Sequences and Series

Name: ____________________ Date: ____________________

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. If the fourth term of an arithmetic sequence is 200 and the eighth term is 500, what is the sixth term? (8)

2. If the fourth term of a geometric sequence of positive numbers is 200 and the eighth term is 800, what is the sixth term? (8)

3. A geometric sequence has common ratio r, where $r \neq 0$, and the n^{th} term is b. Find an expression for the first term of the sequence in terms of r, n, and b. (8)

4. An infinite geometric series has common ratio $-\frac{1}{2}$ and sum 45. What is the first term of the series? (10)

5. (a) What is the sum of the first 50 positive integers? (b) The sum of the first k positive integers is 990. What is k? (12)

6. (a) Evaluate $\sum_{i=1}^{10}(2i-5)$. (12)

(b) Evaluate $\sum_{i=1}^{72} 5$.

(c) Evaluate $\sum_{i=1}^{7} 3^i$.

7. If the sum of the first $3n$ positive integers is 150 more than the sum of the first n positive integers, then what is the sum of the first $4n$ positive integers? (12)

8. In this problem we evaluate the series (12)

$$\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{98 \cdot 100}$$

(a) Notice that each fraction in the sum has the form $\frac{1}{n(n+2)}$ for some positive integer n. Find constants A and B such that

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

(b) Use your answer to part (a) to find the desired sum.

9. Evaluate the sum $\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \ldots$ (12)

10. In this problem we derive a formula for the sum of the first n perfect squares. Let n be a positive integer, and let $S = 1^2 + 2^2 + 3^2 + \cdots + n^2$. (16)

(a) Prove that $1 + 3 + 5 + \cdots + (2k - 1) = k^2$.

(b) Use part (a) to show that $S = (1)(n) + (3)(n-1) + (5)(n-2) + \cdots + (2n-1)(1)$
Hint: If we write each square as the sum of odd numbers as described in part (a), for how many of the n squares will 1 be among the odd numbers in the sum? For how many of them will 3 be among the odd numbers in the sum?

(c) Show that $2S = (2)(1) + (4)(2) + (6)(3) + \cdots + (2n)(n)$

(d) Add the equations in part (b) and (c) to conclude that $S = \frac{n(n+1)(2n+1)}{6}$. *Hint: Find a clever way to combine each term from the series in part (a) with a term in part (b).*