6:07-6:27 and 8:28 to 9:50 1 hour and 42 min

Integrated Algebra 2 and Precalculus

Exam: Chapter 8 of Algebra 2

Polynomial Division and Roots

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Find a constant c such that there is no remainder when $x^3 + cx^2 + 4x - 21$ is divided by $x - 3$. (8)

   Hint: You may use the Remainder Theorem or polynomial long division.

   $(3)^3 + c(3)^2 + 4(3) - 21 = 0$

   $27 + 9c + 12 - 21 = 0$

   $9c = -18$

   $c = -2$

2. Find the quotient and remainder when $x^4 - 23x^3 + 11x^2 + 14x + 20$ is divided by $x + 5$. (10)

   Hint: Consider using synthetic division for this problem.

   ~~$-5 \mid 1 \quad -23 \quad 11 \quad 14 \mid 20$~~

   ~~$-5 \quad 140 \quad 645$~~

   ~~$1 \quad -28 \quad -129 \quad 659$~~

   $x^3 - 28x^2 + 151x - 741 + \frac{3725}{x+5}$

   $-5 \mid 1 \quad -23 \quad 11 \quad 14 \mid 20$

   $-5 \quad 140 \quad -755 \mid 3705$

   $1x^3 \quad -28x^2 + 151x \quad -741 \mid +\frac{3925}{x+5}$

3. Find the quotient and remainder when $x^4 + 3x^3 - x^2 + 7x - 1$ is divided by $2 - x$. (8)

   Hint: Rewrite the divisor in standard form first.

   $-x^3 - 5x^2 - 9x - 25$

   $-x + 2 \overline{) x^4 + 3x^3 - x^2 + 7x - 1}$

   $-(x^4 - 2x^3)$

   $5x^3 - x^2 + 7x - 1$

   $-(5x^3 - 10x^2)$

   $9x^2 + 7x - 1$

   $-(9x^2 - 18x)$

   $25x - 1$

   $25x - 1$

   $-(25x - 50)$

   49

   Q: $-x^3 - 5x^2 - 9x - 25$

   R: 49

4. Find all roots of the following polynomial: (10)

$$g(y) = 12y^3 - 28y^2 - 9y + 10$$

Hint: Look for rational roots first using the Rational Root Theorem.

$y = \frac{5}{2}$

$y = \frac{1}{2}$

$y = -\frac{2}{3}$

$12y^2$
$-6y$
$+8y$
-4

10: 1, 2, 5
12: 1, 2, 3, 4, 6

1, 1/2, 1/3, 1/4, 1/6
2, 2/3,
5, 5/2, 5/3, 5/4, 5/6

$96 - 112 - 18 + 10$ X

$1500 - 700 - 45 + 10$ X

$(y - \frac{5}{2})(12y^2 + 2y - 4)$

$(y - \frac{5}{2})(2y - 1)(6y + 4)$

$12\frac{125}{8} - 28\frac{25}{4} - 9\frac{5}{2} + 10 = 1.5(125) - 7(25) - 5(4.5) + 10$

$125 + 62.5 - 175 - 22.5 + 10$

$-50 + 40 + 10$

$y = \frac{-2 \pm \sqrt{4 + 192}}{24} = -\frac{1}{24} \pm \frac{13}{24} = \{-\frac{15}{24}, \frac{11}{24}\}$

5. When $y^2 + my + 2$ is divided by $y - 1$, the quotient is $f(y)$ and the remainder is R_1. When $y^2 + my + 2$ is divided by $y + 1$, the quotient is $g(y)$ and the remainder is R_2. If $R_1 = R_2$, then find m. (12)

Hint: Use the Remainder Theorem to find expressions for R_1 and R_2.

$R_1 = (1)^2 + m(1) + 2 = m + 3$

$R_2 = (-1)^2 + m(-1) + 2 = -m + 3$

$m + 3 = -m + 3$

$2m = 0$

$m = 0$

6. Suppose $q(x)$ and $r(x)$ are the quotient and remainder, respectively, when the polynomial $f(x)$ is divided by the polynomial $d(x)$. Show that if $x = a$ is a root of $d(x)$, then $r(a) = f(a)$. (10)

   Hint: Use the division algorithm for polynomials.

