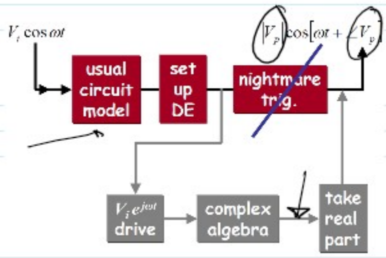
Lecture 22

12 September 2024 22:23



$$\cos(\omega t) = \frac{1}{2}\left[e^{j\omega t} + e^{-j\omega t}\right]$$

$$\sin(\omega t) = \frac{1}{2}\left[e^{j\omega t} - e^{-j\omega t}\right]$$

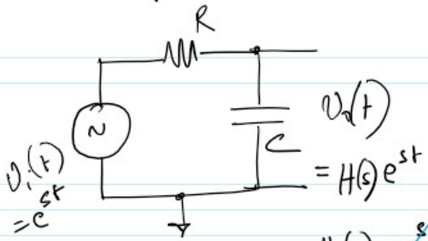
Can be applied through signal gen.

Cannot be applied through sig. gen BUT convinient for analysis.

$$e^{j\omega t} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(j\omega)\, e^{j\omega t} = |H(j\omega)|\, e^{j\phi}\, e^{j\omega t}$$

$$\cos(\omega t) \longrightarrow \boxed{\text{LTI}} \longrightarrow |H(j\omega)| \cos(\omega t + \phi(\omega))$$

Example:



$$\frac{dv_o}{dt} + \frac{v_o}{RC} = \frac{v_i}{RC}$$

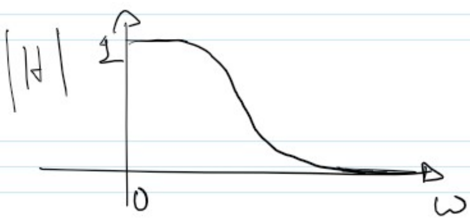
$$s H(s) \cancel{e^{st}} + \frac{1}{RC} H(s) \cancel{e^{st}} = \frac{\cancel{e^{st}}}{RC}$$

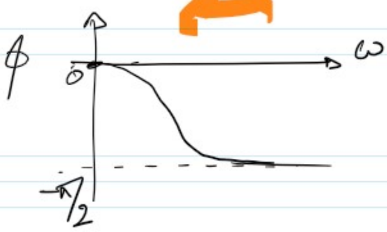
$$\Rightarrow \boxed{H(s) = \frac{1}{1 + sRC}}$$

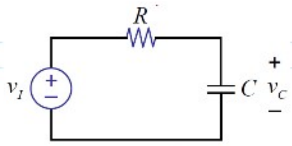
$$\boxed{H(j\omega) = \frac{1}{1 + j\omega RC}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\phi = \tan^{-1} \frac{\mathrm{Im}(H)}{\mathrm{Re}(H)} = -\tan^{-1}(\omega RC)$$







$$\boxed{|H|_{dB} = -20\log_{10}\sqrt{1+(\omega RC)^2}}$$

$$H(j\omega) = \frac{1}{1+j\omega RC}$$

$$\omega = \frac{1}{RC} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{1+(1)^2}} = \frac{1}{\sqrt{2}} = -3dB$$

ω_{3dB}

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

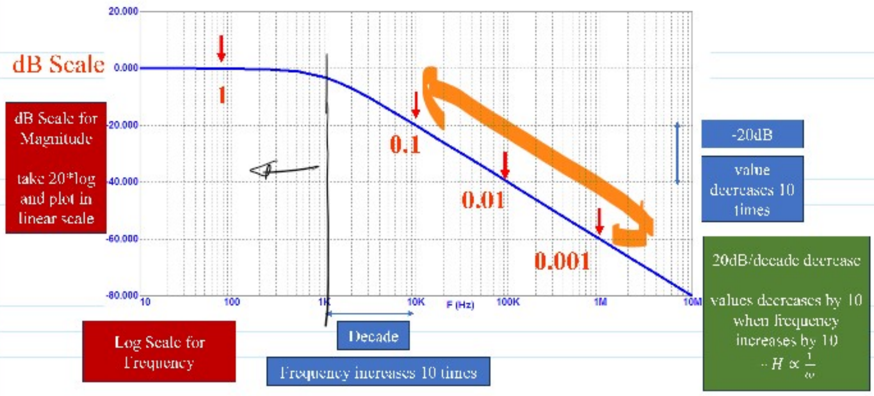
$$\boxed{|H|_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}}$$

