

4:10 - 4:57 and 6:30 - 7:00

2 hour and 17 min

# Rational Functions and Expressions

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**Instructions:** Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. Solve the equation  $\frac{2x}{x-5} = 3 + \frac{1-x}{x-3}$ .

$x = \frac{10}{3}$

$2x = 3(x-5) + \frac{(x-5)(1-x)}{x-3}$

$2x(x-3) = 3(x-5)(x-3) + (x-5)(1-x)$  (8)

$2x^2 - 6x = 3(x^2 - 8x + 15) + x - x^2 - 5 + 5x$

$2x^2 - 6x = 2x^2 - 18x + 40$

$12x = 40 \quad x = \frac{20}{6} = \frac{10}{3}$

(12)

2. Consider the function  $g(x) = \frac{3-2x}{x-7}$ .

(a) Find the domain of  $g$ .

$x \in \mathbb{R}, x \neq 7$

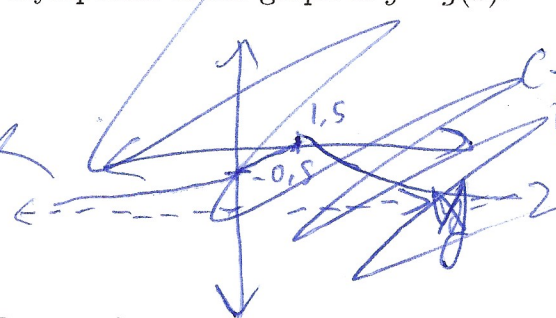
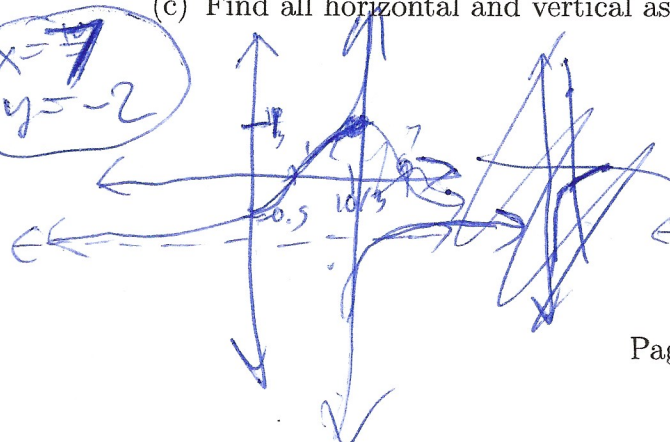
(b) Find the range of  $g$ .

