Roll No. 2K22/EE/295

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Elective 2

B.Tech.

End Term Examination

Dec–2024

CO205: DISCRETE STRUCTURES

Time: 3:00 Hours **Max. Marks: 50**

Note: *Answer **any five** questions. All questions carry equal marks. Assume suitable missing data, if any.*

Q. No. 1

A Test the validity of the following argument relating to the students of Delhi Technological University (DTU) using Rules of Inference: - **(5) [CO1] {L5}**

(i) A student of the DTU is eligible to apply for placement only if the student has CGPA more than 6.0.

(ii) If a student has not obtained passing grade in a subject that a student registered for, then the student cannot appear for the internship test.

(iii) If a student is eligible to apply for placement, then the student can appear for the internship test.

(iv) There are some students who have CGPA less than 6.0

Therefore, it can be concluded that at least one student did not obtain passing grade in a subject that the student registered.

B Convert the statement $(x \to (y \wedge w)) \wedge (z \to (y \wedge w))$ into PDNF forms **(5) [CO1] {L2}**

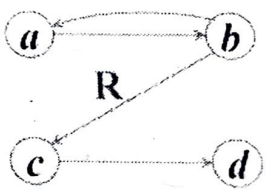
Q. No. 2

A Prove or disprove that for any integer $n \geq 1$, $n^3 - n$ is divisible by 3. **(5) [CO2] {L5}**

B Solve the recurrence relation represented by $a_n = 5a_{n-1} - 6a_n - 2$ for $n \geq 2$, $a_0 = 1$, $a_1 = 2.5$ and determine the value of a_{25} **(5) [CO2] {L3}**

Q. No. 3

A Let $A = \{a, b, c, d\}$ and R a relation on A defined by the digraph shown in the figure. **(5) [CO3] {L3}**



Is R a transitive relation? If not, identify the transitive closure of R.

B If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 1 & 2 & 6 & 5 & 7 & 8 \end{pmatrix}$; **(5) [CO3] {L2}**

$p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 4 & 6 & 7 & 8 \end{pmatrix}$; $p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 2 & 5 & 4 & 1 & 7 & 8 \end{pmatrix}$.

Outline the expression for $(p_2 \circ p_1) \circ p_3$. Is the resulting permutation odd or even?

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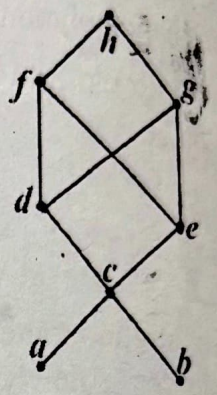
Q. No. 4

A If $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ with the partial order $\leq$ of divisibility on A, *i.e* $a \leq b$ if and only if a *divides* b ; and $B = P(S)$, the power set of S, where $S = \{e, f, g\}$; and $(B, \subseteq)$ be a poset, where $\subseteq$ is the with partial order of containment. Analyse if $(A, \leq)$ and $(B, \subseteq)$ are isomorphic posets. Justify your answer. (5) [CO3] {L4}

B Consider the poset $(A, \leq)$. If $A = \{a, b, c, d, e, f, g, h\}$ and $\leq$ is the partial order on A with Hasse diagram as shown in Figure. $B = \{a, b\}$ and $C = \{c, d, e\}$ are subsets of A. Interpret what are the following: - (5) [CO3] {L2}

