

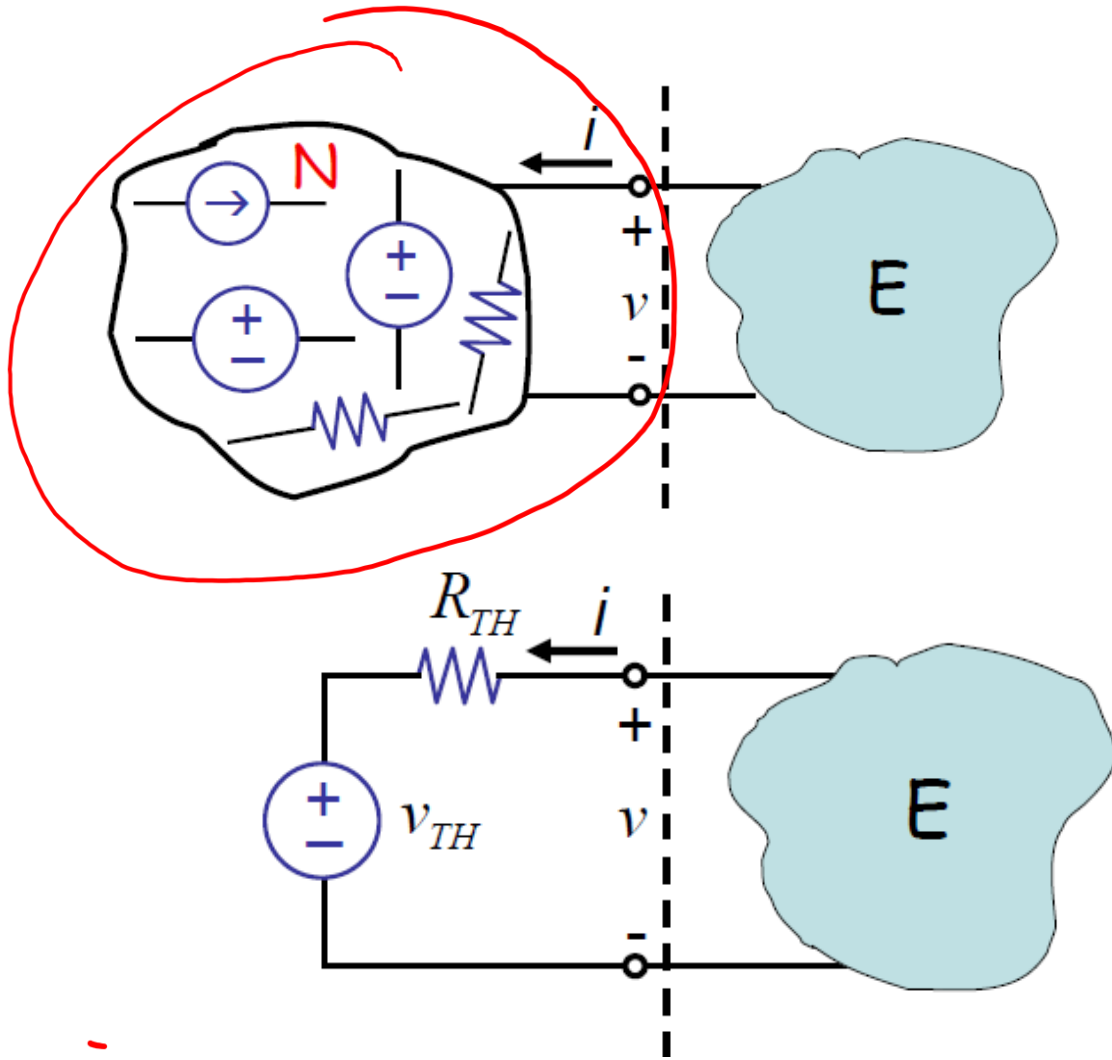
ESC201: Lecture 6



Dr. Imon Mondal

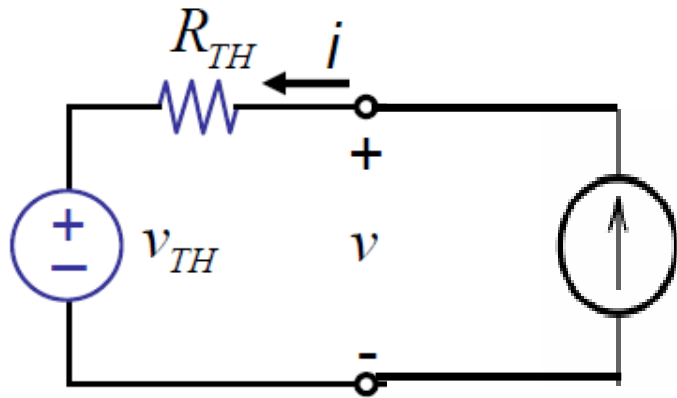
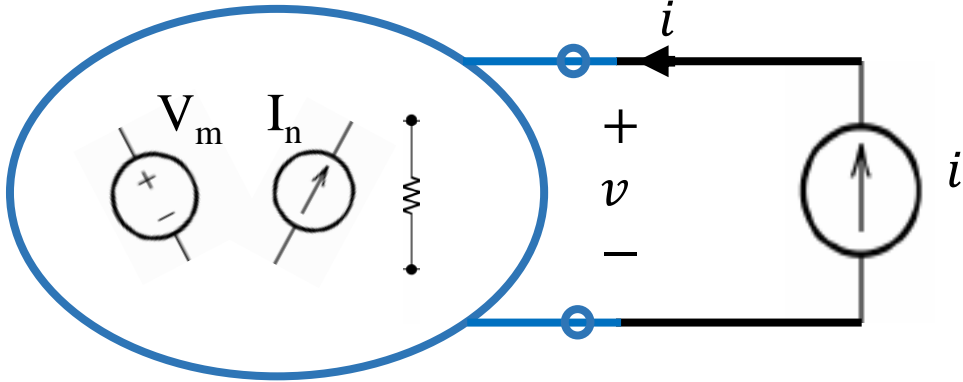
ASSISTANT PROFESSOR,
ELECTRICAL ENGINEERING, IIT KANPUR

Thevenin Equivalent of Linear Sub-Circuits

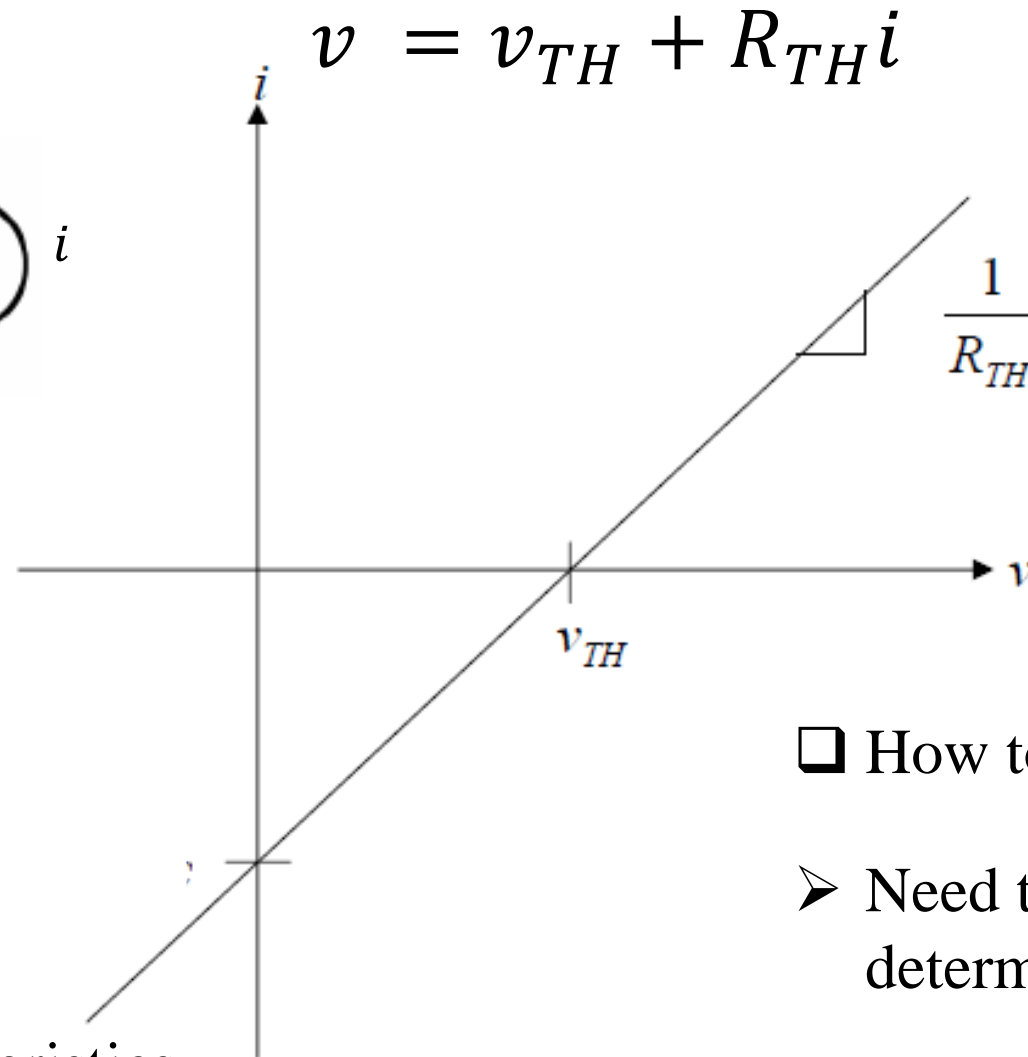


As far as the external world is concerned
(for the purpose of IV relationship)
“arbitrary linear network N ”
is
indistinguishable from its
Thevenin equivalent.

I-V Characteristics

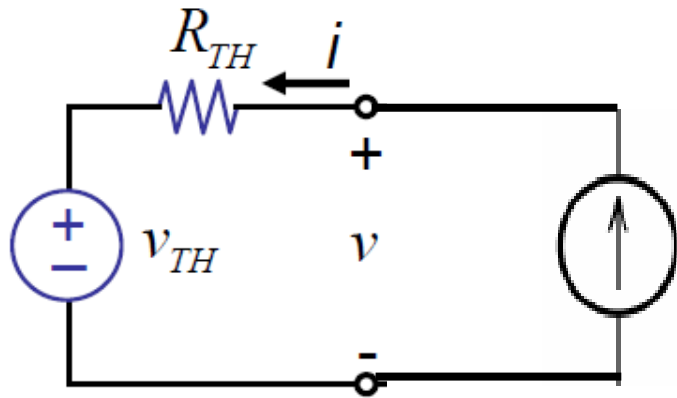
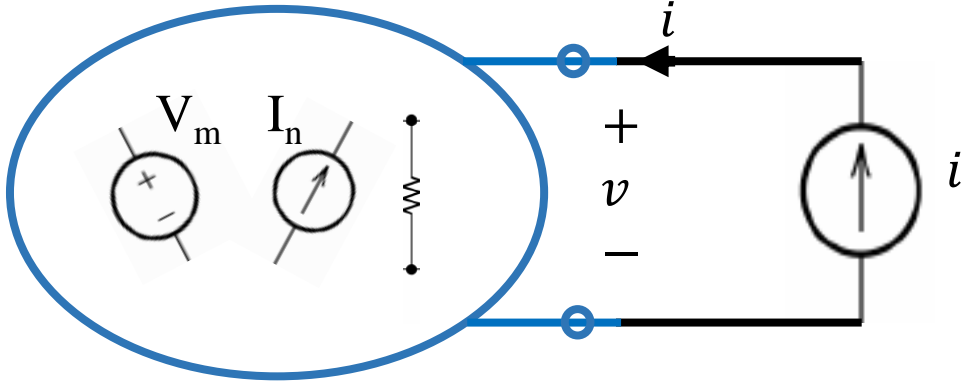


Equal Two Terminal IV Characteristics

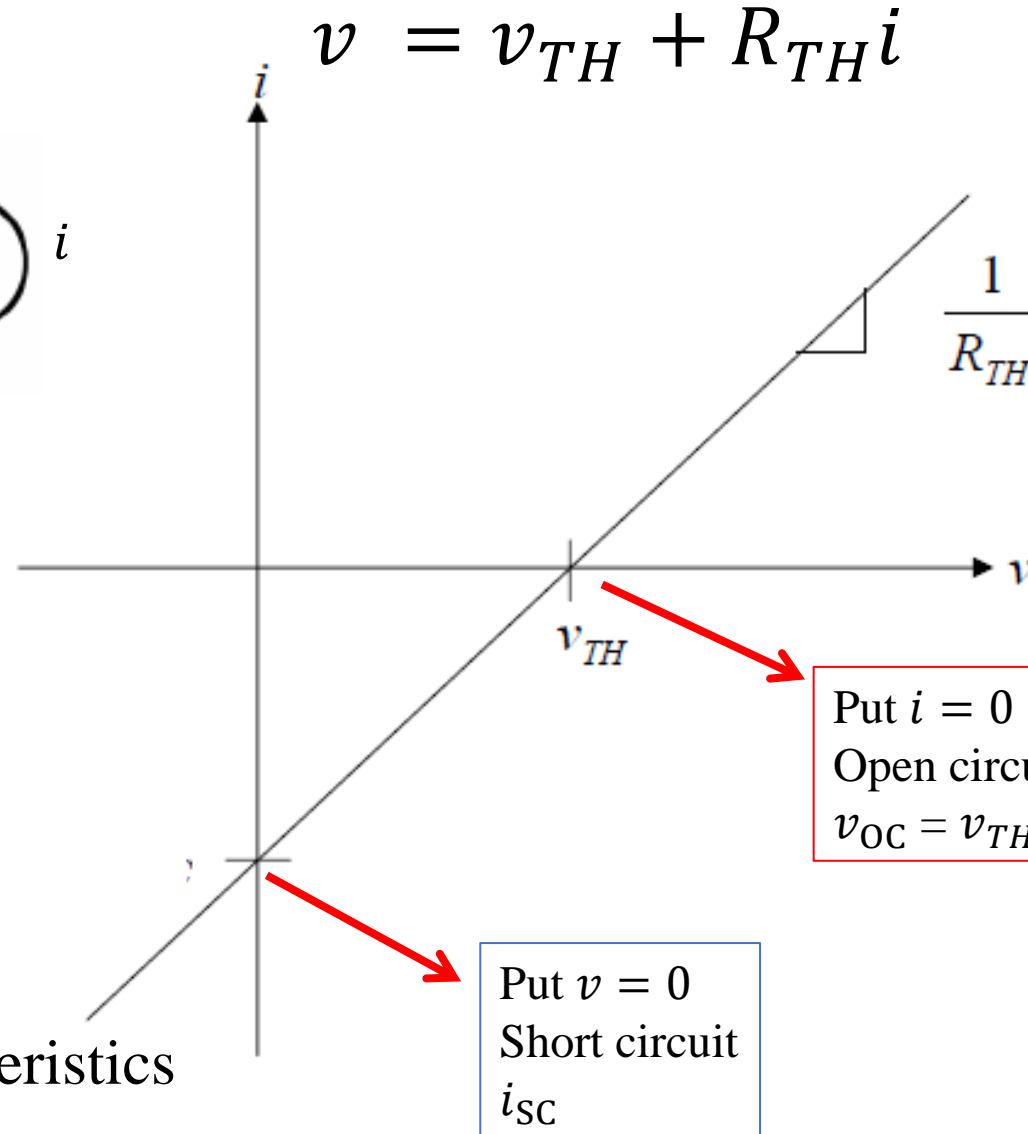


- ❑ How to compute these two?
- Need two points to determine the values

V_{TH} and R_{TH} from I-V Characteristics



Equal Two Terminal IV Characteristics

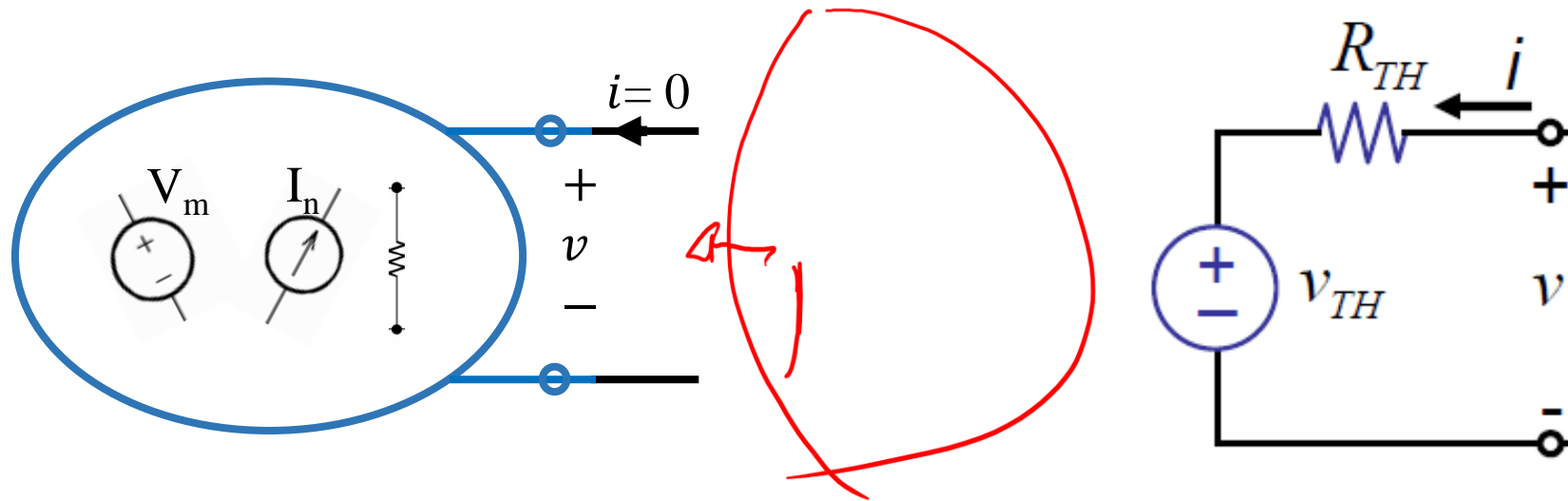


$$0 = v_{TH} + R_{TH}i_{SC}$$

$$-v_{TH} = R_{TH}i_{SC}$$

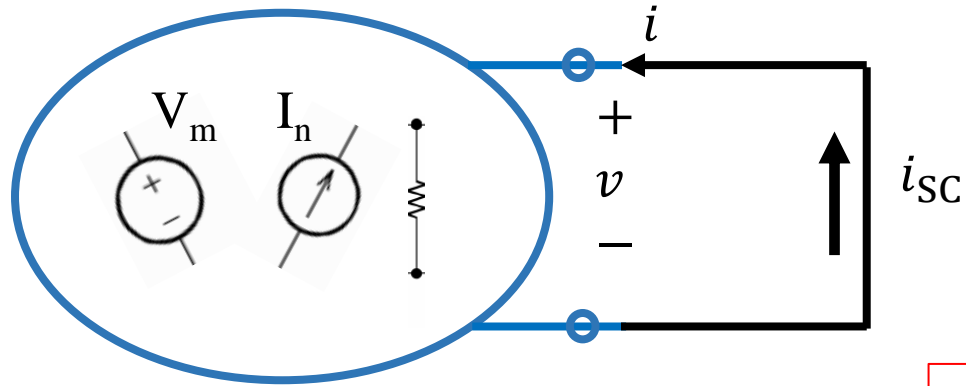
$$R_{TH} = \frac{v_{TH}}{-i_{SC}} = \frac{v_{OC}}{-i_{SC}}$$

