Assignment 3
MSO-201: Probability and Statistics

In all the questions we denote Ω as the sample space and $\mathcal{F}$ is a σ-field.

1. Prove that if $A_1, A_2 \ldots \in \mathcal{F}$, then $\bigcap_{i=1}^{\infty} A_i' \in \mathcal{F}$.

2. Prove that if $A_1, A_2 \ldots \in \mathcal{F}$, then $\bigcap_{i=1}^{\infty} A_i \in \mathcal{F}$.

3. Prove that if B_i is either A_i or A_i', then (a) $\bigcap_{i=1}^{\infty} B_i \in \mathcal{F}$, (b) $\bigcup_{i=1}^{\infty} B_i \in \mathcal{F}$, (c) $\bigcap_{i=1}^{\infty} B_i' \in \mathcal{F}$, (d) $\bigcup_{i=1}^{\infty} B_i' \in \mathcal{F}$.

4. If $\mathcal{F}_1$ and $\mathcal{F}_2$ are σ-fields which are subsets (means the elements are subsets) of Ω, prove that $\mathcal{F} = \mathcal{F}_1 \bigcap \mathcal{F}_2$ is also a σ-field.

5. If $\mathcal{F}_1, \mathcal{F}_2, \ldots$ are σ-fields which are subsets of Ω, prove that $\mathcal{F} = \bigcap_{i=1}^{\infty} \mathcal{F}_i$ is also a σ-field.

6. Suppose $\Omega = \{1,2,3,4,5\}$, $A = \{1,2,3\}$ and $B = \{1,2\}$, find the minimum σ-field which contains A and B.

7. Find a probability function which can be defined on the above σ-field. Find the most general probability function which can be defined on the above σ-field.

8. Let $\Omega = (0,\infty)$, and $\mathcal{F}$ is the class of all subsets of Ω. Let $p_i \geq 0$, for $i = 1,2,3\ldots$, and $\sum_{i=1}^{\infty} p_i = 1$. Let us define a set function $P(A) = \sum_{i \in A} p_i$, for any $A \subset (0,\infty)$. Show that $P(\cdot)$ is a probability function.

9. Suppose $\Omega = (-\infty,\infty)$, and $\mathcal{F}$ is a σ-field defined on Ω, and it contains all the intervals of the form (a,b), where $-\infty < a < b < \infty$. Show that $\{a\} \in \mathcal{F}$, for any $-\infty < a < \infty$.

10. In the same problem as Question 9, prove that $\mathcal{F}$ contains all the intervals of the form $[a,b]$, where $-\infty < a < b < \infty$.