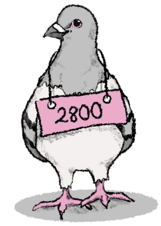
Discrete Structures

CS 2800 Spring 2024 Syllabus



General Information

Brief description: This course covers the mathematics that underlies most of computer science. Topics include mathematical induction; logical proof; propositional and predicate calculus; sets, functions, and relations; graphs; combinatorics and discrete mathematics; basic probability theory; and finite-state machines. These topics are discussed in the context of applications to many areas of computer science.

Course Staff:

Instructor: Anke van Zuylen

- **Office hours:** Monday 11am-noon, and Wednesday 2-3pm in Gates 447.
- **Email:** avz2@cornell.edu. Please do not use email to ask questions about the material, policies, exam dates, etc. You can submit questions like these to Ed Discussion, where everyone can view the reply as well (and don't forget to search whether a question has been asked already).
- **1-on-1 Appointment:** You can make an appointment with the instructor using Calendly. Appointments slots are limited, and are meant for discussing topics you cannot discuss in public office hours (so not for homework questions!)

Course Admin: Sara Perkins, sep247@cornell.edu.

Sara is the person to contact about logistical issues such as exam conflicts, Canvas access, etc.

Teaching Assistants: We are lucky to have more than 50 TAs for the course. You can find more information about them, and their office hours, on Canvas.

Credit Hours: This is a 4-credit course, which can only be taken for a letter grade.

Time and Location:

- Lectures are held MWF 10:10am-11:00am, in the Statler Auditorium (Statler 185) from January 22 until May 6 (with the exception of university-scheduled breaks).
- Discussion sections are 50-minute meetings, held at different times and locations on WThF, that allow you to work in small groups and with course TAs on problems similar to those in the homework assignments.

Pre/corequisites:

- The only formal corequisite for CS 2800 is one programming course, for example CS 1110 or CS 2110. This course can be taken concurrently with CS 2800.
- Some level of mathematical familiarity (at the level of a high school curriculum) will be assumed. You should be comfortable working with familiar functions (e.g. polynomial and exponential functions) and performing simple arithmetic without a calculator.
- The last module of the course assumes familiarity with differential and integral calculus at the level of AP Calculus AB or MATH 1110. The calculus content required for the courae will be introduced and/or reviewed where necessary.

Course description

In this course, we will study the mathematical foundations of computer science starting at the most basic level. The focus of the course will be on many *discrete structures* (i.e. objects that are characterized by distinct, separated values as opposed to the *continuum* of real numbers) such as logical formulas, sets, functions, relations, graphs, and automata. These topics are discussed in the context of applications to many areas of computer science. An emphasis of this course, in contrast to other mathematics classes that you may have taken, will be writing rigorous justification (or a *proof*) to fully explain your mathematical reasoning. As such, this course not only has the goal of introducing you to important definitions and theorems but also of teaching you how to think critically and gain confidence when solving new mathematical or computational problems.

Learning objectives

On completing this course, students should be able to:

- Use **logical notation** to define and reason about statements expressed in informal language.
- Write **mathematical proofs** using clear and precise reasoning; recognize when to use various proof techniques; find errors in faulty proofs.
- Precisely state induction hypotheses and write **inductive proofs**; write and use inductive definitions.
- Reason about **fundamental mathematical concepts** such as sets, functions, relations, graphs and their properties.
- Apply **combinatorial reasoning** to solve counting problems arising in real-world applications; use the pigeonhole principle to prove statements.

- Use **probability concepts and notation** such as conditional probability, Bayes' rule, random variables, and tail bounds to model and analyze real-world applications; compute associated quantitaties such as probabilities, expectations, variance, covariance, and correlation.

- Prove properties of **regular languages and automata**; design automata and regular expressions accepting or denoting a certain regular language; prove that a language is not regular.

Course Textbook and Other Materials

Course Text

A beautiful set of comprehensive course notes, written by Matt Eichhorn, will be available on Canvas. These notes are broken down by lecture topic, and include many worked examples and additional exercises for you to gain mastery of the material.

Optional Textbooks

There are many excellent books about discrete mathematics that offer an alternative presentation of the material, which may be helpful in your studies. Some additional references that we recommend are:

- *Discrete Mathematics and its Applications* by Kenneth Rosen. Refer to this text (any edition is fine) for many great exercises with solutions.

