***RECOMMENDED MARKING SCHEME (END TERM CS102 DISCRETE STRUCTURES***

***(The answers are only representative)***

***Question 1(A) and recommended marking scheme***

Test the validity of the following argument relating to the students of $VII^{th}$ and $VIII^{th}$ Semester of DTU using ***Rules of Inference***: -

(i) There is a student in the $VII^{th}$ semester who is not eligible to file for nomination to the post of class placement representative but is doing internship at Google.

(ii) If a student has CGPA of at least 9.0, then the student is eligible to file for nomination to the post of class placement representative.

(iii) If a student has CGPA less than 9.0, then the student has to clear the screening test for placement.

(iv) If a student has cleared the screening test for placement, the student is eligible for placement in any company.

Therefore, it can be concluded that a few students are eligible for placement in any company.

| **Aspects to be evaluated** | **[5]** |
|---|---|
| Defining predicates and domain of discourse correctly | $(0.25 \times 6 = 1.5)$ |
| Correct use of Rules of Inference | (3.0) |
| Correct conclusion | (0.5) |

*Let* $x \in$ *{students of* $VII^{th}$ *and* $VIII^{th}$ *semester of DTU}*

*Let the predicates* $p(x)$, $q(x)$, $r(x)$, $s(x)$ *and* $t(x)$ *represent the following: -*

| | | | |
|---|---|---|---|
| $p(x)$ : | *x can file for nomination to the post of class placement coordinator* | $q(x)$ : | *x is doing internship in Google* |
| $r(x)$ : | *x has CGPA of at least 9.0* | $s(x)$ : | *x has to clear screening test for placement* |
| $t(x)$ : | *x is eligible for placement in any company* | | |

$\therefore$ *the premises are*

| | | |
|---|---|---|
| $P_1$ : | $\exists x \left(\neg p(x) \land q(x)\right)$ | *(There is a student in the* $VII^{th}$ *semester who is not eligible to file for nomination to the post of class placement representative but is doing internship.)* |
| $P_2$ : | $\forall x \left(r(x) \to p(x)\right)$ | *(If a student has CGPA of at least 9.0 , then the student is eligible to file for nomination to the post of class placement representative.)* |
| $P_3$ : | $\forall x \left(\neg r(x) \to s(x)\right)$ | *(If a student has CGPA less than 9.0 , then the student has to clear the screening test for placement.)* |
| $P_4$ : | $\forall x \left(s(x) \to t(x)\right)$ | *(If a student has cleared the screening test for placement, the student is eligible for placement in any company.)* |
| *and the presumed conclusion is* | | |
| $\therefore$ | $\exists x\, t(x)$ | *(A few students are eligible for placement in any company)* |
| | | |
| ***Sl. No.*** | ***Step*** | ***Reason*** |
| *1.* | $\neg p(a) \land q(a)$ | *Existential Instantiation of Premise 1 using arbitrary student* $a$ |
| *2.* | $\neg p(a)$ | *Simplification of 1* |
| *3.* | $r(a) \to p(a)$ | *Universal Instantiation of Premise 2 using arbitrary student* $a$ |
| *4.* | $\neg r(a)$ | *Modus Tollens using 2 & 3* |
| *5.* | $\neg r(a) \to s(a)$ | *Universal Instantiation of Premise 3 using arbitrary student* $a$ |
| *6.* | $s(a)$ | *Modus Ponens using 4 & 5* |
| *7.* | $s(a) \to t(a)$ | *Universal Instantiation of Premise 4 using arbitrary student* $a$ |
| *8.* | $t(a)$ | *Modus Ponens using 6 & 7* |
| *9.* | $\exists x\, t(x)$ | *Existential generalization of 8; which is the desired conclusion* |

## Question 1(B) and recommended marking scheme

Convert the statement $(x \to (y \wedge w)) \wedge (z \to (y \wedge w))$ into PDNF forms

| Aspect to be evaluated | Marks |
| --- | --- |
| *Correct use of Logical Equivalences to form PDNF* **OR** *Correctly formed truth table with 16 rows and appropriate columns as per propositional statement, correct construction of max-terms to form PDNF* | *5* |