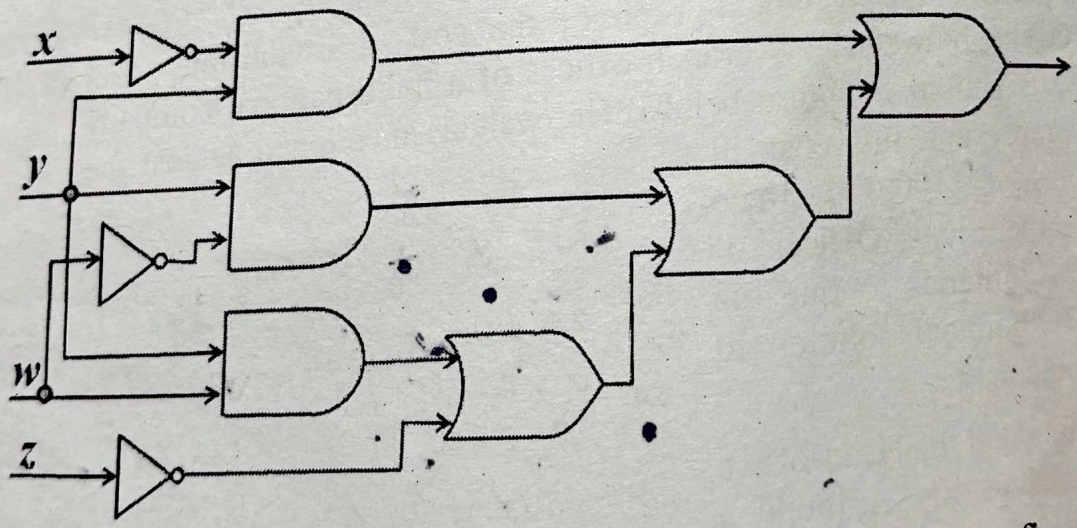


(i) Lower Bounds of B and C.
(ii) Upper Bounds of B and C
(iii) Greatest Lower Bound of B and C
(iv) Least Upper Bound of B and C

Q. No. 5

A Given D_n is the set of positive integers that divides any integer $n \geq 1$ and p is a prime number, prove or disprove that if p^2 divides n , then D_n is not a Boolean Algebra. (5) [CO3] {L5}

B Develop the Boolean polynomial for the function, $f: B_4 \rightarrow B$ given by logic diagram show in the figure given below. (5) [CO3] {L3}

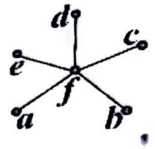


Use the properties of Boolean algebra (or any graphical method) to refine the polynomial to use minimal number of variables and operators.

Draw logic diagram for the new Boolean polynomial.

Q. No. 6

A Draw the digraphs of the possible spanning trees for the symmetric relation whose graph is given alongside. How many spanning trees are there? **(5) [CO4] {L2}**



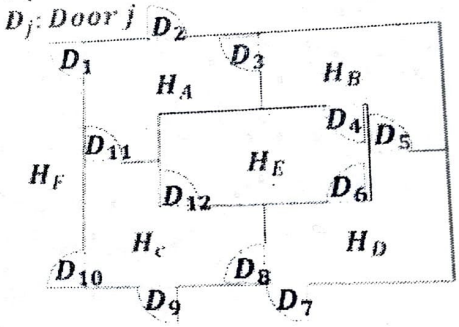
B For the graph H shown in figure, construct the chromatic polynomial, $P_H(x)$. Use $P_H(x)$ to find the chromatic number, $\chi(H)$, of graph H. **(5) [CO4] {L6}**

Graph H

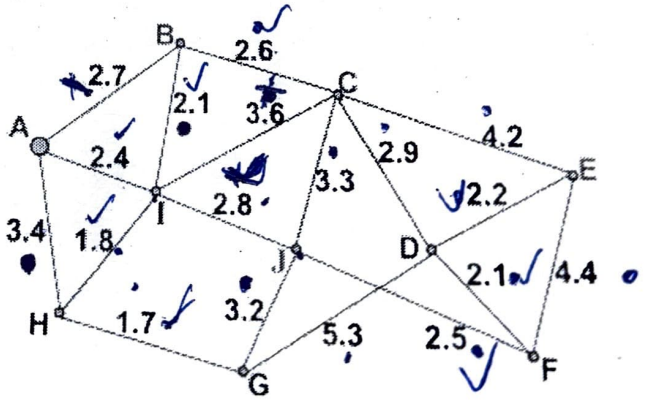
Q. No. 7

A An exhibition route map is required to be made for a newly renovated museum with five indoor Exhibit Halls and one outdoor Exhibit Stand. The floor plan of the museum is given in Figure.

Plan a route to visit every exhibit hall / stand in the museum by passing through each door exactly once? Justify your answer. **(5) [CO4] {L3}**



B The representative weighted-graph of the cost of establishing data connectivity between new branch offices of a multi-national company (MNC) is shown in figure below. The weights mentioned against the edges represent the cost in multiples of ₹100,000/-. The Chief Operations Officer of the MNC intends to find the most economical solution to the problem. **(5) [CO5] {L3}**



Assuming that the Head-office is located at city A, build a minimal spanning tree (root as city A) with vertices representing various branch offices of the company.

What will be the minimum cost for establishing this data network?