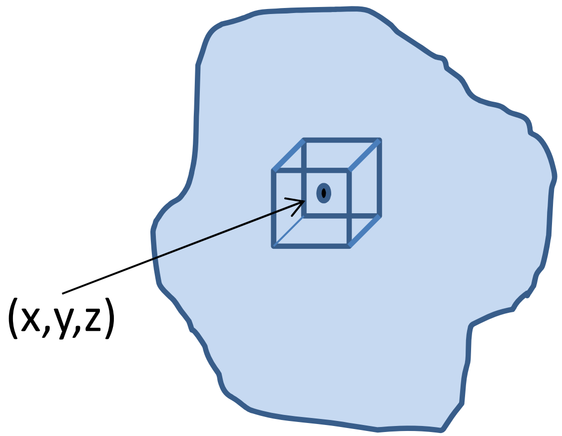
Rate form of First law for a control mass :

Recall that the rate form of the first law as applied to a control mass is

$$\frac{dE_{CM}}{dt} = \dot{Q}_{in} - \dot{W}_{out,total}$$

Here : E_{CM} is the total energy of a given control mass system and it can be expressed as

$$E_{CM} = \int_{CM} e\, \rho\, dV$$



Control mass (CM)

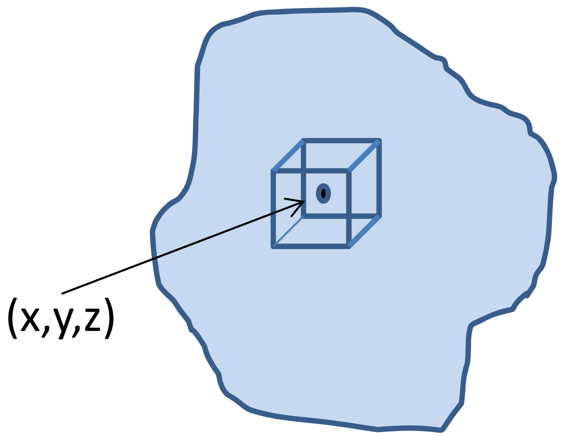
$\dot{Q}_{in}$ = Rate of energy transfer to the control mass as heat

$\dot{W}_{out,total}$ = Rate at which work is done by the control mass. The subscript 'total' is used because work is a sum of two parts as shown in later slide.

First law as applied to a control mass :

The total energy per unit mass is given by $$e = u + \frac{1}{2}V^2 + gZ$$

Here : u(x,y,z) is the specific internal energy of the fluid at (x,y,z), V is the magnitude of the velocity of the fluid at (x,y,z), and Z is the height of the volume element dV about the ground level. The second and third terms in the equation for E_{CM} (see last slide) are the kinetic and potential energies of the fluid per unit mass at (x,y,z).



Control mass (CM)

This integration in the expression for E_{CM} (see last slide) can be rationalized as follows : We consider a volume element dV around a point (x,y,z) within the body of the fluid (control mass). Here e(x,y,z,t) is the energy per unit mass and ρ(x,y,z,t) is the mass density of the fluid. Then, (e ρ dV) = total energy of the fluid inside the volume element dV. The entire control mass is divided into a large number of such volume elements. The energies (e ρ dV) are summed over all the volume elements, this summation yields E_{CM} in the limit as $dV \rightarrow 0$ (i.e., integration is equal to the summation in the limit as the size of the volume elements approaches zero).

IMPORTANT : Note that the heat and work terms in the equation on the last slide can be interpreted as heat received and work done per unit time by the body of the fluid (control mass) that occupies the control volume at the given time t*. Thus,

$$\dot{Q}_{in} = \text{Heat input per unit time to the body of the fluid (control mass) that occupies the control volume at given time } t^{*}$$