- *Discrete Mathematics* ZY Book by Sandy Irani. Get (paid) access to this if you find it helpful to answer brief autograded questions that check your understanding while reading.

- *A course in Discrete Structures*, a concise set of lecture notes by Rafael Pass and Wei-Lung Dustin Tseng.

- *Mathematics for Computer Science* by Eric Lehmann, Albert Meyer, and Tom Leighton.

- *Essential Discrete Mathematics for Computer Science* by Harry Lewis and Rachel Zax.

The topics covered by these texts differ (to varying degrees) from our course. Also, the levels of formality and difficulty of these texts vary, and some of the notational choices differ from or conflict with ours. For consistency, we will expect you to use the notation presented in lectures and the course notes.

Course Websites

We will be using Canvas for distributing handwritten lecture notes, homework solutions, and discussion assignments. You should be enrolled automatically into Canvas, but if not (for example, because your registration has not been processed yet), please contact Sara Perkins to get added manually. All other websites we use (Gradescope for homework submission, Ed Discussions for Q&A, etc.) are linked from the course Canvas site; you should automatically get added to the rosters for these other sites if you access them through the Canvas link.

You are expected to check the Ed Discussion site regularly for announcements, and keep an eye on the Canvas site for useful materials.

Grading

Your grade will based on weekly homework and discussion exercises, participation, two prelims, and one final exam. Each of these components will be given a weight in the following ranges:

- homework: 20%
- discussion: 0 to 5%
- participation: 0 to 3%
- quiz: 5%
- lower prelim score: 10%
- higher prelim score: 25%
- final exam (cumulative): 32% to 40%

At the end of the semester, we will compute the overall score for each individual student using weights in the indicated ranges (so discussion and participation only count if the scores you get for these components is higher than your final exam score).

Homework

This course has weekly homework assignments. All homework assignments will be posted on Gradescope, and typically, homework assignments will be due the next Wednesday at 11:59pm. You can view a (tentative) schedule of the assignments on the Course Calendar. Homework needs to submitted on Gradescope. There will be roughly 11 homework assignments, and the lowest assigment grade is dropped.

It is OK if you handwrite your answers and upload a photo (converted to a single pdf), but we encourage you to learn how to typeset them using LaTeX. We will post resources to help you get familiar with LaTeX, and the LaTeX source files of the homework so you can quickly learn the important syntax and commands.

Participation and Discussion

The learning activities during lectures and discussion sections are meant to help you check your understanding, and to provide you the opportunity to work with others to synthesize concepts and apply what you have learned. For the activities we do in lecture and discussions, you are not expected to give the correct answer, but we do expect you to make an effort to learn.

Your **participation** grade consists of lecture participation (66%) and completion of the mid-semester and end-of-semester course evaluations (17% each). Lecture participation is measured using PollEverywhere; there will be approximately 1-4 polls per lecture, and you get credit for each poll you complete (regardless of whether your answer is correct).

Similarly, your **discussion** grade will be determined based on the *completion* (not correctness) of discussion exercises. You will hand in the discussion exercises on gradescope by Friday 8:00pm. Unlike the homework, discussion exercises are graded as "good effort" (full credit), "mediocre effort" (half credit) or "no effort" (no credit), and the TAs are encouraged to discuss the solution of the discussion exercises in detail before you submit your work.

Exams and Quizzes

There will be one in-class quiz, two prelims and a final exam. The in-class quiz is scheduled for Wednesday February 21, the prelim exams are scheduled for Thursday March 14, 7:30pm and Thursday April 25, 7:30pm. The date for the final exam is to be determined. The exams will be held in-person.

Exams for CS2800 are closed-book and closed-notes and do not require any equipment except pen and paper. You can expect questions on the exam to be similar to the homework problems, so a good understanding of the key ideas used in the homework problems is necessary to do well. Review materials that include questions from previous years' exams will be provided a week before the exam.

Exams for students with SDS accommodations are administered by SDS. Please see the Accommodations section later in this syllabus for detailed instructions.

Course Policies

90% is full credit for homework, participation, and discussions

Your scores for homework and discussions, and the percentage of completed polls for lecture participation each get multiplied by 10/9 (rounded down to 100 if necessary). So if your score for a particular homework is x, it contributes $\min\{100, x/0.9\}$ to your overall homework grade; a similar adjustment is made for the participation and discussion grading.