Open Circuit

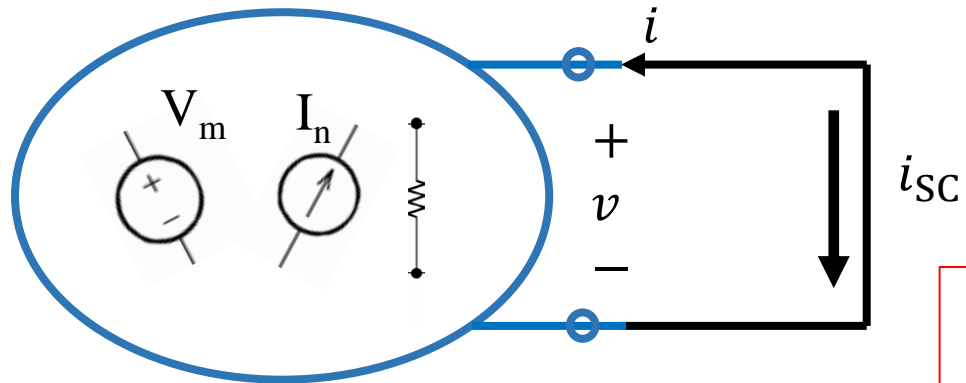


- You may use any circuit analysis method to determine the open circuit voltage

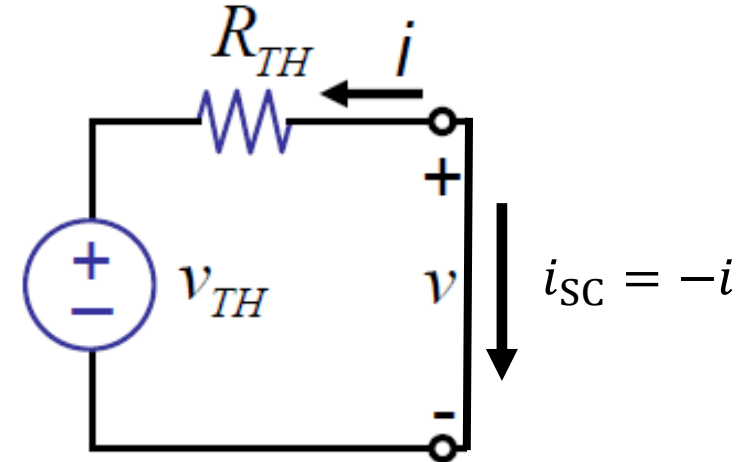
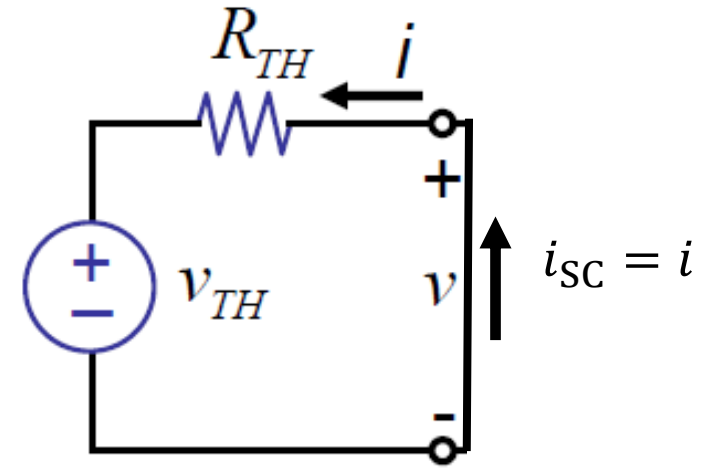
Short Circuit



$$R_{TH} = \frac{v_{OC}}{-i_{SC}}$$

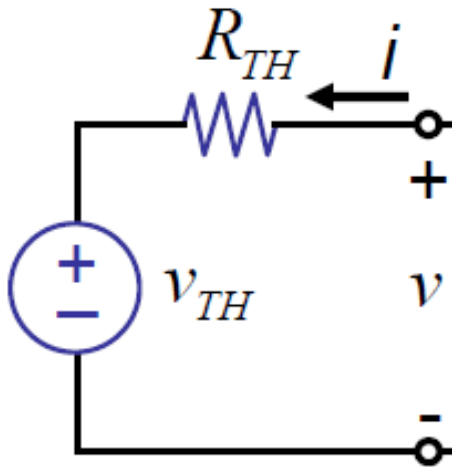
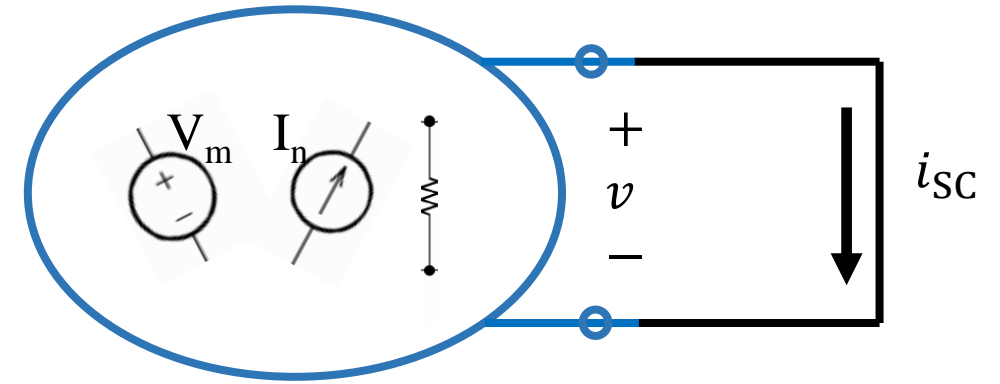
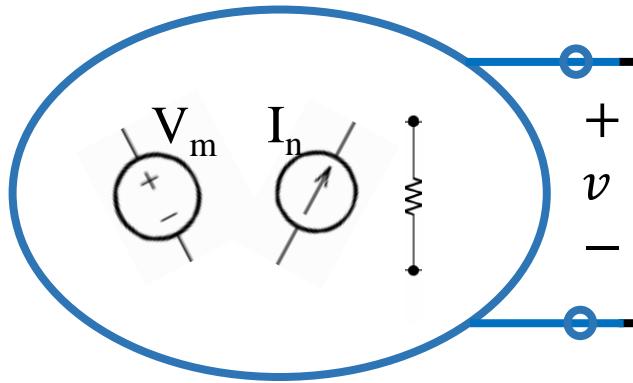


$$R_{TH} = \frac{v_{OC}}{i_{SC}}$$

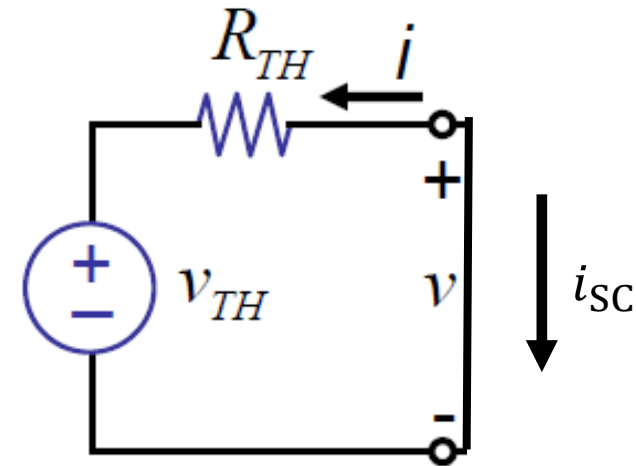


➤ You may use any circuit analysis method to determine the short circuit current

Thevenin Equivalent Computation



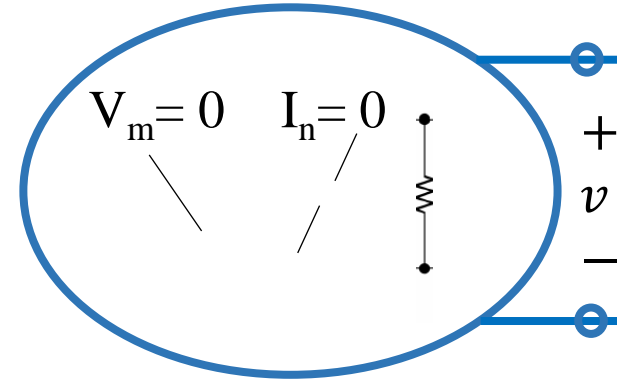
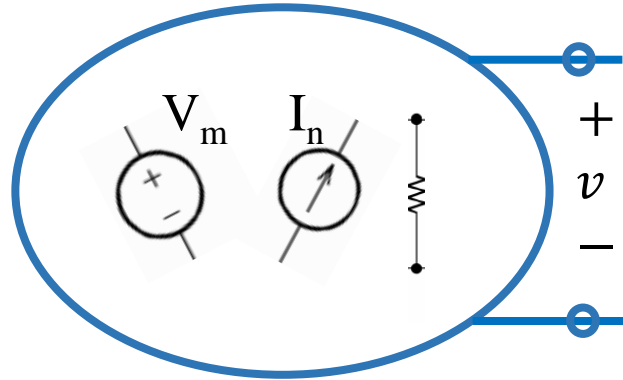
v_{TH} is open circuit voltage v_{OC}



i_{SC} is short circuit current

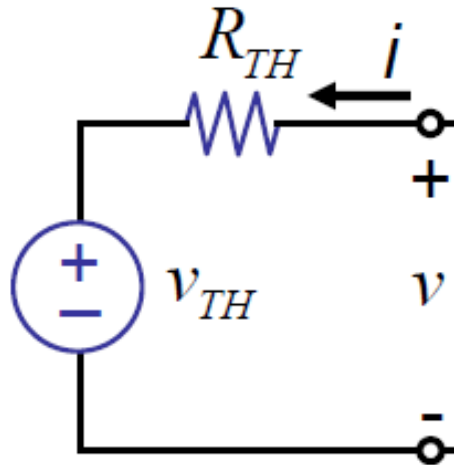
$$R_{TH} = \frac{v_{OC}}{i_{SC}}$$

Finding R_{TH} Directly

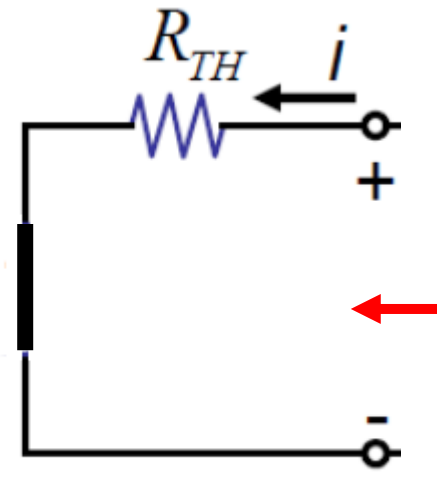


Null all independent sources in the circuit

$$v_{TH} = \sum_m \alpha_m V_m + \sum_n \beta_n I_n$$

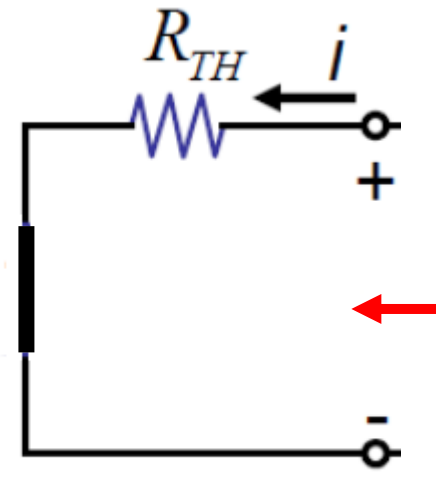
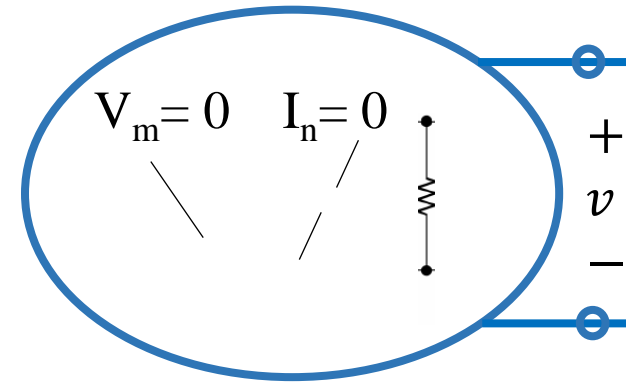


Equal IV Characteristics

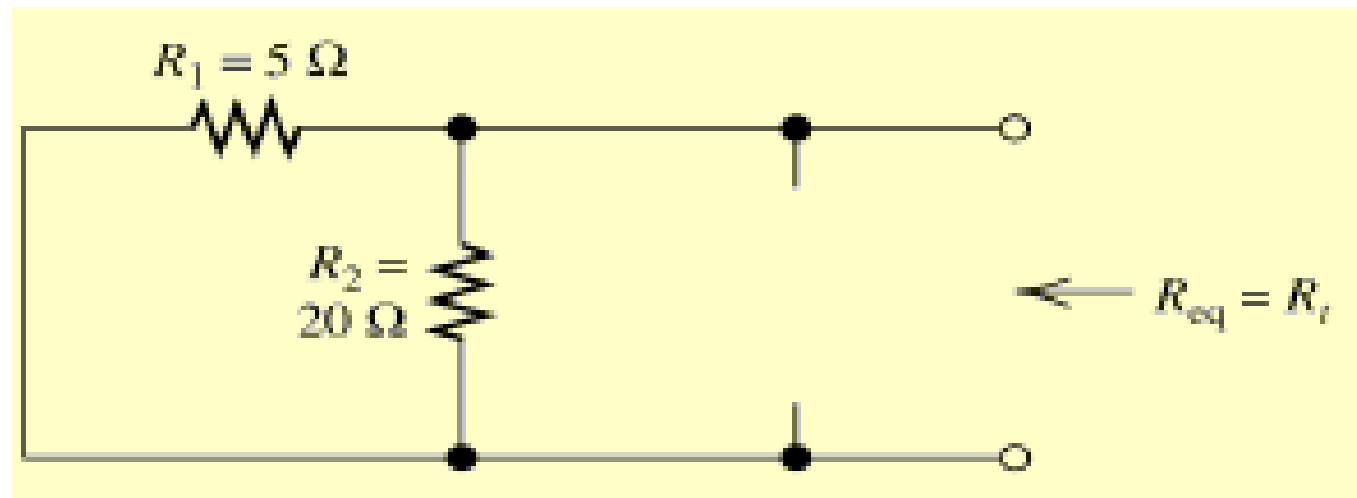
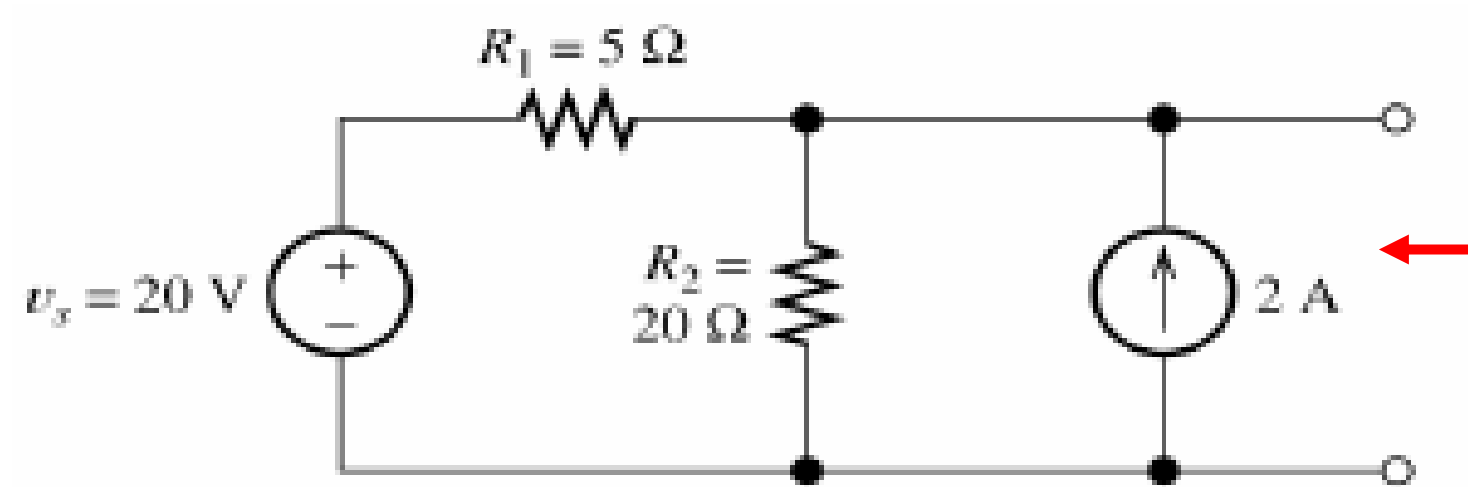


Steps for Finding R_{TH} Directly

- Null all independent sources in the original network
 - A voltage source becomes a short circuit
 - A current source becomes an open circuit
- Compute the resistance between the terminals
 - You may use any circuit analysis method to determine the equivalent resistance

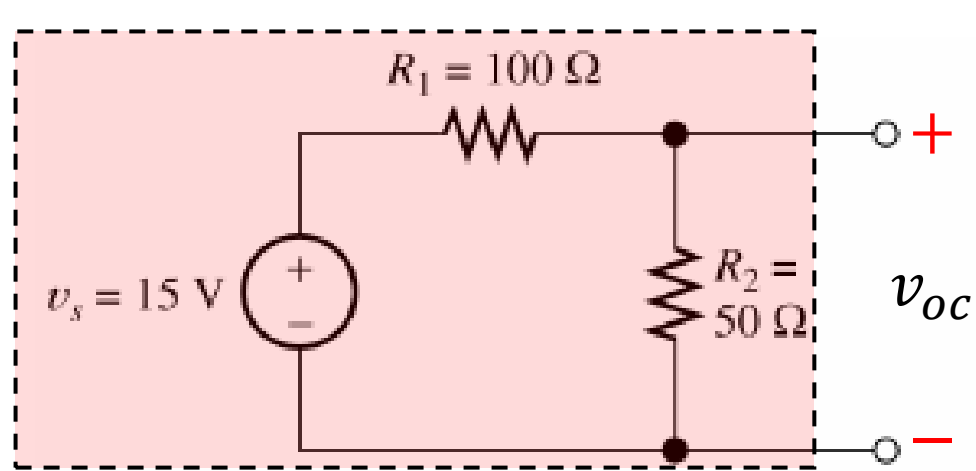


Thevenin Equivalent: Example of R_{TH} Directly

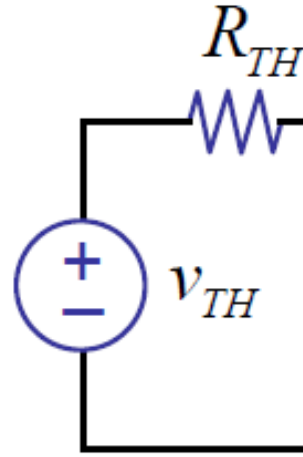


$$R_{eq} = \frac{5 \times 20}{5 + 20} = 4 \Omega$$

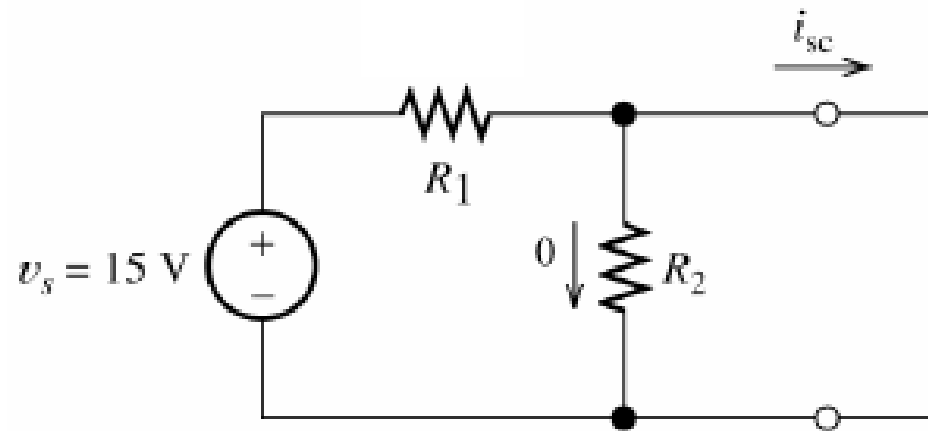
Thevenin Equivalent: Example



$$V_{TH} = v_{oc}$$

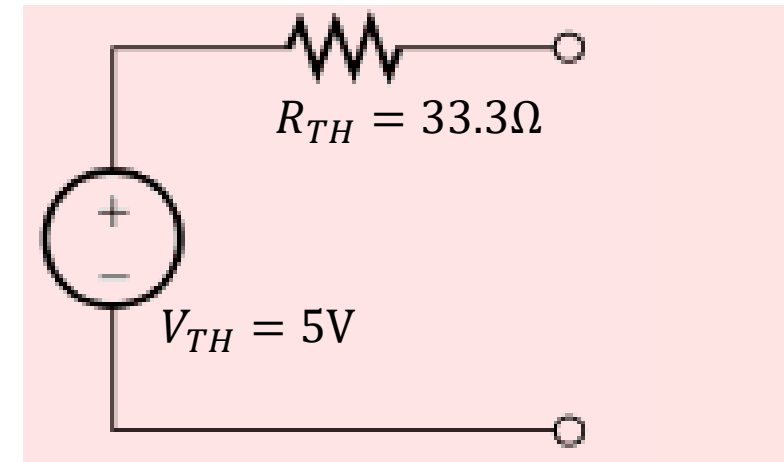


$$V_{TH} = \frac{R_2}{R_2 + R_1} \times 15V = 5V$$

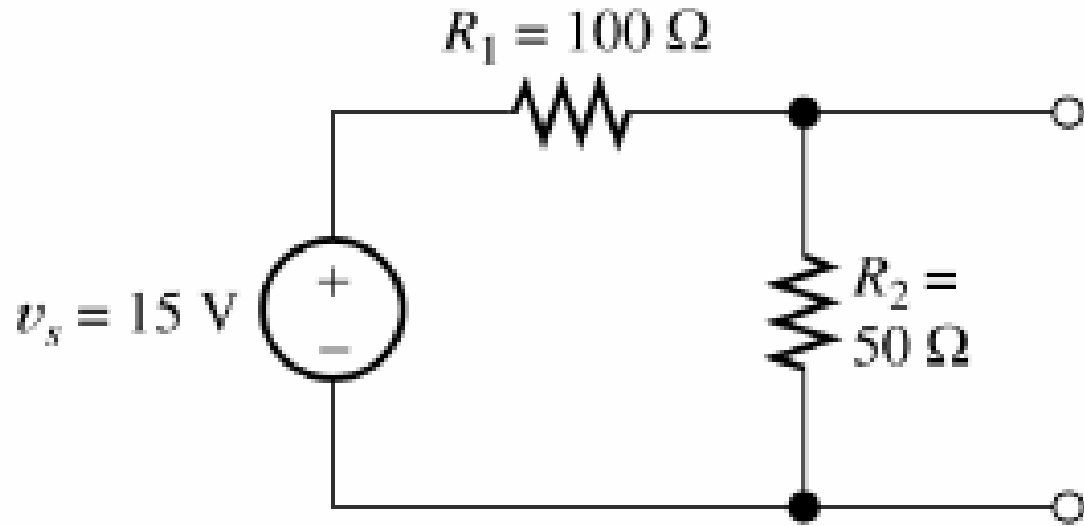


$$i_{sc} = \frac{v_s}{R_1} = 0.15A$$

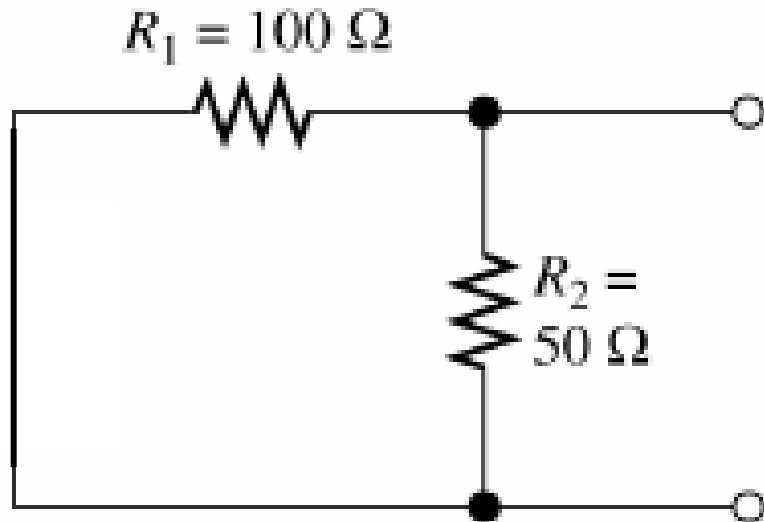
$$R_{TH} = \frac{v_{oc}}{i_{sc}} = 33.3\Omega$$



