

Combinatorics Basics

9:05 pm

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Notes

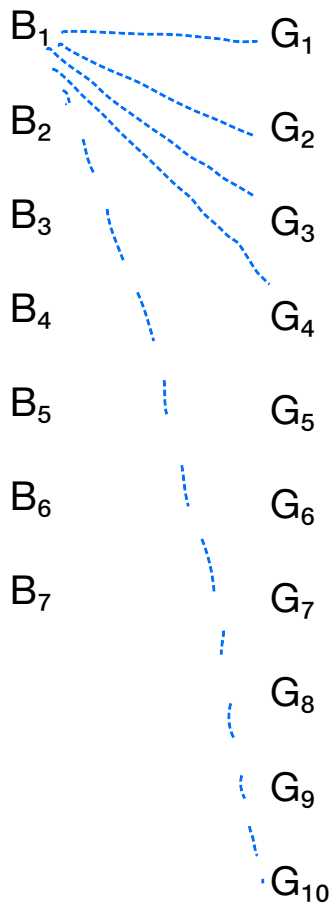


Example1: Given 10 Girls and 7 Boys. How many different pairs?

1 Girl and 1 Boy

Boys

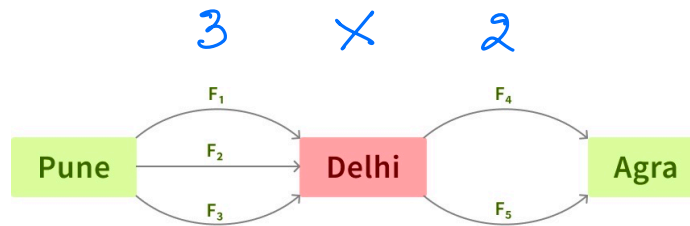
Girls



1 boy \Rightarrow 10 pairs.
7 boys = $7 \times 10 = 70$ pairs



Example-2 :



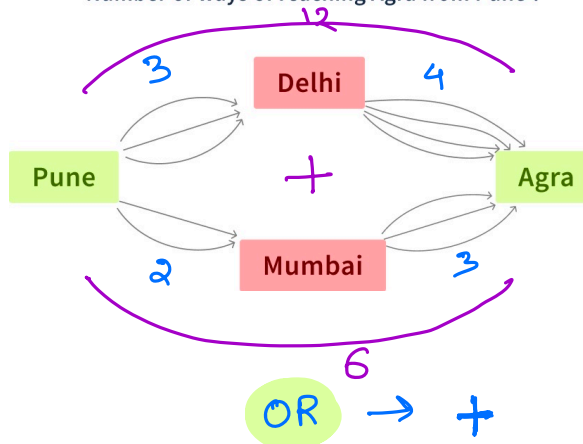
AND \Rightarrow * (multiplication)

Number of ways to reach Agra from Pune via Delhi

$$\begin{aligned} \text{Pune} \rightarrow \text{Delhi} &= 3 \text{ flights} \\ \text{Delhi} \rightarrow \text{Agra} &= 2 \text{ flights} \\ 3 \times 2 &= 6 \end{aligned}$$

Example-3

Number of ways of reaching Agra from Pune ?



$$\text{Pune} \rightarrow \text{Delhi} \rightarrow \text{Agra} \Rightarrow 3 \times 4 = 12$$

$$\text{Pune} \rightarrow \text{Mumbai} \rightarrow \text{Agra} \Rightarrow 2 \times 3 = 6$$

OR $\rightarrow +$

$$\begin{aligned} \text{Total number of ways} &= 12 + 6 \\ &= 18 \end{aligned}$$

Problem

You're tasked with helping **Zomato** identify which restaurant offers the most variety in its meal combos. You're provided with a list, shaped like a grid or a 2D matrix **A**, where each row corresponds to a different restaurant's offerings.

Each row is divided into three parts:

1. $A[i][0]$ tells you the number of main courses,
2. $A[i][1]$ the number of desserts, and
3. $A[i][2]$ the number of beverages a restaurant offers.

Your challenge is to analyze this data and pinpoint which restaurant gives its customers the most options to mix and match their meal combo.