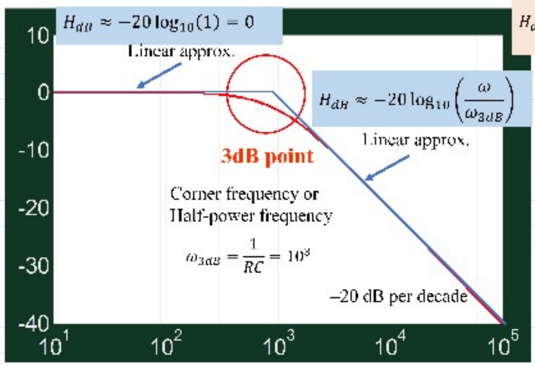
$\omega << \omega_{3dB}$

$$|H|_{dB} \approx -20\log_{10}(1) = 0$$

$\omega >> \omega_{3dB}$

$$\boxed{|H|_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right)}$$





$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

ω	H_{dB}
ω_{3dB}	0
$10\ \omega_{3dB}$	−20
$100\ \omega_{3dB}$	−40

20dB/ decade decrease

$H \propto \frac{1}{\omega}$

$H(\text{in dB}) \propto -20\log\omega$

ω (rad/s)

ALGO for BODE Plot:

Find $H(j\omega) \Rightarrow$ Find the roots of $H(j\omega)$

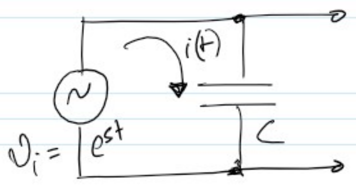
For plotting $|H(j\omega)|$ make the following appox.

$1 + j\omega\tau \approx 1$ (if $\omega << 1/\tau$)

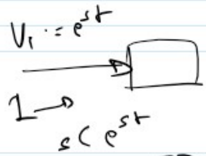
$1 + j\omega\tau \approx j\omega\tau$ (if $\omega >> 1/\tau$)

Plot in log-scale using straight line segments.

Impedance of a Capacitor



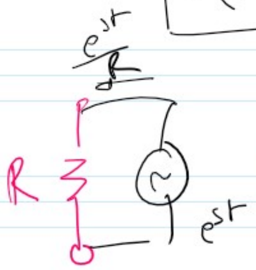
$$\boxed{i(t) = C\frac{dV}{dt}} = sC\,e^{st}$$



$$Z(s) = \frac{V_C(s)}{I_C(s)} = \frac{1}{sC}$$

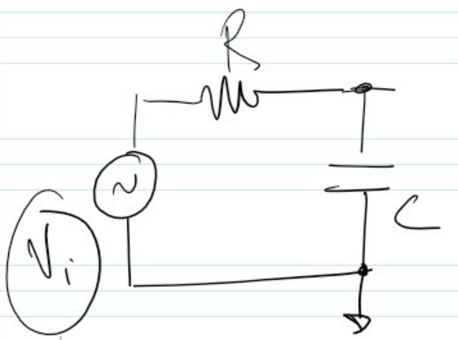
$$\boxed{Z(j\omega) = \frac{1}{j\omega C}}$$

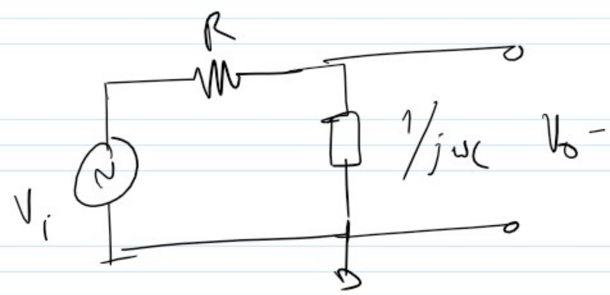
(For Sinusoidal Steady State analysis a Capacitor can be replaced with an impedance $= \left(\frac{1}{j\omega C}\right)$)



