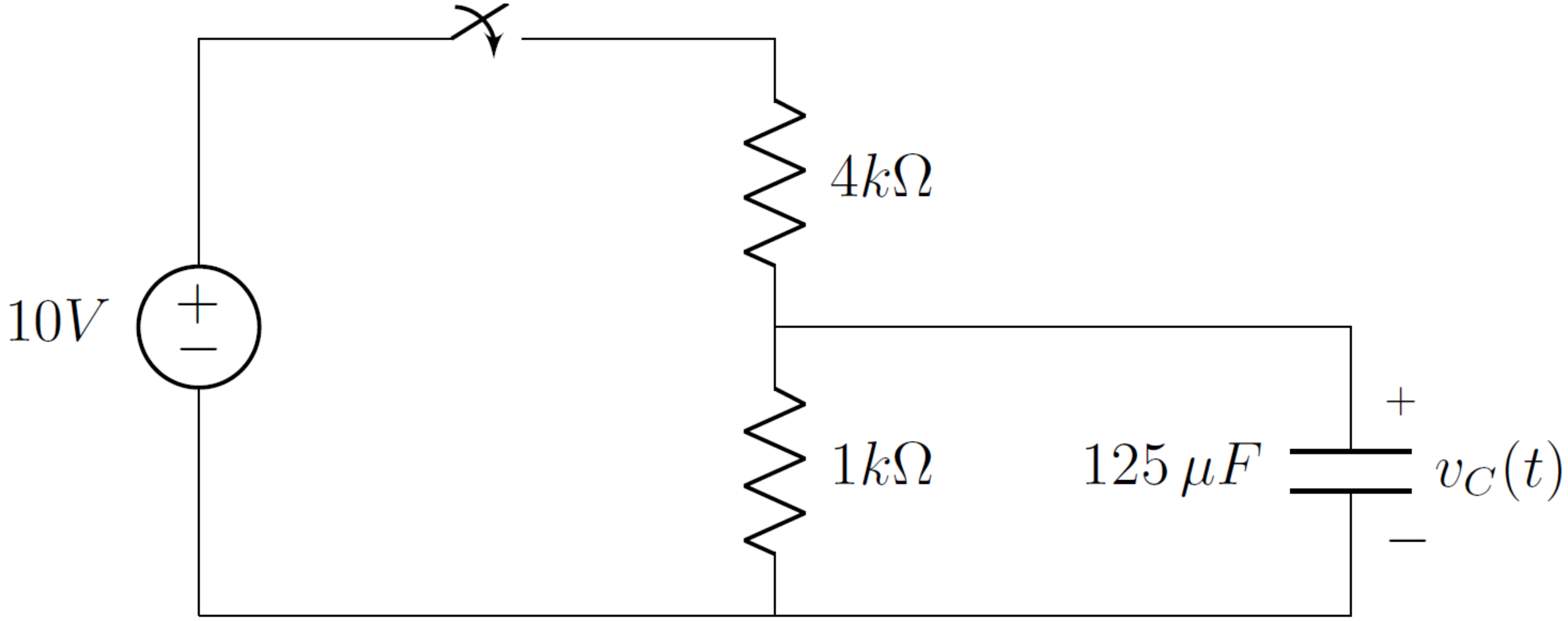
Esc201T: Introduction to Electronics

Mid-semester Examination: Solutions

Q1: In the circuit shown below, the switch is closed at t=0 second and opened at t=2 second. Assume the initial voltage across the 125 μF capacitor to be zero.

a) Find the expression for $v_c(t)$ for all times.

b) Sketch v_c(t) for all times. Clearly label your axes.



Solution:

When the switch is closed at $t = 0^+$, we have $v_c(0^+) = v_c(0^-) = v_c(0) = 0\ V$.

[1 mark]

$v_c(\infty)$ can be found by noting that capacitor acts as open circuit when $t \to \infty$

Hence,

$$v_c(\infty) = \frac{10 \text{ x } 1}{10+4} = 2\ V$$ [1 mark]

The charging time constant is given as

$$\tau_1 = (1K\Omega||4K\Omega) \text{ x } 125\ \mu F = 0.1\text{s}$$ [1 mark]

For $0 \leq t \leq 2$ seconds , we have

$$v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)]e^{-t/\tau_1}$$

$$=> v_c(t) = 2[1 - e^{-10t}]\text{ V} \ , \forall\ 0 \leq t \leq 2.$$ [1 mark]

At t = 2s , the switch is opened, and we have

$$\mathrm{v_c}(2^-) = \mathrm{v_c}(2^+) = 2[1-\mathrm{e}^{-20}]\ \mathrm{V} \approx 2\mathrm{V}$$ [1 mark]

After the switch is opened the capacitor discharges and the discharging time constant is given as

[1 mark]

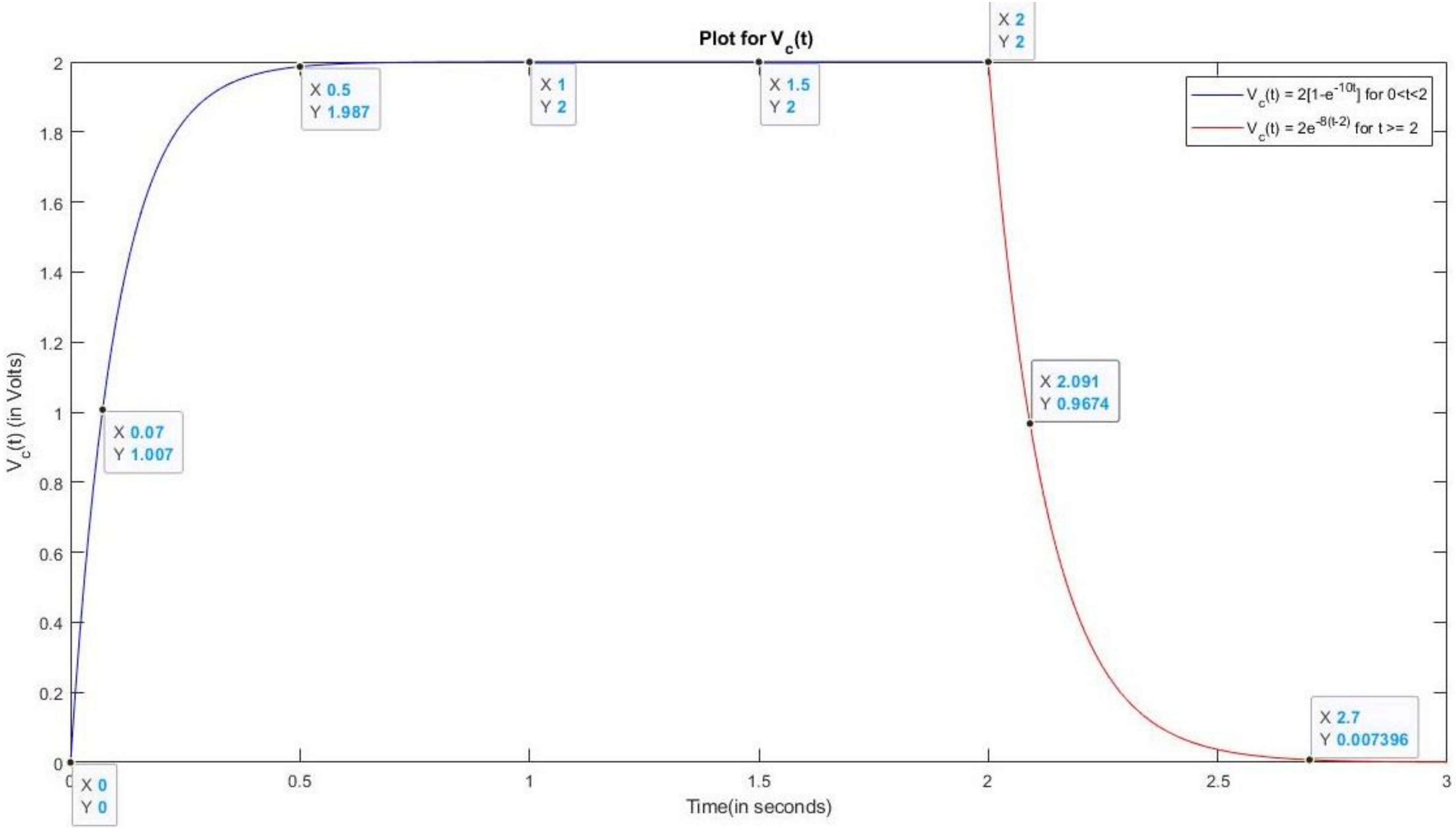
$$\tau_2 = 1K\Omega \text{ x } 125\ \mu F = 0.125 \text{ s}$$

For t $\geq$ 2 sec , we have

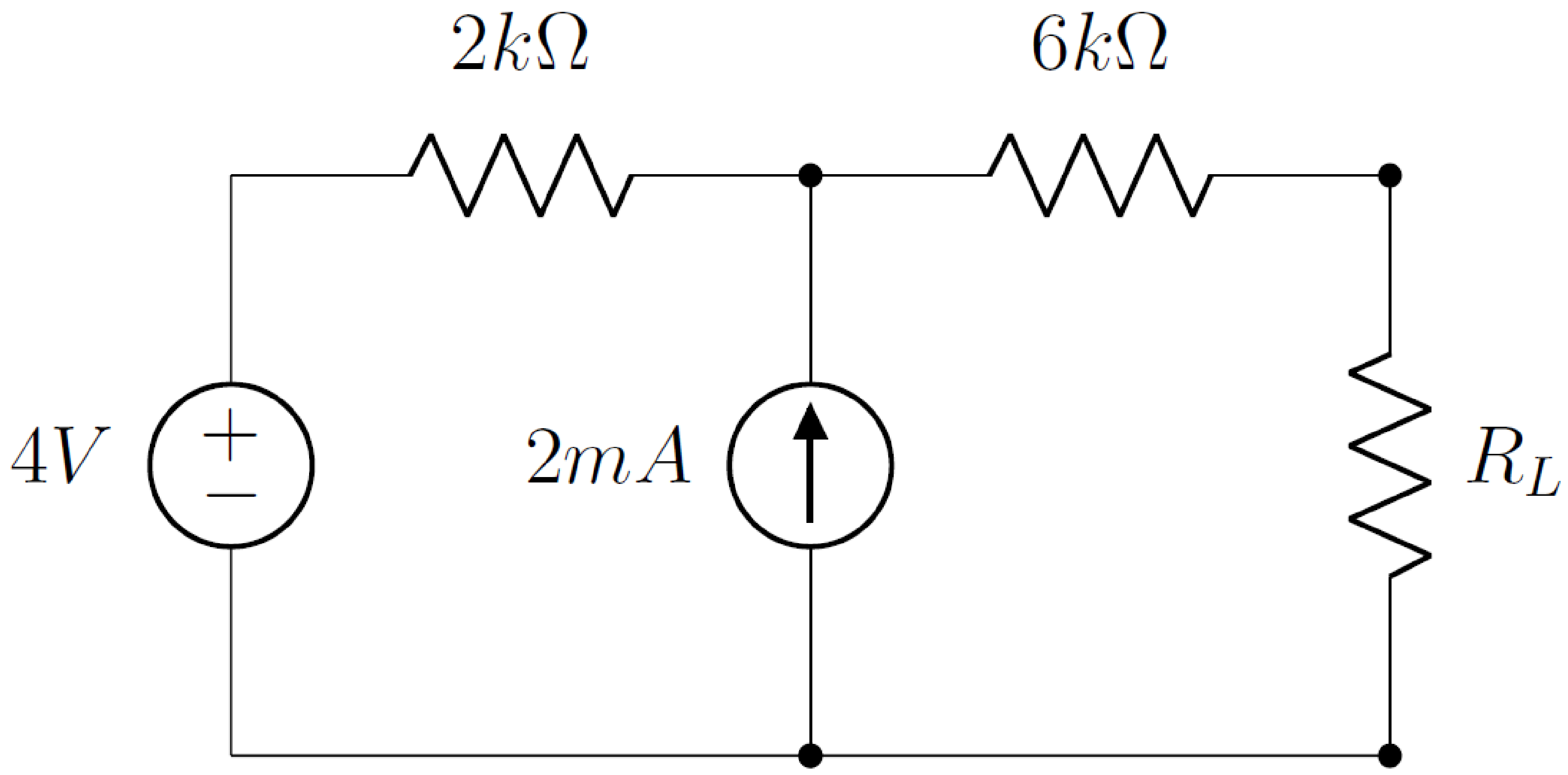
$$v_c(t) = v_c(2^+)e^{-\frac{t-2}{\tau_2}}$$

$$v_c(t) = 2e^{-8(t-2)}V\ , \forall\ t \geq 2$$ [2 marks]

