

4:10 - 4:57 and 6:30 - 7:00

2 hour and 17 min

Rational Functions and Expressions

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. Solve the equation $\frac{2x}{x-5} = 3 + \frac{1-x}{x-3}$.

$x = \frac{10}{3}$

$2x = 3(x-5) + \frac{(x-5)(1-x)}{x-3}$

$2x(x-3) = 3(x-5)(x-3) + (x-5)(1-x)$ (8)

$2x^2 - 6x = 3(x^2 - 8x + 15) + x - x^2 - 5 + 5x$

$2x^2 - 6x = 2x^2 - 18x + 40$

$12x = 40 \quad x = \frac{20}{6} = \frac{10}{3}$

(12)

2. Consider the function $g(x) = \frac{3-2x}{x-7}$.

(a) Find the domain of g .

$x \in \mathbb{R}, x \neq 7$

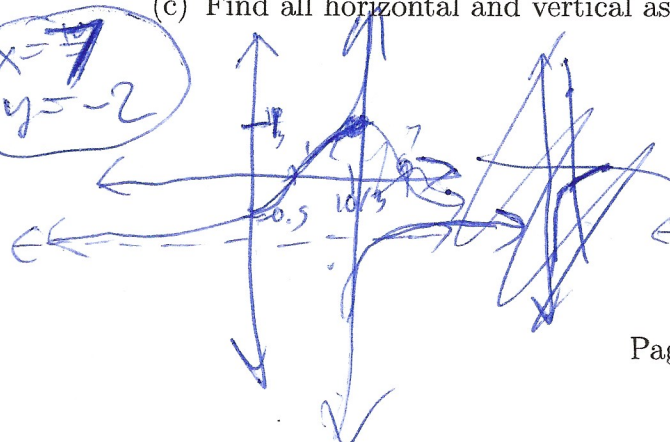
(b) Find the range of g .