Examples

Example 1 :

A = [
[3, 2, 2],	# Restaurant 1	$3 \times 2 \times 2 = 12$			
[4, 3, 3],	# Restaurant 2	$4 \times 3 \times 3 = 36$			
[1, 1, 1]	# Restaurant 3	$1 \times 1 \times 1 = 1$			
]					

ans = 36 Restaurant 2



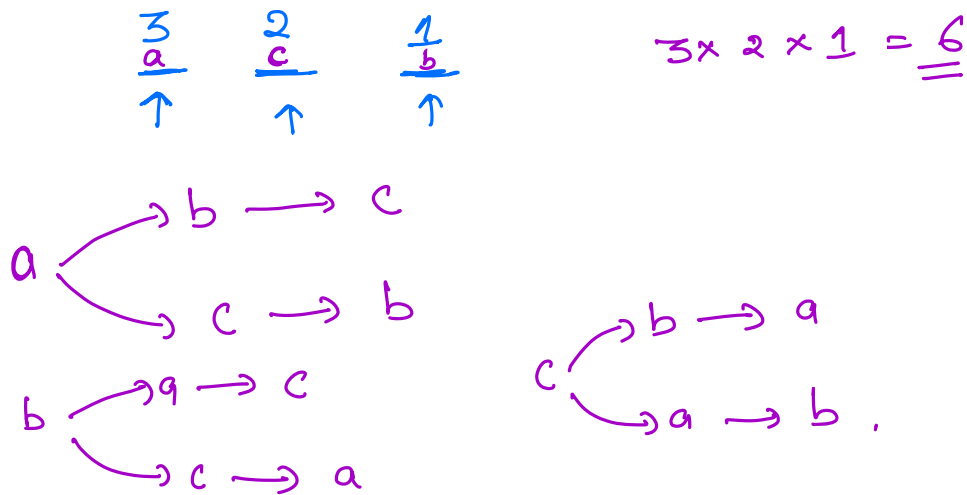
Permutations

arrangement of objects. * order matters.

$$(i, j) \neq (j, i)$$

< Question-1 > : Given 3 distinct characters. In how many ways, we can arrange them?

S = "a b c"



< Question-2 > : In how many ways, you can arrange 4 distinct characters?

$$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = \underline{\underline{24}}$$



< **Question-3** > : In how many ways n distinct characters can be arranged?

$$\underline{N} \times \underline{(N-1)} \times \underline{(N-2)} \times \dots \times \underline{1} = \underline{\underline{n!}}$$

< **Question-4** > : Given 5 distinct characters, in how many ways can we arrange 2 characters?

$$\begin{array}{c} 5 \\ \hline \uparrow \end{array} \times \begin{array}{c} 4 \\ \hline \uparrow \end{array} = \underline{\underline{20 \text{ ways}}}$$

< **Question-5** > : 3 distinct characters?

$$\underline{3} \times \underline{2} = 6$$

< **Question-6** > : 4 characters out of N distinct characters?

$$\underline{N} \times \underline{(N-1)} \times \underline{(N-2)} \times \underline{(N-3)} \quad \underline{[N - (4-1)]} \checkmark$$



< Question > : Given N distinct characters, in how many ways can we arrange r characters?

\hookrightarrow^N
 6 distinct char.
 \hookrightarrow^3 3 places.

$$\begin{array}{ccccccc}
 \underline{6} & \underline{6-1} & \underline{6-2} & \dots & \dots & \dots & \underline{[6-(3-1)]} \\
 \downarrow & & \downarrow & & & & \uparrow \\
 \underline{6} & \underline{5} & \underline{4} & \dots & \dots & \dots & \underline{[3-1]} \\
 \downarrow & & \downarrow & & & & \downarrow \\
 \underline{6} & \underline{5} & \underline{4} & \dots & \dots & \dots & \underline{[3-1]} \\
 & & \downarrow & & & & \downarrow \\
 & & \underline{6-(3-1)} & & & & \underline{[3-1]} \\
 & & = \underline{4} & & & & \underline{[3-1]}
 \end{array}$$

$$\frac{a \times b}{b}$$

$$\frac{N \times (N-1) \times (N-2) \times (N-3) \times \dots \times [N-(r-1)] \times [N-(r-2)] \times \dots \times 1}{(n-r) \times (n-r-1) \times \dots \times 1}$$

$$\Rightarrow \frac{N!}{(n-r)!} \Rightarrow {}^n P_r$$

No. of ways to arrange r places with N distinct characters,

10:05 pm break



Combinations

combination is defined as number of ways to select something.