- Plot of v_c(t) [2 marks]

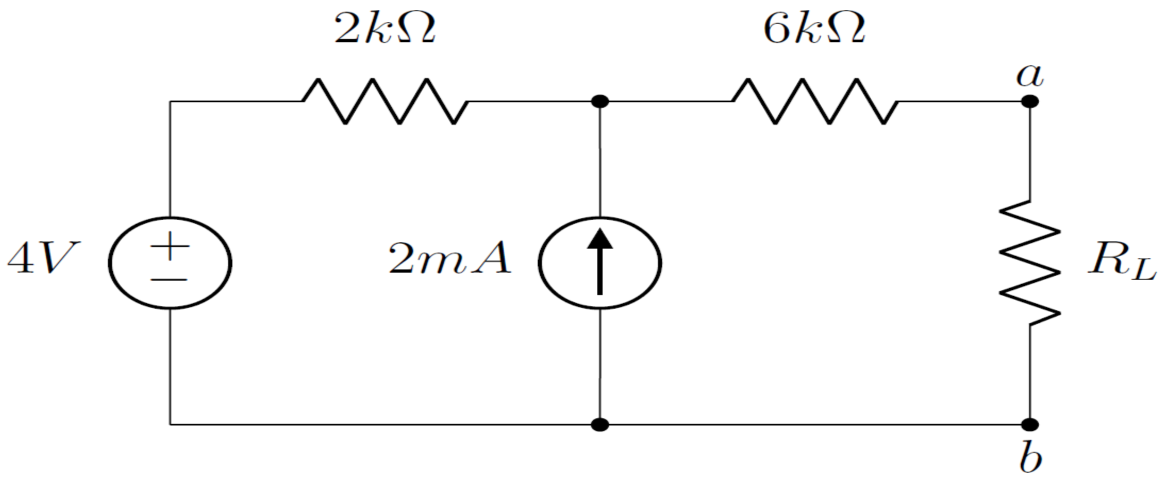


Q2: In the given circuit, find the value of resistor R_L such that maximum power is delivered to it. Also, compute the value of maximum power delivered.

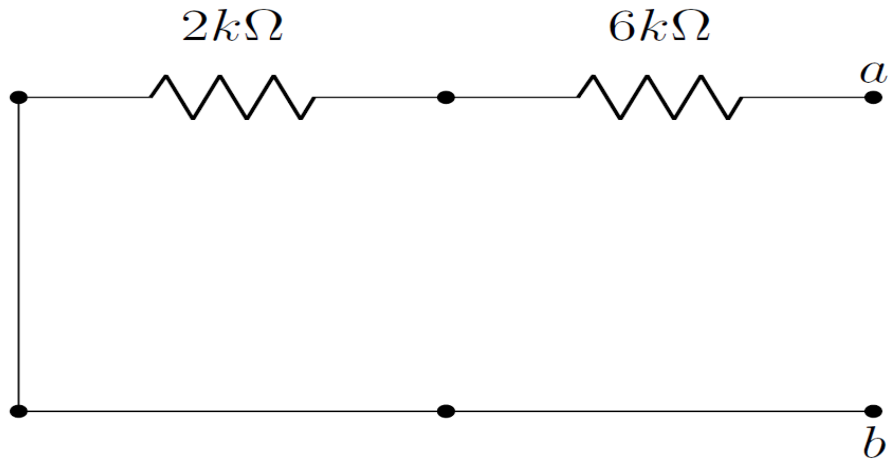


Solution:

Consider the terminals a and b . To apply Thevenin's method across the terminals , we must find R_{TH} and v_{TH} .

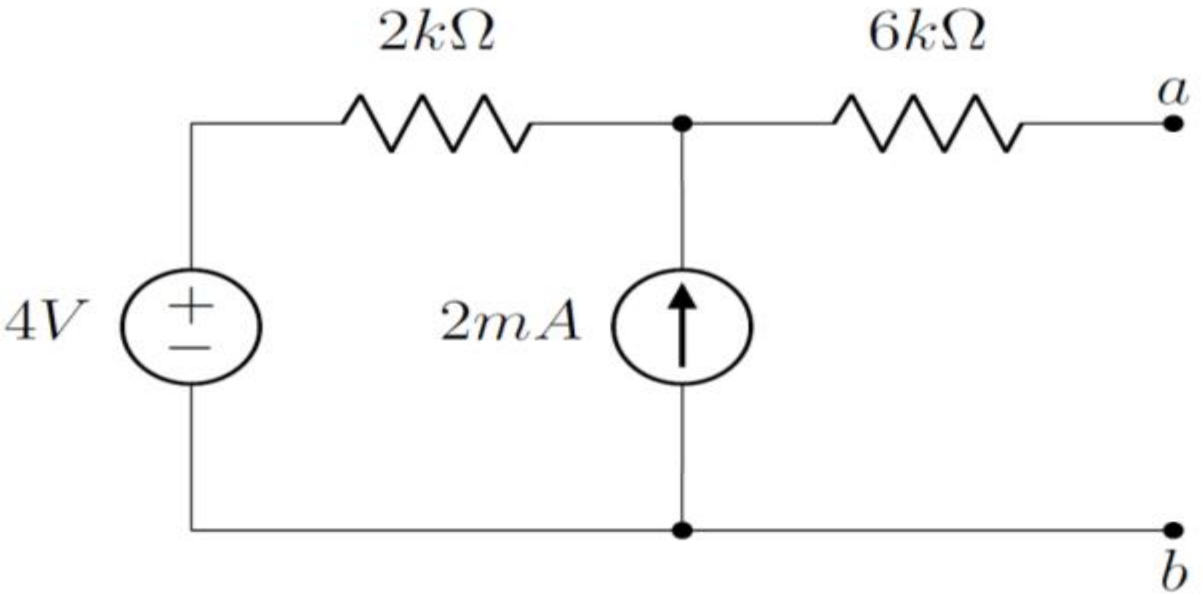


For R_{TH} calculations short circuit the voltage source and open the current source



Hence $R_{TH} = 2K\Omega + 6K\Omega = 8K\Omega$. [1 mark]

For v_{TH} computation, we have to find the open circuit voltage $v_{ab} = v_a - v_b$ in the following figure



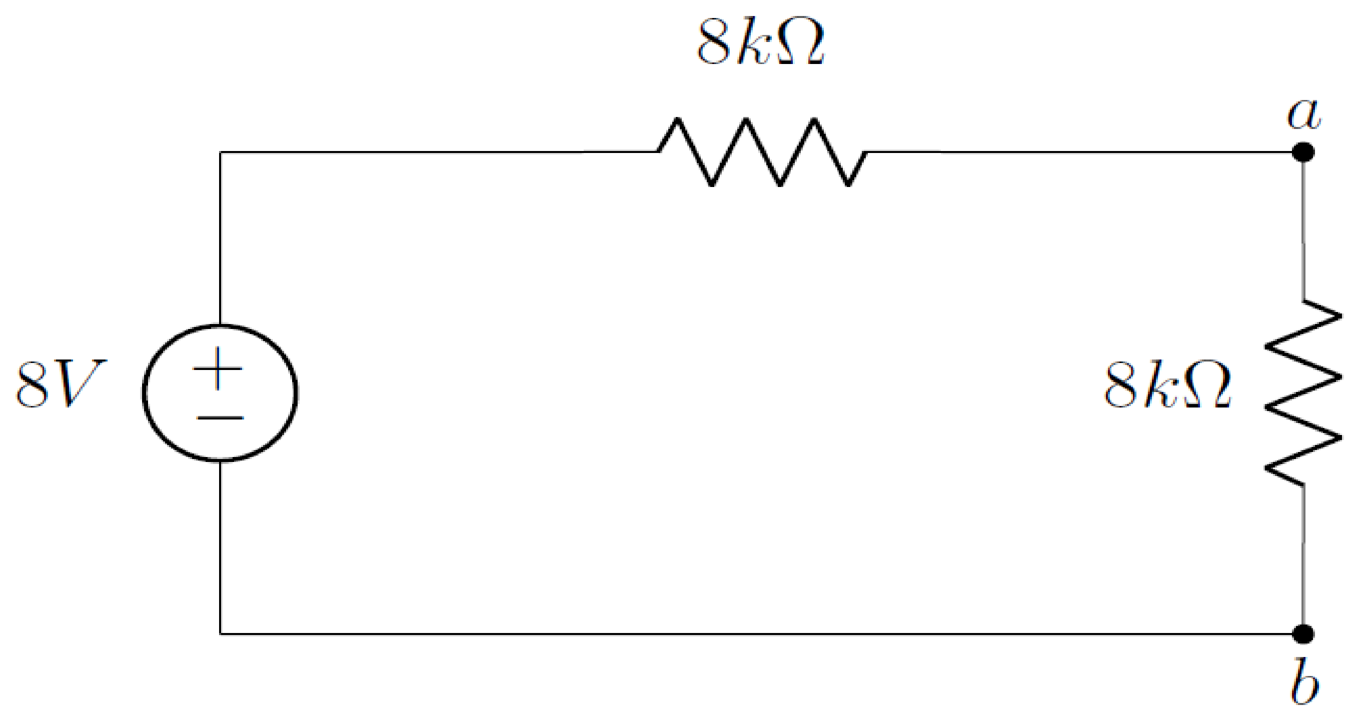
Using KVL, we have

$$v_{TH} = v_a - v_b = 2mA \text{ x } 2k\Omega + 4\text{V} = 8\text{V}.$$ [1 mark]

For maximum power transfer, we have

$$R_L = R_{TH} = 8K\Omega$$ [1 mark]

The equivalent circuit for the maximum power transfer is shown below.

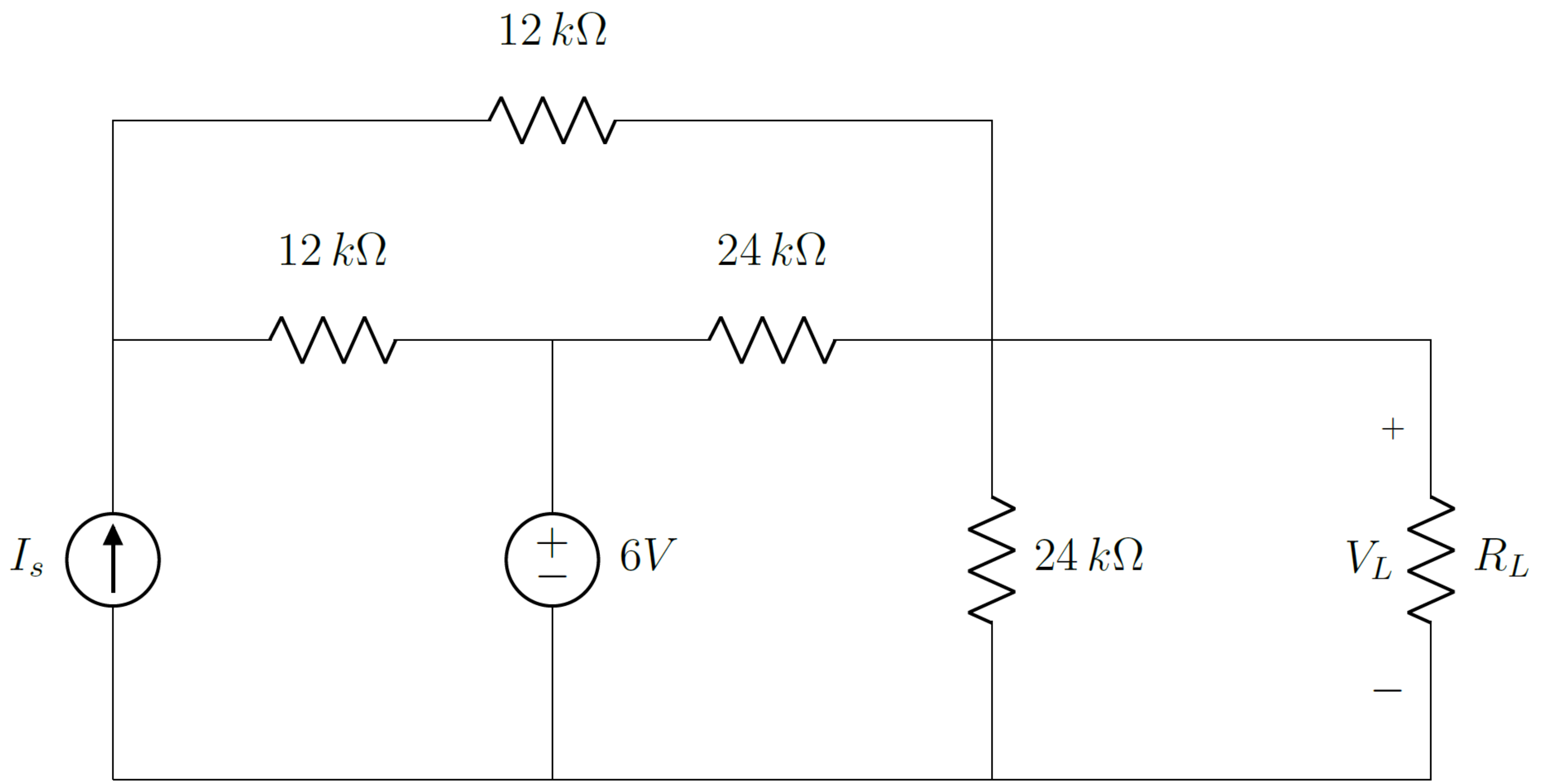


The maximum power is computed as

$$P_{max} = \frac{v_{TH}{}^2}{4R_{TH}} = \frac{64}{32x10^3} = 2mW.$$

[2 marks]

