

## Maximum work in a process between two given states :

Lets consider a closed (fixed mass) system which undergoes a process between states with internal energies  $U$  and  $U+dU$ . The system absorbs heat ( $dQ_{in}$ ) from a thermal reservoir at temperature  $T_R$  and performs works ( $dW_{out}$ ) during the process . There are no changes in kinetic and potential energies. As per the first law,

$$dU = dQ_{in} - dW_{out}$$

As per the second law (increase in entropy principle)

$$dS_{total} = dS + dS_R \geq 0$$

Clausius equation is applicable to reservoir even if process is not reversible.

Thus, we have :

$$dS_R = \frac{-dQ_{in}}{T_R}$$

By combining the above equations (as done in class), it can be shown that heat absorbed and work done are maximum if the process is reversible. Thus for an actual (irreversible process) we have

$$\boxed{dQ_{in} \leq dQ_{in}^{rev}} \quad \text{and} \quad \boxed{dW_{out} \leq dW_{out}^{rev}}$$

## Adiabatic steady flow equipment :

General equation of rate of change of entropy in a control volume :

$$\frac{d}{dt}(S_{cv}) = \sum_{in} \dot{m} s - \sum_{out} \dot{m} s + \sum_k \left( \frac{\dot{Q}_k}{T_k} \right) + \dot{S}_{gen}$$

For a steady-flow equipment,

$$\frac{d}{dt}(S_{cv}) = 0$$

Lets consider an adiabatic flow device with a single inlet and single outlet. The entropy equation reduced to :

$$\dot{m}(s_1 - s_2) + \dot{S}_{gen} = 0$$

Applying first law (in rate form) we get

$$\dot{m} \left[ h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \right] + \dot{W}_{in} = 0$$

## Adiabatic steady flow equipment :

Re-arranging first law equation for adiabatic flow equipment, we get

$$w_{in} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

Here,  $w_{in}$  is the work done per unit mass of the fluid flowing through the equipment,

$$w_{in} = \frac{\dot{W}_{in}}{\dot{m}}$$

Integrating the relation  $dh = T ds + v dP$ , we get

$$h_2 - h_1 = \int_1^2 T ds + \int_1^2 v dP$$

Substituting the expression enthalpy difference in first law equation above, we get

$$w_{in} = \int_1^2 T ds + \int_1^2 v dP + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

## Reversible – adiabatic (isoentropic) steady flow equipment :

If the flow equipment is reversible,

$$\dot{S}_{\text{gen}} = 0$$

Substituting in the entropy balance equation (see slide 2), we get

$$s_1 = s_2$$

Therefore the first term in the right hand side of the first law equation will be zero, i.e.,

$$\int_1^2 T \, ds = 0 \quad \text{since } s_1 = s_2$$

Substituting in the first law equation (last equation of previous slide), we get

$$w_{\text{in}}^{\text{rev}} = \int_1^2 v \, dP + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

The superscript 'rev' indicates that process is reversible.

## Reversible – adiabatic (isoentropic) steady flow equipment :

If the equipment involves no work (for example in a pipe) and for liquid flow (considering specific volume constant), we get

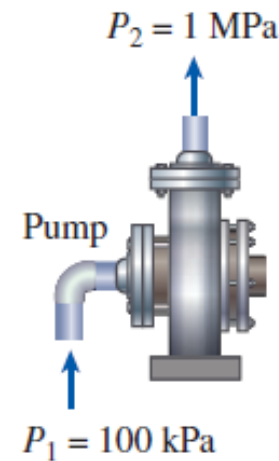
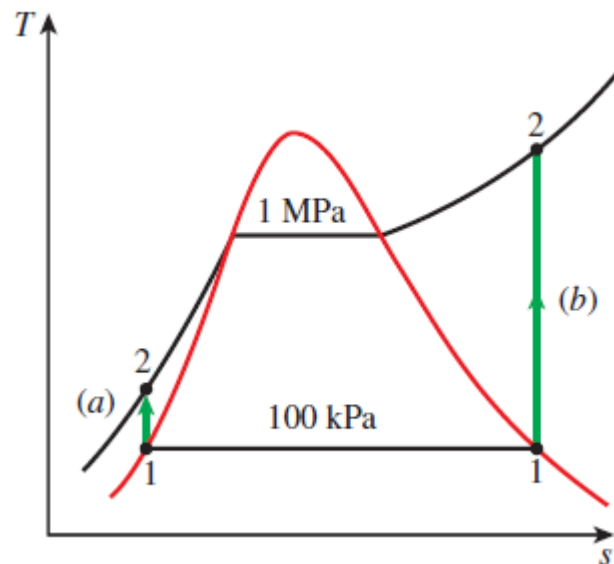
$$v (P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

This equation is known as “Bernoulli equation” in fluid mechanics.

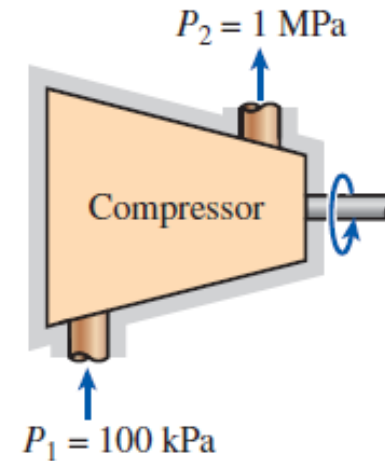
### **EXAMPLE 7–12**      **Compressing a Substance in the Liquid versus Gas Phases**

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (a) saturated liquid and (b) saturated vapor at the inlet state.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 The process is given to be isentropic.



(a) Compressing a liquid



(b) Compressing a vapor

(a) In this case, steam is a saturated liquid initially, and its specific volume is

$$v_1 = v_{f @ 100 \text{ kPa}} = 0.001043 \text{ m}^3/\text{kg} \quad (\text{Table A-5})$$

which remains essentially constant during the process. Thus,

$$\begin{aligned} w_{\text{rev, in}} &= \int_1^2 v dP \cong v_1 (P_2 - P_1) \\ &= (0.001043 \text{ m}^3/\text{kg}) [(1000 - 100) \text{ kPa}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= \mathbf{0.94 \text{ kJ/kg}} \end{aligned}$$

(b) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how  $\nu$  varies

with  $P$  to perform the integration

This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the second  $T ds$  relation by setting  $ds = 0$ :

Thus,

$$w_{\text{rev,in}} = \int_1^2 \nu dP = \int_1^2 dh = h_2 - h_1$$

This result could also be obtained from the energy balance relation for an isentropic steady-flow process. Next we determine the enthalpies:

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 100 \text{ kPa} \\ \text{(sat. vapor)} \end{array} \right\} \quad \left. \begin{array}{l} h_1 = 2675.0 \text{ kJ/kg} \\ s_1 = 7.3589 \text{ kJ/kg}\cdot\text{K} \end{array} \right\} \quad \text{(Table A-5)}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} \quad h_2 = 3194.5 \text{ kJ/kg} \quad \text{(Table A-6)}$$

Thus,

$$w_{\text{rev,in}} = (3194.5 - 2675.0) \text{ kJ/kg} = \mathbf{519.5 \text{ kJ/kg}}$$

**Discussion** Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.