2:25 - 3:20 55 minutes

Integrated Algebra 2 and Precalculus

Exam: Chapter 2 of Algebra 2

Inequalities and Proof

Name: Krish Arora Date: 7/11/25

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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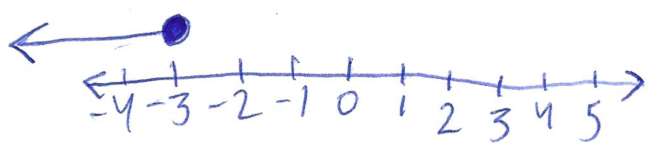
1. For each of the following inequalities, graph the solution to the inequality on the number line and write the solution to the inequality using interval notation. (15)

   (a) $2 - 3x \geq 11$



$3x \leq -11+2$

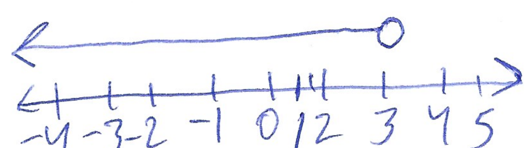
$3x \leq -9$

$x \leq -3$

   (b) $3 + 2x < 30 - 7x$

$9x < 27$

$x < 3$



   (c) $8 - 2x \leq 5 - 5x < 23 - 2x$

$8-2x \leq 5-5x$

$3x \leq -3$

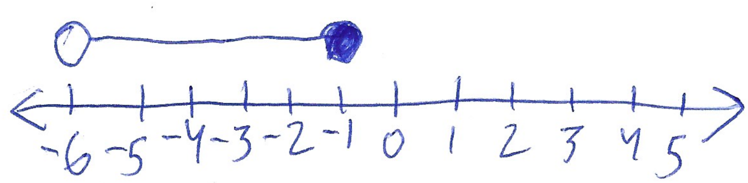
$x \leq -1$

$5-5x < 23-2x$

$-3x < 18$

$x > -6$

$-6 < x \leq -1$



2. What is the smallest positive integer that has a square root that is greater than 10? (10)

$\sqrt{x} > 10$

$x > 100$

The smallest positive integer (bigger than 100) is 101.

3. Solve the equation $|2z - 9| + |z - 3| = 15$. (10)

Case 1: $z < 4.5$;

Case 1: $z \geq 3$;

$-2z + 9 + z - 3 = 15$; $6 - z = 15$; $z = -9$

no sol.

Case 2: $z < 3$;

$-2z + 9 - z + 3 = 15$; $-3z + 12 = 15$

$z = -1$

Case 2: $z \geq 4.5$

Case 1: $z \geq 3$;

$2z - 9 + z - 3 = 15$; $3z - 12 = 15$

$z = 9$

~~Case 2: $z < 3$~~

$z \geq 4.5$ no sol.

$$\boxed{z \in \{-1, 9\}}$$

4. Solve the inequality $\left|\frac{1}{4}t - 2\right| \leq \frac{3}{4}$. (10)

Case 1: $t < 8$

$-\frac{1}{4}t + 2 \leq \frac{3}{4}$

$-t + 8 \leq 3$

$-t \leq -5$

$t \geq 5$

Case 2: $t \geq 8$

$\frac{1}{4}t - 2 \leq 3/4$

$t - 8 \leq 3$

$t \leq 11$

$5 \leq t < 8$

and

$8 \leq t \leq 11$

$5 \leq t \leq 11$

5. Betty goes to the store to get flour and sugar. The amount of flour she buys, in pounds, is at least 6 pounds more than half the amount of sugar, and is no more than twice the amount of sugar. Find the least number of pounds of sugar that Betty could buy. (10)

let f = amount of flower; let s = amount of sugar

$f \geq 6 + \frac{s}{2}$

$f \leq 2s$

$6 + \frac{s}{2} \leq f \leq 2s$

$6 + \frac{s}{2} \leq 2s$

$12 + s \leq 4s$

$3s \geq 12$

$s \geq 4$

$\boxed{\text{The least amount of sugar is 4 pounds.}}$

6. Give a counterexample (with $a, b \in \mathbb{R}$) to show that each statement is false. (10)

(a) If $a^4 = b^4$, then $a - b = 0$.