Q3: In the following circuit, determine the value of I_s which would make the output voltage $V_L = 0$ volts for any value of the resistor R_L

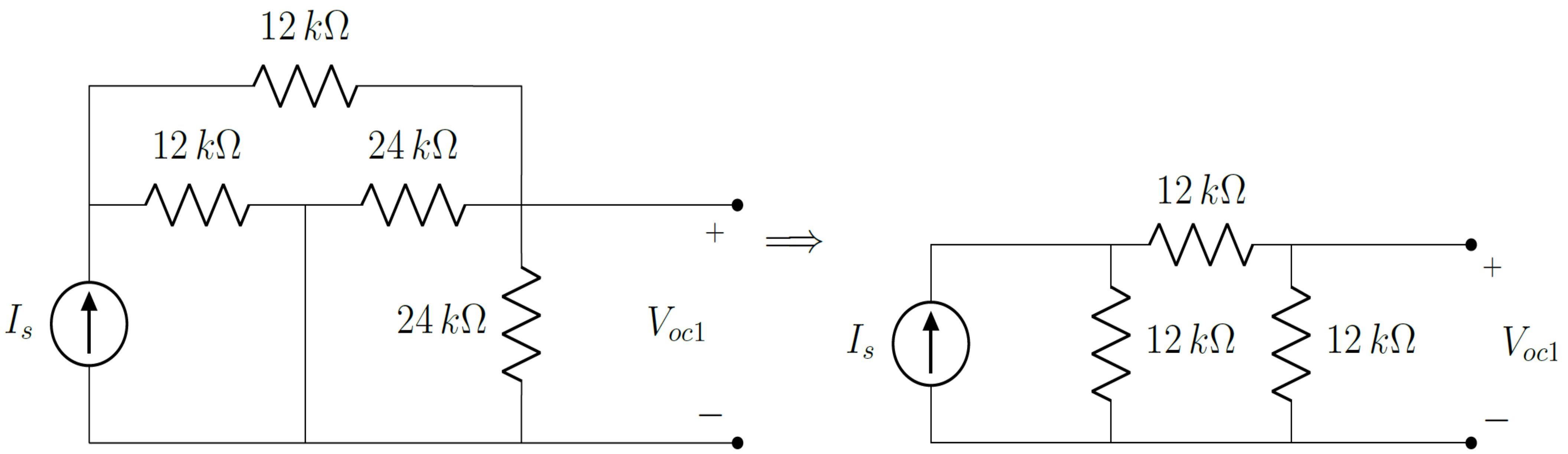


Solution:

We can determine the Thevenin's equivalent of the circuit across R_L . To satisfy the condition given in the question , v_{TH} must be 0V. [1 mark]

Using superposition theorem, we can find V_{TH} .

Case 1: When only source I_s is considered, we have



From the circuit shown in the previous slide , we have

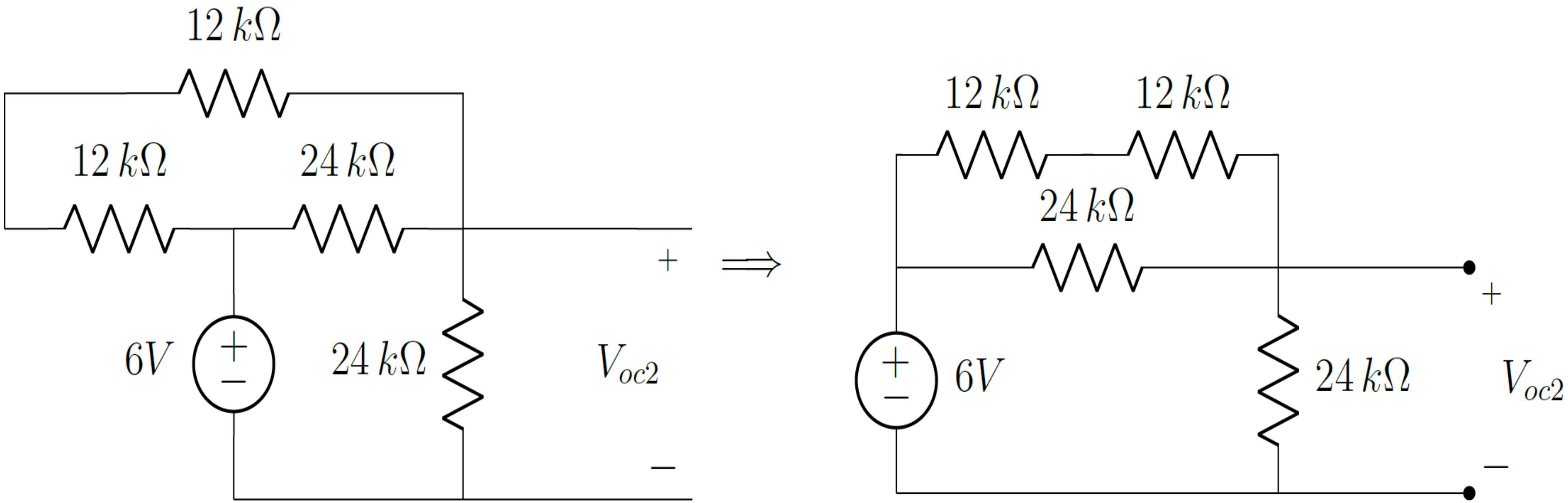
$$V_{oc1} = I_s \text{ x } (12K\Omega || (12K\Omega + 12K\Omega))$$

$$= I_s \text{ x } \frac{12K\Omega \text{ x } 12K\Omega}{12K\Omega + 12K\Omega} = 4I_s.$$

[1 mark]

Assuming I_s is in mA.

Case 2: When only 6V source is considered , we have



From the circuit , we have

$$V_{oc2} = \frac{6V}{((12K\Omega+12K\Omega)||24K\Omega)+24K\Omega} \text{ x } 24K\Omega$$

$$= 4V.$$

[1 mark]

Finally, from superposition, we have

$$V_{TH} = V_{oc1} + V_{oc2} = 4I_s + 4$$

[1 mark]

Since , V_{TH} must be zero, we have

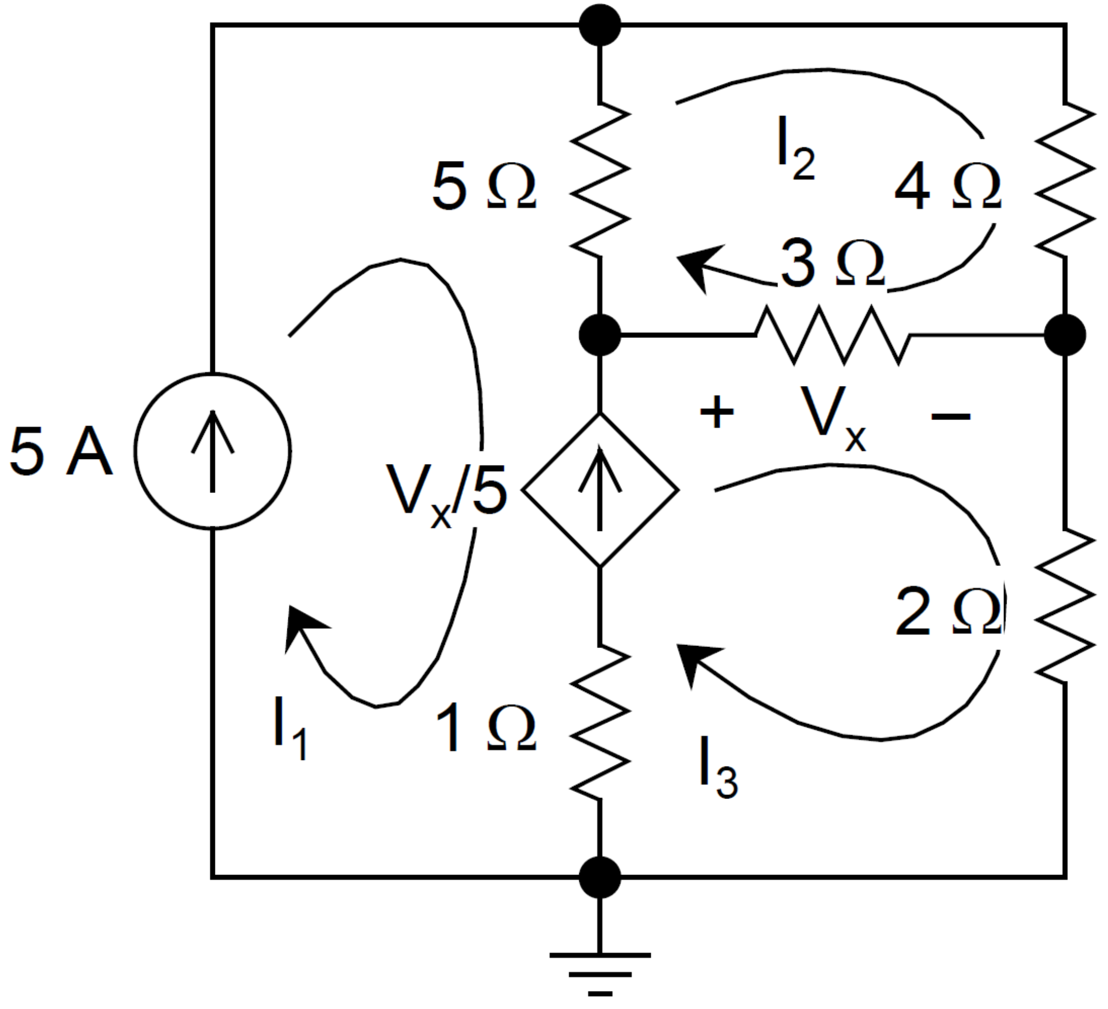
$$4I_s + 4 = 0$$

Or

$$I_s = -1\text{mA}.$$

[1 mark]

Q4: For the circuit shown below, calculate the mesh currents I_1, I_2 and I_3.



