ESC201: Lecture 15

Dr. Imon Mondal

Assistant Professor,
Electrical Engineering, IIT Kanpur

Goal of Simplification

In the SOP expression:

1. Minimize number of product terms

2. Minimize number of literals in each term

$$\text{Simplification} \Rightarrow \text{Minimization}$$

Minimization

$$y = \overline{x_1}\,.\,\overline{x_2}\,.\,x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

$$y = \overline{x_1}\,.x_3.(\overline{x_2} + x_2) + x_1.x_3.(\overline{x_2} + x_2)$$

$$y = \overline{x_1}\,.x_3 + x_1.x_3$$

$$y = (\overline{x_1} + x_1).x_3$$

$$y = x_3$$

Principle used: $x + \overline{x} = 1$

$$f = \bar{x}.\bar{y} + \bar{x}.y + x.\bar{y}$$

Apply the Principle: $x + \bar{x} = 1$ to simplify

$$f = \bar{x}.(\bar{y} + y) + x.\bar{y}$$

$$f = \bar{x} + x.\bar{y}$$

How do we simplify further?

$$f = \bar{x}.\bar{y} + \bar{x}.y + x.\bar{y} = \bar{x}.\bar{y} + \bar{x}.\bar{y} + \bar{x}.y + x.\bar{y}$$

Principle used : $x + x = x$

$$f = \bar{x}.\bar{y} + \bar{x}.y + \bar{x}.\bar{y} + x.\bar{y}$$

$$= \bar{x}.(\bar{y} + y) + (\bar{x} + x).\bar{y} = \bar{x} + \bar{y}$$

Simplify

$$f = \overline{x_1}.x_2.\overline{x_3}.x_4 + \overline{x_1}.x_2.x_3.x_4 + x_1.x_2.\overline{x_3}.x_4 + x_1.x_2.x_3.x_4 +$$
$$x_1.\overline{x_2}.\overline{x_3}.x_4 + x_1.\overline{x_2}.x_3.x_4$$

Principle: $x + \overline{x} = 1$ and $x + x = x$

Need a systematic and simpler method for applying these two principles

Karnaugh Map (K map) is a popular technique for carrying out simplification

It represents the information in problem in such a way that the two principles become easy to apply

K-map representation of truth table

x	y	min term
0	0	$\bar{x} . \bar{y}$ m0
0	1	$\bar{x} . y$ m1
1	0	$x . \bar{y}$ m2
1	1	$x . y$ m3

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x \ y	0	1
0	0	1
1	1	0

$$f_2 = \sum(0,2,3)$$

$\Rightarrow$

x \ y	0	1
0	~~0~~ 1	1 0
1	1	1

x \ y	0	1
0	1	0
1	0	1

$\Rightarrow$

$$f = \bar{x}.\bar{y} + x.y$$

Minimization using Kmap

$$f_2 = \sum(2,3)$$

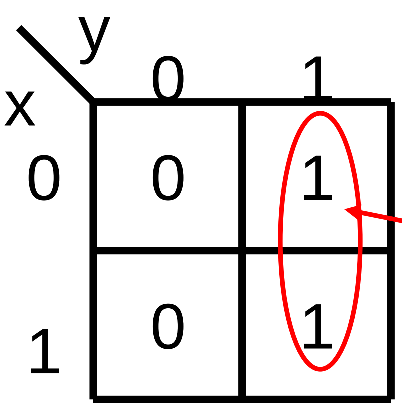
x \ y	0	1
0	0	0
1	1	1

$$f = x.\overline{y} + x.y$$

$$f = x.(\overline{y} + y)$$

$$f = x$$

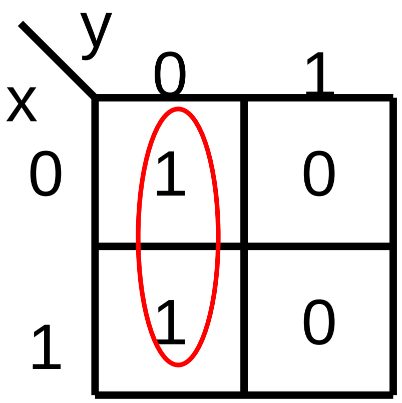
Combine terms which differ in only one bit position. As a result, whatever is common remains.



