ESC201: Lecture 6

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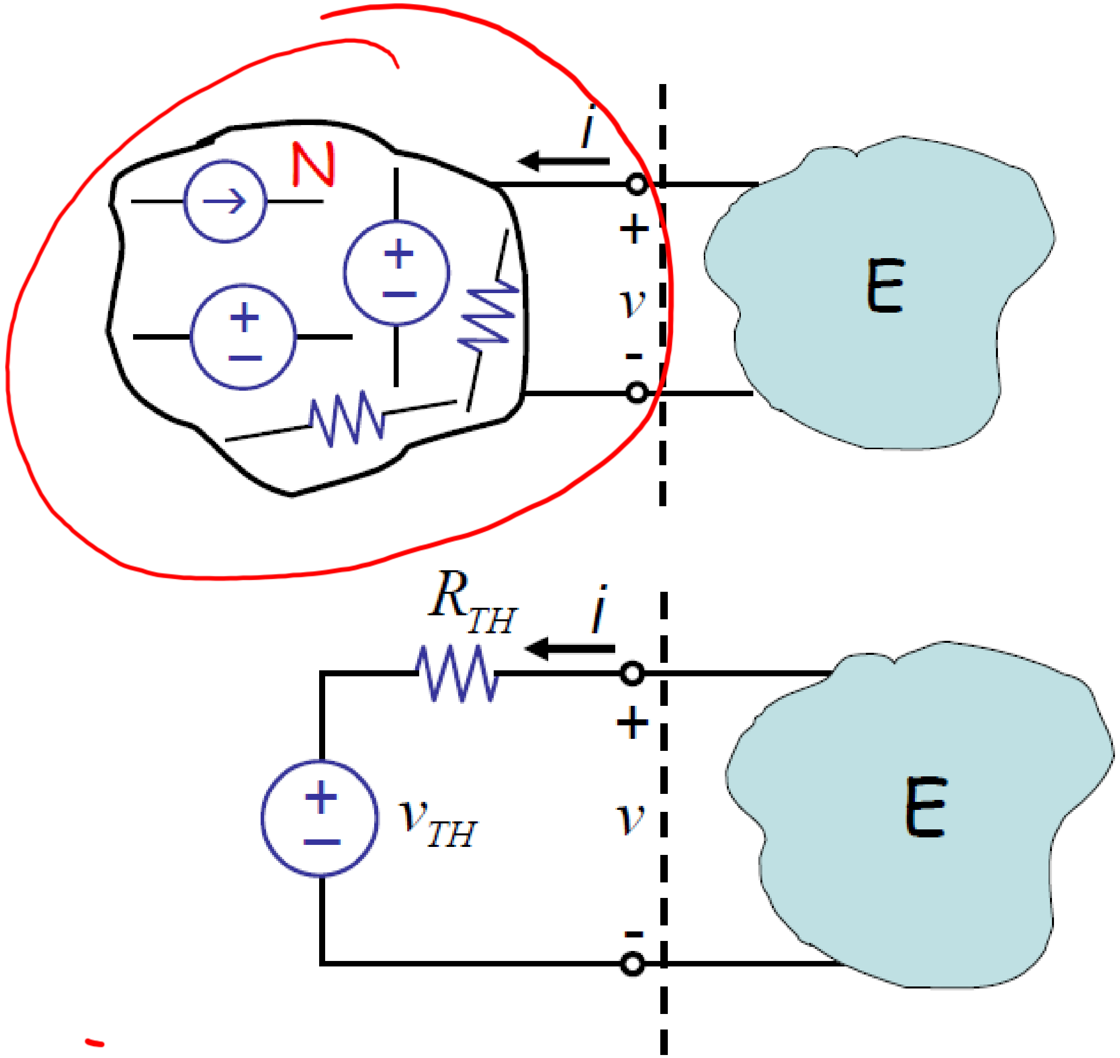
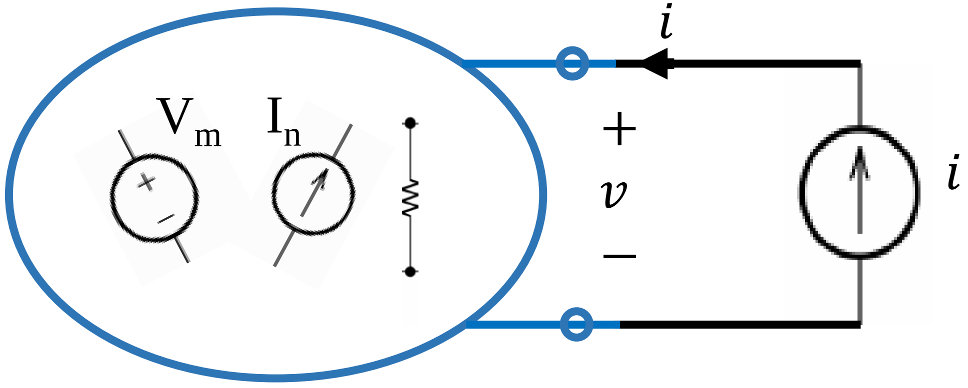
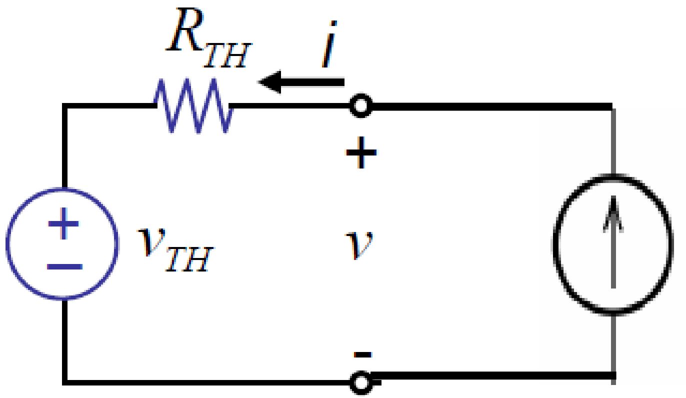
Thevenin Equivalent of Linear Sub-Circuits



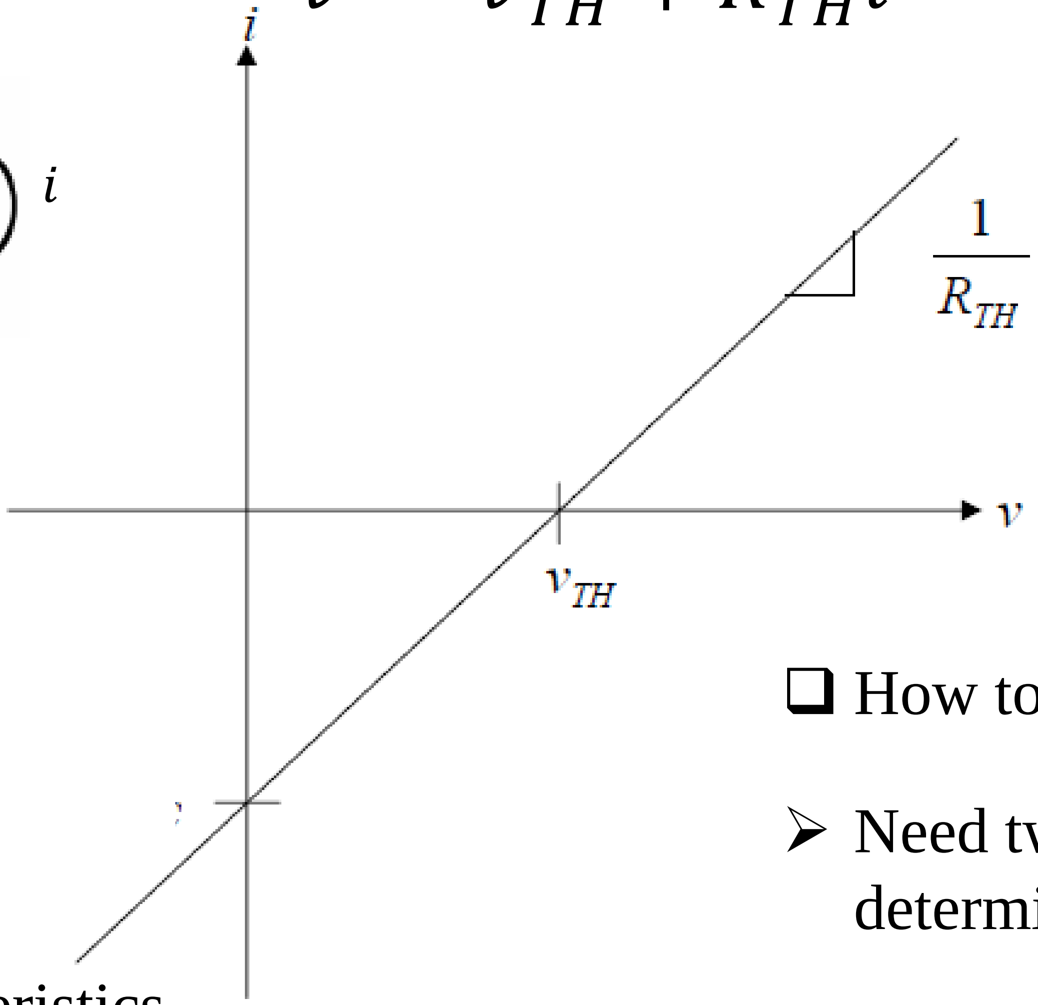
As far as the external world is concerned
(for the purpose of IV relationship)
“arbitrary linear network N”
is
indistinguishable from its
Thevenin equivalent.

I-V Characteristics





$$v = v_{TH} + R_{TH}i$$

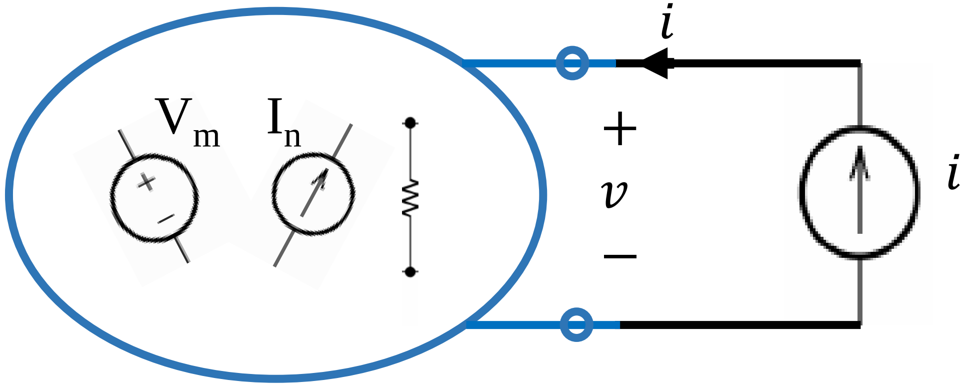


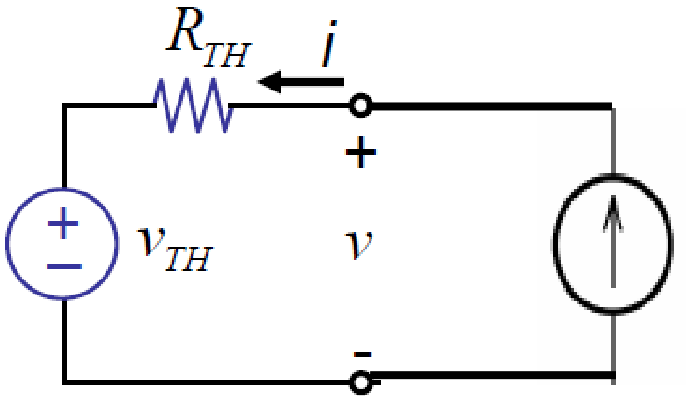
Equal Two Terminal IV Characteristics

- ❑ How to compute these two?
- ➢ Need two points to determine the values

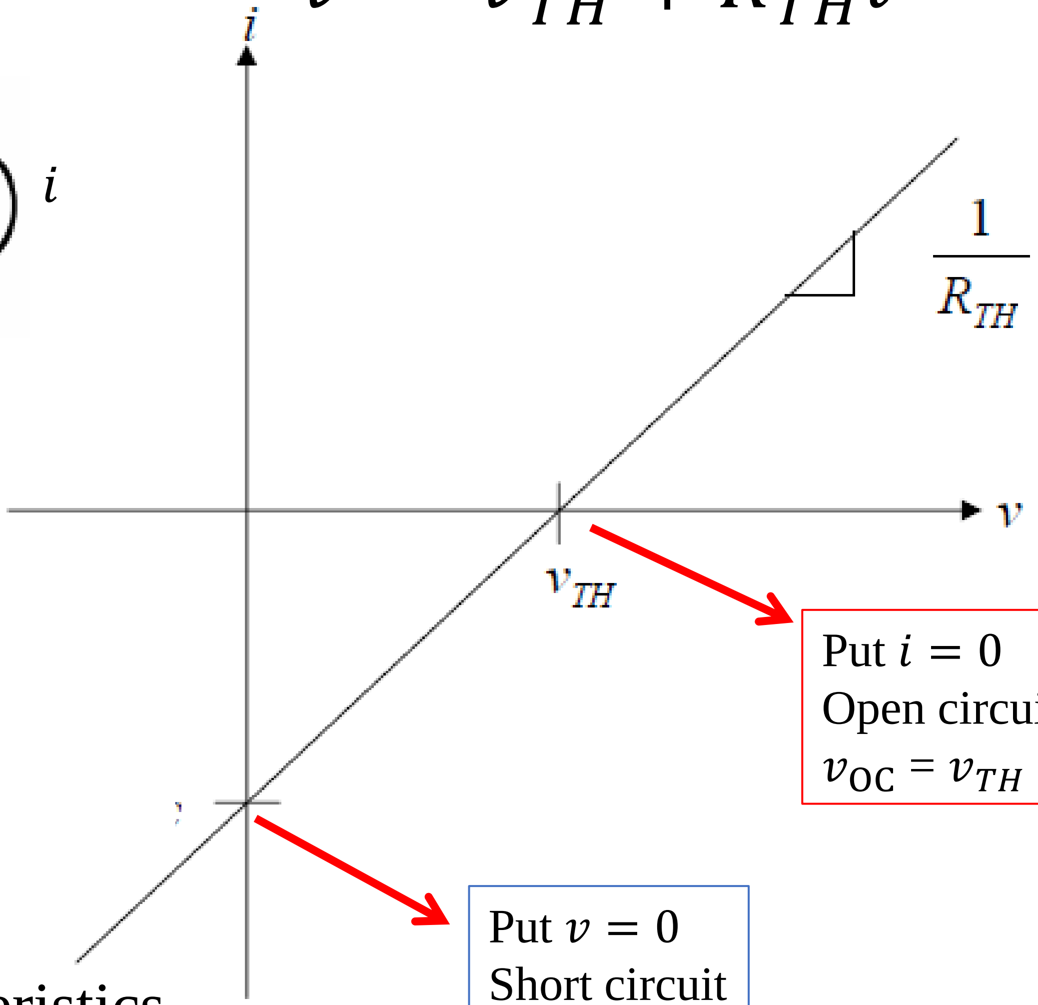
V_{TH} and R_{TH} from I-V Characteristics

$$v = v_{TH} + R_{TH} i$$





Equal Two Terminal IV Characteristics



Put $i = 0$
Open circuit
$v_{OC} = v_{TH}$

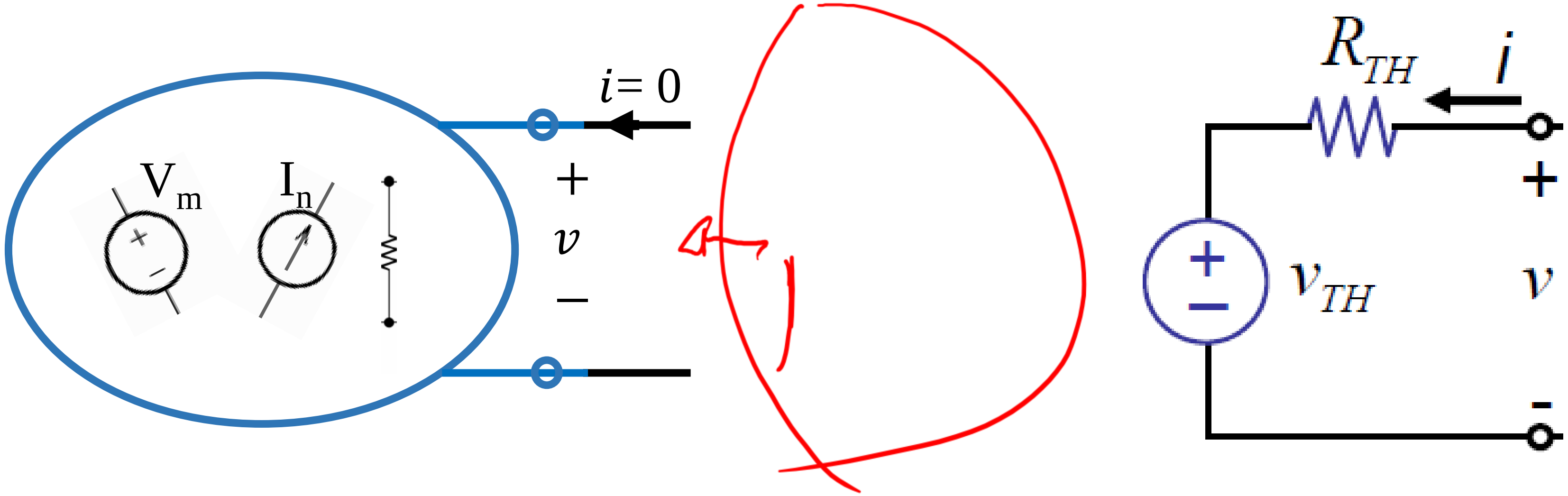
Put $v = 0$
Short circuit
i_{SC}

$$0 = v_{TH} + R_{TH} i_{SC}$$

$$-v_{TH} = R_{TH} i_{SC}$$

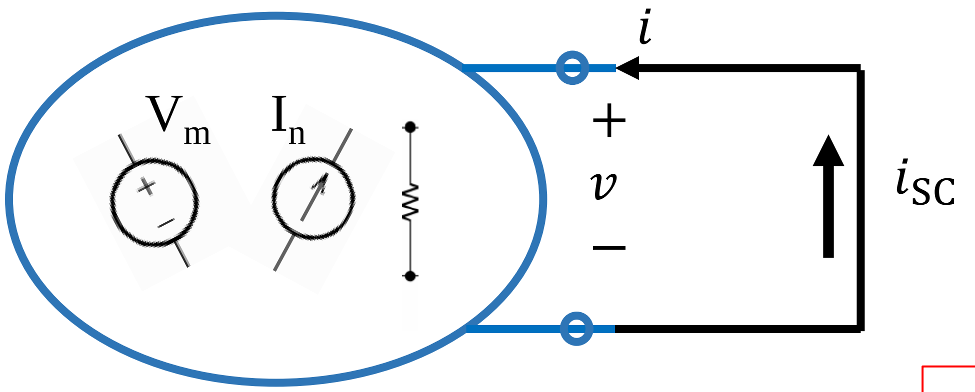
$$\boxed{R_{TH} = \frac{v_{TH}}{-i_{SC}} = \frac{v_{OC}}{-i_{SC}}}$$

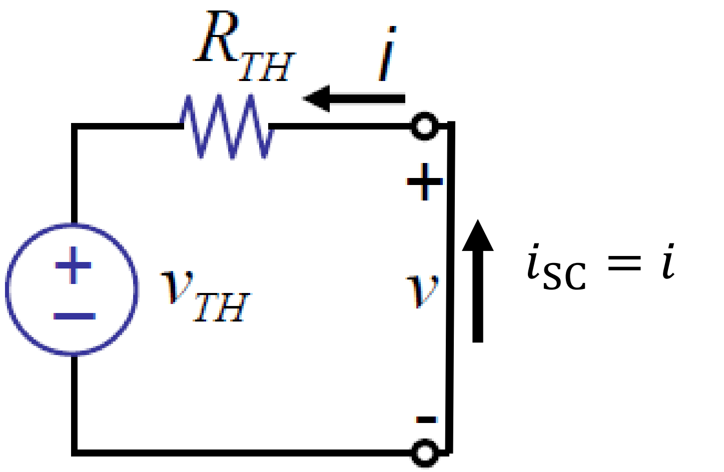
Open Circuit



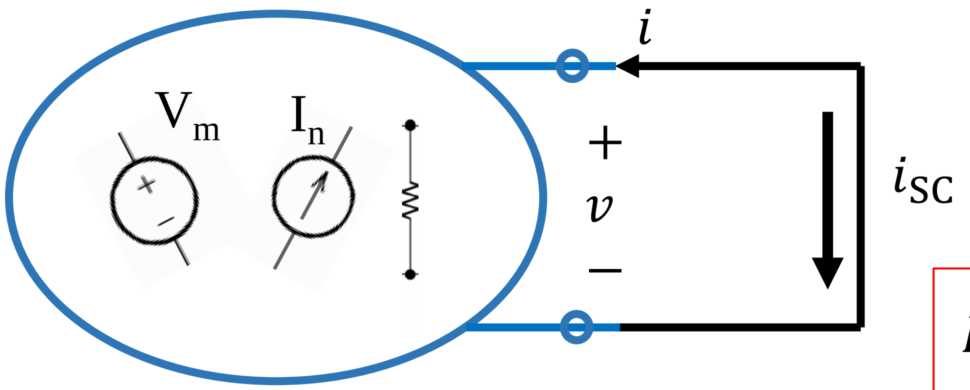
- You may use any circuit analysis method to determine the open circuit voltage