Solution:

$$\mathrm{I_1} = 5\mathrm{A}, \text{and mesh equations for } \mathrm{I_2} \text{ and } \mathrm{I_3} \text{ cannot be written. [1 mark]}$$

Note: $I_3 - I_1 = \frac{V_x}{5}$ [1 mark]

Also, $V_x = 3(I_3 - I_2)$ [1 mark]

$I_3 - 5 = \frac{3}{5}(I_3 - I_2)$

$3I_2 + 2I_3 = 25$ ....(1) [1 mark]

Around the I_2 mesh :

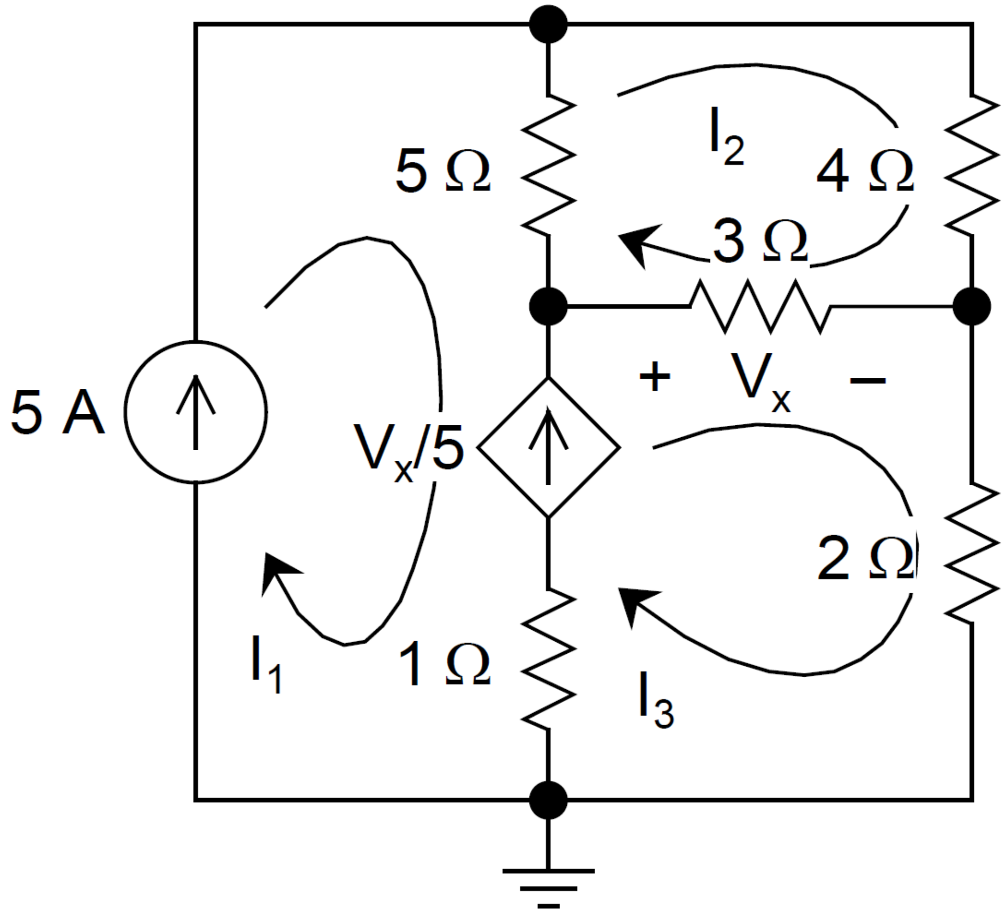
$5(I_2 - I_1) + 4I_2 + 3(I_2 - I_3) = 0$

$12\ I_2 - 3I_3 = 25$ ......(2) [2 marks]

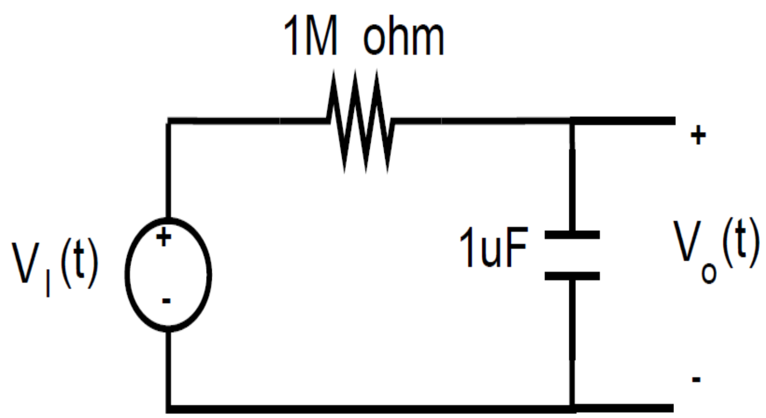
On solving (1) and (2) , we get

$I_2 = 3.788A$ and $I_3 = 6.818A$

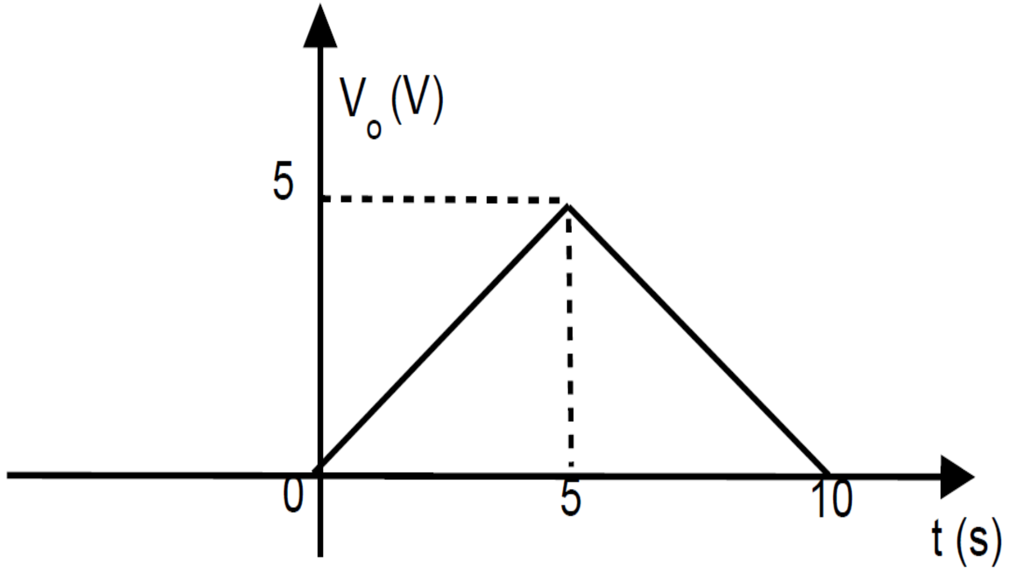
[2 marks] [2 marks]



Q5: You are given an RC circuit as shown in Figure below. Suppose you observe that V_0(t) is a triangular pulse as shown in Figure. Find and draw the waveform V_I(t) that must be applied to produce this output signal. Label time, magnitude and other significant parameters of the function. (Capacitance is 1μF, resistance is 1 MΩ)



(a)



(b)

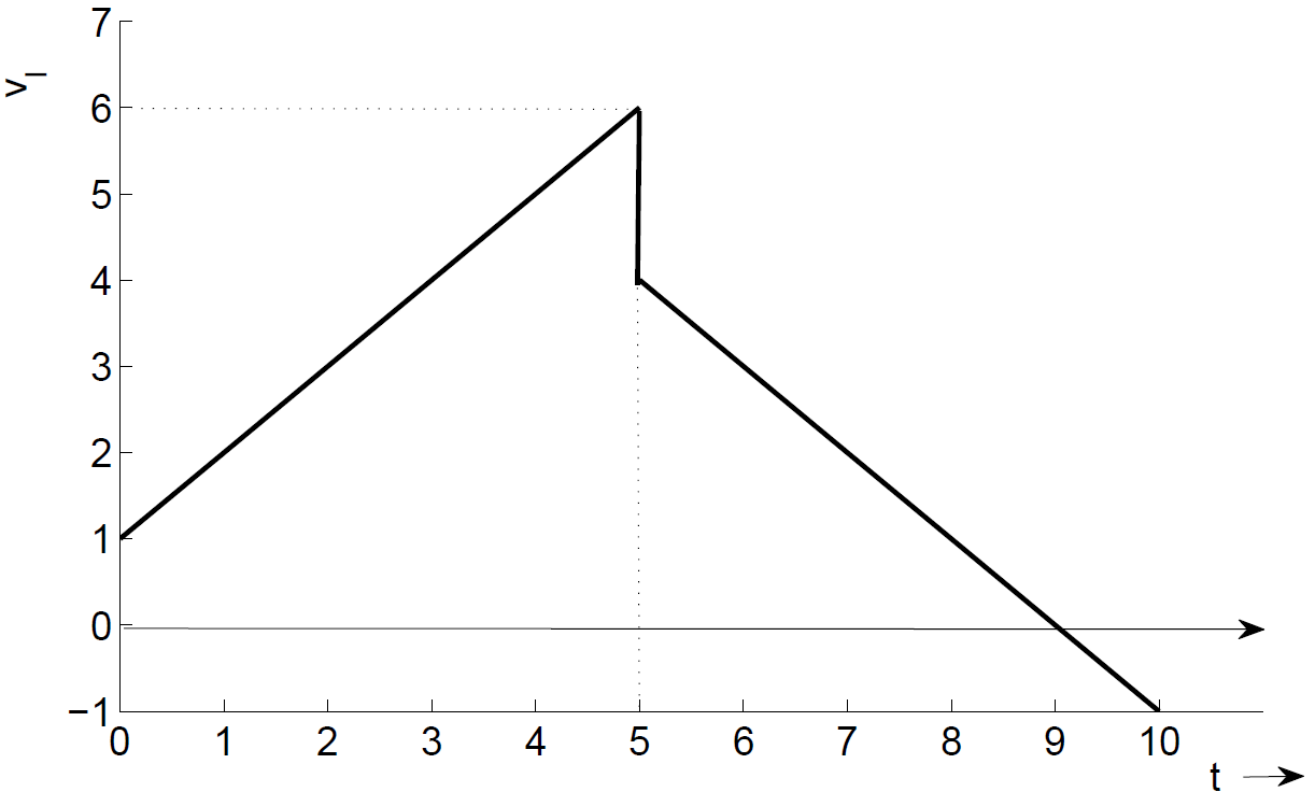
We have $V_I = RC\frac{dV_0}{dt} + V_0,$ [1 mark]

$RC = 1sec$ [1 mark]

For 0 < t < 5, we have $V_0 = t \implies V_I = t + 1$ [1 mark]

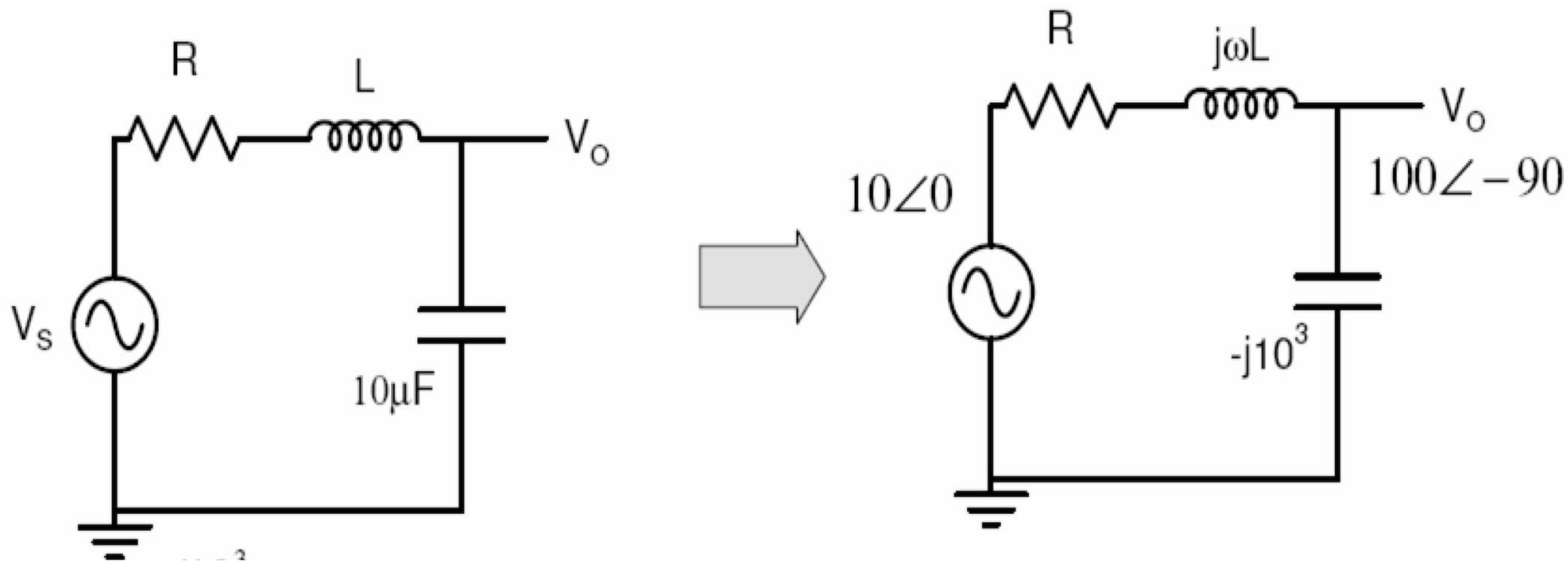
For 5 < t < 10, we have $V_0 = 10 - t \implies V_I = 9 - t$ [1 mark]

$V_I(t)$ is



[1 mark]

Q6: Ira applied an input voltage $Vs = 10V\ \cos(100t)$ as an input to a series RLC circuit and claimed that she measured $V_0 = 100\ V\ \sin(100t)$ across the capacitor of $10\ \mu F$. Is that possible? If so, determine suitable values for inductor and resistor for such a condition to occur? What should be the colour code of the resistor if it has a tolerance of $\pm 5\%$.



