Basic Concepts of Algebra

Name: Krish Arora Date: 7/8/2025

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

---

1. Evaluate each of the following expressions when $a = -8$ and $b = \frac{1}{2}$. (16)

   (a) $ab + 2b + 3a$

   $(-8)(\frac{1}{2}) + 2(\frac{1}{2}) + 3(-8) = -4 + 1 - 24 = \boxed{-27}$

   (b) $\dfrac{a}{b}$

   $(-8) \div \dfrac{1}{2} = \boxed{-16}$

   (c) $4(a^2b + b^2a)$

   $4((-8)^2 \cdot (0.5) + (0.5)^2 \cdot (-8)) = 4(32 - 2) = \boxed{120}$

   (d) $(a - 2)\sqrt{-ab}$

   $((-8) - 2)\sqrt{-(-8)}\sqrt{0.5} = -10 \cdot \sqrt{8} \cdot \dfrac{1}{\sqrt{2}} = -10 \cdot 2 \cdot \sqrt{2} \cdot \dfrac{1}{\sqrt{2}} = \boxed{-20}$

2. Simplify the product as much as possible. (5)

$$\frac{2}{3a^2 - 6b} \cdot \frac{9a^3 - 18ab}{10a^2}$$

$$\frac{2}{3(a^2-2b)} \cdot \frac{9a(a^2-2b)}{10a^2}$$

$$\frac{\cancel{2}\,1}{\cancel{3(a^2-2b)}} \cdot \frac{\cancel{9a(a^2-2b)}}{\cancel{10}\,5a^2}$$

$$\frac{9a}{15a^2} = \boxed{\frac{3}{5a}}$$

3. Decide whether each set is a field under the operations of addition and multiplication. If the set is not a field, name at least one field property that does not hold. (12)

   (a) The natural numbers ($\mathbb{N}$) ✗
   No Multiplicative Inverse

   (b) The integers ($\mathbb{Z}$) ✗
   No Multiplicative Inverse

   (c) The rational numbers ($\mathbb{Q}$) ✓
   Closed Under Addition & Multiplication

   Has Inverses

   Distributive and Associative Property

   (d) The negative rational numbers ✗
   Not Closed Under Multiplication: $-3 \cdot (-2) = 6$ (6 not in set).

4. Solve for the variable in each equation. (10)

   (a) Find all values of y such that $\dfrac{3}{2+\sqrt{y}} + \dfrac{4}{2+\sqrt{y}} = 1$.

$$\frac{7}{2+\sqrt{y}} = 1$$

$$2+\sqrt{y} = 7$$

$$\sqrt{y} = 5$$

$$\boxed{y = 25}$$

(b) What values of x satisfy $\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = 3$?

$$\sqrt{x+1}+\sqrt{x-1} = 3\sqrt{x+1}-3\sqrt{x-1}$$

$$4\sqrt{x-1} = 2\sqrt{x+1}$$

$$\sqrt{x+1} = 2\sqrt{x-1}$$

$$\sqrt{x+1} = \sqrt{4}\sqrt{x-1}$$

$$x+1 = 4x-4$$

$$3x = 5; \boxed{x = \frac{5}{3}}$$

5. (a) Let x be the middle integer of three consecutive integers. What is the sum of these three integers in terms of x? (8)

$(x-1)+x+(x+1) = \boxed{3x}$

(b) The sum of 23 consecutive integers is 2323. What is the largest of the integers? (Hint: Use the result from the first part.)

$(x-11)+(x-10)+(x-9)...+x+...(x+9)+(x+10)+(x+11) = 23x = 2323$

$2323/23 = 101$

$\boxed{x = 101}$

6. A grocer wants to mix peanuts and cashews to produce 20 lb of mixed nuts worth \$6.20/lb. How many pounds of each kind of nut should she use if peanuts cost \$4.80/lb (6)

and cashews cost \$8.00/lb?

$4.8x + 8(20 - x) = 20(6.2)$

$160 - 3.2x = 124$

$3.2x = 36$

$16x = 180$

$\boxed{x = \frac{45}{4}}$