Exponential and Logarithmic Functions

Name: _____ Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Find the base n such that $\log_n 4\sqrt{2} = 10$. (8)

2. Find x if $\log_9(2x - 7) = \frac{3}{2}$. (10)

3. For how many positive integers b is $\log_b 729$ a positive integer? (8)

4. If $\log_3(\log_3(\log_3(x))) = 2$, then how many digits are in x? (10)

5. Find the domain and range of $f(x) = 2\log_3(x^2 - 4x - 5)$. (12)

6. Let $f(x) = 3x^2 - 7$ and $g(f(4)) = 9$. What is $g(f(-4))$? (10)

7. Let $f(x) = 1 - \frac{1}{x}$. (12)

   (a) Find $f(f(x))$.

   (b) Find $f(f(f(x)))$.

   (c) Find $f(f(f(f(x))))$.

   (d) Find $f^{34}(5)$.

   Hint: Compare $f^4(x)$ to $f(x)$. Notice anything interesting? If you don't, then you should find $f^4(x)$ again.

8. If $8^x = 27$, then what is 4^{2x-3}? (10)

   Hint: Express both sides in terms of powers of 2 and 3.

9. Let f be a function whose graph passes through the points $(2, 3)$, $(4, 7)$, and $(8, 12)$. Suppose f has an inverse. Name three points that must be on the graph of $y = f^{-1}(x)$. (8)

10. I have just won a lottery that will pay me $\$1,000,000$ in 10 years. A company offers to buy my winning ticket today for $\$300,000$. (10)

    (a) If the annually compounded interest rate is 9%, should I take the offer?

    (b) For what annually compounded interest rate is my lottery ticket worth $\$300,000$ today?

11. Evaluate $\log_2 8$, $\log_2 16$, and $\log_2(8 \cdot 16)$. (12)

(a) Evaluate $\log_3 \frac{1}{9}$, $\log_3 \sqrt{3}$, and $\log_3 \left(\frac{1}{9} \cdot \sqrt{3}\right)$.

(b) Do you notice a relationship among $\log_a b$, $\log_a c$, and $\log_a(bc)$? Can you prove it?
Hint: Let $x = \log_a b$, $y = \log_a c$, *and* $z = \log_a(bc)$. *Use exponential notation.*

12. At how many points does the parabola $y = x^2$ intersect the exponential curve $y = 2^x$? (8)