$V_0 = \frac{-j10^3}{R+j\omega L-j10^3}$ x 10∠0 = 100 ∠-90^0 [1 mark]

$\Rightarrow \frac{-j10^3}{R+j\omega L-j10^3}$ = -10j

$\Rightarrow \frac{-j10^3}{R}$ = -10j

$\Rightarrow$ R = 100Ω

[1 mark]

$j\omega L - j10^3 = 0$ [1 mark]

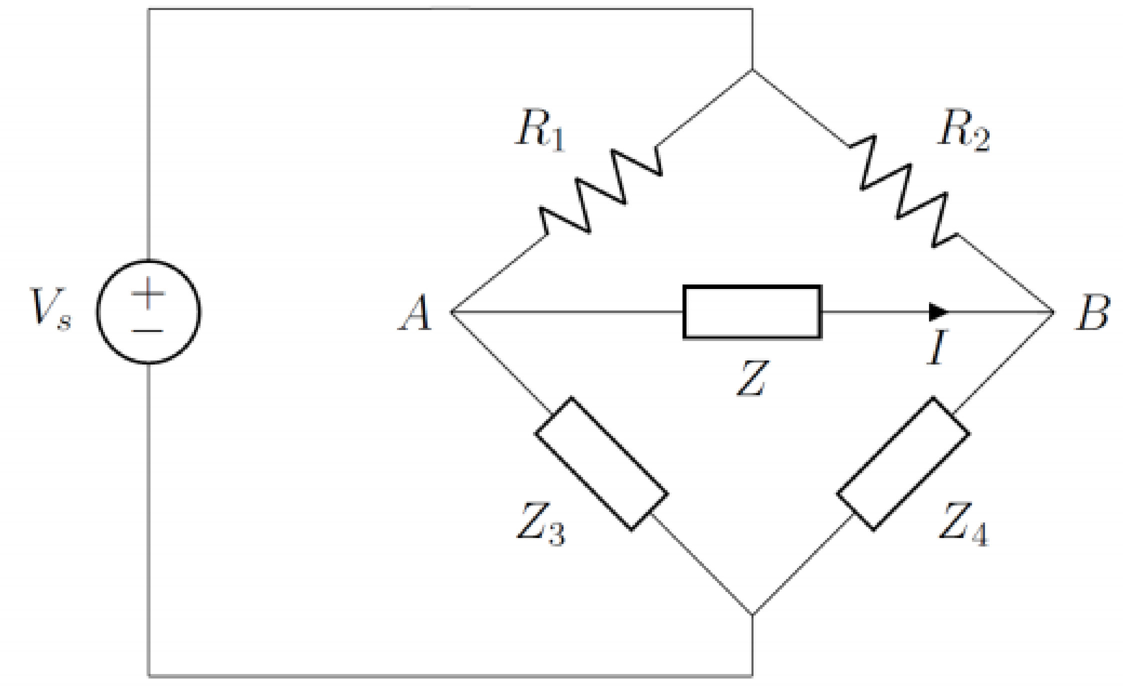
L = 10H

[1 mark]

Resistance should be color coded as Brown Black Brown Gold [1 mark]

In a resistive series circuit, one would expect voltage across any resistor to be less than the source voltage. However, due to negative impedance of the capacitor this does not holds for circuits containing capacitors and inductors. Hence voltage across a series element can be larger than source voltage !

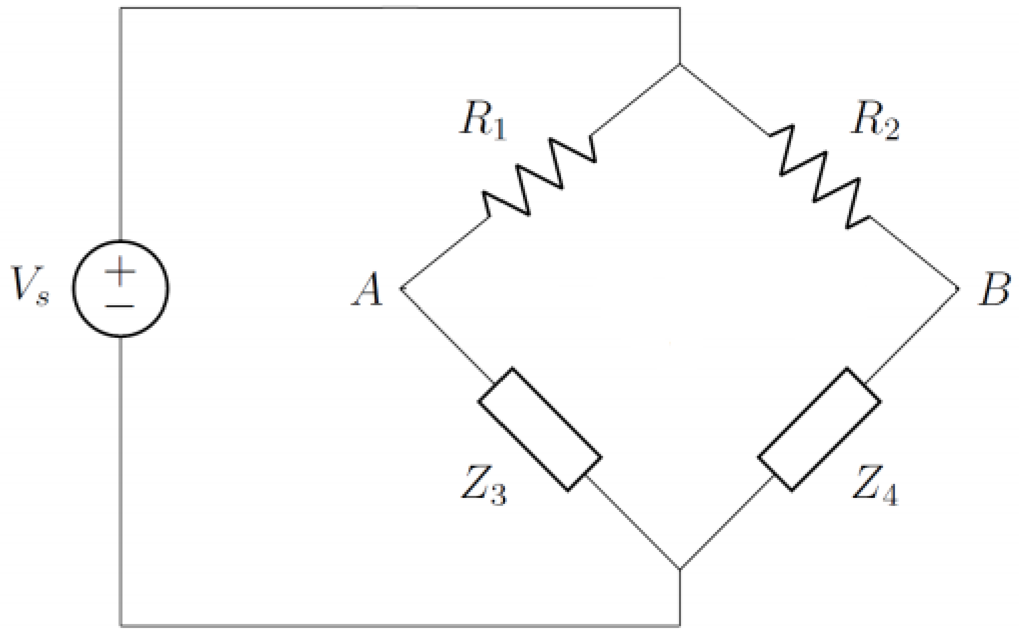
Q7: In the circuit shown below, $V_s = 25\angle 0$ V , R_1 = 25Ω , R_2 = 20Ω, Z_3 = (15 + j30) Ω and Z_4 = (20 + j30) Ω. If the current through Z from A to B is I = (0.12 − j0.16) A, find the value of the unknown impedance Z .



Solution:

First, we will find the Thevenin Equivalent model of the circuit between the terminals A and B.

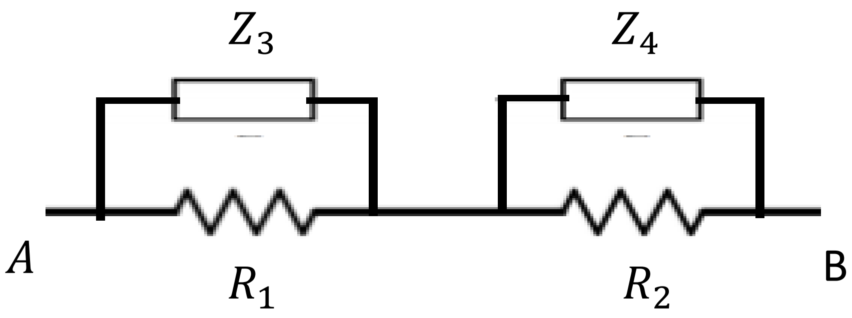
Open circuit voltage V_{AB} can be obtained from the following circuit.



$$V_{TH} = V_{AB} = V_A - V_B$$
$$= V_S \frac{Z_3}{R_1+Z_3} - V_S \frac{Z_4}{R_2+Z_4}$$
$$= V_S [\frac{15+j30}{40+j30} - \frac{20+j30}{40+j30}]$$
$$= V_S \left[\frac{-5}{40+j30}\right] = \frac{-V_S}{8+j6}$$
$$= \frac{25\angle 180}{10\angle 36.87} = 2.5 \angle 143.13$$
$$= -2 + j1.5$$

[2 marks]

Now let us calculate the Thevenin's Impedance.



$$Z_{TH} = R_1||Z_3 + R_2||Z_4$$

$$= \frac{25 \text{ x } (15+j30)}{40+j30} + \frac{20 \text{ x } (20+j30)}{40+j30}$$

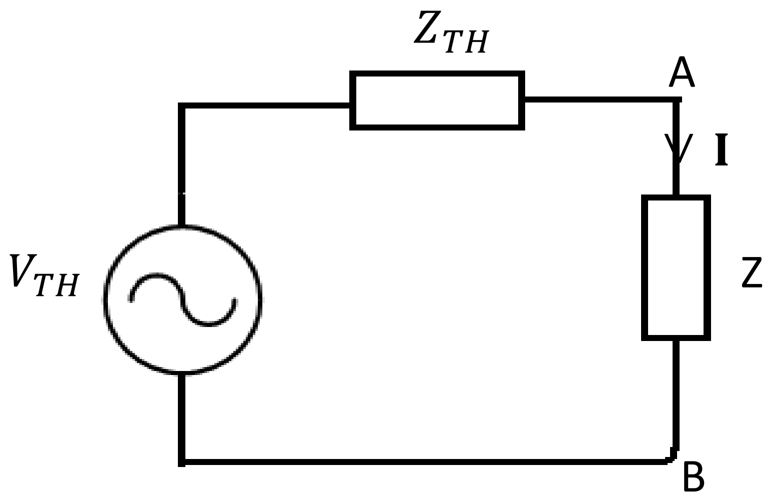
$$= \frac{(775+j1350)}{40+j30} = \frac{1556.64\angle 60.14^0}{50\angle 36.87^0}$$

$$= 31.12 \angle 23.27^0$$

$$= 28.6 + \text{j}12.3$$

[2 marks]

The Thevenin's equivalent circuit can be written as:



Current through the unknown impedance is given by

$$I = \frac{V_{TH}}{Z + Z_{TH}} \Rightarrow Z = \frac{V_{TH}}{I} - Z_{TH}$$ [2 marks]

$$Z = \frac{-2+j1.5}{0.12-j0.16} - (28.6 + j12.3)$$

$$= \frac{2.5\angle 143.13}{0.2\angle -53.13} - (28.6 + j12.3)$$

$$= 12.5\angle 196.26 - (28.6 + j12.3)$$

$$= 12 - j3.5 - 28.6 - j12.3$$

$$= -40.6 - j15.8$$

$$= 43.566\angle 201.264^0$$ [4 marks]

Q8: Draw the Bode plot (magnitude and phase response) of the transfer function given by

$$H(\omega) = \frac{100(j\omega)^2}{(10000 + j\omega)(1 + \frac{j\omega}{1000})}$$ in semi log scale ($1 \leq \omega \leq 10^5$).

First plot the individual contributions of each term before plotting the combined response. Label the graph properly, clearly labelling the corner frequencies.