$y \in \mathbb{R}$

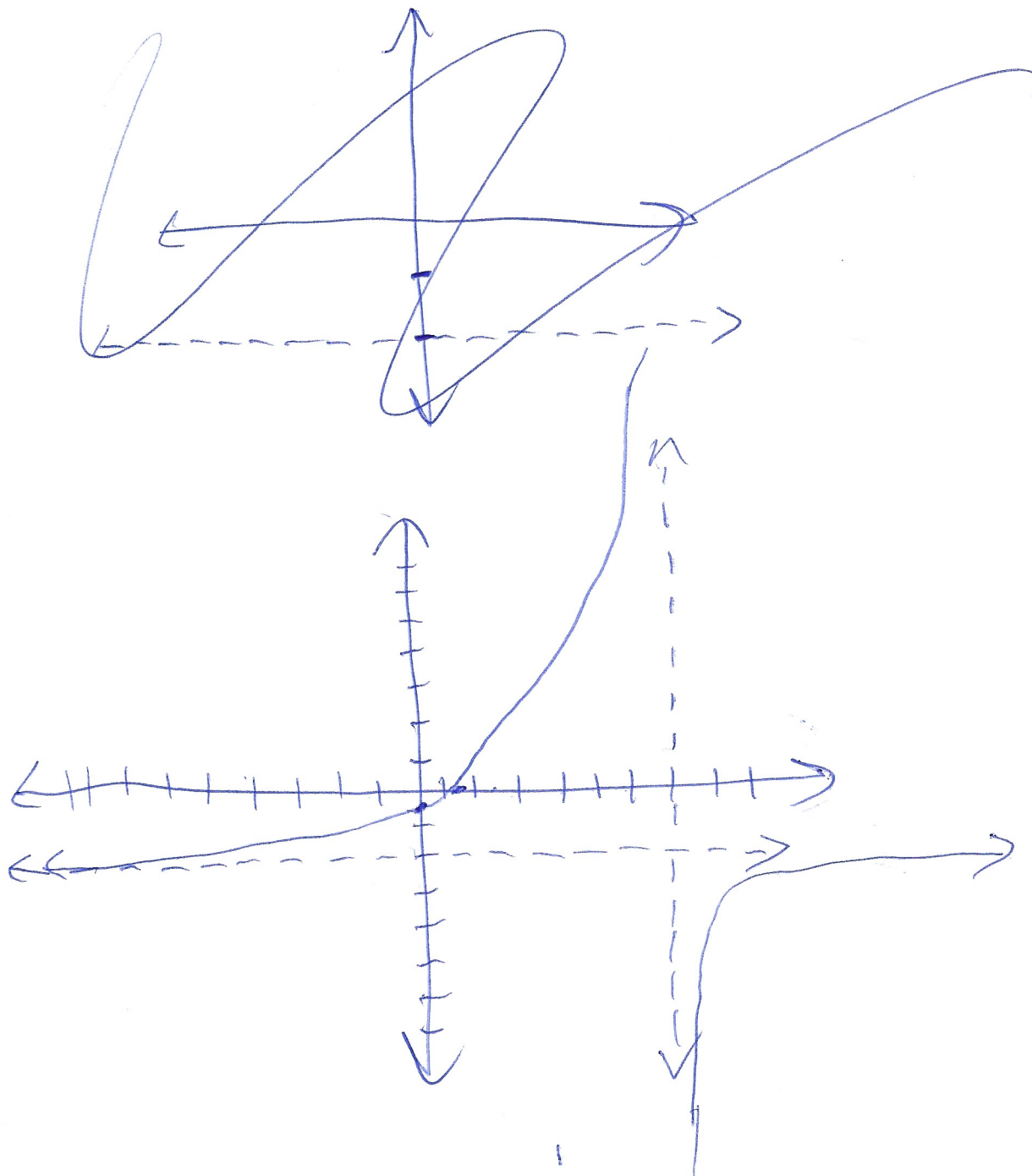
$y \in \mathbb{R}, y \neq -2$

(c) Find all horizontal and vertical asymptotes of the graph of $y = g(x)$.

$x = 7$
 $y = -2$



(d) Graph $y = g(x)$.



3. Find constants A and B such that $\frac{x+7}{x^2-2x-35} = \frac{A}{x-7} + \frac{B}{x+5}$ for all x .

Handwritten work for problem 3:

$$\frac{2}{13} = B$$

and

$$\frac{1}{13} = A$$

or

$$A = \frac{7}{6}$$

$$B = -\frac{1}{6}$$

Handwritten work for problem 3:

$$\frac{x+7}{(x-7)(x+5)} = \frac{A}{x-7} + \frac{B}{x+5}$$

$$x+7 = A(x+5) + B(x-7)$$

$$x+7 = Ax + 5A + Bx - 7B$$

Handwritten work for problem 3:

$$5A - 7B = 7$$

$$A + B = 1$$

$$5A - 7B = 7$$

$$A = 1 - B$$

$$5(1-B) - 7B = 7$$

$$5 - 5B - 7B = 7$$

$$-12B = 2$$

$$B = -\frac{1}{6}$$

$$A = 1 - (-\frac{1}{6}) = \frac{7}{6}$$

4. Find the constants A , B , and C such that

$$\frac{3x-2}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Handwritten work for problem 4:

$$\frac{3x-2}{x(x^2+2x+1)} = \frac{3x-2}{x(x+1)(x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Handwritten work for problem 4:

$$(x+1)(3x-2) = \frac{A}{x} + B(x+1)^2 + C(x^2+x)$$

Handwritten work for problem 4:

$$3x^2+x-2 = A(x^3+2x^2+x) + B(x^3+2x^2+x) + C(x^3+x^2+x)$$

$$3x^2+x-2 = Ax^3+2Ax^2+Ax + Bx^3+2Bx^2+Bx + Cx^3+Cx^2+Cx$$

$$Ax^3+2Ax^2+3Ax+A+Bx^3+2Bx^2+Bx+Cx^3+Cx^2+Cx$$

Handwritten work for problem 4:

$$A+B=0$$

$$3A+2B+C=3$$

$$3A+B+C=1$$

$$A=0$$

$$3A+B+C=1$$

$$2B+C=1$$

$$B=-2$$

Handwritten work for problem 4:

$$B+C=3$$

$$B=-2$$

$$C=5$$

$$A=0$$

$$B=-2$$

$$C=5$$

Handwritten work for problem 4:

$$(x^3+2x^2+x) = x^3+2x^2+x+x^2+2x+1$$

5. Find all values of t that satisfy $\frac{t-7}{t^2-2t-3} = \frac{t+2}{t^2+2t-15}$.

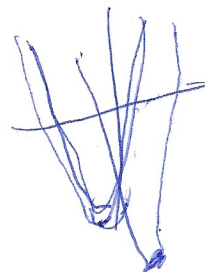
$$\frac{t-7}{(t-3)(t+1)} = \frac{t+2}{(t+5)(t-3)}$$

$$\frac{t-7}{t+1} = \frac{t+2}{t+5}$$

$$t^2 - 2t - 35 = t^2 + 3t + 2$$

$$(t-7)(t+5) = (t+1)(t+2)$$

$$t \neq \{3, -5\}$$

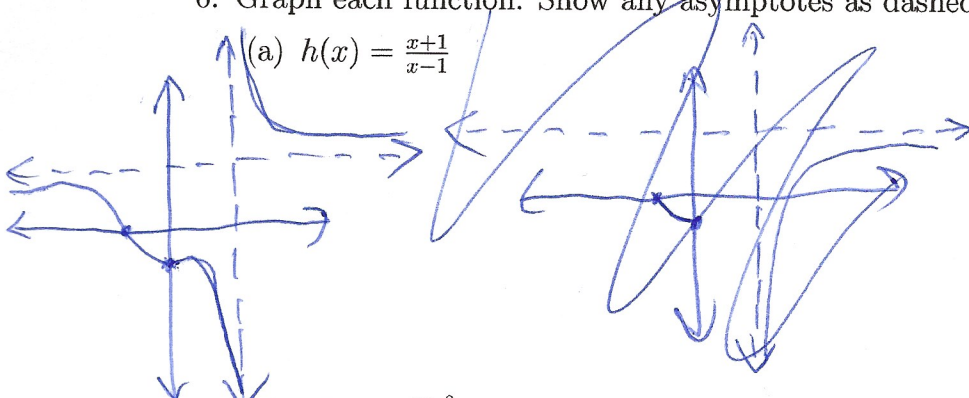


(8)