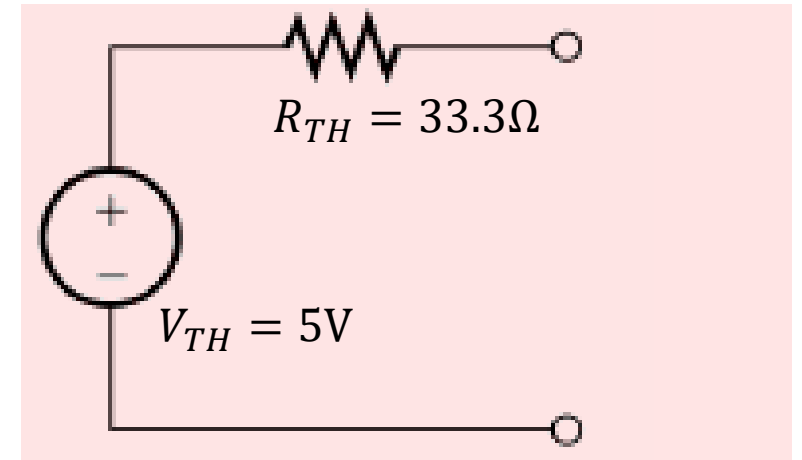
Thevenin Equivalent: Example of R_{TH} Directly



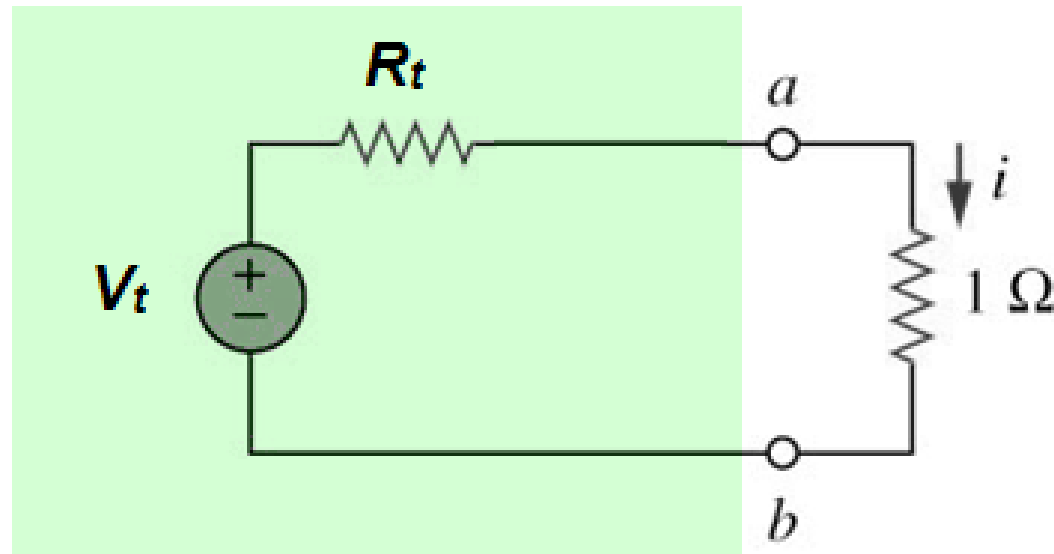
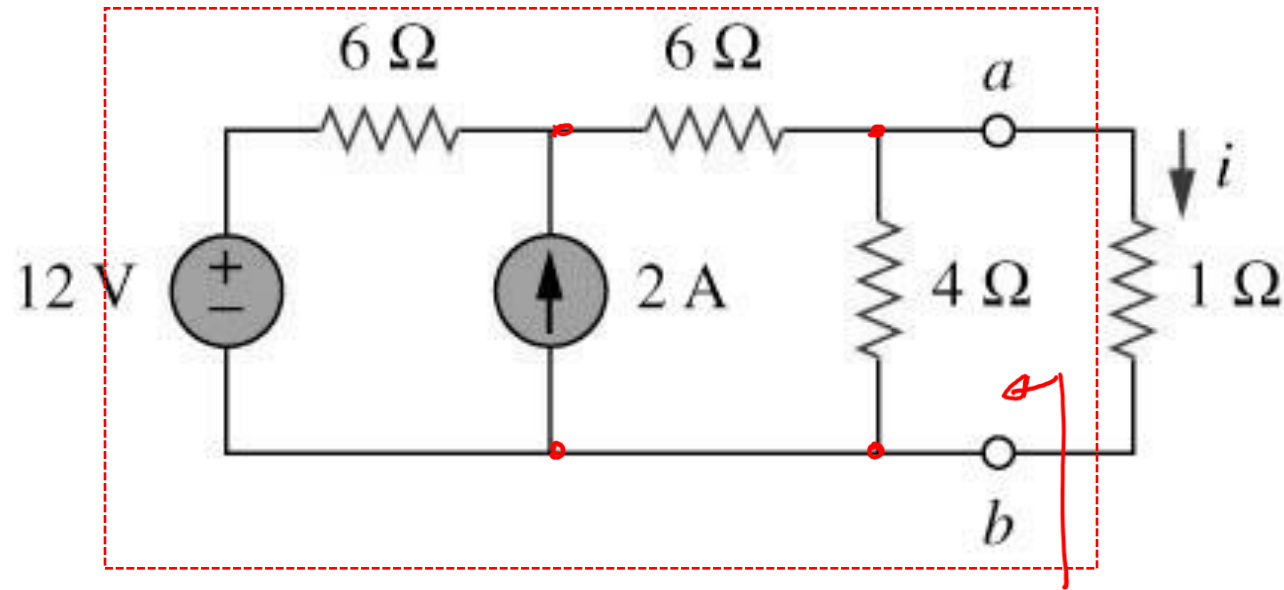
$$V_{TH} = V_{OC} = \frac{R_2}{R_2 + R_1} \times 15\text{V} = 5\text{V}$$



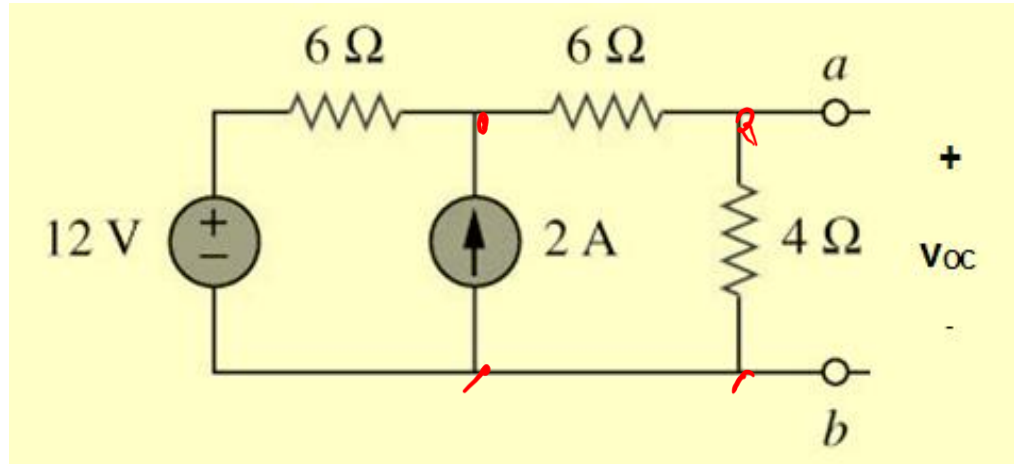
$$R_{TH} = 100 || 50 = 33.3\ \Omega$$



Thevenin Method: Example



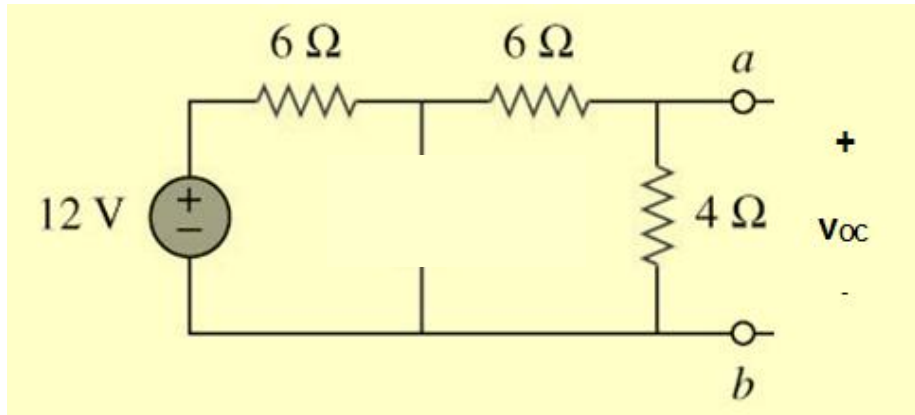
Thevenin Method: Example



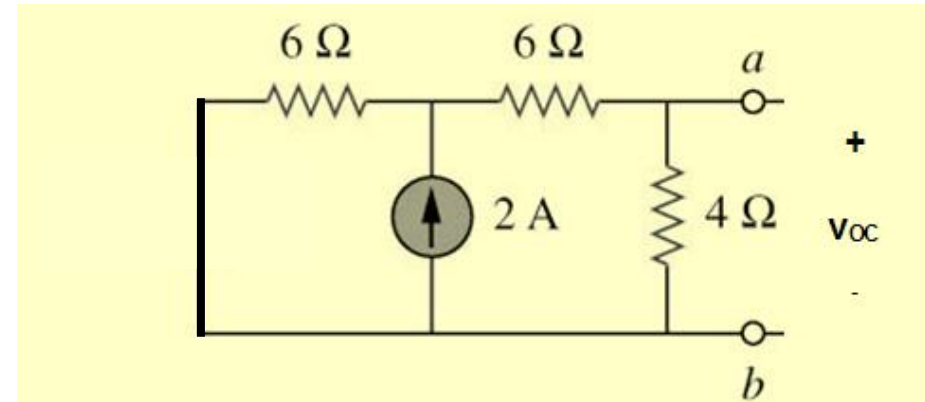
$$V_{oc} = V_{oc1} + V_{oc2} = 6$$

$$v_{oc} = 6V$$

Use Superposition Method

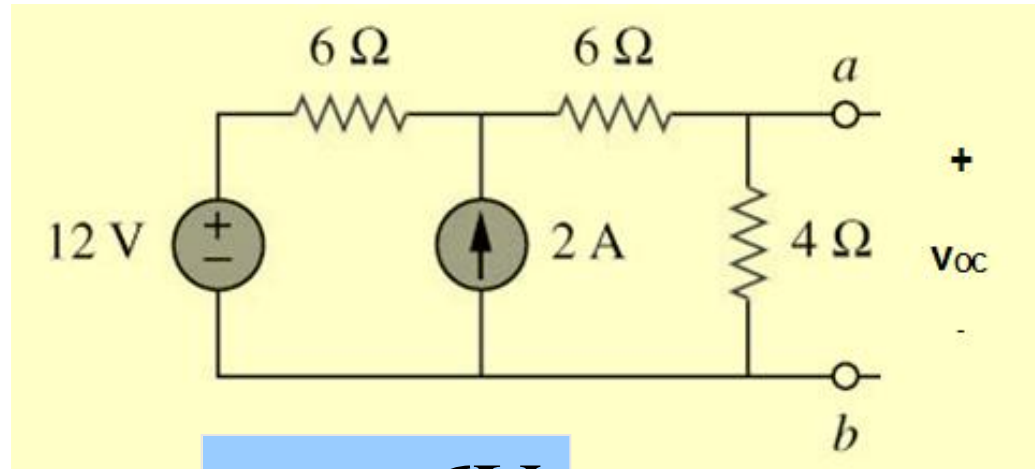
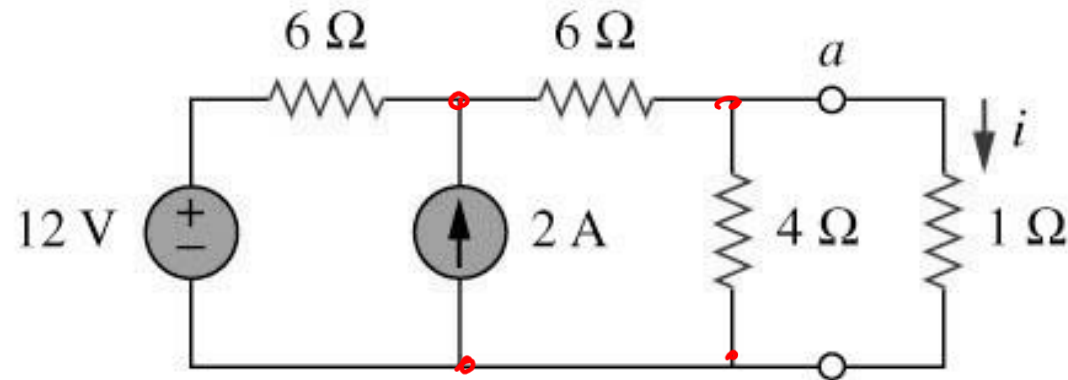


$$V_{oc1} = \frac{4}{4+12} \times 12 = 3$$

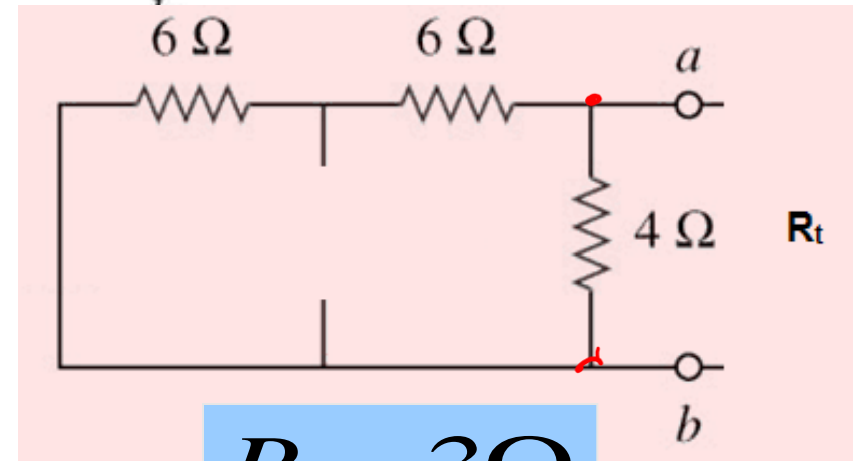


$$V_{oc2} = 4 \times \left(2 \times \frac{6}{6+10} \right) = 3$$

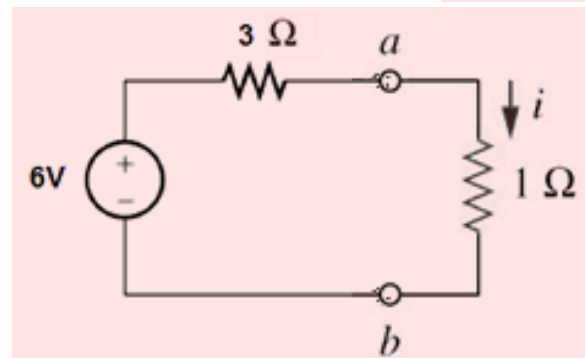
Thevenin Method: Example



$$V_{oc} = 6V$$

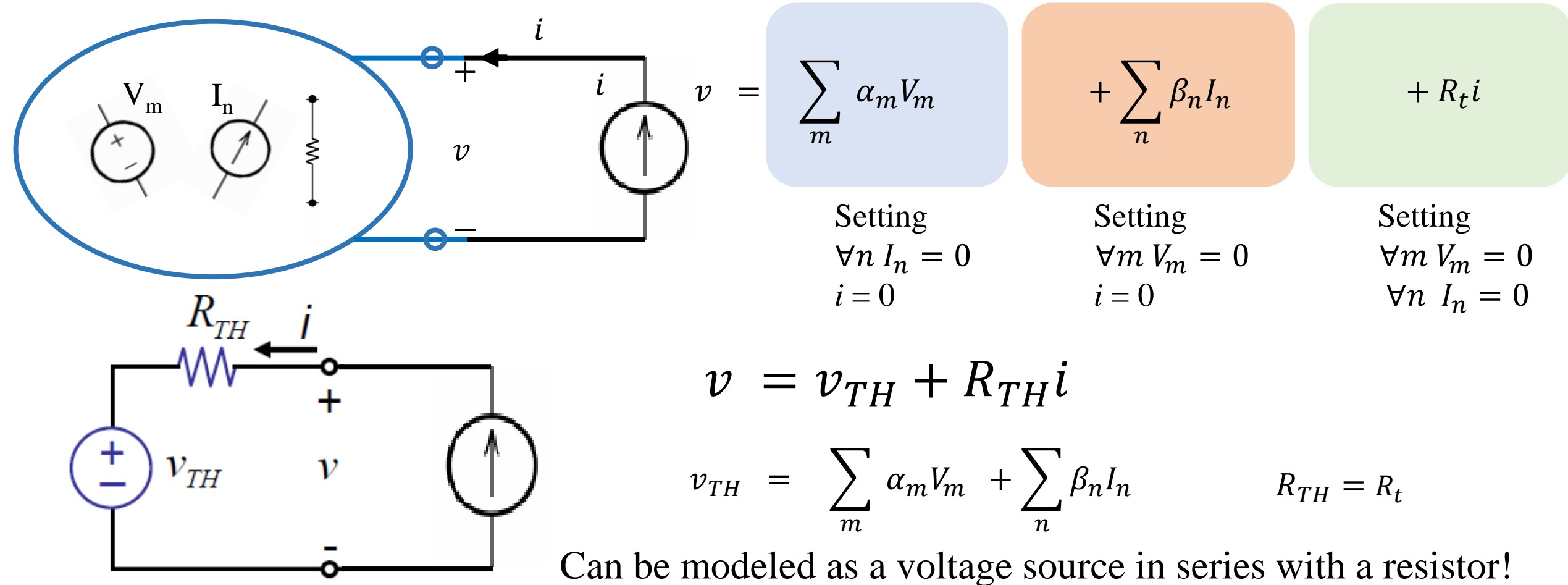


$$R_t = 3\Omega$$



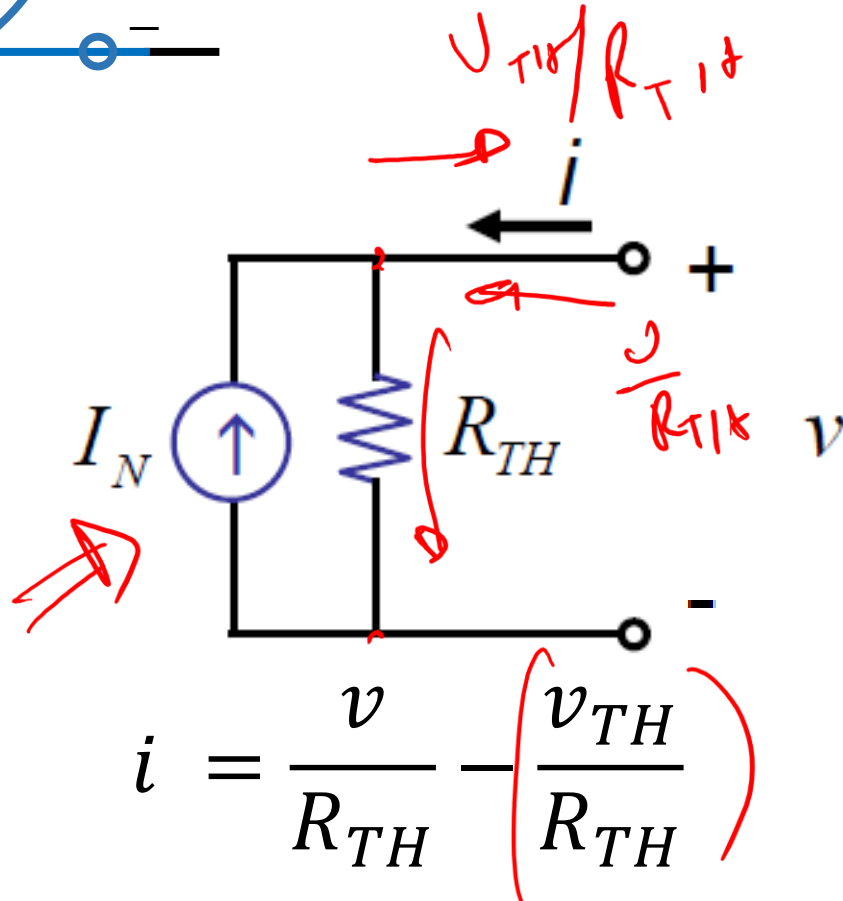
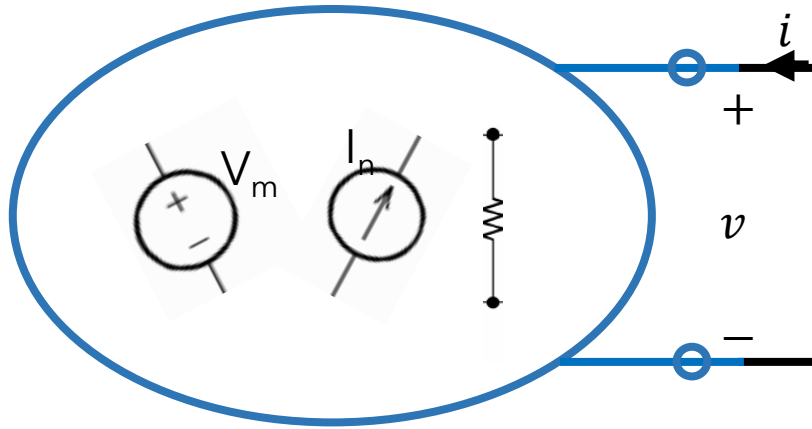
$$i = 1.5A$$