Submission Deadlines

The submission deadlines are handled automatically by Gradescope, and it is your responsibility to ensure that your submission is turned in on time. As technical difficulties (network connection, errors on the website, problems with scanning or file conversion, etc.) may arise, it is advised that you submit your homework well in advance of the deadline. Submission issues that are brought to our attention after the deadline will *not* be accommodated.

Collaboration

Collaboration and conversation is an important part of how ideas get generated. We encourage you to discuss with a small group of your peers in the course to brainstorm ideas for how to solve the assigned exercises. However, **your solution must be written up completely on your own; you are not allowed to share digital or written notes or images of your work in any form with each other**.

Just like in research, you must include a collaborator statement on each homework assignment, including a list of all students who you discussed the homework with.

Admissible Resources

The only resources that you may use for your homework are things that are linked on Canvas (the lecture notes/slides, discussion worksheets, and past assignments) and the optional texts listed in the syllabus. Do not consult outside websites to help prepare your solutions. Using other resources is a violation of academic integrity. If you feel the resources available to you are insufficient, talk to course staff or ask questions on Ed.

Regrades

Regrade requests for homework assignments and exams should be submitted on Gradescope. Regrade requests are accepted for four days, starting one day after grades are released. We may regrade the entire exam or problem set, and your score may go up or down. When submitting a regrade request, be specific about which rubric items were misapplied, and how you feel the grading should be corrected. Point to specific parts of your answer.

Academic integrity

Each student in this course is expected to abide by the Cornell University Code of Academic Integrity. Any work submitted by a student in this course for academic credit will be the student's own work. The complete code is available at http://cuinfo.cornell.edu/Academic/AIC.html.

This course is participating in Accepting Responsibility (AR), which is a pilot supplement to the Cornell Code of Academic Integrity (AI). For details about the AR process and how it supplements the AI Code, see the AR website.

Academic integrity violations of the homework collaboration policy or accessing homework solutions found online result in a score of up to *negative* 50% (-50%) for the assignment. Communicating with any person or accessing unauthorized resources *during an exam or quiz* for CS2800 results in an F in the course.

Advice for Success

Here are a few tips for succeeding in this course:

- **Use the homework to make mistakes and learn!** We see many students each semester who do great on the homework, and who realize midsemester (often during a prelim) that they have no idea how to solve, or even read, the problems on their own. Very often, these students have been relying too much on their collaborators or TAs to solve the homework problems. On the other side of the coin, we also see many students who don't do that well on the homework, and who end up with an A in the course. So please take advantage of the fact that a single assignment counts for only 2% of your grade and allow yourself to make your own mistakes. When you do the homework the right way, it will make sure you are ready to do well on an exam!

- **Start your assignments early**. Even if you aren't writing anything down yet, looking over the problem set well in advance of the due date can ensure you have enough time to brainstorm possible solutions, and to clear up confusion about how to interpret a problem. Solving homework problems requires thought and creativity, and this doesn't work well on a deadline.

- **Use your discussion section to gain confidence and get feedback**. The exercises assigned each week to be solved during your discussion section are at a similar level as the homework exercises. They give you the opportunity to work with your peers and the TAs on all aspects of solving the problems.

- **Talk with classmates at a similar level about ideas for the homework**. As previously stated, while you cannot share physical or digital solutions of any kind to these problems, we actively encourage you to talk to classmates while you work through them. In particular, we recommend finding a group of students to meet with throughout the semester to talk about ideas. For best results, make sure those students are at the same level of understanding of the material as you; talking through your ideas with colleagues with a similar level of understanding will make talking through ideas with each other easier and more equitable, and is more likely to leave you prepared for course exams.

- **Ask questions in class, in discussions, in office hours, and on Ed Discussions**. The material in this class moves quickly and is often cumulative. In addition, in mathematics, you need to understand every single symbol of a statement, in order to make sure you are not considering a completely different problem. This takes some getting used to! So if you find yourself scratching your head after a lecture, even after consulting the textbook and course notes, you're certainly not alone, and it's better to seek help than to wait until you are more confused.

- **Don't panic if you're stuck and confused.** It is normal to struggle, and to be stuck sometimes and have no idea what to do! Struggling with the material helps you get a deeper understanding, so rather than panic, take a break, try again, and find help if you need it!

Academic Excellence Workshop ENGRG 1028

Academic Excellence Workshop (AEW) sections are available to be taken in conjunction with this course. AEWs are optional 1-credit supplemental courses which meet for one 2-hour collaborative problem-solving session each week throughout the semester. Designed to enhance student understanding, the workshops feature group work on problems at or above the level of course instruction. In the workshops, small-group problem-solving is directed by undergraduate peer educators called facilitators. The AEWs are graded S/U, based on attendance.

