Assignment 2

MSO-201: Probability and Statistics

1. Show that if $A \subset B$, then $P(A) \leq P(B)$

2. Show that
$$P\left(A \bigcup B\right) = P(A) + P(B) - P\left(A \bigcap B\right)$$

3. Show that
$$P\left(A \bigcup B \bigcup C\right) = P(A) + P(B) + P(C) - P\left(A \bigcap B\right) - P\left(A \bigcap C\right) - P\left(B \bigcap C\right) + P\left(A \bigcap B \bigcap C\right)$$

4. Show that if $A_1 \subset A_2 \subset A_3 \ldots$, then
$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \lim_{n \to \infty} P(A_i).$$

5. Show that if $A_1 \supset A_2 \supset A_3 \ldots$, then
$$P\left(\bigcap_{i=1}^{\infty} A_i\right) = \lim_{n \to \infty} P(A_i).$$

6. If the sample space $\Omega = A_1 \cup A_2$, and if $P(A_1) = 0.8$ and $P(A_2) = 0.5$, find $P(A_1 \cap A_2)$.

7. Suppose the sample space $\Omega = \{x; -\infty < x < \infty\}$. Let $\{r_1, r_2, \ldots\}$ be the enumeration of all the rational numbers in $(-\infty, \infty)$. Let $p_1, p_2, \ldots$ be the set of non-negative real numbers such that $0 \leq p_i \leq 1$, for all $i = 1, 2, \ldots$ and $\sum_{i=1}^{\infty} p_i = 1$. For any set $A \subset \Omega$, we define
$$P(A) = \sum_{i: r_i \in A} p_i.$$
Show that the above set function $P(\cdot)$ is a proper probability function. Hint: Show that it satisfies all the three properties of a probability function.

8. Suppose
$$\Omega = \{H, TH, TTH, TTTH, TTTTH, \ldots\},$$
and $P(H) = 1/2$, $P(TH) = 1/4$, $P(TTH) = 1/8$ and so on. Further, for any other subsets of Ω it is being extended in such a manner so that it satisfies the properties of a probability function. If A is the event which denotes at least 10 'T's. Find $P(A)$.

9. A random experiment consists of drawing a card from an ordinary deck of 52 playing cards. Let the probability function P assign a probability of $1/52$ to each of the 52 possible outcomes. Let C_1 denote the collection of the 13 hearts and let C_2 denote the collection of the 4 kings. Compute $P(C_1)$, $P(C_2)$, $P(C_1 \cap C_2)$ and $P(C_1 \cup C_2)$.

10. Let $\Omega = \{0, 1, 2, \ldots\}$ and $p_i = \dfrac{e^{-\lambda}\lambda^i}{i!}$, for $i = 0, 1, \ldots$, where $\lambda > 0$. If we define a set function on any subset of Ω as follows: $P(A) = \sum_{i \in A} p_i$. Show that $P(\cdot)$ is a probability function.