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Workshop 2

Commands | + Code | + Text

```
[1] import numpy as np
    import pandas as pd
    import matplotlib.pyplot as plt
```

```
df = pd.read_csv("/content/drive/MyDrive/AI and Manchine Learning/mnist_dataset.csv")
```

```
[3] from google.colab import drive
    drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

```
[6] print("Dataset preview:")
    df.head()
```

Dataset preview:

	label	pixel_0	pixel_1	pixel_2	pixel_3	pixel_4	pixel_5	pixel_6	pixel_7	pixel_8	...	pixel_774	pixel_775	pixel_776	pixel_777	pixel_778	pixel_779	pixel_780	pixel_781	pi
0	5	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	
2	4	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	
3	1	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	
4	9	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	

5 rows × 785 columns

```
[7] print("Dataset info:")
    print(df.info())
```

```
Dataset info:
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 60000 entries, 0 to 59999
Columns: 785 entries, label to pixel_783
dtypes: int64(785)
memory usage: 359.3 MB
None
```

```
[8] #extracting features (x) and target values (y)
    X = df.iloc[:, 1:].values
    y = df.iloc[:, 0].values
```

```
X.shape
```

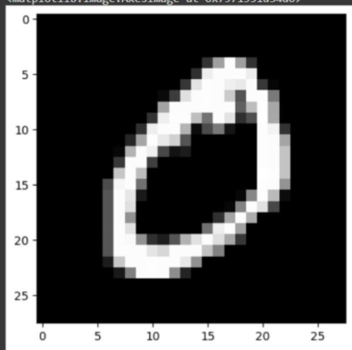
```
(60000, 784)
```

```
[10] y.shape
```

```
(60000,)
```

```
plt.imshow(X[1].reshape(28, 28), cmap='gray')
```

```
<matplotlib.image.AxesImage at 0x7971951d34d0>
```




```
[12] y[1]
```

```
0
```

```
[13] X[0].shape
```

```
(784,)
```

```
[14] from sklearn.model_selection import train_test_split
     from sklearn.preprocessing import OneHotEncoder

     # Splitting
     X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)
```

```
[15] y_train.shape
```

```
(48000,)
```

```
[16] # Reshape y_train and y_test to be column vectors
     y_train = y_train.reshape(-1, 1)
     y_test = y_test.reshape(-1, 1)
```

```
[17] y_train.shape
```

```
(48000, 1)
```

```
# Initialize the OneHotEncoder with correct parameter name
encoder = OneHotEncoder(sparse_output=False, categories='auto')

# Fit and transform the training labels, then transform the test labels
y_train = encoder.fit_transform(y_train)
y_test = encoder.transform(y_test)
```

```
[19] print("\nShapes:")
     print("X_train:", X_train.shape, "y_train:", y_train.shape)
     print("X_test:", X_test.shape, "y_test:", y_test.shape)
```

```
Shapes:
X_train: (48000, 784) y_train: (48000, 10)
X_test: (12000, 784) y_test: (12000, 10)
```

```
[20] y_train[0]
```

```
array([0., 0., 0., 0., 0., 1., 0., 0., 0., 0.])
```

Some Helper Function:

Softmax Function:

```python
import numpy as np

def softmax(z):
    """
    Compute the softmax probabilities for a given input matrix.

    Parameters:
    z (numpy.ndarray): Logits (raw scores) of shape (m, n), where
                       - m is the number of samples.
                       - n is the number of classes.

    Returns:
    numpy.ndarray: Softmax probability matrix of shape (m, n), where
                   each row sums to 1 and represents the probability
                   distribution over classes.

    Notes:
    - The input to softmax is typically computed as: z = XW + b.
    - Uses numerical stabilization by subtracting the max value per row.
    """

    # Your Code Here.
    z_shifted = z - np.max(z, axis=1, keepdims=True)
    exp_z = np.exp(z_shifted)
    softmax_probs = exp_z / np.sum(exp_z, axis=1, keepdims=True)
    return softmax_probs
```

Softmax Test Case:

This test case checks that each row in the resulting softmax probabilities sums to 1, which is the fundamental property of softmax.