Solution:

$$H(\omega) = \frac{100(j\omega)^2}{(10000 + j\omega)(1 + \frac{j\omega}{1000})}$$

$$= \frac{(j\omega)^2}{100(1 + \frac{j\omega}{10^4})(1 + \frac{j\omega}{1000})}$$

Magnitude: $20\log|H(\omega)|$

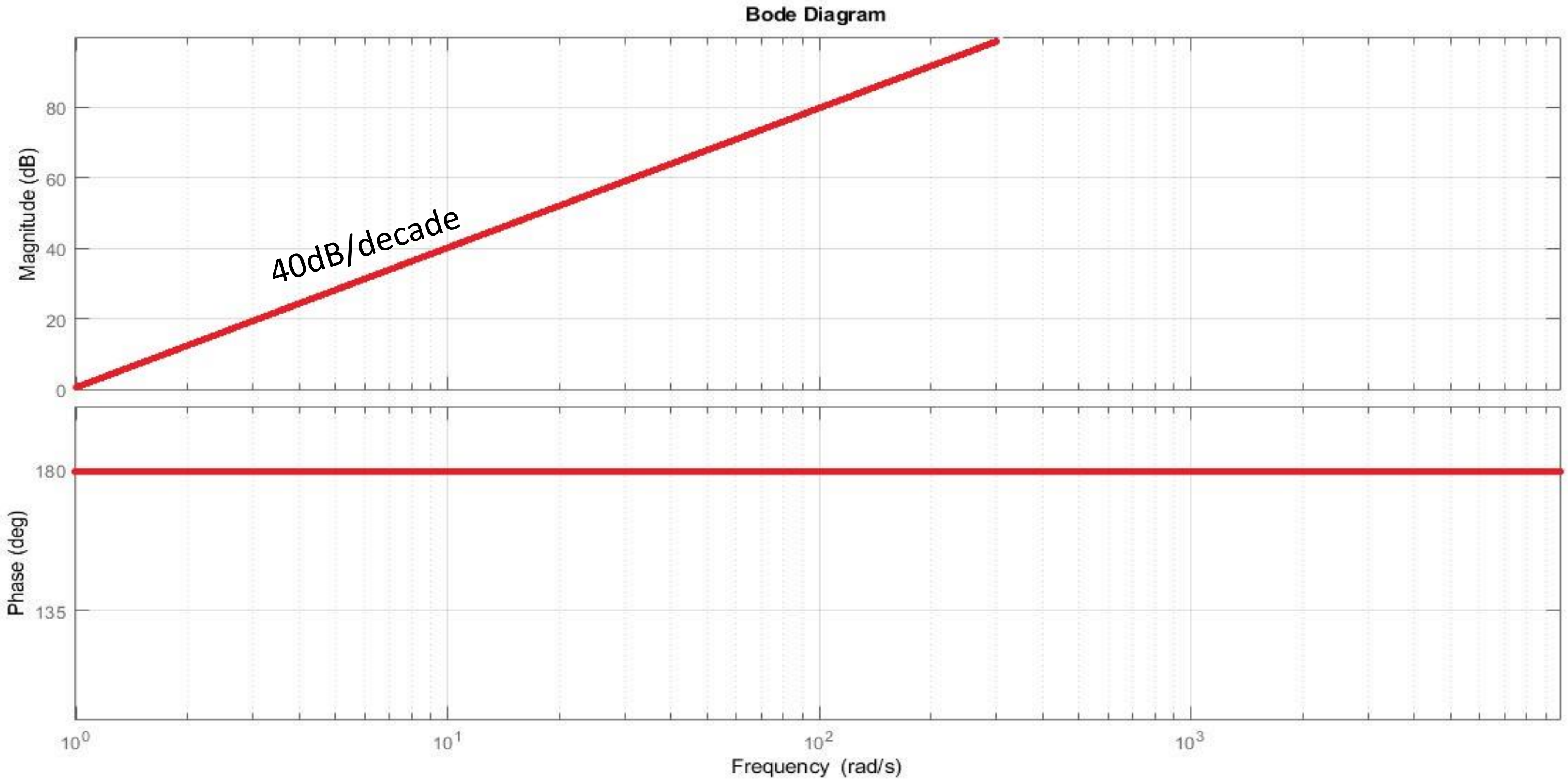
=> $20\log\omega^2 - 40 - 10\log(1 + (\frac{\omega}{10^4})^2) - 10\log(1 + (\frac{\omega}{10^3})^2)$

Phase: $\angle H(\omega)$

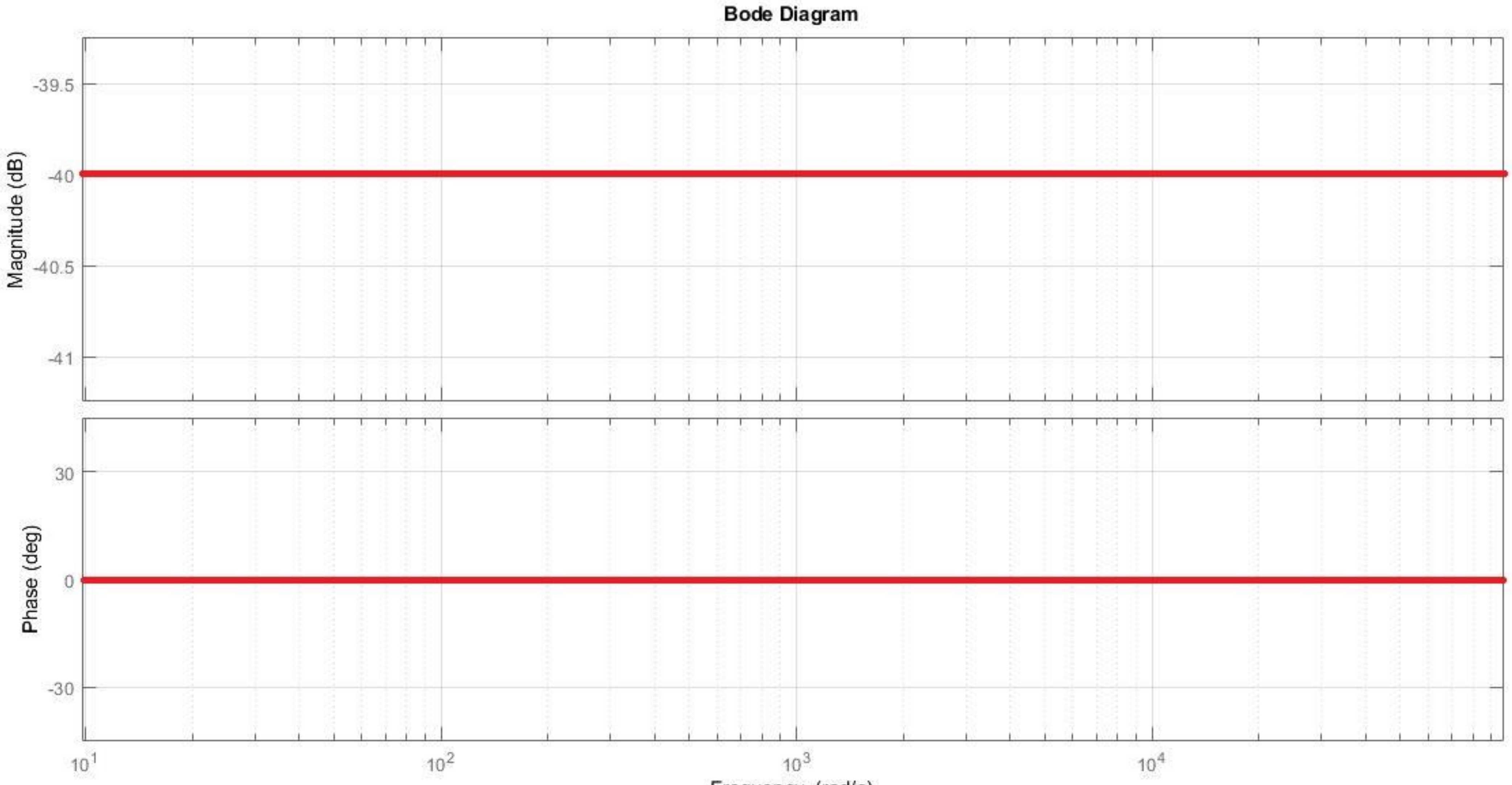
=> $180^0 - \tan^{-1}\frac{\omega}{10^4} - \tan^{-1}\frac{\omega}{10^3}$

Contribution of the term $(j\omega)^2$

[1 mark each for magnitude and phase plot]