$(x \to (y \wedge w)) \wedge (z \to (y \wedge w))$

$\equiv (\neg x \vee (y \wedge w)) \wedge (\neg z \vee (y \wedge w))$

$\equiv (\neg x \wedge \neg z) \vee (y \wedge w)$ **DNF**

$\equiv (\neg x \wedge (\neg y \vee y) \wedge \neg z \wedge (\neg w \vee w)) \vee ((\neg x \vee x) \wedge y \wedge (\neg z \vee z) \wedge w)$

$\equiv (\neg x \wedge \neg y \wedge \neg z \wedge \neg w) \vee (\neg x \wedge \neg y \wedge \neg z \wedge w) \vee (\neg x \wedge y \wedge \neg z \wedge \neg w) \vee (\neg x \wedge y \wedge \neg z \wedge w) \vee$
$(\neg x \wedge y \wedge z \wedge w) \vee (x \wedge y \wedge \neg z \wedge w) \vee (x \wedge y \wedge z \wedge w)$ **PDNF**

## Question 2(A) and recommended marking scheme

Assume that the computational complexity of the divide and conquer algorithm is determined using recurrence relation given below. What is the big theta $(\Theta)$ estimate for the complexity of the algorithms?

$f(n) = 2f(\sqrt[2]{n}) + 1$, where n is a perfect square, $f(1) = 1$

| Aspect to be evaluated | Marks |
| --- | --- |

Application of Master's Theorem for analysis of recursive algorithms

{ If $T(n)$ is defined by recursive relation $T(n) = a \cdot T(n/b) + cn^d$ *for* $n > 1$, *where* $a \geq 1$, $b \geq 2$, $c > 0$ *and* $d \geq 0$ *are constants*, $\frac{n}{b}$ *is either* $\left\lfloor \frac{n}{b} \right\rfloor$ *or* $\left\lceil \frac{n}{b} \right\rceil$ *; then one of the following holds:*
$T(n) = \Theta(n^d)$ *if* $a < b^d$ ; $T(n) = \Theta(n^d \log n)$ *if* $a = b^d$ ; $T(n) = \Theta(n^{\log_b a})$ if $a > b^d$ }

Given $f(n) = 2f(\sqrt{n}) + 1, f(1) = 1$ where $n$ is a perfect square

Assume $2^m = n$ or $m = \log_2 n$

$\therefore f(n) = f(2^m) = 2f((2^m)^{1/2}) + 1$

or $f(2^m) = 2f\left(2^{\frac{m}{2}}\right) + 1$ 2.5

Let $g(m) = f(2^m)$

$\therefore g(m) = 2g(m/2) + 1$

Using Master's Theorem; here $a = 2$, $b = 2$, $c = 1$ and $d = 0$ so $a > b^d$ 2.5

$\therefore g(m) = \Theta(m^{\log_2 2}) = \Theta(m)$ but $m = \log_2 n$; hence $f(n) = \Theta(\log_2 n)$

## Question 2(B) and recommended marking scheme

If $A = \{1,2,3,4,5,6,7,8\}$ , $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 1 & 2 & 6 & 5 & 7 & 8 \end{pmatrix}$ ; $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 4 & 6 & 7 & 8 \end{pmatrix}$; $p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 2 & 5 & 4 & 1 & 7 & 8 \end{pmatrix}$. Outline the expression for $(p_2 \circ p_1) \circ p_3$ . Is the resulting permutation odd or even?

| Aspects to be evaluated | Marks |
| --- | --- |
| Correct composition of permutations | (2.5) |
| Correct identification of odd/even permutations | (2.5) |

Given $= \{1,2,3,4,5,6,7,8\}$ ,

$p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 1 & 2 & 6 & 5 & 7 & 8 \end{pmatrix}$ ; $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 4 & 6 & 7 & 8 \end{pmatrix}$;

$p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 2 & 5 & 4 & 1 & 7 & 8 \end{pmatrix}$.

