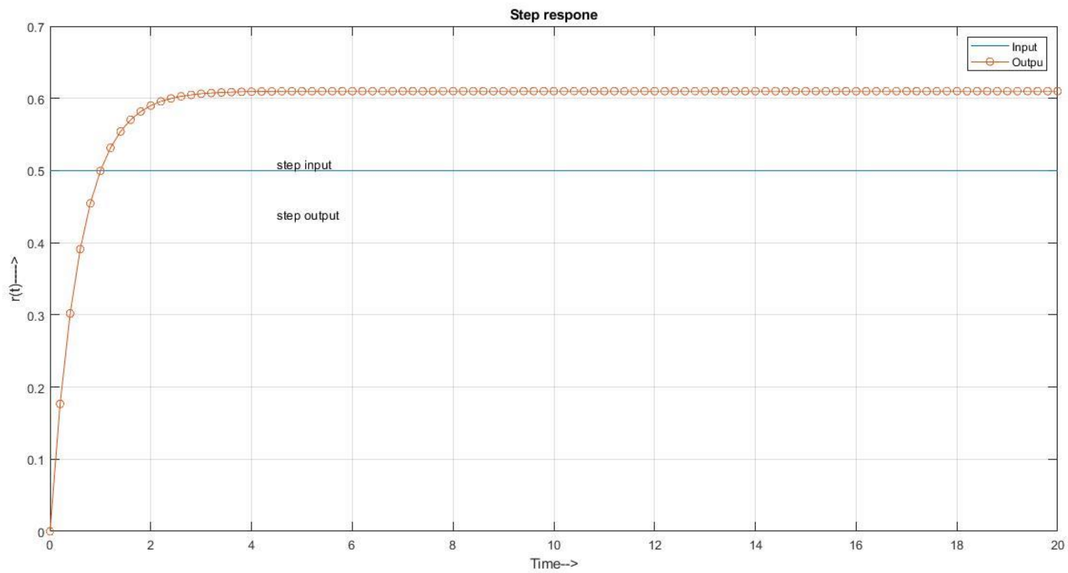
System is observable

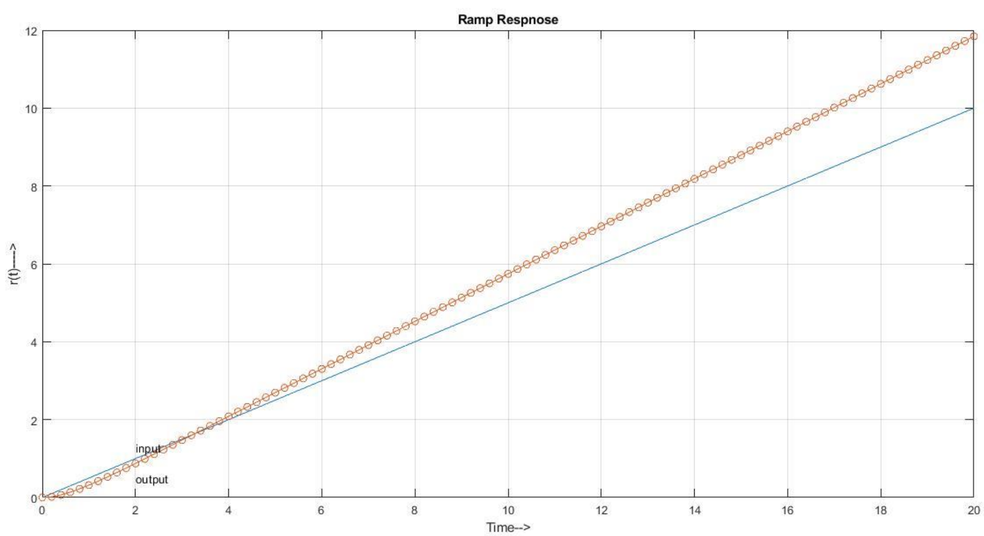
- Step, ramp and Parabolic response of Transfer function
  For Velocity transfer function
  Code

