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35 minutes

Integrated Algebra 2 and Precalculus

Exam: Chapter 1 of Algebra 2

Basic Concepts of Algebra

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. Evaluate each of the following expressions when $a = -8$ and $b = \frac{1}{2}$.

(16)

(a) $ab + 2b + 3a$

$$(-8)(\frac{1}{2}) + 2(\frac{1}{2}) + 3(-8) = -4 + 1 - 24 = -27$$

(b) $\frac{a}{b}$

$$(-8) / (\frac{1}{2}) = -16$$

(c) $4(a^2b + b^2a)$

$$4((-8)^2 \cdot (\frac{1}{2}) + (\frac{1}{2})^2 \cdot (-8)) = 4(32 + 0.25(-8)) = 120$$

(d) $(a - 2)\sqrt{-ab}$

$$(-8 - 2)\sqrt{-(-8)(\frac{1}{2})} = -10 \cdot \sqrt{4} = -10 \cdot 2 = -20$$

2. Simplify the product as much as possible.

$$\frac{1}{3(a^2 - 2b)} \cdot \frac{9a(a^2 - 2b)}{5a^2}$$

$$\frac{9a}{15a^2} = \frac{3}{5a}$$

$$\frac{2}{3a^2 - 6b} \cdot \frac{9a(a^2 - 2b)}{5a^2}$$

$$\frac{3}{5a}$$

(5)

3. Decide whether each set is a field under the operations of addition and multiplication. (12)
If the set is not a field, name at least one field property that does not hold.

(a) The natural numbers (\mathbb{N})

No multiplicative inverse X

(b) The integers (\mathbb{Z})

No multiplicative inverse X

(c) The rational numbers (\mathbb{Q})

Closed under addition & multiplication, ✓

Has inverses

Distributive and Associative Property

(d) The negative rational numbers

Not closed under multiplication: $-3 \cdot (-2) = 6$ X
not in the set

4. Solve for the variable in each equation.

(10)

- (a) Find all values of y such that $\frac{3}{2+\sqrt{y}} + \frac{4}{2+\sqrt{y}} = 1$.

$$\frac{7}{2+\sqrt{y}} = 1 \quad 2+\sqrt{y} = 7$$

$$\sqrt{y} = 5$$

$$y = 25$$

- (b) What values of x satisfy $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$?

$$\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x+1} - 3\sqrt{x-1}$$

$$4\sqrt{x-1} = 2\sqrt{x+1}$$

$$\sqrt{x+1} = 2\sqrt{x-1}$$

$$\sqrt{x+1} = \sqrt{4}\sqrt{x-1}$$

$$x+1 = 4(x-1)$$

$$x+1 = 4x-4$$

$$3x = 5$$

$$x = \frac{5}{3}$$

5. (a) Let x be the middle integer of three consecutive integers. What is the sum of these three integers in terms of x ? (8)

$$3x$$

$$(x+1) + x + (x-1) = 3x$$

- (b) The sum of 23 consecutive integers is 2323. What is the largest of the integers? (Hint: Use the result from the first part.)

$$(x-11) + (x-10) + (x-9) + \dots + x + \dots + (x+9) + (x+10) + (x+11) = 2323$$

$$23x = 2323$$

$$x = 101$$

$$201 \text{ is the largest integer}$$

6. A grocer wants to mix peanuts and cashews to produce 20 lb of mixed nuts worth \$6.20/lb. How many pounds of each kind of nut should she use if peanuts cost \$4.80/lb (6)

and cashews cost \$8.00/lb?

$$\cancel{6.2x} \quad 4.8x + 8(20-x) = 20 \cdot 6.2$$

$$160 - 3.2x = 124$$

$$3.2x = 36$$

$$16x = 180$$

$$\cancel{8x=240} \quad 4x=95$$

$$x = \frac{95}{4}$$