Trigonometric Graphs and Identities

Name: _____   Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. Convert the following angle measures as indicated: (10)

   (a) Convert $\frac{2\pi}{3}$ radians to degrees.

   (b) Convert $150°$ to radians.

   (c) Convert $\frac{5\pi}{6}$ radians to degrees.

   (d) Convert $240°$ to radians.

2. The graph of a sinusoidal function of the form $y = a\cos(b(x - c)) + d$ has a maximum   (10)
   point at $(\pi/3, 5)$ and a subsequent minimum point at $(\pi, 1)$. Find the values for $a, b, c,$

and d, assuming $a > 0$, $b > 0$, and c is the smallest possible positive value.

3. Determine the equations of all vertical asymptotes for the function $f(x) = 2\sec(3x - \frac{\pi}{2})$ (10)
   on the interval $[0, 2\pi]$.

4. Prove the following trigonometric identity: (10)

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A$$

5. Given that $\sin \alpha = \frac{4}{5}$ with α in Quadrant II, and $\cos \beta = \frac{5}{13}$ with β in Quadrant IV, find   (10)
the exact value of $\cos(\alpha - \beta)$.

6. Solve the equation $\cos(2x) + 3 \sin x - 2 = 0$ for all values of x in the interval $0 \leq x < 2\pi$.   (10)

7. Use a half-angle formula to find the exact value of $\tan(105°)$.   (10)

8. Prove the identity $\tan(4\theta) = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$. (Hint: Use the double angle formula for tangent twice.)   (10)

9. Solve the equation $\sin(3\theta) + \sin(\theta) = 0$ for all values of θ in the interval $[0, 2\pi]$.   (10)

10. The height, H, in meters, of the tide in a certain harbor is modeled by the equation   (10)
    $H(t) = 10 + 4\sin(\frac{\pi}{6}t)$, where t is the number of hours after midnight.

    (a) What is the maximum and minimum height of the tide?

    (b) At what times during a 24-hour day is the tide at its maximum height?

    (c) For how many hours is the tide's height greater than 12 meters during a 24-hour period?