Algebra 2 Chapter 2 Assignment (Focus on Proofs)

Problems

21. Prove: If $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.
    (Hint: Use Exercise 20. We will assume Exercise 20 is the property that for $b \neq 0, d \neq 0$, it has been proven that $\frac{1}{b} \cdot \frac{1}{d} = \frac{1}{bd}$.)

    1. Definition of Division

    $\frac{a}{b} \cdot \frac{c}{d} = a \cdot \frac{1}{b} \cdot c \cdot \frac{1}{d}$

    2. Commutative Property

    $a \cdot \frac{1}{b} \cdot c \cdot \frac{1}{d} = ac \cdot \frac{1}{b} \cdot \frac{1}{d}$

    3. Previous Property (From Exercise 20)

    $ac \cdot \frac{1}{b} \cdot \frac{1}{d} = ac \cdot \frac{1}{bd}$

    4. Commutative Property

    $ac \cdot \frac{1}{bd} = \boxed{\dfrac{ac}{bd}}$

22. Prove: If $c \neq 0$ and $d \neq 0$, then $\frac{1}{\frac{c}{d}} = \frac{d}{c}$.

    1. Definition of Division

    $\frac{1}{\frac{c}{d}} = \frac{1}{c \cdot d^{-1}}$

    2. Previous Property (From Exercise 20)

    $\frac{1}{c \cdot d^{-1}} = \frac{1}{c} \cdot \frac{1}{d^{-1}} = \frac{1}{c} \cdot (d^{-1})^{-1}$

    3. Proving Inverse of an Inverse

    $d^{-1} \cdot (d^{-1})^{-1} = 1$ (Multiplicative Inverse Property)

    $d^{-1} \cdot d = 1$ (Multiplicative Inverse Property)

    Thus, $(d^{-1})^{-1}$ and d are equivalent.

    4. Inverse of an Inverse and Commutative Property

    $\frac{1}{c} \cdot (d^{-1})^{-1} = \frac{1}{c} \cdot d = \boxed{\dfrac{d}{c}}$

23. Prove: If $b \neq 0$, $c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$.

    1. Commutative Property

    $\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{1}{\frac{c}{d}}$

    2. Previous Property (From Exercise 22)

    $\frac{a}{b} \cdot \frac{1}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$

    3. Previous Property (From Exercise 21)

    $\frac{a}{b} \cdot \frac{d}{c} = \boxed{\dfrac{ad}{bc}}$

Solutions

Proof for Exercise 21

Statements	Reasons
1. $b \neq 0$ and $d \neq 0$	1. Given
2. $\frac{a}{b} = a \cdot \frac{1}{b}$ and $\frac{c}{d} = c \cdot \frac{1}{d}$	2. Definition of division
3. $\frac{a}{b} \cdot \frac{c}{d} = \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{d}\right)$	3. Substitution
4. $= (a \cdot c) \cdot \left(\frac{1}{b} \cdot \frac{1}{d}\right)$	4. Commutative and Associative properties of multiplication
5. $= ac \cdot \frac{1}{bd}$	5. Assumed from Exercise 20
6. $= \frac{ac}{bd}$	6. Definition of division
7. $\therefore \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	7. Transitive property of equality

Proof for Exercise 22

Statements	Reasons
1. $c \neq 0$ and $d \neq 0$	1. Given
2. $\frac{1}{\frac{c}{d}} = 1 \div \frac{c}{d}$	2. Definition of fraction bar as division
3. The reciprocal of a nonzero number x is $\frac{1}{x}$. The reciprocal of $\frac{c}{d}$ is $\frac{1}{\frac{c}{d}}$.	3. Definition of a reciprocal.
4. $\frac{c}{d} \cdot \frac{d}{c} = \frac{cd}{dc} = 1$	4. Proof from exercise 21.
5. The reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$	5. Definition of a reciprocal $(a \cdot b = 1)$
6. $\frac{1}{\frac{c}{d}} = \frac{d}{c}$	6. Substitution

Proof for Exercise 23

Statements	Reasons
1. $b \neq 0$, $c \neq 0$, $d \neq 0$	1. Given
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{1}{\frac{c}{d}}$	2. Definition of division
3. $\frac{1}{\frac{c}{d}} = \frac{d}{c}$	3. Result from Exercise 22
4. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	4. Substitution
5. $= \frac{ad}{bc}$	5. Result from Exercise 21
6. $\therefore \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$	6. Transitive property of equality