(a) i. During a month with 30 days a baseball team plays at least 1 game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

ii. Find the bitwise OR, bitwise AND, and bitwise XOR of the bit string 0110110110 and 1100011101.

(b) Suppose that the relations R_1 and R_2 on a set A are represented by matrices

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Give graphical representation of $M_{R_1 \cup R_2}$, $M_{R_1 \cap R_2}$ and $M_{R_1 \circ R_2}$.

(c) Check whether the following statement is a tautology or not

$$([P \to Q] \leftrightarrow [\sim (P \wedge (\sim Q))]) \to [(\sim P) \vee Q]$$

Q 3. Attempt any TWO questions out of the following: (2x7.5=15)

(a) Find the conjunctive normal form of the function

$$f = [x \wedge (y' \vee z)] \vee z' \to x$$

and hence find its disjunctive normal form from it.

(b) Define Ring. Prove or disprove that $(R, +, *)$ is a commutative ring with identity. R is a set of rational numbers, $+$ and $*$ denote usual addition and multiplication .

(c) Solve the following Recurrence Relation:

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

Q 4. Attempt any TWO questions out of the following: (2x7.5=15)

(a) Give short note on the following:

i. Complete Bi partite Graph
ii. Rule of inference
iii. Connected Graph

(b) Write the given argument in words and determine whether the argument is valid or not. Let

p : Ramesh goes to school.

q : Ramesh plays.

r : Ramesh is smart.

$p \to (r \vee q)$

$r \to \sim q$

Therefore

$p \to r$

(c) Prove that subgroup of a Cyclic Group is Cyclic.

Q 5. Attempt any TWO questions out of the following: (2x7.5=15)

(a) State and prove Lagrange's Theorem for a Group G.

(b) Prove that tree has no cycle.

(c) Prove that cartesian product of two or more Lattices is a Lattice. Give example also.

III Semester
End Semester Examination
Paper Code: CO-205(OLD SCHEME)
Max Marks: 70

Roll No.: ....
B.Tech., November-2019
Discrete Mathematics
Time: 3:00 Hours

- NOTE: Answer ALL questions. All questions carry equal marks. Assume suitable missing data, if any.

Q 1. Answer all the following questions: (10x1=10)

For (a) -(e) claim, determine whether it is always true or else false is some cases. Give justification for your answer.

(a) $|\{\emptyset\}| = 1$ but $|\emptyset| = 0$

(b) $\overline{\overline{A}} = \overline{(\overline{A})}$, i.e. complement of the complement of A equals A.

(c) $P(\emptyset) = P(\{\emptyset\})$, where $P(\,)$ is power set.

(d) Suppose that A is any set and $B = \{A, \{A\}\}$. Then $A \subseteq B$.

(e) If A and B are finite sets with $P(A) = P(B)$. Then $A = B$.

(f) Let a and b be elements in a Boolean Algebra. Prove that

$$\left(a + \left(\bar{a}.\left(\bar{b} + b\right)\right)\right).b = b$$

(g) Let a and b be elements in a Boolean Algebra. Prove that

$$\left(a.\left(\bar{a} + \left(\bar{b}.b\right)\right)\right) + b = b$$

(h) Give degree of following Recurrence Relation:

$$a_n = a_{n-1} + a_{n-2} + a_{n-3} + \dots\dots + e^n$$

(i) Give order of following Recurrence Relation:

$$a_n = a_{n-1} + a_{n-2} + a_{n-3} + \dots\dots + e^n$$

(j) What is a cyclic group. Give example of a cyclic group as well as non cyclic group.

Q 2. Attempt any TWO questions out of the following: (2x7.5=15)