ESC201: Lecture 9

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Complement of a binary number

Binary system:

- 1's complement
- 2's complement

1's complement of n-bit number x is $2^n - 1 - x$

2's complement of n-bit number x is $2^n - x$

1's complement of 1011 ? $2^4 - 1 - 1011$ $1111 - 1011 = 0100$

1's complement is simply obtained by flipping a bit (changing 1 to 0 and 0 to 1)

$$1\text{'s complement of } 1001101 = ?$$

$$0110010$$

$$2\text{'s complement of } 1010 = 1\text{'s complement of } 1010 + 1 = 0110$$

$$2\text{'s complement of } 110010 =$$

Leave all least significant 0’s as they are, leave first 1 unchanged and then flip all subsequent bits

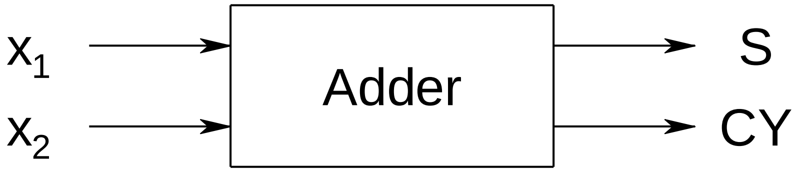
$$001110$$

$$1011 \rightarrow 0101$$

Flipped

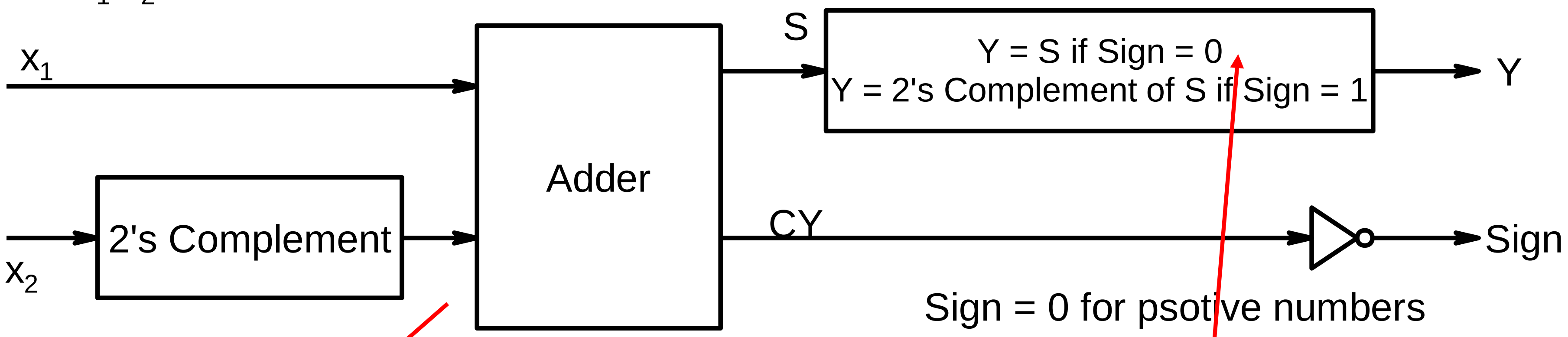
$$101101100 \rightarrow 010010100$$

Advantages of using 2's complement



Can we carry out Y = $X_1 - X_2$ using such an adder?