Thevenin Equivalent

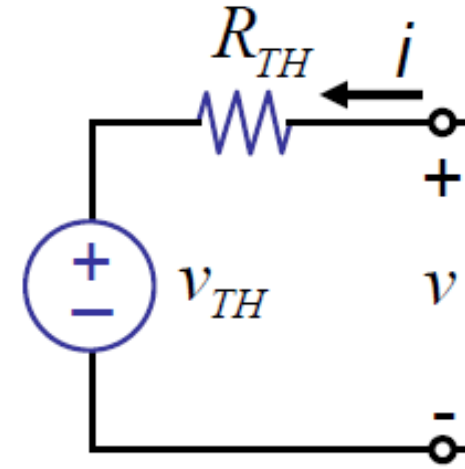


Equal Two Terminal IV Characteristics

Norton Equivalent



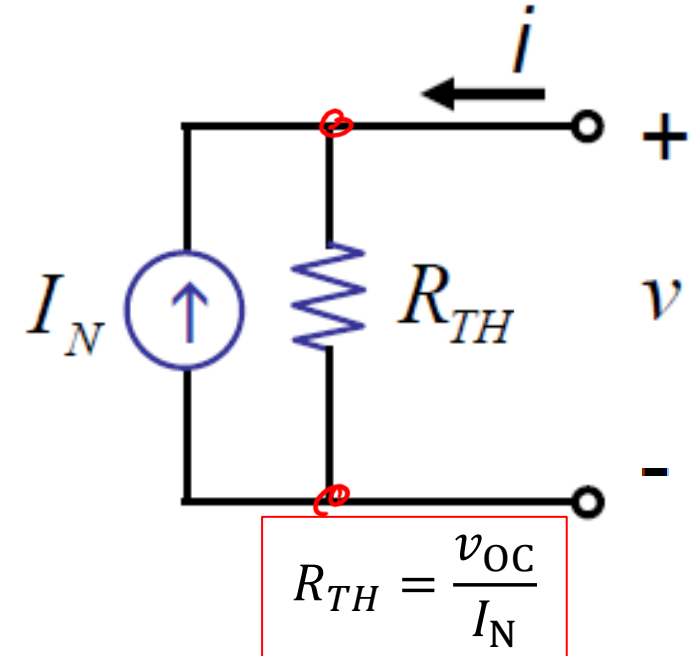
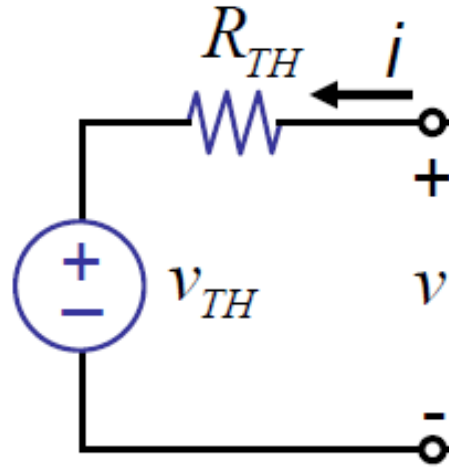
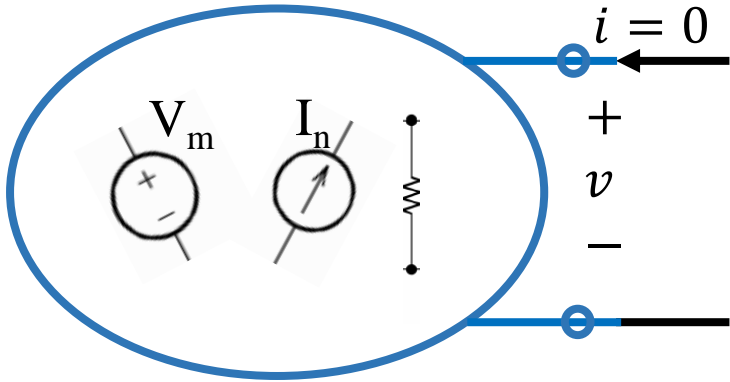
$$i = \frac{v}{R_{TH}} - \left(\frac{v_{TH}}{R_{TH}} \right)$$



$$v = v_{TH} + R_{TH}i$$

$$I_N = \frac{v_{TH}}{R_{TH}}$$

Recall: Open Circuit Voltage

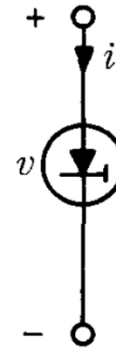
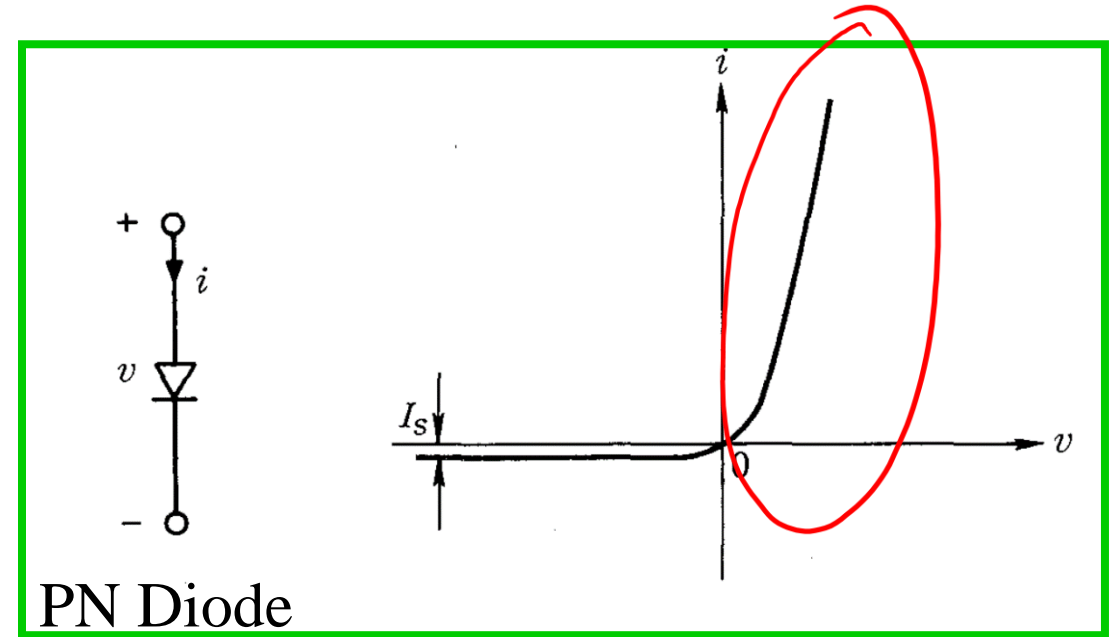
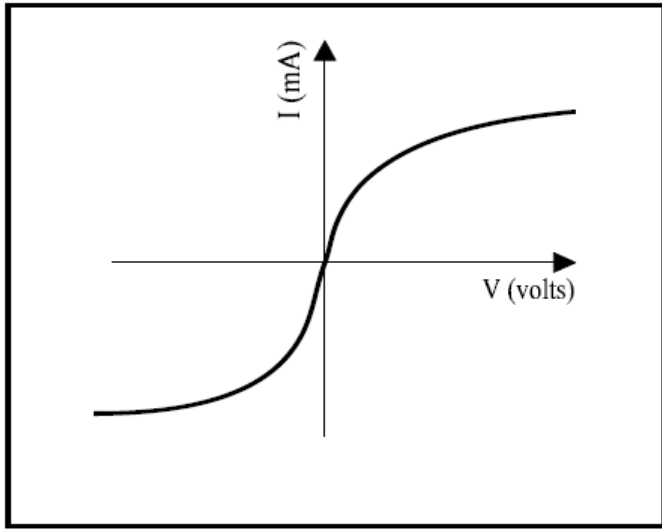


- You may use any circuit analysis method to determine the open circuit voltage

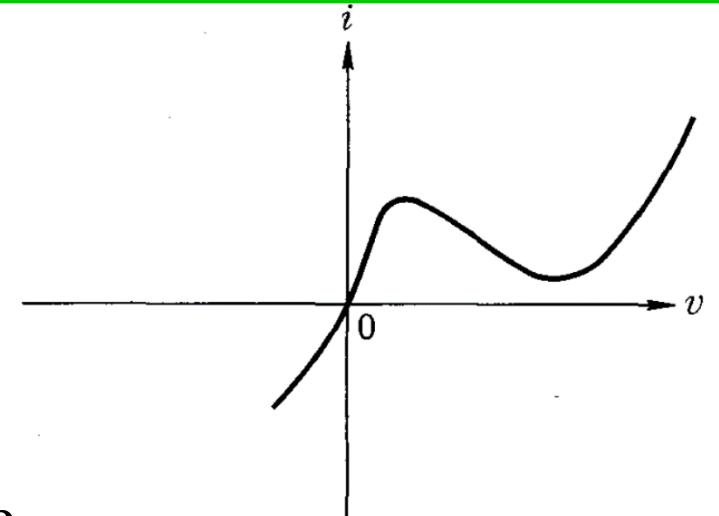
Generalized Resistor



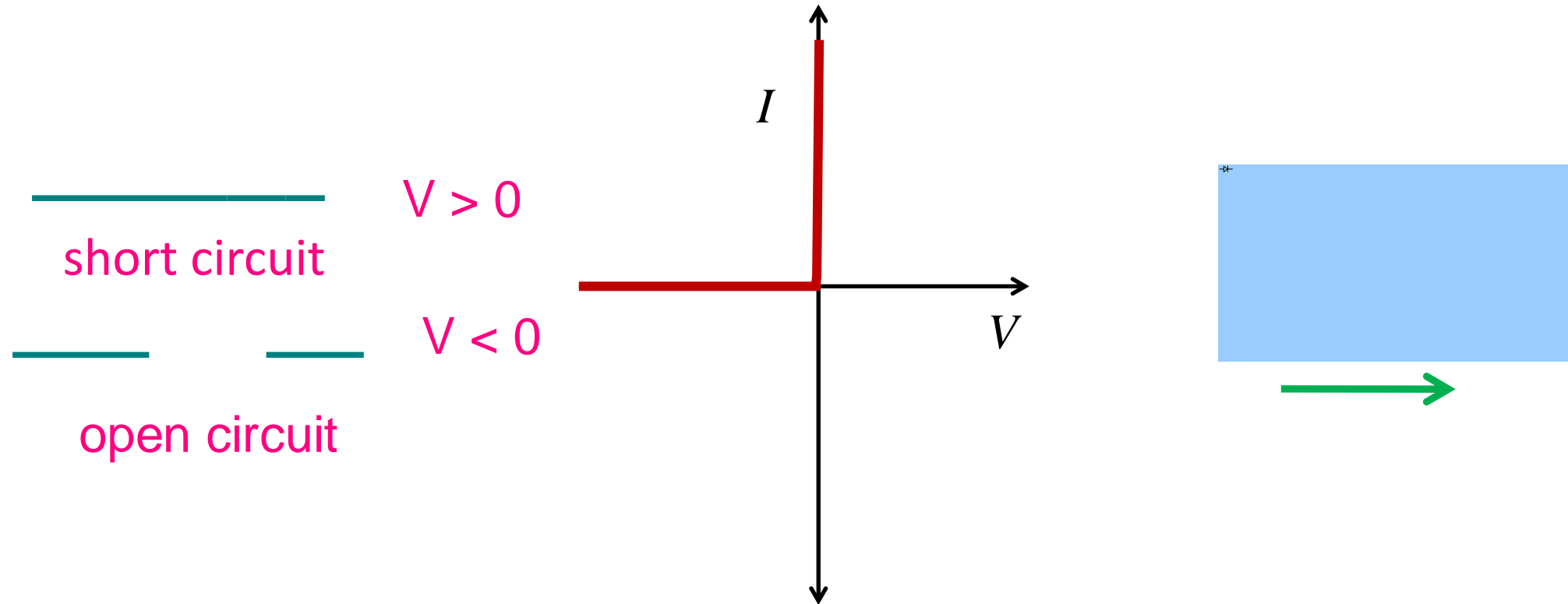
Bulb



Tunnel Diode

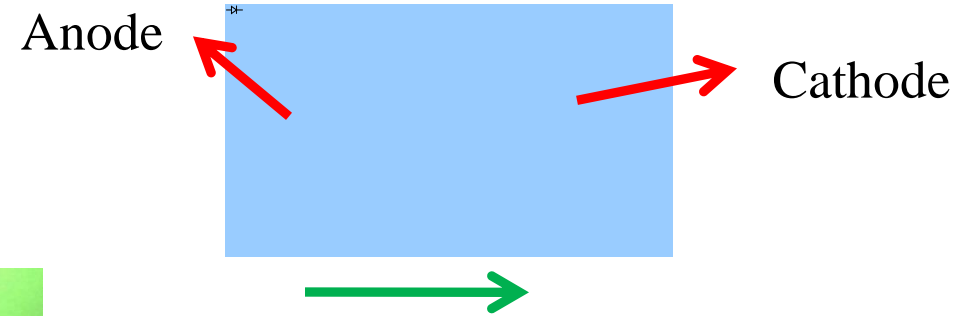


Unidirectional Device: Non-linear Time-invariant Resistor



Diodes

- Diodes allows current in only one direction
 - from anode to cathode terminal
- Non-linear behaviour
- Applications:
 - One way valve
 - AC to DC converter
 - Voltage regulator
 - LED
 - Logic operations
- Passive device: only consumes power



Vacuum diode
used in radio
as a rectifier

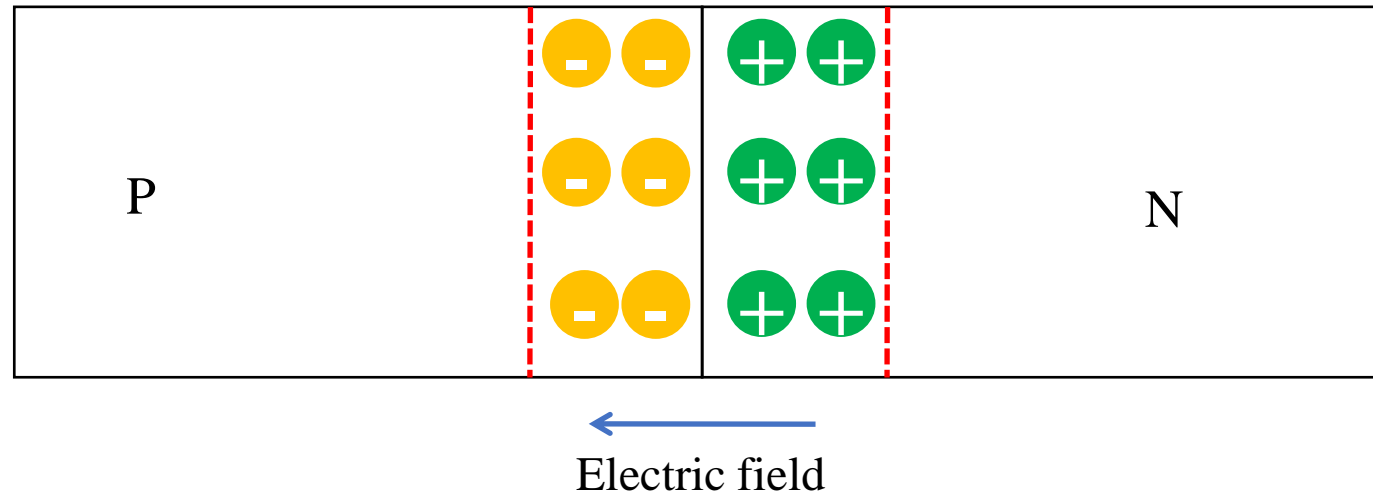
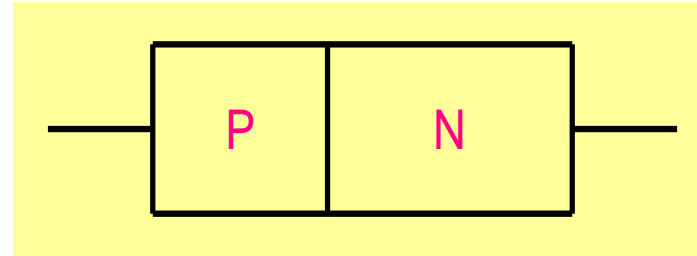
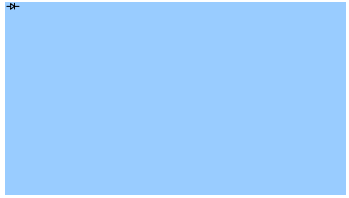


Semiconductor Diode
(PN Junction Diode)



Light Emitting Diode
(LED)

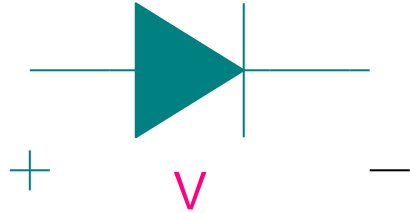
PN Junction Diode



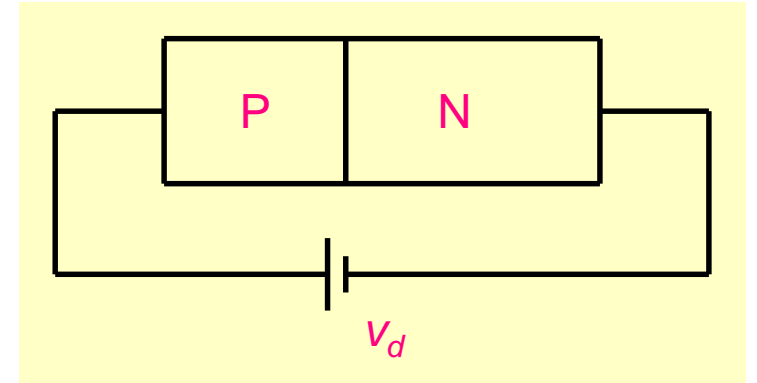
- Inside a PN junction at equilibrium (zero applied voltage), there is a built-in voltage.
- N region being positive and P-region negative, electric field from +ve to -ve
- The built-in voltage (potential barrier) prevents electrons and holes to give rise to current.

Forward and Reverse Bias

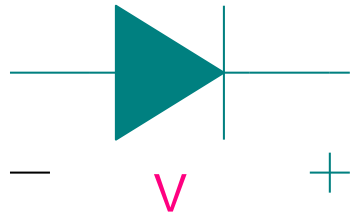
Forward Bias: P is biased at a higher voltage compared to N.



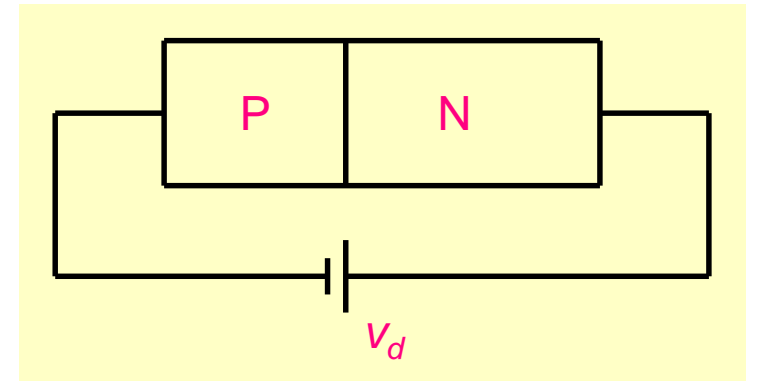
➤ This lowers the built-in potential and allows current to flow.



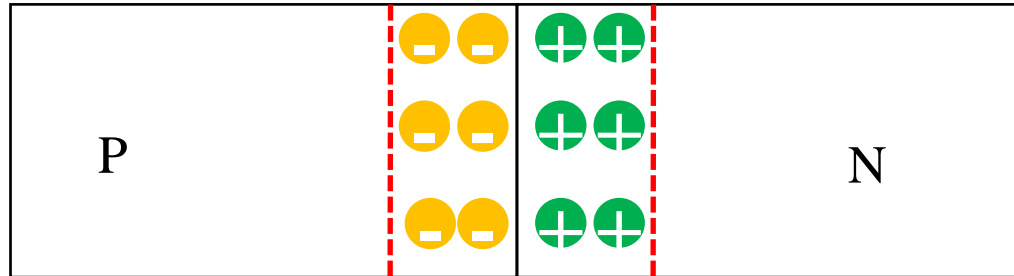
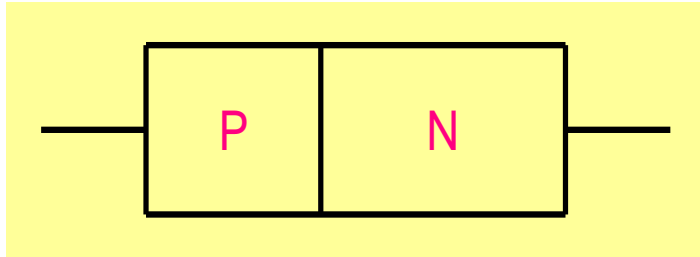
Reverse Bias: N is biased at a higher voltage compared to P.



