Example problem :

A piston-cylinder device contains helium gas initially at 100 kPa, 10^0C and 0.2 m^3. The Helium is now compressed in a polytropic process to 700 kPa and 290^0C. Determine the heat loss or gain during this process.

Data for Helium : R = 2.077, Cv = 3.1156 (at 300 K) from Table A-2(a)

Based on initial and final conditions, polytropic index can be obtained as

$$n = 1.5464$$

The work done in compression can be obtained as : $W_{in} = 36.19$ kJ

From the initial and final conditions, change in internal energy is calculated as

$$\Delta U = 29.66 \text{ kJ}$$

Applying 1^{st} law we get $Q_{out} = 6.53$ kJ (heat loss during the process)

Example problem:

A 0.3 L glass of water at 20^0C is to be cooled with ice to 5^0C. Determine how much ice needs to be added to the water, in grams, if the ice is at (a) 0^0C and (b) -20^0C. Also determine how much water would be needed if the cooling is to be done with cold water at 0^0C. The melting temperature and heat of fusion of ice at atmospheric pressure are 0^0C and 333.7 kJ/kg, respectively, and the density of water is 1 kg/L.

TABLE A–3

Properties of common liquids, solids, and foods

(*b*) Solids (values are for room temperature unless indicated otherwise)

Substance	Density, ρ kg/m^3	Specific heat, c_p kJ/kg·K
Ice		
200 K		1.56
220 K		1.71
240 K		1.86
260 K		2.01
273 K	921	2.11

<u>Note</u> : specific heat of water at 0 C is about 4.22 kJ/kg.K

Why is specific heat of ice lower than that of water ?

Example problem:

A 0.3 L glass of water at 20^0C is to be cooled with ice to 5^0C. Determine how much ice needs to be added to the water, in grams, if the ice is at (a) 0^0C and (b) -20^0C. Also (c) determine how much water would be needed if the cooling is to be done with cold water at 0^0C. The melting temperature and heat of fusion of ice at atmospheric pressure are 0^0C and 333.7 kJ/kg, respectively, and the density of water is 1 kg/L.

Application of first law yields : $Q_{in} = \Delta H = 0$

Based on the above equation, solutions for the three cases are obtained as follows :