   $f(x) = q(x)d(x) + r(x)$

   If $x = a$ is a root of $d(x)$, then

   $f(a) = q(a)d(a) + r(a) \Rightarrow f(a) = q(a)\cdot 0 + r(a) \Rightarrow f(a) = r(a)$

7. Find all roots of each of the following polynomials: (8)

   (a) $f(x) = x^3 - 4x^2 - 11x + 30$ $\rightarrow$ 1, 2, 3, 5, 6, 15, 30

   $125 - 4(25) - 11(5) + 30$ $\quad x = 2$

   $8 - 4(4) - 11(2) + 30$ $\quad x = 5$

   $-27 - 4(9) + 33 + 30$ $\quad x = -3$

   $x - 2 \,\overline{)\, x^3 - 4x^2 - 11x + 30}$ $\quad x^2 - 2x + 10$

   $-(x^3 - 2x^2 + 10x)$

   $-2x^2 - 21x + 30$

   $-(-2x^2 + 4x - 20)$

   $-25x + 50$

   $(x-2)(x^2 - 2x - 15) =$

   $(x-2)(x-5)$ $\quad x^2 - 7x + 10$

   (b) $g(t) = t^4 + 5t^3 - 19t^2 - 65t + 150$

   $16 + 24 - 76 - 130 + 150$ ✗

   $81 + 135 - 171 - 195 + 150$ ✓

   $x - 3 \,\overline{)\, x^4 + 5x^3 - 19x^2 - 65x + 150}$ $\quad x^3 + 8x^2 + 5x - 50$

   $-(x^4 - 3x^3)$

   $8x^3 - 19x^2 - 65x + 150$

   $-(8x^3 - 24x^2)$

   $5x^2 - 65x + 150$

   $-(5x^2 - 15x)$

   $-50x + 150$

   $-(-50x + 150)$

   $125 + 200 + 25 - 50$

   $8 + 32 + 10 - 50$ ✓

   $x - 2 \,\overline{)\, x^3 + 8x^2 + 5x - 50}$ $\quad x^2 + 10x + 25$

   $-(x^3 - 2x^2)$

   $10x^2 + 5x - 50$

   $-(10x^2 - 20x)$

   $25x - 50$

   $-(25x - 50)$

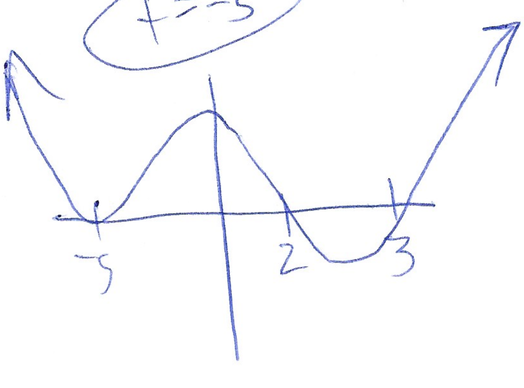
   $x = \dfrac{-10 \pm \sqrt{100 - 100}}{2} = -5$

   $(t-3)(t-2)(t+5)^2$

   $t = 3$

   $t = 2$

   $t = -5$



8. Find the remainder when $x^{100} - 4x^{50} + 5x + 6$ is divided by $x^3 - 2x^2 - x + 2$. (10)

Hint: Can you factor the cubic? Try factoring x^3 out of the first two terms. Can you then factor further?

$$\frac{x^{100} - 4x^{50} + 5x + 6}{(x-1)(x-2)(x+1)} = \frac{\frac{\frac{x^{100} - 4x^{50} + 5x + 6 \;0}{x-1}}{x-2}}{x+1}$$

1, 2, -1, -2

$(x-1)(x-2)(x+1)$

$$\frac{x^{50}(x^{25}-2)(x^{25}+2)}{(x-1)(x+1)(x-2)} + \frac{5x+6}{(x-1)(x+1)(x-2)}$$

$x^{50}(x^{50} - 4) + 5x + 6$

$x^{50}(x^{25}-2)(x^{25}+2)$

$x^3(x^{97} - 4x^{57})$

no answer

9. Suppose that $f(x)$ is a polynomial with integer coefficients such that $f(2) = 3$ and $f(7) = -7$. Show that $f(x)$ has no integer roots. (8)

Hint: Note that 3 and −7 are both odd.