x_1, x_2: N bit numbers



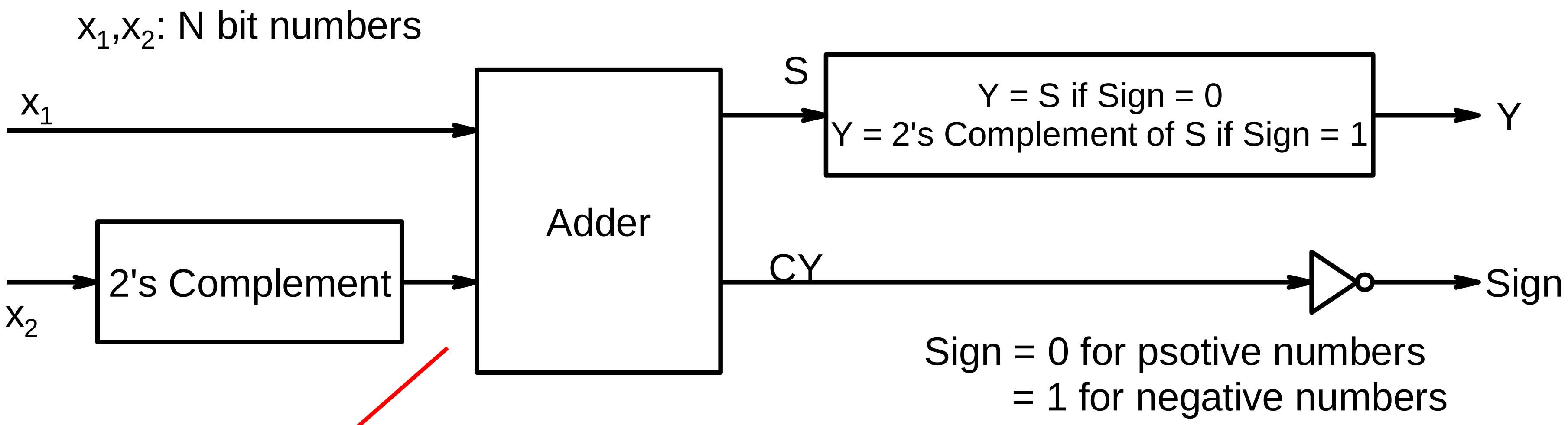
Sign = 0 for psotive numbers
= 1 for negative numbers

$$2^N - x_2$$

$$(CY, S) = x_1 + 2^N - x_2$$

Note that carry will be there only if $x_1 - x_2$ is positive as 2^N is N+1 bits (1 followed by N zeros)

Advantages of using 2's complement



$$2^N - x_2 \qquad (CY, S) = x_1 + 2^N - x_2$$

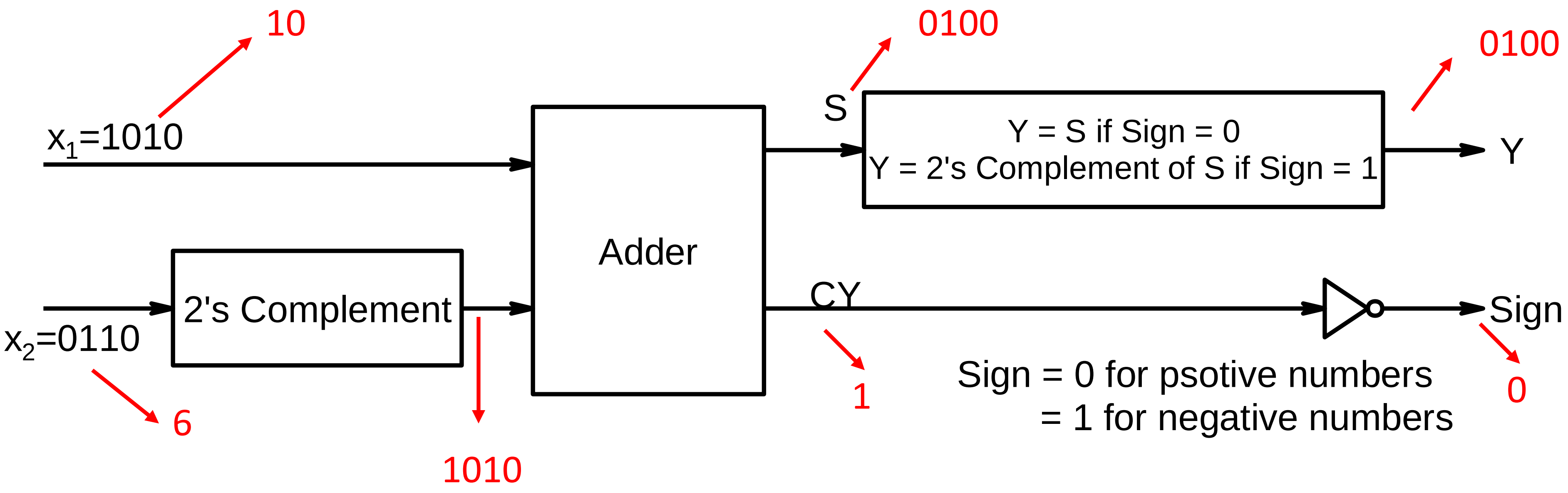
Note that carry will be there only if $x_1 - x_2$ is positive as 2^N is N+1 bits (1 followed by N zeros)