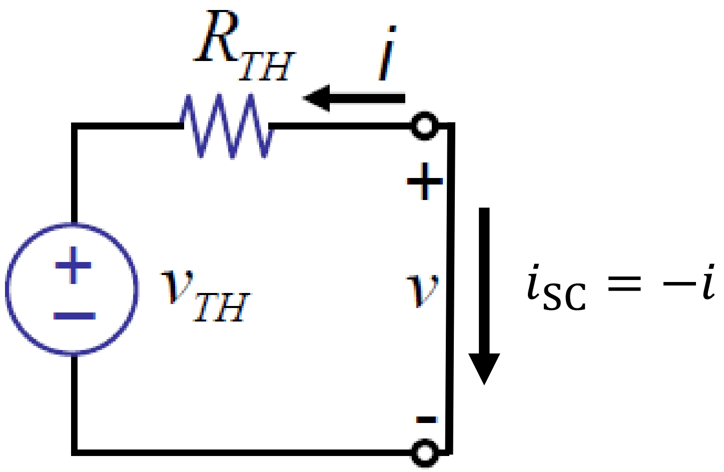
Short Circuit





$$\boxed{R_{TH} = \frac{v_{OC}}{-i_{SC}}}$$

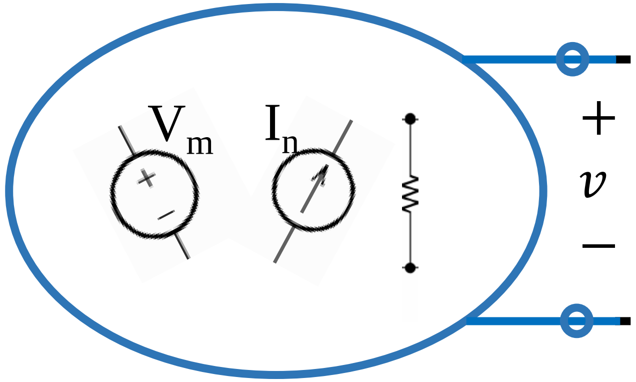


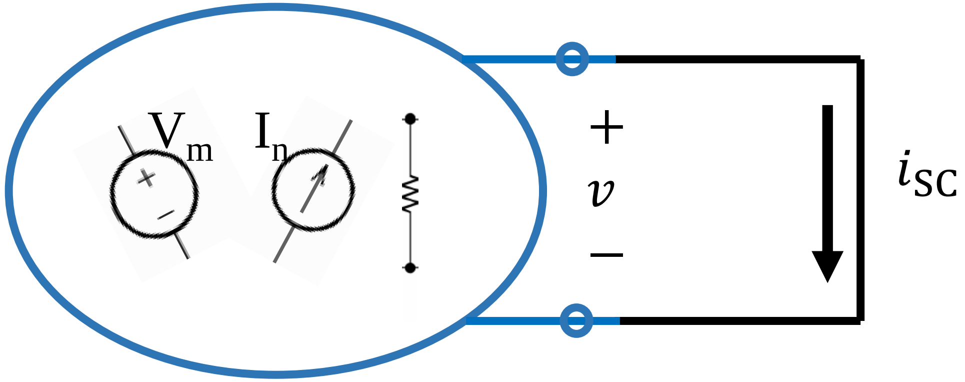


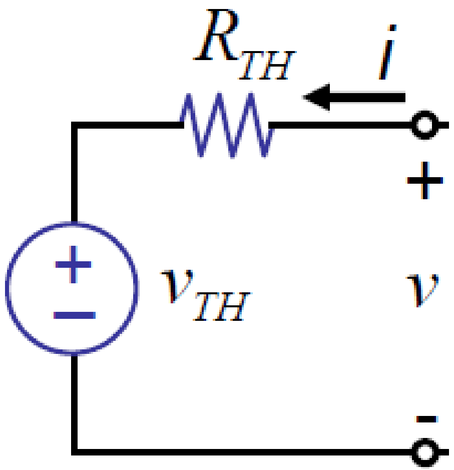
$$\boxed{R_{TH} = \frac{v_{OC}}{i_{SC}}}$$

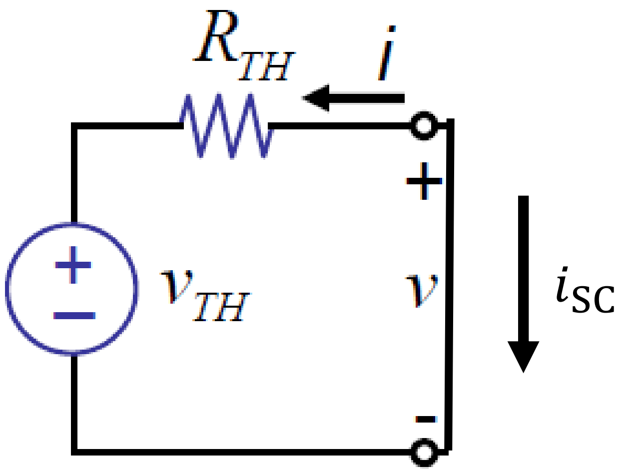
- ➢ You may use any circuit analysis method to determine the short circuit current

Thevenin Equivalent Computation







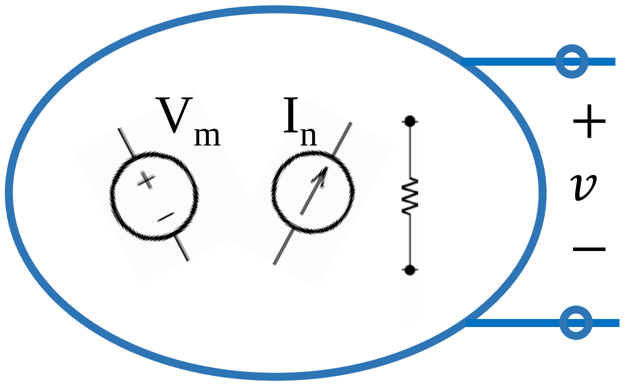


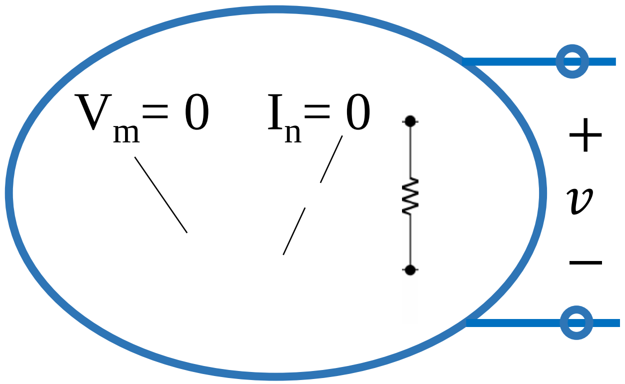
v_{TH} is open circuit voltage v_{OC}

i_{SC} is short circuit current

$$R_{TH} = \frac{v_{OC}}{i_{SC}}$$

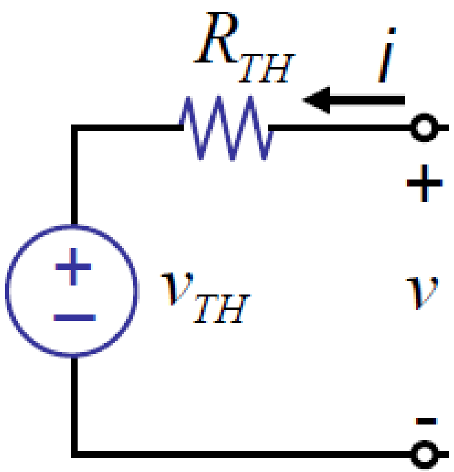
Finding R_{TH} Directly



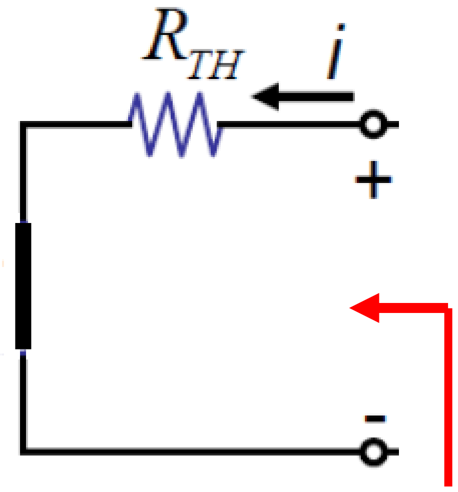


Null all independent sources in the circuit

$$v_{TH} = \sum_m \alpha_m V_m + \sum_n \beta_n I_n$$

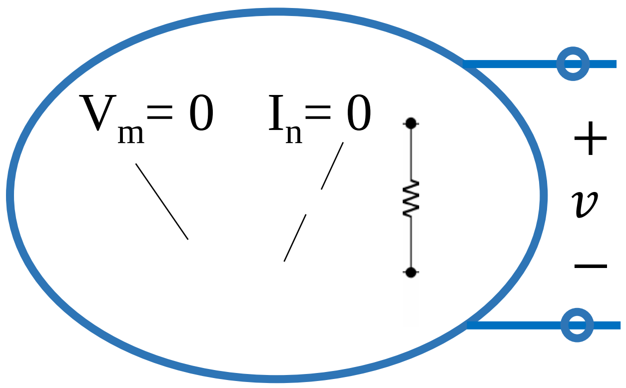


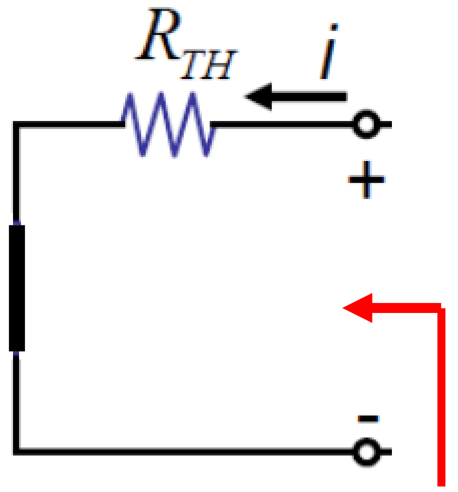
Equal IV Characteristics



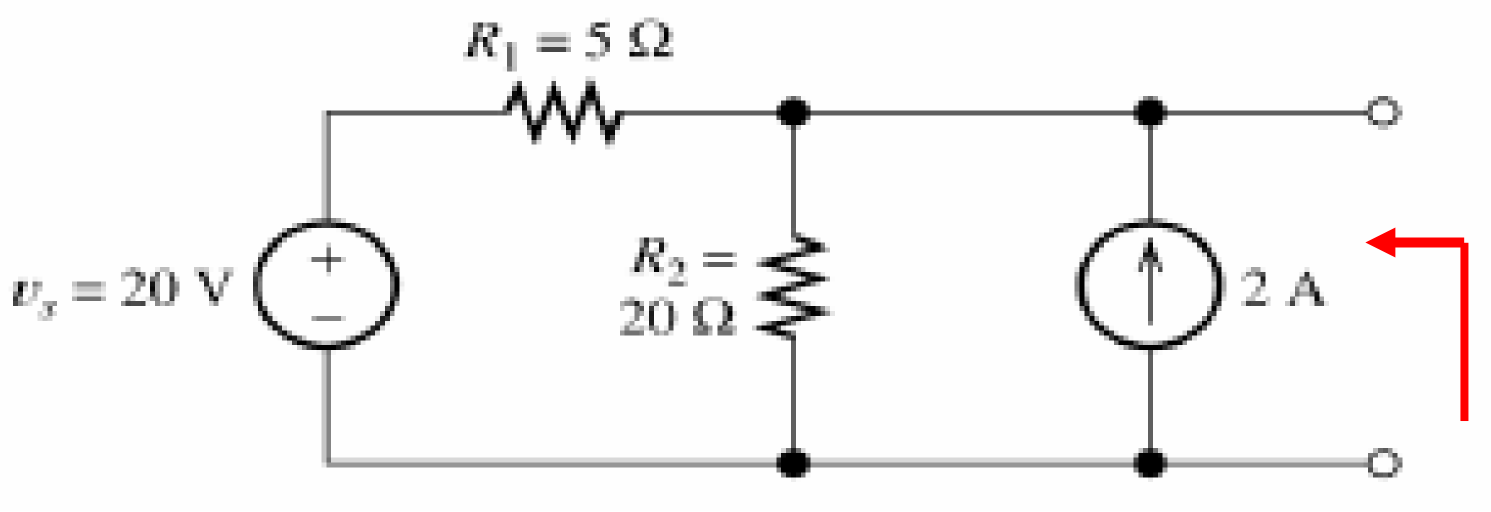
Steps for Finding R_{TH} Directly

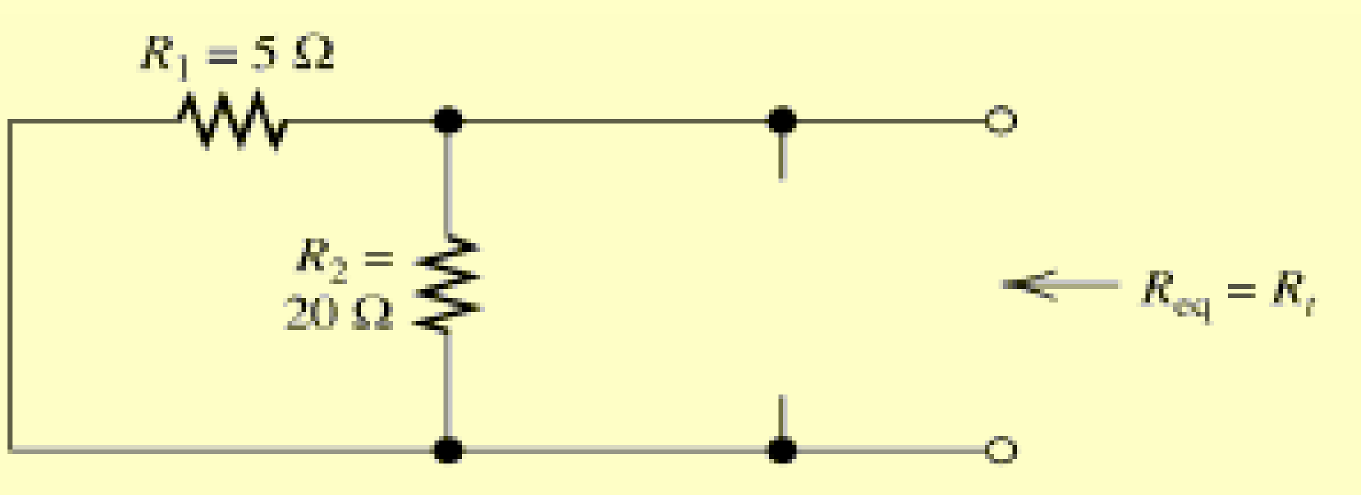
- Null all independent sources in the original network
  - A voltage source becomes a short circuit
  - A current source becomes an open circuit
- Compute the resistance between the terminals
  - ➢ You may use any circuit analysis method to determine the equivalent resistance





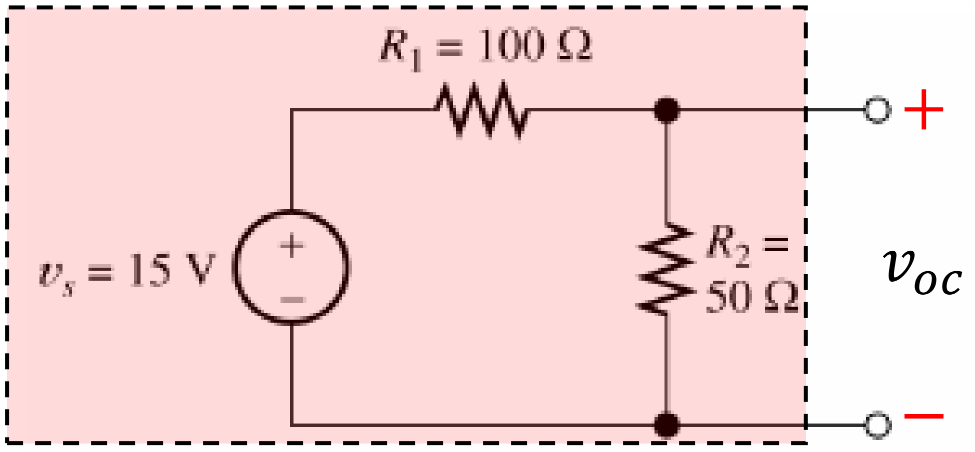
Thevenin Equivalent: Example of R_{TH} Directly

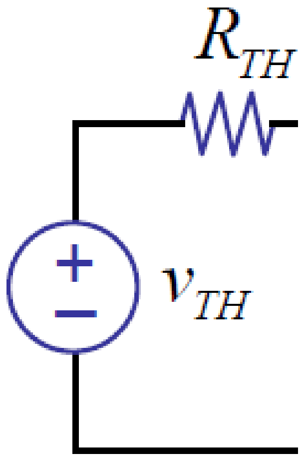




$$R_{eq} = \frac{5 \times 20}{5 + 20} = 4\Omega$$

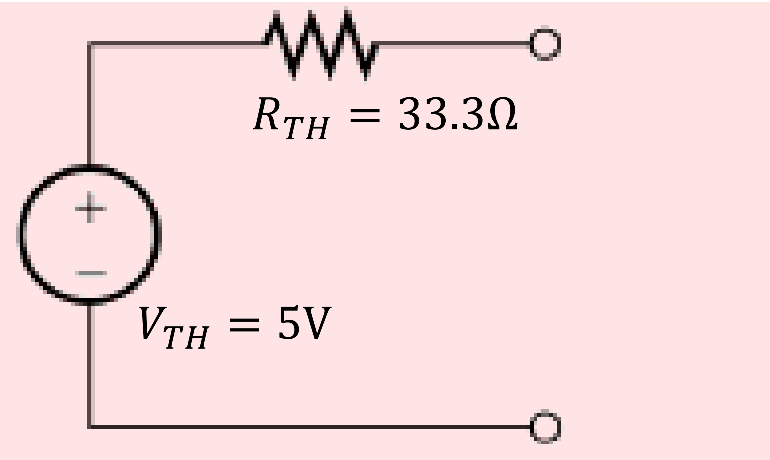
Thevenin Equivalent: Example

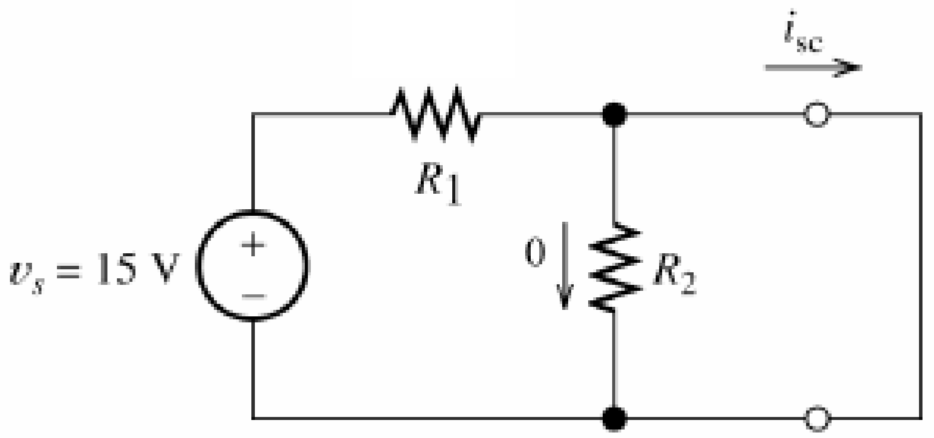




$V_{TH} = v_{oc}$

$$V_{TH} = \frac{R_2}{R_2 + R_1} \times 15\text{V} = 5\text{V}$$

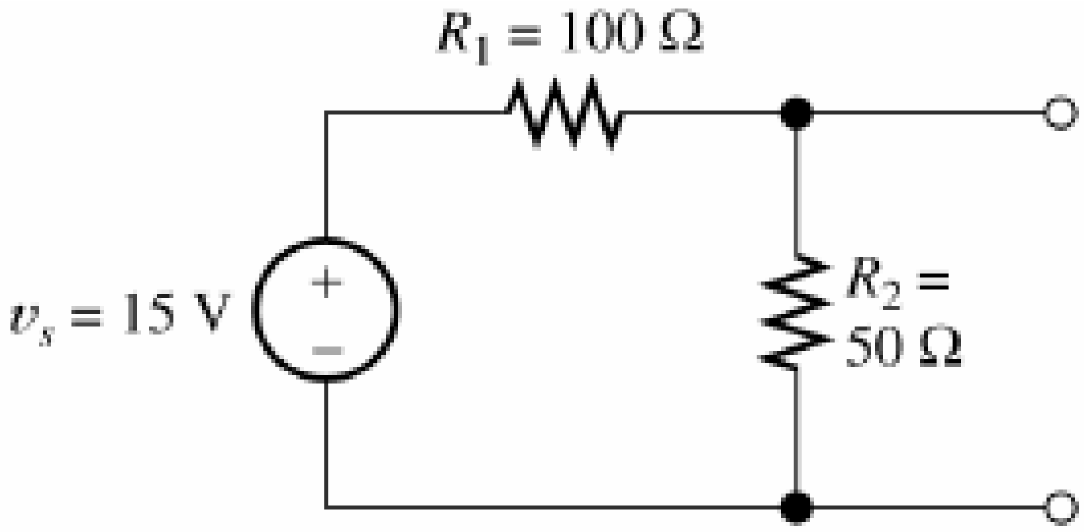




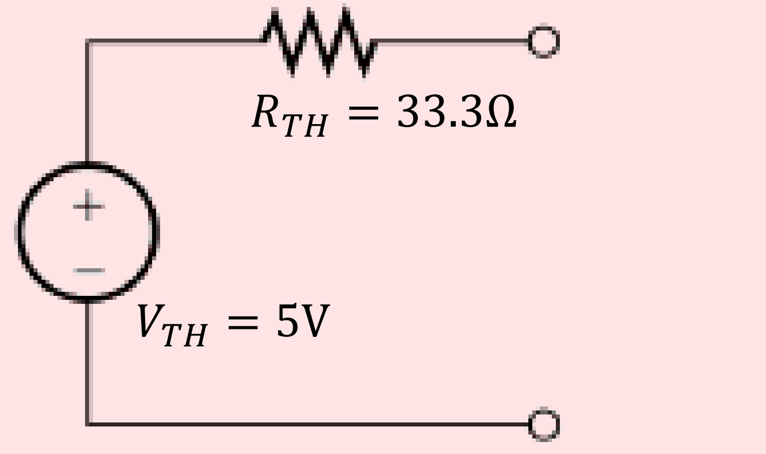
$$i_{sc} = \frac{v_s}{R_1} = 0.15\text{A}$$

