11:10 - 12:32 and 1:18 - 1:22

1 hour and 26 minutes

Polynomial Operations and Factoring

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Find all solutions to each of the following equations: (10)

   (a) $r^2 - 7r = 0$

   $\frac{7 \pm \sqrt{49 - \cancel{4 \cdot 1 \cdot 0}}}{2} = \frac{14}{2}$ and $\frac{0}{2}$

   $r \in \{7, 0\}$

   (b) $x^2 + 3x = 7x - x^2$

   $2x^2 - 4x = 0$

   $2(x^2 - 2x) = 0 \quad x^2 - 2x = 0$

   $x \in \{0, 2\}$

   (c) $2t^2 = 242$

   $t^2 = 121$

   $t = 11$

   (d) $16 - y^2 = -4$

   $y^2 - 20 = 0$

   $y^2 = 20$

   $y = \sqrt{20}$

   20 → 2, 10; 10 → 5, 2

2. Find all solutions to the equation $t^4 - 11t^2 + 18 = 0$. (8)

   Hint: Let $u = t^2$ and solve for u first.

   $u^2 - 11u + 18 = 0$

   18: −2, −9; −11

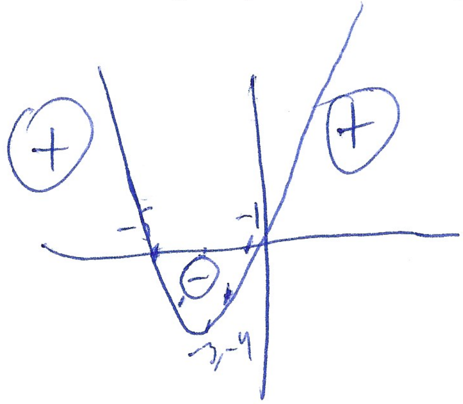
   $(u-2)(u-9) = 0$

   $(t^2 - 2)(t^2 - 9) = 0$

   $(t + \sqrt{2})(t - \sqrt{2})(t + 3)(t - 3) = 0$

   $t \in \{-3, -\sqrt{2}, \sqrt{2}, 3\}$

3. Solve the inequality $m^2 + 6m + 5 \geq 0$, and graph the solutions on the number line. (10)



$\frac{-b}{2a} = -3$

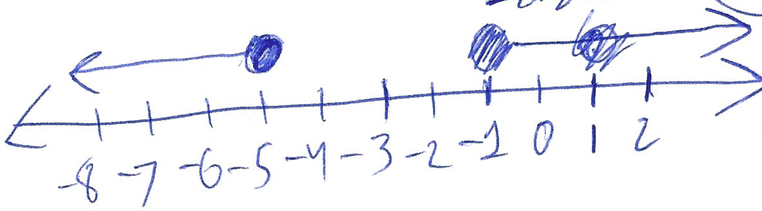
$9-18+5=-4$

$m^2+6m+5=0$

$m \in \{-5,-1\}$

$m^2+6m+5 \geq 0$

$m \leq -5$ or $m \geq -1$



4. For what values of r is $2r^2 - 3r > -7$? (8)

$2r^2 - 3r + 7 > 0$

$2r^2 - 3r + 7 = 0$

$\frac{3 \pm \sqrt{9-}}{4}$

Discriminant negative

$r \in \mathbb{R}$

5. Factor $x^4 + 4y^4$ (completely). (12)

Complete the square

$(x^2 + 2y^2)^2 - (2xy)^2$

$(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$

6. Factor each of the following: (15)

(a) $a^3 + 27$

$a^3 + 3^3$

$(a+3)(a^2 - 3a + 9)$

(b) $a^3b^3 + 8c^3$

$(ab)^3 + (2c)^3 = (ab + 2c)(a^2b^2 - 2abc + 4c^2)$

(c) $2r^3 - 16$

$2(r^3 - 8)$

$2(r-2)(r^2 + 2r + 4)$

(d) $1000 - x^6y^3$

$10^3 - (x^2y)^3$

$(10 - x^2y)(100 + 10x^2y + x^4y^2)$

$1000 + 100x^2y + 10x^4y^2 - 100x^2y - 10x^4y^2 - x^6y^3$

7. (a) The expression $x^5 + y^5$ can be written as the product of $x + y$ and another factor. Find that other factor. (12)

~~x^4~~

$$x^4 - x^3y + x^2y^2 - xy^3 + y^4$$

(b) The expression $x^7 + y^7$ can be written as the product of $x + y$ and another factor. Find that other factor.

$$x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6$$

(c) Write $x^{2n+1} + y^{2n+1}$ as the product of two factors.