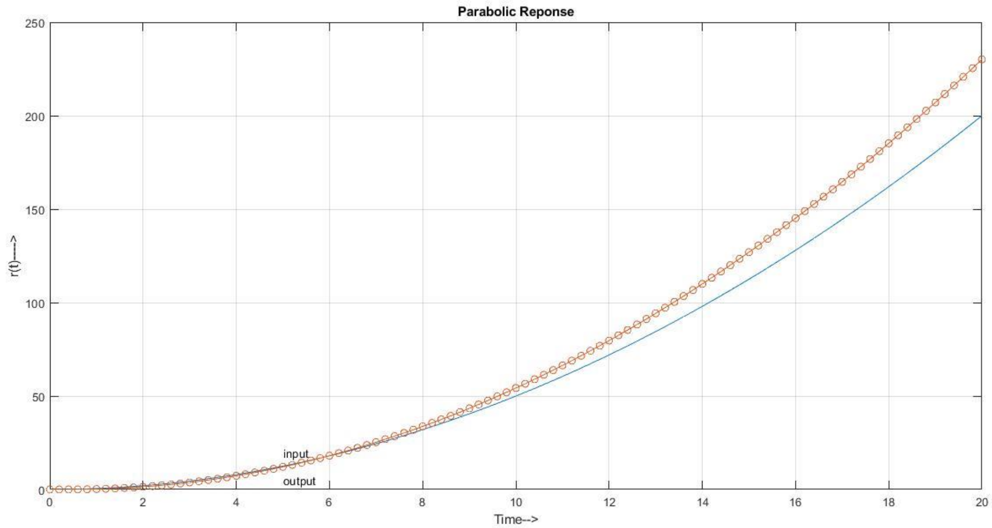
```
clc;
t=[0:0.2:20];

r1=0.5*(t.^0); %Step Input
r2=0.5*(t.^1); %Ramp Input
r3=0.5*(t.^2); %Parabolic input


num=1.22;
den=[0.5854 1];
y0=lsim(num,den,r1,t);
plot(t,r1,'-',t,y0,'-o');
grid on;
title('Step respone');
xlabel('Time-->');
ylabel('r(t)---->');
text(4.5,0.51,'step input');
text(4.5,0.44,'step output');

figure;
y0=lsim(num,den,r2,t);
plot(t,r2,'-',t,y0,'-o');
grid on;
title('Ramp Respnose');
xlabel('Time-->');
ylabel('r(t)---->');
text(2,1.3,'input');
text(2,0.5,'output');

figure;
y0=lsim(num,den,r3,t);
plot(t,r3,'-',t,y0,'-o');
```

```
grid on;
title('Parabolic Reponse');
xlabel('Time-->');
ylabel('r(t)---->');
text(5,20,'input');
text(5,5,'output');
```

