Chapter 1 Assignment — Fundamentals: Strategies, Practice, and Challenges

Name: ____________________ Date: ____________________

How to use this set: Each topic begins with *Strategy Notes* and a *Worked Example* to model thinking for harder problems. Then try the *Practice* (skill-building) and the *Challenge* (beyond-exam) items. Show full reasoning and state domain restrictions when relevant.

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1. **Topic 1: Absolute Value Inequalities and Interval Reasoning**

   Strategy Notes. Use $|A| < k$ $(k > 0)$ $\iff$ $-k < A < k$ and $|A| > k$ $\iff$ $A < -k$ or $A > k$. For expressions like $|\dfrac{ax+b}{cx+d}| \,\square\, k$, also track the sign of the denominator and *forbid* where it is zero.

   Worked Example. Solve $\left|\dfrac{2x-5}{x+1}\right| \geq 3$, $x \neq -1$.

   We solve two cases: $\dfrac{2x-5}{x+1} \geq 3$ or $\dfrac{2x-5}{x+1} \leq -3$.

   Case 1: $\dfrac{2x-5}{x+1} - 3 \geq 0 \;\Rightarrow\; \dfrac{2x-5-3x-3}{x+1} \geq 0 \Rightarrow \dfrac{-x-8}{x+1} \geq 0$. Critical points: $x = -1, -8$. Sign chart gives $[-8, -1)$.

   Case 2: $\dfrac{2x-5}{x+1} + 3 \leq 0 \;\Rightarrow\; \dfrac{2x-5+3x+3}{x+1} \leq 0 \Rightarrow \dfrac{5x-2}{x+1} \leq 0$. Critical points: $x = -1, \frac{2}{5}$. A sign chart gives the solution $(-1, \frac{2}{5}]$.

   Combine: $[-8, -1) \cup (-1, \frac{2}{5}]$, with $x \neq -1$.

   Practice.

   (a) Solve $|2x - 3| \leq 5$ and write interval notation.

   (b) Write $(-6, 2] \cup [5, \infty)$ using inequalities; sketch.

   (c) Find all x such that $|\dfrac{x+4}{x-2}| < 2$.

2. **Topic 2: Exponents and Radicals at an Advanced Level**

   Strategy Notes. Convert roots to rational exponents when helpful: $\sqrt[n]{a^m} = a^{m/n}$ with principal roots. Factor radicands to expose perfect powers. Rationalize with conjugates for multi-term denominators.

   Worked Example. Simplify $\left(81a^4b^{-2}\right)^{3/4} \cdot \left(\sqrt{18} - \sqrt{8}\sqrt{2}\right)$.

   First factor: $(81)^{3/4}a^3b^{-3/2} = 3^3a^3b^{-3/2} = 27a^3b^{-3/2}$. Second factor: $\sqrt{18} - \sqrt{16} = 3\sqrt{2} - 4$. Final:

   $$\boxed{27a^3\,b^{-3/2}\,(3\sqrt{2} - 4) \;=\; \frac{27a^3(3\sqrt{2} - 4)}{b^{3/2}}}$$

   Practice.

   (a) $\left(\dfrac{32x^{-5}y^{12}}{2x^3y^{-6}}\right)^{-2/5}$

   (b) $\dfrac{3\sqrt{5}}{2\sqrt{2} + \sqrt{3}}$ (rationalize fully)

   (c) $\dfrac{\sqrt[3]{54x^5}}{\sqrt[3]{2x}}$

   Challenge.

   (a) Simplify $\dfrac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} + \dfrac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$ for $a, b > 0$.

   (b) Simplify $\left(\dfrac{x^{1/3} - x^{-1/3}}{x^{1/6}}\right)^6$ for $x > 0$.

3. **Topic 3: Rational Expressions — Factor, Restrict, Simplify**

Strategy Notes. Always factor first and state restrictions from original denominators. For complex rational expressions, clear the small fractions by multiplying numerator and denominator by the least common denominator.

Worked Example. Simplify and state restrictions:

$$\frac{x^2-9}{x^2-4x+3}\cdot\frac{x-3}{x+1}.$$

Factoring gives $\dfrac{(x-3)(x+3)}{(x-1)(x-3)}\cdot\dfrac{x-3}{x+1} = \dfrac{x+3}{x-1}\cdot\dfrac{x-3}{x+1} = \dfrac{(x+3)(x-3)}{(x-1)(x+1)} = \dfrac{x^2-9}{x^2-1}$. Restrictions: $x \neq -1, 1, 3$.

Practice.

(a) Simplify $\dfrac{x^2+3x+2}{x^2-x-2}$ and state restrictions.