$R(j\omega) = R$

$$\boxed{\text{Transformed Network in phasor domain.}}$$

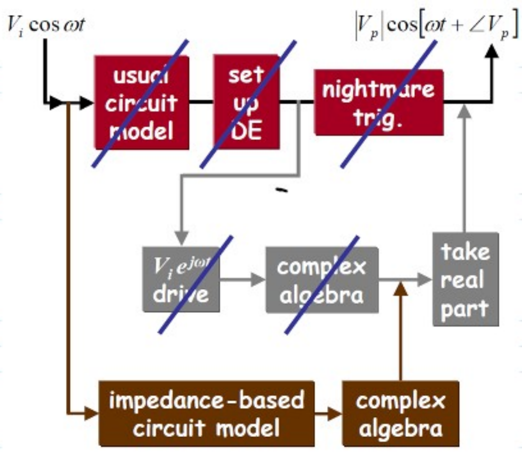




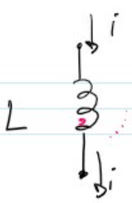
V_i → Represents amplitude and phase of i/p. Not a time dependent source any more

$$V_o = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

$$V_o = \frac{\quad}{R + 1/j\omega C} = \frac{\quad}{1 + j\omega RC}$$

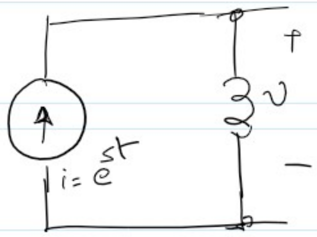


Inductor



$$\Phi = L i$$

$$v = L \frac{di}{dt}$$



$$v = sL\, e^{st}$$

$$\Rightarrow \frac{v}{i} = sL = j\omega \Big|_{s=j\omega}$$

C	L
$i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$
$I(j\omega) = j\omega C\, V(j\omega)$	$V(j\omega) = j\omega L\, I(j\omega)$

Mathematically there is $\boxed{\text{no}}$ difference between an L and a C if the variables are hidden from us,

as long as I and V are interchanged,

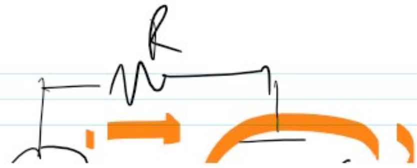
$\Rightarrow$ Behavior of voltage in L will be IDENTICAL to the " " current through C iff all the L's and C's are interchanged.

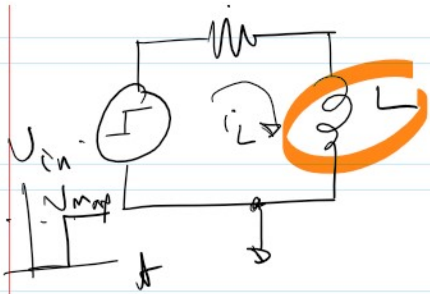
To transform a network with C and R with L and resistances.

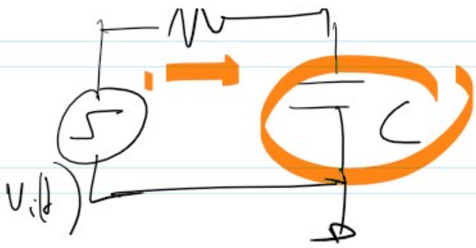
$$C \text{ or } \omega C \xRightarrow{\text{transformation}} L \text{ or } \omega L$$

$$V \Longrightarrow I$$

$$R \Longrightarrow 1/R$$







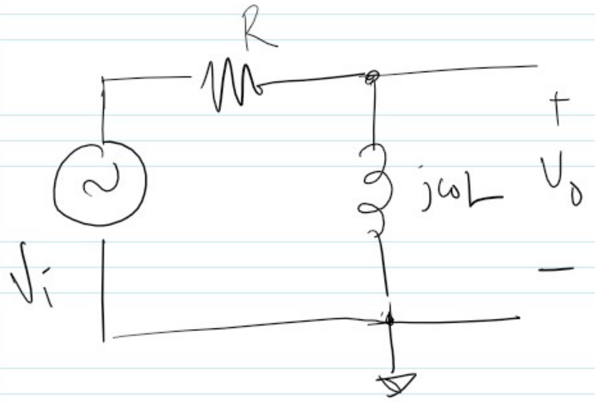
$$i_L(t) = i_{max}\left(1-e^{-t/(L/R)}\right) \qquad v_C(t) = V_{max}\left(1-e^{-t/RC}\right)$$

$$= i_{max}\left(1-e^{-\left(\frac{tR}{L}\right)}\right) \qquad \left(\tau = L/R\right)$$