$y \in \mathbb{R}$

$y \in \mathbb{R}, y \neq -2$

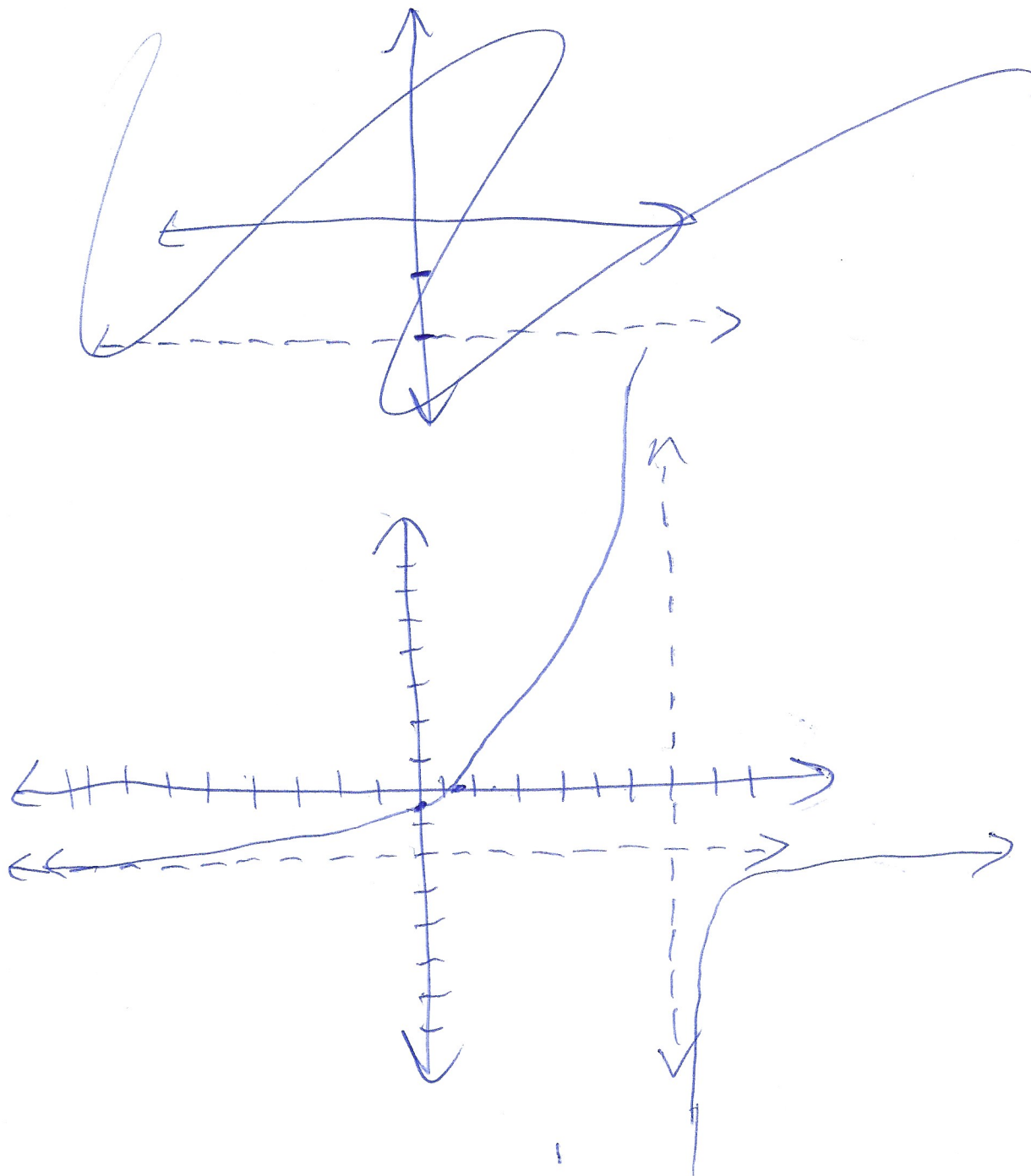
(c) Find all horizontal and vertical asymptotes of the graph of  $y = g(x)$ .

$x = 7$   
 $y = -2$



$(-2)(x-7) = (3-2x)$   
 $-2x + 14 = 3 - 2x$   
 $11 = 0$   
 $x^2 - 14x + 49$

(d) Graph  $y = g(x)$ .



3. Find constants  $A$  and  $B$  such that  $\frac{x+7}{x^2-2x-35} = \frac{A}{x-7} + \frac{B}{x+5}$  for all  $x$ .

Handwritten work for problem 3:

$$\frac{2}{13} = B$$

and

$$\frac{1}{13} = A$$

or

$$B = \frac{2}{13}$$

$$A = \frac{1}{13}$$

Handwritten work for problem 3:

$$\frac{x+7}{(x-7)(x+5)} = \frac{A}{x-7} + \frac{B}{x+5}$$

$$x+7 = A(x+5) + B(x-7)$$

$$x+7 = Ax + 5A + Bx - 7B$$

Handwritten work for problem 3:

$$5A - 7B = 7$$

$$A + B = 1$$

$$5A - 7B = 7$$

$$A = 1 - B$$

$$5(1-B) - 7B = 7$$

$$5 - 5B - 7B = 7$$

$$-12B = 2$$

$$B = -\frac{1}{6}$$

$$A = 1 - (-\frac{1}{6}) = \frac{7}{6}$$

4. Find the constants  $A$ ,  $B$ , and  $C$  such that

$$\frac{3x-2}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Handwritten work for problem 4:

$$\frac{3x-2}{x(x^2+2x+1)} = \frac{3x-2}{x(x+1)(x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Handwritten work for problem 4:

$$(x+1)(3x-2) = \frac{A}{x} + B(x+1) + \frac{C}{x+1}$$

Handwritten work for problem 4:

$$3x^2+x-2 = A(x+1)^2 + Bx(x+1) + C(x+1)$$

$$3x^2+x-2 = A(x^2+2x+1) + B(x^2+x) + C(x+1)$$

$$3x^2+x-2 = Ax^2+2Ax+A+Bx^2+Bx+Cx+C$$

$$3x^2+x-2 = (A+B)x^2 + (2A+B+C)x + (A+C)$$

Handwritten work for problem 4:

$$A+B=0$$

$$3A+2B+C=3$$

$$3A+B+C=1$$

$$A=0$$

$$3A+B+C=1$$

$$2B+C=1$$

$$B=-2$$

Handwritten work for problem 4:

$$B+C=3$$

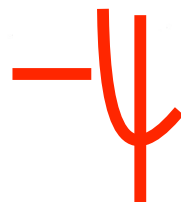
$$B=-2$$

$$C=5$$

$$A=0$$

$$B=-2$$

$$C=5$$



Handwritten work for problem 4:

$$(x+1)(3x-2) = x^3+2x^2+x+x^2+2x+1$$



5. Find all values of  $t$  that satisfy  $\frac{t-7}{t^2-2t-3} = \frac{t+2}{t^2+2t-15}$ .

$$\frac{t-7}{(t-3)(t+1)} = \frac{t+2}{(t+5)(t-3)}$$

$$\frac{t-7}{t+1} = \frac{t+2}{t+5}$$

$$t^2 - 2t - 35 = t^2 + 3t + 2$$

$$(t-7)(t+5) = (t+1)(t+2)$$

-2

~~$t = 3, -5$~~

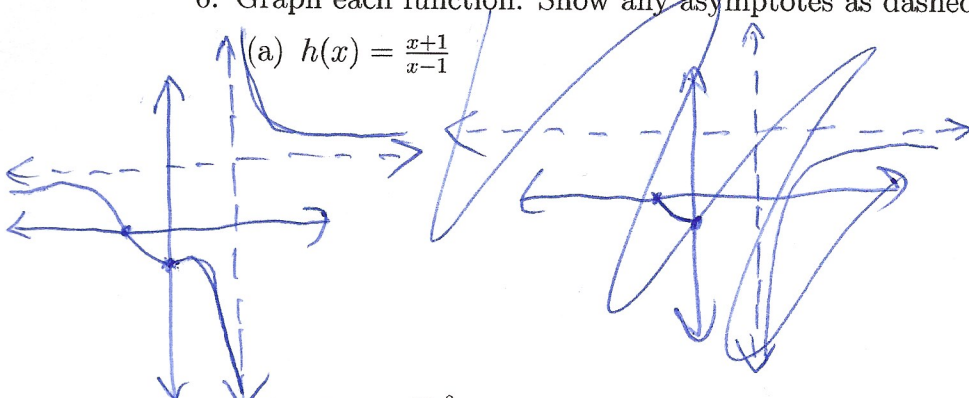


(8)

6. Graph each function. Show any asymptotes as dashed lines.

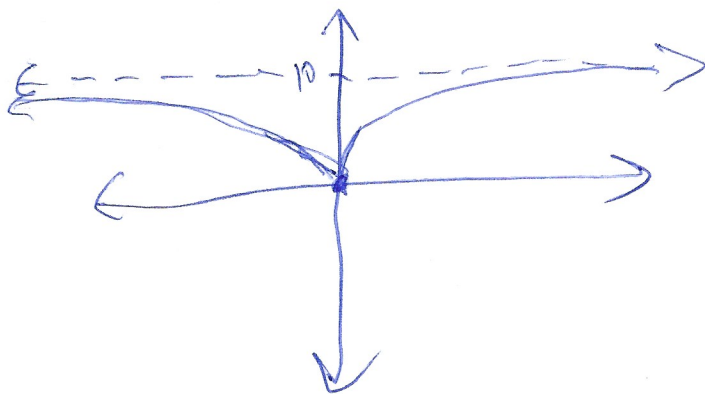
(10)