$(p_2 \circ p_1) \circ p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 2 & 3 & 6 & 4 & 7 & 8 \end{pmatrix} \circ p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 2 & 5 & 6 & 3 & 1 & 7 & 8 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 2 & 5 & 6 & 3 & 1 & 7 & 8 \end{pmatrix} = (3 \quad 5) \circ (1 \quad 4 \quad 6) = (3 \quad 5) \circ (1 \quad 6) \circ (1 \quad 4)$ , which is a composition of odd number of transpositions. $\therefore (p_2 \circ p_1) \circ p_3$ results in an odd permutation

## Question 3(A) and recommended marking scheme

Let $A = \{a, b, c, d, e\}$ , $R$ and $S$ be equivalence relations on $A$ whose matrices are given below. Compute the matrix of the smallest relation containing $R$ and $S$ (including their properties). Also list the elements of this relation.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}; M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

| Aspects to be evaluated | Marks |
|---|---|
| *Correct application of the concept of joint of two Boolean matrices.*<br>*Application of Warshall's Algorithm for finding the Transitive Closure of a relation.* | |

$$\text{Given } M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}; M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} M_{R\cup S} = M_R \vee M_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$ 0.5

;

the matrix of the smallest relation containing $R$ and $S$ is $M_{(R\cup S)^\infty}$

Compute $M_{(R\cup S)^\infty}$ using Warshall's Algorithm

$$W_0 = M_{R\cup S} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

for $k = 1$ examine $W_0$ — 0.5

| | |
|---|---|
| Column 1 has 1s at positions | {1,2} |
| Row 1 has 1s at positions | {1,2} |

∴ add new 1 in locations (1,1), (1,2), (2,1), (2,2) if they do not exist to generate $W_1$

$$W_0 = W_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

for $k = 2$ examine $W_1$ — 0.5

| | |
|---|---|
| Column 2 has 1s at positions | {1,2,3} |
| Row 2 has 1s at positions | {1,2,3} |

∴ add new 1 in locations (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) if they do not exist to generate $W_2$

$$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

for $k = 3$ examine $W_2$ — 0.5

| | |
|---|---|
| Column 3 has 1s at positions | {1,2,3} |
| Row 3 has 1s at positions | {1,2,3} |

∴ add new 1 in locations (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) if they do not exist to generate $W_3$

$$W_2 = W_3 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

for $k = 4$ examine $W_3$ — 0.5

| | |
|---|---|
| Column 4 has 1s at positions | {4,5} |
| Row 4 has 1s at positions | {4,5} |

∴ add new 1 in locations (4,4), (4,5), (5,4), (5,5) if they do not exist to generate $W_4$

$$W_2 = W_3 = W_4 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

for $k = 5$ examine $W_4$ — 0.5

| | |
|---|---|
| Column 5 has 1s at positions | {4,5} |
| Row 5 has 1s at positions | {4,5} |

∴ add new 1 in locations (4,4), (4,5), (5,4), (5,5) if they do not exist to generate $W_5$

$$W_2 = W_3 = W_4 = W_5 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \therefore M_{(R\cup S)^\infty} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$ 0.5

The elements of the smallest relation containing $R$ and $S$ is 1.5

$(R \cup S)^\infty = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,d), (d,e), (e,d), (e,e)\}$

## Question 3(B) and recommended marking scheme

If $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ with the partial order $\leq$ of divisibility on $A$, $i.e\ a \leq b$ if and only if $a\ divides\ b$ ; and $B = P(S)$, the power set of $S$, where $S = \{e, f, g\}$; and $(B, \subseteq)$ be a poset, where $\subseteq$ is the partial order of set containment. Analyse if $(A, \leq)$ and $(B, \subseteq)$ are isomorphic posets. Justify your answer.

| Aspects to be evaluated | Marks |
|---|---|
| *Understanding of the definition of partial order set and drawing the Hasse Diagram for the poset* | |
| *Understanding of the definition of isomorphic posets* | (2+2) |

Given $A = \{1,2,3,5,6,10,15,30\}$ with the partial order $\leq$ of divisibility on A, the Hasse Diagram of the poset $(A, \leq)$ is shown in Fig. 1.