A zero carry implies a negative number whose magnitude $(x_2 - x_1)$ can be found as follows:

$$S = x_1 + 2^N - x_2$$

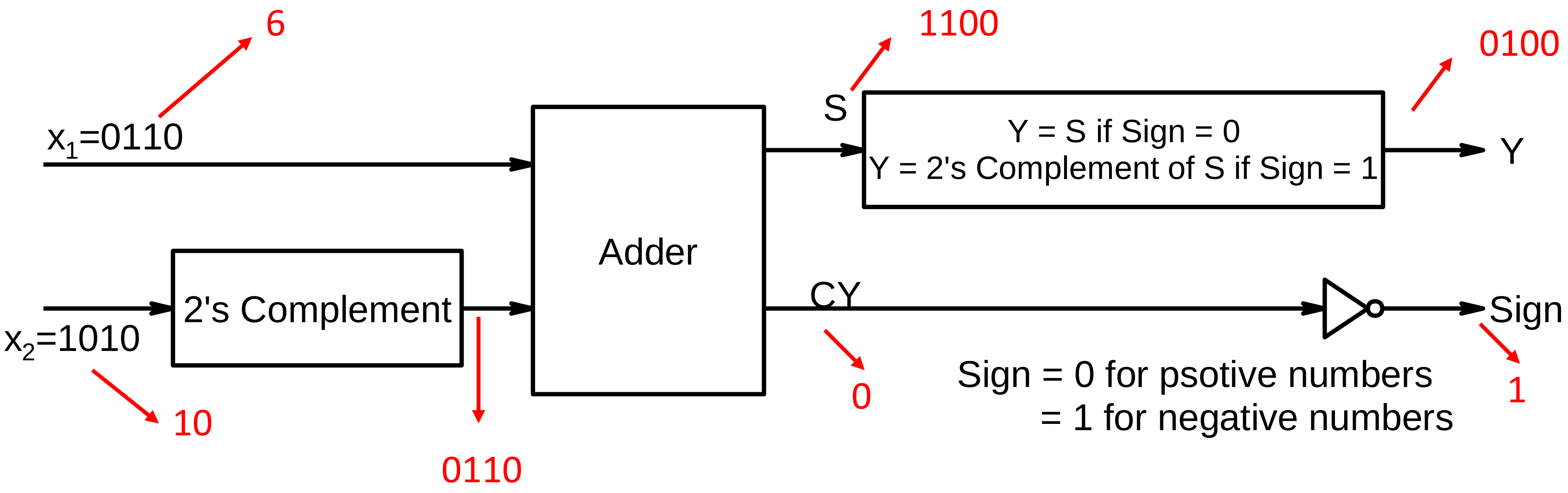
$$\text{2'scomplement of } S = 2^N - (x_1 + 2^N - x_2) = x_2 - x_1$$

Example



$$
\begin{array}{r}
1\,0\,1\,0 \\
+\ 1\,0\,1\,0 \\
\hline
1\,0\,1\,0\,0 \\
\hline
\end{array}
$$