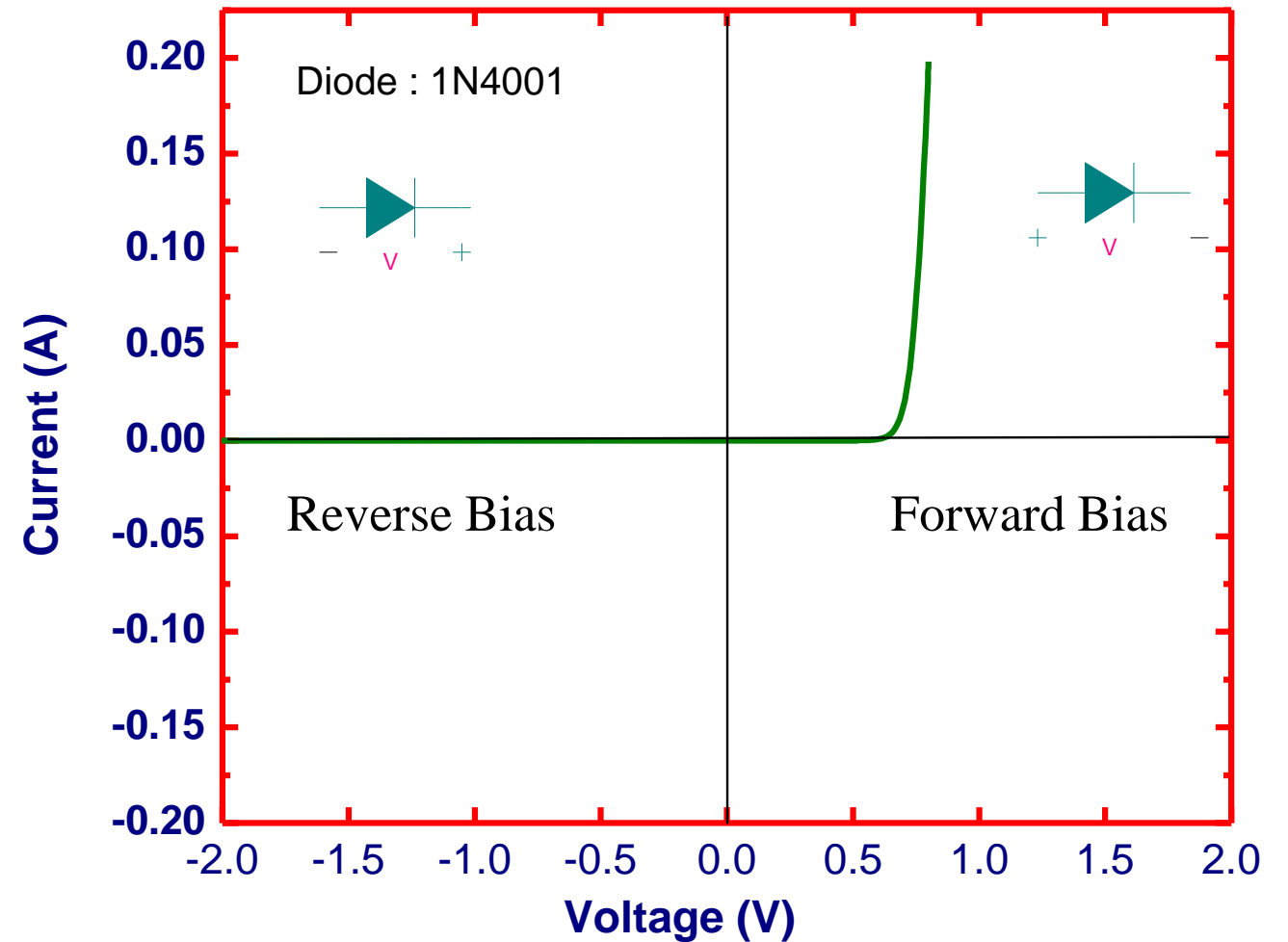
➤ This increases built-in potential and very little current flows.



PN Junction Diode: IV Characteristic



- The p-n junction only conducts significant current in forward-bias.



I-V Characteristics: Non-linear Behavior

Applied voltage = v_D

Diode current:

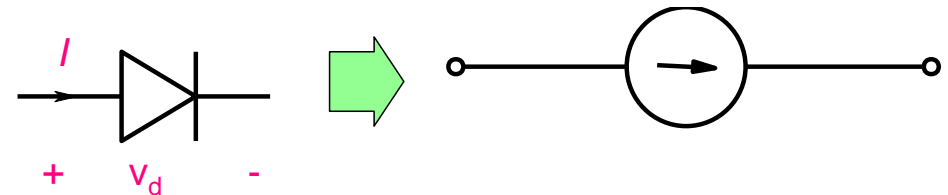
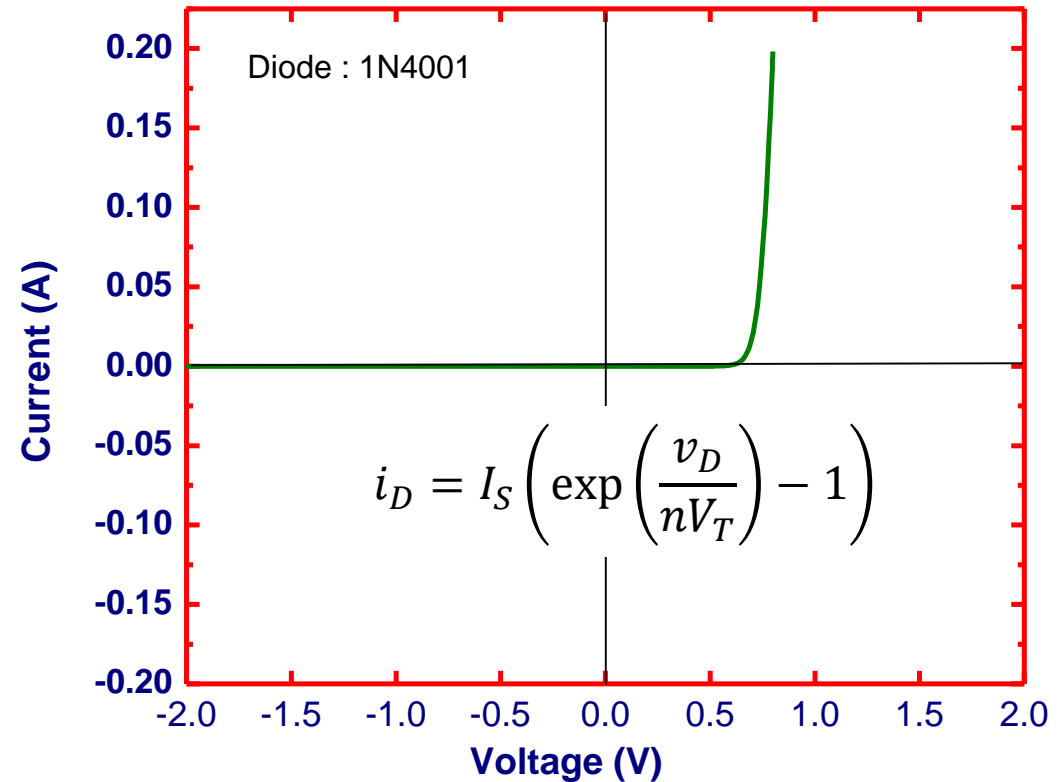
$$i_D = I_S \left(\exp \left(\frac{v_D}{nV_T} \right) - 1 \right)$$

I_S : Reverse saturation current

n : ideality factor (= 1 for ideal diodes)

$$V_T = \frac{kT}{q} \approx 26\text{mV at } T = 300\text{K}$$

□ How to analyze circuits containing diodes?



Forward and Reverse Bias

$$I_D = I_S \left(\exp \left(\frac{v_D}{V_T} \right) - 1 \right)$$

□ Forward Bias:

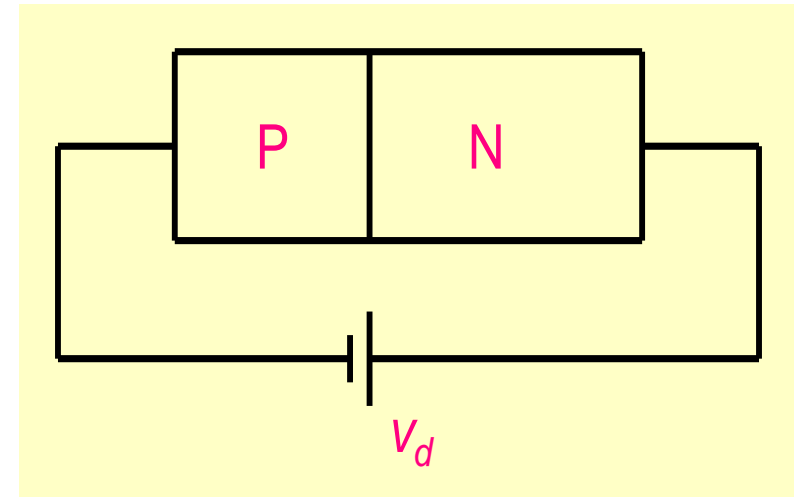
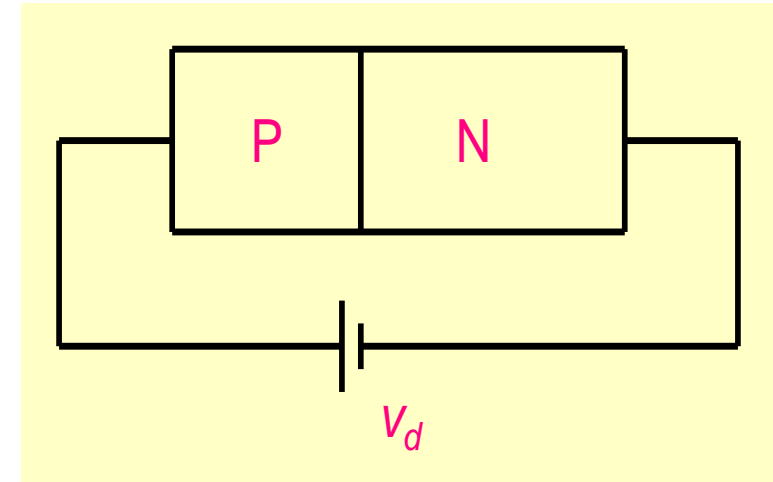
$$v_d \gg V_T = 26mV$$

$$i_D \approx I_S \times \exp \left(\frac{v_d}{V_T} \right)$$

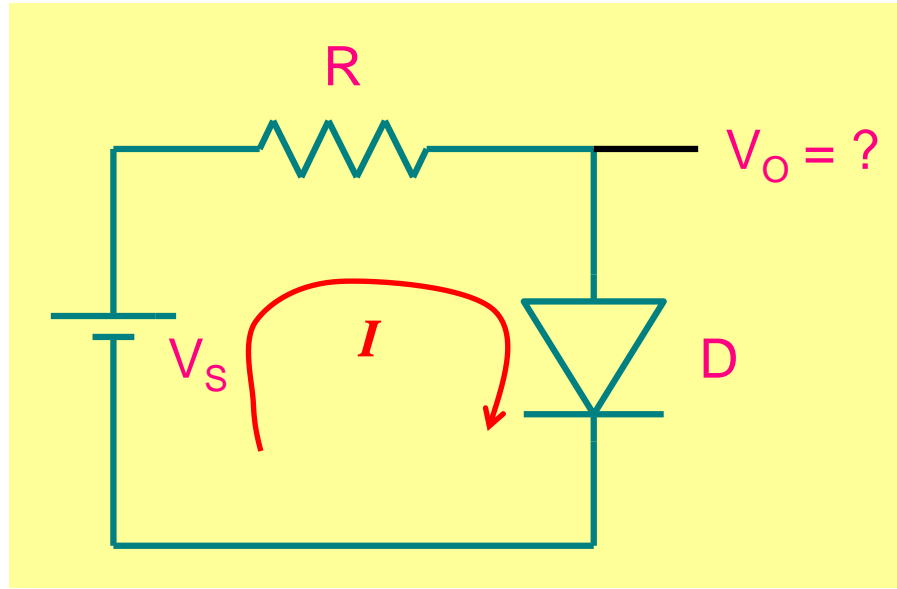
□ Reverse Bias:

$$v_d = -v_R \quad |v_R| \gg V_T$$

$$i_D = I_S \left(\exp \left(-\frac{v_R}{V_T} \right) - 1 \right) \approx -I_S$$



Circuit Analysis



$$V_S = IR + V_D$$

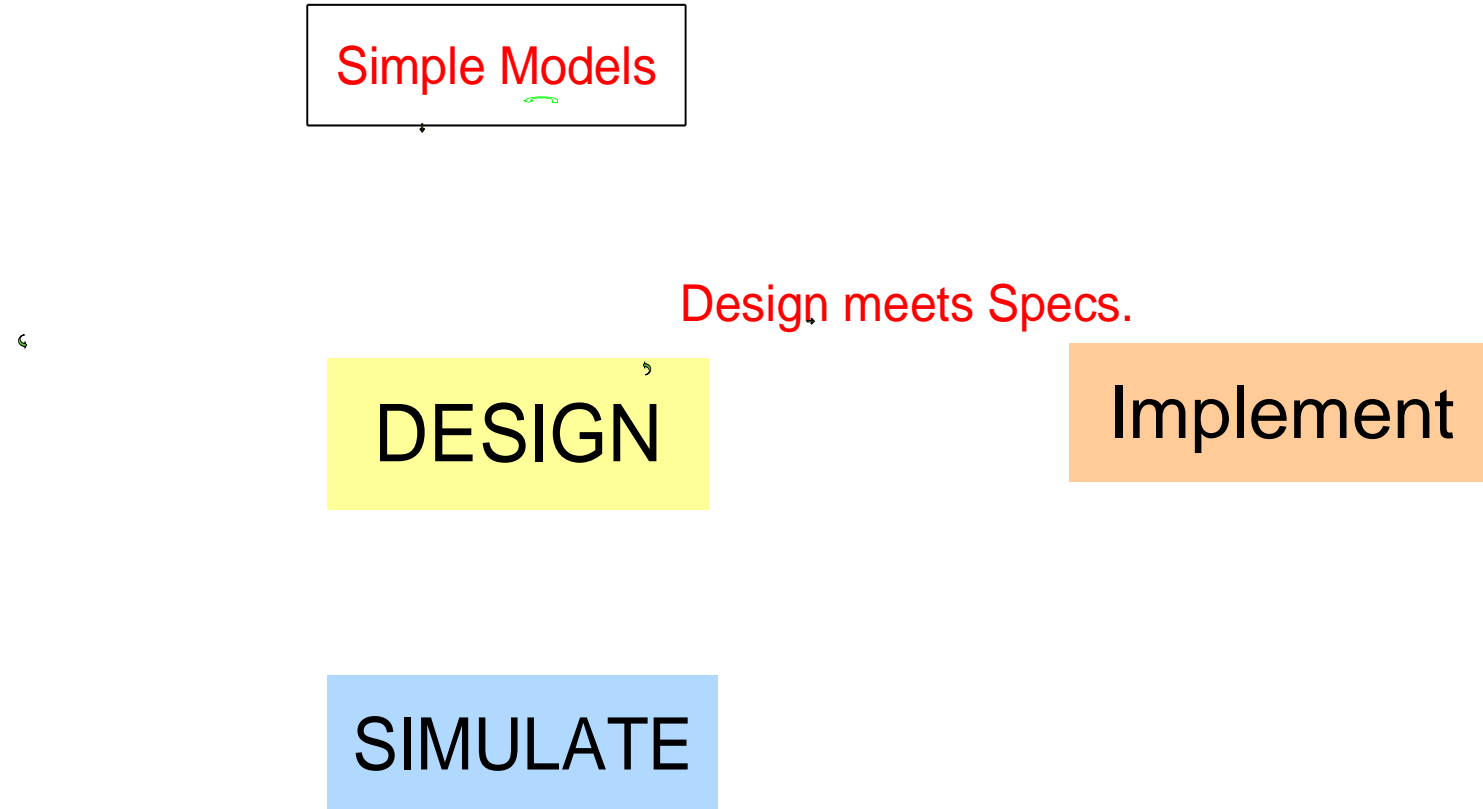
$$I = I_S \left(\exp \left(\frac{V_D}{V_T} \right) - 1 \right)$$

2 equations, 2 variables

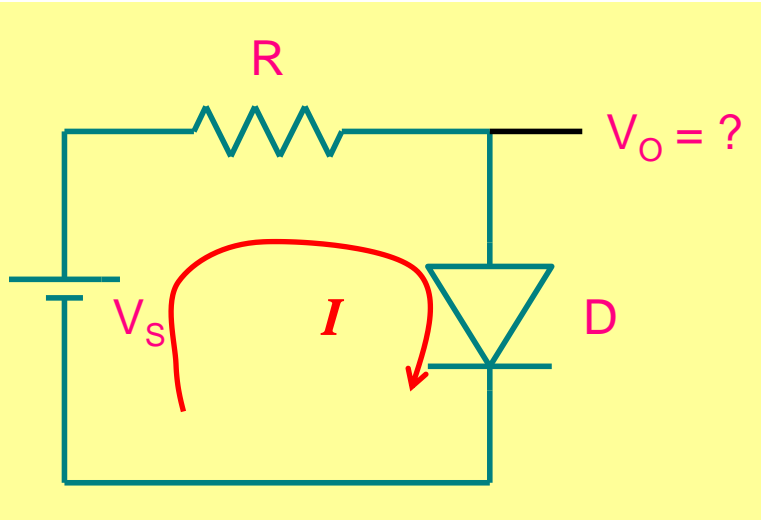
Numerical methods can be used to solve

- Analysis using a non-linear diode model is relatively difficult and time consuming.
- It also does not always give a symbolic expression that can provide insight.
- Need SIMPLER and LINEAR Device Models.

The Design Style



Method of Approximation

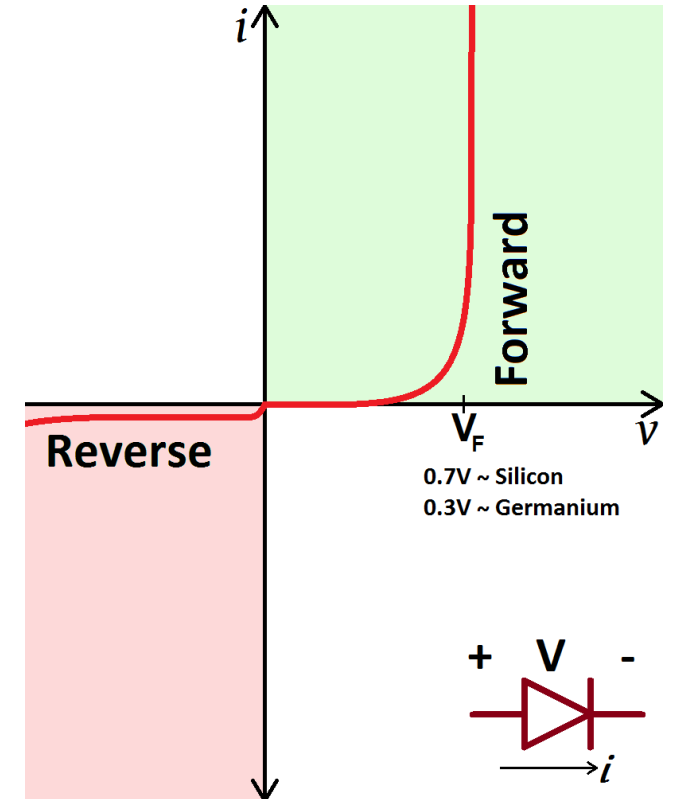


$$V_S = R I + V_D \quad I = I_S \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

2 equations, 2 variables

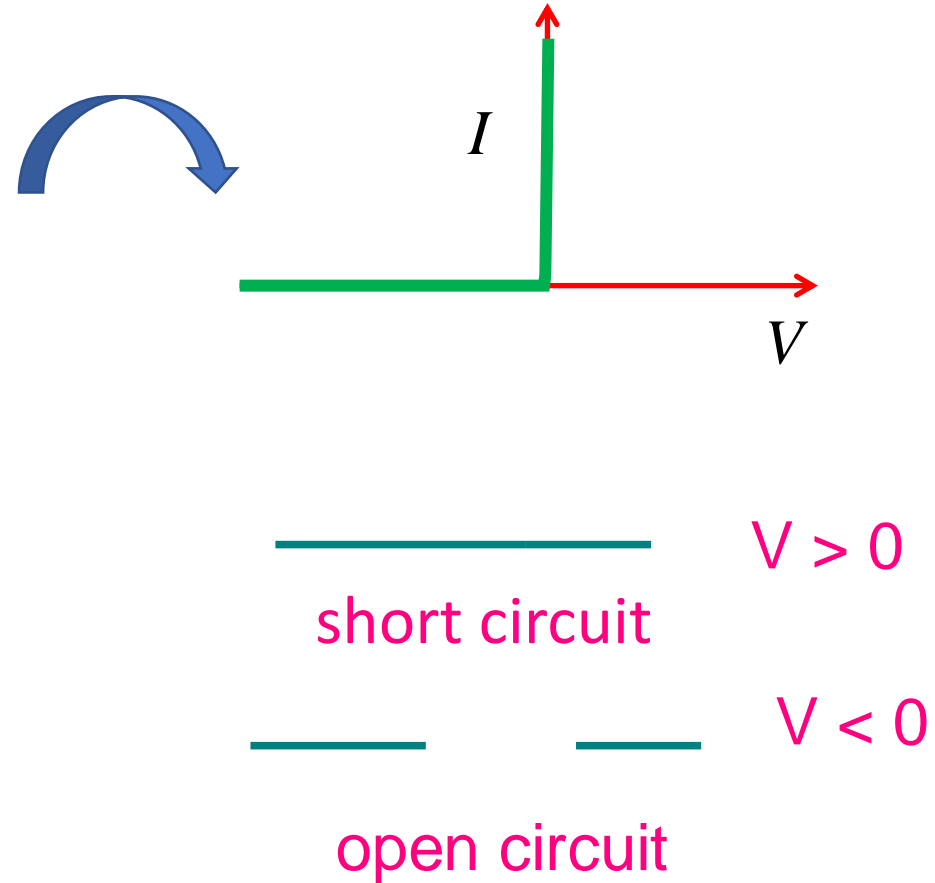
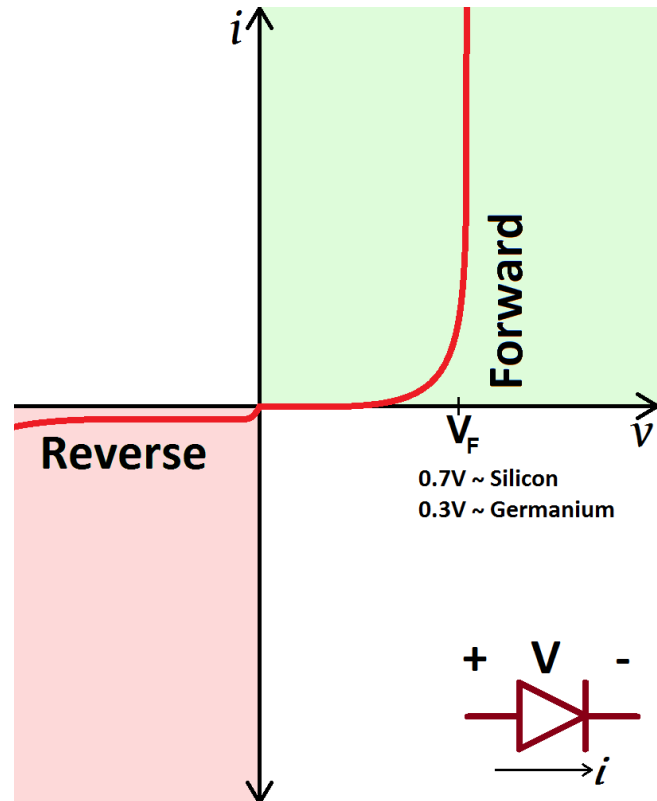
$$V_S = R I_S \left(e^{\frac{V_D}{V_T}} - 1 \right) + V_D$$