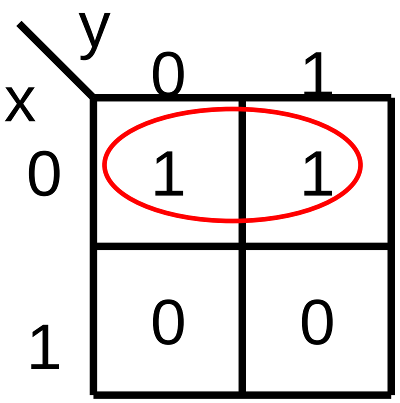
$$f = \bar{x}.y + x.y$$

$$f = (\bar{x} + x).y$$

$$\Rightarrow f = y$$



$$\Rightarrow f = \bar{y}$$



$$\Rightarrow f = \bar{x}$$

$$f_2 = \sum(\overset{1}{\cancel{0}}, 2, 3)$$

x \ y	0	1
0	0	1
1	1	1

$A + A = A$

$$f = x.\bar{y} + x.y + \bar{x}.y$$

$$f = x.(\bar{y} + y) + \bar{x}.y$$
$$= x + \bar{x}.y$$

$$f = x + \bar{x}.y + x.y$$
$$= x + (\bar{x} + x).y$$
$$= x + y$$

The idea is to cover all the 1's with as few and as simple terms as possible

3-variable K-map representation

x	y	z	min terms	
0	0	0	$\bar{x} . \bar{y} . \bar{z}$	m0
0	0	1	$\bar{x} . \bar{y} . z$	m1
0	1	0	$\bar{x} . y . \bar{z}$	m2
0	1	1	$\bar{x} . y . z$	m3
1	0	0	$x . \bar{y} . \bar{z}$	m4
1	0	1	$x . \bar{y} . z$	m5
1	1	0	$x . y . \bar{z}$	m6
1	1	1	$x . y . z$	m7

x \ yz	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$\mathrm{f} = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.z + x.\bar{y}.z + x.y.z$$

3-variable minimization

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$f = \overline{x}.\overline{y}.\overline{z} + \overline{x}.y.z + x.y.z + x.\overline{y}.z$$

$y.z$

$x.z$

$$f = \overline{x}.\overline{y}.\overline{z} + y.z + x.z$$

3-variable minimization

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$f = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.\bar{z} + x.y.z + x.\bar{y}.z$$

$\bar{x}.\bar{z}$

$x.z$

$$f = \bar{x}.\bar{z} + x.z$$

3-variable minimization

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$f = x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z + x.y.\bar{z}$$

$x.\bar{y}$

$x.y$

$$f = x.\bar{y} + x.y$$

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$f = x.(\bar{y} + y) = x$$

x

x \ yz	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$\bar{x}$

x \ yz	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$\bar{z}$

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

z

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

$\bar{z}$

x

$$f = x + \bar{z}$$

Can we do this ?

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Note that each encirclement should represent a single product term. In this case it does not.

$$f = x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z$$
$$= x.\bar{y} + x.z$$

We do not get a single product term. In general we cannot make groups of 3 terms.

Can we use kmap with the following ordering of variables?

x \ yz	00	01	10	11
0	0	0	0	0
1	0	1	1	0

Can we combine these two terms into a single term ?

$$f = x.\bar{y}.z + x.y.\bar{z}$$

$$= x.(\bar{y}.z + y.\bar{z})$$

Note that no simplification is possible. Kmap requires information to be represented

x \ yz	00	01	10	11
0	0	1	0	1
1	0	0	0	0

These two terms can be combined into a single term but it is not easy to show that on the diagram.

$$f = \bar{x}.\bar{y}.z + \bar{x}.y.z$$
$$= \bar{x}.(\bar{y} + y).z = \bar{x}.z$$

Kmap requires information to be represented in such a way that it is easy to apply the principle $x + \bar{x} = 1$

4-variable K-map representation

w	x	y	z	min terms
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
⋮	⋮	⋮	⋮	⋮
1	1	1	0	m_{14}
1	1	1	1	m_{15}

wx \ yz	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$f = \overline{w}.\overline{x}.\overline{y}.\overline{z} + \overline{w}.\overline{x}.y.z + \overline{w}.x.\overline{y}.z + \overline{w}.x.y.z$$
$$+ w.x.\overline{y}.\overline{z} + w.x.y.\overline{z} + w.\overline{x}.\overline{y}.\overline{z}$$

4-variable minimization

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$\overline{w}.y.z$

$\overline{w}.x.z$

$w.\overline{y}.\overline{z}$

$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + \overline{w}.\overline{x}.\overline{y}.\overline{z} + w.x.y.\overline{z}$$

But is this the simplest expression ?