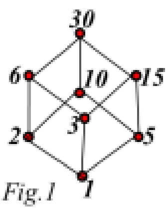


Fig.1

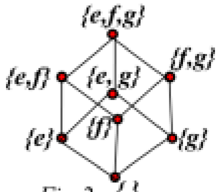


Fig.2

Given $B = P(S)$, where $S = \{e, f, g\}$

$\therefore$ B $= \{\{\,\}, \{e\}, \{f\}, \{g\}, \{e, f\}, \{e, g\}, \{f.g\}, \{e, f, g\}\}$

The Hasse Diagram of the poset $(B, \subseteq)$ is shown in Fig.2.

It is possible to define a function $f: A \to B$, a one-to-one correspondence, from $A$ to $B$ as: (1)

$f(1) = \{\,\}$; $f(2) = \{e\}$; $f(3) = \{f\}$; $f(5) = \{g\}$; $f(6) = \{e, f\}$; $f(10) = \{e, g\}$; $f(15) = \{f, g\}$; $f(30) = \{e, f, g\}$;

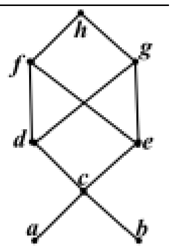
The posets $(A, \leq)$ and $(B, \subseteq)$ are therefore isomorphic.

**OR**

The Hasse Diagrams of posets $(A, \leq)$ and $(B, \subseteq)$ are of the same form except for the labels of the vertices. $\therefore$ posets $(A, \leq)$ and $(B, \subseteq)$ are isomorphic

## Question 4(A) and recommended marking scheme

$(A, \leq)$ is a poset, where $A = \{a, b, c, d, e, f, g, h\}$ and $\leq$ is the partial order on A with Hasse diagram as shown in Figure. If $B = \{a, b\}$ and $C = \{c, d, e\}$ are subsets of A. Interpret what are the following: -




(i) Lower Bounds of subsets B and C.
(ii) Upper Bounds of subsets B and C
(iii) Greatest Lower Bound of subsets B and C
(iv) Least Upper Bound of subsets B and C

**Aspects to be evaluated** **[5]**

*Interpretation / identification of Lower Bound, Upper Bound, GLB, LUB of a subset of the poset$(A, \leq)$* $(1.25 \times 4 = 5)$

(i) Lower Bounds (B) = { }, Lower Bound of (C) = $\{a, b, c\}$
(ii) Upper Bounds (B) = $\{c, d, e, f, g, h\}$ , Upper Bounds (C) = $\{f, g, h\}$
(iii) GLB (B) does not exist, GLB (C) = $c$
(iv) LUB (B) = $c$, LUB (C) does not exist

## *Answer 4(B) and recommended marking scheme*

If $n$ is a positive integer, $p$ a prime number and $p^2$ divides $n$. Is $D_n$, the set of positive integers that divides $n$, a lattice under the relation of divisibility? Prove or disprove that $D_n$ is not a Boolean Algebra.

| Aspects to be evaluated | Marks |
|---|---|
| *Understanding of the definition of lattice* | (2) |
| *Understanding of the definition of Boolean Algebra and correct application of methods of proof to establish that $D_n$ is not a Boolean Algebra* | (3) |

By definition of $D_n$ (*i.e. set of positive integers that divides the positive integer n*), $D_n$ is a lattice under the relation of divisibility, in which $x_1 \wedge x_2 = HCF\ (x_1,\ x_2)$ and $x_1 \vee x_2 = LCM\ (x_1,\ x_2)\ \forall \{x_1, x_2\} \in D_n$

Let $a$ be the compound proposition: $p$ is a prime number AND $p^2$ divides $n$ AND $n$ is a positive integer, and $b$ be the proposition: $D_n$ is not a Boolean Algebra.

