Inequalities and Proof

Name: _____ Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. For each of the following inequalities, graph the solution to the inequality on the number line and write the solution to the inequality using interval notation. (15)

   (a) $2 - 3x \geq 11$

   (b) $3 + 2x < 30 - 7x$

   (c) $8 - 2x \leq 5 - 5x < 23 - 2x$

2. What is the smallest positive integer that has a square root that is greater than 10? (10)

3. Solve the equation $|2z - 9| + |z - 3| = 15$. (10)

4. Solve the inequality $\left|\frac{1}{4}t - 2\right| \leq \frac{3}{4}$. (10)

5. Betty goes to the store to get flour and sugar. The amount of flour she buys, in pounds, is at least 6 pounds more than half the amount of sugar, and is no more than twice the amount of sugar. Find the least number of pounds of sugar that Betty could buy. (10)

6. Give a counterexample (with $a, b \in \mathbb{R}$) to show that each statement is false. (10)

   (a) If $a^4 = b^4$, then $a - b = 0$.

   (b) If $a < b$, then $b - a < 0$.

7. Prove the cancellation property of multiplication: If $c \neq 0$ and $ca = cb$, then $a = b$. (12)

8. Prove: For all real numbers a and b, $|ab| = |a| \cdot |b|$. (Hint: Consider the different cases for the signs of a and b.) (10)

9. (a) Describe the difference between an axiom (or postulate) and a theorem. (8)

(b) Describe what a corollary of a theorem is.

10. Determine whether each statement is true or false. If it is false, give a counterexample. If it is true, give a brief explanation of why it is true. (16)

(a) If $a < b$, then $a^2 < b^2$.

(b) If $a \neq 0, b \neq 0$, and $a > b$, then $\frac{1}{a} > \frac{1}{b}$.

(c) For any number k, the equation $|k| = -k$ is always false.

(d) If $c < 0$, then $ac > bc$ implies $a < b$.