Output-





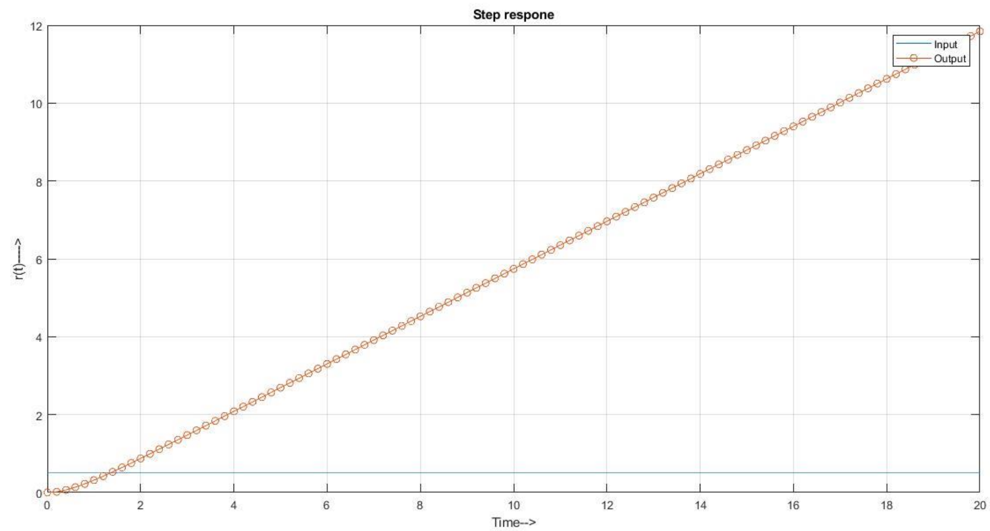


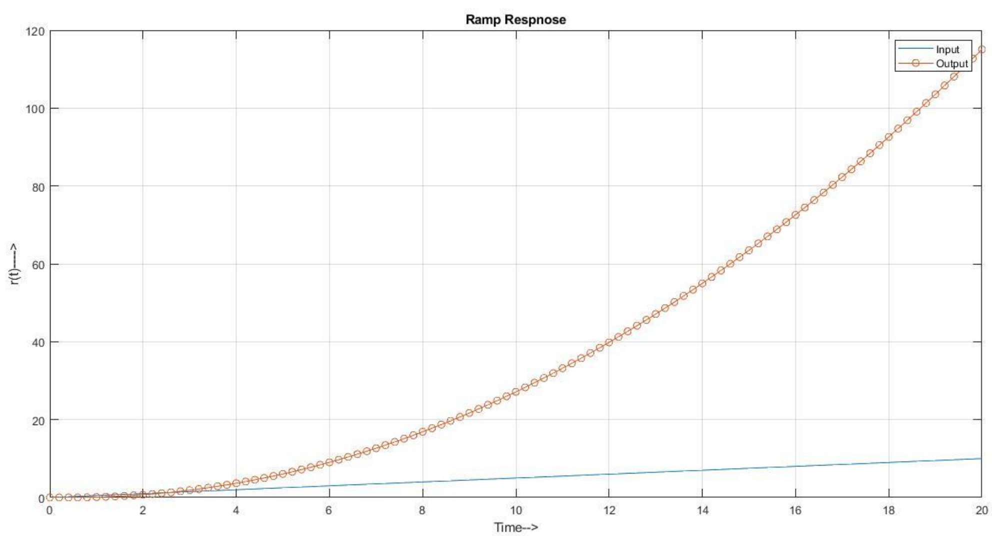
For Position transfer function
Code

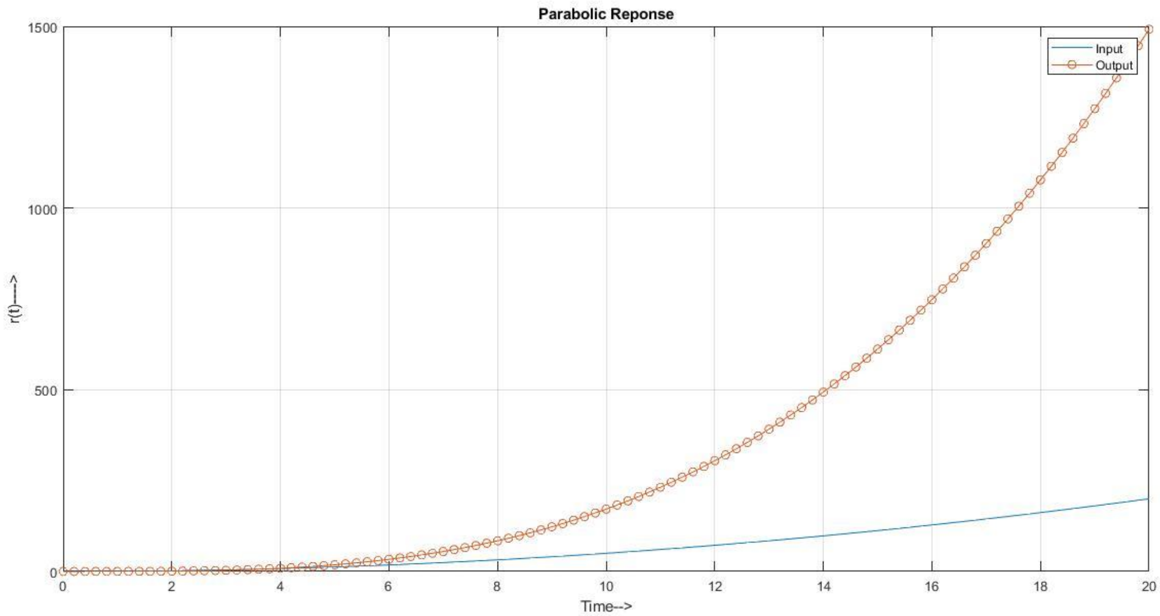
```
clc;
t=[0:0.2:20];

r1=0.5*(t.^0); %Step Input
r2=0.5*(t.^1); %Ramp Input
r3=0.5*(t.^2); %Parabolic input


num=1.22;
den=[0.5854 1 0];
y0=lsim(num,den,r1,t);
plot(t,r1,'-',t,y0,'-o');
grid on;
title('Step respone');
xlabel('Time-->');
ylabel('r(t)---->');


figure;
y0=lsim(num,den,r2,t);
plot(t,r2,'-',t,y0,'-o');
grid on;
title('Ramp Respnose');
xlabel('Time-->');
ylabel('r(t)---->');

figure;
y0=lsim(num,den,r3,t);
plot(t,r3,'-',t,y0,'-o');
grid on;
title('Parabolic Reponse');
```

```
xlabel('Time-->');
ylabel('r(t)---->');
```