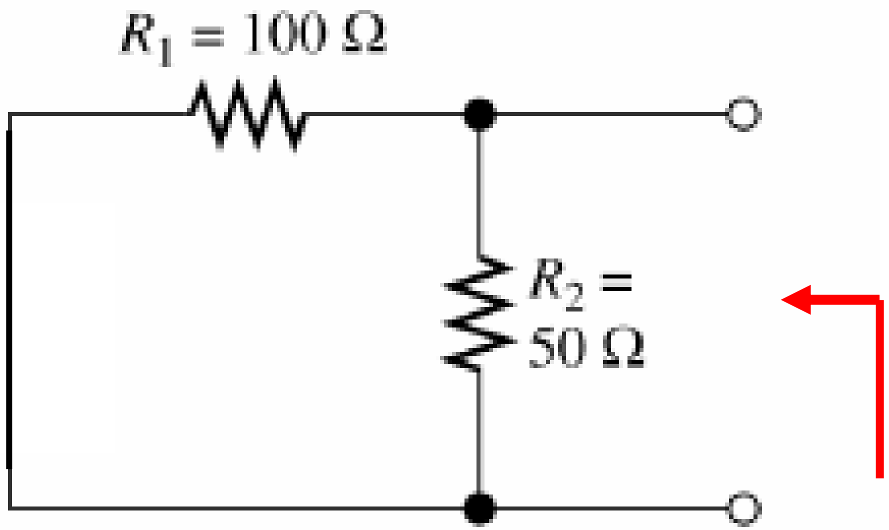
$$R_{TH} = \frac{v_{oc}}{i_{sc}} = 33.3\Omega$$

Thevenin Equivalent: Example of R_{TH} Directly



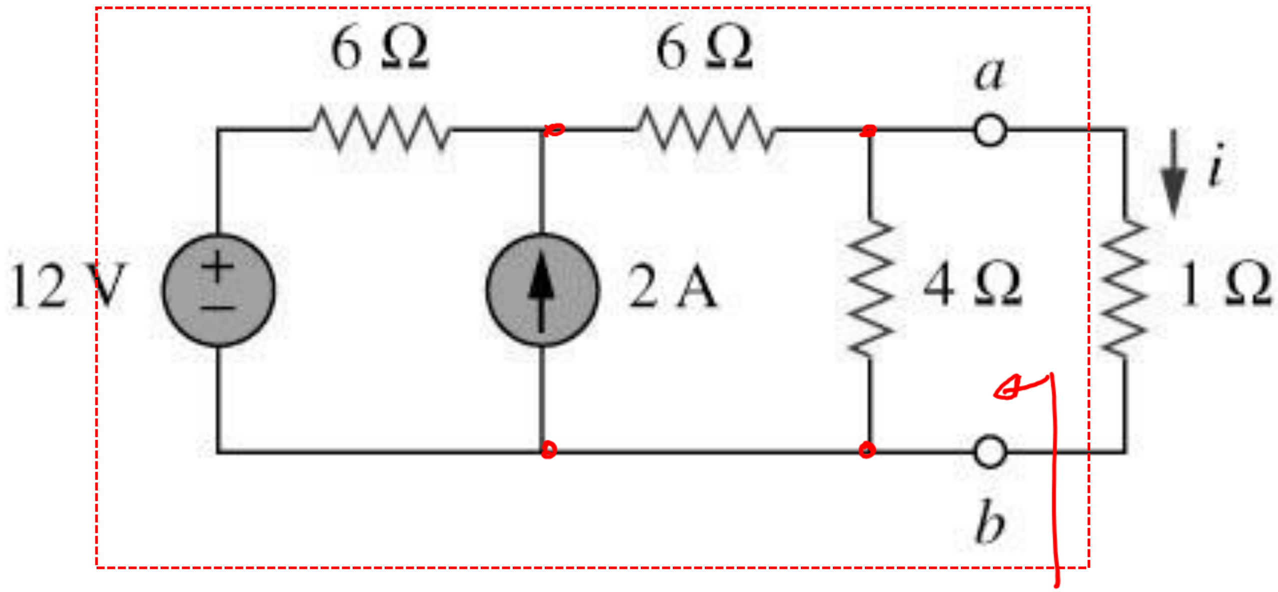
$$V_{TH} = V_{OC} = \frac{R_2}{R_2 + R_1} \times 15\text{V} = 5\text{V}$$

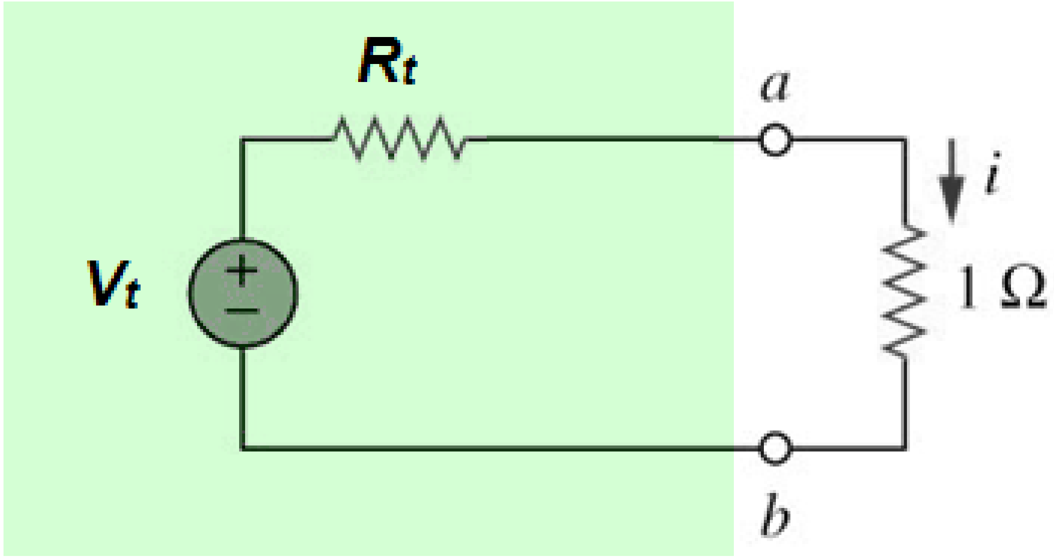




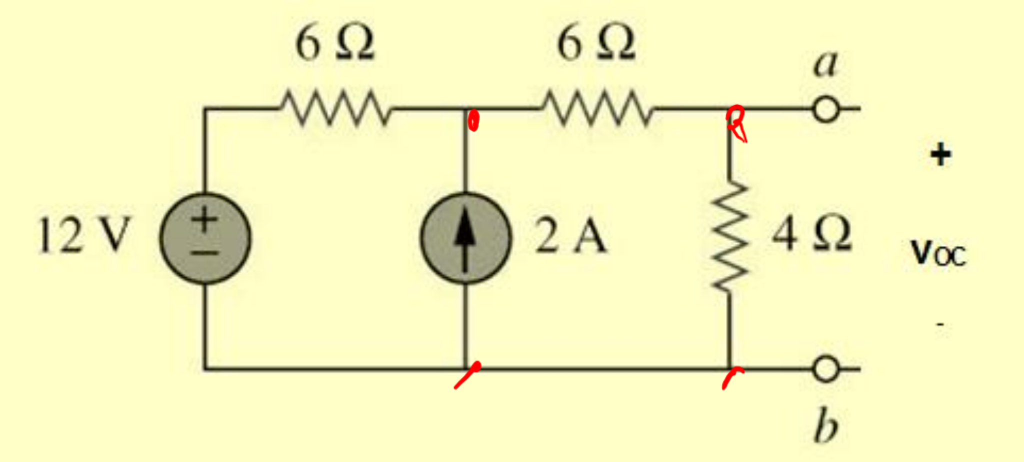
$$R_{TH} = 100||50 = 33.3\Omega$$

Thevenin Method: Example





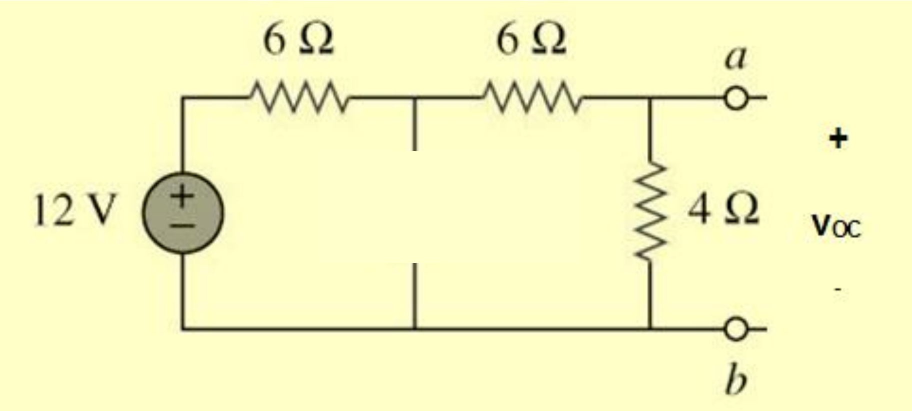
Thevenin Method: Example



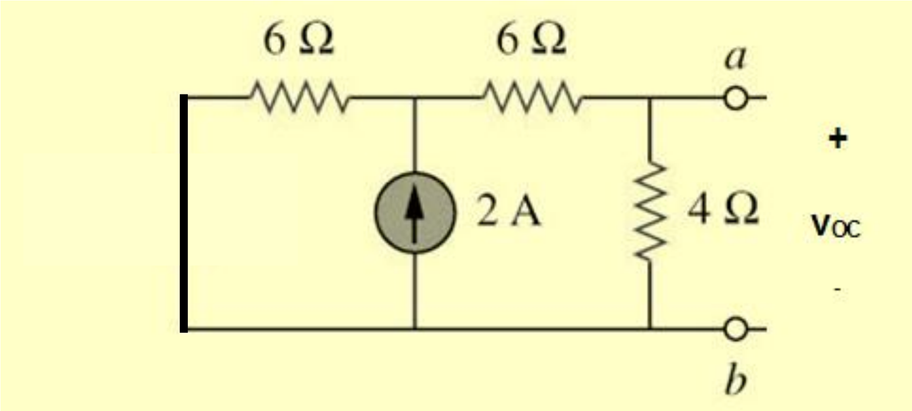
$$V_{oc} = V_{oc1} + V_{oc2} = 6$$

$$v_{oc} = 6V$$

Use Superposition Method

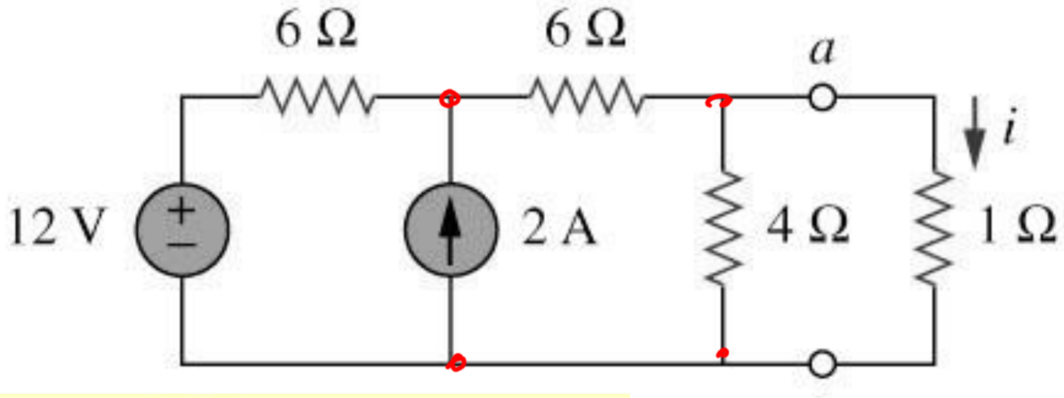


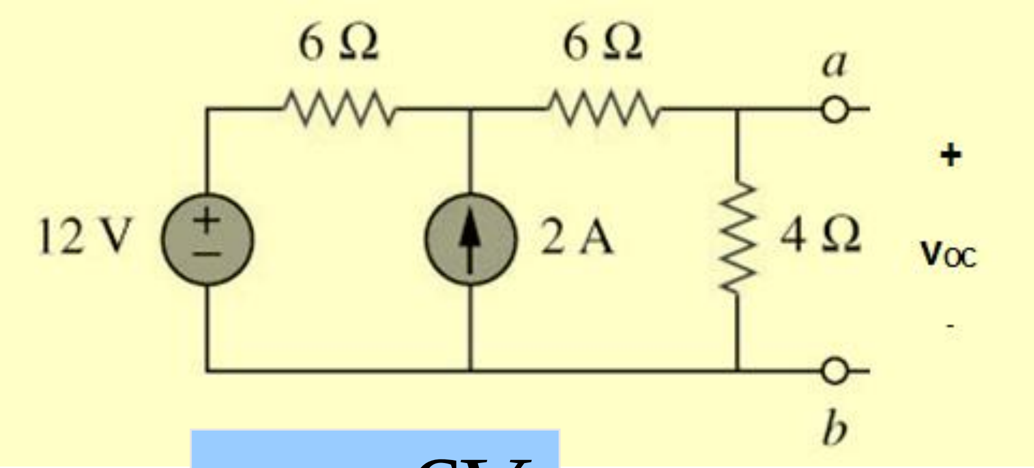
$$V_{oc1} = \frac{4}{4+12} \times 12 = 3$$

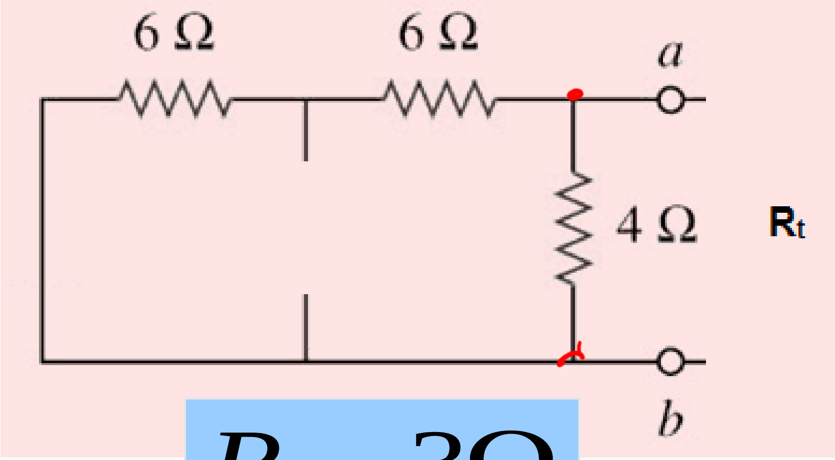


$$V_{oc2} = 4 \times \left(2 \times \frac{6}{6+10} \right) = 3$$

Thevenin Method: Example

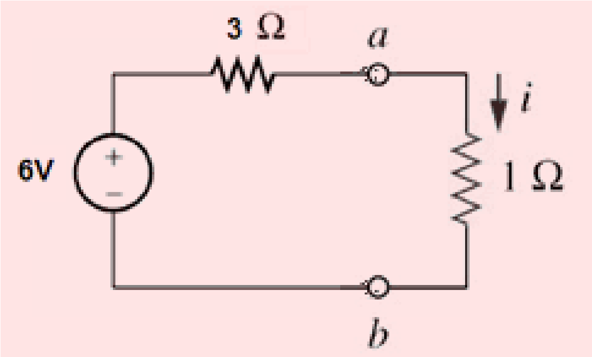






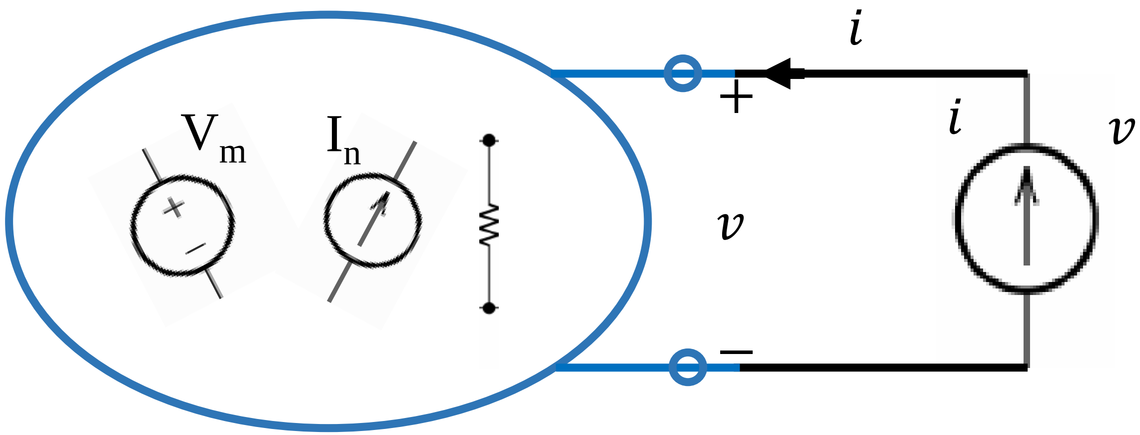
$$v_{oc} = 6V$$

$$R_t = 3\Omega$$



$$i = 1.5A$$

Thevenin Equivalent

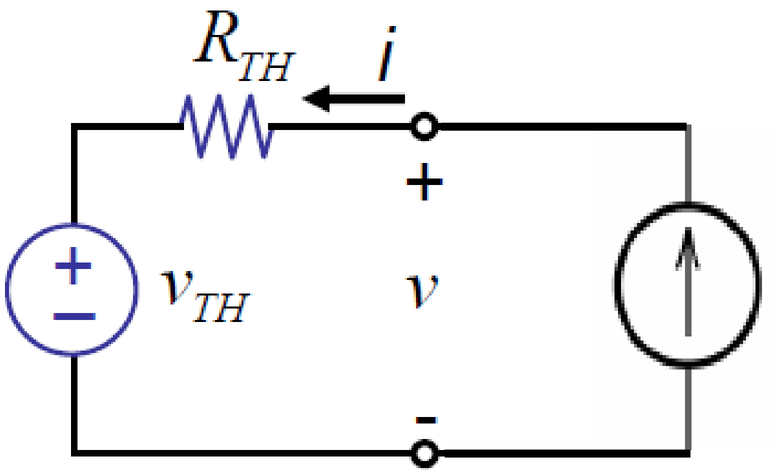


$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + R_t i$$

Setting
$\forall n \ I_n = 0$
$i = 0$

Setting
$\forall m \ V_m = 0$
$i = 0$

Setting
$\forall m \ V_m = 0$
$\forall n \ I_n = 0$



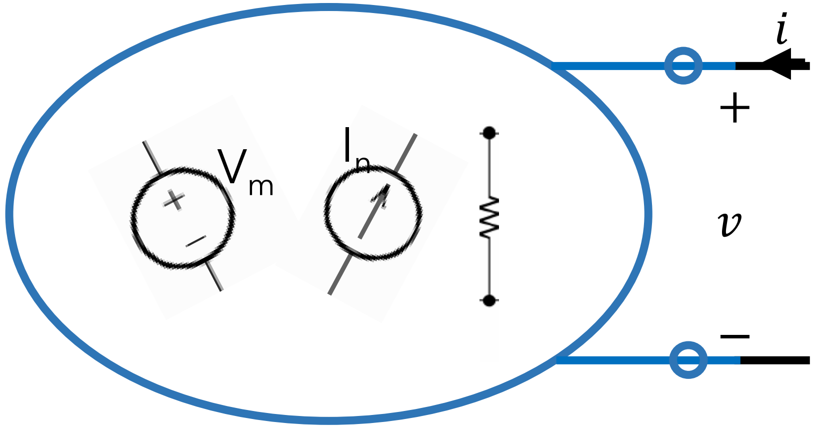
$$v = v_{TH} + R_{TH} i$$

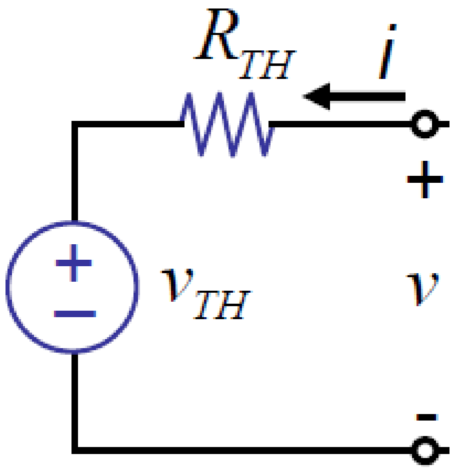
$$v_{TH} = \sum_m \alpha_m V_m + \sum_n \beta_n I_n \qquad R_{TH} = R_t$$

Can be modeled as a voltage source in series with a resistor!