(a) $m_{ice} = 53.5$ g

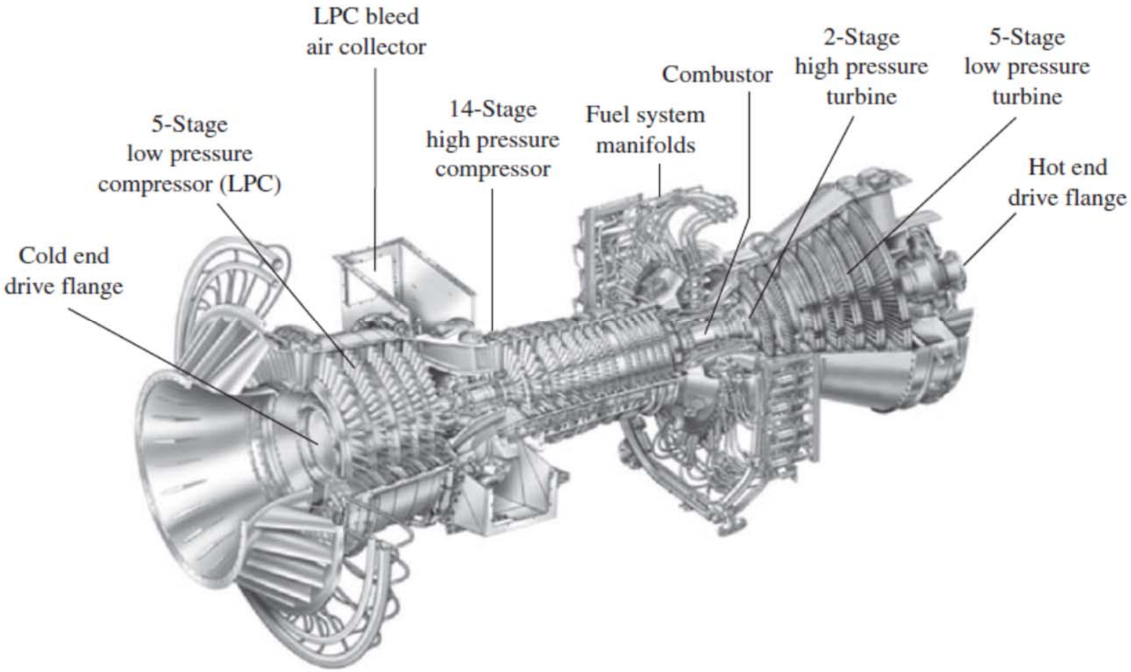
(b) $m_{ice} = 47.8$ g

(c) $m_{water} = 900$ g

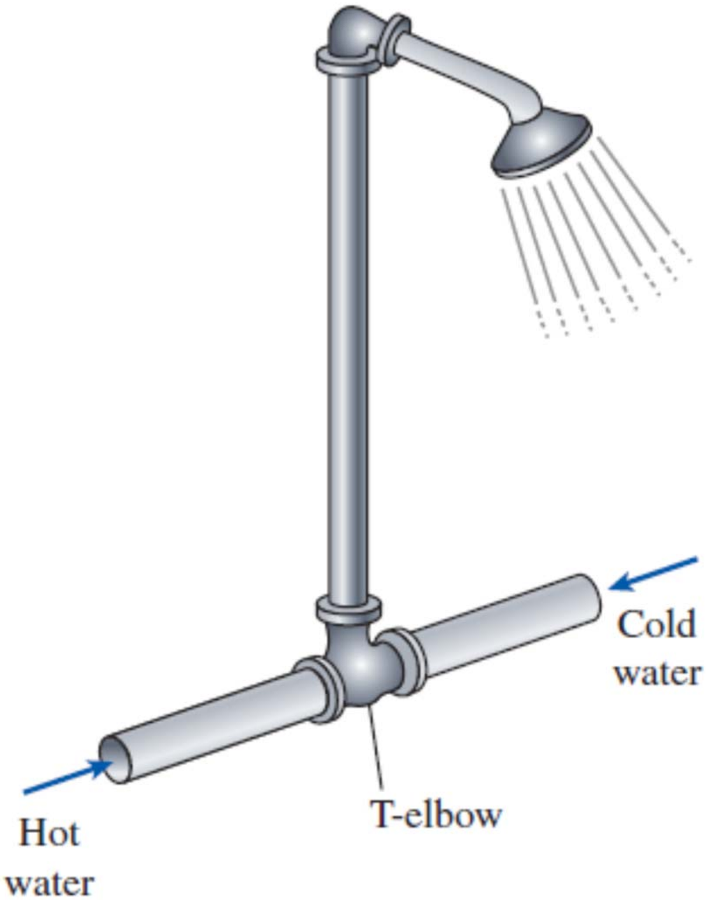
Note that comparing (a) and (c), almost 17 times water is needed for the same cooling load !!

Flow (open) systems :

