

2:50 - 3:36

46 minutes

Linear Functions and Relations

Name: Krish AroraDate: 7/13/25

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

1. Compute $\frac{x}{y}$ if $x + \frac{1}{y} = 4$ and $y + \frac{1}{x} = \frac{1}{4}$. (8)

$$x + \frac{1}{y} = 4$$

$$\cancel{xy+1=4y} \quad xy+1=4y \Rightarrow y = \frac{xy+1}{4}$$

$$y + \frac{1}{x} = \frac{1}{4}$$

$$xy+1 = \frac{x}{4} \Rightarrow x = 4xy+4$$

$$\frac{x}{y} = \frac{4xy+4}{\frac{xy+1}{4}} = 4 \frac{4xy+4}{xy+1} = \boxed{16}$$

2. Two lines with nonzero slope and the same y -intercept have the property that the sum of their slopes is 0. What is the sum of the x -coordinates of their x -intercepts? (10)

Hint: Turn the words into math. Let the common y -intercept be $(0, b)$ and the two x -intercepts be $(c, 0)$ and $(d, 0)$.

$$y = mx + b \Rightarrow 0 = mx + b \Rightarrow x = \frac{-b}{m}$$

$$y = -mx + b \Rightarrow 0 = -mx + b \Rightarrow x = \frac{-b}{-m} \Rightarrow \frac{b}{m}$$

$$\frac{-b}{m} + \frac{b}{m} = \boxed{0}$$

3. A line passing through the points $(2a + 1, 3a^2)$ and $(3a + 1, 5a^2)$ has slope $a + 3$, where a is nonzero. Find the value of a . (12)

$$a+3 = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5a^2 - 3a^2}{3a+1 - (2a+1)} = \frac{2a^2}{a}$$

$$\frac{2a^2}{a} = 2a$$

$$\begin{aligned} a+3 &= 2a \\ a &= 3 \end{aligned}$$

4. Find A if the graph of the equation $Ax + 3y = 5$ is parallel to the graph of $5x - 2y = 4$. (12)

$$y = \frac{-A}{3}x + \frac{5}{3}$$

$$m_1 = \frac{-A}{3}$$

$$y = \frac{5}{2}x - 2$$

$$m_2 = \frac{5}{2}$$

$$m_1 = m_2$$

$$\frac{-A}{3} = \frac{5}{2}$$

$$A = \frac{15}{2}$$

$$A = 7.5$$

5. Find B if the graph of the equation $3x - By = 2$ is perpendicular to the graph of $3y = -2x + 4$. (12)

~~3x~~

$$y = \frac{3}{B}x + \frac{2}{-B}$$

$$m_1 = \frac{3}{B}$$

$$y = \frac{-2}{3}x + \frac{4}{3}$$

$$m_2 = \frac{-2}{3}$$

$$m_1 = \frac{1}{-m_2}$$

$$\frac{3}{B} = \frac{1}{2/3} = \frac{3}{2}$$

$$B = 2$$

6. Find A and B if the graph of the equation $Ax + 3y = B$ produces the same line as the graph of the equation $2x + 6y = 7$. (12)

$$y = -\frac{A}{3}x + \frac{B}{3}$$

$$m_1 = -\frac{A}{3}$$

$$b_1 = \frac{B}{3}$$

$$y = -\frac{2}{6}x + \frac{7}{6}$$

$$m_2 = -\frac{1}{3}$$

$$b_2 = \frac{7}{6}$$

$$m_1 = m_2 \quad b_1 = b_2$$

$$\frac{A}{3} = -\frac{1}{3} \quad \frac{B}{3} = \frac{7}{6}$$

$$A = 1$$

$$b_1 = b_2$$

$$B = 3.5$$

7. Consider the points $(1, -2)$ and $(-5, 6)$.

- (a) Find the distance between the two points. (10)

$$d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2} = \sqrt{(8)^2 + (-6)^2} = \sqrt{100} = 10$$

- (b) What are the coordinates of the midpoint of the segment connecting these points?

