Polynomial Division and Roots

Name: ______________________ Date: ______________________

Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Find a constant c such that there is no remainder when $x^3 + cx^2 + 4x - 21$ is divided by $x - 3$. (8)

   Hint: You may use the Remainder Theorem or polynomial long division.

2. Find the quotient and remainder when $x^4 - 23x^3 + 11x^2 + 14x + 20$ is divided by $x + 5$. (10)

   Hint: Consider using synthetic division for this problem.

3. Find the quotient and remainder when $x^4 + 3x^3 - x^2 + 7x - 1$ is divided by $2 - x$. (8)

   Hint: Rewrite the divisor in standard form first.

4. Find all roots of the following polynomial: (10)

$$g(y) = 12y^3 - 28y^2 - 9y + 10$$

Hint: Look for rational roots first using the Rational Root Theorem.

5. When $y^2 + my + 2$ is divided by $y - 1$, the quotient is $f(y)$ and the remainder is R_1. When $y^2 + my + 2$ is divided by $y + 1$, the quotient is $g(y)$ and the remainder is R_2. If $R_1 = R_2$, then find m. (12)

Hint: Use the Remainder Theorem to find expressions for R_1 and R_2.

6. Suppose $q(x)$ and $r(x)$ are the quotient and remainder, respectively, when the polynomial $f(x)$ is divided by the polynomial $d(x)$. Show that if $x = a$ is a root of $d(x)$, then $r(a) = f(a)$. (10)

   Hint: Use the division algorithm for polynomials.

7. Find all roots of each of the following polynomials: (8)

   (a) $f(x) = x^3 - 4x^2 - 11x + 30$

   (b) $g(t) = t^4 + 5t^3 - 19t^2 - 65t + 150$

8. Find the remainder when $x^{100} - 4x^{50} + 5x + 6$ is divided by $x^3 - 2x^2 - x + 2$. (10)

   Hint: Can you factor the cubic? Try factoring x^3 out of the first two terms. Can you then factor further?

9. Suppose that $f(x)$ is a polynomial with integer coefficients such that $f(2) = 3$ and $f(7) = -7$. Show that $f(x)$ has no integer roots. (8)

   Hint: Note that 3 and -7 are both odd.

   Hint: Is it possible for $f(0)$ to be even?

10. How can we quickly tell that $x - 1$ is a factor of $x^5 + 6x^4 - 7x^3 + 2x^2 - 2$ without performing the long division? (8)

    Hint: Use the Factor Theorem.

11. Find the quotient and remainder for the following polynomial division: (10)

$$x^2 - 19x + 17 \text{ divided by } x + 7$$

    Hint: Use polynomial long division or synthetic division.

12. Teresa divides $3x^4 + 2x^3 - 7x^2 + 4x - 1$ by $x + 2$ and gets a quotient of $3x^3 - 4x^2 + x + 2$ and a remainder of 5. How can Teresa quickly realize that she made a mistake without performing the division again, and without multiplying $x + 2$ by the quotient? (12)

    Hint: Use the Remainder Theorem to check her work.

13. The polynomial $p(x) = 3x^3 - 20x^2 + kx + 12$ is divisible by $x - 3$ for some constant k. Factor $p(x)$ completely. (8)

Hint: Use the Factor Theorem to find k first, then factor completely.