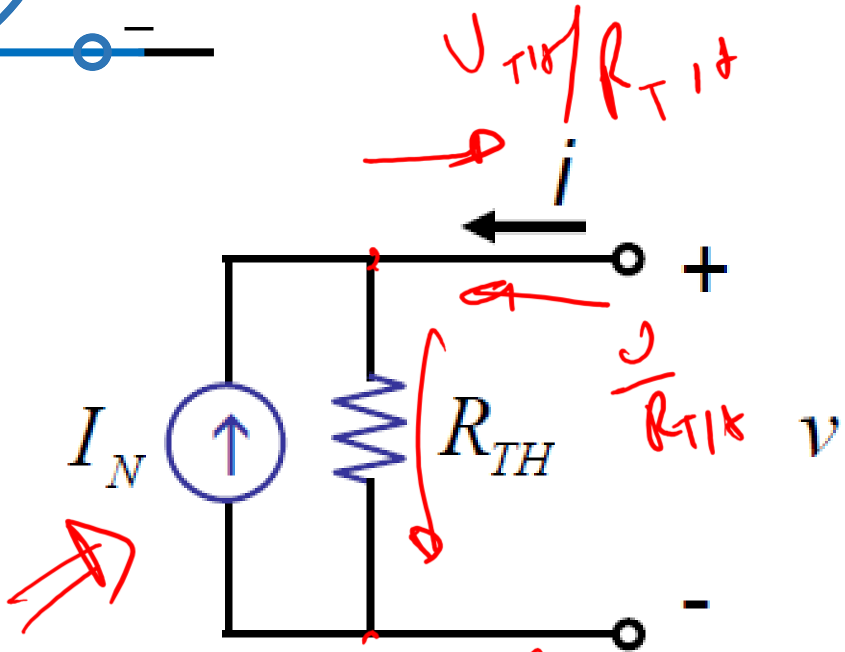
Equal Two Terminal IV Characteristics

Norton Equivalent





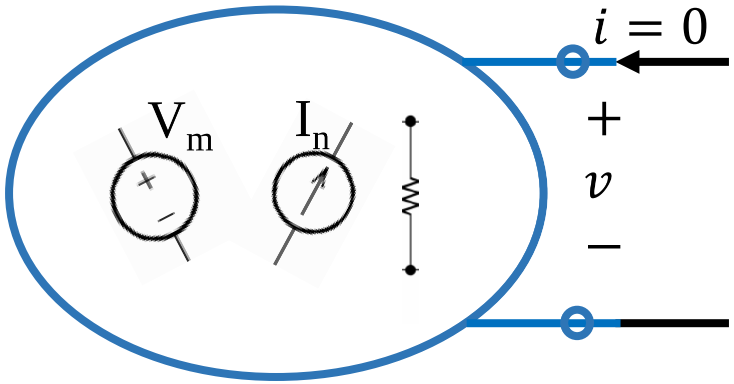
$$v = v_{TH} + R_{TH} i$$

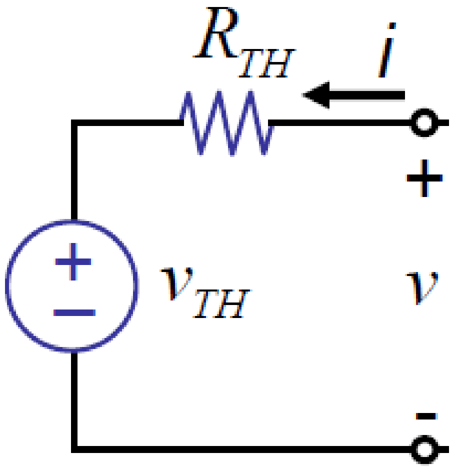


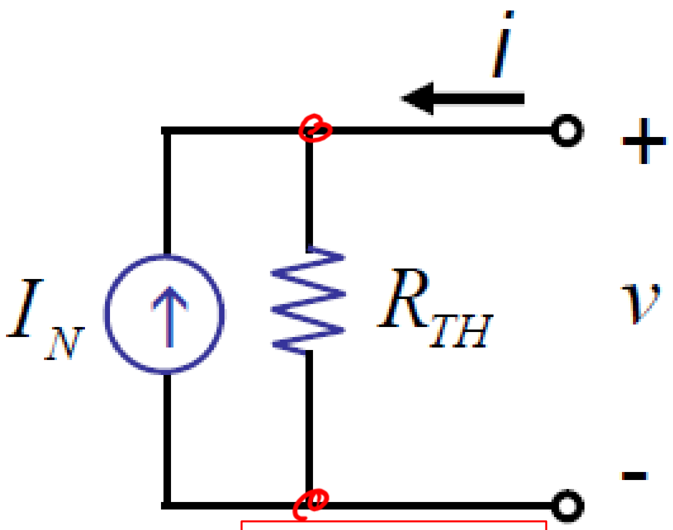
$$I_N = \frac{v_{TH}}{R_{TH}}$$

$$i = \frac{v}{R_{TH}} - \left(\frac{v_{TH}}{R_{TH}}\right)$$

Recall: Open Circuit Voltage





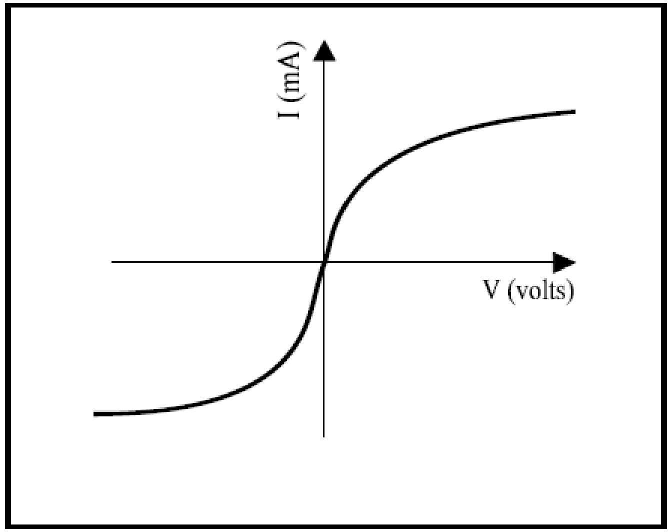


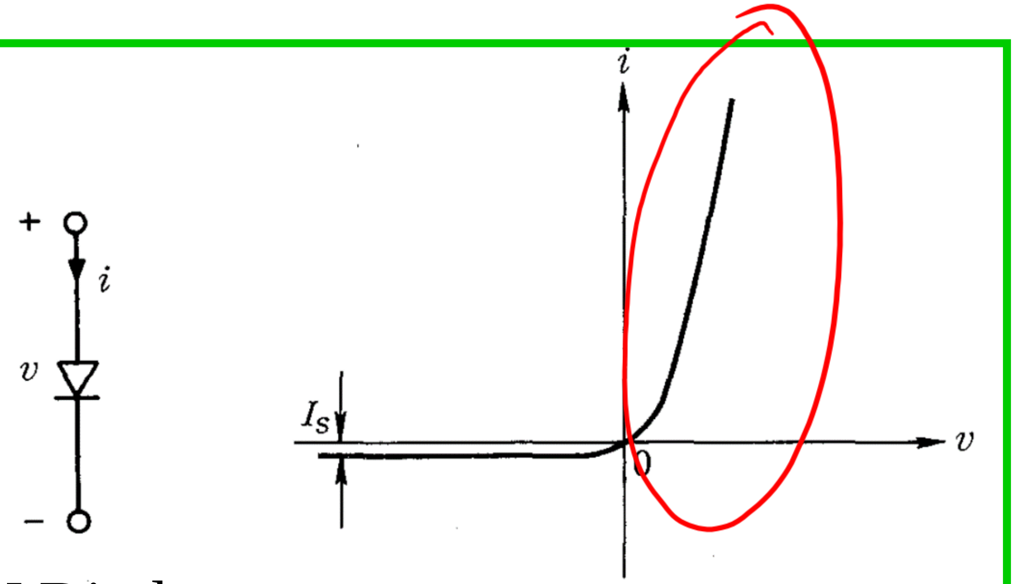
$$\boxed{R_{TH} = \frac{v_{\mathrm{OC}}}{I_{\mathrm{N}}}}$$

- You may use any circuit analysis method to determine the open circuit voltage

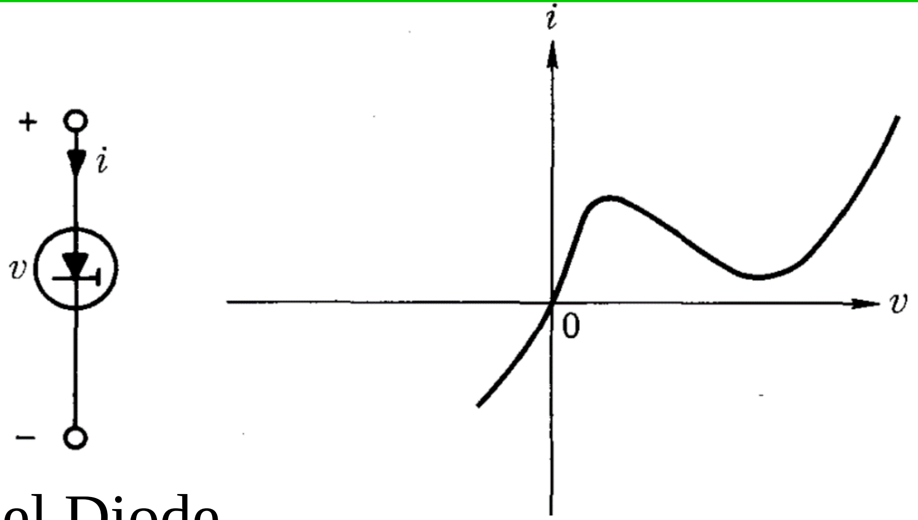
Generalized Resistor

Bulb





PN Diode



Tunnel Diode

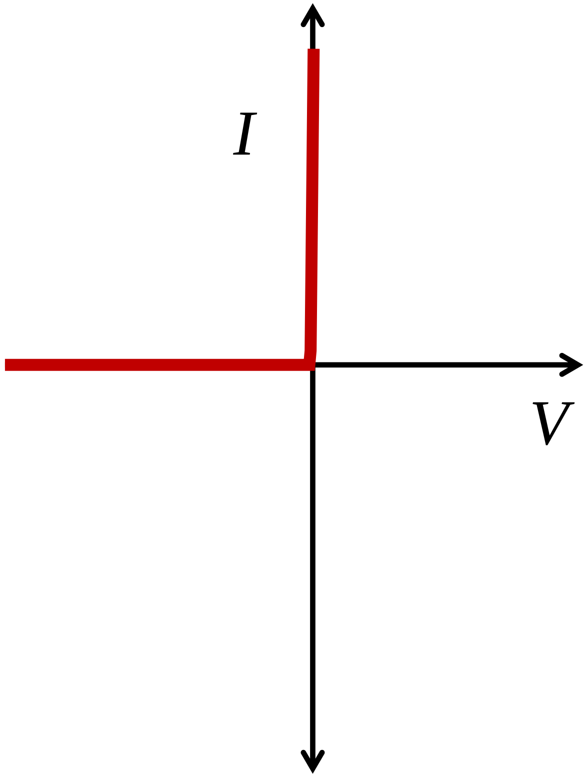
Unidirectional Device: Non-linear Time-invariant Resistor

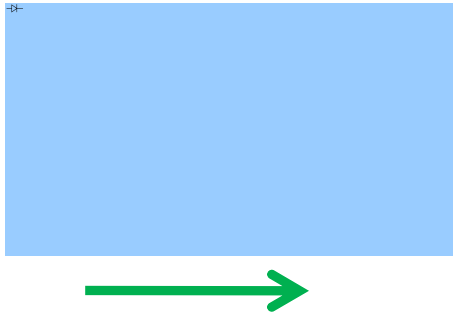
V > 0

short circuit

V < 0

open circuit





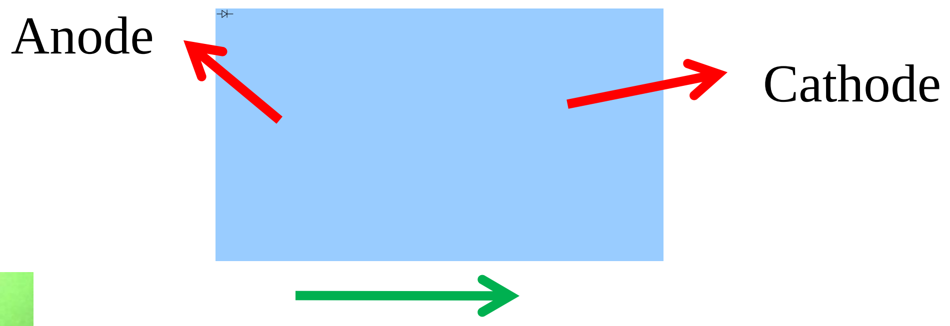
Diodes

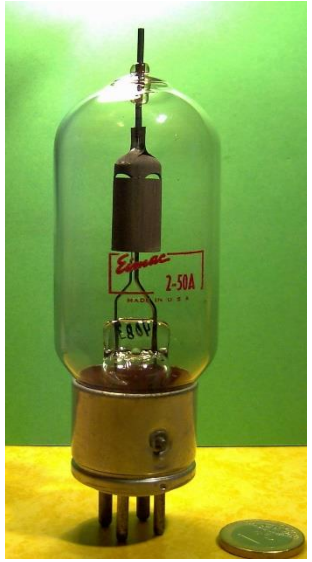
- Diodes allows current in only one direction
  - ➢ from anode to cathode terminal

- Non-linear behaviour

- Applications:
  - One way valve
  - AC to DC converter
  - Voltage regulator
  - LED
  - Logic operations

- Passive device: only consumes power



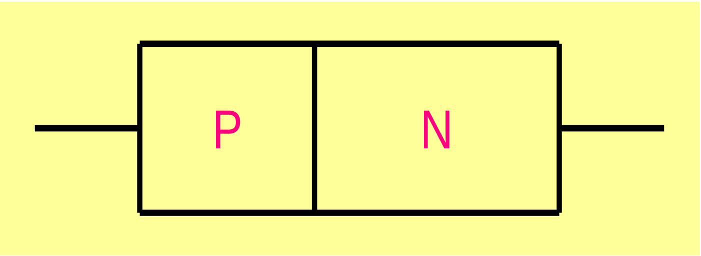


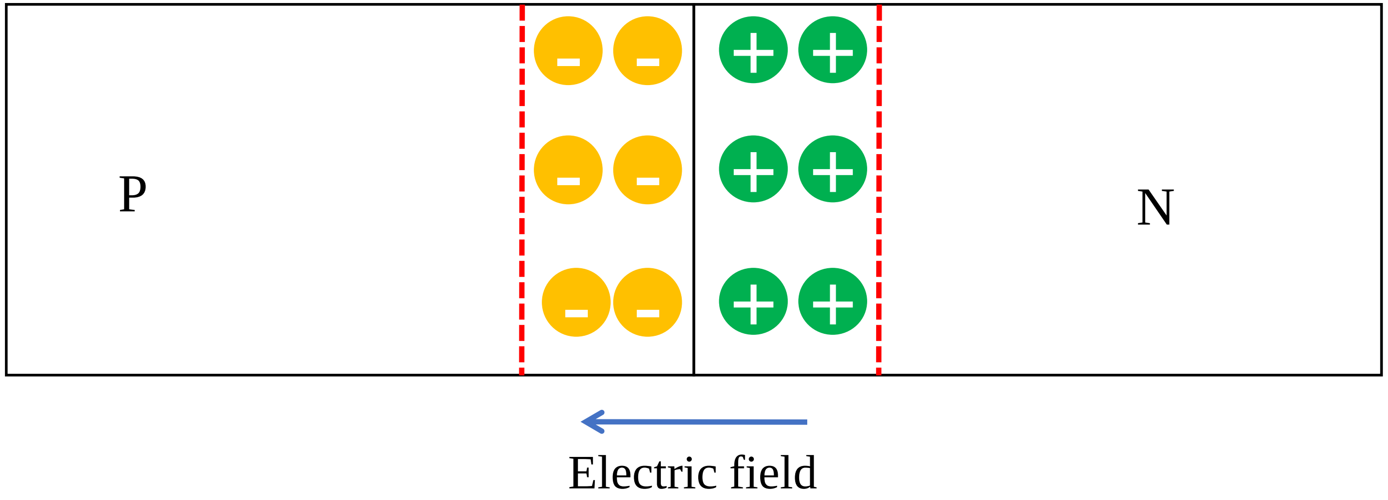
Vacuum diode used in radio as a rectifier

Semiconductor Diode (PN Junction Diode)

Light Emitting Diode (LED)

PN Junction Diode

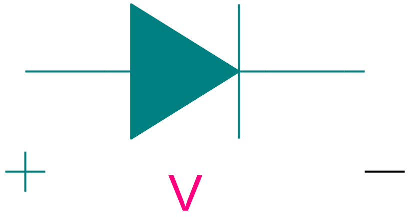


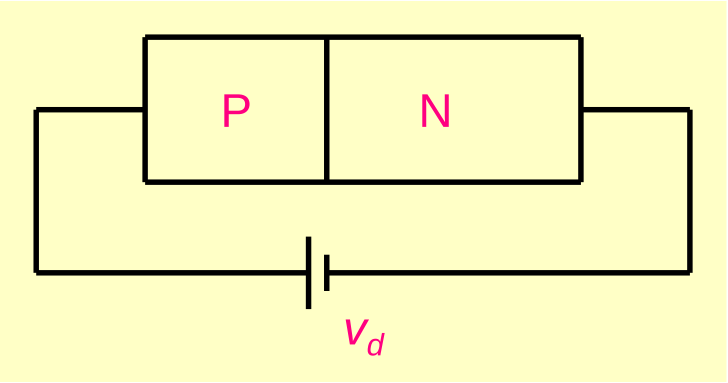


- Inside a PN junction at equilibrium (zero applied voltage), there is a built-in voltage.
- N region being positive and P-region negative, electric field from +ve to –ve
- The built-in voltage (potential barrier) prevents electrons and holes to give rise to current.

Forward and Reverse Bias

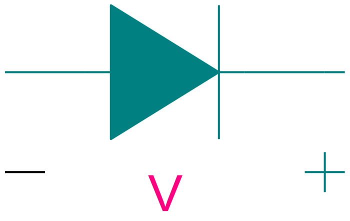
Forward Bias: P is biased at a higher voltage compared to N.

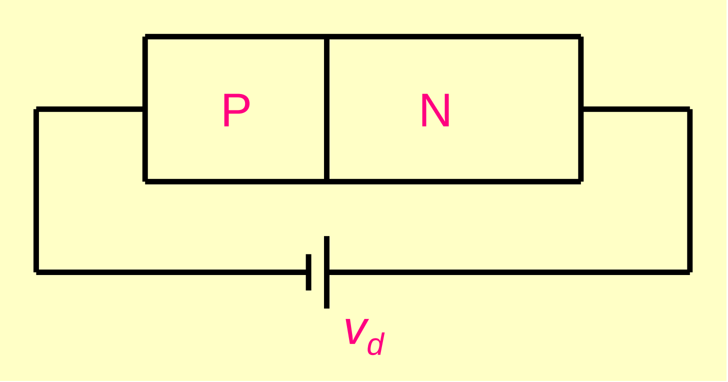




- This lowers the built-in potential and allows current to flow.

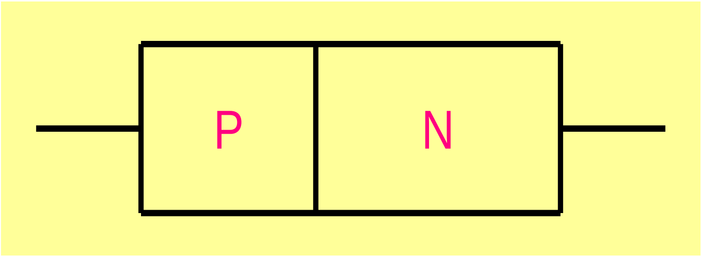
Reverse Bias: N is biased at a higher voltage compared to P.