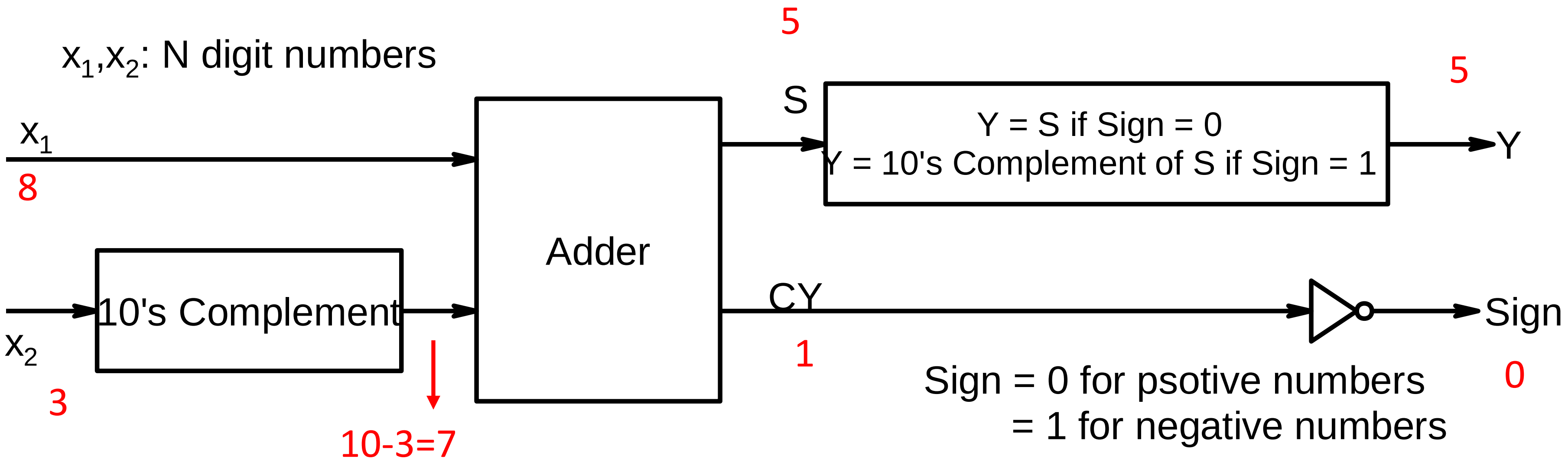
Example



$$
\begin{array}{r}
0\,1\,1\,0 \\
+\,0\,1\,1\,0 \\
\hline
1\,1\,0\,0 \\
\hline
\end{array}
$$

It makes sense to use adder as a subtractor as well provided additional circuit required for carrying out 2’s complement is simple

Subtraction using 10's complement



This way of subtraction would make sense only if subtracting a number x_2 from 10^N is much simpler than directly subtracting it directly from x_1

Representing positive and negative binary numbers

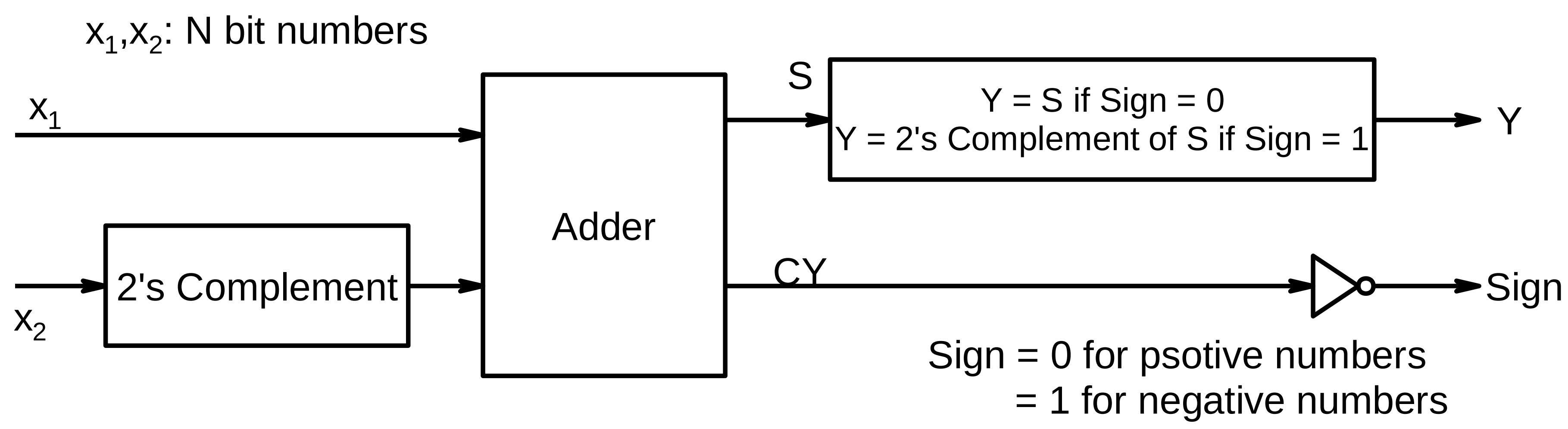
One extra bit is required to carry sign information. Sign bit = 0 represents positive number and Sign bit = 1 represents negative number