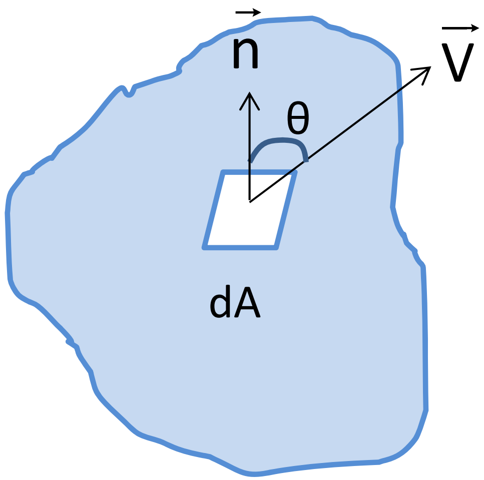
$$\dot{W}_{out,total} = \dot{W}_{out}^{non-flow} + \dot{W}_{out}^{flow}$$
$$= \text{total work done per unit time by the body of the fluid (control mass) that occupies the control volume at a given time } t^{*}$$

Flow work = force x displacement

$$\dot{W}_{out}^{flow} = \int_{CS} P\,(\vec{V}.\vec{n})\,dA$$

PdA = Force acting on the differential area element dA in the direction **opposite** of $\vec{n}$

$\vec{V}.\vec{n}$ = displacement per unit time in the direction of $\vec{n}$



$\dot{W}_{out}^{flow}$ = Work done by the control mass (which is entirely occupying the control volume) due to displacement of control mass **against** pressure

Example :

Steam is leaving a 4-L pressure cooker which is operating at 150 kPa. It is observed that the amount of liquid in cooker has decreased by 0.6 L in 40 minutes. The cross-sectional area of the exit opening is 8 mm^2.

$v_f = 0.001053$ m^3/kg $\qquad v_g = 1.1594$ m^3/kg

Total mass lost in 40 min. = 0.0006/0.001053
= 0.570 kg

$$\dot{m}_{out} = 2.37 \text{ x } 10^{-4} \text{ kg/s}$$

$$\dot{W}_{out}^{flow} = P v_g . \dot{m}_{out} = 0.0412 \text{ kJ/s}$$

First law as applied to a control mass :

Splitting work term in two parts and writing the expression for flow work as in the previous slide, we get

$$\frac{dE_{CM}}{dt} = \dot{Q}_{in} - \dot{W}_{out}^{non-flow} - \int_{CS} P\,(\vec{V}.\vec{n})\,dA$$

The **non-flow work** includes work done due to moving parts inside the control volume such as shaft work, moving boundary work, as well as non-mechanical forms of work such as electrical work

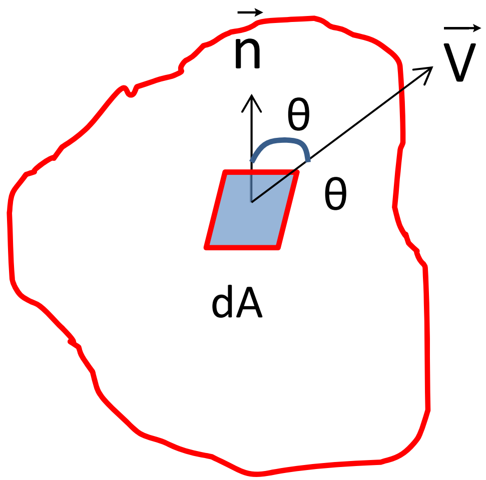
First law as applied to a control volume :

Applying Reynolds transport theorem to the left hand side of the rate form of first law (on previous slide), we get

$$\frac{dE_{CV}}{dt} + \int_{CS} e\,\rho\,(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non-flow} - \int_{CS} P\,(\vec{V}.\vec{n})\,dA$$

Here : E_{CV} is the total energy of a the fluid inside the control volume at a given time t, expressed as

$$E_{CV} = \int_{CV} e\,\rho\,dV$$



$\int_{CS} e\,\rho\,(\vec{V}.\vec{n})\,dA =$ Rate of loss of energy associated with the material flowing out of the control volume

First law as applied to control volumes :

The first law equation on the last slide can be re-arranged as follows :

$$\frac{dE_{CV}}{dt} + \int_{CS} e\,\rho\,(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow} - \int_{CS} P\,(\vec{V}.\vec{n})\,dA$$

$$\frac{dE_{CV}}{dt} + \int_{CS} (e\,\rho + P)\,(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

$$\frac{dE_{CV}}{dt} + \int_{CS} (e + Pv)\,\rho(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

Choosing the control volume such that the control surface is perpendicular to the direction of flow, we have $\vec{V}.\vec{n}=V$ (the magnitude of the velocity) for outlets and $\vec{V}.\vec{n}= - V$ for inlets.