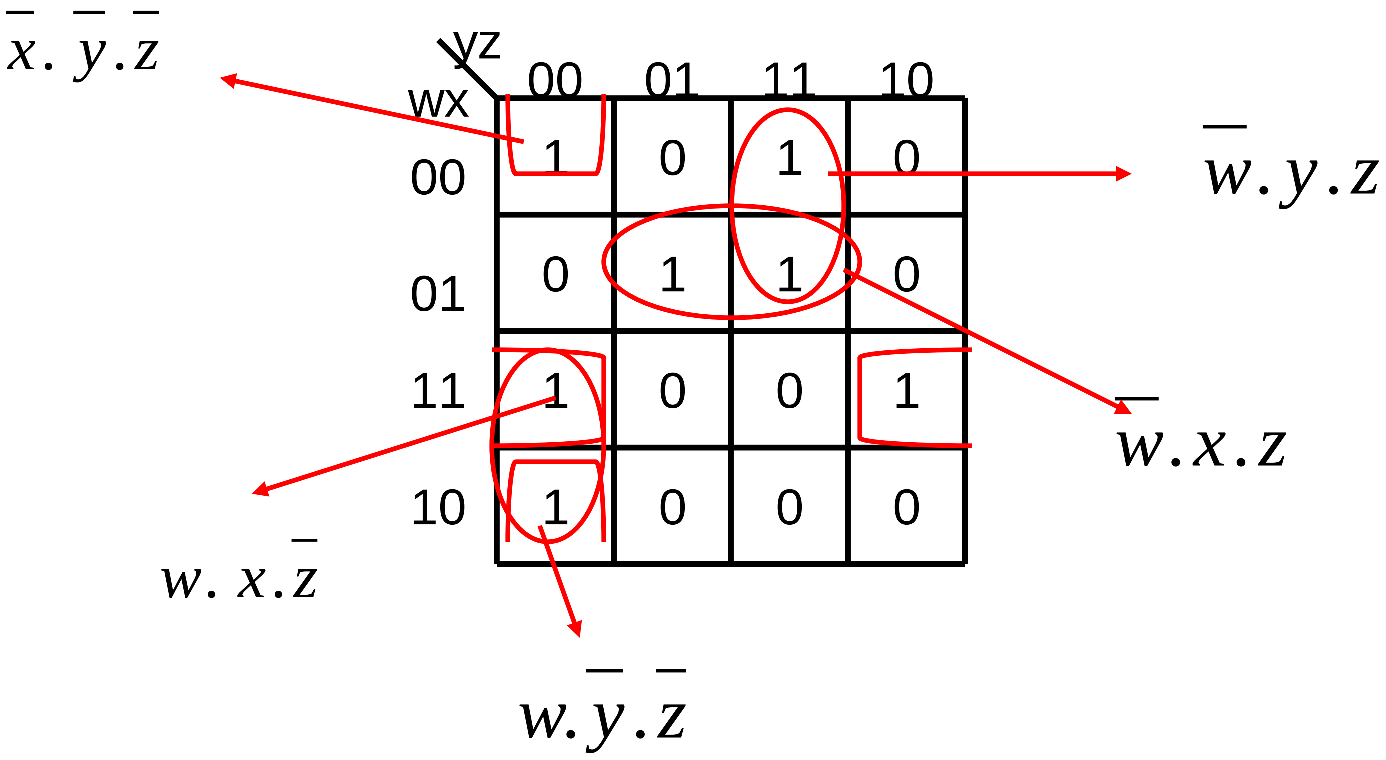
wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.x.\bar{y}.\bar{z}+w.x.y.\bar{z}=w.x.\bar{z}$$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.\bar{x}.\bar{y}.\bar{z}+\bar{w}.\bar{x}.\bar{y}.\bar{z}=\bar{x}.\bar{y}.\bar{z}$$

4-variable minimization



$$f = \bar{w}.y.z + \bar{w}.x.z + w.\bar{y}.\bar{z} + w.x.\bar{z} + \bar{x}.\bar{y}.\bar{z}$$

Is this the best that we can do ?