Output –







- Root Locus Plot
  Code

```
clc;close all;clear all;
syms s;
k=5;
h=1;
num1 = input('enter num = ');
den1 = input('enter den = ');
num=sym2poly(num1);
den=sym2poly(den1);
g=tf(num,den)
rlocus(g*h*k)
grid on
```

Output-

enter num = 1.22*s^0

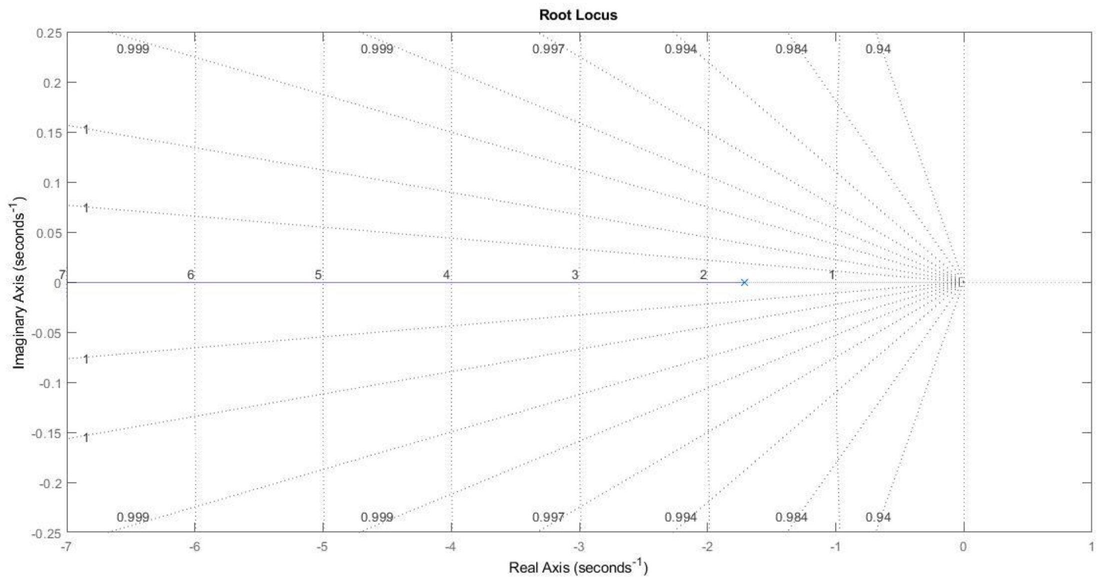
enter den = 0.5854*s+1

g =

1.22

0.5854 s + 1

Continuous-time transfer function.