```python
# Example test case
z_test = np.array([[2.0, 1.0, 0.1], [1.0, 1.0, 1.0]])
softmax_output = softmax(z_test)

# Verify if the sum of probabilities for each row is 1 using assert
row_sums = np.sum(softmax_output, axis=1)

# Assert that the sum of each row is 1
assert np.allclose(row_sums, 1), f"Test failed: Row sums are {row_sums}"

print("Softmax function passed the test case!")
```

```
Softmax function passed the test case!
```

```python
def cross_entropy_loss(y_pred, y):
    """
    Compute the cross-entropy loss for a single sample.

    Parameters:
    y_pred (numpy.ndarray): Predicted probabilities of shape (c,) for a single sample,
                            where c is the number of classes.
    y (numpy.ndarray): True labels (one-hot encoded) of shape (c,), where c is the number of classes.

    Returns:
    float: Cross-entropy loss for the given sample.
    """

    # Avoid log(0) by adding a small value (epsilon)
    epsilon = 1e-15  # for not making (log(0))
    y_pred = np.clip(y_pred, epsilon, 1 - epsilon)

    # Calculate cross-entropy loss
    loss = -np.sum(y * np.log(y_pred))

    return loss
```

```python
def cost_softmax(X, y, W, b):
    """
    Compute the softmax regression cost (cross-entropy loss).

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of samples and d is the number of features.
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c), where c is the number of classes.
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    float: The softmax cost (cross-entropy loss).
    """
    n = X.shape[0]  # Number of samples
    z = np.dot(X, W) + b
    y_pred = softmax(z)
    cost = cross_entropy_loss(y_pred, y)
    return cost
```

```python
[25] def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """
    n, d = X.shape #(total_number_of_row, total_columns)
    z = np.dot(X, W) + b
    y_pred = softmax(z)
```

```python
def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """
    n, d = X.shape #(total_number_of_row, total_columns)
    z = np.dot(X, W) + b
    y_pred = softmax(z)

    grad_W = np.dot(X.T, (y_pred - y)) / n  # Gradient with respect to weights
    grad_b = np.sum(y_pred - y, axis=0) / n  # Gradient with respect to biases

    return grad_W, grad_b
```

[26]
```python
def gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False):
    """
    Perform gradient descent to optimize the weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    alpha (float): Learning rate.
    n_iter (int): Number of iterations.
    show_cost (bool): Whether to display the cost at intervals.

    Returns:
    tuple: Optimized weights, biases, and cost history.
    """
    cost_history = []

    for i in range(n_iter):
        # Compute gradients
        grad_W, grad_b = compute_gradient_softmax(X, y, W, b)

        # Update weights and biases using the gradients
        W -= alpha * grad_W
        b -= alpha * grad_b

        # Compute and store cost
        cost = cost_softmax(X, y, W, b)
        cost_history.append(cost)

        # Print cost at regular intervals
        if show_cost and (i % 100 == 0 or i == n_iter - 1):
            print(f"Iteration {i}: Cost = {cost:.6f}")