Pressure cooker

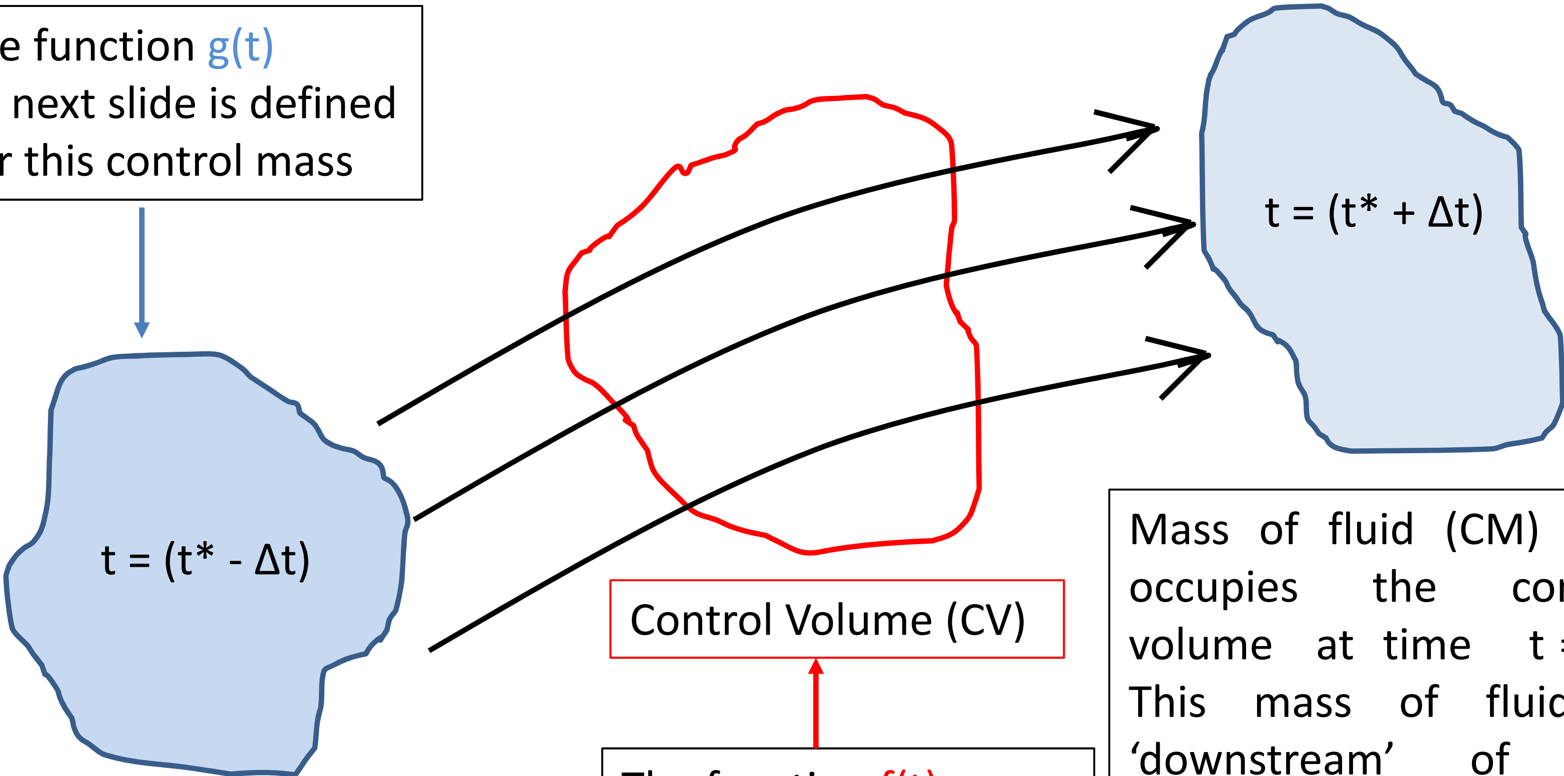


Flow (open) systems :



Reynolds Transport Theorem:

The function g(t)
on next slide is defined
For this control mass



Control Volume (CV)

The function f(t)
on next slide is defined
For this control volume

Mass of fluid (CM) that occupies the control volume at time t = t*. This mass of fluid is ‘upstream’ of the control volume at t = (t* - Δt), i.e., it has not yet entered the CV at t = (t* - Δt).

Mass of fluid (CM) that occupies the control volume at time t = t*. This mass of fluid is ‘downstream’ of the control volume at t = (t* + Δt), i.e., it has now come out of the control volume. Note that it has a different shape and volume than what it had at earlier times.

Reynolds Transport Theorem:

Consider a scalar quantity b which is a function of coordinates and time, i.e., b=b(x,y,z,t). As an example, this scalar quantity can be density of the fluid. Then we define two functions of time f(t) and g(t) as follows :

