Rational Functions and Expressions

Name: _____   Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Solve the equation $\frac{2x}{x-5} = 3 + \frac{1-x}{x-3}$.

   (8)

2. Consider the function $g(x) = \frac{3-2x}{x-7}$.

   (12)

   (a) Find the domain of g.

   (b) Find the range of g.

   (c) Find all horizontal and vertical asymptotes of the graph of $y = g(x)$.

(d) Graph $y = g(x)$.

3. Find constants A and B such that $\frac{x+7}{x^2-2x-35} = \frac{A}{x-7} + \frac{B}{x+5}$ for all x. (10)

4. Find the constants A, B, and C such that (12)

$$\frac{3x-2}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}.$$

5. Find all values of t that satisfy $\frac{t-7}{t^2-2t-3} = \frac{t+2}{t^2+2t-15}$.   (8)

6. Graph each function. Show any asymptotes as dashed lines.   (10)

   (a) $h(x) = \frac{x+1}{x-1}$

   (b) $h(x) = \frac{10x^2}{x^2+1}$

7. For the rational function $f(x) = \frac{x^2-4}{x^2-3x-4}$:   (15)

   (a) Find the domain of f.

   (b) Find all vertical asymptotes.

   (c) Find all horizontal asymptotes.

   (d) Find all holes (removable discontinuities), if any.

   (e) Find the x-intercepts and y-intercept.

8. Find the oblique asymptote for $f(x) = \frac{2x^2+3x-1}{x+2}$ using polynomial long division.   (10)

9. Simplify each rational expression: (8)

(a) $\frac{x^2-9}{x^2+6x+9}$

(b) $\frac{2x^3-8x}{x^2-4}$

(c) $\frac{x^3+8}{x^2-4}$

(d) $\frac{6x^2-24}{3x^2-6x-24}$

10. Perform the indicated operations and simplify: (7)

$$\frac{x^2-1}{x+2} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3}$$