    return W, b, cost_history
```

Prediction Function:

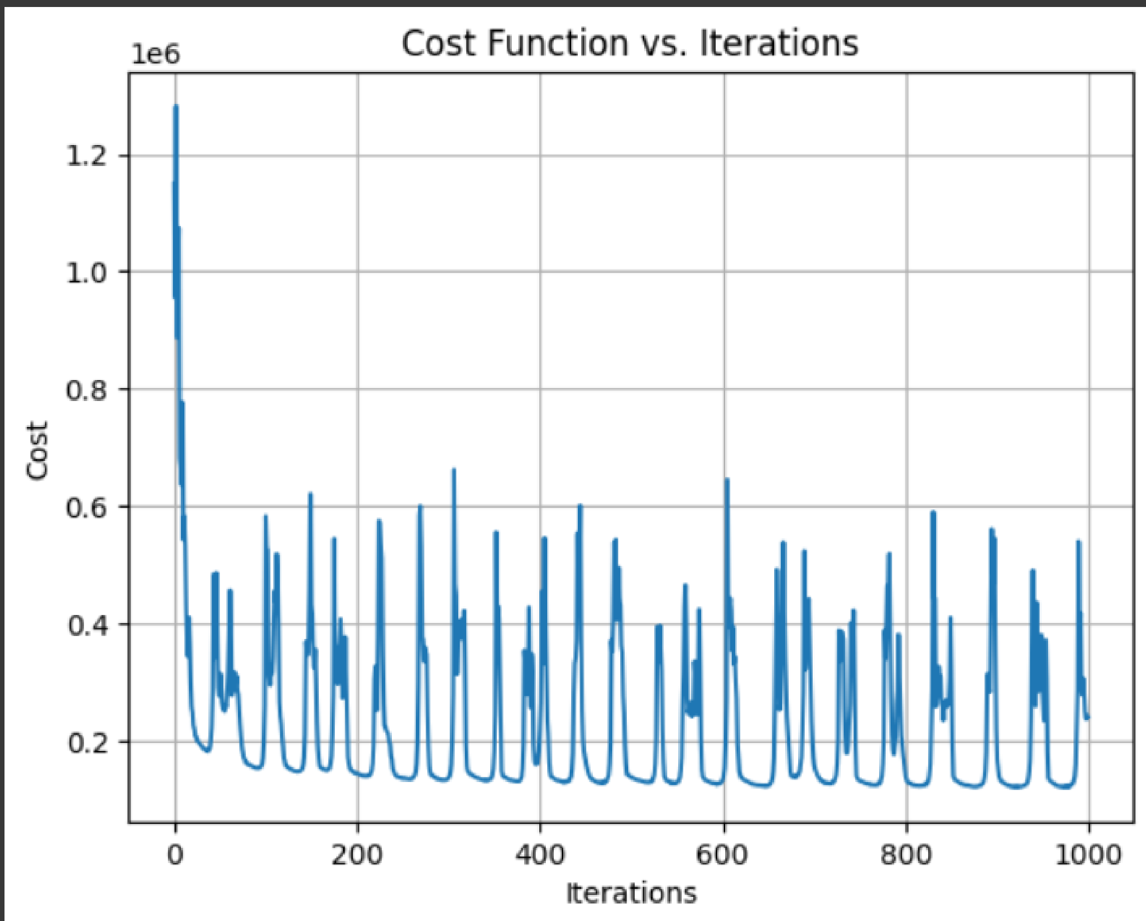
```
[27] def predict_softmax(X, W, b):
         """
         Predict the class labels for a set of samples using the trained softmax model.

         Parameters:
         X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of samples and d is the number of features.
         W (numpy.ndarray): Weight matrix of shape (d, c), where c is the number of classes.
         b (numpy.ndarray): Bias vector of shape (c,).

         Returns:
         numpy.ndarray: Predicted class labels of shape (n,), where each value is the index of the predicted class.
         """
         z = np.dot(X, W) + b  # Compute the scores (logits)
         y_pred = softmax(z)  # Get the probabilities using the softmax function

         # Assign the class with the highest probability
         predicted_classes = np.argmax(y_pred, axis=1)

         return predicted_classes
```

```
[28] y_train.shape
```

```
(48000, 10)
```

```
[29] # Initialize the weights and biases
     d = X_train.shape[1]  # Number of features
     c = y_train.shape[1]  # Number of classes
     W = np.random.randn(d, c) * 0.01  # Small random weights
     b = np.zeros(c)  # Bias initialized to 0

     # Set hyperparameters
     alpha = 0.1  # Learning rate
     n_iter = 1000  # Number of iterations

     # Train the model using gradient descent
     W_opt, b_opt, cost_history = gradient_descent_softmax(X_train, y_train, W, b, alpha, n_iter, show_cost=True)

     # Plot the cost history to visualize the convergence
     plt.plot(cost_history)
     plt.title('Cost Function vs. Iterations')
     plt.xlabel('Iterations')
     plt.ylabel('Cost')
     plt.grid(True)
     plt.show()

