9:06 - 10:30

2 hour and 24 min

Integrated Algebra 2 and Precalculus

Exam: Chapter 9 of Algebra 2

Systems of Linear Equations: From Two Variables to Matrices

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Date: 7/25/25

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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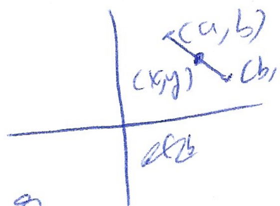
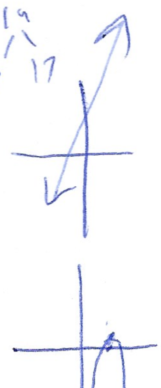
1. The midpoint of the segment connecting (a, b) and (b, a) is (x, y). Express y in terms of x. (8)



(not accurate)

$$\boxed{x = \frac{a+b}{2} \qquad y = \frac{a+b}{2}}$$

2. At what two points do the graphs of $y = -2x^2 + 5x - 3$ and $4x - y = -12$ intersect? (10)



$4x+12 = -2x^2+5x-3$

$2x^2 - x + 15 = 0$

$x = \frac{1 \pm \sqrt{1-120}}{4} = \frac{1}{4} \pm \frac{\sqrt{119}\, i}{4}$

~~$y = \sqrt{119}i + 12$~~

$y = 4x+12$

$y = 4\left(\frac{1}{4} \pm \frac{\sqrt{119}\, i}{4}\right) + 12 = 13 \pm \sqrt{119}\, i$

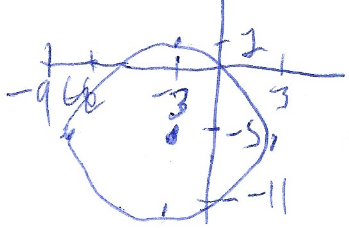
$\left(0.25 + \frac{\sqrt{119}\, i}{4},\ 13 + \sqrt{119}\, i\right)$

$\left(0.25 - \frac{\sqrt{119}\, i}{4},\ 13 - \sqrt{119}\, i\right)$

3. At how many points does the graph of the equation $3x + y = 7$ intersect the graph of the equation $x^2 + y^2 + 6x + 10y = 2$? (8)

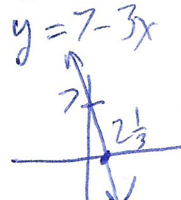
$x^2 + 6x + 9 + y^2 + 10y + 25 - 36 = 0$

$(x+3)^2 + (y+5)^2 = 6^2$



$-10y = 30x$

$y = 7 - 3x$



$(x+3)^2 + (-3x+12)^2 = 6^2$

$x^2+6x+9+9x^2-72x+144 = 36$

$10x^2 - 66x + 117 = 0$

$5x^2 - 33x + 57 = 0$

$x = \frac{33 \pm \sqrt{1089-1140}}{10}$

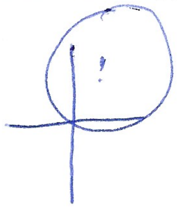
$x = 3.3 \pm \frac{\sqrt{51}\, i}{10}$

$y = 7 - 9.9 \mp \frac{3\sqrt{51}\, i}{10}$

$= -2.9 \mp \frac{3\sqrt{51}\, i}{10}$

No real times, 2 complex

4. A diameter of a circle is a segment with endpoints on the circle such that the segment passes through the center of the circle. Find the equation of the circle that has center $(3, 5)$ and a diameter with length 6 units. (10)



$(x-3)^2+(y-5)^2=6^2$

or

$x^2-6x+9+y^2-10y+25=36$

$\Rightarrow$

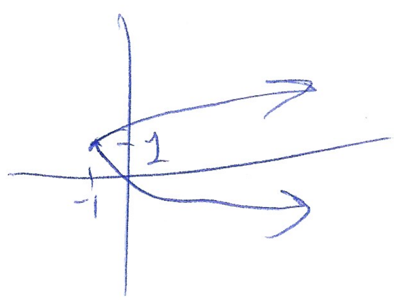
$x^2+y^2-6x-10y=2$

5. Graph the equation $x = 2y^2 - 4y + 1$. What is its vertex? (12)

$y=\frac{4}{4}=1$

$x=2(1)^2-4(1)+1=2-4+1=-1$

$(-1,1)$



6. A parabola has its vertex at $(4, -5)$ and meets the x-axis at two points that are on opposite sides of the y-axis. Suppose this parabola is the graph of $y = ax^2 + bx + c$. Determine which of a, b, and c must be positive, which must be negative, and which could be either. (10)

Hint: What direction does the parabola open? What is the vertex of $y = ax^2 + bx + c$ in terms of a, b, and c?

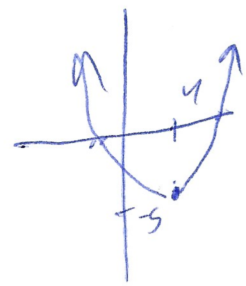
Hint: What does the fact that the parabola meets the x-axis on opposite sides of the y-axis tell us about the roots of $ax^2 + bx + c = 0$?

