Integrated Algebra 2 and Precalculus

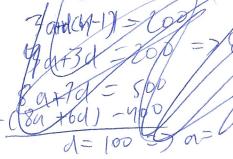
Exam: Chapter 11 of Algebra 2

Sequences and Series

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. If the fourth term of an arithmetic sequence is 200 and the eighth term is 500, what is

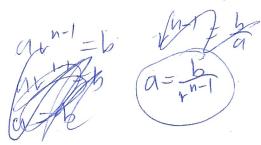
the sixth term?



2. If the fourth term of a geometric sequence of positive numbers is 200 and the eighth (8)term is 800, what is the sixth term?

$$0.1^{3} = 200$$
  $0.5^{2} = 50\sqrt{2}$   
 $0.1^{3} = 200$   $0.1^{8} = 50\sqrt{2}$ ,  $1.1^{2} = 100$   
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3. A geometric sequence has common ratio r, where  $r \neq 0$ , and the  $n^{\text{th}}$  term is b. Find an (8)expression for the first term of the sequence in terms of r, n, and b.



(10)

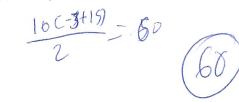
- 4. An infinite geometric series has common ratio  $-\frac{1}{2}$  and sum 45. What is the first term of the series?
- Algebra  $\frac{1}{2} = \frac{1}{2} \text{ and sum 45. What is the first ter}$   $\frac{1}{2} = \frac{1}{2} = \frac{1}{$
- 5. (a) What is the sum of the first 50 positive integers? (b) The sum of the first k positive (12)

integers is 990. What is k?

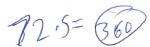
LChel) = 1980 L2+ L= 1980=0 L= -1 = 1+7920 = -2 ± 44,5 € 2-48 (443)

(12)

6. (a) Evaluate  $\sum_{i=1}^{10} (2i-5)$ .



(b) Evaluate  $\sum_{i=1}^{72} 5$ .



(c) Evaluate  $\sum_{i=1}^{7} 3^i$ .

0-1 L=3 \$1=3 \$m=8 1-3 = 3280 1-3 = 3280 -1 = 3230

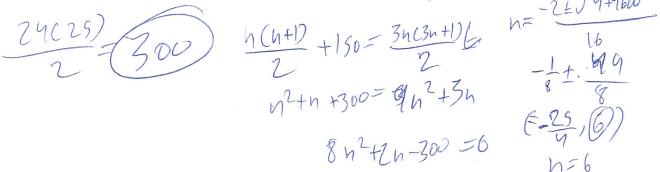
(n+2) + B(h) (n+2(n))

F(n+1) + B(n)

Au+2A+Bn=/

(12)

7. If the sum of the first 3n positive integers is 150 more than the sum of the first n positive (12)integers, then what is the sum of the first 4n positive integers?



8. In this problem we evaluate the series

$$\frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \dots + \frac{1}{98\cdot 100}$$

(a) Notice that each fraction in the sum has the form  $\frac{1}{n(n+2)}$  for some positive integer n. Find constants A and B such that

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

(b) Use your answer to part (a) to find the desired sum.

The state of the state of

$$\frac{1}{2n} + \frac{1}{2(n+2)} = \frac{1}{2n} + \frac{1}{2n+2} = \frac{1}{$$

$$= \frac{1}{2} \left( \sum_{n=a_n}^{\infty} \frac{1}{n} \right)$$

$$=\frac{1}{2}\left(\frac{9900+99507}{9900}\right)\times(0.74)$$

9. Evaluate the sum  $\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{4}{2^4}$ (12)

10. In this problem we derive a formula for the sum of the first n perfect squares. Let n be a positive integer, and let  $S = 1^2 + 2^2 + 3^2 + \cdots + n^2$ . (16)

(a) Prove that  $1 + 3 + 5 + \cdots + (2k - 1) = k_{-}^{2}$ 

(b) Use part (a) to show that  $S = (1)(n) + (3)(n-1) + (5)(n-2) + \cdots + (2n-1)(1)$ Hint: If we write each square as the sum of odd numbers as described in part (a), for how many of the n squares will 1 be among the odd numbers in the sum? For how many of them will 3 be among the odd numbers in the sum?

Sin +(3)(n+1)+5 (2n-1)+(2n-1)  $(1)=\sum_{k=1}^{n}(2k-1)(n-k+1)+\sum_{k=1}^{n}\sum_{j=1}^{n}(2k-1)(n-k+1)+\sum_{k=1}^{n}\sum_{j=1}^{n}(2k-1)(n-k+1)+\sum_{k=1}^{n}\sum_{j=1}^{n}(2k-1)(n-k+1)+\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{j=1}^{n}(2k-1)(n-k+1)+\sum_{k=1}^{n}\sum_{j=1}^{n}$ 

T= 2(1)+4(2)+6(3)+...+(21)(4) T= 2+8+18+212=2(1)2+2(2)2+3(3)2+2(1)2=25

(d) Add the equations in part (b) and (c) to conclude that  $S = \frac{n(n+1)(2n+1)}{S}$ . Hint: Find a clever way to combine each term from the series in part (a) with a term in part (b).

n+1)+3(n)(n+1)-1/2 (n+1)=

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