(a)  $h(x) = \frac{x+1}{x-1}$



Asymptotes:  
 $x = 1$   
 $y = 1$

(b)  $h(x) = \frac{10x^2}{x^2+1}$



Asymptotes  
 $y = 10$

~~$(x+i)(x-i)$~~   $\frac{10x^2}{(x+i)(x-i)}$

Poles:  
 $i, -i$  (not important)  
 defined for all  
 real  $x$

7. For the rational function  $f(x) = \frac{x^2-4}{x^2-3x-4}$ :  $\frac{(x+2)(x-2)}{(x-4)(x+1)}$  (15)

(a) Find the domain of  $f$ .

$$x \in \mathbb{R}, x \neq 4, x \neq -1$$

(b) Find all vertical asymptotes.

$$x=4, x=-1$$

(c) Find all horizontal asymptotes.

$$y=1$$

(d) Find all holes (removable discontinuities), if any.

None

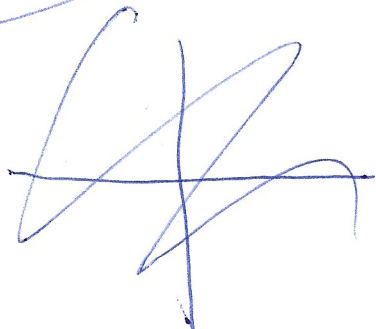
(e) Find the  $x$ -intercepts and  $y$ -intercept.

~~$x$ -intercept:  $(0, 1)$~~   
 ~~$y$ -intercept:~~

$y$ -intercept:  $(0, 1)$   
 $x$ -intercepts:  $(-2, 0)$   
 $(2, 0)$

8. Find the oblique asymptote for  $f(x) = \frac{2x^2+3x-1}{x+2}$  using polynomial long division.  $2x-1$  (10)

$$y=2x-1$$



$$\begin{array}{r} 2x-1 \overline{) 2x^2+3x-1} \\ \underline{2x^2+4x} \phantom{-1} \\ x-1 \end{array}$$

$$\begin{array}{r} x+2 \overline{) 2x^2+3x-1} \\ \underline{2x^2+4x} \phantom{-1} \\ -x-1 \phantom{-1} \\ \underline{-(-x-2)} \\ 1 \end{array}$$

9. Simplify each rational expression:

(8)

(a)  $\frac{x^2-9}{x^2+6x+9}$

$$\frac{(x+3)(x-3)}{(x+3)^2} = \frac{x-3}{x+3}$$

(b)  $\frac{2x^3-8x}{x^2-4}$

$$\frac{2x(x^2-4)}{(x+2)(x-2)} = 2x$$

(c)  $\frac{x^3+8}{x^2-4}$

$$\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)} = \frac{x^2-2x+4}{x-2}$$

(d)  $\frac{6x^2-24}{3x^2-6x-24}$

$$\frac{6(x^2-4)}{3(x^2-2x-8)} = \frac{2(x+2)(x-2)}{(x-4)(x+2)} = \frac{2(x-2)}{(x-4)}$$

10. Perform the indicated operations and simplify:

(7)

$$\frac{x^2-1}{x+2} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3}$$

$$\frac{(x+1)(x-1)}{x+2} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3}$$

$$\frac{(x+1)(x-1)(x+2)(x-2)}{(x+2)(x+2)(x-1)} \cdot \frac{x+2}{x-1} \div \frac{x+1}{x-3} = \frac{(x+1)(x-1)(x+2)(x-2)}{(x+2)(x+2)(x-1)} \cdot \frac{x+2}{x-1} \cdot \frac{x-3}{x+1} = \frac{(x-2)(x-3)}{(x+2)(x-1)} = \frac{x-3}{x+2}$$