$a = -8$
$b = 8$

$(-8)^4 = (8)^4$

$65536 = 65536$ ✓

$-8 - 8 \neq 0$ X

(b) If $a < b$, then $b - a < 0$.

$a = -2$
$b = 5$

$a < b$ ✓

$5 - (-2) = 7 > 0$ X

7. Prove the cancellation property of multiplication: If $c \neq 0$ and $ca = cb$, then $a = b$. (12)

$c \cdot \frac{1}{c} = 1$	(Multiplicative Inverse Property)
$\frac{1}{c} \cdot ca = a$	(Substitution)
$\frac{1}{c} \cdot cb = b$	(Substitution)
$ca = cb$	(given)
$\frac{1}{c} \cdot ca = b$	(Substitution)
$a = b$	(Mutiplicative Inverse Property)

8. Prove: For all real numbers a and b, $|ab| = |a| \cdot |b|$. (Hint: Consider the different cases for the signs of a and b.) (10)

Case 1: $a \geq 0$

Case 1: $b \geq 0$

~~$|ab|$~~ $a \cdot b \geq 0 \therefore |ab| = ab$

$a \geq 0 \therefore |a| = a$; $b \geq 0 \therefore |b| = b$

$ab = ab$ ✓

Case 2: $b < 0$

$a \geq 0 \therefore |a| = a$; $b < 0 \therefore |b| = -b$

$ab \leq 0 \therefore |ab| = -ab$

$-ab = a \cdot (-b) = -ab$ ✓

Case 2: $a < 0$

Case 1: $b \geq 0$

$a < 0 \therefore |a| = -a$; $b \geq 0 \therefore |b| = b$

$ab < 0 \therefore |ab| = -ab$

$-ab = -a \cdot b = -ab$ ✓

Case 2: $b < 0$

$a < 0 \therefore |a| = -a$; $b < 0 \therefore |b| = -b$

~~$ab \geq 0 \therefore |ab| = ab$~~

$ab = (-a) \cdot (-b) = ab$ ✓

True, this holds for all cases.

aditional proof: ~~$\sqrt{x}$~~

$|x| = \sqrt{x^2}$

$\sqrt{a^2b^2} = \sqrt{a^2} \cdot \sqrt{b^2} = \sqrt{a^2b^2}$ ✓

9. (a) Describe the difference between an axiom (or postulate) and a theorem. (8)

We take axioms as given facts that must be true, and construct theorems from axioms using formal proofs.

(b) Describe what a corollary of a theorem is.

A corollary is a statement that can be trivially derived from a theorem, such as if $ca = cb$ then $a = b$ (theorem) then if ~~a~~ ~~$abcd = ebcd$ then a=~~ $c^2 a = c^2 b$ then $a = b$ (Corollary).

10. Determine whether each statement is true or false. If it is false, give a counterexample. If it is true, give a brief explanation of why it is true. (16)

(a) If $a < b$, then $a^2 < b^2$.

$a = -4$
$b = -2$

False.

$a < b$ ✓
$a^2 = 16$ $b^2 = 4$ $a^2 > b^2$ X

(b) If $a \neq 0, b \neq 0$, and $a > b$, then $\frac{1}{a} > \frac{1}{b}$.

$a = 5$
$b = 3$

$a > b$ ✓

$\frac{1}{a} = \frac{1}{5}$
$\frac{1}{b} = \frac{1}{3}$
$\frac{1}{b} > \frac{1}{a}$ X

False.

(c) For any number k, the equation $|k| = -k$ is always false.

$k = 4$
$|k| = 4 \neq -k$

False.

(d) If $c < 0$, then $ac > bc$ implies $a < b$.

True.

Because of the sign flip property, dividing both sides by c, a negative number, flips the sign, resulting in $a < b$.