For this course there are multiple sections of ENGRG 1028 available, please see student center for details. You can enroll online during the add period. Space may fill up quickly – if there are no spots available in a section that fits your schedule, use the link included with the course listing in the registration system to indicate your interest and availability. For more information about AEWs, visit `http://www.engineering.cornell.edu/aew`.

Course Management

Inclusiveness

We understand that our members represent a rich variety of backgrounds and perspectives. Cornell University is committed to providing an atmosphere for learning that respects diversity. We expect students to communicate in a respectful manner with the instructors, course staff, and fellow students, in a way the honors the unique experiences, values, and beliefs represented by different members of our community.

You should expect and demand to be treated by your classmates and the course staff with respect. You belong here, and we are here to help you learn and enjoy this course. If any incident occurs that challenges this commitment to a supportive and inclusive environment, please let the instructor or a TA know so that we can address the issue. We are personally committed to this, and subscribe to the Computer Science Department's Values of Inclusion.

What to do if you get sick

If you are sick, you're first priority should be getting better; that means getting a lot of rest, drinking fluids, going to see a doctor if needed, etc. If you feel well enough to come to lecture or discussion, please make sure to wear a mask so you don't infect others.

Zoom recordings of lectures are not available for absences, including absences due to illness. For any illness, you are expected to keep up with course material by working with a peer in the course and/or accessing the lecture notes on canvas. You are allowed to miss 10% of the lectures and discussions for any reason without affecting your lecture participation and discussion credit. We do not excuse additional lecture absences for any reason, except in the case of long term illness. Please note that a single lecture counts for at most 0.05 (out of 100) towards your final grade, so absences because of a short illness will not affect your grade.

Getting support

We urge you to talk to the instructor or any of the TAs if you have any concerns about your learning or progress in the course, if you are sick for more than a week, or if you have other personal difficulties. Resources at Cornell are also available (`caringcommunity.cornell.edu`).

In case you experience major unexpected events (for instance, you being hospitalized unexpectedly, death of a close family member), please set up a meeting using calendly or send an email to the course staff, and we will figure out how to handle the situation together.

We cannot accommodate requests for exceptions for minor issues (leaving early for a break, forgetting to submit a homework, for example) — the course policies and grading scheme have been set up to take care of this.

Accommodations

This course complies with the university policies and equal access laws, and we provide accommodations for disability, religious observance, Title IX, varsity athletes, medical emergencies, and military service. Requests for academic accommodations should be made during no later than the drop deadline, except for unusual circumstances, so arrangements can be made as soon as possible. Testing-related academic accommodations are organized by SDS; please carefully read the next subsection for details. If you require accommodations that are not testing related, please email the course admin to discuss the logistics of your accommodation(s).

Testing accommodations

This course is participating in the SDS Alternative Testing Program (ATP) for the Spring 2024 semester. All exams will be centrally managed and supported by the ATP Testing Coordinator in the Office of Student Disability Services (SDS) and accessible in your SDS student portal. Exam logistics will be communicated to you via email from sds-testing@cornell.edu and accessible in your SDS student portal. The ATP will automatically send an email with the exam date, time, location 10 days before the exam, and a reminder email 48 hours prior to the exam date.

Course Material Copyright

Course materials posted on Canvas, gradescope, or Ed Discussions are intellectual property belonging to the author. Students are not permitted to buy or sell any course materials without the express permission of the instructor. Such unauthorized behavior constitutes academic misconduct.

Learning Outcomes

The following is a list of skills that you will have after taking this course. This list is not meant to be exhaustive, and some topics may need to be reduced or omitted based on our pacing during the semester. A lot of the outcomes use terminology that we will introduce during the course, so the list is most useful in retrospect.

