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Integrated Algebra 2 and Precalculus

Exam: Chapter 10 of Algebra 2

1 hour and 23 min

Exponential and Logarithmic Functions

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Instructions: Answer all questions to the best of your ability. Show all your work in the space provided for full credit.

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1. Find the base n such that $\log_n 4\sqrt{2} = 10$. (8)

$n^{10} = 4\sqrt{2}$

$n = (4\sqrt{2})^{\frac{1}{10}} = 32^{\frac{1}{20}} = \sqrt{2}$

2. Find x if $\log_9(2x-7) = \frac{3}{2}$. (10)

$\sqrt{9^3} = 2x - 7 = 27$

$9^3 = 4x^2 - 28x + 49$

$2x = 34$

$x = 17$

3. For how many positive integers b is $\log_b 729$ a positive integer? (8)

for all $1 < b \leq 729$

$b \in \{2, 3, 6\}$

4. If $\log_3(\log_3(\log_3(x))) = 2$, then how many digits are in x? (10)

$\log_3(\log_3(x)) = 3^2$

$\log_3(x) = 3^{3^2}$

$x = 3^{3^{3^2}}$

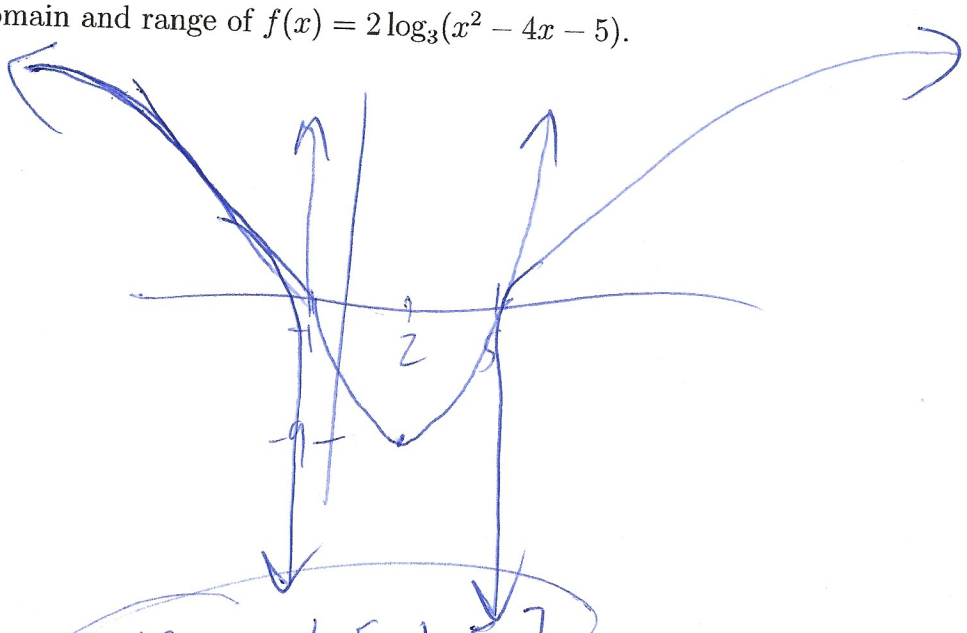
$\lfloor 3^9 \cdot \log_{10} 3 \rfloor + 1$

$\lfloor 19683 \cdot \log_{10} 3 \rfloor + 1$

9392

$\lfloor 3^9 \cdot \log_{10}(3^9) \rfloor + 1$

5. Find the domain and range of $f(x) = 2\log_3(x^2 - 4x - 5)$. (12)



Domain: $x \in \mathbb{R}, x \notin [-1, 5]$

Range: $\mathbb{R}$

6. Let $f(x) = 3x^2 - 7$ and $g(f(4)) = 9$. What is $g(f(-4))$? (10)

9

7. Let $f(x) = 1 - \frac{1}{x}$. (12)

(a) Find $f(f(x))$.

$1 - \frac{1}{1-\frac{1}{x}} = 1 - \frac{1}{\frac{x-1}{x}} = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{-1}{x-1}$

$1 - \frac{1}{\frac{x-1}{x}} = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{-1}{x-1}$

(b) Find $f(f(f(x)))$.

$f(f^2(x)) = f\left(1 - \frac{x}{x-1}\right) = 1 - \frac{1}{1-\frac{x}{x-1}} = 1 - \frac{1}{\frac{x-1-x}{x-1}} = 1 + x - 1 = x$

$1 - \frac{1}{\frac{x-1}{x-1} - \frac{x}{x-1}}$

(c) Find $f(f(f(f(x))))$.