(b) Simplify $\dfrac{1}{x-3}-\dfrac{1}{x+3}$ and state restrictions.

(c) Simplify $\dfrac{2x}{x^2-9}+\dfrac{3}{x+3}$ and state restrictions.

Challenge.

(a) Simplify $\dfrac{\frac{1}{x}-\frac{1}{x-2}}{\frac{1}{x+2}}$ and give all restrictions.

(b) Solve $\dfrac{x^2-5x+6}{x-2}=x$ and reject any extraneous solutions.

4. **Topic 4: Equations with Radicals/Rationals — Extraneous Solutions**

   Strategy Notes. Isolate a radical before squaring. Enforce domain restrictions (radicands ≥ 0, denominators $\neq 0$). After squaring, *check* candidates in the original equation.

   Worked Example. Solve $\sqrt{2x+3} - \sqrt{x-1} = 1$.

   Let $\sqrt{2x+3} = 1 + \sqrt{x-1}$. Square: $2x+3 = 1 + 2\sqrt{x-1} + x - 1 \Rightarrow x+3 = 2\sqrt{x-1}$. Square again: $(x+3)^2 = 4(x-1)$. Then $x^2+6x+9 = 4x-4 \Rightarrow x^2+2x+13 = 0$, which has no real solutions. *Conclusion:* **No real solution** (the attempt produced complex roots). Domain check prevents spurious answers.

   Practice.

   (a) $\sqrt{x+4} = x - 2$ (check for extraneous solutions)

   (b) $\dfrac{2}{x-3} + \dfrac{1}{x+3} = 1$

   (c) $\sqrt{3x+4} \geq x - 1$ (solve as an inequality; show domain first)

   Challenge.

   (a) Solve $\sqrt{x+7} + \sqrt{2x-1} = 6$.

   (b) Solve $\sqrt{2x-3} = \dfrac{x}{x-2}$ with domain restrictions.

5. **Topic 5: Advanced Inequalities — Mixed Forms (Consolidated)**

Strategy Notes. For $\dfrac{N(x)}{D(x)} \square 0$: find real zeros of N and D; plot as critical points; test intervals. Denominator zeros are *excluded* from the solution. For radicals, add the domain endpoints to the chart.

Worked Example. Solve $\sqrt{2x+5} < x+1$.

Domain to allow squaring with inequality preserved: $x \geq -1$ (so $x+1 \geq 0$) and $x \geq -\frac{5}{2}$ from the radicand. Work on $x \geq -1$. Square to get $2x+5 < (x+1)^2 = x^2+2x+1$, hence $0 < x^2-4$ and $|x| > 2$. Combine with $x \geq -1$ to obtain $(2, \infty)$.

Challenge.

(a) Solve $\left|\dfrac{x-2}{x+1}\right| + \left|\dfrac{x+3}{x-4}\right| \geq 3$ (state restrictions and justify interval testing).

(b) Solve $\dfrac{x^4-5x^2+4}{x^2-4x+3} > 0$ (factor completely; distinguish cancellations from domain restrictions; use a sign chart).

(c) Solve the system $\begin{cases} \sqrt{2x+5} \leq x-1, \\ |x-3| \geq 4. \end{cases}$

(d) Parameter analysis: find all real k for which $|x-1| \leq k(x+2)$ has a real solution; for maximal such k, describe the solution set.

6. **Topic 6: Coordinate Geometry and Lines**

Strategy Notes. Distance $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Slope $m = \frac{y_2-y_1}{x_2-x_1}$. A perpendicular bisector has slope $-\frac{1}{m}$ and passes through the midpoint. A circle with diameter $\overline{PQ}$ has center at the midpoint and radius $\frac{1}{2}d$.

Worked Example. Let $P(-2, 5)$ and $Q(6, -1)$. Find the circle for which $\overline{PQ}$ is a diameter.

Midpoint $M(2, 2)$, distance $d = \sqrt{(8)^2 + (-6)^2} = \sqrt{100} = 10$, radius $= 5$. Equation: $(x - 2)^2 + (y - 2)^2 = 25$.

Practice.

(a) Find the perpendicular bisector of the segment joining $(3, -6)$ and $(-1, 2)$.

(b) Find the line through $(-2, 5)$ perpendicular to $y = -2x + 7$ and its intercepts.

(c) Through $(3, -6)$ parallel to $3x+y-10 = 0$: find slope-intercept form and intercepts.

Challenge.

(a) Find all points on the circle $(x - 2)^2 + (y + 1)^2 = 13$ whose tangent line has slope $-\frac{3}{4}$.