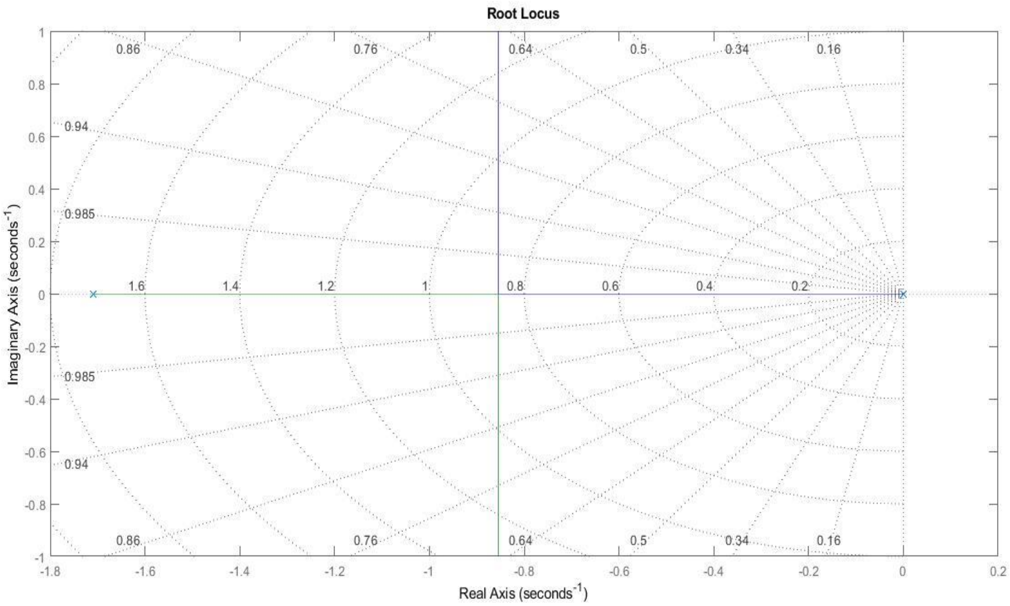
For Position transfer function
enter num = s^0*1.22
enter den = 0.5854*s^2+s

g =

1.22

0.5854 s^2 + s

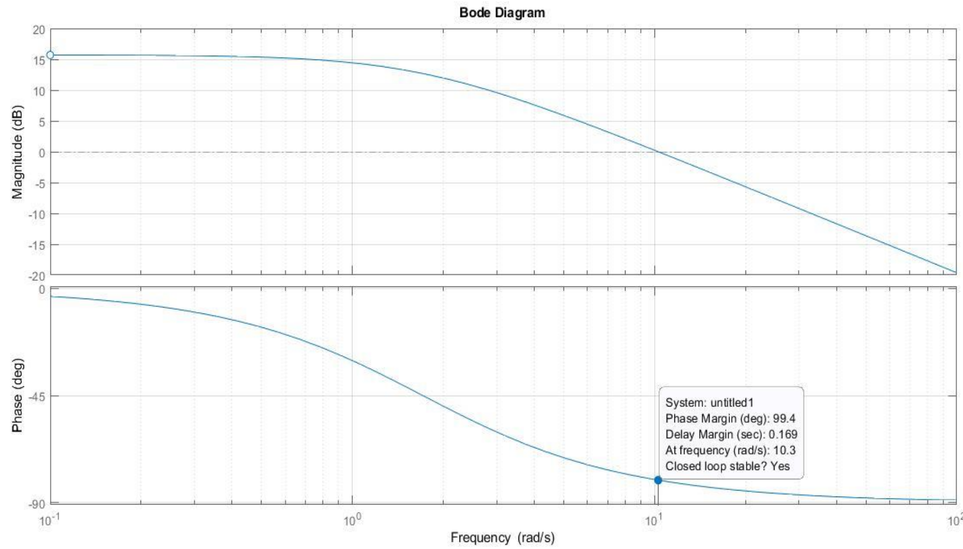
Continuous-time transfer function.