$(-2, 2)$

$$(x, y) = (-3, 2)$$

$$x = \frac{x_1 + x_2}{2} = \frac{1 + (-5)}{2} = -2$$

$$y = \frac{y_1 + y_2}{2} = \frac{-2 + 6}{2} = 2$$

- (c) Find the slope of the line through both points.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8}{-6} = -\frac{4}{3}$$

- (d) Find the standard form of the equation whose graph is the line through both points.

$$m = -\frac{4}{3} \quad b = y_1 - x_1 m = -2 + \frac{4}{3} = -\frac{6}{3} + \frac{4}{3} = -\frac{2}{3}$$

$$y = -\frac{4}{3}x - \frac{2}{3}$$

$$4x + 3y = -2$$

8. If
- $f(x) = 3x - 2$
- and
- $g(x) = x^2 + 1$
- , find:

(10)

(a) $f(g(2))$

$$3(x^2 + 1) - 2$$

$$3x^2 + 1$$

$$3(2)^2 + 1 = 12 + 1 = \boxed{13}$$

(b) $g(f(-1))$

$$3(3x - 2)^2 + 1$$

~~$$9x^2 - 12x + 4$$~~

$$9(-1)^2 - 12(-1) + 5 = 9 + 12 + 5 =$$

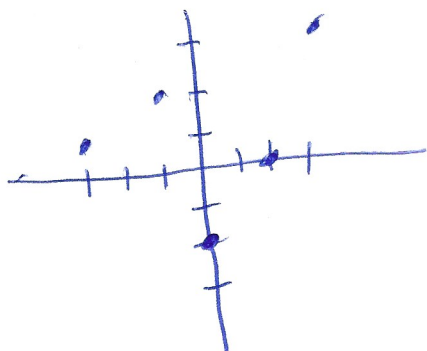
$$\boxed{26}$$

$$9x^2 - 12x + 4 + 1 = 9x^2 - 12x + 5$$

9. Graph the relation
- $\{(-3, 1), (-1, 2), (0, -2), (2, 0), (3, 3)\}$
- .

(8)

On your coordinate plane, plot each point. Then answer: Is this relation a function?



Yes, this is a function.

~~Yes~~

10. Give the domain of each relation.

(12)

(a) $\{(x, y) : y = \frac{2x+1}{x-4}\}$

$$D = \{x \in \mathbb{R} : x \neq 4\}$$

(b) $\{(x, y) : y \leq \frac{3}{x^2-1}\}$

$$D = \{x \in \mathbb{R} : x \neq 1 \text{ and } x \neq -1\}$$

(c) $\{(x, y) : y = \frac{x}{|x|+5}\}$

~~$D = \{x \in \mathbb{R} : x \neq 5\}$~~ ~~$D = \mathbb{R}$~~

(d) $\{(x, y) : |2x - 3| + |y - 1| = 2\}$

~~$D = \mathbb{R}$~~ $D = \mathbb{R}$ $|2x - 3| < 2$ $-2x + 3 < 2$
 $2x - 3 < 2$ $2x > 1$
 $2x < 5$ $x > 0.5$
 $x < 2.5$
 $D = \{x \in \mathbb{R} : 0.5 < x < 2.5\}$

11. True or False?

(a) All relations are functions.

False X

(b) All functions are relations.

True ✓

12. Find the range of each function with domain D .

(a) $h : x \mapsto -2x + 5, D = \{1, 3, 5\}$

$R = \{-5, -1, 3\}$

(b) $j : x \mapsto x^2 - 1, D = \{-2, 0, 2\}$

$R = \{-1, 3\}$

(c) $k : x \mapsto |x + 2|, D = \{-4, -2, 1\}$

$R = \{0, 3\}$

~~X~~ $\{0, 2, 3\}$

(d) $m : x \mapsto 4x - 3, D = \{0, 2, 4\}$

$R = \{-3, 5, 13\}$