

## Isoentropic processes :

In a quasi-static and adiabatic process final entropy is the same as initial entropy. A process in which final entropy is same as initial entropy is typically referred to as an “isoentropic process”.

Lets consider isoentropic process of an ideal gas. Assuming specific heats  $C_V$  and  $C_P$  to be constant, we get

$$s_2 - s_1 = C_V \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{V_2}{V_1} \right) = 0$$

Re-arranging the above equation,  $\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{V_2}{V_1} \right)^{-\left( \frac{R}{C_V} \right)}$

We denote ratio of specific heats as ‘k’ (recall lecture 7) :

$$k = \frac{C_P}{C_V}$$

## Isoentropic processes for ideal gas :

Recall that specific heats of ideal gas are related by the expression :

$$C_P - C_V = R$$

Substituting  $C_P = k C_V$  in the above equation, we get

$$(k - 1)C_V = R \quad (\text{OR}) \quad (k - 1) = \frac{R}{C_V}$$

Substituting above in the equation for the ratio of temperatures (see last slide), we get

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^{(k-1)}$$

Substituting ideal gas equation of state in the above equation and re-arranging, we get

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}}$$

## Isoentropic processes for ideal gas :

If we do not consider specific heats to be constant, then we can use the following relation for isentropic process of an ideal gas :

$$s_2 - s_1 = [s^o(T_2) - s^o(T_1)] - R \ln \left( \frac{P_2}{P_1} \right) = 0$$

Re-arranging the above equation,

$$\left( \frac{P_2}{P_1} \right)_{s=\text{const.}} = \exp \left( \frac{s_2^o - s_1^o}{R} \right)$$

where,  $s_1^o = s^o(T_1)$  and  $s_2^o = s^o(T_2)$

In the ideal gas tables, the values of 'relative pressure' denoted as  $P_r$  are listed. The relative pressure is defined as :

$$P_r(T) = \exp \left( \frac{s^o(T)}{R} \right)$$

## Isoentropic processes for ideal gas :

Table A-17 to A-24 (ideal gas properties) list the values of  $P_r(T)$  at different  $T$ .

**TABLE A-17**

Ideal-gas properties of air

$T$ K	$h$ kJ/kg	$P_r$	$u$ kJ/kg	$v_r$	$s^\circ$ kJ/kg·K
200	199.97	0.3363	142.56	1707.0	1.29559
210	209.97	0.3987	149.69	1512.0	1.34444
220	219.97	0.4690	156.82	1346.0	1.39105
230	230.02	0.5477	164.00	1205.0	1.43557
240	240.02	0.6355	171.13	1084.0	1.47824

relative  
pressure ( $P_r$ )

relative  
volume ( $v_r$ )

In the ideal gas tables, the values of 'relative volume' (denoted as  $v_r$ ) is defined as :

$$v_r(T) = \frac{T}{P_r(T)}$$

## Isoentropic processes for ideal gas :

Using relative pressures listed in Table A-17 to A-24 (ideal gas properties) the pressure ratio in an isentropic process can be obtained as,

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

From the ideal gas equation of state, we get

$$\left(\frac{V_2}{V_1}\right)_{s=\text{const.}} = \left(\frac{T_2 P_1}{T_1 P_2}\right)_{s=\text{const.}} = \left(\frac{T_2 P_{r1}}{T_1 P_{r2}}\right)_{s=\text{const.}} = \left[\frac{(T_2/P_{r2})}{(T_1/P_{r1})}\right]_{s=\text{const.}}$$

Thus, we get the following simplified form :

$$\left(\frac{V_2}{V_1}\right)_{s=\text{const.}} = \frac{V_{r2}}{V_{r1}}$$

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio  $V_1/V_2$  of this engine is 8, determine the final temperature of the air.

Using variable specific heats (rigorous method)

Ideal-gas properties of air

$T$ K	$h$ kJ/kg	$P_r$	$u$ kJ/kg	$v_r$	$s^\circ$ kJ/kg·K
295	295.17	1.3068	210.49	647.9	1.68515
660	670.47	23.13	481.01	81.89	2.50985
670	681.14	24.46	488.81	78.61	2.52589

For closed systems:

$$\frac{V_2}{V_1} = \frac{v_2}{v_1}$$

At  $T_1 = 295$  K:

$$v_{r1} = 647.9$$

$$v_{r2} = v_{r1} \left( \frac{v_2}{v_1} \right) = (647.9) \left( \frac{1}{8} \right) = 80.99 \rightarrow T_2 = \mathbf{662.7 \text{ K}}$$

Using constant specific heats  
(approximate method)

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

We know that the temperature of the air will rise considerably during this adiabatic compression process, so we *guess* the average temperature to be about 450 K. The  $k$  value at this anticipated average temperature is determined from Table A-2b to be 1.391. Then, the final temperature of air becomes

$$T_2 = (295 \text{ K})(8)^{1.391-1} = 665.2 \text{ K}$$

This gives an average temperature value of 480.1 K, which is sufficiently close to the assumed value of 450 K. Therefore, it is not necessary to repeat the calculations by using the  $k$  value at this average temperature.