$$f(t) = \int_{CV} b(x, y, z, t)dV$$

$$g(t) = \int_{CM} b(x, y, z, t)dV$$

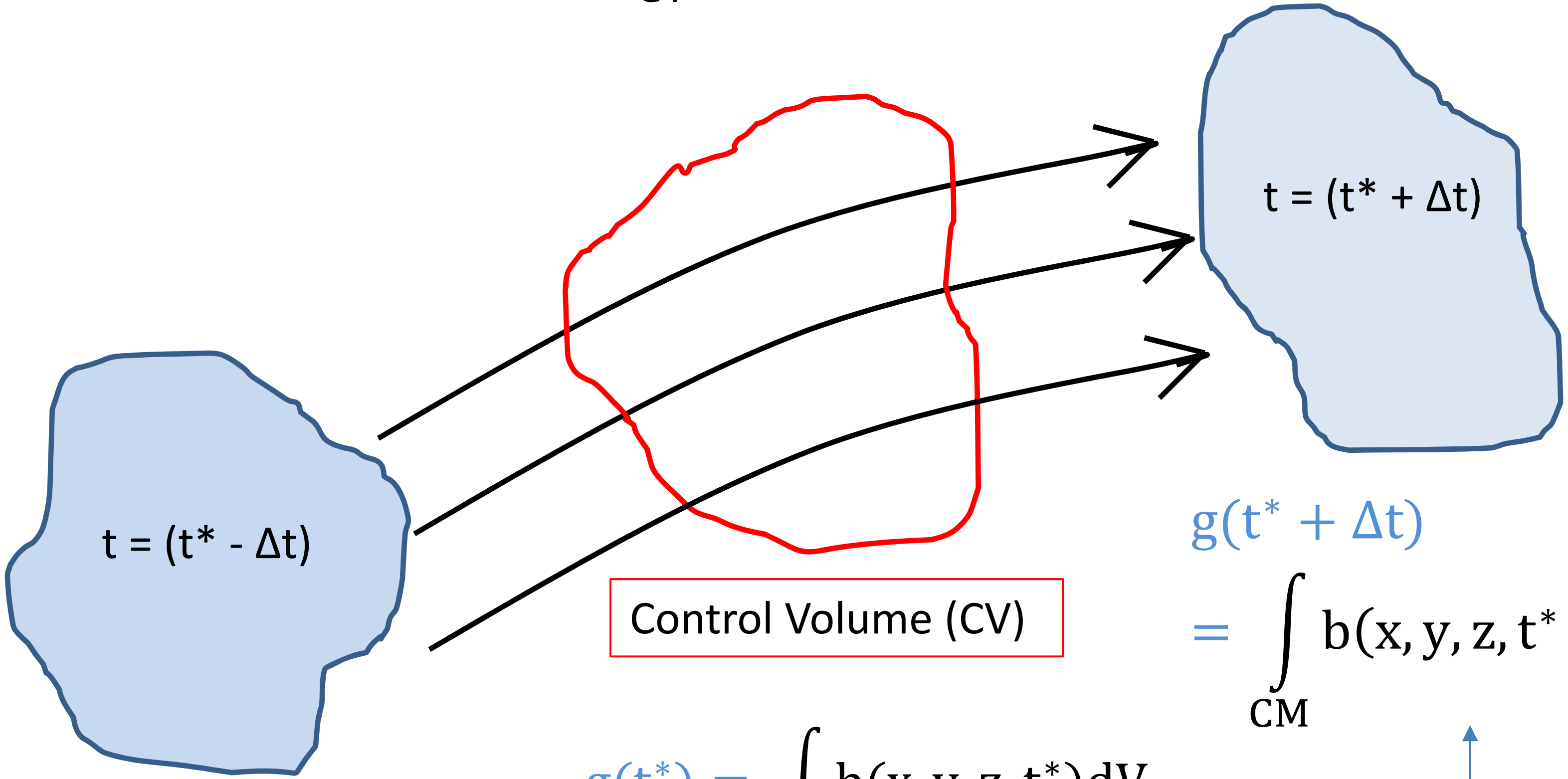
In defining f(t), integration is taken over the control volume (CV) [see figure in the last slide]

In defining g(t), integration is taken over the mass of fluid (CM) that occupies the control volume at t = t* [see last slide]

Reynolds Transport Theorem:

$$f(t) = \int_{CV} b(x, y, z, t)\,dV$$

The limits on the integrals in f(t) are always over the control volume. These limits may vary with time if control surface moves



