

Basic Concepts of Algebra

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. Evaluate each of the following expressions when $a = -8$ and $b = \frac{1}{2}$. (16)

(a) $ab + 2b + 3a$

$$(-8)\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) + 3(-8) = -4 + 1 - 24 = \boxed{-27}$$

(b) $\frac{a}{b}$

$$(-8) \div \frac{1}{2} = \boxed{-16}$$

(c) $4(a^2b + b^2a)$

$$4((-8)^2 \cdot (0.5) + (0.5)^2 \cdot (-8)) = 4(32 - 2) = \boxed{120}$$

(d) $(a - 2)\sqrt{-ab}$

$$((-8) - 2)\sqrt{-(-8)}\sqrt{0.5} = -10 \cdot \sqrt{8} \cdot \frac{1}{\sqrt{2}} = -10 \cdot 2 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = \boxed{-20}$$

2. Simplify the product as much as possible. (5)

$$\frac{2}{3a^2 - 6b} \cdot \frac{9a^3 - 18ab}{10a^2}$$

$$\frac{2}{3(a^2 - 2b)} \cdot \frac{9a(a^2 - 2b)}{10a^2}$$

$$\frac{\cancel{2} 1}{3(\cancel{a^2 - 2b})} \cdot \frac{9a(\cancel{a^2 - 2b})}{10 5a^2}$$

$$\frac{9a}{15a^2} = \boxed{\frac{3}{5a}}$$

3. Decide whether each set is a field under the operations of addition and multiplication. (12)
If the set is not a field, name at least one field property that does not hold.

(a) The natural numbers (\mathbb{N}) **X**
No Multiplicative Inverse

(b) The integers (\mathbb{Z}) **X**
No Multiplicative Inverse

(c) The rational numbers (\mathbb{Q}) **✓**
✓ Closed Under Addition & Multiplication
✓ Has Inverses
✓ Distributive and Associative Property

(d) The negative rational numbers **X**
Not Closed Under Multiplication: $-3 \cdot (-2) = 6$ (6 not in set).

goal! ✓

4. Solve for the variable in each equation. (10)

(a) Find all values of y such that $\frac{3}{2 + \sqrt{y}} + \frac{4}{2 + \sqrt{y}} = 1$.

$$\frac{7}{2 + \sqrt{y}} = 1$$

$$2 + \sqrt{y} = 7$$

$$\sqrt{y} = 5$$

$$\boxed{y = 25}$$



(b) What values of x satisfy $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$?

$$\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x+1} - 3\sqrt{x-1}$$

$$4\sqrt{x-1} = 2\sqrt{x+1}$$

$$\sqrt{x+1} = 2\sqrt{x-1}$$

$$\sqrt{x+1} = \sqrt{4}\sqrt{x-1}$$

$$x+1 = 4x-4$$

$$3x = 5; \boxed{x = \frac{5}{3}}$$



5. (a) Let x be the middle integer of three consecutive integers. What is the sum of these three integers in terms of x ? (8)

$$\underline{(x-1)} + \underline{x} + \underline{(x+1)} = \boxed{3x}$$



- (b) The sum of 23 consecutive integers is 2323. What is the largest of the integers? (Hint: Use the result from the first part.)

$$(x-11) + (x-10) + (x-9) \dots + x + \dots (x+9) + (x+10) + (x+11) = 23x = 2323$$

$$2323/23 = 101$$

$$\boxed{x = 101}$$



6. A grocer wants to mix peanuts and cashews to produce 20 lb of mixed nuts worth \$6.20/lb. How many pounds of each kind of nut should she use if peanuts cost \$4.80/lb (6)



and cashews cost \$8.00/lb?

$$4.8x + 8(20 - x) = 20(6.2)$$

$$160 - 3.2x = 124$$

$$3.2x = 36$$

$$16x = 180$$

$$x = \frac{45}{4}$$

missing how much
each nut costs
calculations are correct