     # Predict on the test set
     #y_pred_test = predict_softmax(X_test, W_opt, b_opt)

     # Evaluate accuracy
     #y_test_labels = np.argmax(y_test, axis=1)  # True labels in numeric form
     #accuracy = np.mean(y_pred_test == y_test_labels)
     #print(f"Test accuracy: {accuracy * 100:.2f}%")
```

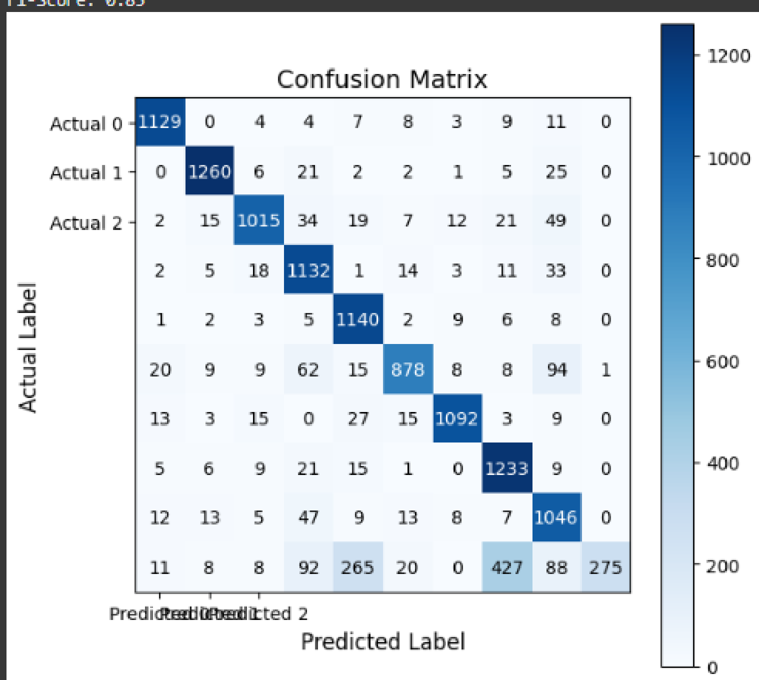
```
Iteration 0: Cost = 1149736.612632
Iteration 100: Cost = 582747.027293
Iteration 200: Cost = 143895.863400
Iteration 300: Cost = 137412.689552
Iteration 400: Cost = 218113.846292
Iteration 500: Cost = 137911.223933
Iteration 600: Cost = 133376.355944
Iteration 700: Cost = 157737.261412
Iteration 800: Cost = 138856.799485
Iteration 900: Cost = 167943.940920
Iteration 999: Cost = 240358.253716
```



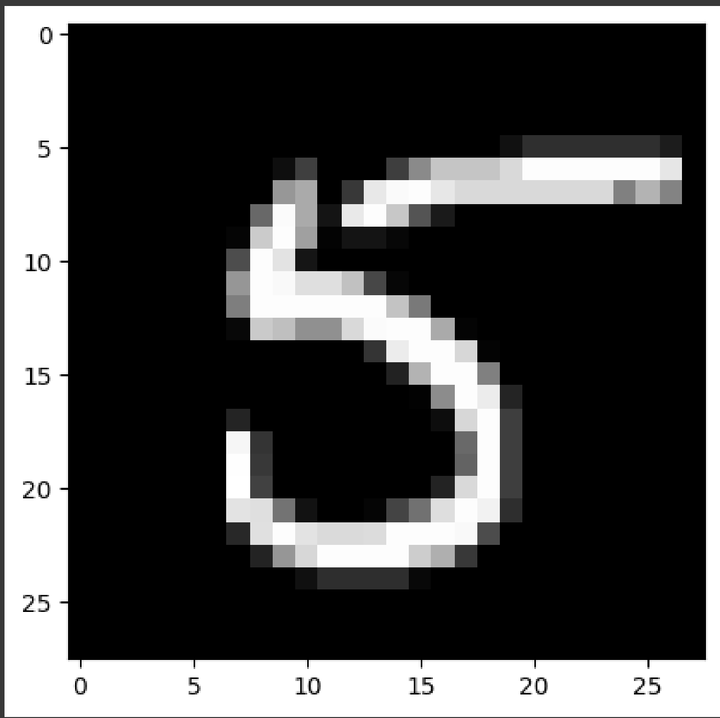
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix, precision_score, recall_score, f1_score

# Evaluation Function
def evaluate_classification(y_true, y_pred):
    """
    Evaluate classification performance using confusion matrix, precision, recall, and F1-score.

    Parameters:
    y_true (numpy.ndarray): True labels
    y_pred (numpy.ndarray): Predicted labels

    Returns:
    tuple: Confusion matrix, precision, recall, F1 score
    """
    # Compute confusion matrix
    cm = confusion_matrix(y_true, y_pred)

    # Compute precision, recall, and F1-score
    precision = precision_score(y_true, y_pred, average='weighted')
    recall = recall_score(y_true, y_pred, average='weighted')
    f1 = f1_score(y_true, y_pred, average='weighted')

    return cm, precision, recall, f1

# Predict on the test set
y_pred_test = predict_softmax(X_test, W_opt, b_opt)
# Evaluate accuracy
y_test_labels = np.argmax(y_test, axis=1)  # True labels in numeric form

# Evaluate the model
cm, precision, recall, f1 = evaluate_classification(y_test_labels, y_pred_test)

# Print the evaluation metrics
print("\nConfusion Matrix:")
print(cm)
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1:.2f}")

# Visualizing the Confusion Matrix
fig, ax = plt.subplots(figsize=(6, 6))