~~assuming~~ assuming $n \in \mathbb{Z}$, $2n+1 \% 2 = 1$ ($2n+1$ is odd)

$(x+y)(x^{2n}$ ~~+~~

(d) Why does the factorization in the previous part fail when the powers of x and y are even? In other words, why can we not factor $x^4 + y^4$ or $x^6 + y^6$ using the patterns we found in the first three parts?

The alternating signs make b^n negative, although all middle terms cancel, you are left with $x^n - y^n$ not $x^n + y^n$.

~~It doesn't cancel out fully~~

$(x+y)(x^3 - x^2y + xy^2 - y^3)$

$x^4 - \cancel{x^3y} + \cancel{x^2y^2} - \cancel{xy^3} + \cancel{x^3y} - \cancel{x^2y^2} + \cancel{xy^3} - y^4$

$\sqrt{-12} = 2\sqrt{3}\,i \qquad \sqrt{-48} = 4\sqrt{3}\,i$

8. Solve each polynomial equation by factoring: (10)

(a) $x^3 - 8 = 0$

$x^3 - 2^3 = 0$

$(x-2)(x^2+2x+4) = 0$

$x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$

$x - 2 = 0$

$x = 2$

$x^2 + 2x + 4 = 0$

neg. discriminant not in reals

~~$x = 2$ if $x \in \mathbb{R}$~~

$x \in \{2, -1 \pm \sqrt{3}i\}$

(b) $x^3 + 64 = 0$

$x^3 + 4^3$

$(x+4)(x^2-4x+16)$

$x^3 - 4x^2 + 16x + 4x^2 - 16x + 64$

$x + 4 = 0$

$x = -4$

$x^2 - 4x + 16$

neg. discriminant not in reals

~~$x = -4$~~

~~if $x \in \mathbb{R}$~~

$x \in \{-4, 2 \pm 2\sqrt{3}i\}$

(c) $2x^3 - 16x = 0$

$2(x^3 - 8) = 0$

$x^3 - 8 = 0$

(from 8a.)

~~$x = 2$ if $x \in \mathbb{R}$~~

$x \in \{2, -1+\sqrt{3}i, -1-\sqrt{3}i\}$

(d) $x^4 - 16 = 0$

$(x^2+4)(x^2-4) = 0$

$(x^2+4)(x+2)(x-2) = 0$

$(x+2i)(x-2i)(x+2)(x-2)$

~~$x \in \{2, -2\}$ if $x \in \mathbb{R}$~~

$x \in \{-2, 2, -2i, 2i\}$

9. Find the GCF of $54x^7t^3$, $90x^5t^2$, and $108x^4t$. (8)

(a) $18x^7t$

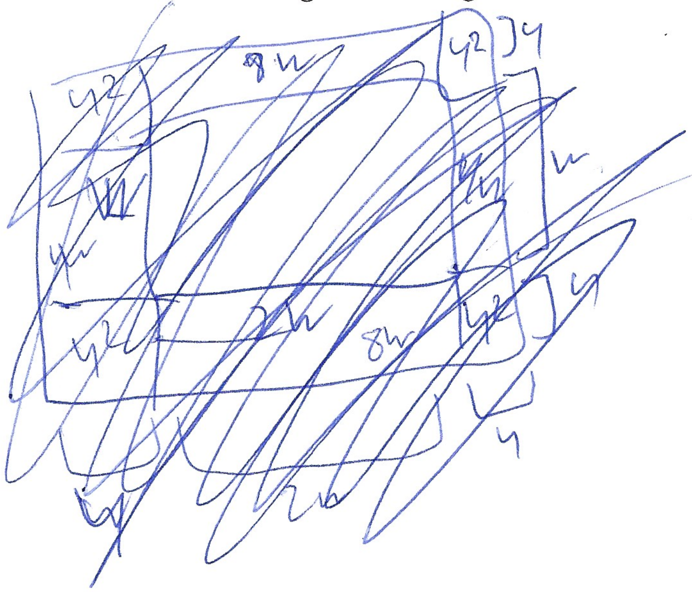
(b) $9x^2t$

(c) $540x^4t^3$

(d) $36xt$

$18x^4t$

10. A rectangle is twice as long as it is wide. If its length is increased by 4 cm and its width is decreased by 3 cm, the new rectangle formed has an area of 100 cm^2. Find the dimensions of the original rectangle. (12)



$(2w+4)(w-3)=100$

$2w^2-6w+4w-12=100$

$\cancel{2w^2-2w+88=0}$

$\cancel{2(w^2-w+44)=0}$

~~negative discriminant~~

$2w^2-6w+4w-12-100=0$

$2w^2-2w-112=0$

$2(w^2-w-56)=0$

$(w-8)(w+7)=0$

$w \in \{-7, 8\}$

W: 8cm
L: 16cm