- Non-linear equation: How to solve?
 - Numerical methods, graphical method, analytical method, etc.
- We can however approximate its behavior with piecewise linear one
 - I-V graph is approximated by joining two or more straight lines



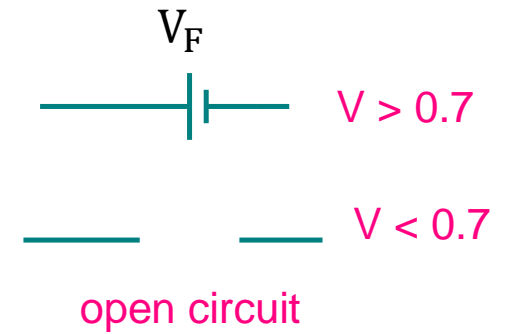
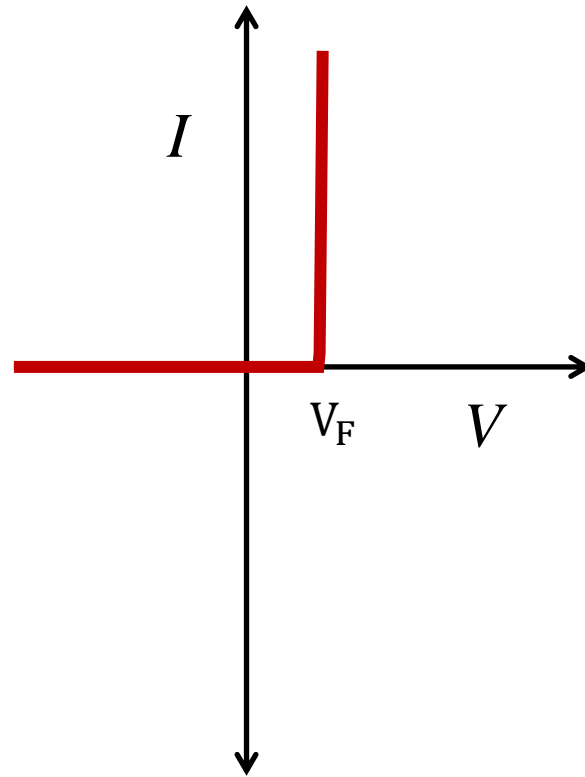
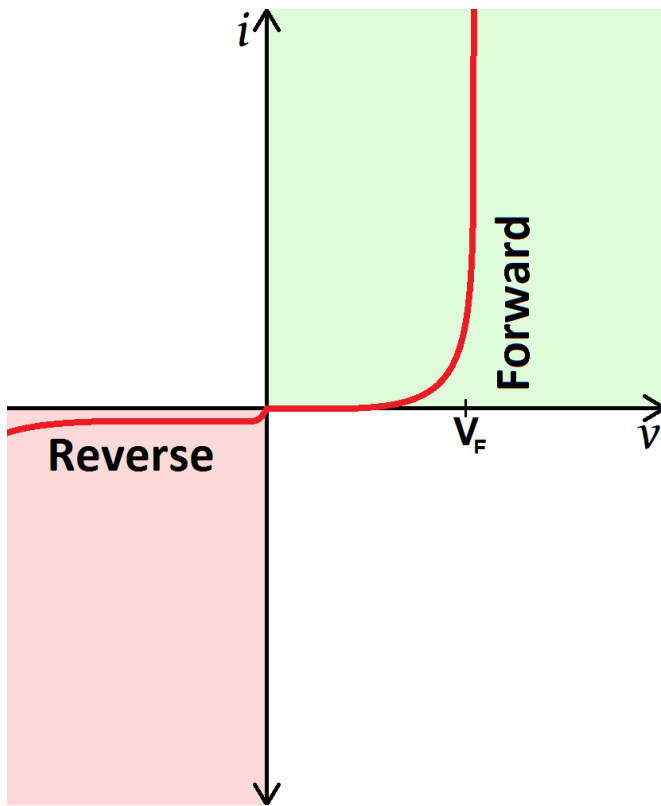
Model A: Ideal Diode (Unidirectional Device)

- Simplest possible approximation: acts as an ideal valve



Model B: Adding a Constant Voltage Drop

- Simplest possible approximation: acts as an ideal valve with a drop in voltage



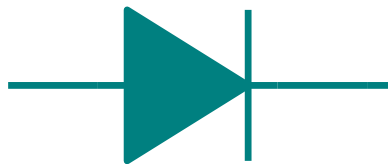
Model C: Adding a Series Resistance

$$V = V_{\gamma} + I r_f$$

V_{γ} is called cut-in or turn-in voltage, depends on nature of diode and range of current considered

For example:

$$V_{\gamma} = 0.7V \text{ and } r_f \sim 10\Omega$$

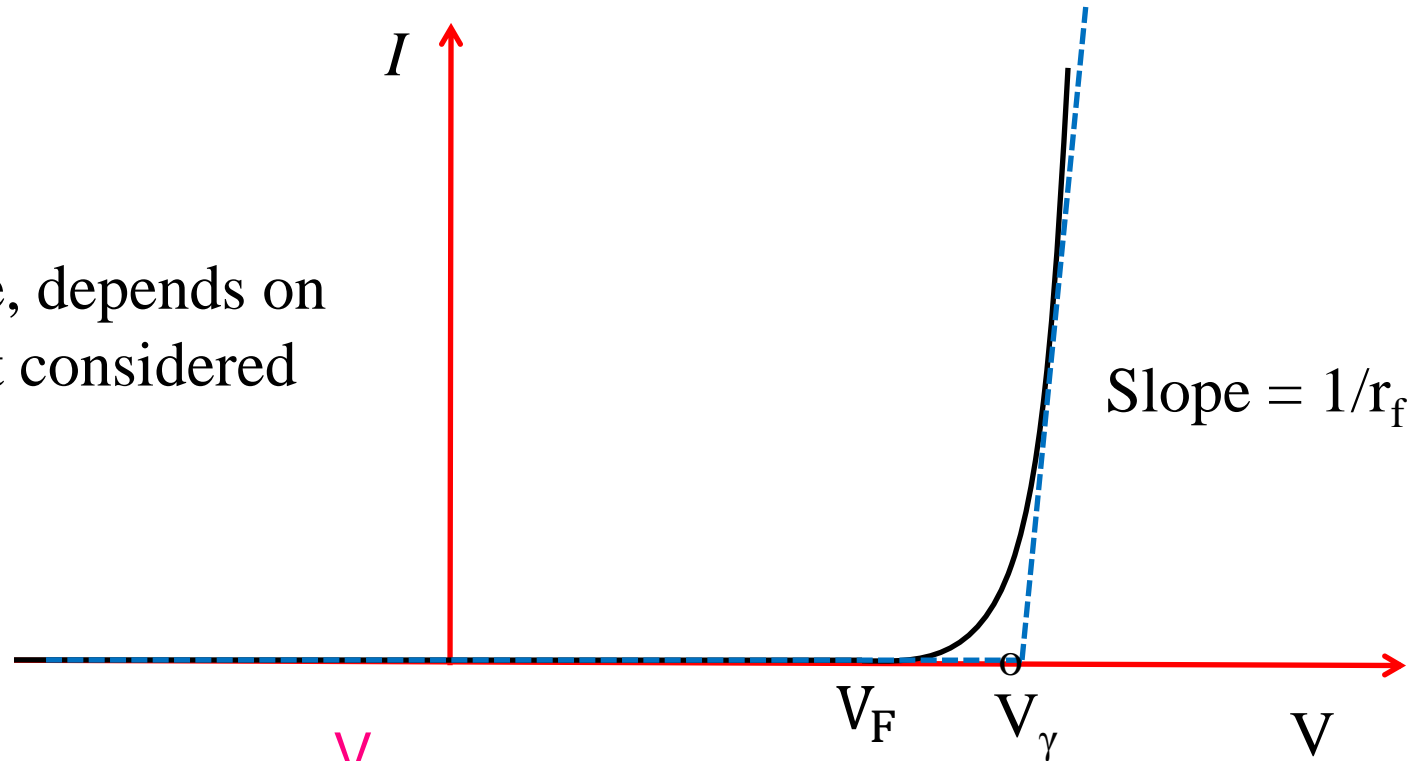


$$V > V_{\gamma}$$



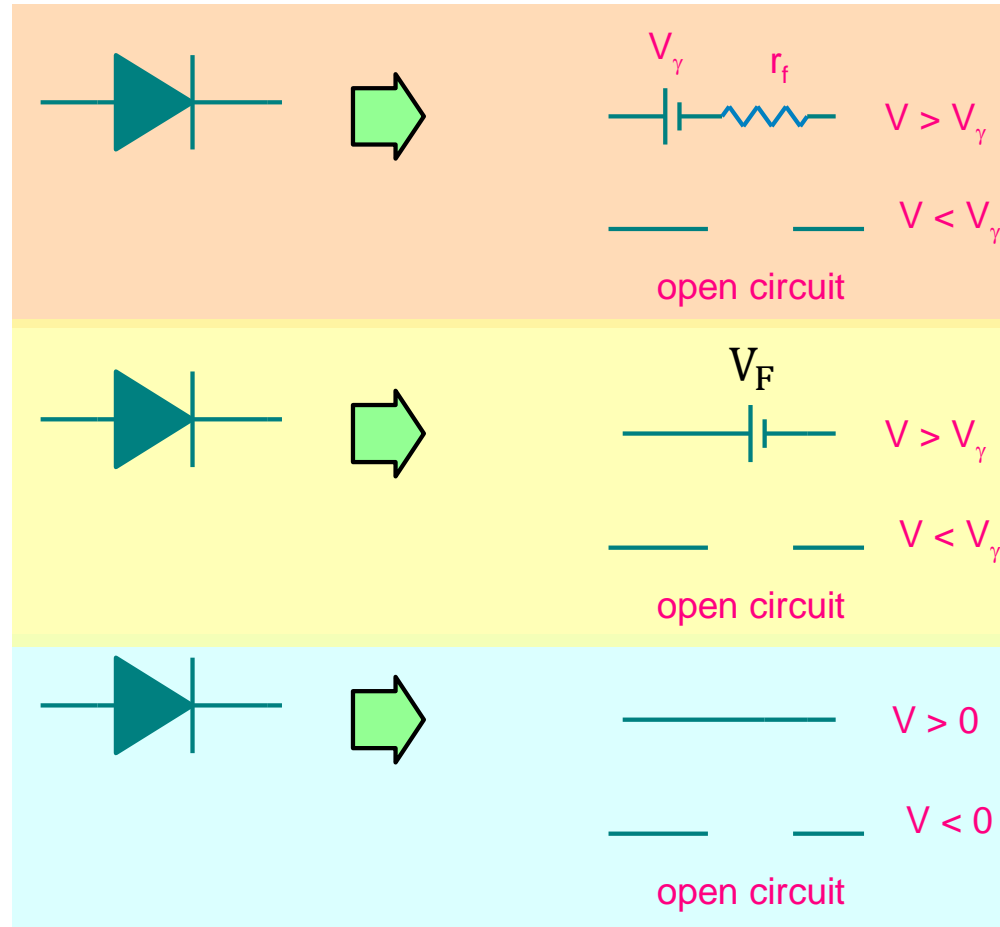
$$V < V_{\gamma}$$

open circuit

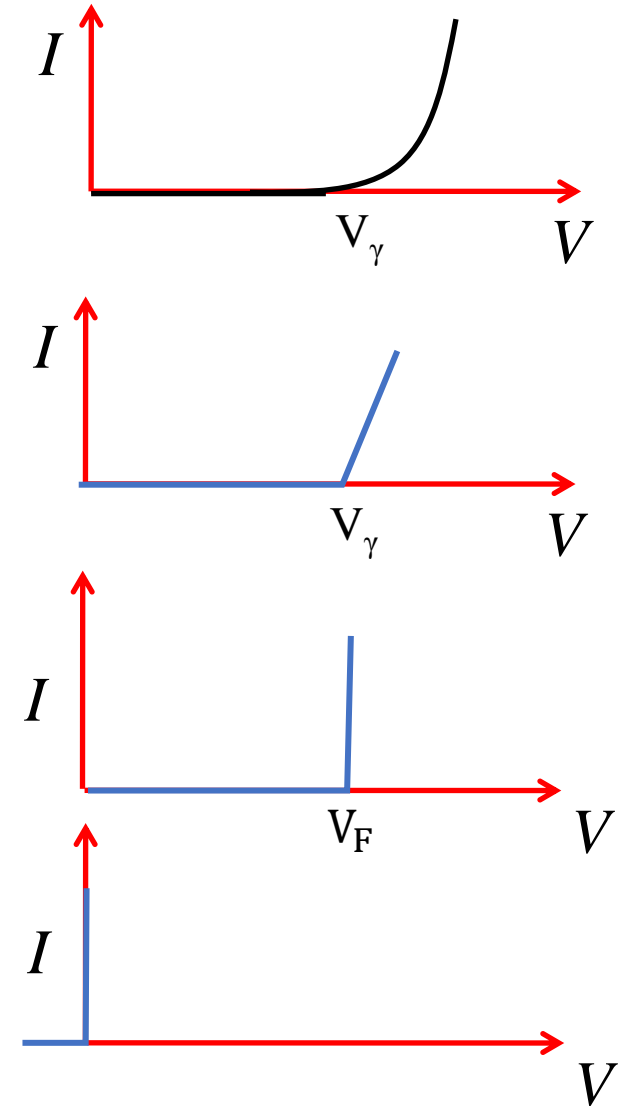


Summary: Approximate Diode Models

Simplicity

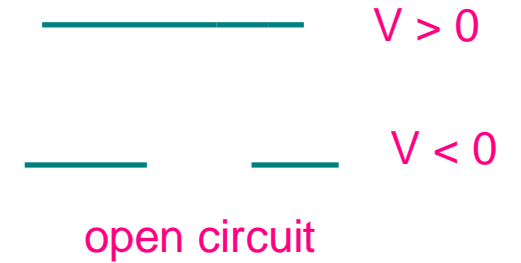
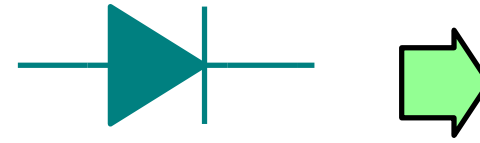
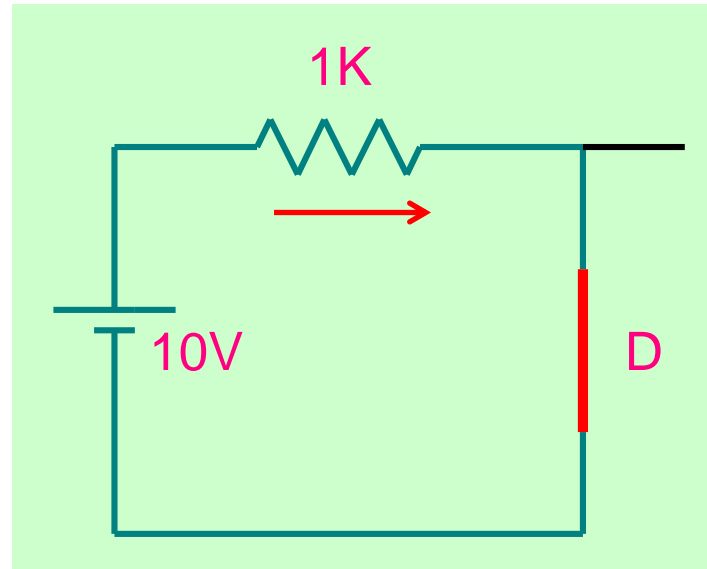
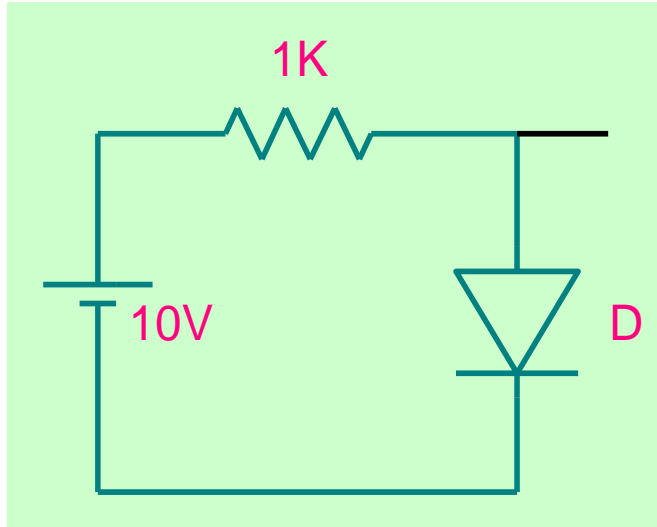


Accuracy



Circuit Analysis: Example 1

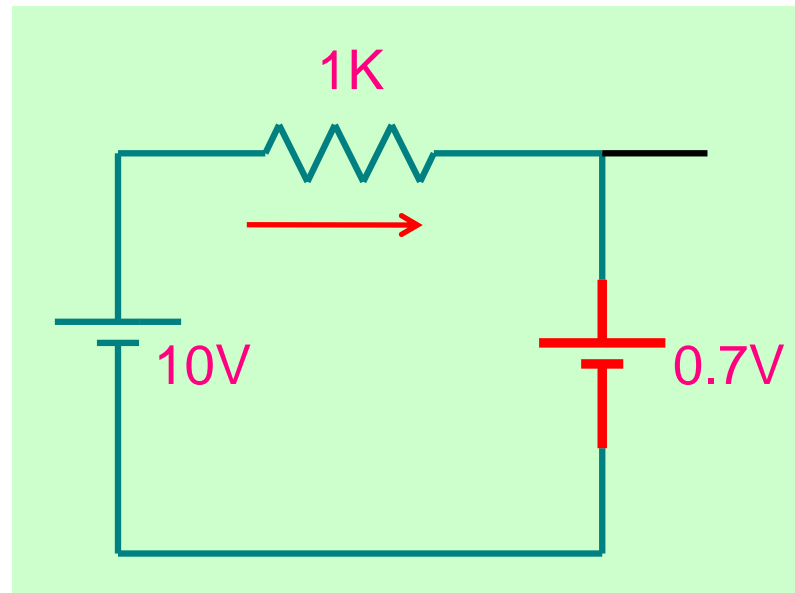
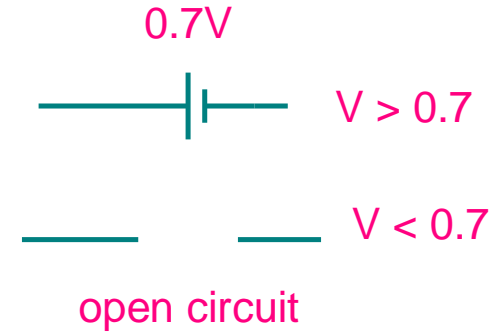
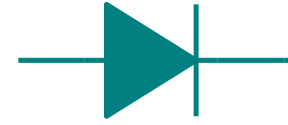
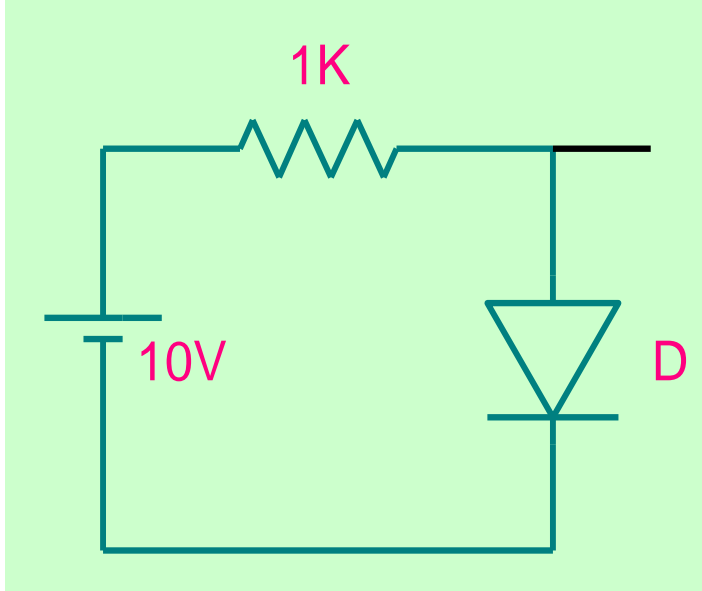
Analysis using ideal diode model