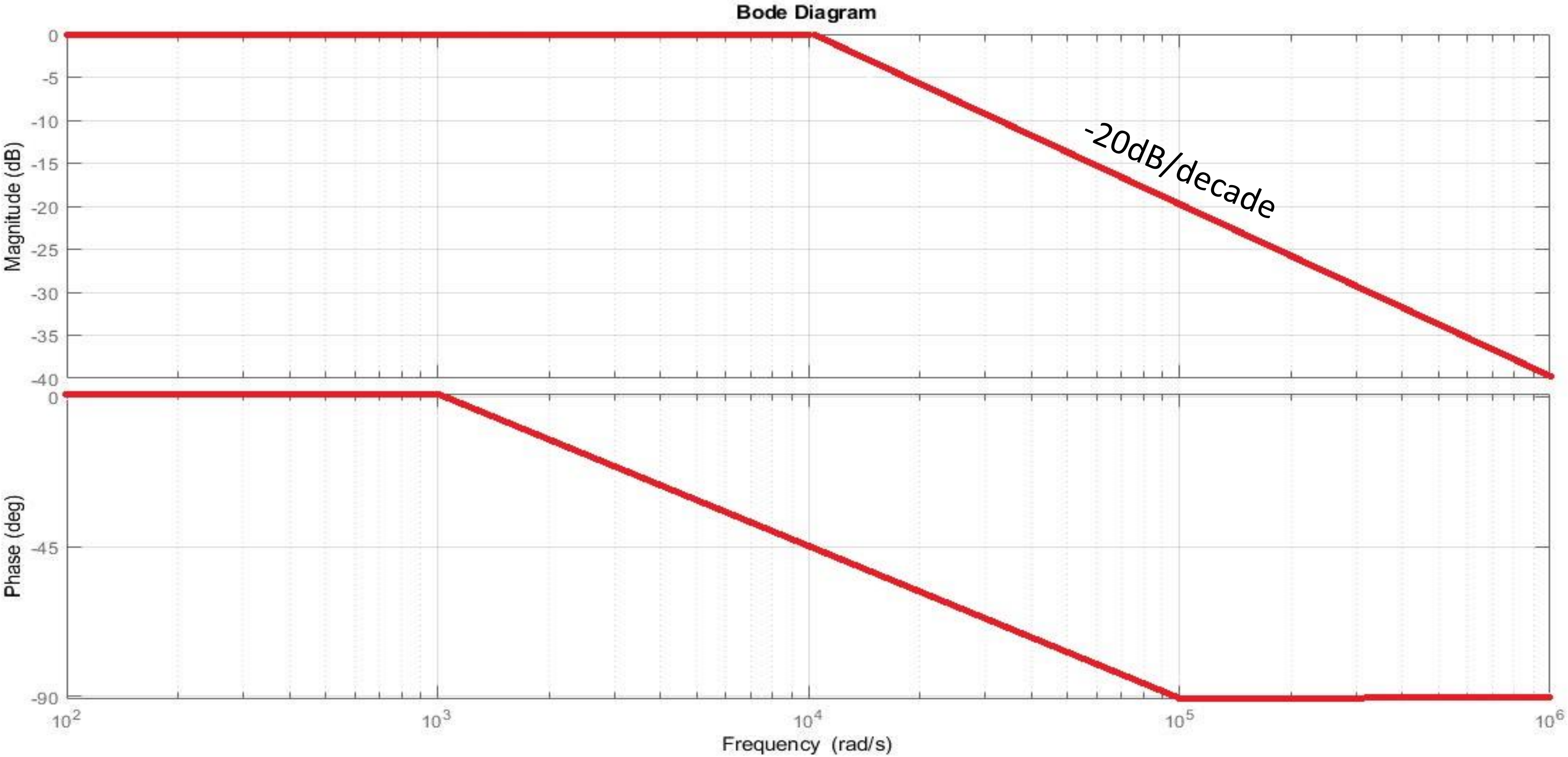


Contribution of the term $\frac{1}{100}$

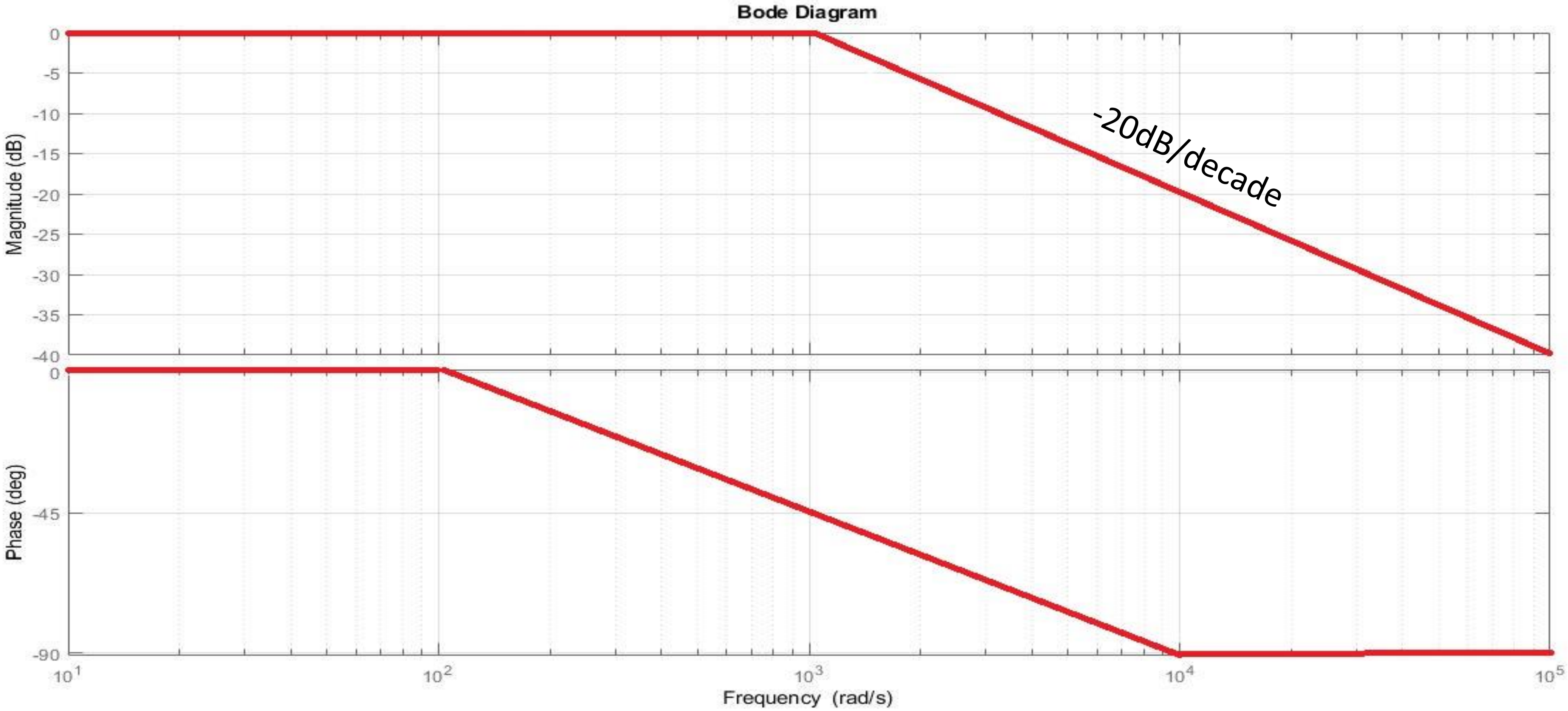


Contribution of the term $\frac{1}{(1+\frac{j\omega}{10^4})}$

[1 mark each for magnitude and phase plot]

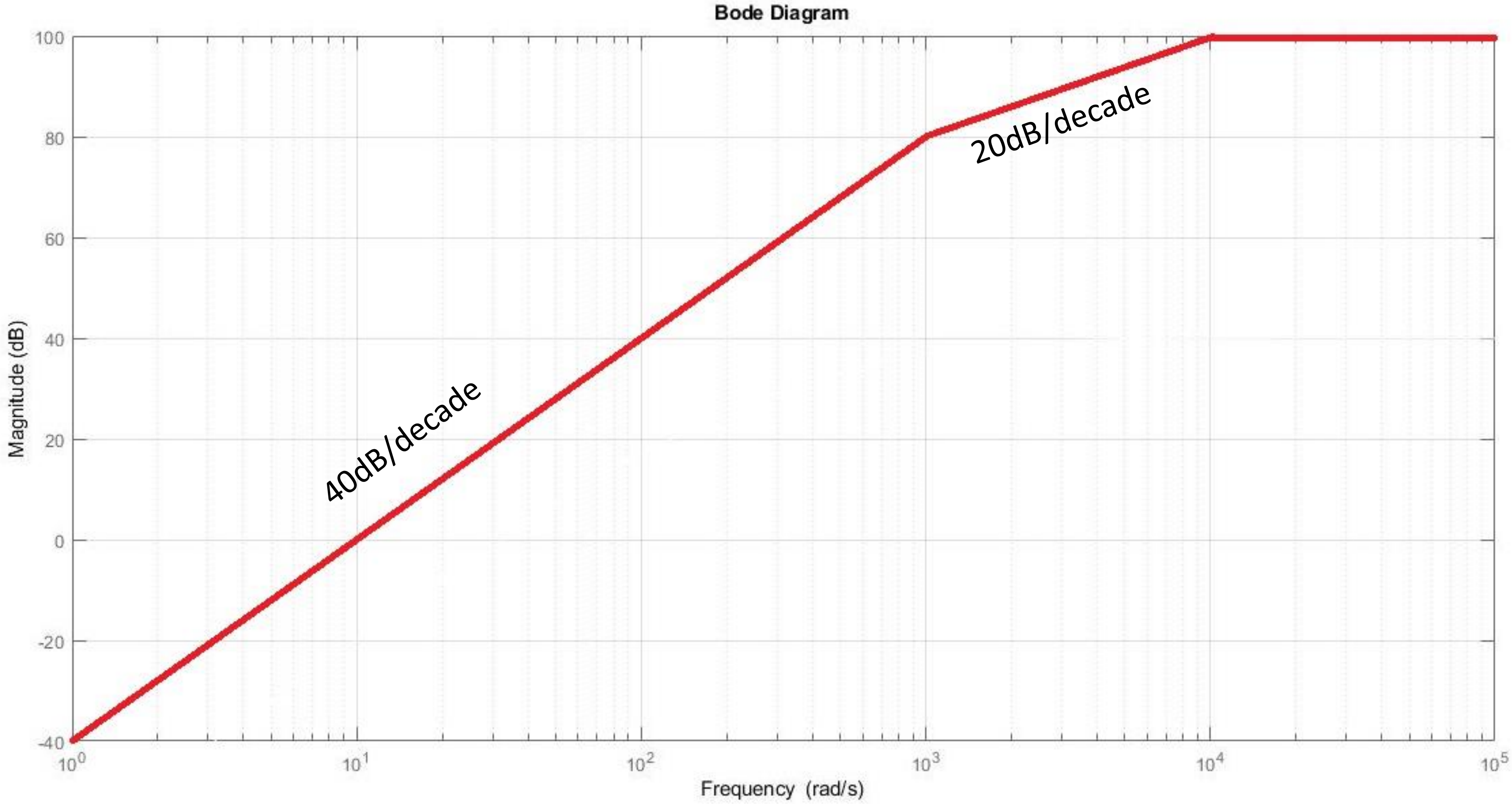


Contribution of the term $\dfrac{1}{(1+\frac{j\omega}{1000})}$ [1 mark each for magnitude and phase plot]



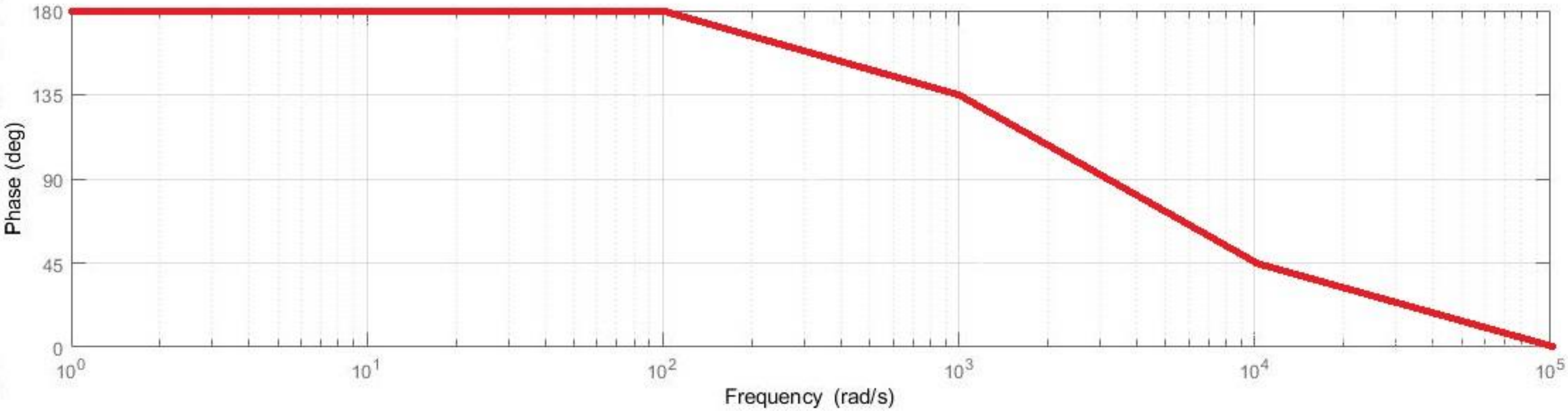
Overall Bode Magnitude plot

[2 mark]



Overall Bode Phase plot

[2 mark]



Q8: Draw the Bode plot (magnitude and phase response) of the transfer function given by

$H(\omega) = \dfrac{100(j\omega)^2}{(10000 + j)(1 + \frac{j}{1000})}$ in semi log scale ($1 \leq \omega \leq 10^5$).

First plot the individual contributions of each term before plotting the combined response. Label the graph properly, clearly labelling the corner frequencies. (If ω is not considered in the denominator)

Solution:

$$H(\omega) = \frac{100(j\omega)^2}{(10000 + j)(1 + \frac{j}{1000})}$$

$$= \frac{100(j\omega)^2}{10000 + j10 + j + \frac{j^2}{1000}}$$

$$= \frac{100(j\omega)^2}{10^4 + j11}$$

$$= \frac{100(j\omega)^2}{10^4\angle 0.063} = \frac{(j\omega)^2}{100\angle 0.063}$$

<u>Magnitude</u>: 20log|H(ω)|

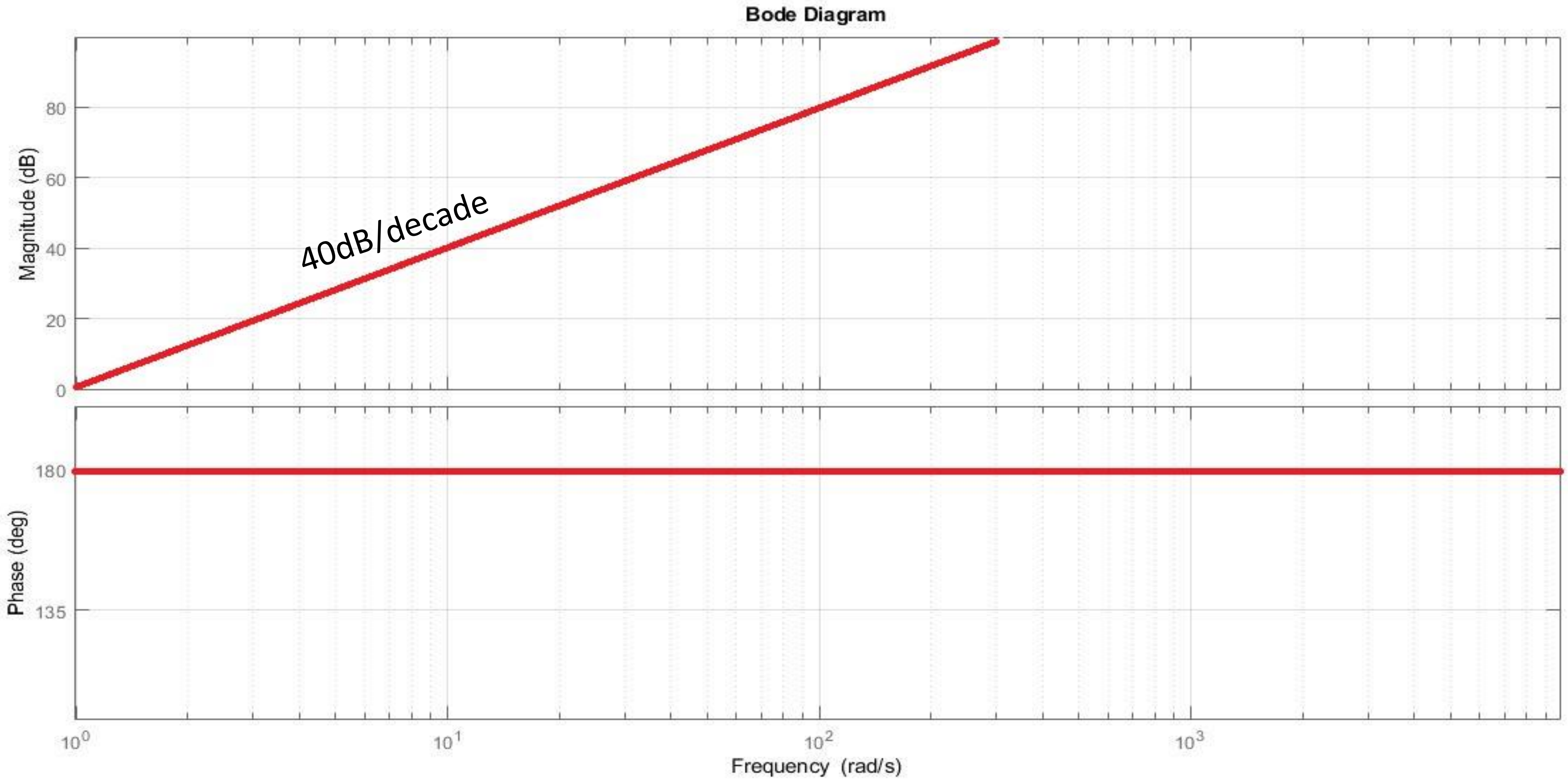
=> 20log$\omega^2 - 40$

<u>Phase:</u> ∠H(ω)