<u>Given</u>:

$p^2$ divides $n$

$\Rightarrow n = p^2 \cdot q$ for some positive integer $q$

$\Rightarrow \frac{n}{p} = p \cdot q$

Since $p^2$ divides $n$ $\therefore$ $p$ divides $n$ , hence $p \in D_n$

<u>RTP</u> : $a \rightarrow b$ is a tautology

<u>Proof by Contradiction:</u> Assume $a$ is true and $b$ is false (i.e. $D_n$ is a Boolean algebra)

If $D_n$ is a Boolean algebra, then $p$ must have a complement, $p'$ and $HCF\ (p,\ p') = 1$ and $LCM\ (p,\ p') = n$

It is known that $a \cdot b = HCF\ (a, b) \cdot LCM(a\ , b)$

$\Rightarrow p \cdot p' = n \cdot 1$ or $p' = \frac{n}{p} = p \cdot q$ This shows that $HCF\ (p, pq) = 1$ , which is not possible since the $HCF\ (p, pq) = q$ . Hence, $D_n$ cannot be a Boolean algebra

## Question 5(A) and recommended marking scheme

*System specification for a 4-input security system is modelled using the function* $\boldsymbol{f}: \boldsymbol{B_4} \rightarrow \boldsymbol{B_1}$ *, which is defined as given below*

$$f(x, y, z, w) = \begin{cases} 1 & \text{as shown in table} \\ \mathbf{0} & \textbf{otherwise} \end{cases}$$

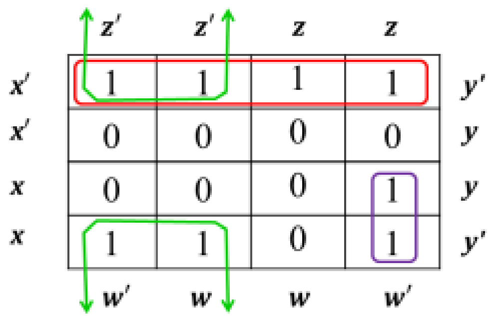
*Use Karnaugh Map to develop a logical circuit diagram with minimum number of logic gates to implement the system specification.*

| x | y | z | w | f(x,y,z,w) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 |

| **Aspects to be evaluated** | **[5]** |
|---|---|
| Identification of minterms, Correct creation and reduction of Karnaugh Map | (2.5) |
| Correct identification of reduced logical expression | (0.5) |
| Correct Logical Circuit Diagram | (2.0) |

The system specification can be expressed in the non-reduced form as

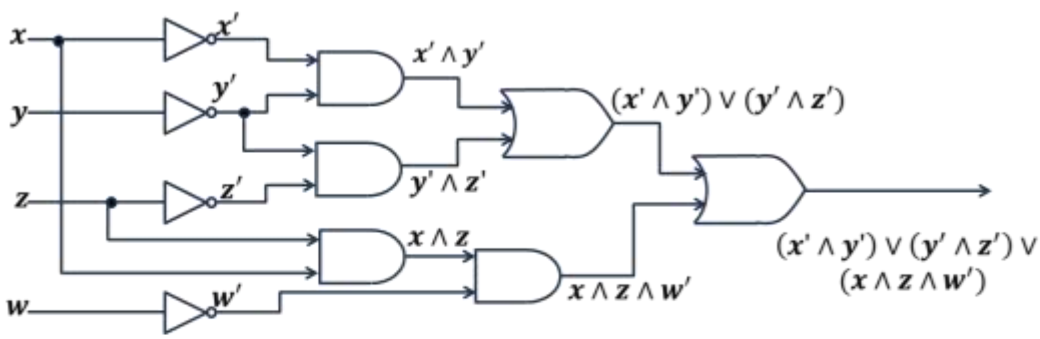
$(x' \wedge y' \wedge z' \wedge w') \vee (x' \wedge y' \wedge z' \wedge w) \vee (x' \wedge y' \wedge z \wedge w') \vee$
$(x' \wedge y' \wedge z \wedge w) \vee (x \wedge y' \wedge z' \wedge w') \vee (x \wedge y' \wedge z' \wedge w) \vee$
$(x \wedge y' \wedge z \wedge w') \vee (x \wedge y \wedge z \wedge w')$




The Karnaugh-map for the same is shown in Figure, using which the system specification gets reduced to