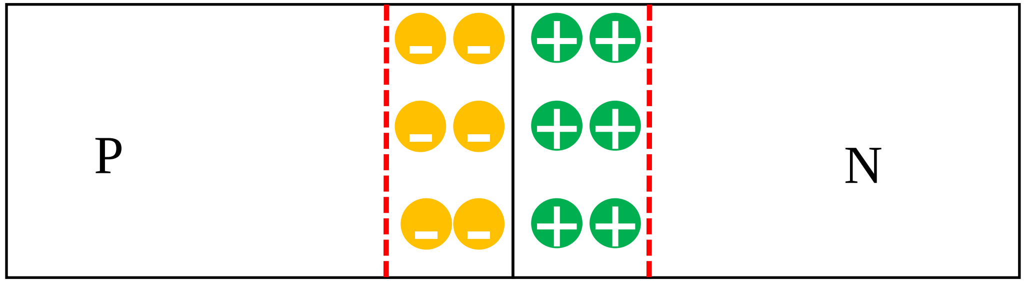


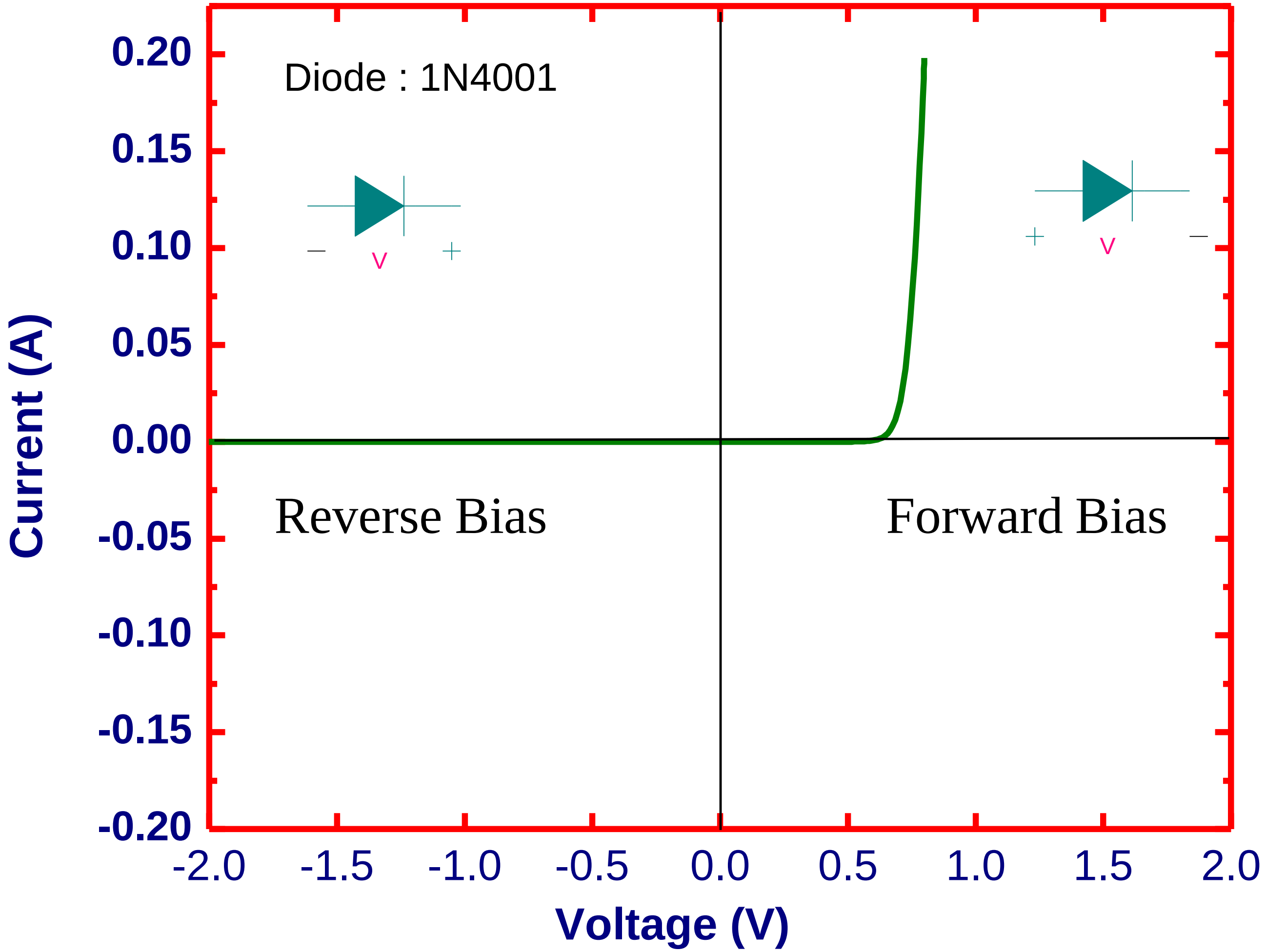


- This increases built-in potential and very little current flows.

PN Junction Diode: IV Characteristic







- The p-n junction only conducts significant current in forward-bias.

I-V Characteristics: Non-linear Behavior

Applied voltage $= v_D$

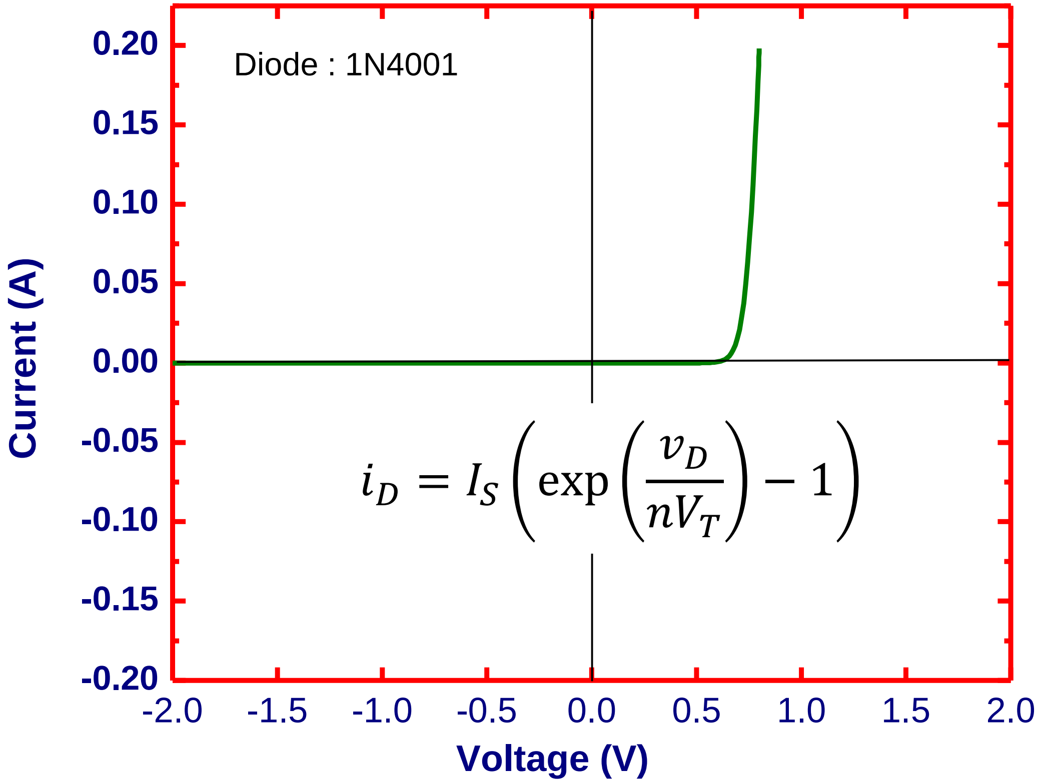
Diode current:

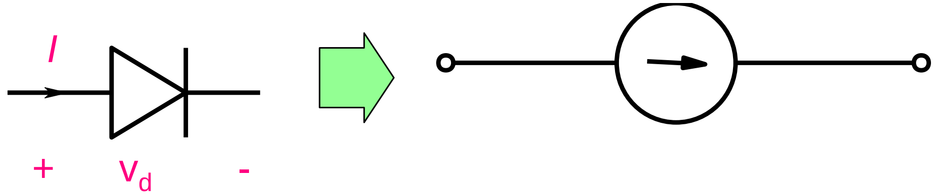
$$i_D = I_S\left(\exp\left(\frac{v_D}{nV_T}\right) - 1\right)$$

I_S : Reverse saturation current

n: ideality factor (= 1 for ideal diodes)

$$V_T = \frac{kT}{q} \approx 26mV \text{ at } T = 300K$$





- ❑ How to analyze circuits containing diodes?

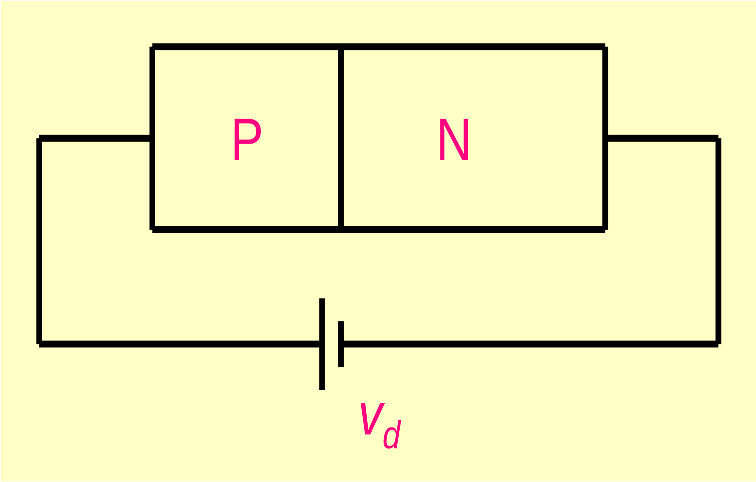
Forward and Reverse Bias

$$I_D = I_S\left(\exp\left(\frac{v_D}{V_T}\right) - 1\right)$$

- Forward Bias:

$$v_d >> V_T = 26mV$$

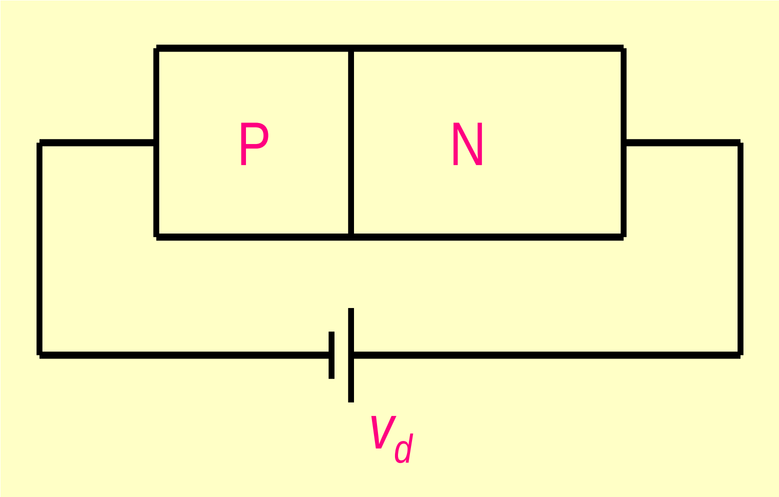
$$i_D \approx I_S \times \exp\left(\frac{v_d}{V_T}\right)$$



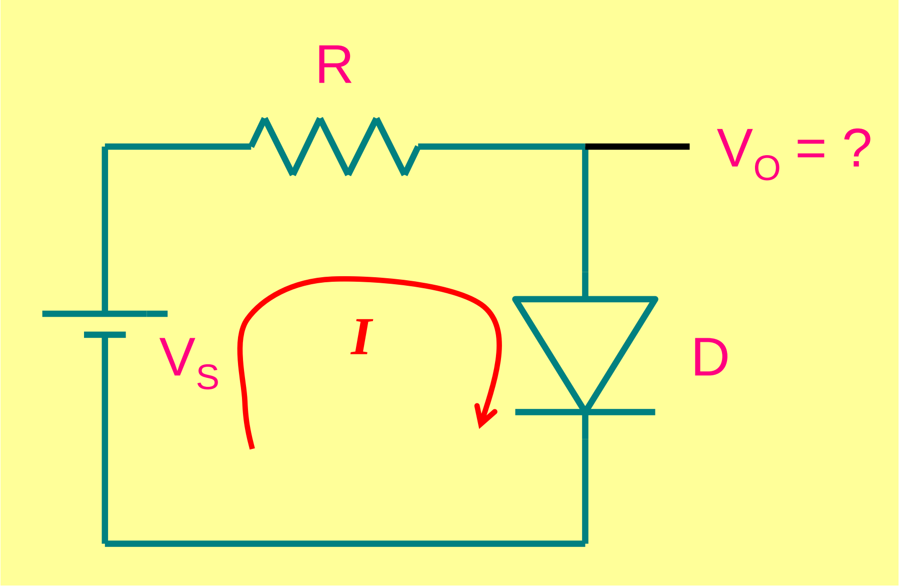
- Reverse Bias:

$$v_d = -v_R \qquad |v_R| >> V_T$$

$$i_D = I_S\left(\exp\left(-\frac{v_R}{V_T}\right) - 1\right) \approx -I_S$$



Circuit Analysis



$$V_S = IR + V_D$$

$$I = I_S\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right)$$

2 equations, 2 variables

Numerical methods can be used to solve

- Analysis using a non-linear diode model is relatively difficult and time consuming.

- It also does not always give a symbolic expression that can provide insight.

➢ Need SIMPLER and LINEAR Device Models.

The Design Style

Simple Models

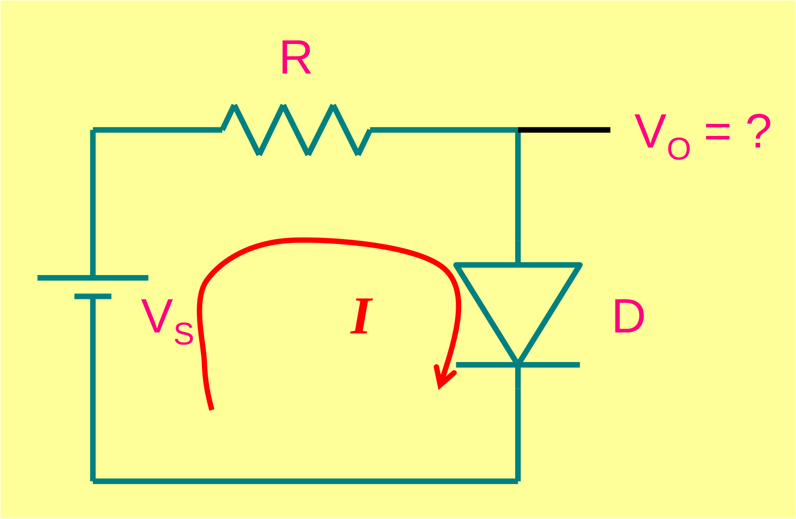
Design meets Specs.

DESIGN

Implement

SIMULATE

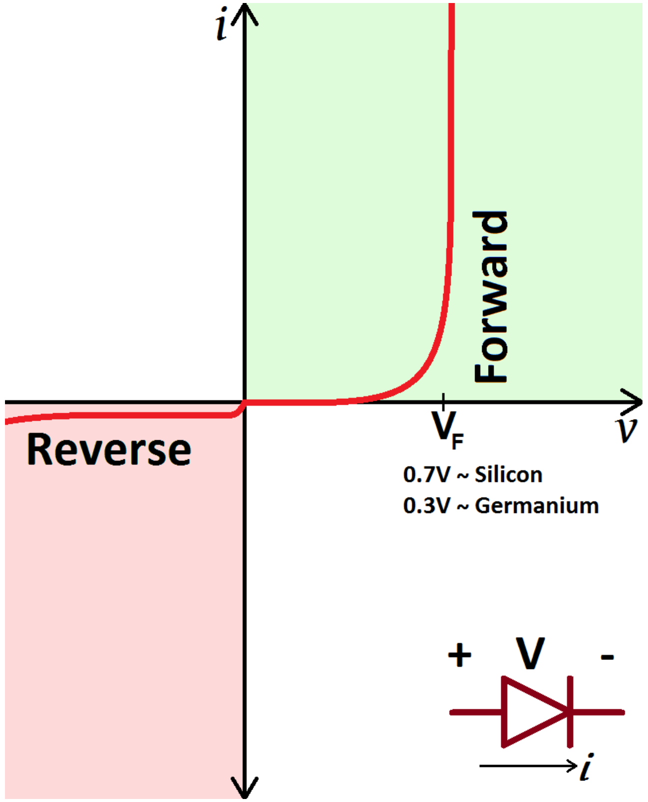
Method of Approximation



$$V_S = R\,I + V_D \qquad I = I_S\left(e^{\frac{V_D}{V_T}} - 1\right)$$

2 equations, 2 variables

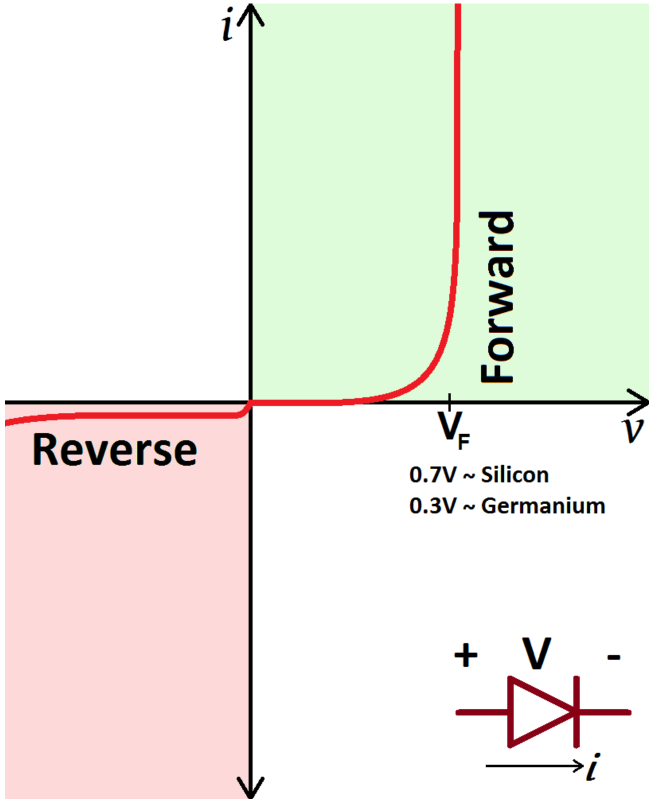
$$V_S = R I_S\left(e^{\frac{V_D}{V_T}} - 1\right) + V_D$$

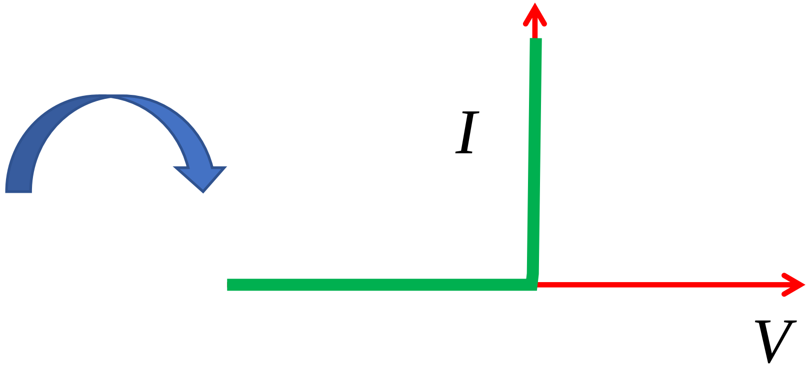


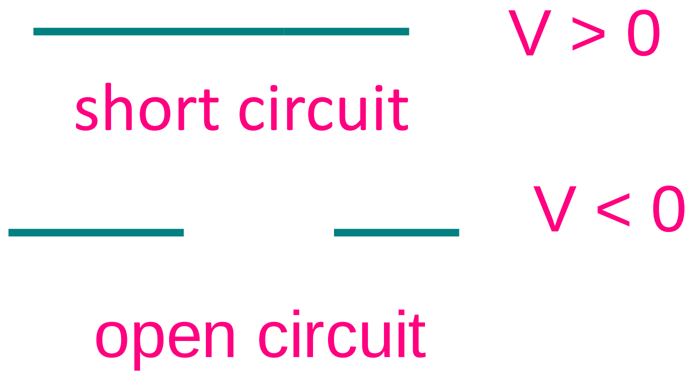
- Non-linear equation: How to solve?
  - Numerical methods, graphical method, analytical method, etc.
- We can however approximate its behavior with piecewise linear one
  - I-V graph is approximated by joining two or more straight lines

Model A: Ideal Diode (Unidirectional Device)

