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Third Semester

End Term Examination

Roll No. 2K24/CO/48

B.Tech. (CO)

Nov/Dec-2023

CO205 DISCRETE STRUCTURES

Time: 3:00 Hours

Max. Marks: 50

Note: Answer any five questions. All questions carry equal marks. Assume suitable missing data, if any.

Q. No. 1

[5x2]

A Test the validity of the following argument: -

[CO#1]

- (i) It is not sunny today and it is colder than yesterday.
- (ii) We will go for swimming only if it is sunny.
- (iii) If we don't go for swimming, we will go for a trekking trip.
- (iv) If we go out trekking, then we will come home by sunset.

Therefore, it can be concluded that we will be home by sunset.

B Convert the statement  $\neg(p \leftrightarrow (q \rightarrow (r \vee p)))$  into PDNF forms

[CO#1]

Q. No. 2

[5x2]

A Prove or disprove that the product of a nonzero rational number and an irrational number is irrational. [CO#1]

B Find the expression for the 50<sup>th</sup> term of the recurrence relation

[CO#2]

$$a_n = -6a_{n-1} - 9a_{n-2} \text{ for } n \geq 2, a_0 = 3, a_1 = -3$$

Q. No. 3

[5x2]

A What is the principle of inclusion and exclusion? Let A, B, C be subsets of the universal set V. Given  $A \cap B = A \cap C$  and  $\bar{A} \cap B = \bar{A} \cap C$ , is it necessary that  $B = C$ ? Justify your answer. ( $\bar{A}$  is complement of the set A). [CO#3]

B Write the program or pseudo-code of a function with time complexity of order  $n$  to find the sum of coefficients of the Binomial Expansion  $(x + y)^n$  [CO#3]

Q. No. 4

[5x2]

A Let  $A = \{a, b, c, d, e\}$  and let R and S be relations on A whose matrices are given below. Compute the matrix of the smallest relation containing R and S. Also list the elements of this relation. (Hint: use Warshall's Algorithm). [CO#4]

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}; \quad M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

B What is the necessary condition for permutation of a function?

[CO#4]

Given  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ,

(i) Compute the product  $(3, 5, 7, 8) \circ (1, 3, 2)$

(ii) Is the permutation  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 5 & 7 & 8 & 4 & 3 & 2 & 1 \end{pmatrix}$  odd or even?

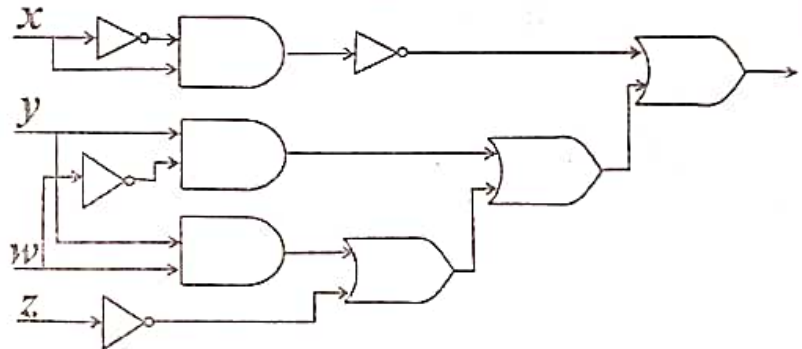
(iii) Find the period of the permutation  $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$

Q. No. 5

[5x2]

A Given  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$  with the partial order  $\leq$  of divisibility [CO#4] on  $A$ , i.e.  $a \leq b$  if and only if  $a$  divides  $b$ .  $B = P(S)$ , the power set of  $S$ , where  $S = \{e, f, g\}$ ; and  $(B, \subseteq)$  be a poset, where  $\subseteq$  is the with partial order of containment. Show that  $(A, \leq)$  and  $(B, \subseteq)$  are isomorphic.

B State the Boolean polynomial for the function,  $f: B_4 \rightarrow B$  given by logic diagram show in the Figure.



[CO#4]

Use the properties of a Boolean algebra (or any graphical method) to refine the polynomial to use minimal number of variables and operators.

Draw logic diagram for the new Boolean polynomial.

Q. No. 6

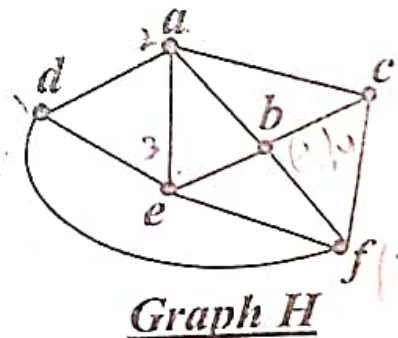
[5x2]

A Construct a labelled positional binary tree for the fully parenthesized algebraic expression  $(3 - (2 - (11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$ .

Assuming that visiting a vertex  $v$  prints the label of  $v$ , show the result of performing a pre-order search on the constructed binary tree.

B For the graph H shown in figure, find the chromatic polynomial,  $P_H(x)$ .

Use  $P_H(x)$  to find the chromatic number,  $\chi(H)$ , of graph H.



[CO#5]

Graph H



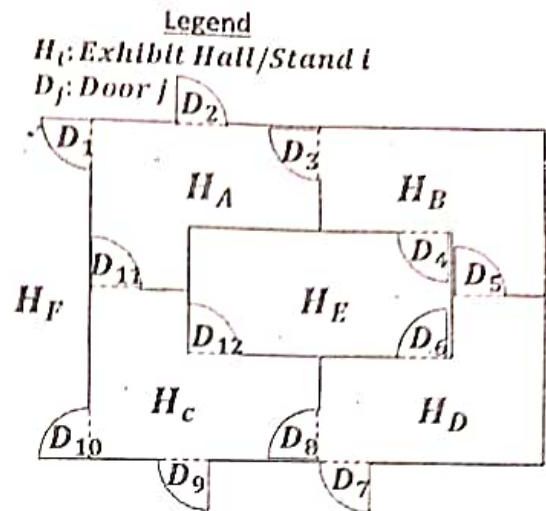
Q. No. 7

[5x2]

[CO#5]

- A An exhibition route map is required to be made for a newly renovated museum with five indoor Exhibit Halls and one outdoor Exhibit Stand. The floor plan of the museum is given in Figure.

Is it possible to visit every exhibit hall / stand in the museum by passing through each door exactly once? Justify your answer.



- B The representative weighted-graph of the cost of establishing data connectivity between new branch offices of a multi-national company (MNC) is shown in figure below. The weights mentioned against the edges represent the cost in multiples of ₹100,000/-. The Chief Operations Officer of the MNC intends to find the most economical solution to the problem.

Assuming that the Head-office is located at city A, find a minimal spanning tree (root as city A) with vertices representing various branch offices of the company.

What will be the minimum cost for establishing this data network?