Cover the 1's with minimum number of terms

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$f = \overline{w}.y.z + \overline{w}.x.z + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

4-variable minimization

wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \overline{w}.x.\overline{y} + w.\overline{x}.\overline{z} + \overline{w}.\overline{y}.\overline{z}$$

wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \overline{w}.x.\overline{y} + w.\overline{x}.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

Groups of 4

wx \ yz	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	0	0

$\overline{w}.x$

$\overline{y}.z$

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$x.z$

$w.z$

Groups of 8

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

z

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

x

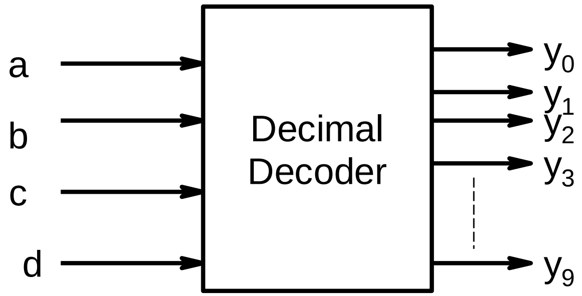
wx \ yz	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$\bar{z}$

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$\bar{x}$

Don’t care terms



Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

$$y_3 = \bar{a}.\bar{b}.c.d$$

$$y_3 = \bar{b}.c.d$$

Don’t care conditions

a	b	c	d	$y_0y_1y_2y_3y_4y_5y_6y_7y_8y_9$
0	0	0	0	1 0 0 0 0 0 0 0 0 0
0	0	0	1	0 1 0 0 0 0 0 0 0 0
0	0	1	0	0 0 1 0 0 0 0 0 0 0
0	0	1	1	0 0 0 1 0 0 0 0 0 0
0	1	0	0	0 0 0 0 1 0 0 0 0 0
0	1	0	1	0 0 0 0 0 1 0 0 0 0
0	1	1	0	0 0 0 0 0 0 1 0 0 0
0	1	1	1	0 0 0 0 0 0 0 1 0 0
1	0	0	0	0 0 0 0 0 0 0 0 1 0
1	0	0	1	0 0 0 0 0 0 0 0 0 1

Don’t care terms can be chosen as 0 or 1. Depending on the problem, we can choose the don’t care term as 1 and use it to obtain a simpler Boolean expression

Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

Decimal Encoding using Binary Coded Decimal (BCD)

$$y_3 = \bar{b}.c.d$$

Don’t care terms should only be included in encirclements if it helps in obtaining a larger grouping or smaller number of groups.

Don’t Care Terms

- Don’t care terms can be chosen as 0 or 1
  - Sometimes choosing them as 1 yields simpler expressions in SOP minimization

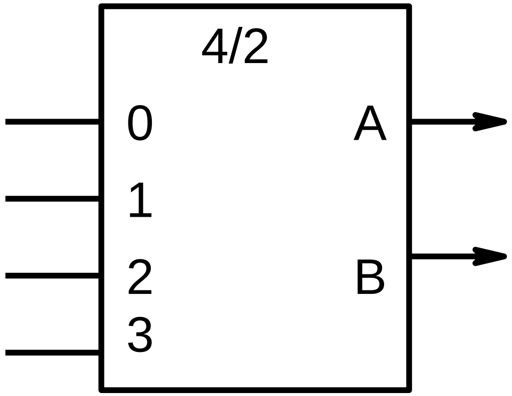
wx \ yz	00	01	11	10
00	1	x	0	1
01	1	0	1	1
11	0	x	1	1
10	1	x	1	x

$$f = \overline{w} \cdot \overline{z} + x \cdot y + w \cdot \overline{x}$$

Don’t care terms should only be included in encirclements if it helps in obtaining a larger grouping or smaller number of groups.

Encoder

- Inverse of decoders



d_3	d_2	d_1	d_0	B	A
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

$$A = \overline{d_2}\ \overline{d_0}$$

$$B = \overline{d_1}\ \overline{d_0}$$

$$A = \overline{d_2}\ \overline{d_0}$$

d_3d_2 \ d_1d_0	00	01	11	10
00	x	0	x	1
01	0	x	x	x
11	x	x	x	x
10	1	x	x	x

A

d_3d_2 \ d_1d_0	00	01	11	10
00	x	0	x	0
01	1	x	x	x
11	x	x	x	x
10	1	x	x	x

B

$$B = \overline{d_1}\ \overline{d_0}$$

You know what to do ☺