11:10 - 12:32 and 1:18 - 1:22

1 hour and 26 minutes

Integrated Algebra 2 and Precalculus | Exam: Chapter 4 of Algebra 2

Polynomial Operations and Factoring

Name: Krish Arora Date: 7/15/25

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Find all solutions to each of the following equations: (10)

   (a) $r^2 - 7r = 0$

   $\frac{7 \pm \sqrt{49-4(1)(0)}}{2} = \frac{14}{2}$ and $\frac{0}{2}$

   $r \in \{7, 0\}$

   (b) $x^2 + 3x = 7x - x^2$

   $2x^2 - 4x = 0$

   $2(x^2 - 2x) = 0 \quad x^2 - 2x = 0$

   $x \in \{0, 2\}$

   (c) $2t^2 = 242$ -1

   $t^2 = 121$

   $t = 11$

   $(-11)^2(2) = 121 \cdot 2 = 242$ ✗

   (d) $16 - y^2 = -4$ -1 ✗

   $y^2 - 20 = 0$

   $y^2 = 20$

   $y = \sqrt{20}$

   remember $\sqrt{x^2} = |x|$ not $= x$

   not equal

   $20 = 2 \cdot 10 = 5 \cdot 2 \cdot 2$

2. Find all solutions to the equation $t^4 - 11t^2 + 18 = 0$. (8)

   Hint: Let $u = t^2$ and solve for u first.