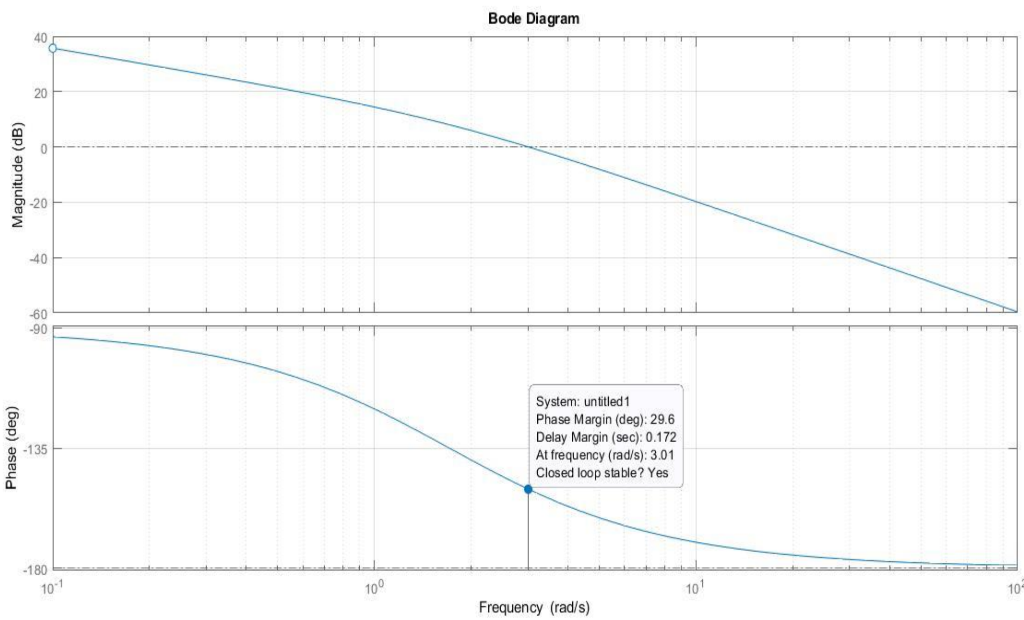
- Bode Plot
  For velocity transfer function
  Code

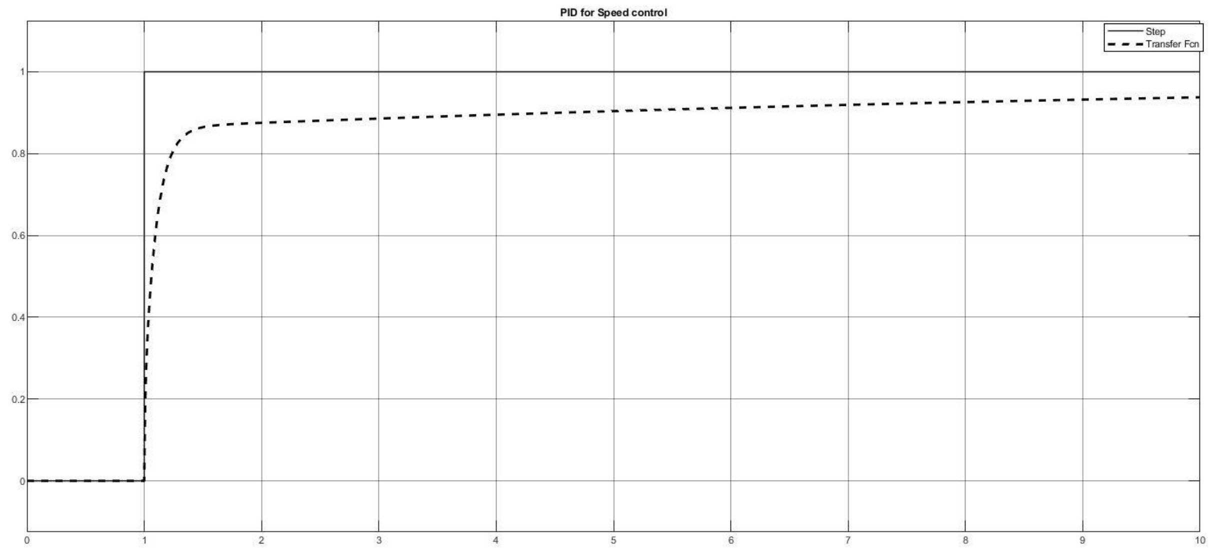
```
clc;close all;clear all;
syms s;
k=5;
h=1;
num1 = input('enter num = ');
den1 = input('enter den = ');
num=sym2poly(num1);
den=sym2poly(den1);
g=tf(num,den);
bode(g*h*k)
grid on;
```