Hint: Is it possible for $f(0)$ to be even?

if a polynomial ~~at x∈Z~~ in $Z[x]$ at an even x is odd, the constant is odd since all powers of x are even, and subsequently all terms, leaving only the constant to be odd. Thus, $f(0)$ is odd, because $f(2)$ is odd,

$f(x) = (x-r)g(x)$

$f(0) = C$

$f(r) = (r-r)g(x) = 0$

For odd values of x, ~~the rest~~ $f(x)$ still remains odd, indicating that all non-constant terms summate to an even value for all x, and that $f(x)$ is subsequently odd because of the odd constant,

Since $f(x)$ is always odd at integer x, it will never be 0, and $f(x)$ thus has no integer roots.

10. How can we quickly tell that $x - 1$ is a factor of $x^5 + 6x^4 - 7x^3 + 2x^2 - 2$ without performing the long division? (8)

Hint: Use the Factor Theorem.

$f(a)$ is the remainder of $\frac{f(x)}{x-a}$

~~$x^5 + 6x^4 - 7x^3 + 2x^2 - 2$~~

~~$x-1$~~

$a = 1$

$f(1) = 1 + 6(1) - 7(1) + 2(1) - 2 = 0$

Since the remainder is 0, it is divisible by $x-1$.

11. Find the quotient and remainder for the following polynomial division: (10)

$$x^2 - 19x + 17 \text{ divided by } x + 7$$

Hint: Use polynomial long division or synthetic division.

-7	1	-19	$+17$
		-7	182
	1	-26	199

~~$x - 26 + \frac{199}{x+7}$~~

Quotient: $x-26$ Remainder: 199

12. Teresa divides $3x^4 + 2x^3 - 7x^2 + 4x - 1$ by $x + 2$ and gets a quotient of $3x^3 - 4x^2 + x + 2$ and a remainder of 5. How can Teresa quickly realize that she made a mistake without performing the division again, and without multiplying $x + 2$ by the quotient? (12)

Hint: Use the Remainder Theorem to check her work.

The remainder is $3(-2)^4 + 2(-2)^3 - 7(-2)^2 + 4(-2) - 1$

$= 3(16) + 2(-8) - 7(4) + 4(-2) - 1$

$= 48 - 16 - 28 - 8 - 1$

$= 48 - 53 = -5$

So she is incorrect.

13. The polynomial $p(x) = 3x^3 - 20x^2 + kx + 12$ is divisible by $x - 3$ for some constant k. Factor $p(x)$ completely. (8)

Hint: Use the Factor Theorem to find k first, then factor completely.

Divisible if

$p(3) = 0$

$3(3)^3 - 20(3)^2 + k(3) + 12 = 0$

$81 - 180 + 3k + 12 = 0$

$-87 + 3k = 0$

$3k = 87$

$k = 29$

1, 2, 3, 4, 6

$\frac{1}{3}, \frac{2}{3}, \frac{4}{3}$

$3x^3 - 20x^2 + 29x + 12$

$(x-3)(3x^2 - 11x - 4x)$

$x = \frac{11 \pm \sqrt{121 + 48}}{6} = \left\{\frac{11}{6} \pm \frac{13}{6} = \left\{-\frac{1}{3}, 4\right\}\right.$

3	3	-20	29	12
		9	-33	-12
	3	-11	-4	0

$(x-3)(x+\frac{1}{3})(x-4)$