Week 1 – Core Vector Concepts for ML

1. Vectors in Machine Learning

- A vector represents a data point in multidimensional space.
- Each dimension corresponds to a meaningful feature.
- Direction encodes feature relationships; magnitude encodes scale/strength.
- Proper representation affects similarity, optimization, and learning stability.

2. Dot Product (Numerical View)

- Computed as sum(a_i * b_i).
- Measures combined contribution of aligned components.
- Positive → components mostly aligned; negative → mostly opposed; zero → cancellations.

3. Dot Product (Geometric View)

- a · b = ||a|| ||b|| cos(theta).
- Reflects angular alignment between vectors.
- Represents projection of one vector onto another.
- Core idea for similarity, PCA, projections, and neural weighted sums.

4. Norms and Magnitude

- L2 Norm: Euclidean length; affects distances, gradients, model stability.
- L1 Norm: Absolute sum; encourages sparsity; robust against outliers.
- Magnitude determines influence in dot products and optimization.

5. Normalization & Cosine Similarity

- Normalization removes magnitude, preserves direction.
- Cosine similarity = (a · b) / (||a|| ||b||).
- Essential for fair comparisons, embedding models, and stable geometry.

6. Failure Cases of Raw Dot Product

- Magnitude trap: large values dominate even if direction differs.

- Cancellation: mixtures of large positive/negative products produce misleading zero.

- High-dimensional accumulation of floating■point errors.

- Cosine similarity fixes these issues by factoring out magnitude.

7. Visual Intuition (Mental Plots)

- Vectors as arrows from origin.

- Dot product as shadow/projection.

- Norm as radius to a hypersphere.

- Normalization as mapping all vectors onto a unit circle.

Use this document as a quick recall reference before implementing Week 2 operations.