$(x' \wedge y') \vee (y' \wedge z') \vee (x \wedge z \wedge w')$

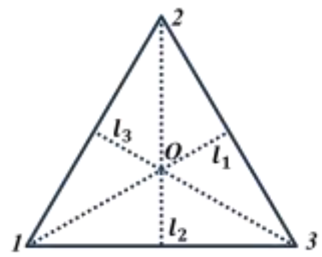
The logical circuit diagram is given below




## Question 5(B) and recommended marking scheme

*Consider the equilateral triangle shown opposite. The symmetries of the triangle are: -*

*Counter-clockwise rotations $f_1$, $f_2$, and $f_3$, about $O$ through $360°$, $120°$ and $240°$, respectively; reflections $g_1$ , $g_2$ and $g_3$ about the respective angular bisectors $l_1$ , $l_2$ and $l_3$ respectively. Develop the multiplication table for the operation *, "followed by" defined on the set of symmetries of the triangle.*




| **Aspects to be evaluated** | **Marks** |
|---|---|

| | |
|---|---|
| Correct identification of set of symmetries of the triangle | (2) |
| Correct generation of multiplication table for the operation *, "*followed by*" | (3) |

$f_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad f_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \quad f_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$g_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad g_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad g_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

Set of all symmetries of the triangle is described by the set of permutations of the set $\{1, 2, 3\}$, let these symmetries be represented by the set $S_3 = \{ f_1, f_2, f_3, g_1, g_2, g_3 \}$

The multiplication table of * may be computed geometrically or algebraically.

(For example, to compute $f_2 * g_2$ geometrically, "*followed by*" refers to the geometric order of the application of transformation. To compute $f_2 * g_2$ algebraically, compute $f_2 \circ g_2$ )

The multiplication table of * is given below

| $*$ | $f_1$ | $f_2$ | $f_3$ | $g_1$ | $g_2$ | $g_3$ |
|---|---|---|---|---|---|---|
| $f_1$ | $f_1$ | $f_2$ | $f_3$ | $g_1$ | $g_2$ | $g_3$ |
| $f_2$ | $f_2$ | $f_3$ | $f_1$ | $g_3$ | $g_1$ | $g_2$ |
| $f_3$ | $f_3$ | $f_1$ | $f_2$ | $g_2$ | $g_3$ | $g_1$ |
| $g_1$ | $g_1$ | $g_2$ | $g_3$ | $f_1$ | $f_2$ | $f_3$ |
| $g_2$ | $g_2$ | $g_3$ | $g_1$ | $f_3$ | $f_1$ | $f_2$ |
| $g_3$ | $g_3$ | $g_1$ | $g_2$ | $f_2$ | $f_3$ | $f_1$ |

## *Question 6(A) and recommended marking scheme*

Let $A = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ be a set and $T$ be a relation on A, defined as

$T = \{ (v_2, v_3), (v_2, v_1), (v_4, v_5), (v_4, v_6), (v_5, v_8), (v_6, v_7), (v_4, v_2), (v_7, v_9), (v_7, v_{10})\}$ .

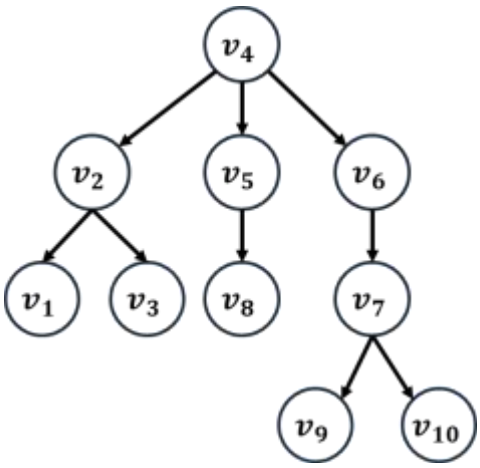
Determine if T is a tree. If yes, draw the digraph of the tree and identify the root, the leaves and the height of the tree. In case $T$ is not a tree identify the properties that the relation satisfies.

| **Aspects to be evaluated** | **Marks** |
|---|---|
| Application of the definition of Tree and identification root, nodes, leaves and height of the tree | (3) |
| Correct generation of digraph of the tree | (2) |