## Specific entropy changes of water and R-134a:

The specific entropy values (per unit mass basis) for subcooled, saturated, and superheated states of water are listed in Property tables. As a reference state, the entropy of the saturated liquid water at triple point ( $s_f$ ) is arbitrarily assigned the value of zero.

Saturated water—Temperature table

Temp., $T$ °C	Sat. press., $P_{\text{sat}}$ kPa	Specific volume, $\text{m}^3/\text{kg}$		Internal energy, $\text{kJ/kg}$			Enthalpy, $\text{kJ/kg}$			Entropy, $\text{kJ/kg}\cdot\text{K}$		
		Sat. liquid, $v_f$	Sat. vapor, $v_g$	Sat. liquid, $u_f$	Evap., $u_{fg}$	Sat. vapor, $u_g$	Sat. liquid, $h_f$	Evap., $h_{fg}$	Sat. vapor, $h_g$	Sat. liquid, $s_f$	Evap., $s_{fg}$	Sat. vapor, $s_g$
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661

Reference value

For a mixture of saturated liquid and saturated vapor, specific entropy can be calculated as :

$$s = (1 - x)s_f + xs_g$$

(OR) 
$$s = s_f + x s_{fg}$$



## Specific entropy changes of water and R-134a:

Similarly, specific entropy changes of R-134a (refrigerant can be obtained) from the corresponding property tables (A-11 to A-13). Here saturated liquid at  $-40^{\circ}\text{C}$  is taken as reference state, i.e., the specific entropy in this state is assigned the value of 0.

TABLE A-11

Saturated refrigerant-134a—Temperature table

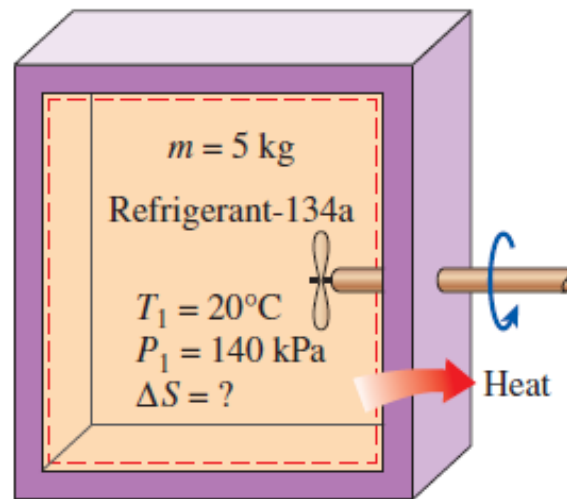
Temp., $T^{\circ}\text{C}$	Sat. press., $P_{\text{sat}}$ kPa	Specific volume, $\text{m}^3/\text{kg}$		Internal energy, $\text{kJ/kg}$			Enthalpy, $\text{kJ/kg}$			Entropy, $\text{kJ/kg}\cdot\text{K}$		
		Sat. liquid, $v_f$	Sat. vapor, $v_g$	Sat. liquid, $u_f$	Evap., $u_{fg}$	Sat. vapor, $u_g$	Sat. liquid, $h_f$	Evap., $h_{fg}$	Sat. vapor, $h_g$	Sat. liquid, $s_f$	Evap., $s_{fg}$	Sat. vapor, $s_g$
-40	51.25	0.0007054	0.36081	-0.036	207.40	207.37	0.000	225.86	225.86	0.00000	0.96866	0.96866
-38	56.86	0.0007083	0.32732	2.475	206.04	208.51	2.515	224.61	227.12	0.01072	0.95511	0.96584
-36	62.95	0.0007112	0.29751	4.992	204.67	209.66	5.037	223.35	228.39	0.02138	0.94176	0.96315
-34	69.56	0.0007142	0.27090	7.517	203.29	210.81	7.566	222.09	229.65	0.03199	0.92859	0.96058
-32	76.71	0.0007172	0.24711	10.05	201.91	211.96	10.10	220.81	230.91	0.04253	0.91560	0.95813

Reference value

As in the case of other properties, we can approximate specific entropy of any compressed liquid (such as water or R-134a) as follows :

$$s_{@T,P}(\text{compressed liquid}) \approx s_{f@T}$$

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.



Recognizing that the specific volume remains constant during this process, the properties of the refrigerant at both states are

$$\begin{array}{ll} \text{State 1:} & \left. \begin{array}{l} P_1 = 140 \text{ kPa} \\ T_1 = 20^\circ\text{C} \end{array} \right\} \begin{array}{l} s_1 = 1.0625 \text{ kJ/kg}\cdot\text{K} \\ v_1 = 0.16544 \text{ m}^3/\text{kg} \end{array} \\ \\ \text{State 2:} & \left. \begin{array}{l} P_2 = 100 \text{ kPa} \\ (v_2 = v_1) \end{array} \right\} \begin{array}{l} v_f = 0.0007258 \text{ m}^3/\text{kg} \\ v_g = 0.19255 \text{ m}^3/\text{kg} \end{array} \end{array}$$