   $u^2 - 11u + 18 = 0$

   $(u-2)(u-9) = 0$

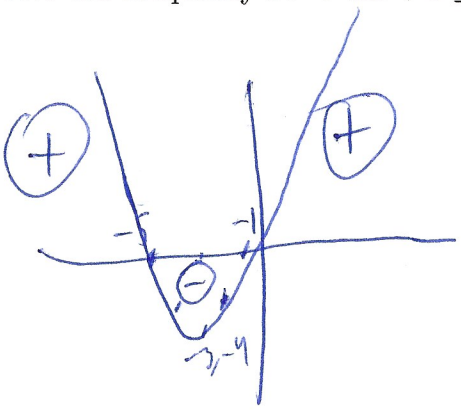
   $-2 \times -9 = 18$, $-2 - 9 = -11$

   $(t^2 - 2)(t^2 - 9) = 0$

   $(t+\sqrt{2})(t-\sqrt{2})(t+3)(t-3) = 0$

   $t \in \{-3, -\sqrt{2}, \sqrt{2}, 3\}$

3. Solve the inequality $m^2 + 6m + 5 \geq 0$, and graph the solutions on the number line. (10)



$\frac{-b}{2a} = -3$

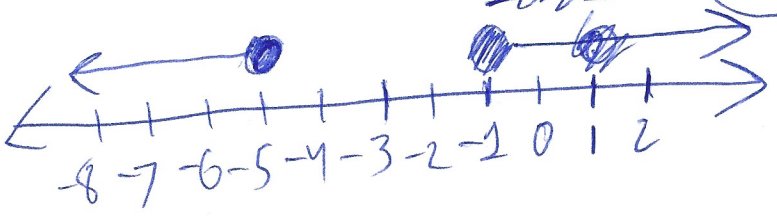
$9 - 18 + 5 = -4$

$m^2 + 6m + 5 = 0$

$m \in \{-5, -1\}$

$m^2 + 6m + 5 \geq 0$

$m \leq -5$ or $m \geq -1$



4. For what values of r is $2r^2 - 3r > -7$? (8)

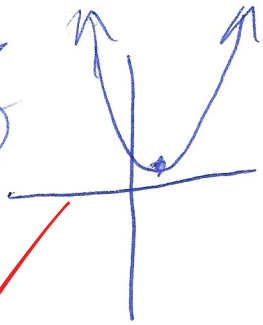
$2r^2-3r+7>0$

$2r^2-3r+7=0$

$\frac{3 \pm \sqrt{9-}}{4}$

Discriminant negative

$r \in \mathbb{R}$



5. Factor $x^4 + 4y^4$ (completely). (12)

Complete the square

$(x^2+2y^2)^2-(2xy)^2$

$(x^2+2xy+2y^2)(x^2-2xy+2y^2)$

$(x^2)^2+(2y^2)^2$

$(x^2-2y^2i)(x^2+2y^2)$

$(x+\sqrt{2}yi)(\sqrt{2}+\sqrt{2}i)$

6. Factor each of the following: (15)

(a) $a^3 + 27$

$(a+3)(a^2-3a+9)$

a^3+3^3

$(a+3)(a^2-3a+9)$

$a^3-3a^2+9a+3a^2-9a+27$

(b) $a^3b^3 + 8c^3$

$(ab)^3+(2c)^3=(ab+2c)(a^2b^2-2abc+4c^2)$

$(ab)^3+(2c)^3$

$(ab+2c)(a^2b^2-2abc+4c^2)$

(c) $2r^3 - 16$

$2(r^3-8)$

$2(r-2)(r^2+2r+4)$

(d) $1000 - x^6y^3$

$10^3-(x^2y)^3$

$(10-x^2y)(100+10x^2y+x^4y^2)$

$1000+100x^2y+10x^4y^2-100x^2y-10x^4y^2-x^6y^3$

7. (a) The expression $x^5 + y^5$ can be written as the product of $x + y$ and another factor. Find that other factor. (12)

$x^4 - x^3y + x^2y^2 - xy^3 + y^4$ ✓

(b) The expression $x^7 + y^7$ can be written as the product of $x + y$ and another factor. Find that other factor.

$x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6$ X

signs need to alternate because how else would you get to x^7+y^7

(c) Write $x^{2n+1} + y^{2n+1}$ as the product of two factors.

assuming $n \in \mathbb{Z}$, $2n+1 \% 2 = 1$ ($2n+1$ is odd)

$(x+y)(x^{2n+} \ldots$ X

$(x+y)\sum_{k=0}^{2n}(-1)^k x^{2n-k} y^k$

(d) Why does the factorization in the previous part fail when the powers of x and y are even? In other words, why can we not factor $x^4 + y^4$ or $x^6 + y^6$ using the patterns we found in the first three parts?

-1/2

The alternating signs make b^n negative, although all middle terms cancel, you are left with $x^n y^n$ not $x^n + y^n$.

$+$ doesn't cancel out fully

$(x+y)(x^3 - x^2y + xy^2 - y^3)$

$x^4 - x^3y + x^2y^2 - xy^3 + x^3y - x^2y^2 + xy^3 - y^4$

no $(x+y)(\ldots)$ factorization exists because $x+y$ does not divide them when exponent is even

8. Solve each polynomial equation by factoring: (10)

(a) $x^3 - 8 = 0$

(b) $x^3 + 64 = 0$

(c) $2x^3 - 16x = 0$

(d) $x^4 - 16 = 0$

9. Find the GCF of $54x^7t^3$, $90x^5t^2$, and $108x^4t$. (8)

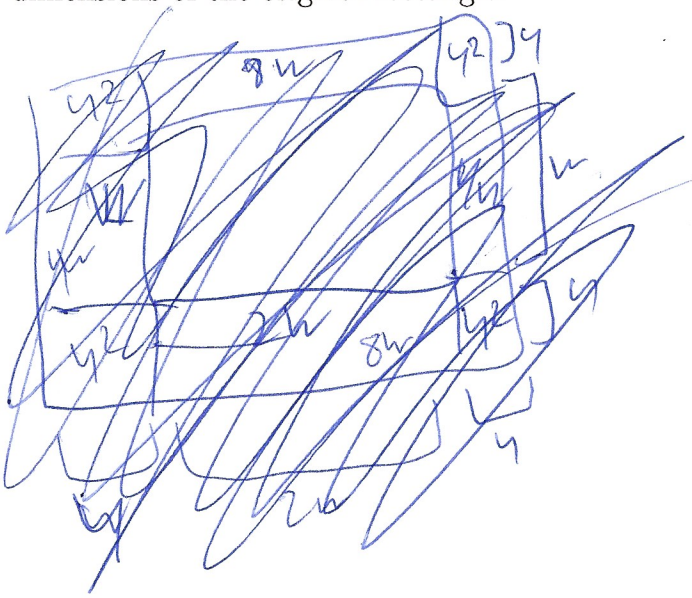
(a) $18x^7t$

(b) $9x^2t$

(c) $540x^4t^3$

(d) $36xt$

10. A rectangle is twice as long as it is wide. If its length is increased by 4 cm and its width is decreased by 3 cm, the new rectangle formed has an area of 100 cm^2. Find the dimensions of the original rectangle. (12)



$(2w+4)(w-3)=100$

$2w^2-6w+4w-12=100$

$2w^2-2w+88=0$

$2(w^2-w+44)=0$

negative discriminant

$2w^2-6w+4w-12-100=0$

$2w^2-2w-112=0$

$2(w^2-w-56)=0$

$(w-8)(w+7)=0$

$w=8, -7$

W: 8cm

L: 16cm