$$g(t^* + \Delta t) = \int_{CM} b(x, y, z, t^* + \Delta t)\,dV$$

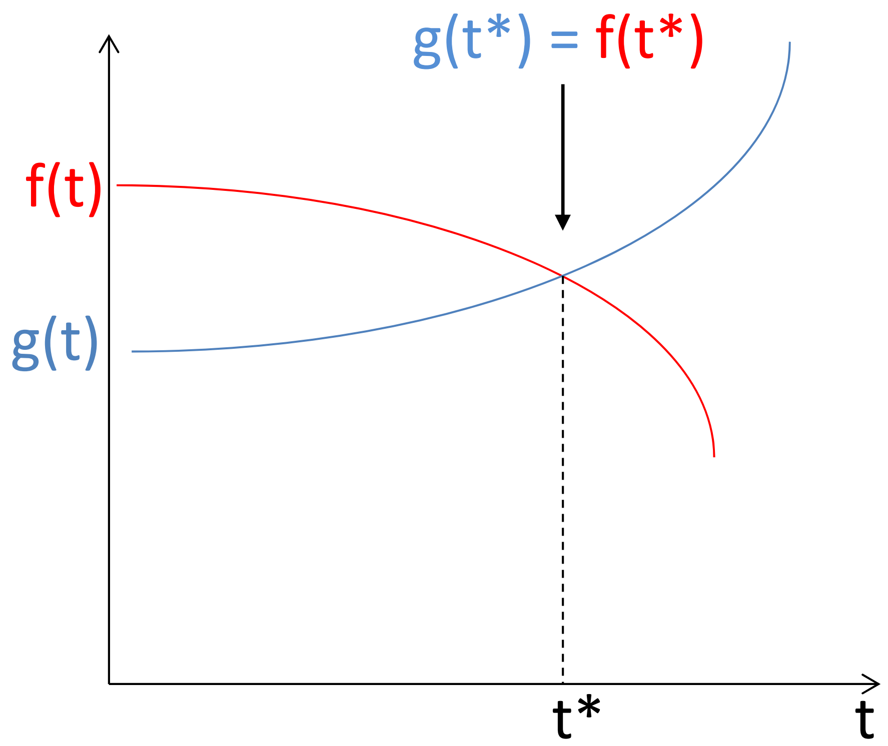
$$g(t^*) = \int_{CM} b(x, y, z, t^*)\,dV$$

$$g(t^* - \Delta t) = \int_{CM} b(x, y, z, t^* - \Delta t)\,dV$$

The limits on the integrals in g(t) may change with time since the the shape and volume of control mass may vary with time

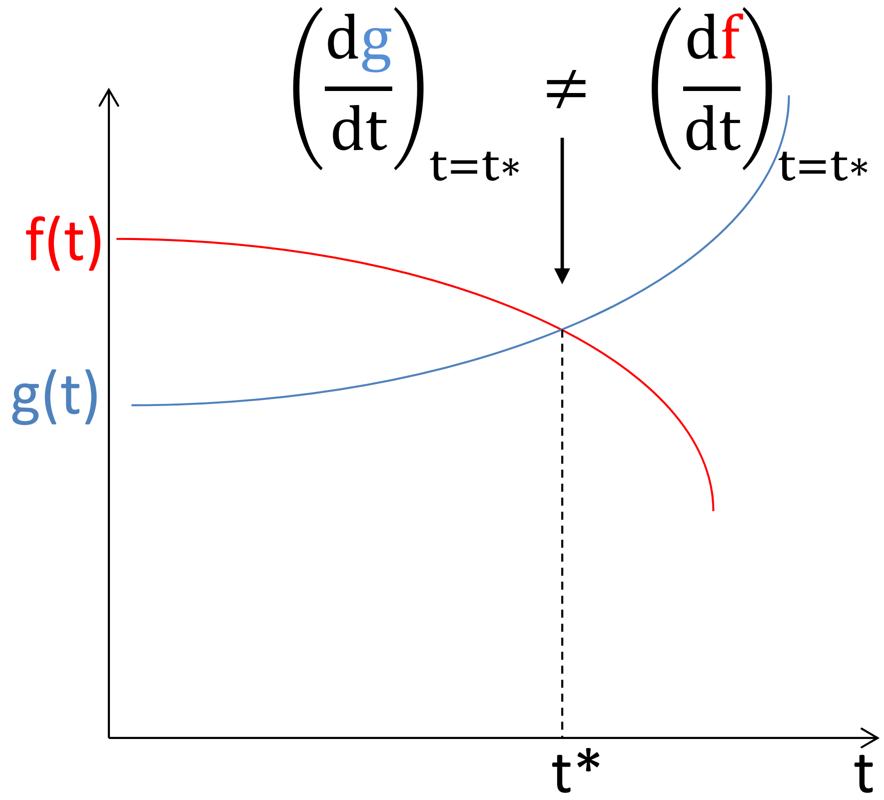
Reynolds Transport Theorem:

The two functions of time can be plotted and the graph might look as depicted below.



Since the mass of the fluid (CM) occupies the control volume CV at t = t*, the two functions of time will have the same value. Thus the two curves intersect at t = t* as seen in above plot.