6. Graph each function. Show any asymptotes as dashed lines.

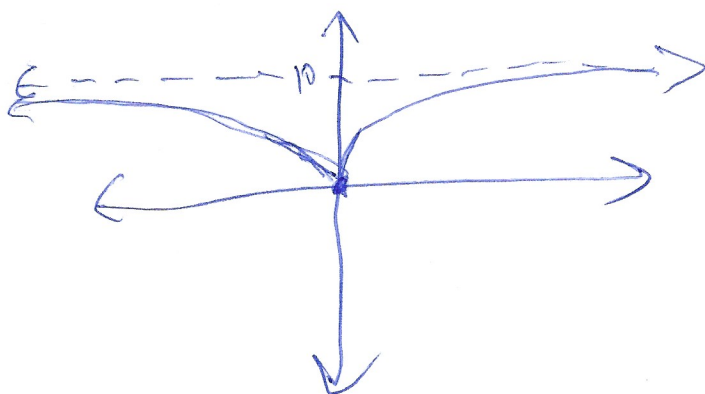
(10)

(a) $h(x) = \frac{x+1}{x-1}$



Asymptotes:
 $x = 1$
 $y = 1$

(b) $h(x) = \frac{10x^2}{x^2+1}$



Asymptotes
 $y = 10$

~~$(x+i)(x-i)$~~ $\frac{10x^2}{(x+i)(x-i)}$

Poles:
 $i, -i$ (not important)
 defined for all
 real x

7. For the rational function $f(x) = \frac{x^2-4}{x^2-3x-4}$: $\frac{(x+2)(x-2)}{(x-4)(x+1)}$ (15)

(a) Find the domain of f .

$$x \in \mathbb{R}, x \neq 4, x \neq -1$$

(b) Find all vertical asymptotes.

$$x=4, x=-1$$

(c) Find all horizontal asymptotes.

$$y=1$$

(d) Find all holes (removable discontinuities), if any.

None

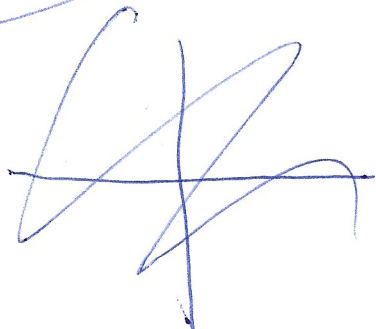
(e) Find the x -intercepts and y -intercept.

~~x -intercept: $(0, 1)$~~
 ~~y -intercept:~~

y -intercept: $(0, 1)$
 x -intercepts: $(-2, 0)$
 $(2, 0)$

8. Find the oblique asymptote for $f(x) = \frac{2x^2+3x-1}{x+2}$ using polynomial long division. $2x-1$ (10)

$$y=2x-1$$



$$\begin{array}{r} 2x^2+3x-1 \div x+2 \\ \underline{-(2x^2+4x)} \\ -1x-1 \\ \underline{-(-x-2)} \\ 3 \end{array}$$

9. Simplify each rational expression:

(8)

(a) $\frac{x^2-9}{x^2+6x+9}$

$$\frac{(x+3)(x-3)}{(x+3)^2} = \frac{x-3}{x+3}$$

(b) $\frac{2x^3-8x}{x^2-4}$

$$\frac{2x(x^2-4)}{(x+2)(x-2)} = 2x$$

(c) $\frac{x^3+8}{x^2-4}$

$$\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)} = \frac{x^2-2x+4}{x-2}$$

(d) $\frac{6x^2-24}{3x^2-6x-24}$

$$\frac{6(x^2-4)}{3(x^2-2x-8)} = \frac{2(x+2)(x-2)}{(x-4)(x+2)} = \frac{2(x-2)}{(x-4)}$$

10. Perform the indicated operations and simplify:

(7)

$$\frac{x^2-1}{x+2} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3}$$

$$\frac{(x+1)(x-1)}{x+2} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3}$$

$$\frac{(x+1)(x-1)(x+2)(x-2)}{(x+2)(x+2)(x-1)} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3} = \frac{(x+1)(x-1)(x+2)(x-2)}{(x+2)(x+2)(x-1)} \cdot \frac{x+2}{x-1} \cdot \frac{x-3}{x+1} = \frac{(x-2)(x-3)}{(x+2)(x-1)} = \frac{x-3}{x+2}$$