The in-degree and out-degree of the vertices is tabulated below: -

| Vertices | In-degree | Out-degree |
|---|---|---|
| $v_1$ | 1 | 0 |
| $v_2$ | 1 | 2 |
| $v_3$ | 1 | 0 |
| $v_4$ | 0 | 3 |
| $v_5$ | 1 | 1 |

| Vertices | In-degree | Out-degree |
|---|---|---|
| $v_6$ | 1 | 1 |
| $v_7$ | 1 | 2 |
| $v_8$ | 1 | 0 |
| $v_9$ | 1 | 0 |
| $v_{10}$ | 1 | 0 |




From the table it is observed that: -

(a) T is a tree with $v_4$ as the root, as in-degree of all vertices is 1 except, root, which is zero.

(b) The leaf nodes are $v_1$ , $v_3$ , $v_8$ , $v_9$ and $v_{10}$, as the out-degree of these vertices is zero.

The digraph of the tree $(T, v_0)$is shown opposite. From the digraph it is observed that the height of the tree is 3. (***OR from the definition of the relation T, it is observed that the longest paths $\pi_1 : v_4, v_6, v_7, v_9$ and $\pi_2 : v_4, v_6, v_7, v_{10}$ in T are of length 3***)

## *Question 6(B) and recommended marking scheme*

A college offers 5 elective subjects (with codes $E01$ to $E05$) for 2nd Sem students to choose 2 from these subjects. The data of the number of students registered for $Subject_i$ and $Subject_j$ is tabulated below.

| | $Subject_i$ | $Subject_j$ | # |
|---|---|---|---|
| a) | E01 | E02 | 19 |
| b) | E01 | E05 | 26 |
| c) | E02 | E03 | 18 |
| d) | E02 | E04 | 24 |

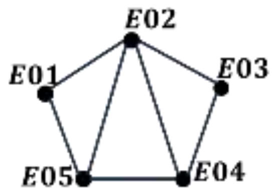
| | $Subject_i$ | $Subject_j$ | # |
|---|---|---|---|
| e) | E02 | E05 | 17 |
| f) | E03 | E04 | 28 |
| g) | E04 | E05 | 35 |

A clash will occur in the timetable if the examination of two subjects is conducted during the same time-slot, resulting in a student not being able to take the examination of one of the subjects for which the student is registered. Assuming that the number of examinations per day for a given student is not more than one

(i) Apply the principle of graph colouring to find the minimum numbers of days required to conduct the End-Semester Examination for the elective subjects without any clash in the timetable?

(ii) How many different clash-free time-tables can be generated for the conduct of End-Semester Examinations of the elective subjects in the minimum number of days?

| Aspects to be evaluated | Marks |
|---|---|
| Generation of graph for the time-table scheduling problem | (2) |
| Correct generation of Chromatic Polynomial and identification of Chromatic Number | (2) |
| Use of chromatic polynomial to identify the minimum number of clash-free timetables | (1) |




The graph $G$, for the time-table scheduling problem with the elective subject codes as vertices and an edge between two vertices if a student has registered for the corresponding $subject_i$ and $subject_j$ is given in figure opposite.

Assuming a set of $n$ colours, and starting from vertex $E01$, for proper colouring: -

Vertex $E01$ can be coloured using any of the $n$ colours

Vertex $E02$ can be coloured using any of the remaining $(n-1)$ colours

Vertex $E05$ can be coloured using any of the remaining $(n-2)$ colours

Vertex $E04$ can be coloured using any of the remaining $(n-2)$ colours

Vertex $E03$ can be coloured using any of the remaining $(n-2)$ colours

Thus, the Chromatic Polynomial of the Graph, $\mathrm{X_G}(n) = n(n-1)(n-2)^3$

$\mathrm{X_G}(0) = \mathrm{X_G}(1) = \mathrm{X_G}(2) = 0 \;;\; \mathrm{X_G}(3) = 6$

Hence, the chromatic number of the graph, $G$ is 3

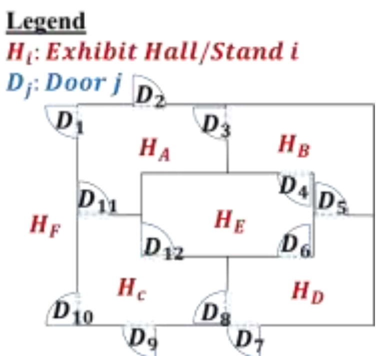
Therefore, the minimum numbers of days required to conduct the End-Semester Examination for the elective subjects without any clash in the timetable is 3 days.