Reynolds Transport Theorem:

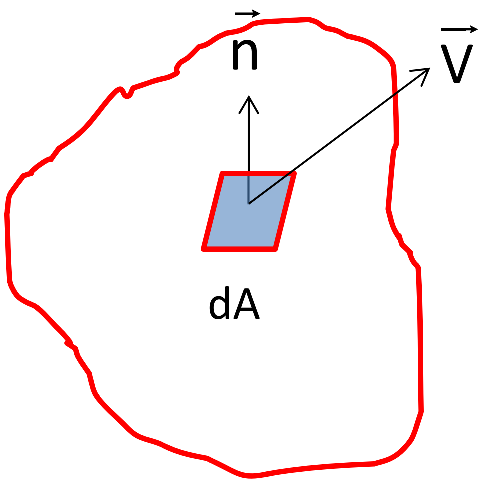


Although the value of the two functions will be the same at t = t*, the slopes of these functions are not the same.

Reynolds Transport Theorem:

The difference between the values of the two derivatives (or the slope of the two functions) at t = t* is given by **Reynolds Transport Theorem** as follows :

$$\left(\frac{dg}{dt}\right)_{t=t*} = \left(\frac{df}{dt}\right)_{t=t*} + \int_{CS} b\,(\vec{V}\,.\,\vec{n}\,)\,dA$$



The integral in the above equation is taken over the entire surface (CS) bounding the control volume (CV). The integration is performed at t = t*.

$\vec{n}$ = normal vector (directed outwards) to the differential area 'dA' on the surface of the control volume

$\vec{V}$= Velocity vector at the differential area 'dA'

Mass balance:

The mass of fluid is obtained by integrating the density ρ(x,y,z,t) over the volume :

$$M_{CM}(t) = \int_{CM} \rho(x,y,z,t)dV$$

$$M_{CV}(t) = \int_{CV} \rho(x,y,z,t)dV$$

Here $M_{CM}(t)$ is the mass of the specific body of the fluid that occupies control volume (CV) at a certain time t*. We are tracing that body with respect to time as it moves across the control volume and $M_{CM}(t)$ is the mass of that body of fluid at time t. $M_{CV}(t)$ is the mass of the body of fluid that occupies the control volume at a given time t. Since for a given body of fluid, its mass is fixed (we are not considering nuclear reactions here !). Hence we have the condition :

$$\frac{dM_{CM}}{dt}=0$$

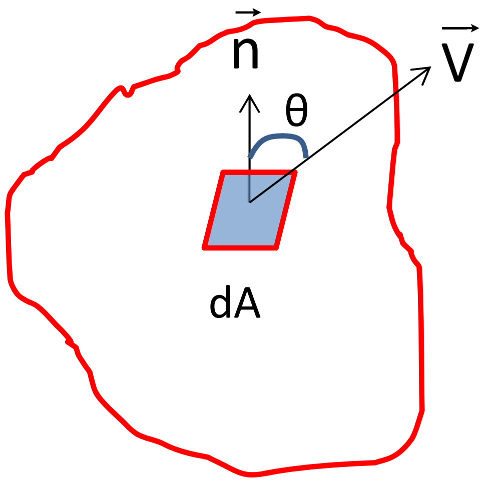
Mass balance:

Applying Reynolds transport theorem to the left hand side of the last equation :

$$\frac{dM_{CV}}{dt} + \int_{CS} \rho\, (\vec{V}.\vec{n})\, dA = 0$$

Note that $(\vec{V}.\vec{n}) = V \cos\theta$

Here θ is the angle between the velocity vector and the normal to the differential Area dA (see figure)



If $\theta = 0^0$, $(\vec{V}.\vec{n}) = V\cos(0^0) = V$

Then the flow is directed **outwards** and along the same direction as that $\vec{n}$

If $\theta = 180^0$, $(\vec{V}.\vec{n}) = V\cos(180^0) = -V$

Then the flow is directed **inwards** and opposite to the direction of $\vec{n}$

How to choose a control volume (CV) ?