$a > 0$

$c < 0$

$\cancel{b \in \mathbb{R}}$

$\frac{-b}{2a} = 4$ so $b < 0$



7. For how many values of a is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$? (8)

$\cancel{x + a = x^2 + a^2}$

$\frac{-b}{2a} = \frac{0}{2}$

$(0, a^2)$

$a^2 = 0 + a$

$a^2 = a$

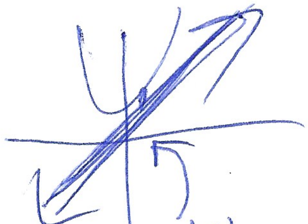
only true for $\cancel{a = 0}$

$a = 0$ or $a = 1$

8. If the parabola defined by $y = ax^2 + b$ is tangent to the line $y = x$, then calculate the constant a. (A line is tangent to a parabola if it touches the parabola at one point, but is otherwise always "outside" the parabola.) (10)

$a = \frac{1}{4b}$

for any $b \in \mathbb{R}$



(not accurate)

vertex $= (0, b)$

If it touches at $x = c$ then

$ac^2 + b = c$

$ac^2 \cancel{+} c + b = 0$

$c = \frac{1 \pm \sqrt{1-4ab}}{2a}$

$1 - 4ab = 0 \Rightarrow 4ab = 1 \Rightarrow a = \frac{1}{4b}$

9. Find the vertex and axis of symmetry of the graph of $x = \frac{1}{6}(y-4)^2 - 1$. Graph the equation. (12)

$x = \frac{1}{6}y^2 - \frac{4}{3}y + \frac{5}{3}$

$y = \frac{4/3}{2/6} = 4$

$x = \frac{1}{6}(4-4)^2 - 1 = -1$

$(-1, 4)$

Axis of symmetry:

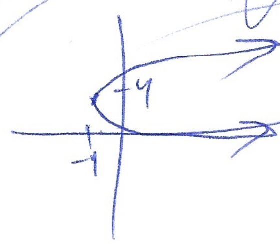
$y = 4$

$x = \frac{1}{6}y^2 - \frac{4}{3}y + \frac{8}{3}$

$y = \frac{4/3}{2/6} = \frac{8}{2} = 4$

$x = \frac{1}{6}(4-4)^2 = 6$

$(0, 4)$



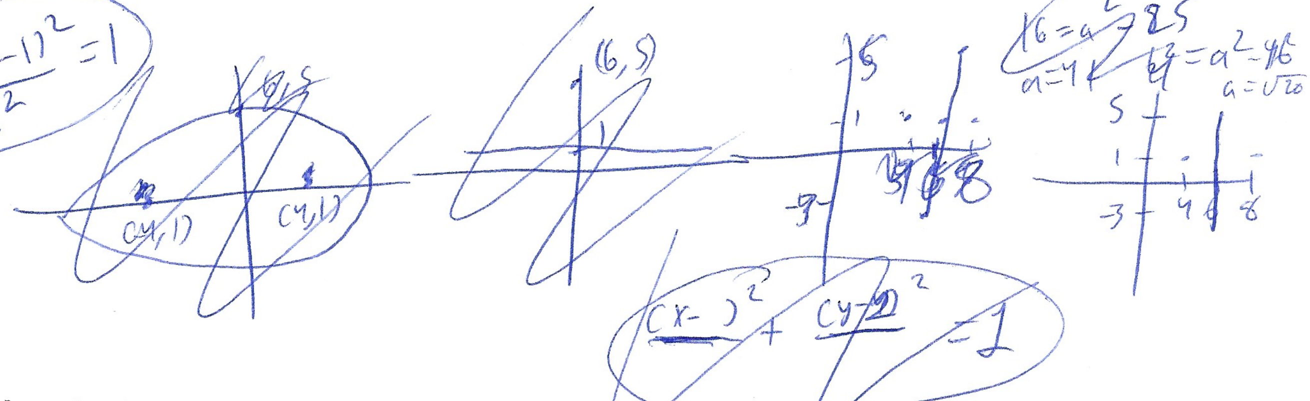
10. Ellipse $\mathcal{E}$ has a horizontal major axis, one focus at $(4, 1)$, and one axis with an endpoint at $(6, 5)$. Find an equation whose graph is this ellipse. (10)

