2nd Semester

B.Tech. (Ist Year)

End Term Examination

May-2024

IT 102 Course Title: Discrete Structures

Time: 3 Hours *Max. Marks: 50*

Note: All questions are compulsory.
All questions carry equal marks.
Assume suitable missing data, if any

Q.1 a. Consider the following logical statement S expressed in conjunctive normal form (CNF): [5] [CO1]

$S=(P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

Now, let P, Q, and R represent three different conditions, each of which can either be true (T) or false (F). Determine under what conditions S evaluates to true.

b. State and Prove the rules of inference for propositional logics. [5] [CO1]

Q.2 a. In a certain town, there are 100 people who like either pizza, burgers, or both. Among them, 60 people like pizza, 70 people like burgers, and 30 people like both pizza and burgers. [5] [CO2]

(i) How many people in the town like neither pizza nor burgers?

(ii) If there are 150 people in total in the town, how many people like neither pizza nor burgers?

(b) Prove that for all positive integers n, the sum of the cubes of the first n positive integers is equal to the square of the sum of the first n^n positive odd integers. [5] [CO2]

In other words, prove that:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 3 + 5 + \cdots . + (2n - 1))^2$$

Q.3 a. Find a_{50}, where a_n is represented by the recurrence relation:

$a_n = -2a_{n-1} - 4a_{n-2}$, n is an integer ≥ 3, $a_1=1$, $a_2=3$

b. Find the generating function for the Fibonacci sequence defined by [5] [CO3]

$a_n = a_{n-1} + a_{n-2}$, $n \geq 2$, with $a_0=0$, $a_1=1$

Q.4 a. Let A = {a, b, c, d, e} and let R and S be relation on A whose matrices are given below. Compute the matrix of the smallest relation containing R and S. Also list the elements of this relation using Warshall's Algorithm. [5] [CO4]

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad ; \quad M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

b. Draw oriented graph for given cut set matrix: [5] [CO6]

$$[C] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Q.5 a. Reduce the Boolean Expression and realize using minimum number of gates: [5] [CO5]

$$A = XY + X(Y + Z) + Y(Y + Z)$$ [5] [CO5]

b. Minimize the following Boolean Function and realize using NOR and NAND gate only. $F = \pi_m(1,3,4,6,8,9,11,13,15) + \Sigma_d(0,2,14)$