What happens to L if there is a sudden change in current?

$$i_{max} = \frac{V_{max}}{R}$$

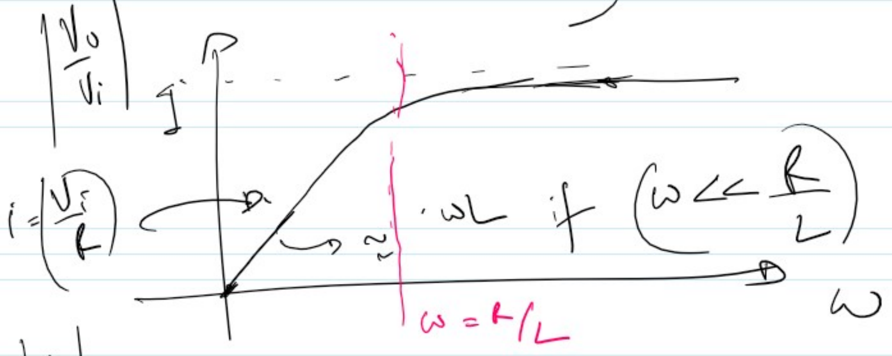
Phasor Representation.

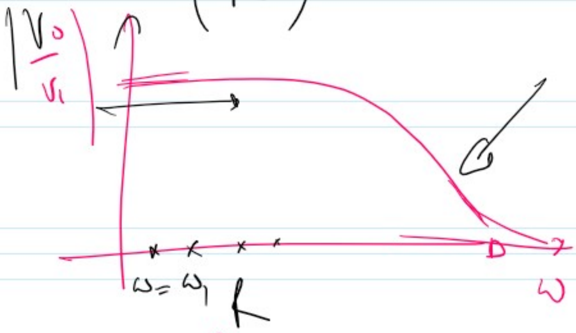


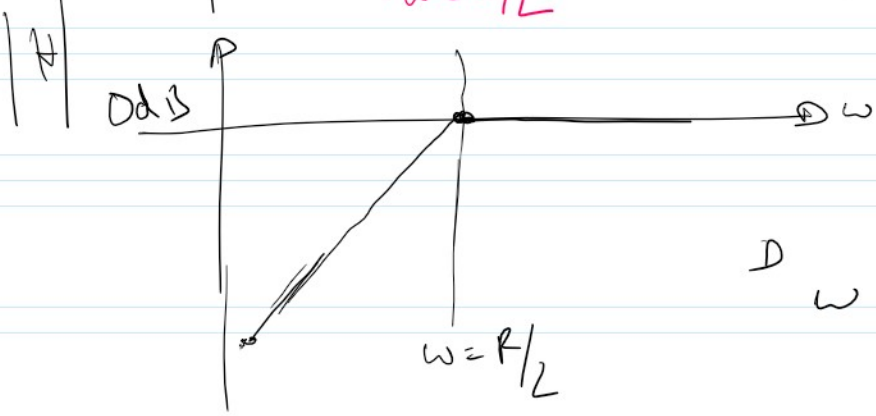
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \boxed{\frac{j\omega L}{(R+j\omega L)}}$$

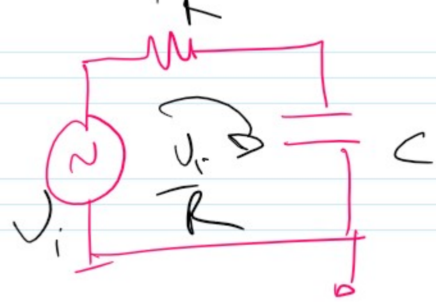
$$\left|\frac{V_o}{V_i}\right| = \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}}$$

$$\angle = +\pi/2 - \tan^{-1}\left(\frac{\omega L}{R}\right)$$









@ very high freq ($\gg \omega_{3dB}$)

$i \approx \frac{V_i}{R}$ $\qquad V_C(j\omega) = \frac{V_i}{j\omega CR}$