cax = ax.imshow(cm, cmap='Blues')  # Use a color map for better visualization

# Set tick labels for the axes
ax.set_xticks(range(3))
ax.set_yticks(range(3))
ax.set_xticklabels([f'Predicted {i}' for i in range(3)])
ax.set_yticklabels([f'Actual {i}' for i in range(3)])

# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha='center', va='center', color='white' if cm[i, j] > np.max(cm) / 2 else 'black')

# Add grid lines and axis labels
ax.grid(False)
plt.title('Confusion Matrix', fontsize=14)
```

```python
# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha='center', va='center', color='white' if cm[i, j] > np.max(cm) / 2 else 'black')

# Add grid lines and axis labels
ax.grid(False)
plt.title('Confusion Matrix', fontsize=14)
plt.xlabel('Predicted Label', fontsize=12)
plt.ylabel('Actual Label', fontsize=12)

# Adjust layout
plt.tight_layout()
plt.colorbar(cax)
plt.show()
```

```
Confusion Matrix:
[[1129    0    4    4    7    8    3    9   11    0]
 [   0 1260    6   21    2    2    1    5   25    0]
 [   2   15 1015   34   19    7   12   21   49    0]
 [   2    5   18 1132    1   14    3   11   33    0]
 [   1    2    3    5 1140    2    9    6    8    0]
 [  20    9    9   62   15  878    8    8   94    1]
 [  13    3   15    0   27   15 1092    3    9    0]
 [   5    6    9   21   15    1    0 1233    9    0]
 [  12   13    5   47    9   13    8    7 1046    0]
 [  11    8    8   92  265   20    0  427   88  275]]
Precision: 0.87
Recall: 0.85
F1-Score: 0.83
```



[33]
```
# Reshape the first image to (28, 28)
image = X_train[0].reshape(28, 28)

# Display the image using plt.imshow
plt.imshow(image, cmap='gray')  # 'gray' colormap for grayscale images
plt.show()
```



```
y_train[0]
```

```
array([0., 0., 0., 0., 0., 1., 0., 0., 0., 0.])
```

[35]
```
single_image = X_train[0].reshape(1, -1)  # Reshape to (1, 784) as it needs to be 2D


predicted_class = predict_softmax(single_image, W_opt, b_opt)
```

[36]
```
print("Predicted class for the single image:", predicted_class[0])
```

```
Predicted class for the single image: 5
```