$$\frac{(x-6)^2}{20}+\frac{(y-1)^2}{4^2}=1$$

$a=\sqrt{20}$

$b=4$

$c=2$

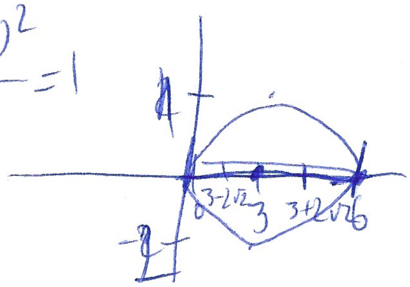


$a=\sqrt{20}$

11. Graph each of the equations below, then find the foci, center, and lengths of the axes of each graph. (8)

(a) $\frac{(x-3)^2}{9}+y^2=1$

$$\frac{(x-3)^2}{3^2}+\frac{(y-0)^2}{1^2}=1$$



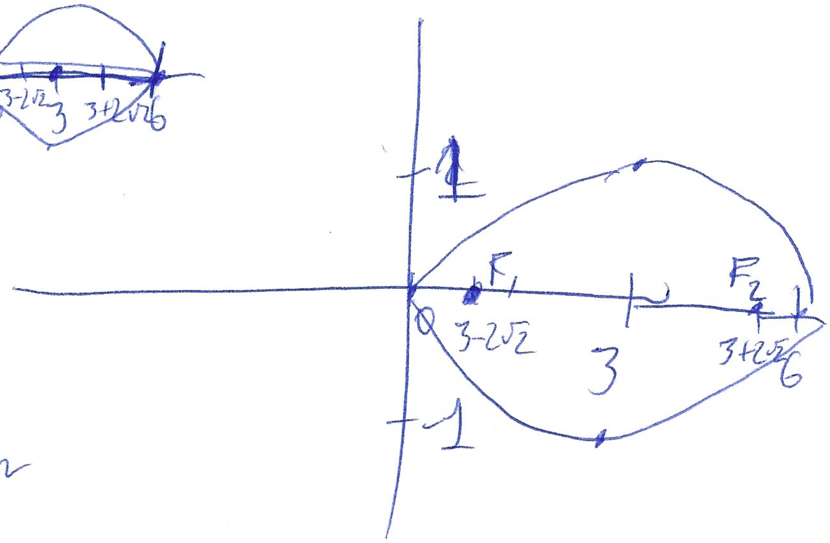
$a=3$

$b=1$

$c^2=9-1=8$

$c=2\sqrt{2}$

$(h,k)=(3,0)$ center



$(3+2\sqrt{2},0)$

$(3-2\sqrt{2},0)$ foci

6 & 2 lengths

Extra Credit: P is a fixed point on the diameter $\overline{AB}$ of a circle. Prove that for any chord $\overline{CD}$ of the circle that is parallel to $\overline{AB}$, we have $PC^2 + PD^2 = PA^2 + PB^2$.

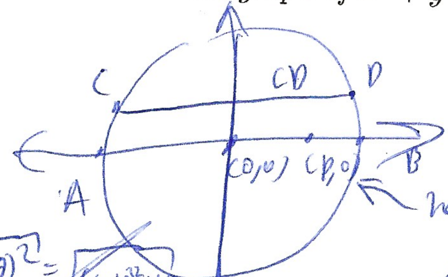
Hint: Suppose the circle is the graph of $x^2 + y^2 = r^2$, and let P be $(p, 0)$.

$C = (-c, b)$

$D = (c, b)$

$A = (-a, 0)$

$B = (a, 0)$



$PD = \sqrt{(c-p)^2 + (b-0)^2} = \sqrt{(c-p)^2 + b^2}$

$PB = \sqrt{(a-p)^2 + (0-0)^2} = |a-p|$

$PA = \sqrt{(-a-p)^2 + (0-0)^2} = |-a-p|$

$PC = \sqrt{(-c-p)^2 + (b-0)^2} = \sqrt{(-c-p)^2 + b^2}$

$(c-p)^2 + b^2 + (-c-p)^2 + b^2 =$

$(a-p)^2 + (-a-p)^2 \Rightarrow$

$P = (p, 0)$

$A = (-a, 0)$

$B = (a, 0)$

$C = (-b, c)$

$D = (b, c)$

$PA^2 = \left(\sqrt{(-a-p)^2 + (0-0)^2}\right)^2 = (-a-p)^2 = a^2 + 2ap + p^2$

$PB^2 = \left(\sqrt{(a-p)^2 + (0-0)^2}\right)^2 = (a-p)^2 = a^2 - 2ap + p^2$

$PC^2 = \left(\sqrt{(-b-p)^2 + (c-0)^2}\right)^2 = b^2 + 2bp + p^2 + c^2$

$PD^2 = \left(\sqrt{(b-p)^2 + (c-0)^2}\right)^2 = b^2 - 2bp + p^2 + c^2$

$(b^2 + \cancel{2bp} + p^2 + c^2) + (b^2 - \cancel{2bp} + p^2 + c^2)$

$= (a^2 + \cancel{2ap} + p^2) + (a^2 - \cancel{2ap} + p^2)$

$\Rightarrow \cancel{2}(b^2 + \cancel{p^2} + c^2) = \cancel{2}(a^2 + \cancel{p^2})$

$\Rightarrow a^2 = b^2 + c^2 \Rightarrow \sqrt{a^2} = \sqrt{b^2 + c^2}$

$\sqrt{a^2}$ is the distance of point A (or B) from the center since $A = (-a, 0)$;

$\sqrt{b^2 + c^2}$ is the distance of point C or D from the center since $C = (-b, c)$

Since all points on the circle are equidistant to the center, this holds true regardless of P and $(p, 0) = P$.