Output –

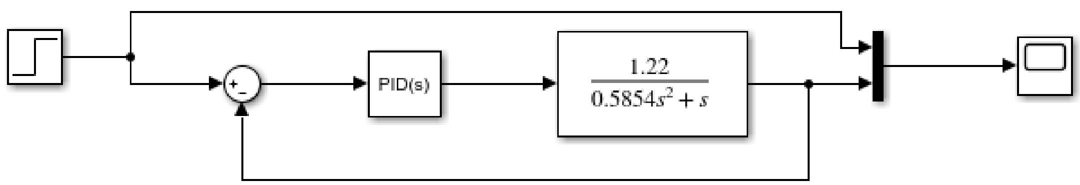


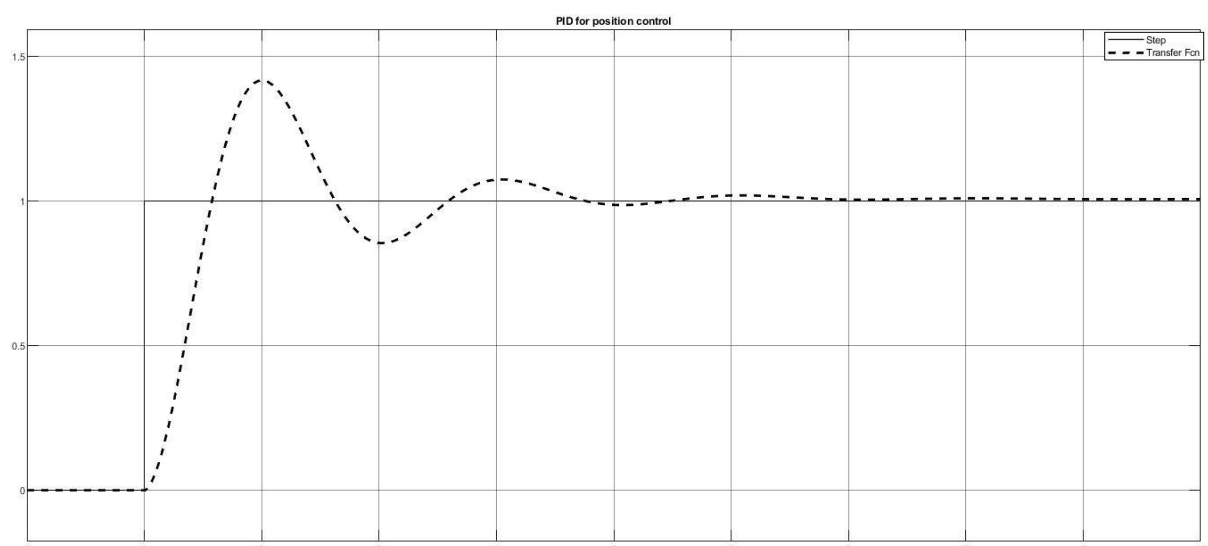
For Position transfer function





For position control





Chapter 5

Conclusion

In this experiment, we calculated all the mathematical modelling and analysis parameters from a DC servo motor circuit.

We derived the transfer function from the DC motor parameters for speed and position control. Also the state space representation of the transfer function using MATLAB.

The system is controllable as well as observable. We performed the observability and controllability check for both the speed control and position control.

We observed the step, ramp and parabolic response for the velocity transfer function and position transfer function.

We plotted the root locus for velocity transfer function and all the poles lie in the left hand plane which tells that the system is stable. Same case was for the position transfer function.

We plotted the bode plot for both the transfer function and found that the system is stable.

At last we did the PID controller design for the motor to improve the steady state error and decreases system settling time while maintaining transient response.