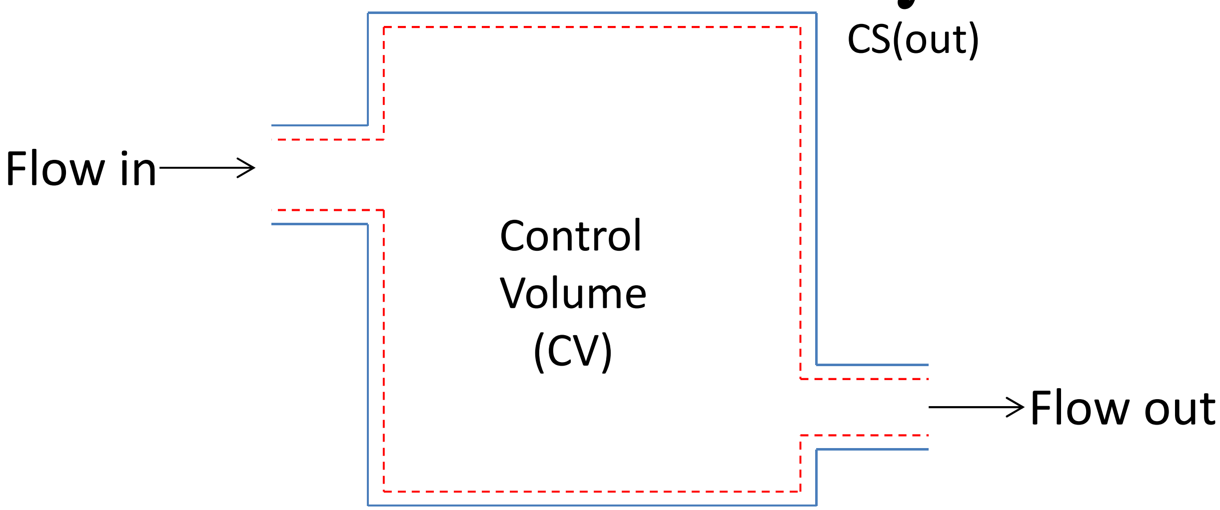
CV is chosen such that a given problem can be simplified as much as possible. As a general rule, CV is chosen such that surface (CS) bounding the control volume is perpendicular to the flow direction along inlets and outlets (see figure below).

Inlet : If the flow direction is perpendicular for an inlet and ρ is taken to be constant over the inlet area, then

$$\int_{CS(in)} \rho(\vec{V}.\vec{n})\,dA = \rho \int_{CS(in)} (-V)\,dA = -\dot{m}_{in}$$

Outlet : If the flow direction is perpendicular to an outlet and ρ is taken to be constant over the outlet area, then

$$\int_{CS(out)} \rho(\vec{V}.\vec{n})\,dA = \rho \int_{CS(out)} V\,dA = \dot{m}_{out}$$



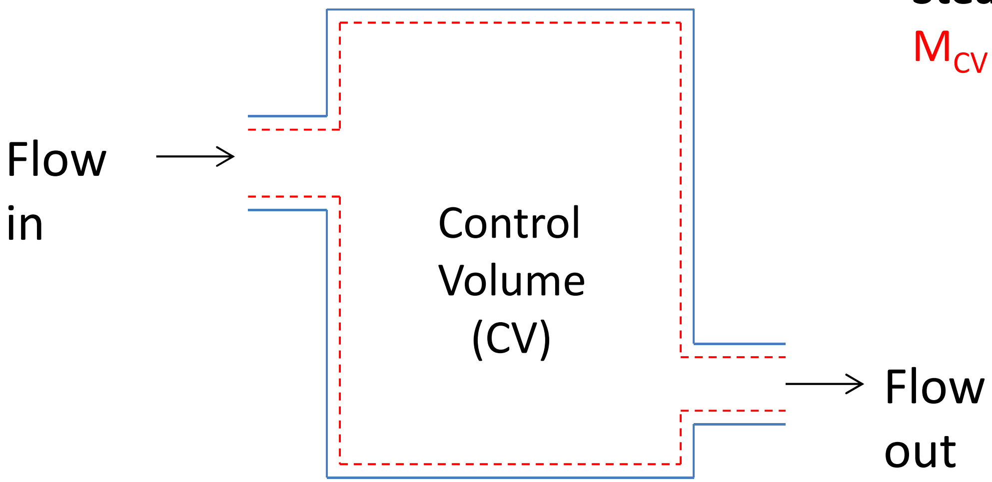
Mass balance :

If the CS is chosen such that direction of flow is perpendicular to inlets and outlets, (see last slide) then

$$\int_{CS} \rho\,(\vec{V}.\vec{n})\,dA = \sum_{out} \dot{m}_{out} - \sum_{in} \dot{m}_{in}$$