$f^4(x) = f(f^3(x)) = f(x) = 1 - \frac{1}{x}$

(d) Find $f^{34}(5)$.

Hint: Compare $f^4(x)$ to $f(x)$. Notice anything interesting? If you don't, then you should find $f^4(x)$ again.

$f^4(x) = f(x)$

$f^5(x) = f(f^4(x)) = f(x)$

$f^{10}(x) = f^5(f^5(x)) = f(x)$

$f^{34}(x) = f^5(f^4(f^4(f^4(f^4(f^4(f^4(f^4(f(x))))))))) = f^2(x) = x$

$f^{1+3(10)}(x) = f(x) = f^{34}(x)$

5

8. If $8^x = 27$, then what is 4^{2x-3}? (10)

Hint: Express both sides in terms of powers of 2 and 3.

$4^{2x-3} = 4^{\log_4(81)} \cdot 4^{-3} = 81 \cdot \frac{1}{43} = \frac{81}{64}$

$2^{3x} = 3^3$

$x = \log_2(3) = \log_4(9) = \frac{1}{2}\log_4(81)$

9. Let f be a function whose graph passes through the points $(2,3)$, $(4,7)$, and $(8,12)$. Suppose f has an inverse. Name three points that must be on the graph of $y = f^{-1}(x)$. (8)

$(3,2)$

$(7,4)$

$(12,8)$

10. I have just won a lottery that will pay me \$1,000,000 in 10 years. A company offers to buy my winning ticket today for \$300,000. (10)

(a) If the annually compounded interest rate is 9%, should I take the offer?

$1.09 \cdot 1.09 = 1.1881$ (in \$1000)

$300\,(109\%)^{10} \approx 300(2.37) = 811$

No

(b) For what annually compounded interest rate is my lottery ticket worth \$300,000 today?

\$1 million

in 10 years

$\log_{10}\left(\frac{10}{3}\right) = \log(10) - \log(3) = 1 - \log(3)$

$a^{10} = \frac{10}{3}$

$a = \frac{10\sqrt{10}}{10\sqrt{3}} \approx 1.128$

$\approx 12.8\%$

$1 - \left(\frac{10}{3}\right)^{\frac{1}{10}}\%$

11. Evaluate $\log_2 8$, $\log_2 16$, and $\log_2(8 \cdot 16)$. → 3, 4, 12 (12)

(a) Evaluate $\log_3 \frac{1}{9}$, $\log_3 \sqrt{3}$, and $\log_3 \left(\frac{1}{9} \cdot \sqrt{3}\right)$.

-2 1/2 -1.5

(b) Do you notice a relationship among $\log_a b$, $\log_a c$, and $\log_a(bc)$? Can you prove it?
Hint: Let $x = \log_a b$, $y = \log_a c$, *and* $z = \log_a(bc)$. *Use exponential notation.* Yes, Yes

$\log_a(b) = x \Leftrightarrow a^x = b$

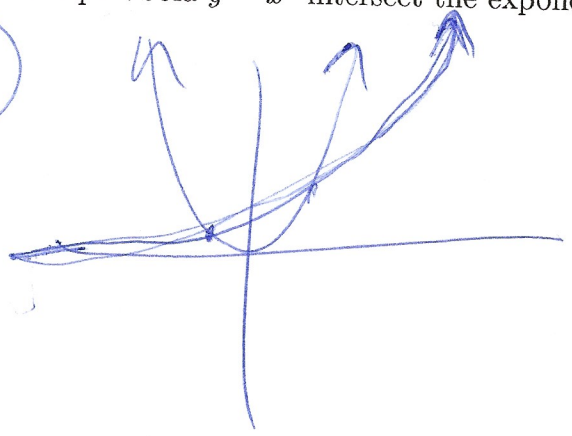
$\log_a(c) = y \Leftrightarrow a^y = c$

$\log_a(bc) = z \Leftrightarrow a^z = bc = a^x a^y = a^{x+y} \Leftrightarrow \log_a(a^x \cdot a^y) = z$

$z = x + y \Rightarrow \log_a(bc) = \log_a(b) + \log_a(c)$

12. At how many points does the parabola $y = x^2$ intersect the exponential curve $y = 2^x$? (8)

Two Points



$2^x = x^2$

$x = \log_2(x^2) \Rightarrow x = 2\log_2(x)$