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Second Semester ***B.Tech. (CSE)***

End Term Examination May–2024

CS102: DISCRETE STRUCTURES

Time: 3:00 Hours **Max. Marks: 50**

Note: Answer any five questions. All questions carry equal marks. Assume suitable missing data, if any.

Q. No. 1 [5x2]

A Test the validity of the following argument relating to the students of Department of Computer Science and Engineering (CSE) using Rules of Inference: - [CO#1]

(i) A student of the CSE will graduate this semester only if the student has passed the subject programming fundamentals.
(ii) If Raju does not study the subject programming fundamentals for 10 hours a week, then he will not pass in programming fundamentals.
(iii) If students study programming fundamentals for 10 hours a week, then they cannot play volleyball.

Therefore, it can be concluded that at least one student will not graduate this semester unless the student does not play volleyball.

B Convert the statement $(x \to (y \wedge w)) \wedge (z \to (y \wedge w))$ into PDNF forms [CO#1]

Q. No. 2 [5x2]

A Show that among any $n+1$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers. [CO#2]

B If the computational complexity of a divide and conquer algorithm is determined using recurrence relation $f(n) = 2f(\sqrt{n}) + 1,\ f(1) = 1$, where n is a perfect square, what is the big theta (Θ) estimate for the complexity of the algorithms? [CO#2]

Q. No. 3 [5x2]

A Let $A = \{a, b, c, d\}$ and R a relation on A defined as $R = \{(a,b), (b,a), (b,c), (c,d)\}$. Is R a transitive relation? If not, find the transitive closure of R. [CO#3]

B If $A = \{1,2,3,4,5,6\}$, $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$; [CO#3]

$p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$; $p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$.

Find $(p_2 \circ p_1) \circ p_3$. Is the resulting permutation odd or even?

Q. No. 4 [5x2]

A Let A = {a, b, c, d, e, f, g, h} and R be the relation defined by [CO#3]

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i) Show that (A, R) is a poset.

(ii) Does the poset (A, R) have a least element and a greatest element? If so, identify them.

(iii) Show that the poset (A, R) is complemented and list all pairs of complements.

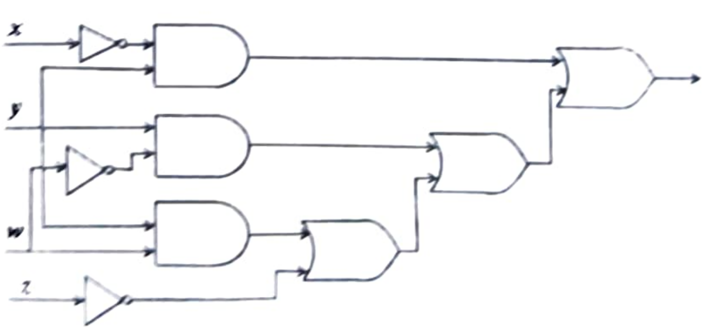
(iv) Prove or disprove that (A, R) is a Boolean algebra.

B Let $*$ be a binary operation on a set A, and suppose that $*$ satisfies the idempotent, Commutative and Associative Properties $\forall a, b, c \in A$. Define a relation $\leq$ on A by $a \leq b$ if and only if $a = a * b$. Show that $(A, \leq)$ is a poset, and for all a, b in A, $GLB(a, b) = a * b$. (*Note:* $GLB(a, b) = a \wedge b$) [CO#3]

Q. No. 5 [5x2]

A Compare isomorphism and homomorphism of semigroups. If Z is the set of integers, T the set of even numbers and $\times$ is the multiplication operator, prove or disprove that the semigroups $(Z, \times)$ and $(T, \times)$ are isomorphic. [CO#3]

B State the Boolean polynomial for the function, $f: B_4 \to B$ given by logic diagram show in the Figure. [CO#3]

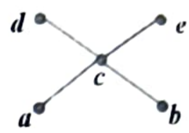


Use the properties of Boolean algebra (or any graphical method) to refine the polynomial to use minimal number of variables and operators.

Draw logic diagram for the new Boolean polynomial.

Q. No. 6 [5x2]

A Draw the digraphs of the possible spanning trees for the symmetric relation whose graph is given alongside. How many spanning trees are there? [CO#4]



B A college offers 8 elective subjects (with codes E01 to E08) for 2nd Semester students to choose from. The data of the number of students registered for both $\boldsymbol{Subject_i}$ and $\boldsymbol{Subject_j}$ is as tabulated below: - [CO#4]

	$\boldsymbol{Subject_i}$	$\boldsymbol{Subject_j}$	#
a)	E01	E04	9
b)	E04	E07	6
c)	E07	E02	8
d)	E02	E05	4

	$\boldsymbol{Subject_i}$	$\boldsymbol{Subject_j}$	#
e)	E05	E08	7
f)	E08	E03	8
g)	E03	E06	5
h)	E06	E01	11

A clash will occur in the timetable if the examination of two subjects, which are conducted during the same time-slot results in a student not being able to take the examination of one of the subjects for which the student is registered.

(i) Assuming that a student takes only one examination per day, in how many minimum numbers of days can the end-semester examination be conducted without any clash in the timetable for these elective subjects?

(ii) How many different *clash-free* time-tables can be generated for the conduct of End-Sem Examination for the elective subjects? (*Hint: Application of graph colouring*)

Q. No. 7 [5x2]

A Construct a labelled positional binary tree for the fully parenthesized algebraic expression $((a \div b) - c) \times (e + (f \div g))$. Assuming that visiting a vertex v prints the label of v, show the result of performing a post-order search on the constructed binary tree. [CO#4]

B The CEO of a newly formed entrepreneurship wished to send out a salesman to visit all major clients across 5 towns, $T = \{t_1, t_2, t_3, t_4, t_5\}$. The travel cost between these towns is given by the matrix C_T, where $c_{ij} = n$ if the cost of travel from town i to j is ₹$(n \times 1000)$. If the headquarters of this establishment is located at t_1, write a program to find the most economical round-strip for the salesman. What will be the minimum cost of the trip? [CO#5]

$$C_T = \begin{bmatrix} 0 & 7 & 50 & 1 & 1 \\ 7 & 0 & 3 & 50 & 8 \\ 50 & 3 & 0 & 6 & 2 \\ 1 & 50 & 6 & 0 & 7 \\ 1 & 8 & 2 & 7 & 0 \end{bmatrix}$$