Chapter 3: Polynomial and Rational Functions

Name: _____ Date: _____

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit. Unless otherwise stated, assume all variables represent real numbers and all functions are defined on their natural domains.

1. Let $P(x) = -(x + 2)^3(x - 1)^2$. (12)

   (a) Determine the end behavior of P.

   (b) Find all intercepts and state whether the graph crosses or only touches the x-axis at each zero.

   (c) Sketch a clean graph, labeling intercepts and identifying the local behavior near each zero.

2. (a) For what value(s) of k is $(x-2)$ a factor of $x^3 + 2kx^2 + k^2x + k - 4$?   (12)

(b) Let $f(x) = x^2 + 4x$. Solve $f(f(x)) = f(x)$.

3. Let $p(x) = 2x^3 - x^2 - 5x + 3$. (12)

   (a) Show that $p(0) > 0$ and $p(1) < 0$, and conclude there is a real zero in $(0, 1)$.

   (b) Use the Rational Root Theorem to list all rational candidates and explain why there is no rational root in $(0, 1)$.

   (c) Approximate this zero to two decimal places (justify your method).

4. Let $w = 2 + 3i$ and $z = 4 - 5i$.　　　　　　　　　　　　　　　　(10)

    (a) Compute $2w - 3z$.

    (b) Find $\dfrac{1}{w}$ in $a + bi$ form.

    (c) Evaluate $\dfrac{2}{\overline{w} + z}$.

    (d) Simplify $\dfrac{(1 - i)^4}{(1 + i)^3}$.

5. (a) Suppose one root of $x^3 + ax^2 - 4x + b = 0$ is $1 + i$, where $a, b \in \mathbb{R}$. Find the other   (12)
        two roots and determine a and b.

    (b) If two factors of $x^3 - t_1 x + t_2$ are $x + 2$ and $x - 1$, find the roots of $x^2 - t_1 x + t_2$.

6. Consider $s(x) = \dfrac{x^3 - 9x}{x^2 - 25}$.   (14)

   (a) Determine the domain and all intercepts.

   (b) Find all vertical asymptotes and the slant (oblique) asymptote.

   (c) On a clean set of axes, sketch the graph, indicating sign on each interval of the domain and behavior near each asymptote.

7. (a) Solve $\dfrac{x^3 - 9x}{x^2 - 25} \geq 0$. Express the solution in interval notation and mark excluded   (12)
   points.

   (b) Solve $\dfrac{x^2(x - 3)(x + 2)}{(x - 1)(x + 2)} < 0$. Carefully distinguish cancellations from domain re-
   strictions.