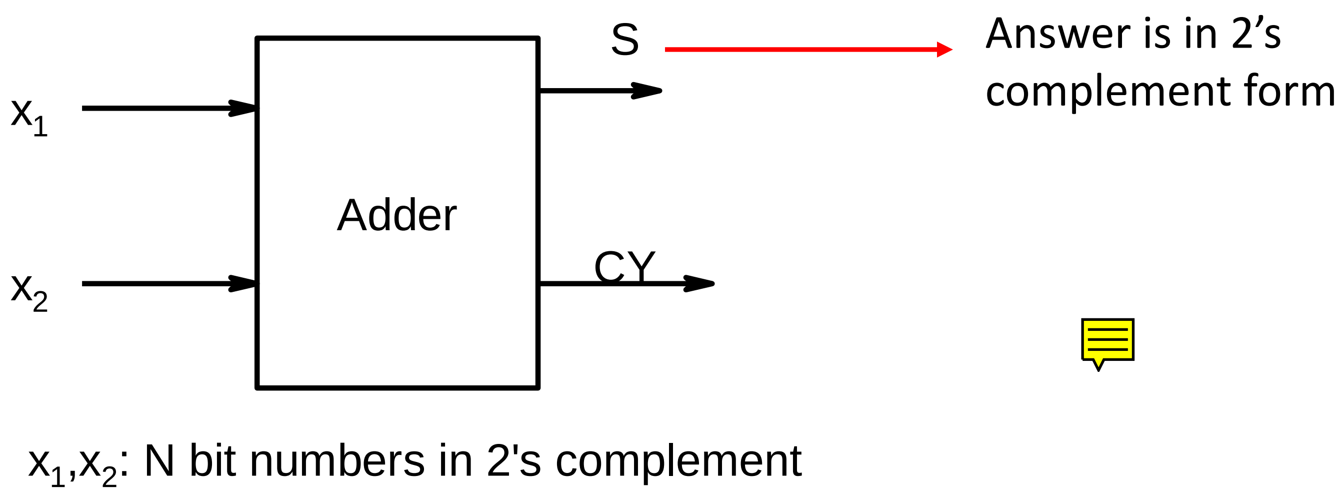
decimal	Signed Magnitude
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1000
-1	1001
-2	1010
-3	1011
-4	1100
-5	1101
-6	1110
-7	1111

decimal	Signed 1's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1111
-1	1110
-2	1101
-3	1100
-4	1011
-5	1010
-6	1001
-7	1000

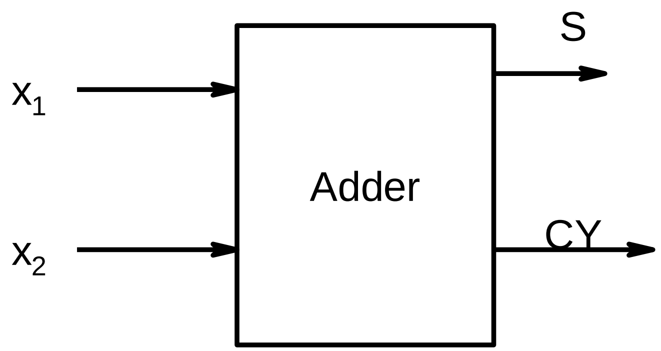
decimal	Signed 2's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-1	1111
-2	1110
-3	1101
-4	1100
-5	1011
-6	1010
-7	1001

If we represent numbers in 2’s complement form carrying out subtraction is same as addition





Example



x_1, x_2: N bit numbers in 2's complement

$$\begin{array}{r} +5 \\ +2 \\ \hline +7 \\ \hline \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,0\,0\,1\,0 \\ \hline 0\,1\,1\,1 \\ \hline \end{array} \qquad \begin{array}{r} +5 \\ -2 \\ \hline +3 \\ \hline \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,1\,0 \\ \hline 0\,0\,1\,1 \\ \hline \end{array}$$

$$\begin{array}{r} -5 \\ +2 \\ \hline -3 \\ \hline \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ +\,0\,0\,1\,0 \\ \hline 1\,1\,0\,1 \\ \hline \end{array} \qquad \begin{array}{r} -5 \\ -2 \\ \hline -7 \\ \hline \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ +\,1\,1\,1\,0 \\ \hline 1\,0\,0\,1 \\ \hline \end{array}$$

2’s complement is 0011 = 3

2’s complement is 0111 = 7

Boolean Algebra

Algebra on Binary numbers

A variable x can take two values {0,1}

0 → False, No, Low voltage

1 → True, Yes, High voltage

Basic operations:

$$\text{AND:} \quad y = x_1 . x_2$$

Y is 1 if and only if both x_1 and x_2 are 1, otherwise zero

Truth Table

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Basic operations:

$$\text{OR:} \quad y = x_1 + x_2$$

Y is 1 if either x_1 and x_2 is 1. Or y= 0 if and only if both variables are zero

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\text{NOT:} \quad y = \overline{x}$$

x	y
0	1
1	0