$$I = \frac{10}{1k} = 10mA$$

Circuit Analysis: Example 1

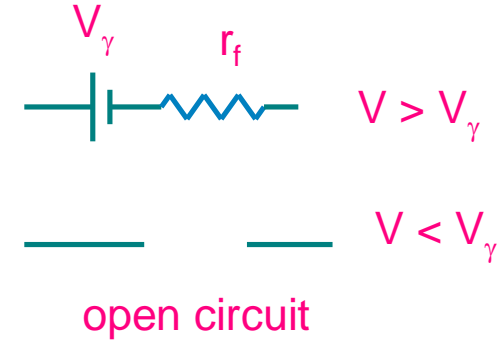
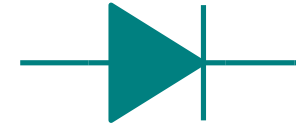
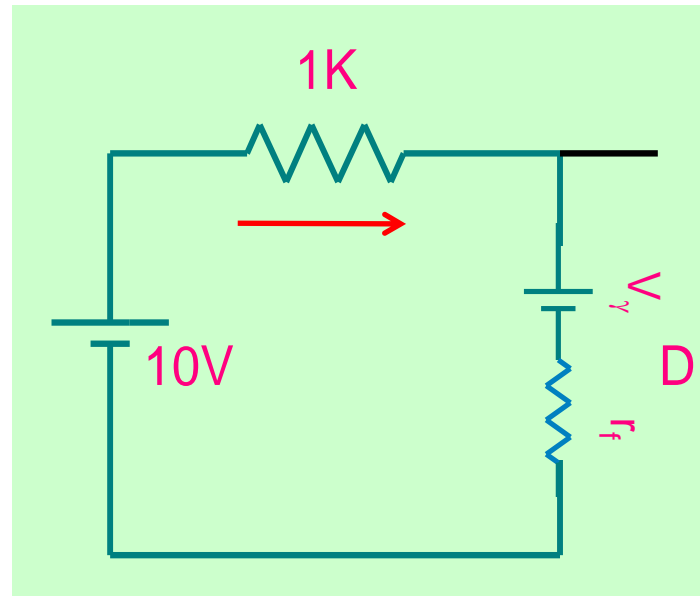
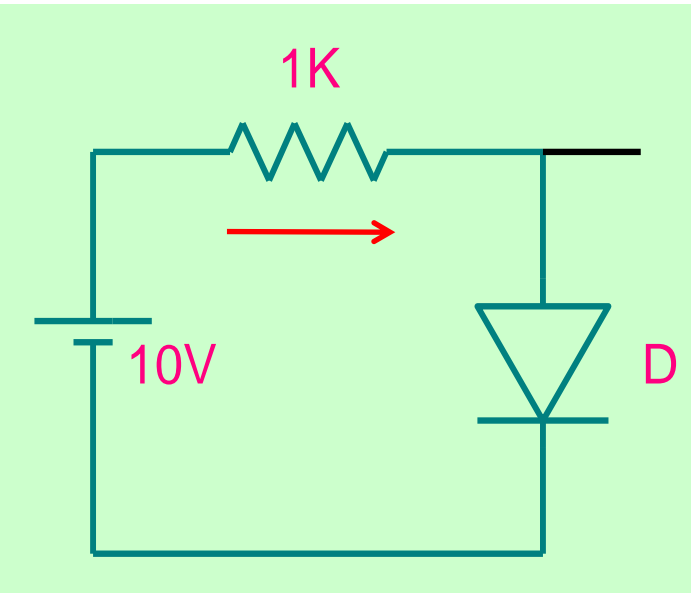
Analysis with a constant voltage diode model



$$I = \frac{10 - 0.7}{1k} = 9.3mA$$

Circuit Analysis: Example 1

Analysis with a constant voltage
plus series resistor diode model



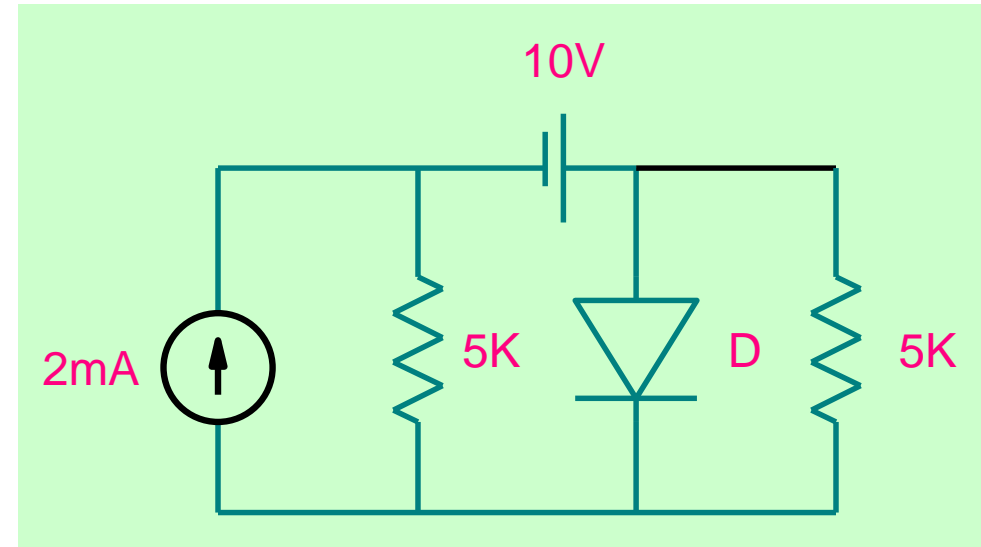
$$I = \frac{10 - 0.7}{1000 + 10} = 9.208mA$$

Self-Consistent Analysis

- How to know in which state diode is?
 - Easier if the voltage is known.
 - Otherwise
 - Analyze circuit assuming diode is forward biased
 - Check assumption ($I > 0$?)
 - Analyze circuit assuming diode is reverse biased
 - Check assumption ($V < 0$?)
 - Select the consistent one.
- What if 2 diodes: 4 possible circuits, only 1 will be valid
- N diodes $\Rightarrow 2^N$ circuits, only one will be valid

Circuit Analysis: Example 2

Find the current through the 5k resistor using ideal diode model



Is the diode forward biased? – Not Sure!!

Assume that it is forward biased

Carry out analysis and then check if current through the diode is in **appropriate** direction.

If not, diode is reverse biased and we carry out the analysis again!!

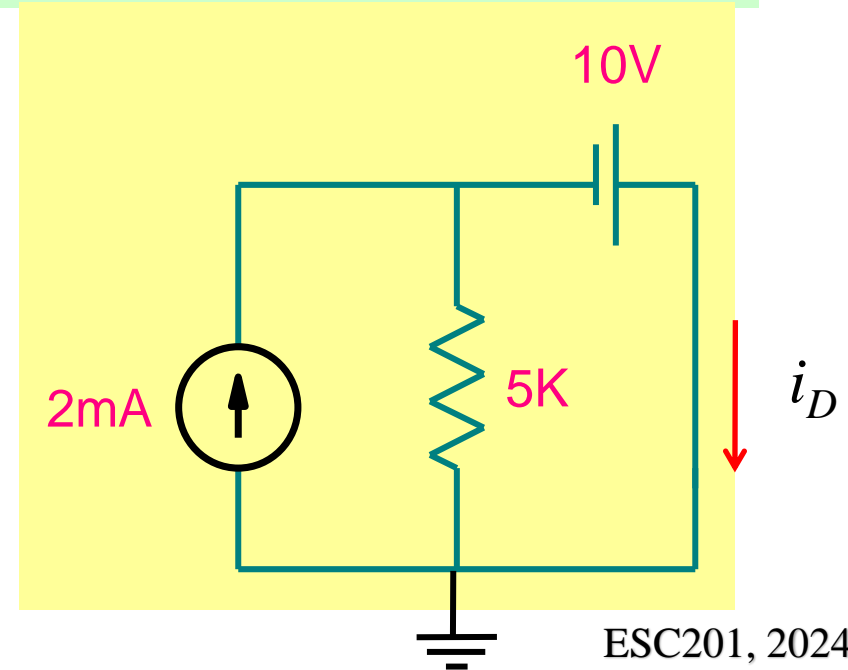
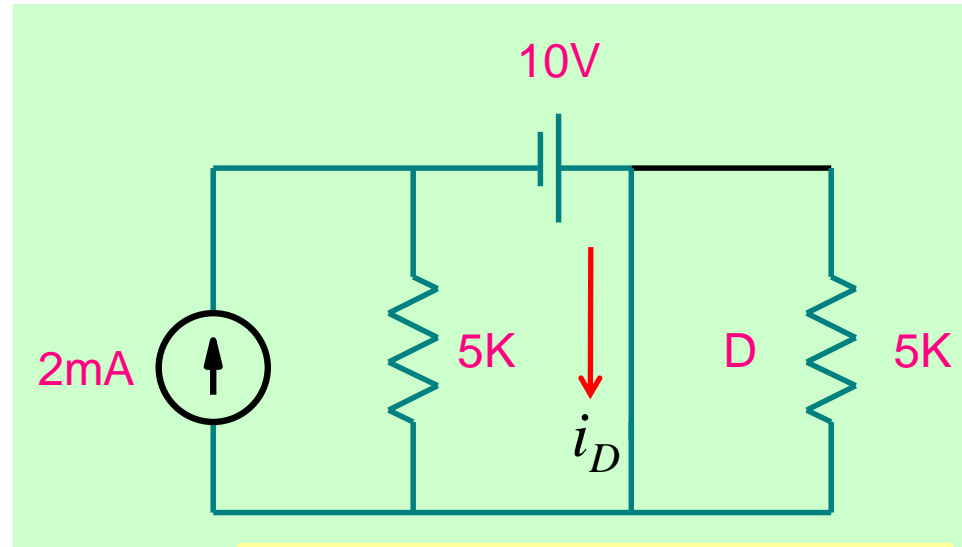
Circuit Analysis: Example 2

Assume forward bias

$$-2 \text{ mA} + \frac{-10 \text{ V}}{5 \text{ k}\Omega} + i_D = 0$$

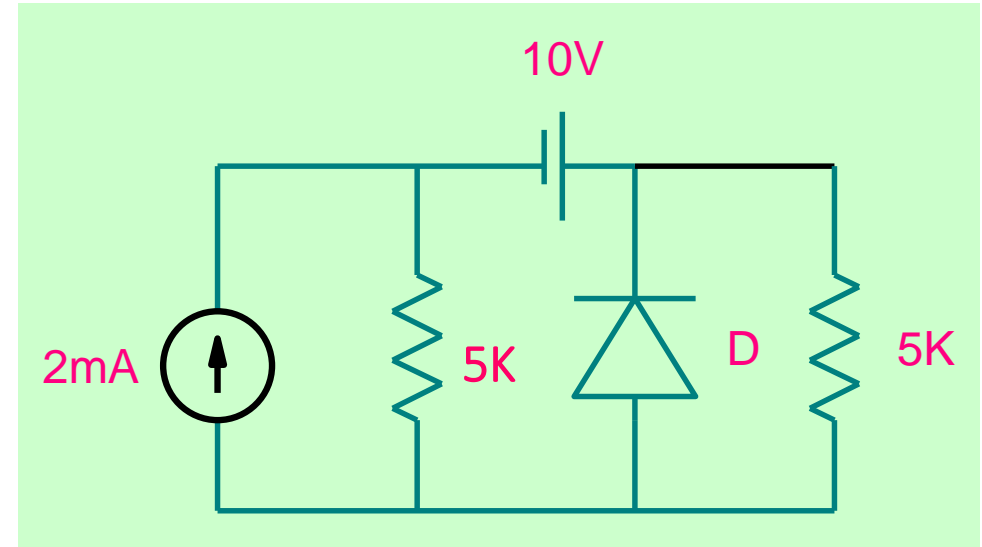
$$i_D = 4 \text{ mA}$$

Current is positive, so our assumption is correct!

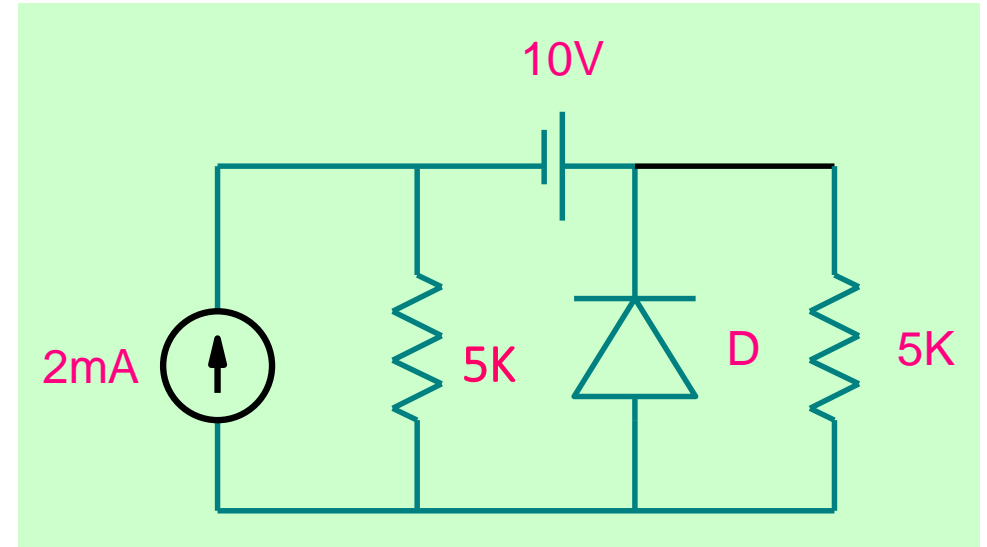


Circuit Analysis: Example 3

Find the current through the 5k resistor using ideal diode model

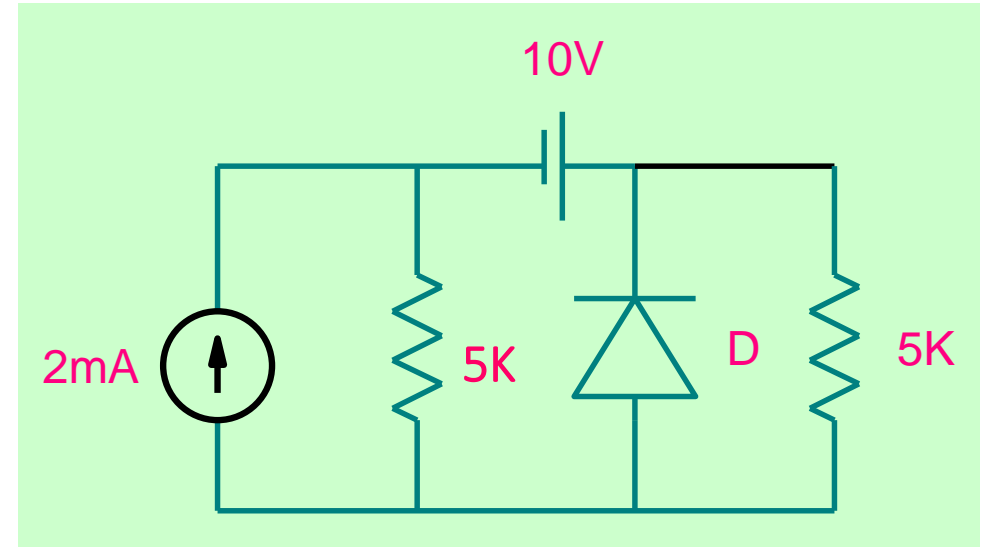


Is the diode forward biased? – Not Sure!!



Circuit Analysis: Example 3

Find the current through the 5k resistor using ideal diode model



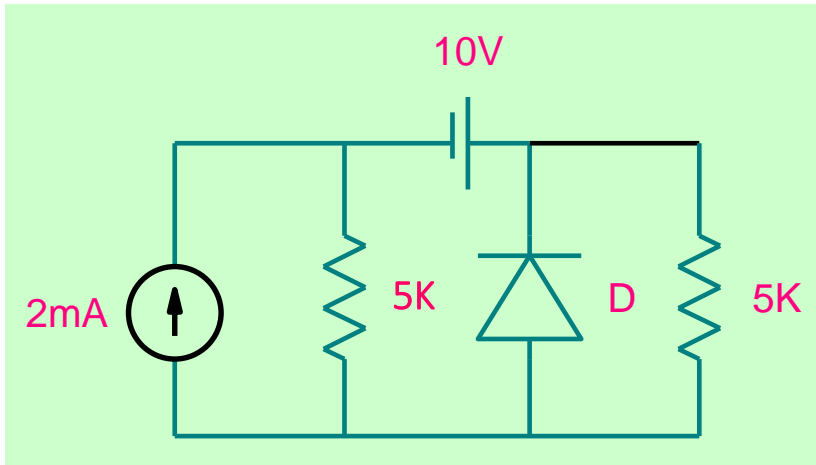
Is the diode forward biased? – Not Sure!!

Assume that it is forward biased

Carry out analysis and then check if current through the diode is in **appropriate** direction.

If not, diode is reverse biased and we carry out the analysis again!!

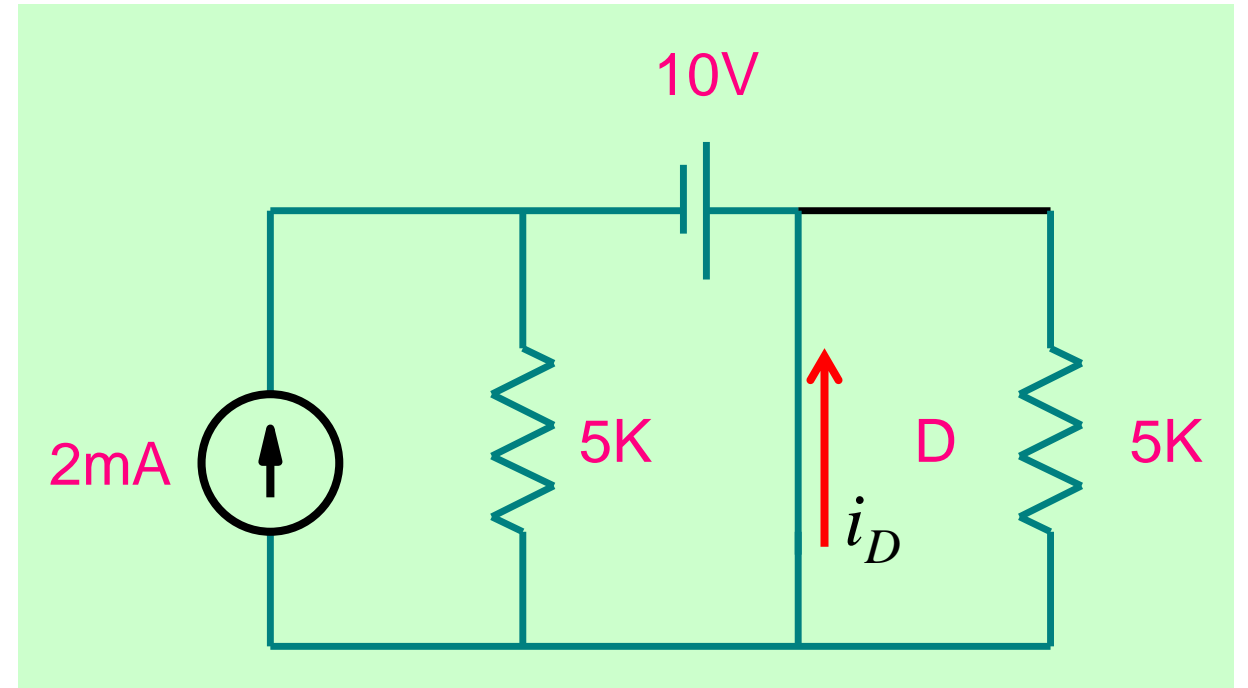
Circuit Analysis: Example 3



Assume forward bias

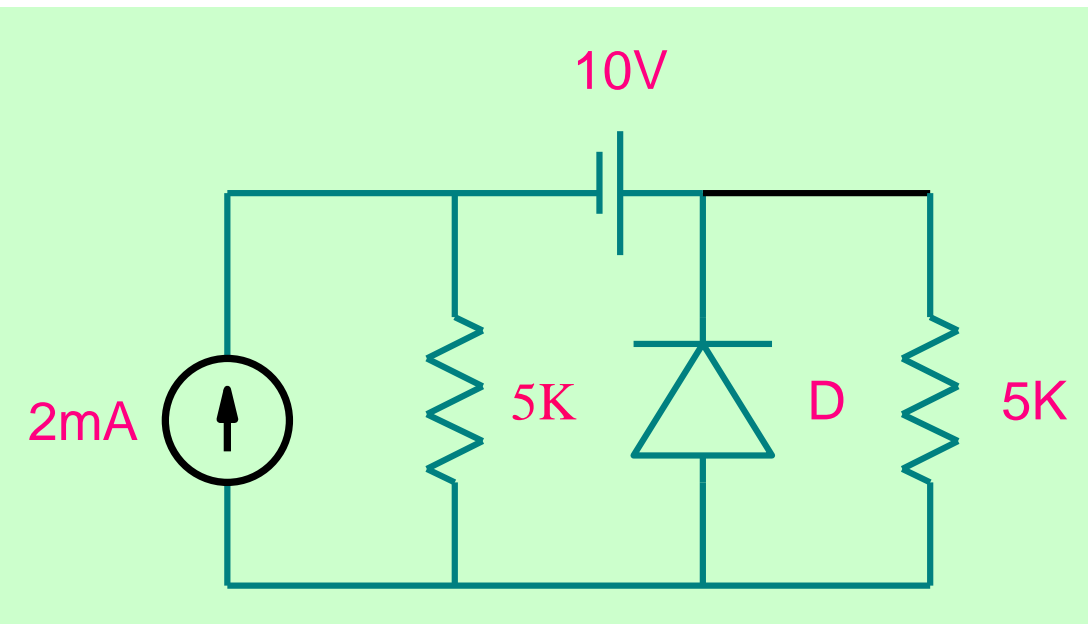
$$-2 \text{ m} + \frac{-10}{5 \text{ k}} - i_D = 0$$