Module 1: Foundations

1. Explain the syntax and semantics of operations in propositional logic
2. Use truth tables to determine whether two compound propositions are logically equivalent
3. Describe sets using set builder notation
4. Demonstrate understanding of predicates with one or more variables
5. Identify the free and bound variables in a quantified logical statement
6. Establish the truth value of statements containing one or more quantifiers
7. Demonstrate understanding of the set intersection, union, difference, Cartesian product, and power set operations
8. Write formal arguments of set containment and equality
9. Explain and carry out direct proofs, proofs by case analysis, proofs by contradiction, and proofs by contraposition
10. Construct counterexamples that refute a proposition
11. Translate between a visual and a formal set description of directed and undirected graphs
12. Identify paths and cycles in a graph

Module 2: Induction

13. Identify the steps of a proof by induction
14. Use ordinary induction to prove statements quantified over the natural numbers
15. Compare and contrast ordinary and strong induction
16. Carry out inductive proofs that require multiple base cases, multiple inductive steps, and/or the strong inductive hypothesis
17. Use induction to establish properties of graphs

Module 3: Functions and Cardinality

18. Invoke the formal definition to determine whether a function is well-defined
19. Compute the composition of two functions
20. Give a rigorous argument that a function is injective, surjective, or bijective
21. Describe the relationship between bijectivity and function inverses
22. Determine whether a given function is invertible
23. State and apply the Pigeonhole Principle
24. Indicate when one proposition is a generalization or specialization of another
25. Use the Pigeonhole Principle to establish properties of graphs
26. Explain how one can use injective functions to define the cardinalities of sets
27. Determine whether a given set is countable or uncountable
28. Carry out proofs by diagonalization

Module 4: Binary Relations

29. Compare and contrast the formal definitions of functions and binary relations
30. Decide whether a binary relation is reflexive, symmetric, anti-symmetric, and/or transitive
31. Justify whether or not a relation is a partial order
32. Visualize the structure of a partially ordered set with a Hasse diagram
33. Justify whether or not a relation is an equivalence relation
34. Find the equivalence classes of an equivalence relation
35. Describe the quotient of an equivalence relation

Module 5: Languages and Automata

36. Differentiate between alphabets, strings, and languages
37. Give a formal description of an automaton from its graph representation
38. Determine whether a given word is in the language of a given automaton

39. Draw a DFA that recognizes a given language
40. Describe the relationship between the Pigeonhole Principle and the Pumping Lemma
41. Use the Pumping Lemma to argue that a language cannot be recognized by a DFA
42. Perform the subset construction to convert an NFA into a DFA that recognizes the same language
43. Construct a regular expression that represents a given language
44. Carry out Thompson's construction to construct an NFA that recognizes the same language as a regular expression
45. Use the state elimination procedure to write a regular expression that represents the language of an automaton
46. Apply Kleene's theorem to reason about properties of regular languages
47. Calculate the Nerode congruence classes of a language
48. Apply the Myhill-Nerode theorem to argue that a language is not regular

Module 6: Combinatorics

49. Describe multi-step processes for enumerating the items in a set
50. Demonstrate understanding of the Multiplication, Addition, Subtraction, Bijection, and Division rules in combinatorics
51. Prove identities using combinatorial reasoning
52. Explain and apply the Binomial theorem
53. Apply the Inclusion-Exclusion rule to calculate the size of a set
54. Differentiate between arrangements with distinguishable/indistinguishable balls/bins
55. Use the stars and stripes formula to calculate the size of a set
56. Demonstrate understanding of the Stirling numbers

Module 7: Probability and Statistics

57. Identify the components of a probability space
58. Utilize techniques from combinatorics to compute probabilities in discrete probability spaces
59. Calculate the probability of an event conditioned on another event

60. Describe how conditioning on an event changes a probability space Determine whether events are independent

61. Explain and apply the Law of Total Probability

62. Use Bayes' Rule to calculate conditional probabilities

63. Describe how to use Bayesian Inference to update our beliefs about a probability based on new observations

64. Explain how the probability mass function and Cumulative Distribution Function describe the distribution of a discrete random variable

65. Identify the pmf, CDF, expectation, and variance of three families of discrete random variables (Bernoulli, binomial, geometric)

66. Use the definition to compute the expectation of a random variable

67. Calculate and interpret the variance of a random variable

68. Demonstrate understanding of joint and marginal distributions of random variables

69. Describe what it means if two random variables are independent and how this assumption can simplify calculations

70. Explain and apply the linearity of expectation

71. Calculate and interpret the covariance and correlation of two random variables

72. Use Markov's, Chebyshev's, and Hoeffding's Inequalities to reason about the distribution of a random variable.

73. Describe when one probability inequality may be preferable to another.

74. Explain the law of large numbers.

75. Explain the information conveyed by the probability density function of a continuous random variable

76. Identify three families of continuous distributions and describe their parameters

77. Calculate statistics of continuous random variables