- Simplest possible approximation: acts as an ideal valve

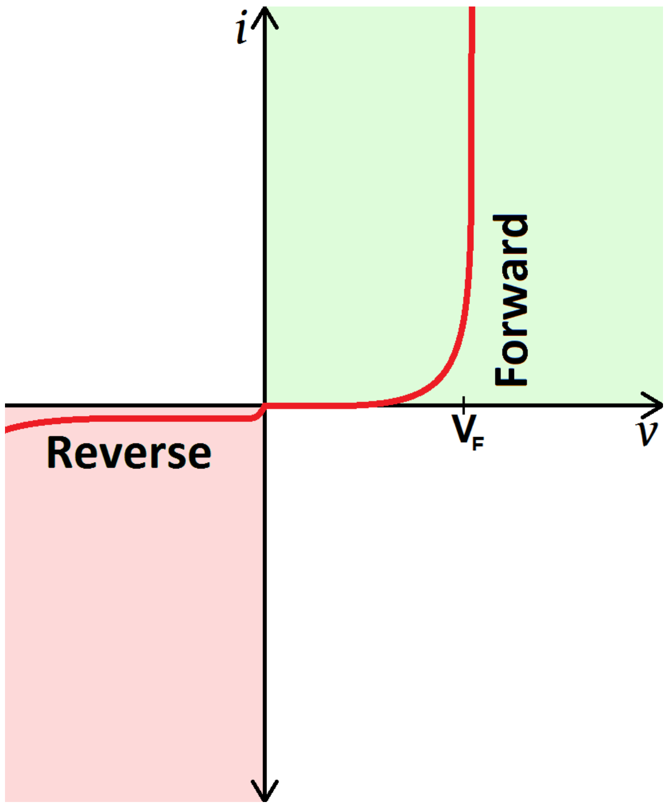


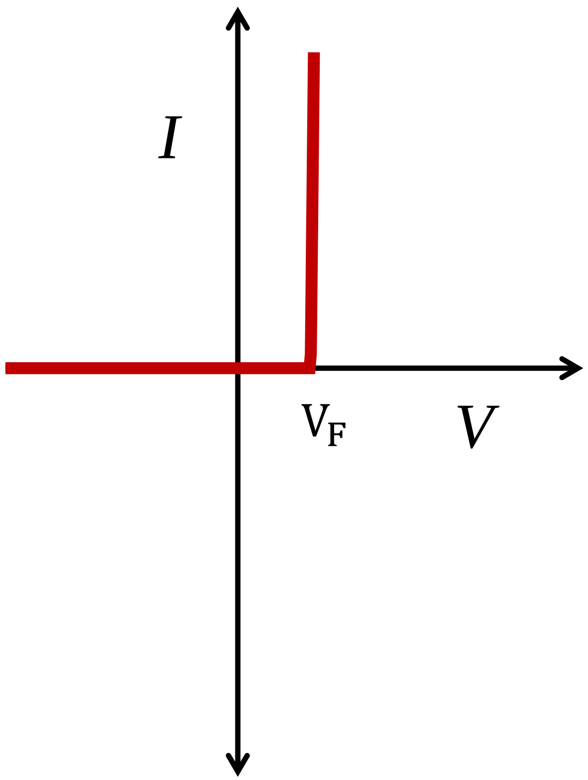


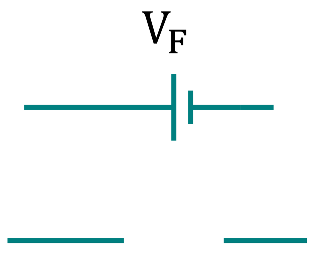


Model B: Adding a Constant Voltage Drop

- Simplest possible approximation: acts as an ideal valve with a drop in voltage







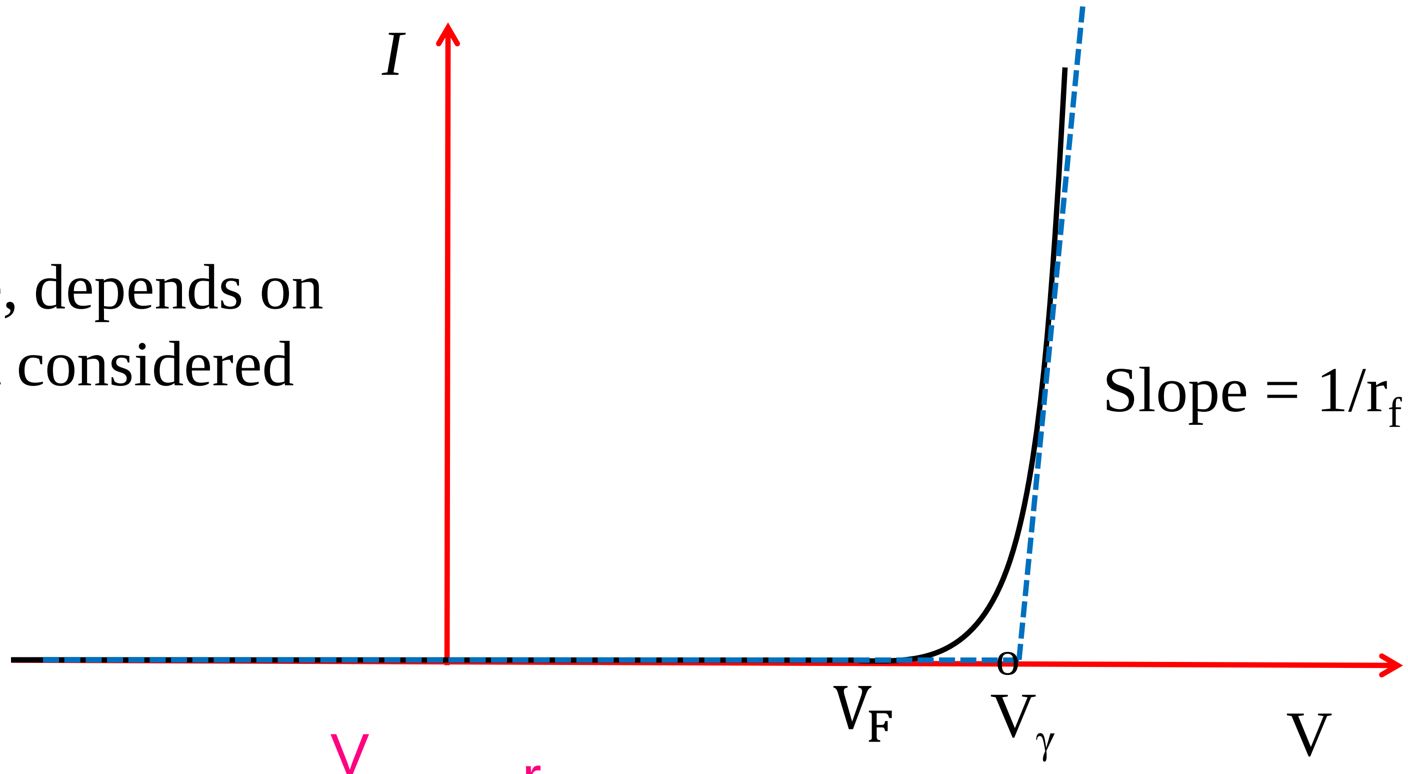
Model C: Adding a Series Resistance

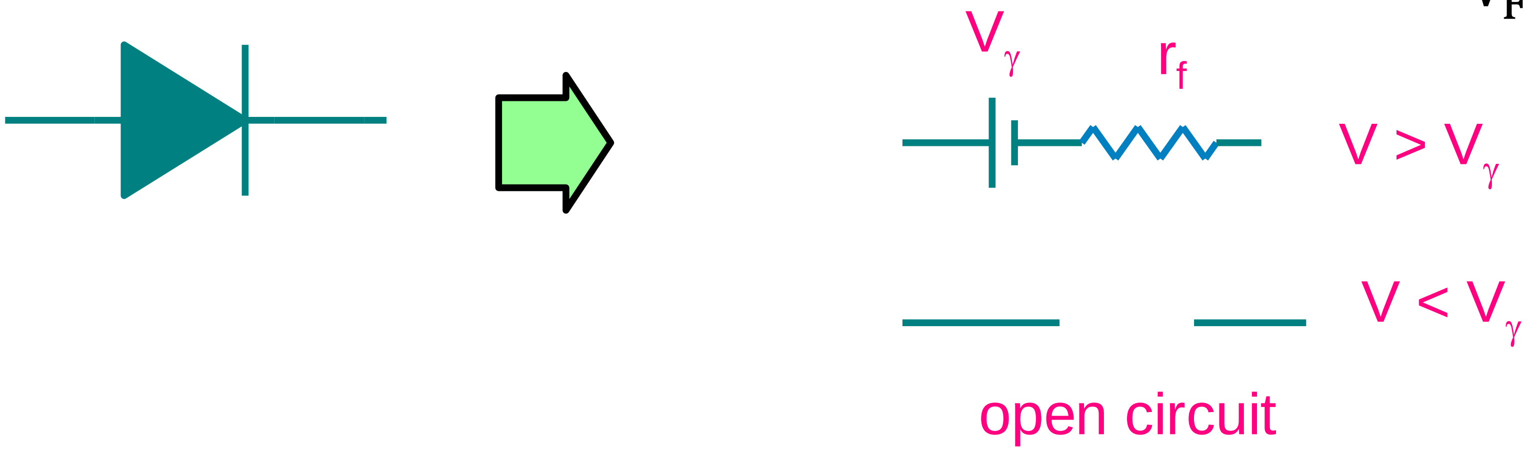
$$V = V_\gamma + I\, r_f$$

V_γ is called cut-in or turn-in voltage, depends on nature of diode and range of current considered

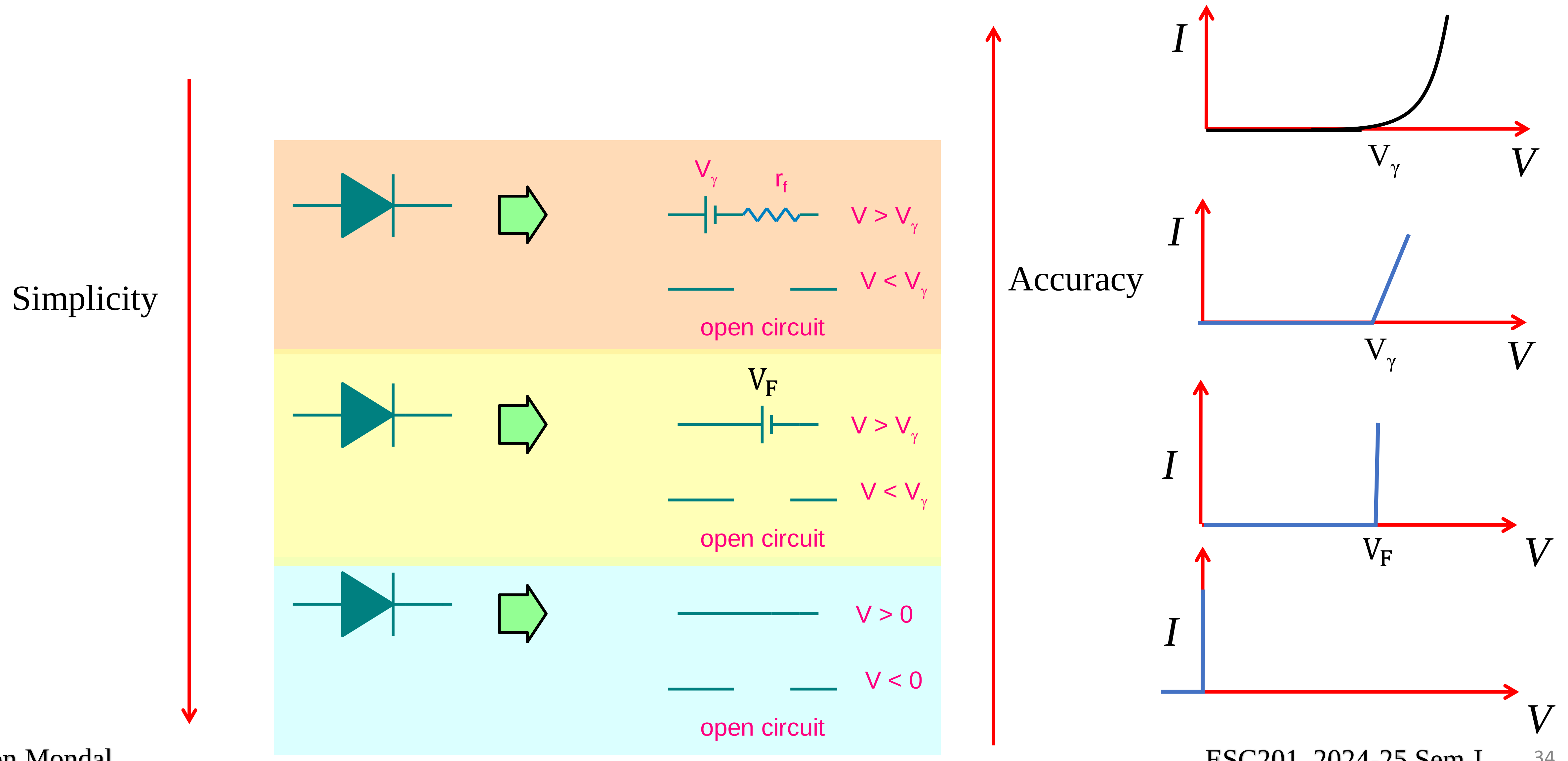
For example:

$V_\gamma = 0.7\text{V}$ and $r_f \sim 10\Omega$



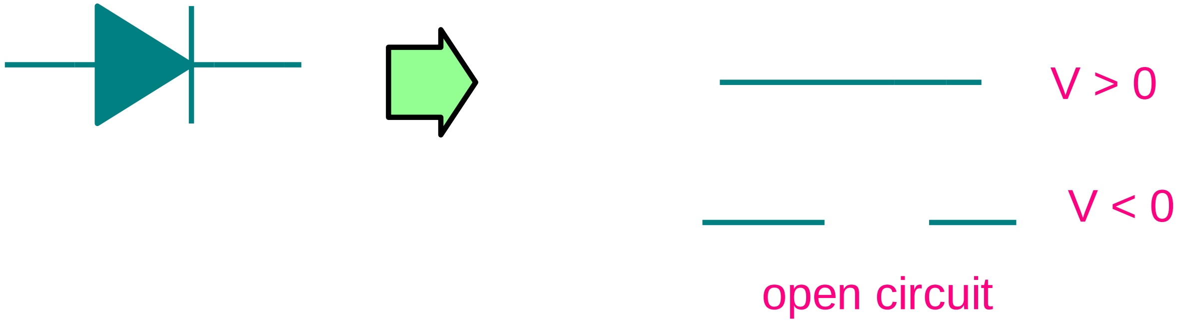


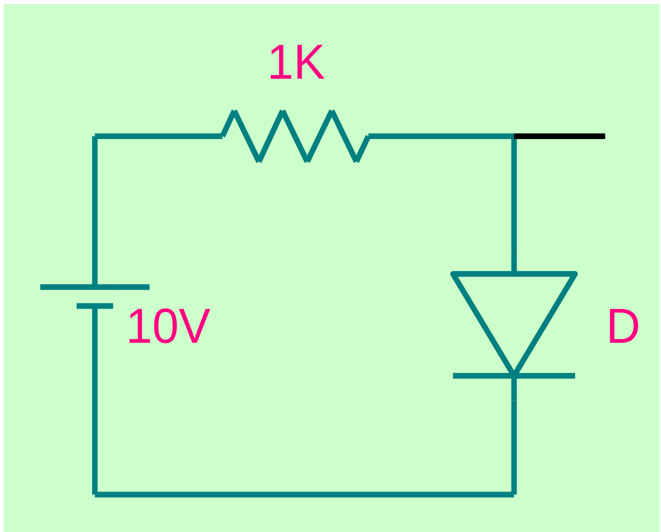
Summary: Approximate Diode Models

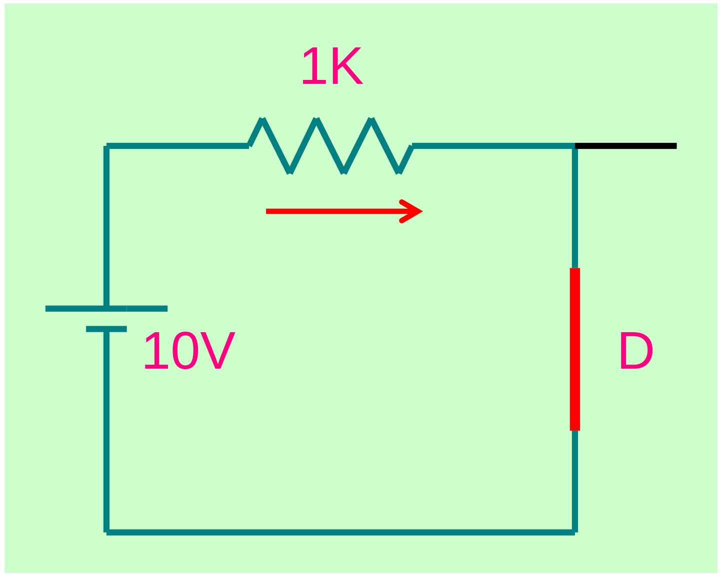


Circuit Analysis: Example 1

Analysis using ideal diode model



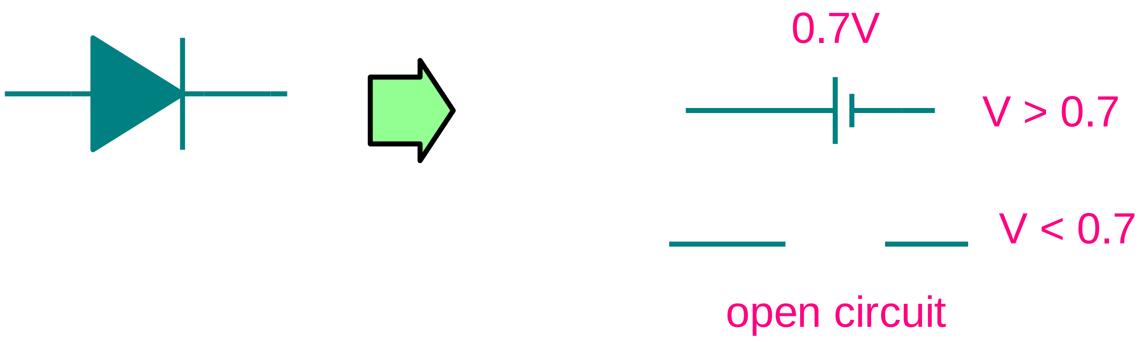


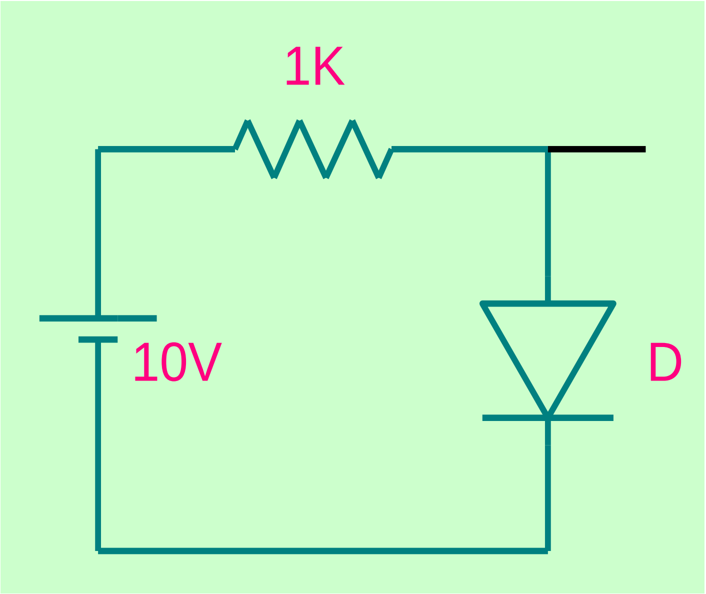


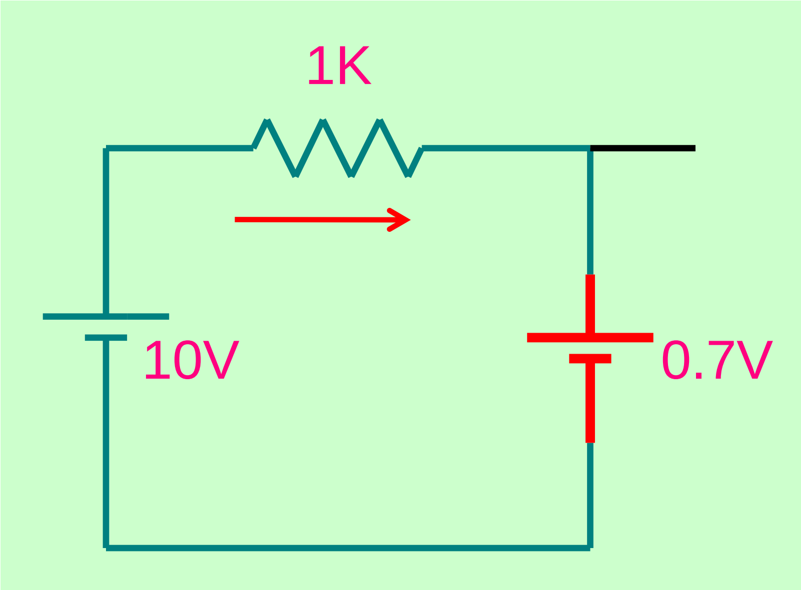
$$I = \frac{10}{1k} = 10mA$$

Circuit Analysis: Example 1

Analysis with a constant voltage diode model



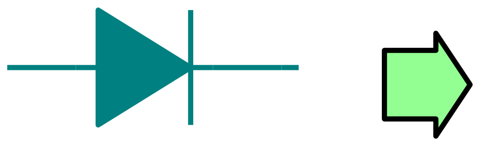


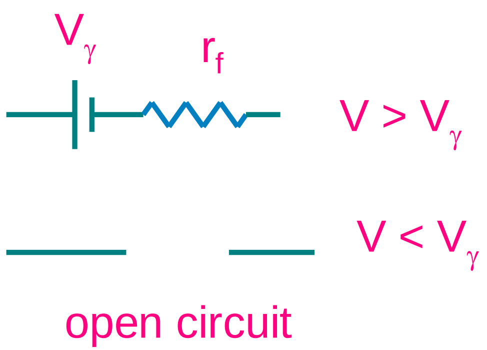


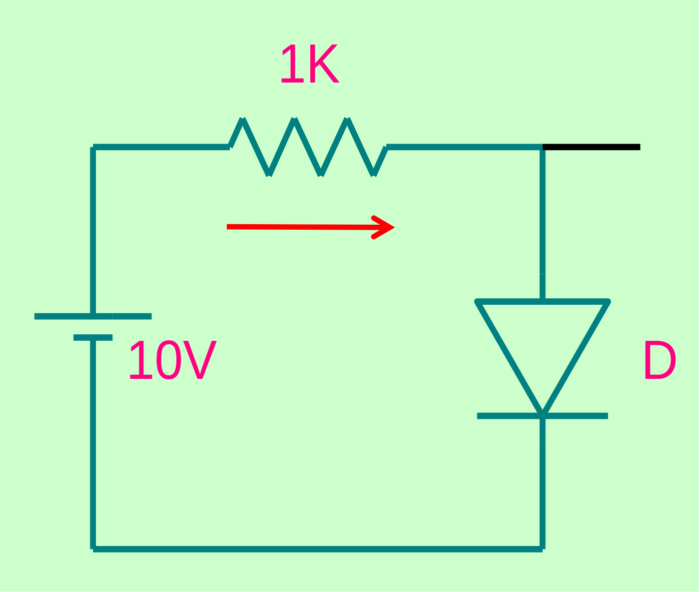
$$I = \frac{10 - 0.7}{1k} = 9.3mA$$

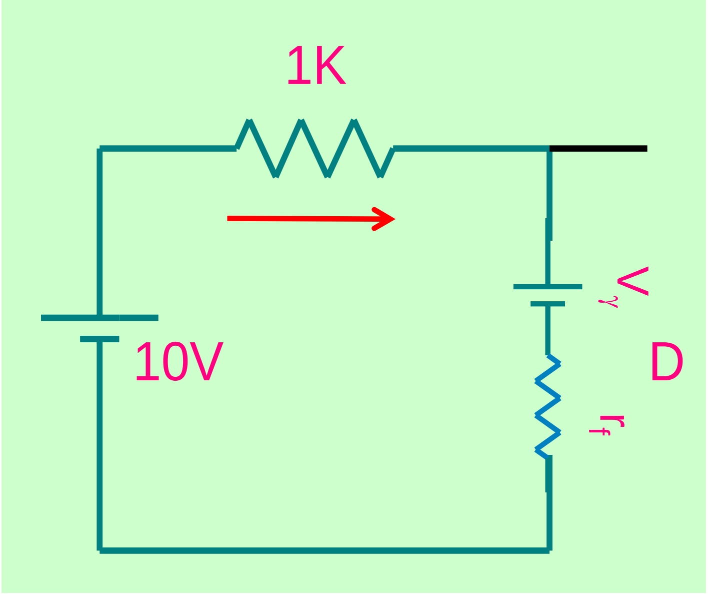
Circuit Analysis: Example 1

Analysis with a constant voltage plus series resistor diode model









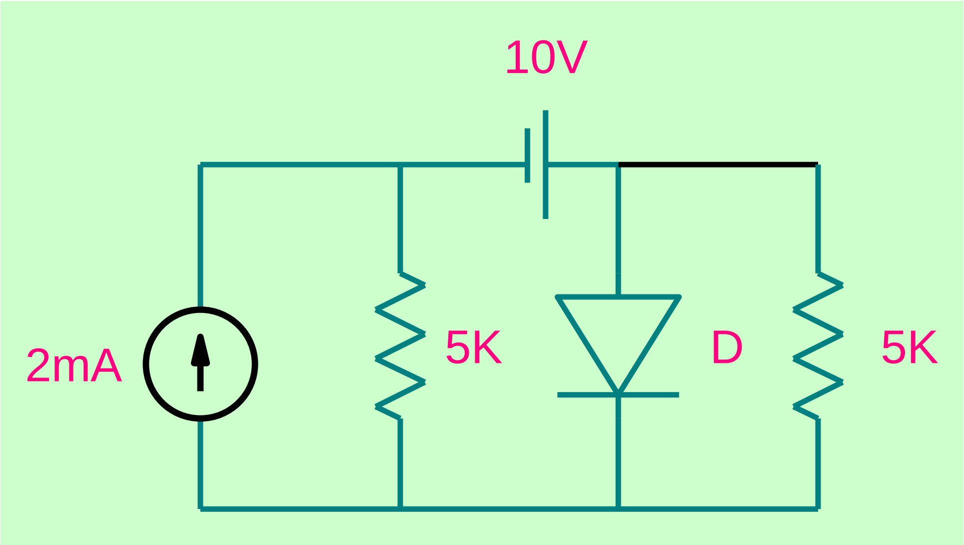
$$I = \frac{10 - 0.7}{1000 + 10} = 9.208mA$$

Self-Consistent Analysis

- How to know in which state diode is?
  - Easier if the voltage is known.
  - Otherwise
  - Analyze circuit assuming diode is forward biased
    - Check assumption ($I > 0$?)
  - Analyze circuit assuming diode is reverse biased
    - Check assumption ($V < 0$?)
  - Select the consistent one.
- What if 2 diodes: 4 possible circuits, only 1 will be valid
- N diodes => 2^N circuits, only one will be valid

Circuit Analysis: Example 2

Find the current through the 5k resistor using ideal diode model



Is the diode forward biased? – Not Sure!!

Assume that it is forward biased

Carry out analysis and then check if current through the diode is in **appropriate** direction.

If not, diode is reverse biased and we carry out the analysis again!!

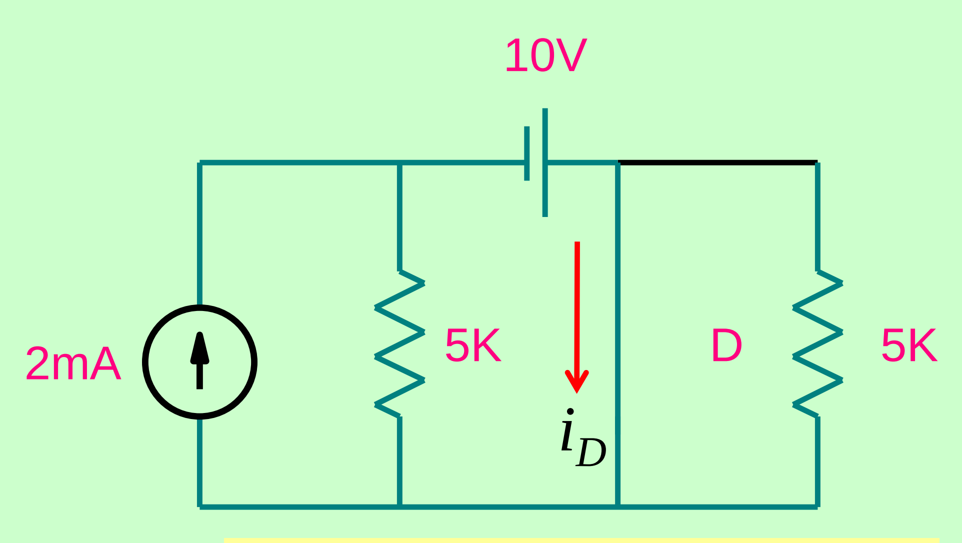
Circuit Analysis: Example 2

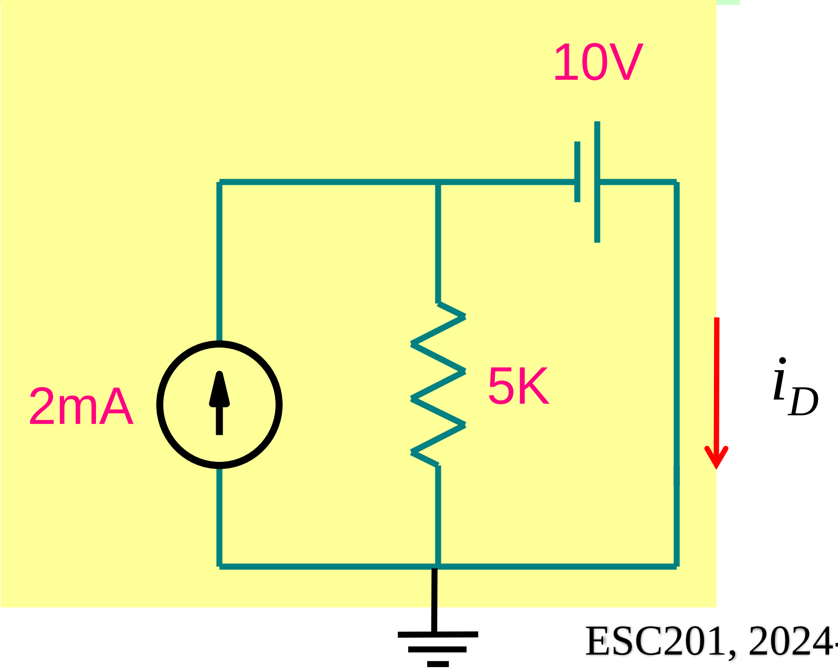
Assume forward bias

$$-2\text{ mA} + \frac{-10\text{ V}}{5\ k\Omega} + i_D = 0$$

$$i_D = 4\text{ mA}$$

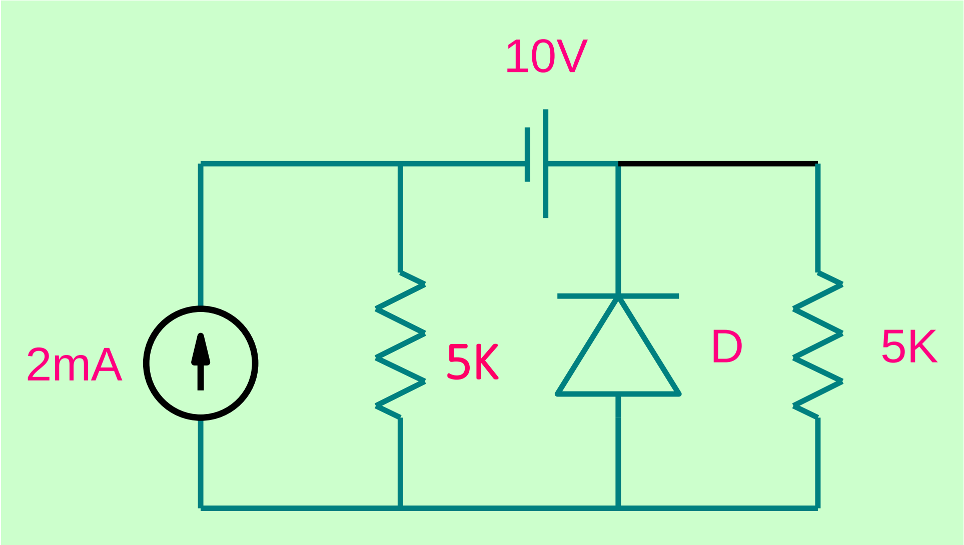
Current is positive, so our assumption is correct!



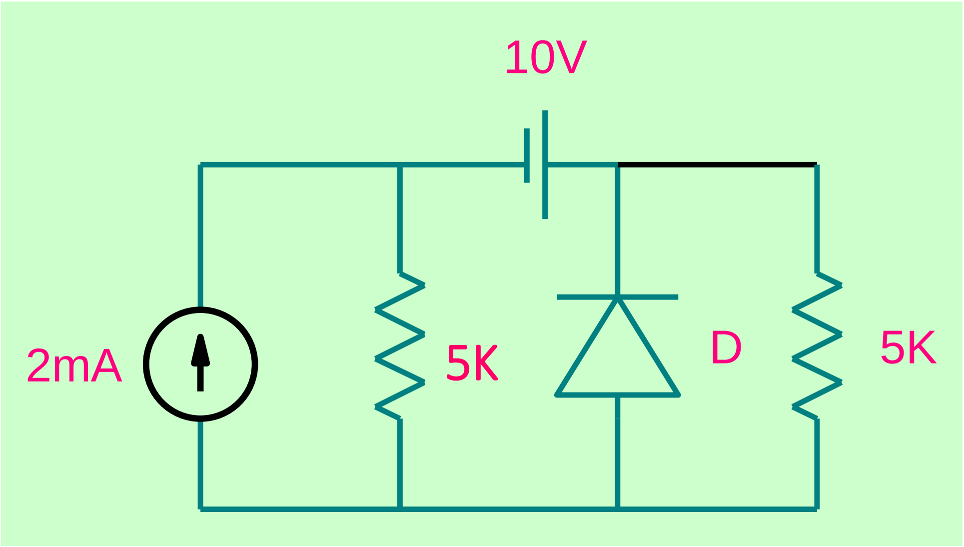


Circuit Analysis: Example 3

Find the current through the 5k resistor using ideal diode model

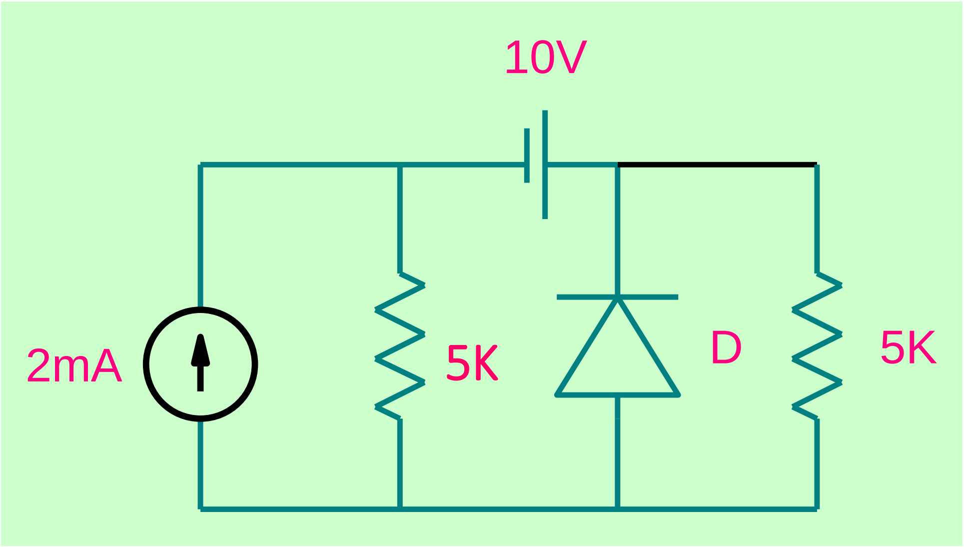


Is the diode forward biased? – Not Sure!!


10V
2mA
5K
D
5K

Circuit Analysis: Example 3

Find the current through the 5k resistor using ideal diode model



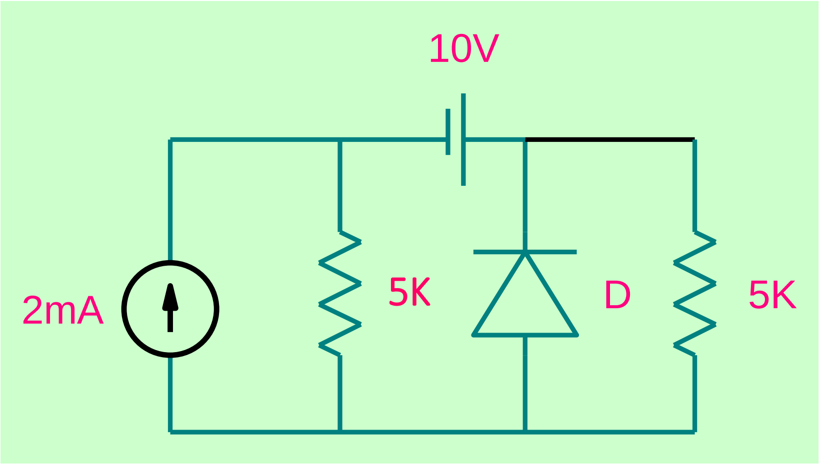
Is the diode forward biased? – Not Sure!!

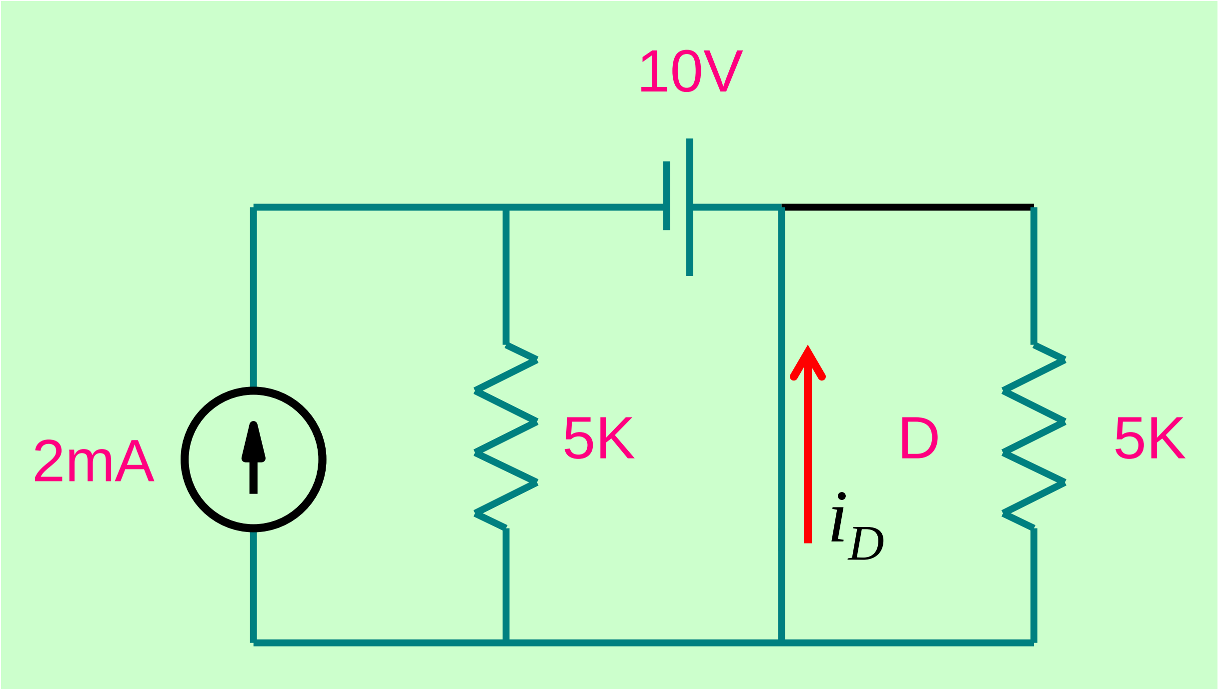
Assume that it is forward biased

Carry out analysis and then check if current through the diode is in **appropriate** direction.

If not, diode is reverse biased and we carry out the analysis again!!