$$i_D = -4 \text{ mA}$$



Therefore, our assumption is incorrect ☹

Circuit Analysis: Example 3

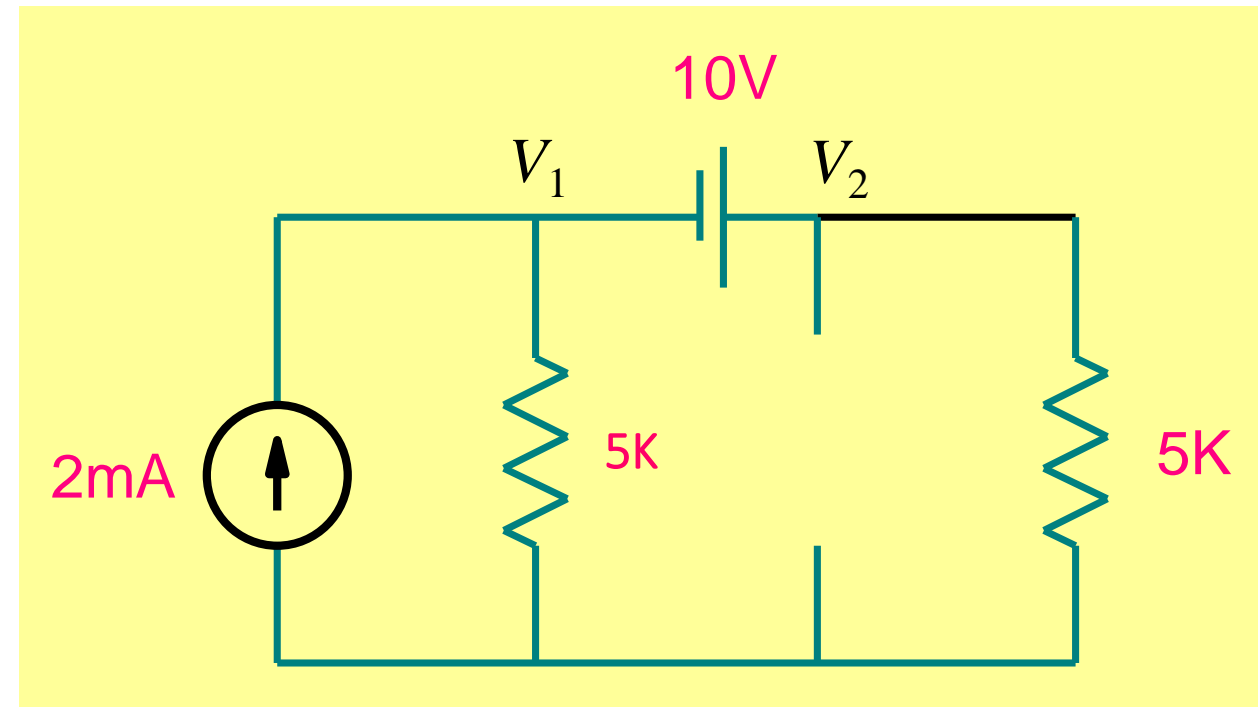


Assume reverse bias

$$-2 \text{ m} + \frac{V_1}{5 \text{ k}} + \frac{V_1 + 10}{5 \text{ k}} = 0$$

$$V_1 = 0$$

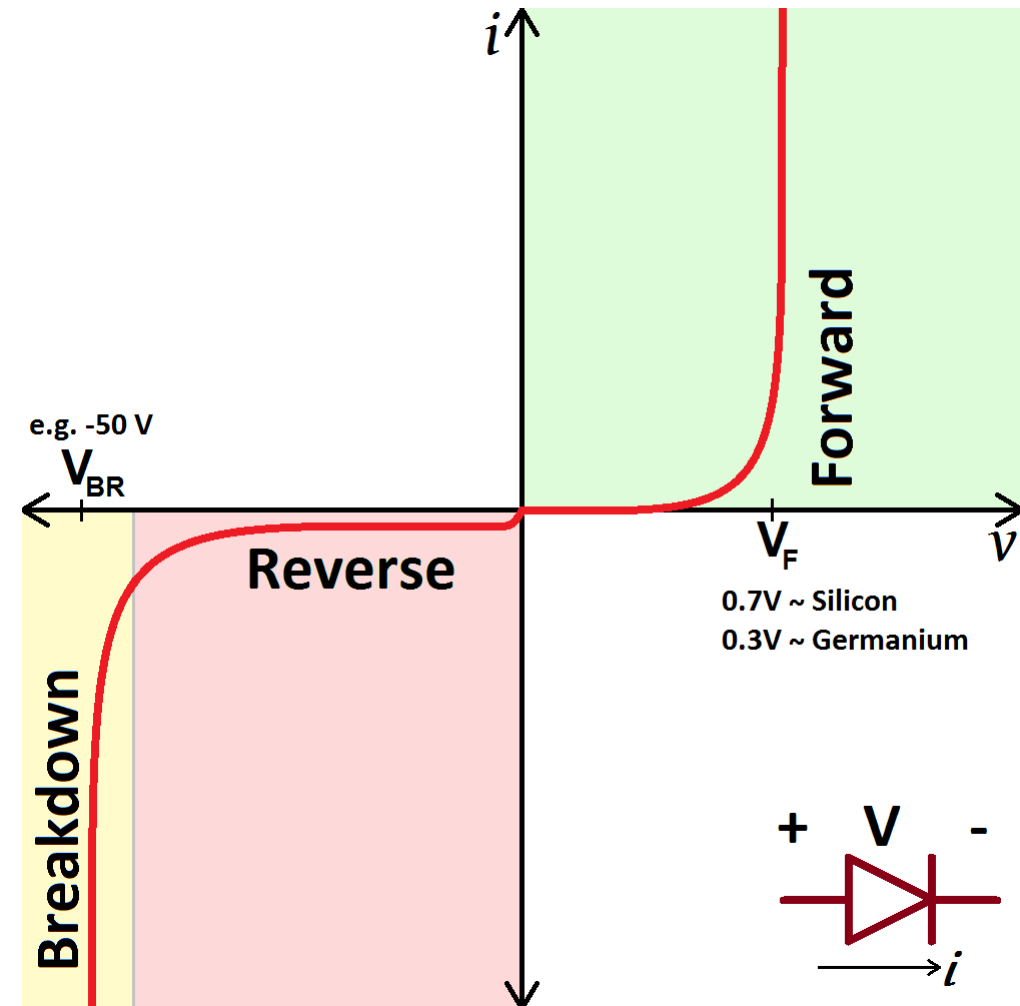
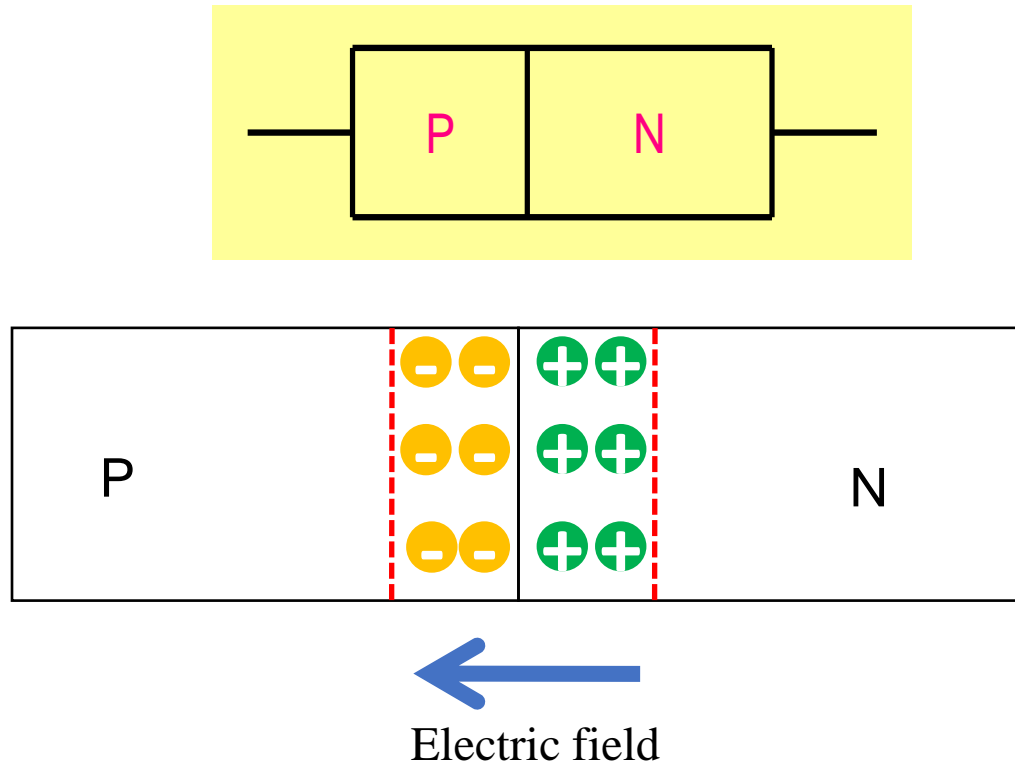
$$V_2 = 10\text{V}$$



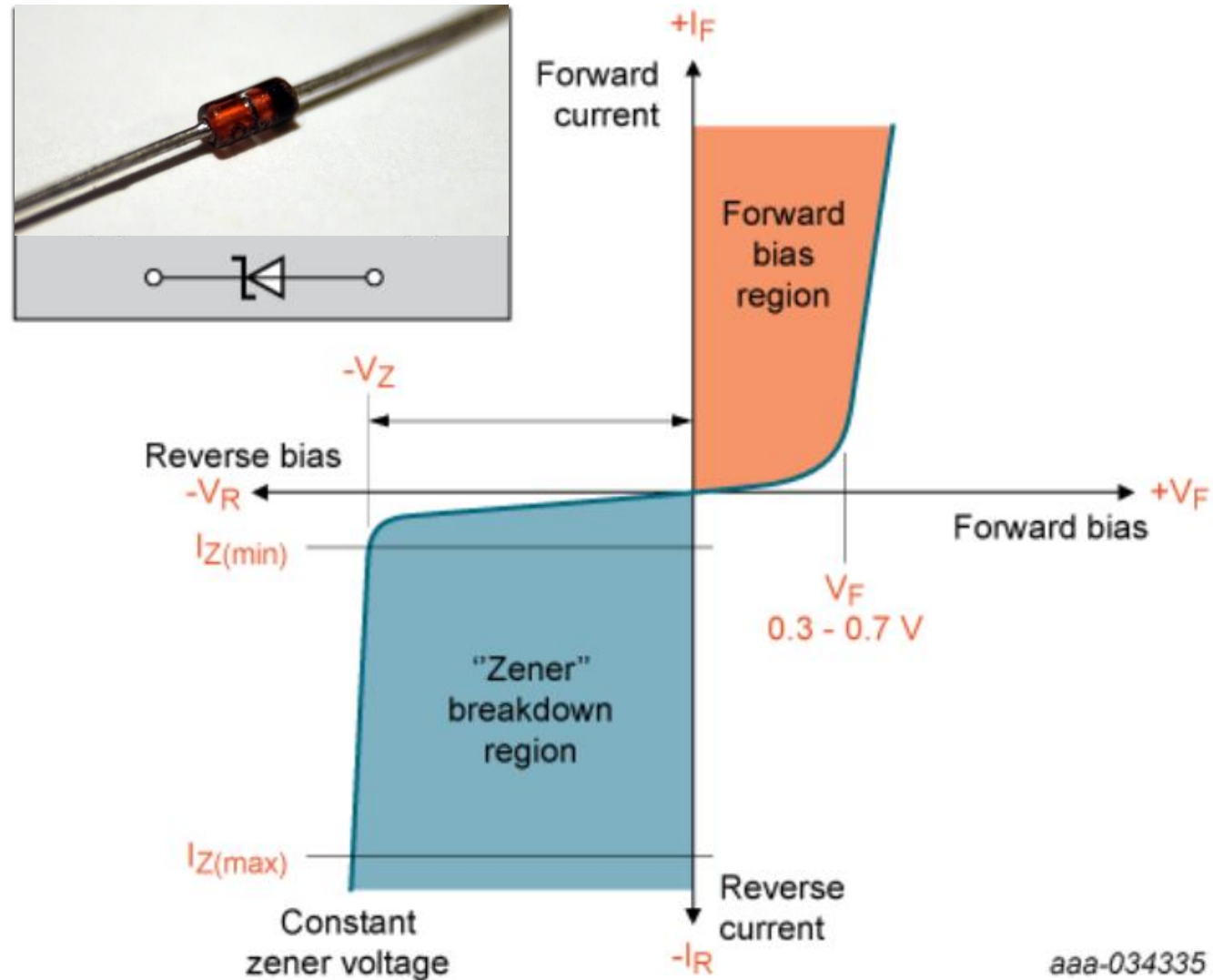
Therefore, our assumption is correct ☺

$$I_{5K} = 2 \text{ mA}$$

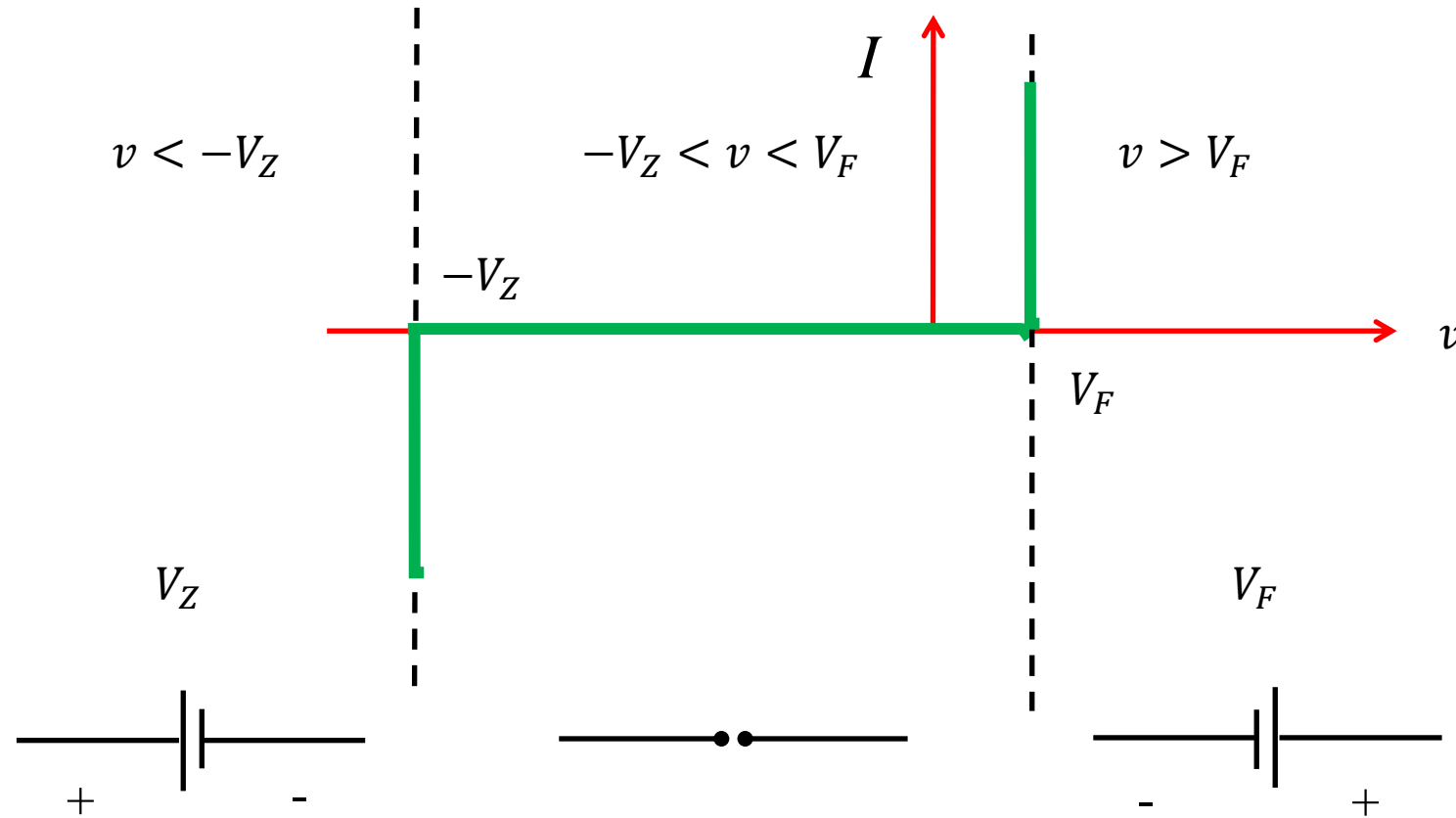
Breakdown



Leveraging Breakdown: Zener Diode



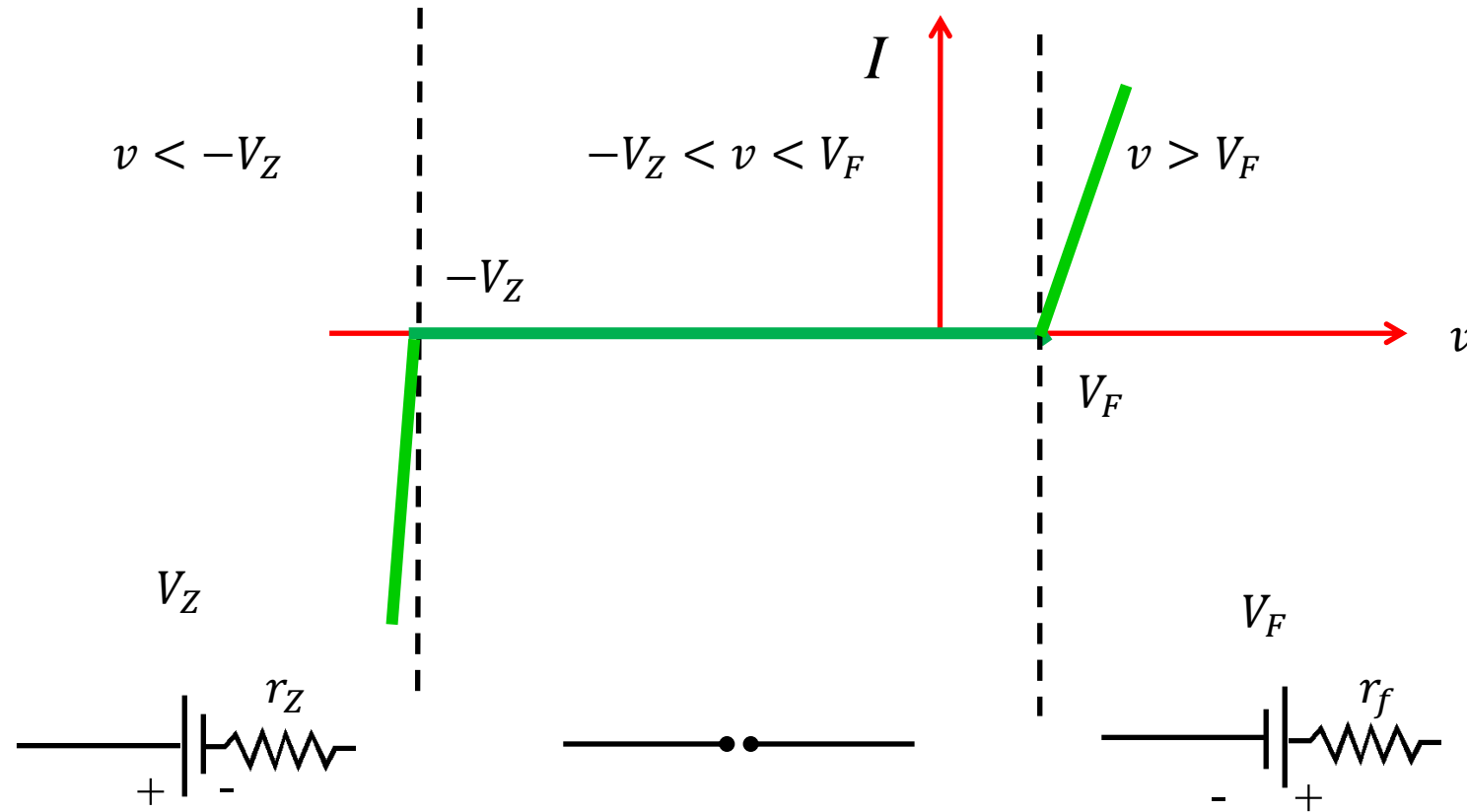
Zener Diode: A Simple Piecewise Linear Model



Zener diode does not allow the voltage across it to go beyond the range $[-V_Z, V_F]$

Method of assumed states: 3 possibilities now

Zener Diode: A Little Complex Piecewise Linear Model



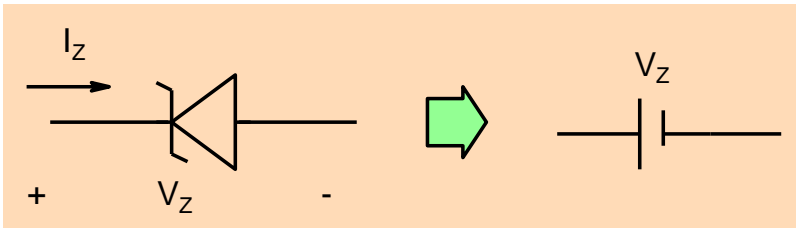
Typically, both forward and breakdown resistances are a few ohms

Zener diodes have limited current carrying capacity: power rating

Circuit Analysis: Example 4

Find R such that current through diode is limited to 3 mA

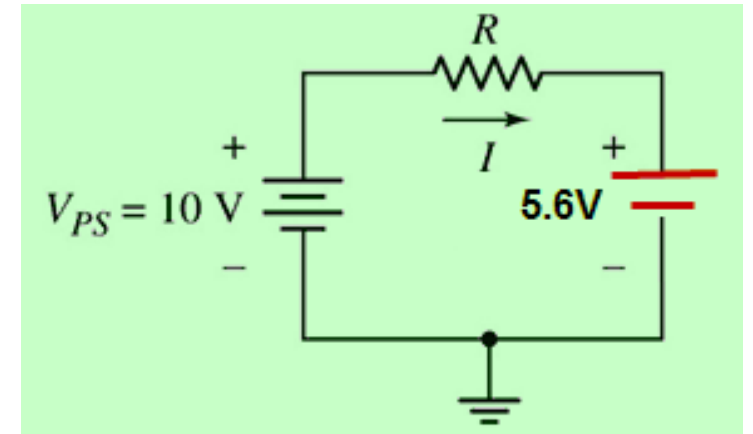
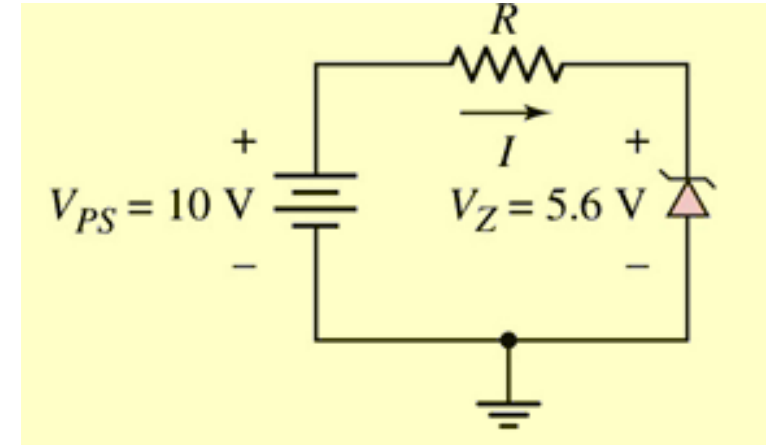
Assume breakdown region



$$RI = V_{PS} - V_Z$$

$$R = \frac{V_{PS} - V_Z}{I} = \frac{(10 - 5.6)V}{3 \text{ mA}} = 1.47 \text{ k}\Omega$$

$$I < 3 \text{ mA} \Rightarrow R > 1.47 \text{ k}\Omega$$



Summary: Zener Diode I-V Characteristics