First law as applied to control volumes :

$$\frac{dE_{CV}}{dt} + \int_{CS(out)} (e + Pv)\rho V \, dA - \int_{CS(in)} (e + Pv)\rho V \, dA$$

$$= \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

Here CS(out) is that part of the control surface where fluid is exiting the control volume.

Similarly, CS(in) is that part of the control surface where fluid is entering the control volume.

Now, we assume that properties e, P, v, and $\rho (= 1 / v)$ are constant over CS(out) and CS(in). We can simplify the terms involving the area integration in the above expression.

First law as applied to control volumes :

Note that using the assumption made in the last slide,

$$\int_{CS(out)} (e+Pv)\,\rho\, V\, dA = \sum_{out}\left[(e+Pv)\,\rho \int_{out} V\, dA\right]$$

$$= \sum_{out}(e+Pv)\,\dot{m}$$

Similarly integration over CS(in) can be simplified.

Thus,the first law expression can be written as

$$\frac{dE_{CV}}{dt} + \sum_{out}\dot{m}(e+Pv) - \sum_{in}\dot{m}(e+Pv)$$

$$= \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

First law as applied to control volumes :

Note that $(e + Pv) = \left(u + \frac{1}{2}V^2 + gZ + Pv \right)$

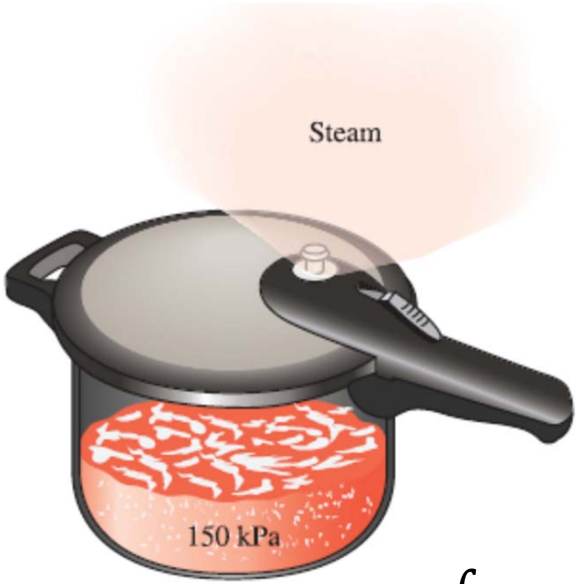
$$= \left(h + \frac{1}{2}V^2 + gZ \right)$$

Substituting this in the previous equation, we get

$$\boxed{\frac{dE_{CV}}{dt} = \sum_{in} \dot{m} \left(h + \frac{1}{2}V^2 + gZ \right) - \sum_{out} \dot{m} \left(h + \frac{1}{2}V^2 + gZ \right) + \dot{Q}_{in} - \dot{W}_{out}}$$

where the superscript "non - flow" on work term is dropped for simplicity. Henceforth, it will be understood that the work term in the above expression is the non - flow work.

Now we revisit the pressure cooker example on slide 5. We have already Shown that the flow work is : $\dot{W}_{out}^{flow} = Pv_g.\dot{m}_{out} = 0.0412\ kJ/s$



Now we consider rate of energy loss associated with exiting Steam.

Neglecting potential energy of exiting steam, we get

Velocity of steam = 34.3 m/s

Neglecting potential energy of exiting steam, we get

$$\int_{CS} e\,\rho\,(\vec{V}.\vec{n})\,dA = e\dot{m}_{out} = \left(u + \frac{1}{2}V^2\right)\dot{m}_{out} = 0.5972\ kJ/s$$

where, we used $u = u_g = 2519.2$ kJ/kg

Substituting in 1st law equation we get

$$\frac{dE_{CV}}{dt} = \dot{Q}_{in} - 0.5972 - 0.0412 = \dot{Q}_{in} - 0.6384\ \ kJ/s$$