Circuit Analysis: Example 3





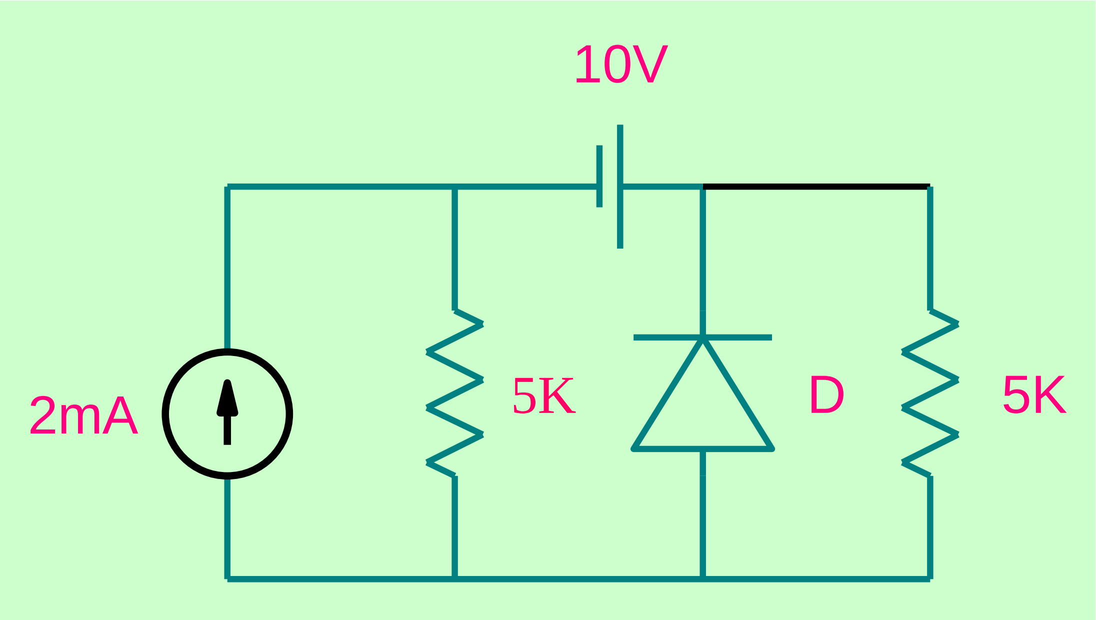
Assume forward bias

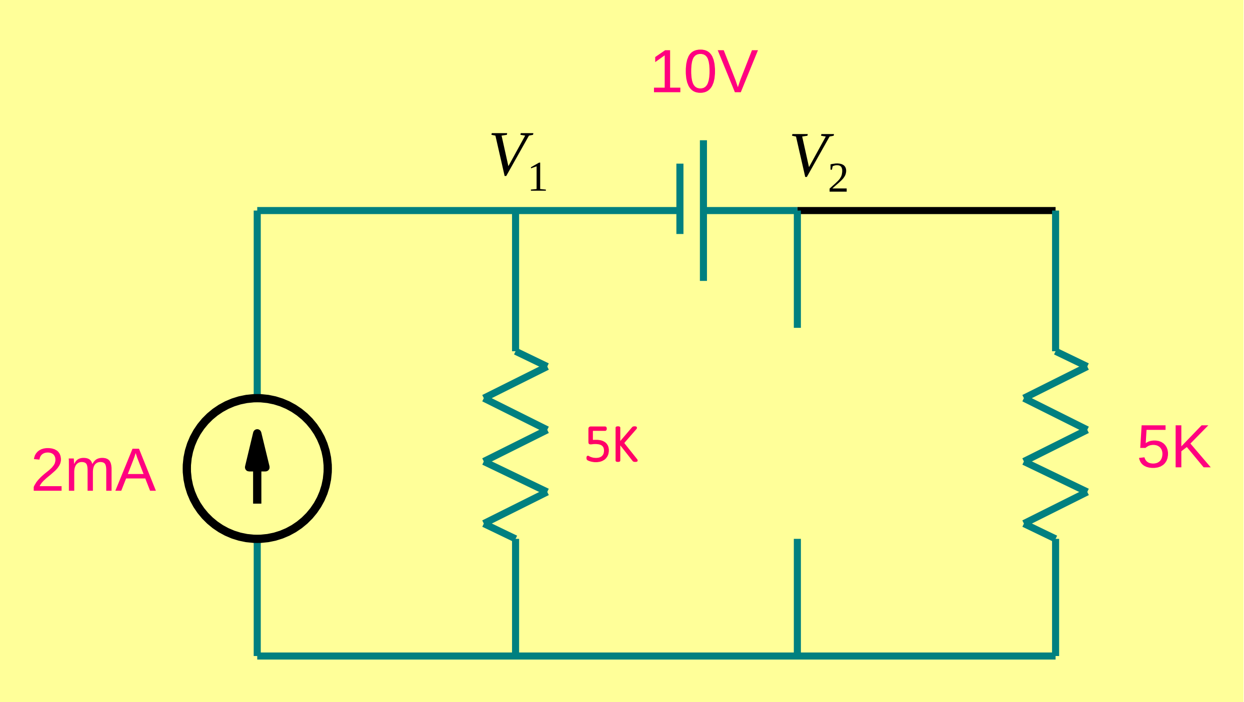
$$-2\,m + \frac{-10}{5\,k} - i_D = 0$$

$$i_D = -4\text{ mA}$$

Therefore, our assumption is incorrect ☹

Circuit Analysis: Example 3





Assume reverse bias

$$-2\,m + \frac{V_1}{5\,k} + \frac{V_1 + 10}{5\,k} = 0$$

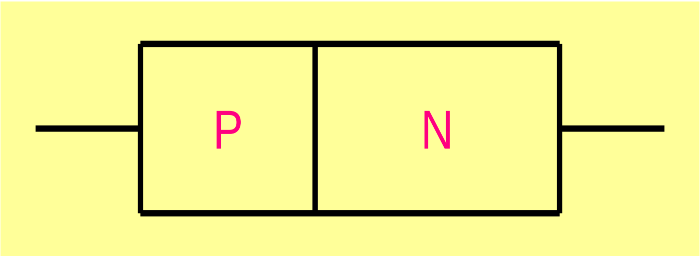
$$V_1 = 0$$

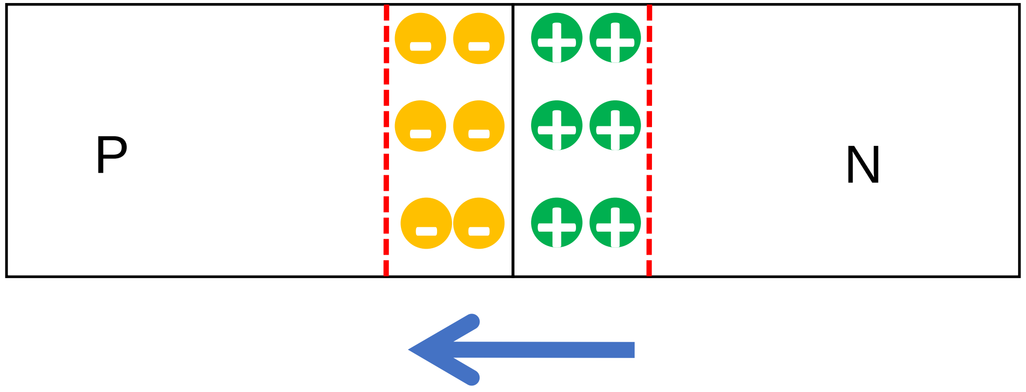
$$V_2 = 10V$$

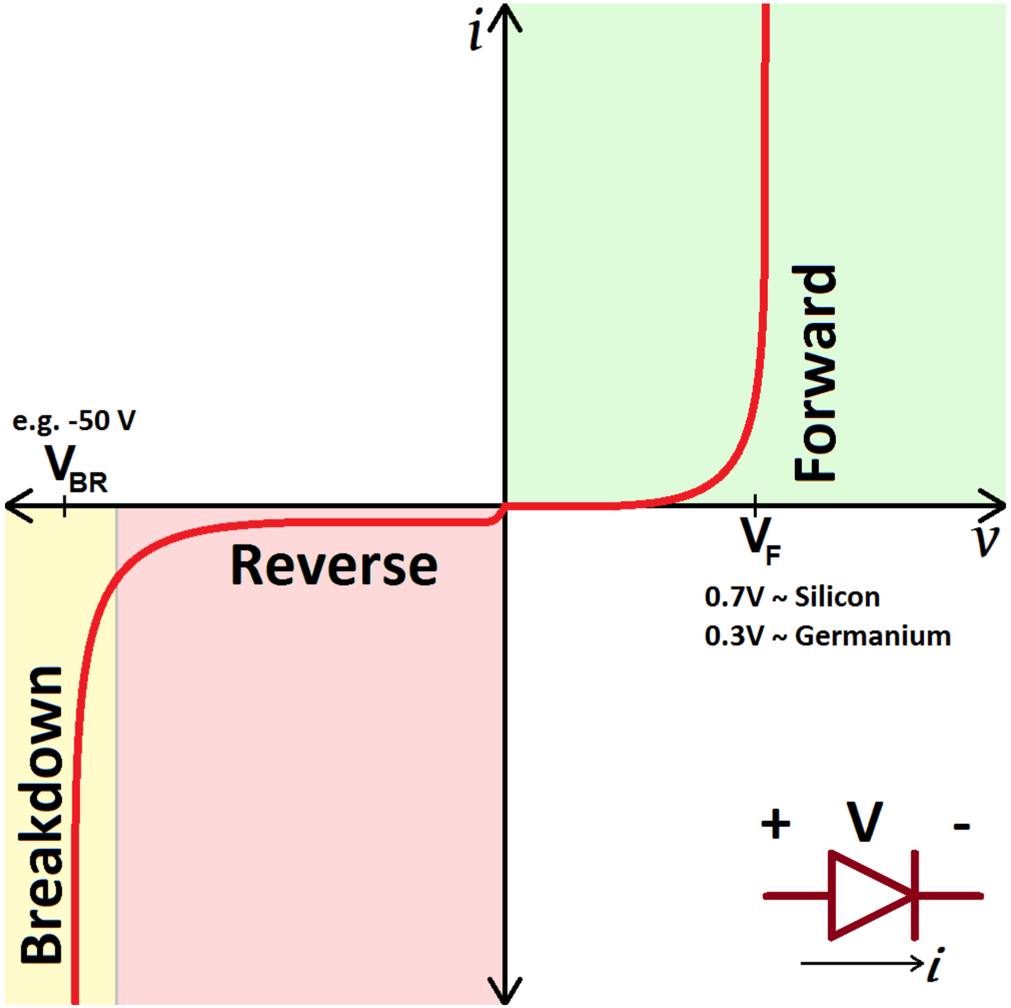
Therefore, our assumption is correct ☺

$I_{5K} = 2$ mA

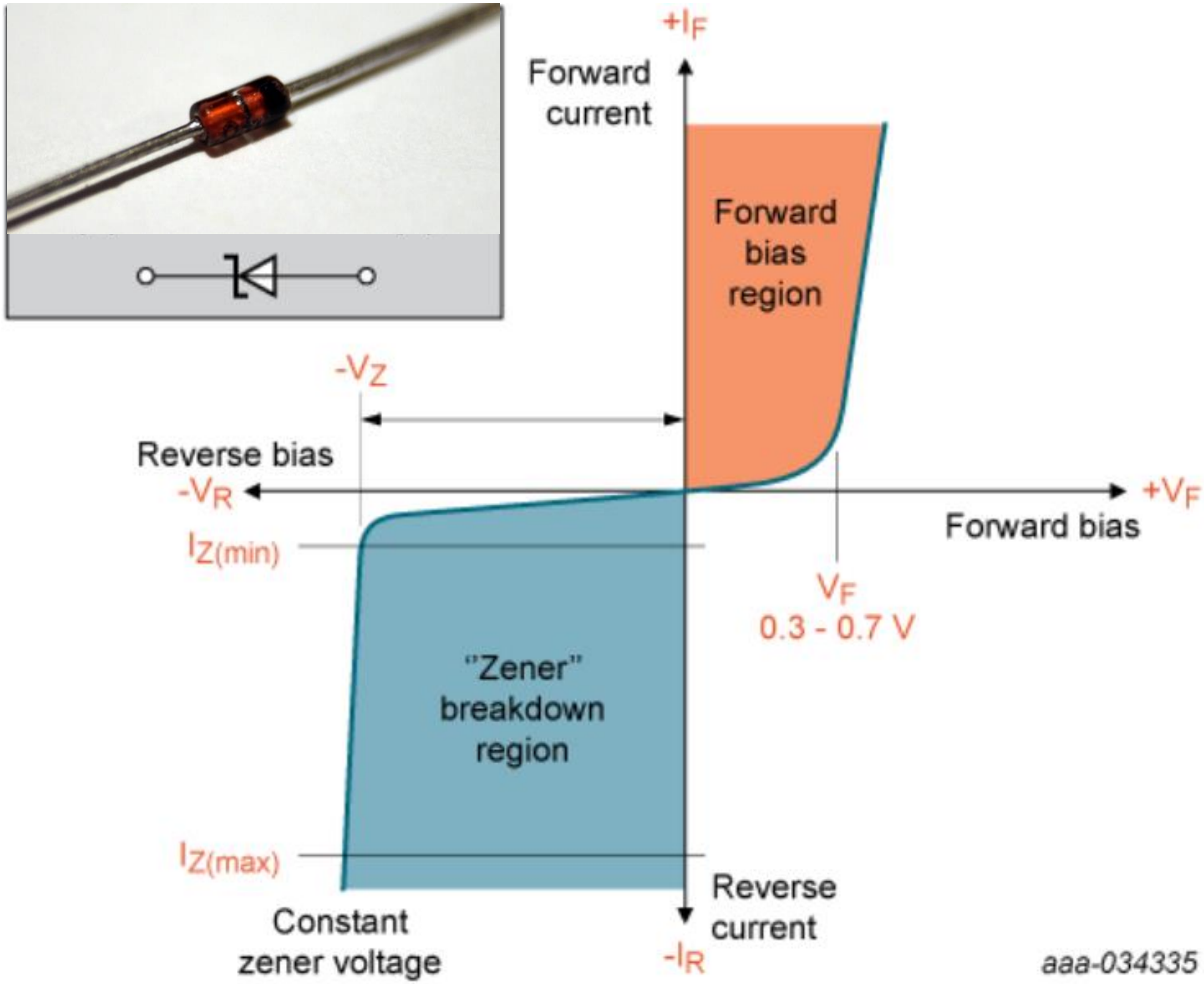
Breakdown


P
N

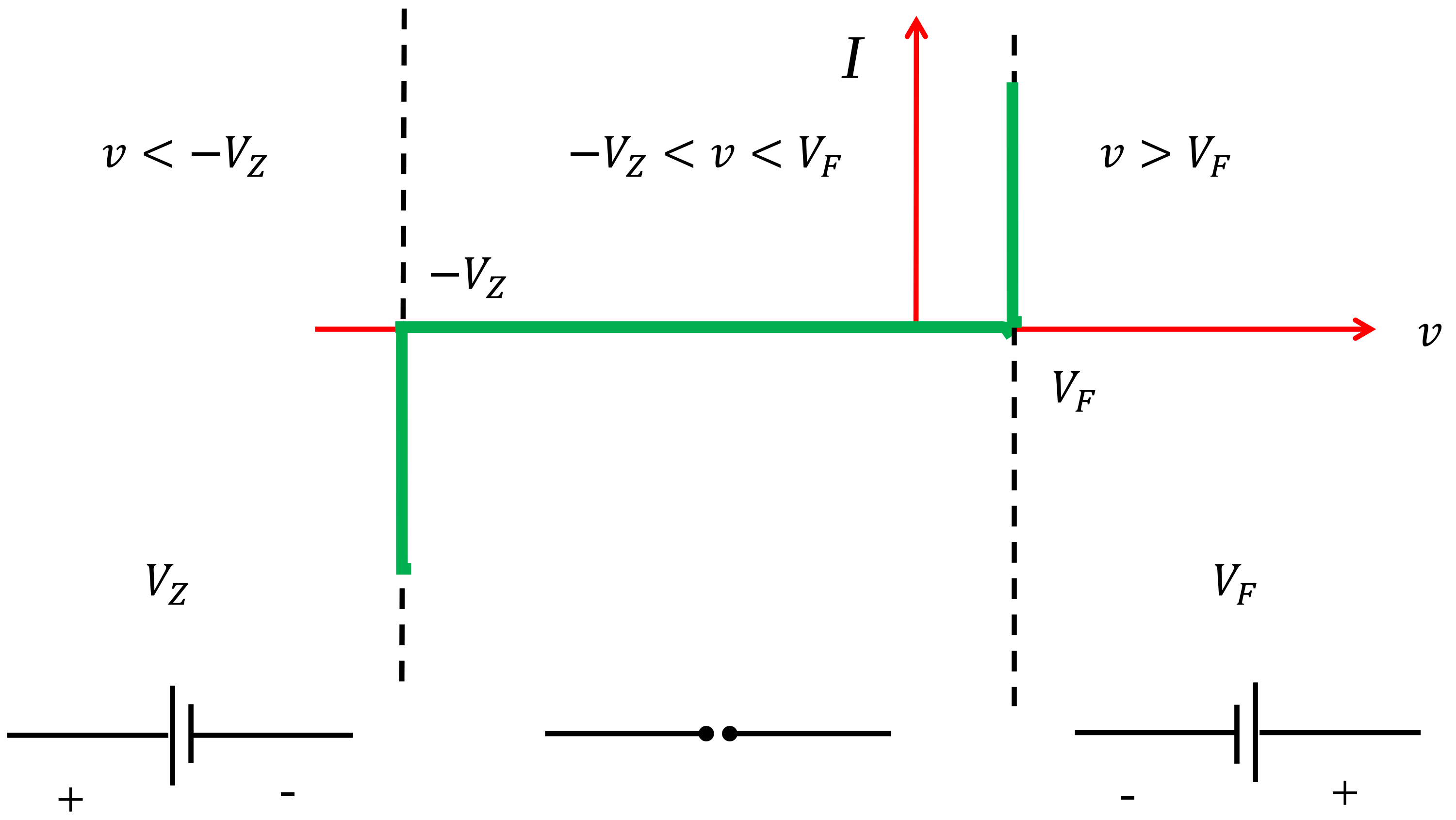

P
N
Electric field


i
e.g. -50 V
V_BR
Forward
Reverse
Breakdown
V_F
v
0.7V ~ Silicon
0.3V ~ Germanium
+ V -
i

Leveraging Breakdown: Zener Diode



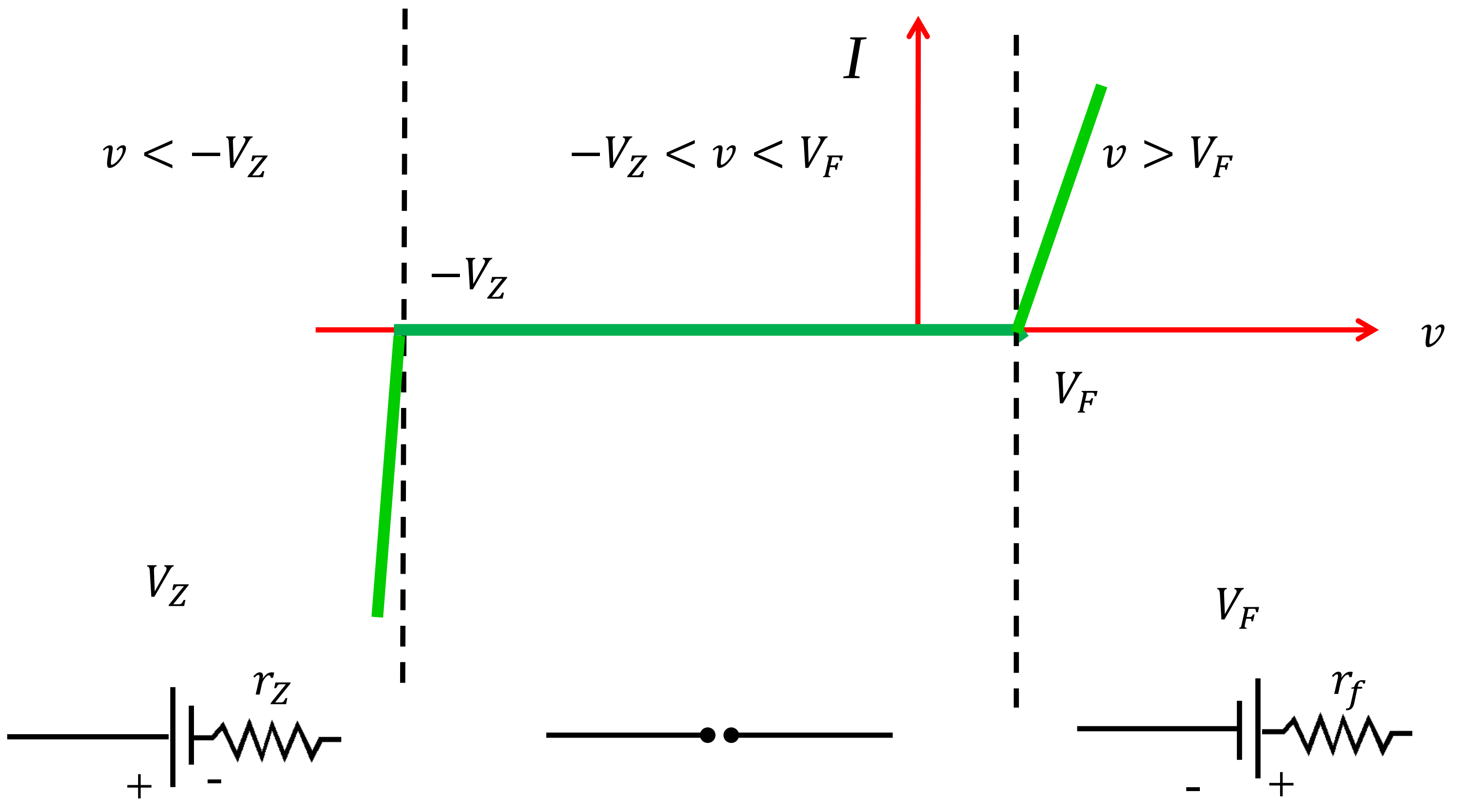
Zener Diode: A Simple Piecewise Linear Model



Zener diode does not allow the voltage across it to go beyond the range $[-V_Z, V_F]$

Method of assumed states: 3 possibilities now

Zener Diode: A Little Complex Piecewise Linear Model



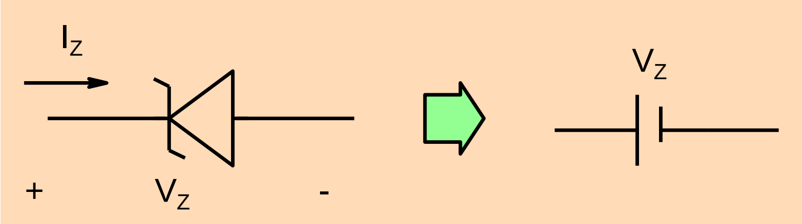
Typically, both forward and breakdown resistances are a few ohms

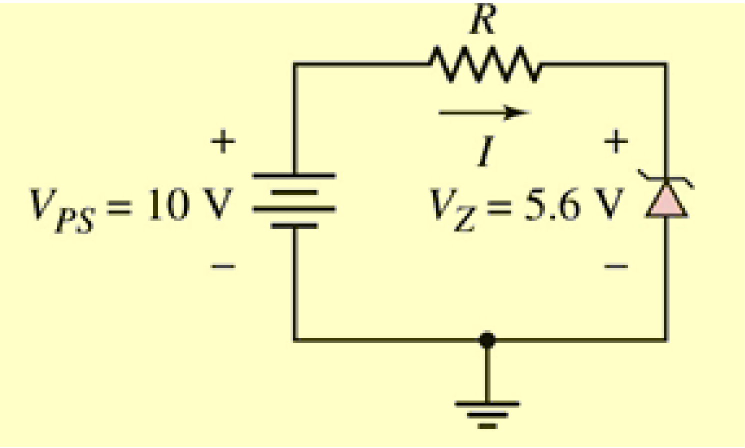
Zener diodes have limited current carrying capacity: power rating

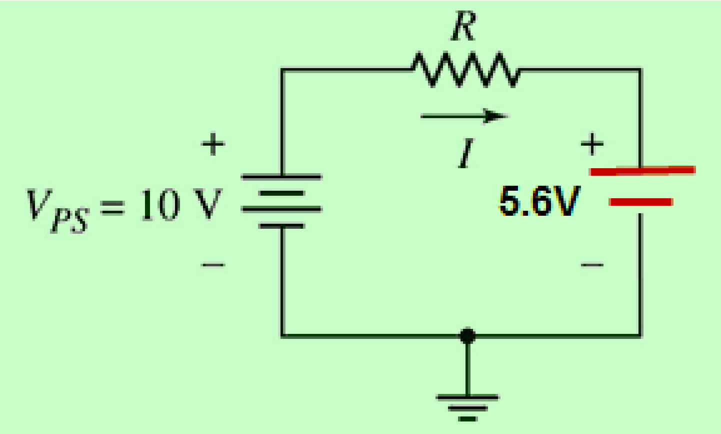
Circuit Analysis: Example 4

Find R such that current through diode is limited to 3 mA

Assume breakdown region







$$RI = V_{PS} - V_z$$

$$R = \frac{V_{PS} - V_z}{I} = \frac{(10 - 5.6)V}{3\text{ mA}} = 1.47\text{ k}\Omega$$

$$I < 3\text{ mA} \Rightarrow R > 1.47\text{k}\Omega$$

Summary: Zener Diode I-V Characteristics