

## Chapter 2: Functions, Domains, Inverses, and Transformations

Name: \_\_\_\_\_

Date: \_\_\_\_\_

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**Instructions:** Answer all questions to the best of your ability. Show all your work in the space provided for full credit. Unless otherwise stated, assume all variables represent real numbers and all functions are defined on their natural domains.

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1. For each relation below, decide whether it defines  $y$  as a function of  $x$ . If it does, state the domain and range; if not, explain why. (12)

(a)  $y^2 = x + 3$

(b)  $x = |y - 2|$

(c)  $y = \begin{cases} 2x + 1, & x \leq 0 \\ \sqrt{x}, & x > 0 \end{cases}$

2. Suppose that a function  $f(x)$  is defined for all real  $x$ . Describe how to obtain the graph of  $f(|x|)$  from the graph of  $f(x)$ . (8)

3. For which constants  $a, b, c$ , and  $d$  does the function  $f(x) = \frac{ax + b}{cx + d}$  have an inverse? (10)  
Justify your answer.

4. Find the domain of each of the following functions.

(18)

(a)  $f(x) = \frac{1}{\sqrt{2x-5}} + \sqrt{9-3x}$

(b)  $f(x) = |\sqrt{x} - 2| + |\sqrt{x} - 2|$

(c)  $g(x) = \sqrt{|x| - 2} + \sqrt{|x - 3|}$

5. Consider  $g(x) = -\frac{2}{3x-6} + 1$ . (14)

(a) Find all vertical and horizontal asymptotes and justify.

(b) Find the  $x$ - and  $y$ -intercepts, if any.

(c) The graph of  $y = \frac{1}{x}$  is transformed to obtain  $y = g(x)$ . Give the mapping rule that sends a generic point  $(x, y)$  on  $y = \frac{1}{x}$  to a point on  $y = g(x)$ .

6. Let  $f(x) = |x - 3| - |x + 1| + 2x$ . (16)
- (a) Determine the intervals on which  $f$  is increasing and decreasing. Justify using a case breakdown.
  - (b) Compute the average rate of change of  $f$  on  $[-4, 5]$ .
  - (c) Find all real  $c$  such that the average rate of change on  $[c, c + 2]$  equals 1.

7. Find all real slopes  $m$  such that the line  $y = mx + 1$  is tangent to the parabola  $y = -x^2 + 8x$ . For each such  $m$ , give the point of tangency. (14)

8. **Maximizing Profit.** A community bird-watching society makes and sells simple bird feeders to raise money for its conservation activities. The materials for each feeder cost \$6, and they sell an average of 20 per week at a price of \$10 each. They have been considering raising the price, so they conduct a survey and find that for every dollar increase they lose 2 sales per week. (12)
- (a) Find a function that models weekly profit in terms of the price per feeder.
  - (b) What price should the society charge for each feeder to maximize profits? What is the maximum profit?



9. Let  $f(x) = \sqrt{x+2}$  with domain  $x \geq -2$  and  $g(x) = \frac{1}{x-1}$  with domain  $x \neq 1$ . (10)

(a) Find the domain of  $h(x) = f(g(x))$ .

(b) Find the domain of  $k(x) = g(f(x))$ .

(c) Decide which, if either, of  $h$  or  $k$  is defined at  $x = -1$  and justify.

10. BONUS QUESTION (VERY HARD): Consider  $H(x) = \frac{ax+b}{cx+a}$  with real parameters (18)  
where  $a^2 - bc \neq 0$  and  $cx + a \neq 0$  on the domain.

- (a) Determine necessary and sufficient conditions on  $a, b, c$  such that  $H \circ H$  is the identity on its domain (that is,  $H(H(x)) = x$ ). Show all steps.
- (b) Under those conditions, find an explicit inverse  $H^{-1}(x)$  and its domain.