Chapter 1: Fundamentals — Real Numbers, Expressions, and Graphs

Name: _____   Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit. Unless otherwise stated, assume all variables represent real numbers and all functions are defined on their natural domains.

1. Answer each part clearly. For interval notation questions, also sketch your answer on a real number line.   (10)

   (a) Solve the inequality $|2x - 3| \leq 7$. Express the solution set in interval notation and sketch it.

   (b) Express the set $(-6, 2] \cup [5, \infty)$ using inequality symbols (use the word "or" if appropriate).

   (c) Find the distance between -11 and 7 on the real number line.

2. Simplify each expression. Write your final answers without negative exponents and rationalize any denominators.   (14)

(a) $\left(\dfrac{32x^{-5}y^{12}}{2x^3y^{-6}}\right)^{-2/5}$

(b) $\dfrac{3\sqrt{5}}{2\sqrt{2}+\sqrt{3}}$

(c) $\left(81a^4b^{-2}\right)^{3/4}$

(d) $\dfrac{\sqrt[3]{54x^5}}{\sqrt[3]{2x}}$

(e) $(-32)^{-3/5}$

(f) $\dfrac{\sqrt{18}-\sqrt{8}}{\sqrt{2}}$

3. Factor and simplify as indicated. State all restrictions on the variable(s) when simplifying   (10)
   rational expressions.

   (a) Factor completely: $x^3 - 3x^2 - 4x + 12$.

   (b) Simplify $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ and state all restrictions.

   (c) Simplify $\dfrac{1}{x - 3} - \dfrac{1}{x + 3}$ and state all restrictions.

4. Solve each equation or inequality. Show algebraic steps and check for extraneous solutions where appropriate.   (10)

   (a) $\sqrt{x+4} = x - 2$

   (b) $\dfrac{2}{x-3} + \dfrac{1}{x+3} = 1$

   (c) $|2x - 5| > 3$

5. Perform the indicated operations and write answers in the form $a + bi$.   (10)

   (a) $(3 - 2i) + (4 + 3i)$

   (b) $(3 - 2i)(4 + 3i)$

   (c) $\dfrac{2 + 5i}{1 - 2i}$

   (d) i^{73}

6. Let $P(-2, 5)$ and $Q(6, -1)$ be points in the coordinate plane.   (14)

    (a) Plot P and Q.

    (b) Find the distance PQ.

    (c) Find the midpoint of $\overline{PQ}$.

    (d) Find the slope of the line through P and Q.

    (e) Find an equation of the perpendicular bisector of $\overline{PQ}$.

    (f) Find an equation of the circle for which $\overline{PQ}$ is a diameter.

7. Consider the function $y = (x - 2)^2 - 9$. 　　　　　　　　　　　(8)

(a) Sketch the graph using transformations. Label the vertex, axis of symmetry, and intercepts.

(b) Find the range and the intervals where the function is increasing and decreasing.

(c) Solve $(x - 2)^2 - 9 \leq 0$ and express the solution in interval notation. Justify briefly.

8. The intensity of light I (in lux) from a point source varies inversely as the square of the   (8)
   distance d (in meters) from the source: $I = \dfrac{k}{d^2}$.

   (a) If $I = 900$ when $d = 2$, determine k.

   (b) Using your model, find the intensity at $d = 5$.

   (c) At what distance will the intensity be 100 lux? Round your answer to the nearest
       tenth of a meter.

9. Find an equation of the line with the given property, then find its x- and y-intercepts.   (8)

   (a) It passes through the point $(3, -6)$ and is parallel to the line $3x + y - 10 = 0$.

   (b) It passes through $(-2, 5)$ and is perpendicular to the line $y = -2x + 7$.

10. Solve the inequalities and express each solution set in interval notation. Indicate critical　　(8)
    points on a sign chart and sketch the solutions on a number line.

    (a) $\dfrac{x^2 - 5x + 6}{x - 1} \leq 0$

    (b) $\sqrt{3x + 4} \geq x - 1$