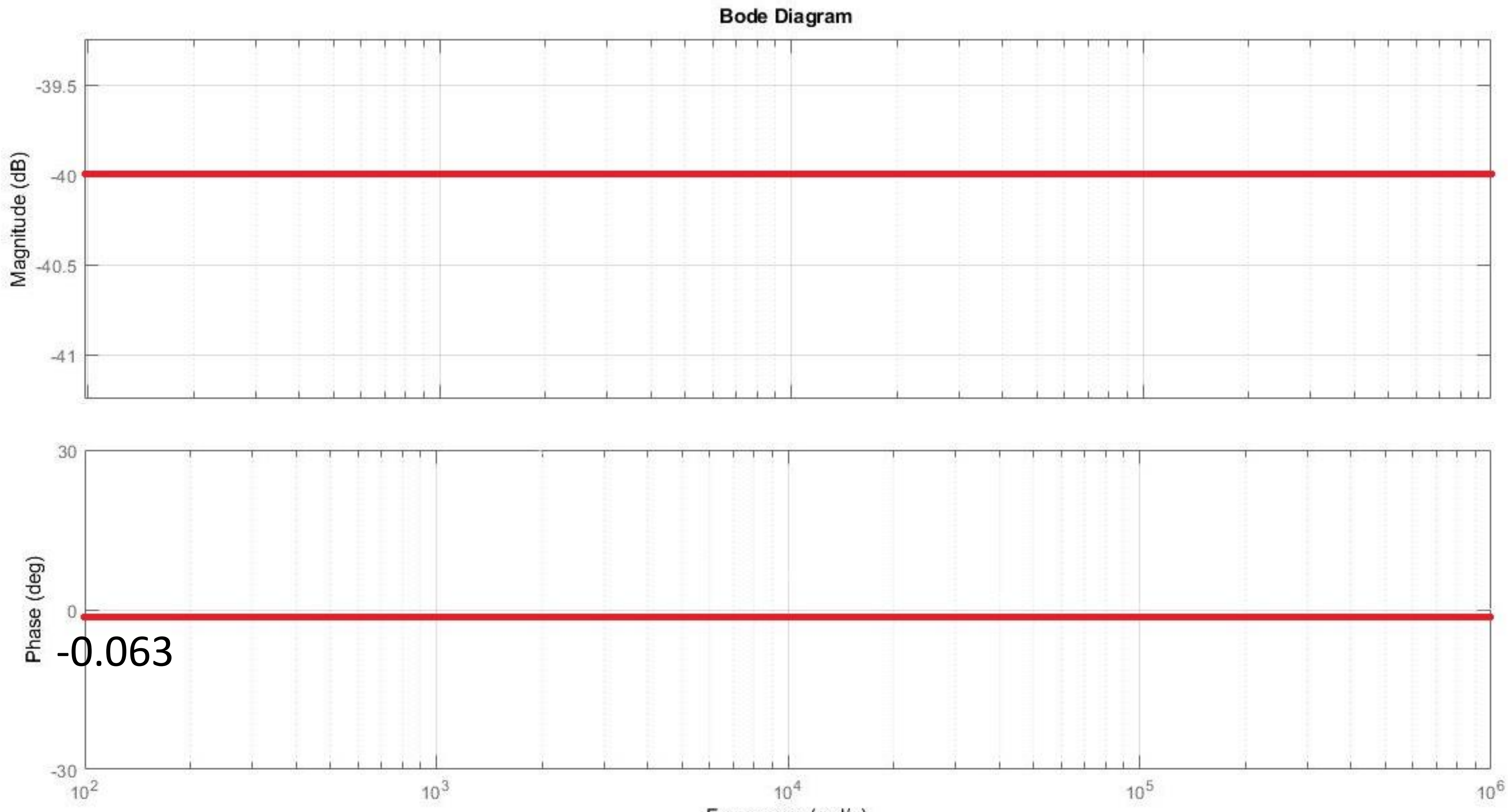
=> $180^0 - 0.063 = 179.937$

Contribution of the term $(j\omega)^2$

[1 marks each for magnitude and phase plot]

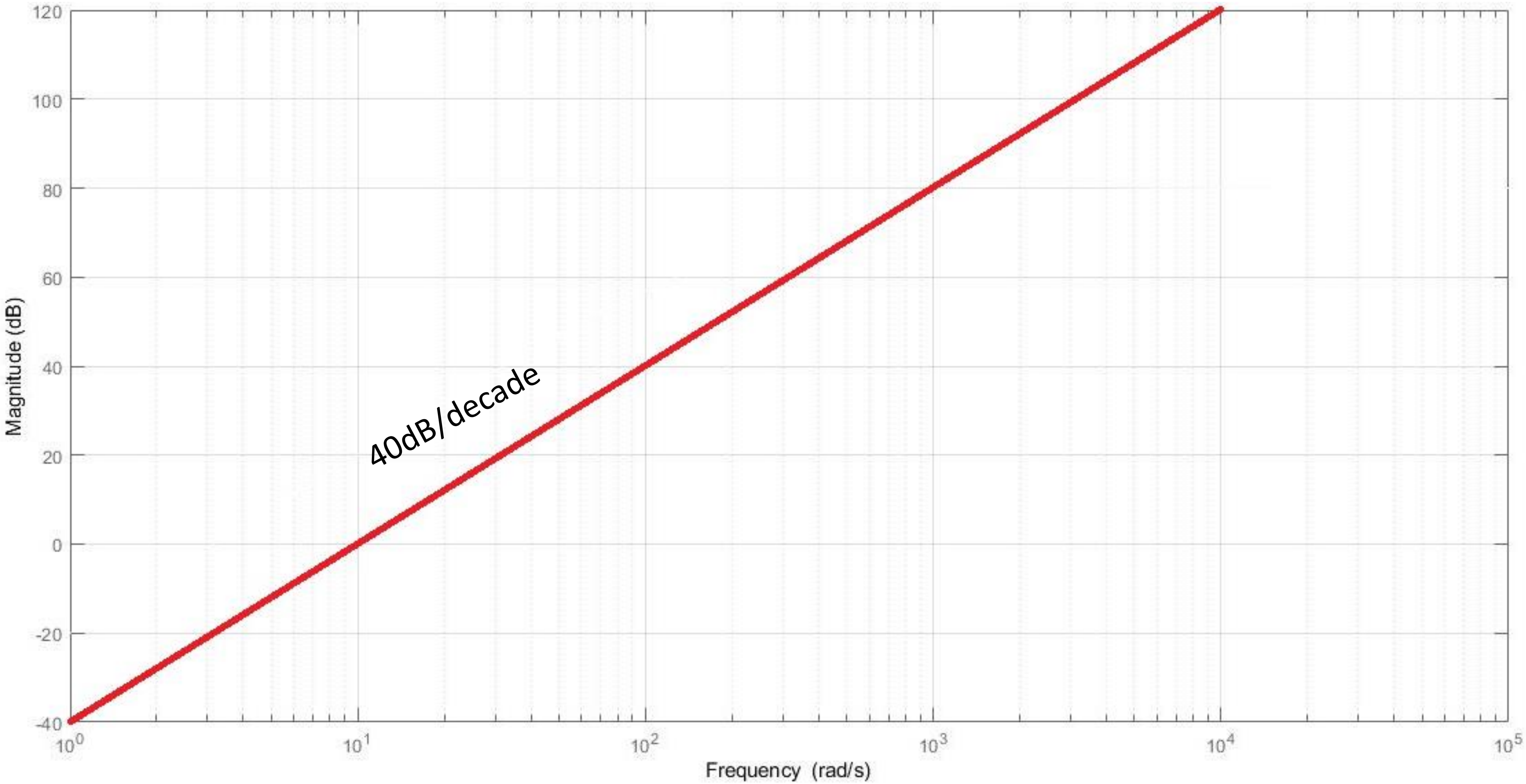


Contribution of the term $\frac{1}{100\angle 0.063}$ [2 mark each for magnitude and phase plot]

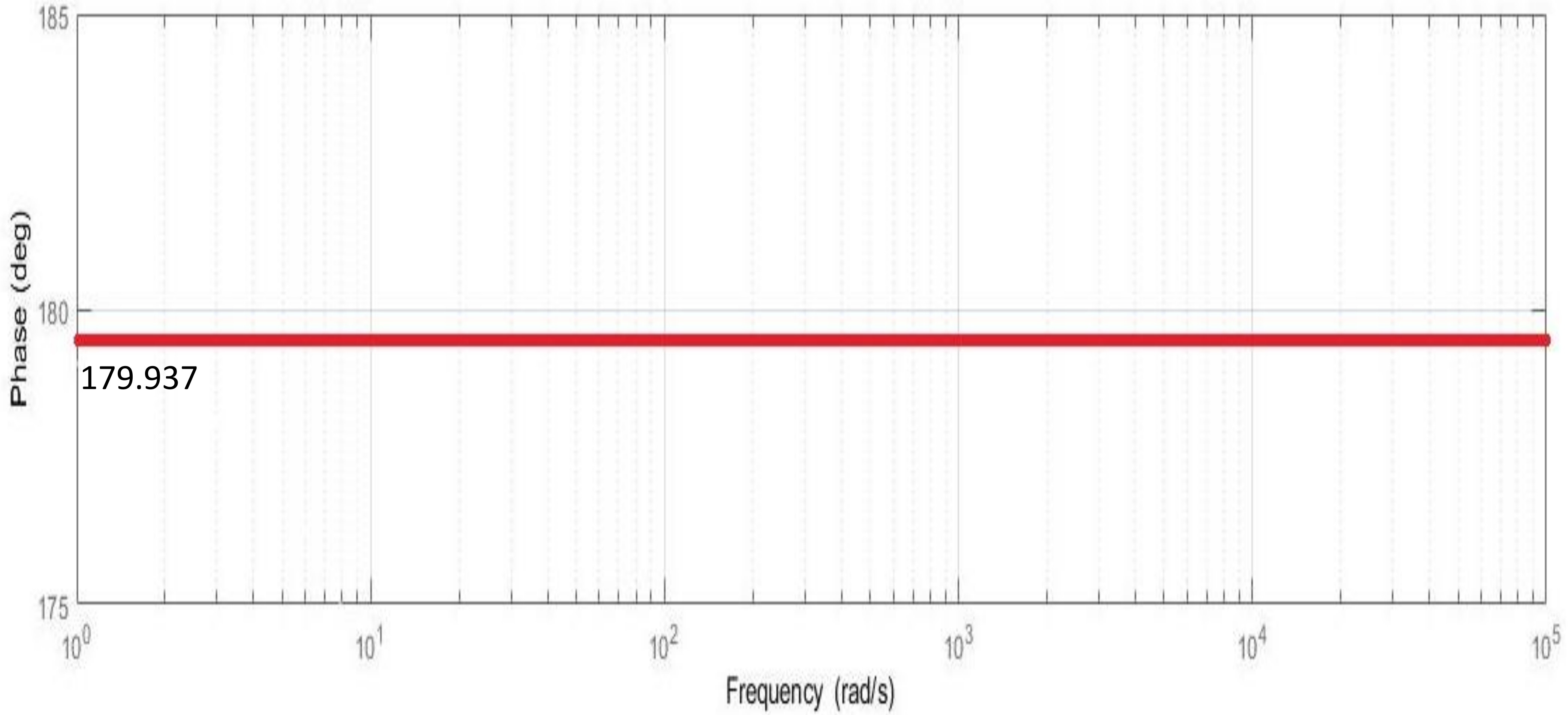


Overall Bode plot

[2 marks]



[2 marks]


185
180
175
Phase (deg)
179.937
10^0
10^1
10^2
10^3
10^4
10^5
Frequency (rad/s)