Irrational and Complex Numbers

Name: _____ Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Rationalize the denominators of the following: (8)

   (a) $\frac{3}{\sqrt{6}}$

   (b) $\frac{2}{\sqrt{9}}$

2. Rationalize the denominator of $\frac{2}{\sqrt{2}-\sqrt{5}+\sqrt{7}}$. (10)

3. Simplify: $\frac{1}{\sqrt{100}+\sqrt{99}} + \frac{1}{\sqrt{99}+\sqrt{98}} + \frac{1}{\sqrt{98}+\sqrt{97}} + \cdots + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{1}}$ (12)

4. Solve $3 - 2\sqrt{x} = 7$. (8)

   (a) $\{-2\}$

   (b) $\{4\}$

   (c) $\{-4\}$

   (d) $\emptyset$

5. Simplify. If no simplification is possible, say so. (10)

$$\sqrt[3]{24} - \sqrt[3]{56} + \sqrt[3]{81}$$

6. Find all complex numbers $a + bi$ such that $a + bi = (a + bi)^2$. (12)

   Hint: Write each side as a complex number in terms of a and b. Build a system of equations by considering the real and imaginary parts of both sides.

7. Simplify $(i - i^{-1})^{-1}$. (8)

8. If $x = \frac{1-i\sqrt{3}}{2}$, then what complex number is equal to $\frac{1}{x^2-x}$? (10)

9. For two positive numbers x and y: (12)

   (a) The arithmetic mean of two numbers x and y is the number $\frac{x+y}{2}$. If x and y are rational numbers, what can you conclude about their arithmetic mean?

   (b) The geometric mean of two positive numbers x and y is the number $\sqrt{xy}$. If x and y are positive rational numbers, can you conclude that their geometric mean is also rational? Explain.

10. Suppose x is rational and z is irrational. Prove that $x + z$ is irrational. (10)

    Hint: Let $x = \frac{a}{b}$ where a and b are integers, and use an indirect proof by assuming that $x + z$ is a rational number $\frac{c}{d}$.