And using $\mathrm{X_G}(3)$, 6 different clash-free time-tables can be generated for the conduct of End-Semester Examinations of the elective subjects in the minimum number of days.

## *Question 7(A) and recommended marking scheme*

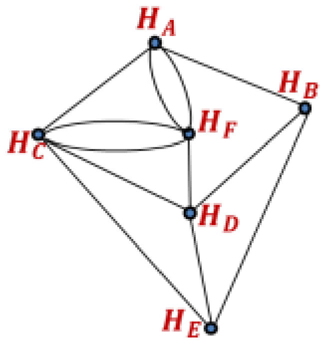
An exhibition route map is required to be made for a newly renovated museum with five indoor Exhibit Halls and one outdoor Exhibit Stand. The floor plan of the museum is given in Figure.

Is it possible to plan a route to visit every exhibit hall / stand in the museum by passing through each door exactly once? If it is possible, enumerate the path. Justify your answer, in case it is not possible to plan such a route..




| Aspects to be evaluated | Marks |
|---|---|
| Generation of the graph for the given problem | (5) |
| Understanding of concepts for identification of existence of Euler Path in a graph | |
| Justification for non-existence of Euler Path | |

Let the Exhibition Hall / Exhibition Stand be represented by the vertices of the graph. Let there be an edge between two vertices if there is a door connecting the corresponding Exhibition Hall / Stand. The graph corresponding to the problem statement is shown in Figure.

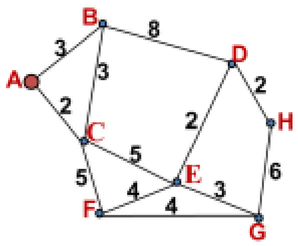


*Graph G*

From the Graph, it is observed that there are four vertices, with odd degree, namely $H_B$ (degree 3), $H_c$ (degree 5), $H_E$ (degree 3) and $H_F$ (degree 5).
Thus a Euler Path does not exist in the graph, as a Euler Path can exist in a graph only if the graph has utmost two vertices with odd degree, with the path starting at one of the vertices with odd degree and ending at the other with odd degree.

## *Question 7(B) and recommended marking scheme*

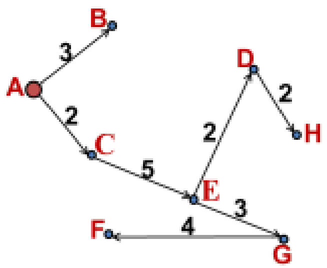
The representative weighted-graph of the cost of establishing high-speed data connectivity between new branch offices of a multi-national company (MNC) is shown in figure below. The weights mentioned against the edges represent the cost in multiples of ₹100,000/-. The Chief Operations Officer of the MNC intends to find the most economical solution to the problem.




Assuming that the Head-office is located at city A, build a minimal spanning tree (root as city A) with vertices representing various branch offices of the company.
What will be the minimum cost for establishing this data network?

| Aspects to be evaluated | Marks |
|---|---|
| Data structure for correct representation of graph and cost matrix / weighted graph. | (5) |
| Any algorithm for Identification of minimal spanning tree (MST) | |
| Identification of minimum cost. | |

One possible minimal spanning trees (MST) with root as vertex A out of the four possibilities is given below. The other possible options are combinations of edge $C \rightarrow B$ instead of $A \rightarrow B$ along-with edge $E \rightarrow F$ instead of $G \rightarrow F$




The minimum cost for establishing this data network will be

$$\sum wieghts\ of\ the\ edges\ of\ the\ MST = (3 + 2 + 5 + 3 + 2 + 2 + 4)$$

$$= ₹21\ Lakhs$$