Basic Concepts of Algebra

Name: _____ Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Evaluate each of the following expressions when $a = -8$ and $b = \frac{1}{2}$. (16)

   (a) $ab + 2b + 3a$

   (b) $\dfrac{a}{b}$

   (c) $4(a^2b + b^2a)$

   (d) $(a-2)\sqrt{-ab}$

2. Simplify the product as much as possible. (5)

$$\frac{2}{3a^2 - 6b} \cdot \frac{9a^3 - 18ab}{10a^2}$$

3. Decide whether each set is a field under the operations of addition and multiplication. If the set is not a field, name at least one field property that does not hold. (12)

   (a) The natural numbers ($\mathbb{N}$)

   (b) The integers ($\mathbb{Z}$)

   (c) The rational numbers ($\mathbb{Q}$)

   (d) The negative rational numbers

4. Solve for the variable in each equation. (10)

(a) Find all values of y such that $\dfrac{3}{2+\sqrt{y}} + \dfrac{4}{2+\sqrt{y}} = 1$.

(b) What values of x satisfy $\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = 3$?

5. (a) Let x be the middle integer of three consecutive integers. What is the sum of these three integers in terms of x? (8)

(b) The sum of 23 consecutive integers is 2323. What is the largest of the integers? (Hint: Use the result from the first part.)

6. A grocer wants to mix peanuts and cashews to produce 20 lb of mixed nuts worth \$6.20/lb. How many pounds of each kind of nut should she use if peanuts cost \$4.80/lb (6)

and cashews cost \$8.00/lb?