The refrigerant is a saturated liquid–vapor mixture at the final state since  $v_f < v_2 < v_g$  at 100 kPa pressure. Therefore, we need to determine the quality first:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.16544 - 0.0007258}{0.19255 - 0.0007258} = 0.859$$

Thus,

$$s_2 = s_f + x_2 s_{fg} = 0.07182 + (0.859)(0.88008) = 0.8278 \text{ kJ/kg}\cdot\text{K}$$

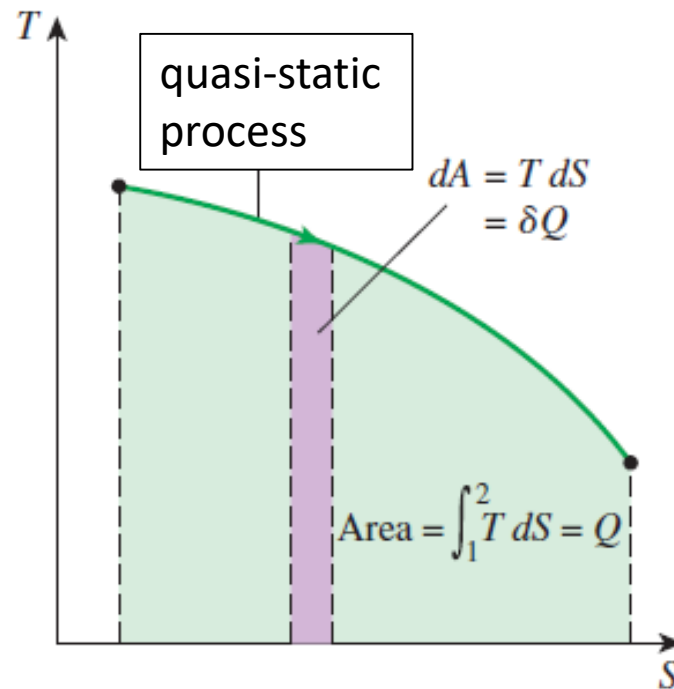
Then, the entropy change of the refrigerant during this process is

$$\begin{aligned}\Delta S &= m(s_2 - s_1) = (5 \text{ kg})(0.8278 - 1.0625) \text{ kJ/kg}\cdot\text{K} \\ &= \mathbf{-1.173 \text{ kJ/K}}\end{aligned}$$

**Discussion** The negative sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the *entropy generation*  $S_{\text{gen}}$  that cannot be negative.

## T-S diagram:

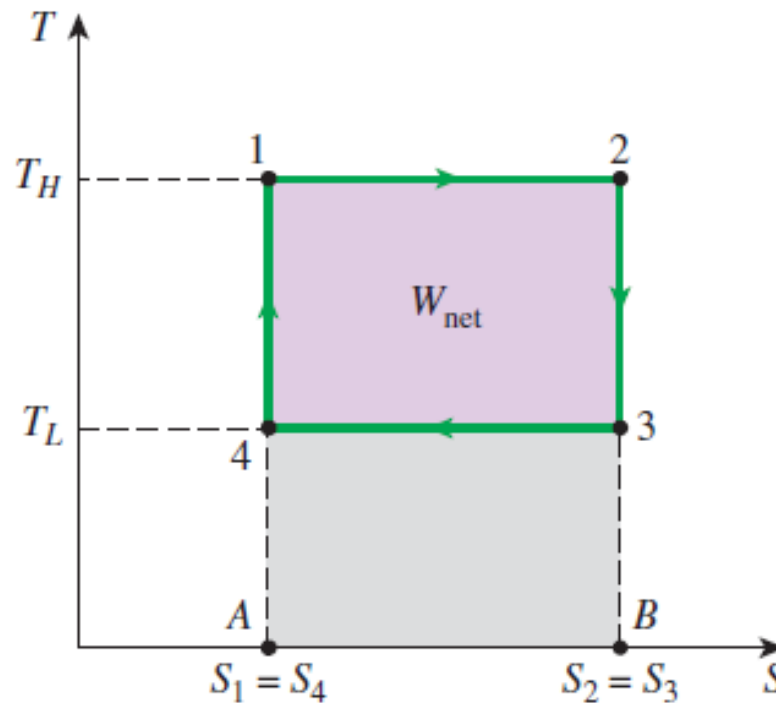
As a visual aid in understanding the process, it is sometimes helpful to plot  $T$  as a function of  $S$  (entropy). Such a diagram is shown in the figure. If the process is quasi-static (in which system is in internal equilibrium), then area under the curve is equal to the heat absorbed by the system (recall Clausius equality).



## T-S diagram for a Carnot cycle :

Show the Carnot cycle on a  $T$ - $S$  diagram and indicate the areas that represent the heat supplied  $Q_H$ , heat rejected  $Q_L$ , and the net work output  $W_{\text{net,out}}$  on this diagram.

T-S diagram for Carnot cycle is shown below. The area under the curve is equal to heat absorbed if  $S$  increase during the process. The area under the curve is equal to heat rejected if  $S$  decrease during the process.



$$W_{\text{net,out}} = Q_H - Q_L$$