Note: In combination order of selection doesn't matter: $[(i,j) = (j,i)]$

< Question > : In how many ways can we select 3 players from a pool of 4 players?

[P1 P2 P3 P4]

P₁ P₂ P₃

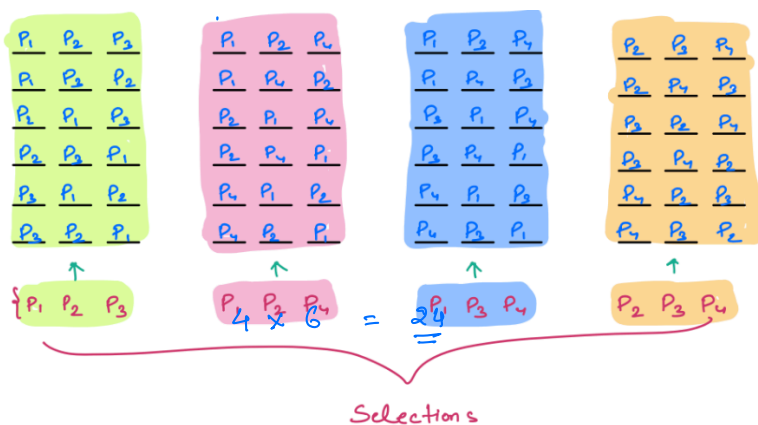
P₁ P₃ P₄

P₁ P₂ P₄

P₂ P₃ P₄

< Question > : Number of ways to arrange the players in 3 slots

Given 4 players → [P1 P2 P3 P4]



⇒ No. of selection (x)
= 3 out of 4 plays
↓
4 ways.

⇒ No. of arrangement
in each selection =
 $3! = \underline{\underline{6}}$

⇒ Total no. of arrangement 4×6
 $= \underline{\underline{24}}$

No. of selection * no. of arrangement of each selection = Total no. of arrangement.



Combinatorics Basics

we can select r elements.

Given n distinct element in how many ways

$$n \text{ distinct arrange } r \text{ element} = {}^n P_r = \frac{n!}{(n-r)!}$$

$$\text{arrange } r \text{ element} = r!$$

No. of selection for r distinct element: 1

$$\begin{aligned} r! &\Rightarrow 1 \\ 1 &= \frac{1}{r!} \end{aligned}$$

Properties

1. ${}^N C_0$



Select 0 element from N elements.

$${}^N C_r = \frac{\text{Total no. of arrange}}{r!} = \frac{n!}{(n-r)! \times r!}$$

$${}^N C_0 = \frac{N!}{(N-0)! \times 0!} \rightarrow 1 = \frac{1}{1}$$

2. ${}^N C_N$

$${}^N C_r = \frac{N!}{(N-r)! \times r!} = \frac{N!}{0! \times N!} = \frac{1}{1}$$

