2:50 - 3:36 46 minutes

Integrated Algebra 2 and Precalculus Exam: Chapter 3 of Algebra 2

Linear Functions and Relations

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Compute $\frac{x}{y}$ if $x + \frac{1}{y} = 4$ and $y + \frac{1}{x} = \frac{1}{4}$. (8)

$x + \frac{1}{y} = 4$

$xy + 1 = 4y \Rightarrow y = \frac{xy+1}{4}$

$y + \frac{1}{x} = \frac{1}{4}$

$xy + 1 = \frac{x}{4} \Rightarrow x = 4xy + 4$

$\frac{x}{y} = \frac{4xy+4}{\frac{xy+1}{4}} = 4\frac{4xy+4}{xy+1} = 16$

2. Two lines with nonzero slope and the same y-intercept have the property that the sum of their slopes is 0. What is the sum of the x-coordinates of their x-intercepts? (10)

Hint: Turn the words into math. Let the common y-intercept be $(0, b)$ and the two x-intercepts be $(c, 0)$ and $(d, 0)$.

$y = mx + b \Rightarrow 0 = mx + b \Rightarrow x = \frac{-b}{m}$

$y = -mx + b \Rightarrow 0 = -mx + b \Rightarrow x = \frac{-b}{-m} \Rightarrow \frac{b}{m}$

$\frac{-b}{m} + \frac{b}{m} = 0$

3. A line passing through the points $(2a+1, 3a^2)$ and $(3a+1, 5a^2)$ has slope $a+3$, where a is nonzero. Find the value of a. (12)

$$a+3 = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5a^2 - 3a^2}{3a+1-(2a+1)}$$

$$\frac{2a^2}{a} = 2a \qquad a+3 = 2a \qquad a = 3$$

4. Find A if the graph of the equation $Ax + 3y = 5$ is parallel to the graph of $5x - 2y = 4$. (12)

$$y = \frac{-A}{3}x + \frac{5}{3} \qquad y = \frac{5}{2}x - 2$$

$$m_1 = \frac{-A}{3} \qquad m_2 = \frac{5}{2}$$

$$m_1 = m_2 \qquad \frac{A}{3} = \frac{5}{2} \qquad A = \frac{15}{2} \qquad A = 7.5$$

5. Find B if the graph of the equation $3x - By = 2$ is perpendicular to the graph of $3y = -2x + 4$. (12)

$$y = \frac{3}{B}x + \frac{2}{-B} \qquad y = \frac{-2}{3}x + \frac{4}{3}$$

$$m_1 = \frac{3}{B} \qquad m_2 = \frac{-2}{3}$$

$$m_1 = \frac{1}{-m_2}$$

$$\frac{3}{B} = \frac{1}{2/3} = \frac{3}{2} \qquad B = 2$$

6. Find A and B if the graph of the equation $Ax + 3y = B$ produces the same line as the graph of the equation $2x + 6y = 7$. (12)

$y = -\frac{A}{3} + \frac{B}{3}$ $\quad m_1 = -\frac{A}{3}$ $\quad b_1 = \frac{B}{3}$

$y = -\frac{2}{6}x + \frac{7}{6}$ $\quad m_2 = -\frac{1}{3}$ $\quad b_2 = \frac{7}{6}$

$m_1 = m_2$ $\quad b_1 = b_2$

$-\frac{A}{3} = -\frac{1}{3}$ $\quad \frac{B}{3} = \frac{7}{6}$

$A = 1$ $\quad B = 3.5$

~~$b_1 = b_2$~~

7. Consider the points $(1, -2)$ and $(-5, 6)$. (10)

(a) Find the distance between the two points.

$d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{100} = 10$

(b) What are the coordinates of the midpoint of the segment connecting these points?

$(x, y) = (-3, 2)$

$(-2, 2)$

(c) Find the slope of the line through both points.

$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8}{-6} = -\frac{4}{3}$

(d) Find the standard form of the equation whose graph is the line through both points.

$m = -\frac{4}{3}$ $\quad b = y_1 - x_1 m = -2 + \frac{4}{3} = -\frac{6}{3} + \frac{4}{3} = -\frac{2}{3}$

$y = -\frac{4}{3}x - \frac{2}{3}$

$4x + 3y = -2$

8. If $f(x) = 3x - 2$ and $g(x) = x^2 + 1$, find: (10)

(a) $f(g(2))$

$3(x^2+1)-2$

$3x^2+1$ $\quad 3(2)^2+1 = 12+1 = 13$

(b) $g(f(-1))$

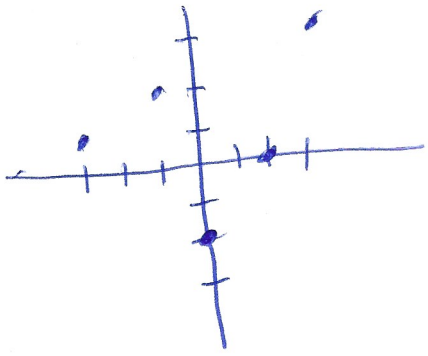
$9(-1)^2-12(-1)+5 = 9+12+5 = 26$

$(3x-2)^2+1$

$9x^2-12x+4+1 = 9x^2-12x+5$

9. Graph the relation $\{(-3,1),(-1,2),(0,-2),(2,0),(3,3)\}$. (8)

On your coordinate plane, plot each point. Then answer: Is this relation a function?



Yes, this is a function.

10. Give the domain of each relation. (12)

(a) $\{(x,y) : y = \frac{2x+1}{x-4}\}$

$D = \{x \in R ; x \neq 4\}$

(b) $\{(x,y) : y \leq \frac{3}{x^2-1}\}$

$D = \{x \in R : x \neq 1 \text{ and } x \neq -1\}$

(c) $\{(x,y) : y = \frac{x}{|x|+5}\}$

$D = \{x \in \mathbb{R} : x \neq 5$ or $D = \mathbb{R}$ $D = R$

(d) $\{(x,y) : |2x-3| + |y-1| = 2\}$

$x \in \mathbb{R}$ $D = \mathbb{R}$ $|2x-3| < 2$ $-2x+3 < 2$

$D = \{x \in \mathbb{R} : 0.5 < x < 2.5\}$ $2x-3 < 2$ $2x > 1$

$2x < 5$ $x > 0.5$

$x < 2.5$

11. True or False? (8)

(a) All relations are functions.

False X

(b) All functions are relations.

True ✓

12. Find the range of each function with domain D. (10)

(a) $h : x \mapsto -2x + 5$, $D = \{1, 3, 5\}$

$R = \{-5, -1, 3\}$

(b) $j : x \mapsto x^2 - 1$, $D = \{-2, 0, 2\}$

$R = \{-1, 3\}$

(c) $k : x \mapsto |x + 2|$, $D = \{-4, -2, 1\}$

$R = \{0, 3\}$

(d) $m : x \mapsto 4x - 3$, $D = \{0, 2, 4\}$

$R = \{-3, 5, 13\}$