Then the mass balance takes a simpler form :

$$\frac{dM_{CV}}{dt} + \sum_{out} \dot{m}_{out} - \sum_{in} \dot{m}_{in} = 0$$

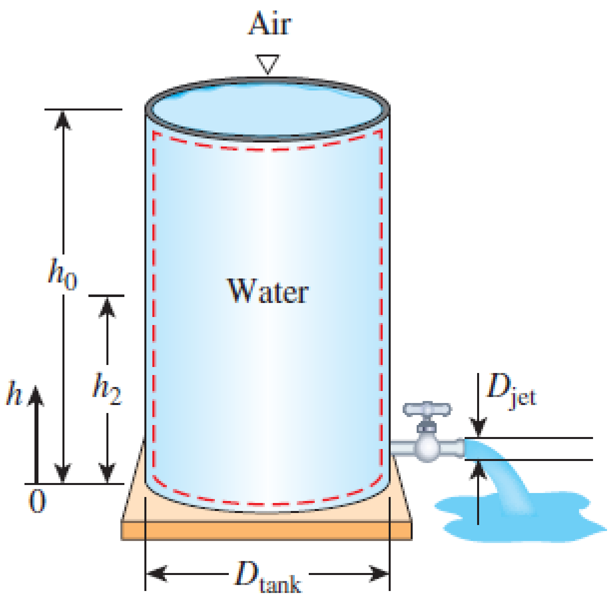


Steady flow problems :
M_{CV} does not change with time.

Example problem :

Discharge of water from a tank (unsteady flow problem)

A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out (Fig. 5–11). The average velocity of the jet is approximated as $V = \sqrt{2gh}$, where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop to 2 ft from the bottom.



SOLUTION The plug near the bottom of a water tank is pulled out. The time it takes for half of the water in the tank to empty is to be determined.

Assumptions **1** Water is a nearly incompressible substance. **2** The distance between the bottom of the tank and the center of the hole is negligible compared to the total water height. **3** The gravitational acceleration is 32.2 ft/s^2.

Analysis We take the volume occupied by water as the control volume. The size of the control volume decreases in this case as the water level drops, and thus this is a variable control volume. (We could also treat this as a fixed control volume that consists of the interior volume of the tank by disregarding the air that replaces the space vacated by the water.) This is obviously an unsteady-flow problem since the properties (such as the amount of mass) within the control volume change with time.

The conservation of mass relation for a control volume undergoing any process is given in rate form as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt} \tag{1}$$

Example (contd.)

During this process no mass enters the control volume ($\dot{m}_{in} = 0$), and the mass flow rate of discharged water is

$$\dot{m}_{out} = (\rho VA)_{out} = \rho\sqrt{2gh}A_{jet} \tag{2}$$

where $A_{jet} = \pi D_{jet}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is

$$m_{CV} = \rho V = \rho A_{tank} h \tag{3}$$

where $A_{tank} = D_{tank}^2/4$ is the base area of the cylindrical tank. Substituting Eqs. 2 and 3 into the mass balance relation (Eq. 1) gives

$$-\rho\sqrt{2gh}A_{jet} = \frac{d(\rho A_{tank}h)}{dt} \rightarrow -\rho\sqrt{2gh}(\pi D_{jet}^2/4) = \frac{\rho(\pi D_{tank}^2/4)dh}{dt}$$

Canceling the densities and other common terms and separating the variables give

$$dt = -\frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

Example (contd.)

Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives

$$\int_0^t dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2\sqrt{2g}}\int_{h_0}^{h_2}\frac{dh}{\sqrt{h}} \rightarrow t = \frac{\sqrt{h_0}-\sqrt{h_2}}{\sqrt{g/2}}\left(\frac{D_{\text{tank}}}{D_{\text{jet}}}\right)^2$$

Substituting, the time of discharge is determined to be

$$t = \frac{\sqrt{4\text{ ft}}-\sqrt{2\text{ ft}}}{\sqrt{32.2/2\text{ ft/s}^2}}\left(\frac{3\times 12\text{ in}}{0.5\text{ in}}\right)^2 = 757\text{ s} = \mathbf{12.6\ min}$$

Therefore, it takes 12.6 min after the discharge hole is unplugged for half of the tank to be emptied.

Discussion Using the same relation with $h_2 = 0$ gives $t = 43.1$ min for the discharge of the entire amount of water in the tank. Therefore, emptying the bottom half of the tank takes much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing h.