3. ${}^N C_{N-r}$

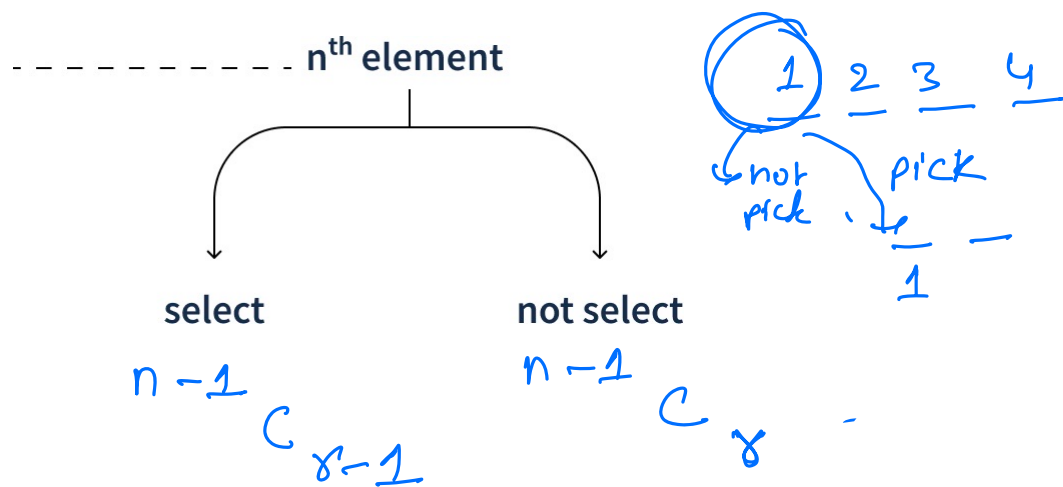
$$\begin{aligned} {}^N C_r &= \frac{N!}{(N-(N-r))! \times (N-r)!} \\ &= \frac{N!}{r! \times (N-r)!} = {}^N C_{N-r} \end{aligned}$$

**

$${}^N C_r = {}^N C_{N-r}$$



< Question > : Given N distinct elements, select r distinct elements.

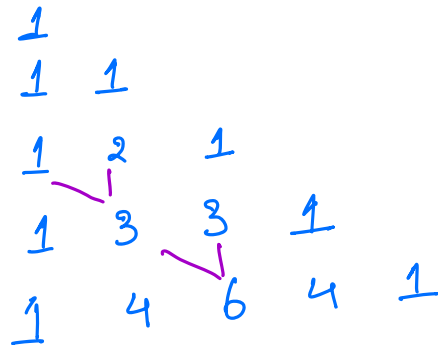


$${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$$



Pascal Triangle

- Generate the Pascal's triangle for given N

[illegible]

$${}^2C_1 = {}^1C_0 + {}^1C_1$$

$$C[i][j] = C[i-1][j] + C[i-1][j-1]$$



</> Code

```
function pascalTriangle ( n ) {  
    ncr [n][n] = {0}  
    for ( i → 0 to n ) {  
        ncr [i][0] = 1  
        ncr [i][i] = 1  
        for ( j → 1 to i - 1 )  
            {  
                ncr[i][j] = ncr[i-1][j]  
                    + ncr[i-1][j-1]  
            }  
        }  
    }  
}
```

T.C: $O(n^2)$
S.C: $O(n^2)$



Nth Column Title

- Find the Nth column title

N = 1 2 3 26 27 28 50 51 52 53 54 ---
 A B C --- Z AA AB ----- AX AY AZ BA BB --- BZ

N = 30 → AD

N = 50 → AX

N = 100 → CV

26. $\left[\begin{array}{l} A \ 27 \ AA \ BA \\ B \ 28 \ AB \\ C \ 29 \ AC \\ D \ 30 \ \underline{AD} \\ \vdots \\ Z \ A2 \ B2 \end{array} \right]$

base 26

N = 30

26	30	1 → A
	4	4 → D
	0	

= AD

Z = 26

N = 50

26	50	1 → A
	24	24 → X
	0	

= AX



$$\underline{N = 52}$$

26	52	2
	0	<u>0</u>

* 1) Create map \rightarrow ✓

string ans = ""

while (¹ ~~N~~₀ = 0)

{ rem = N % 26; = 0 1

if (rem == 0)

ans = 'Z' + ans; N-1;

{ else

{

ans = map.get(^A rem) + ans

} N = N / 26; = 0

return ans;

= AZ

51/26
= 1



</> Code

$$N = \underline{100}$$

28	100	3 → C
	22	22 → V
	0	

~~28~~ 0 ~ 25
 ==>

func columnTitle (n) {

ans = "" ; 27 / 1

while (n > 0)

{

0 % 26 = 0

A

ans = ((n-1) % 26 + 1A) + ans

n = (n-1) / 26 = 1

}

return ans;

}

AA

==