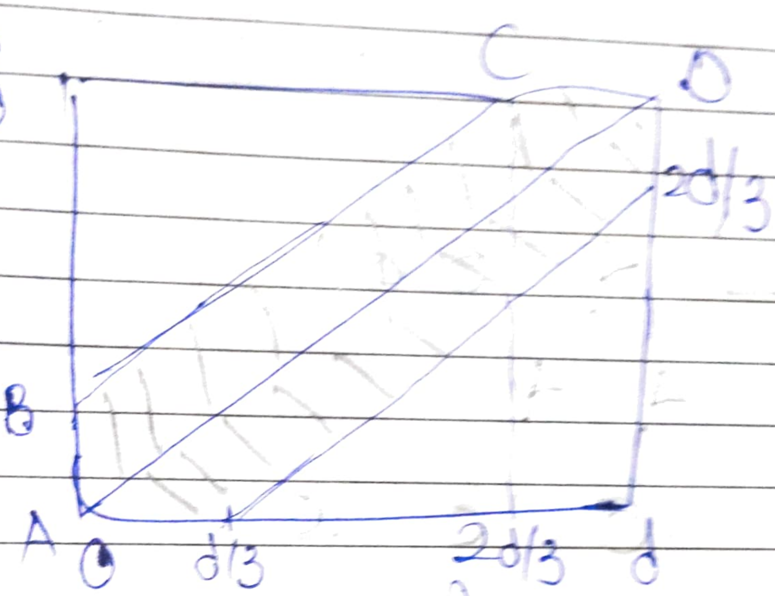
d (Values

$|x-y| \leq \frac{d}{3}$

$x + \frac{d}{3} = y$ $x \geq y \Rightarrow y \geq x$

$x \leq y$

Q6



Values of y ↑

Values of x →

$|x-y| < \frac{d}{3}$

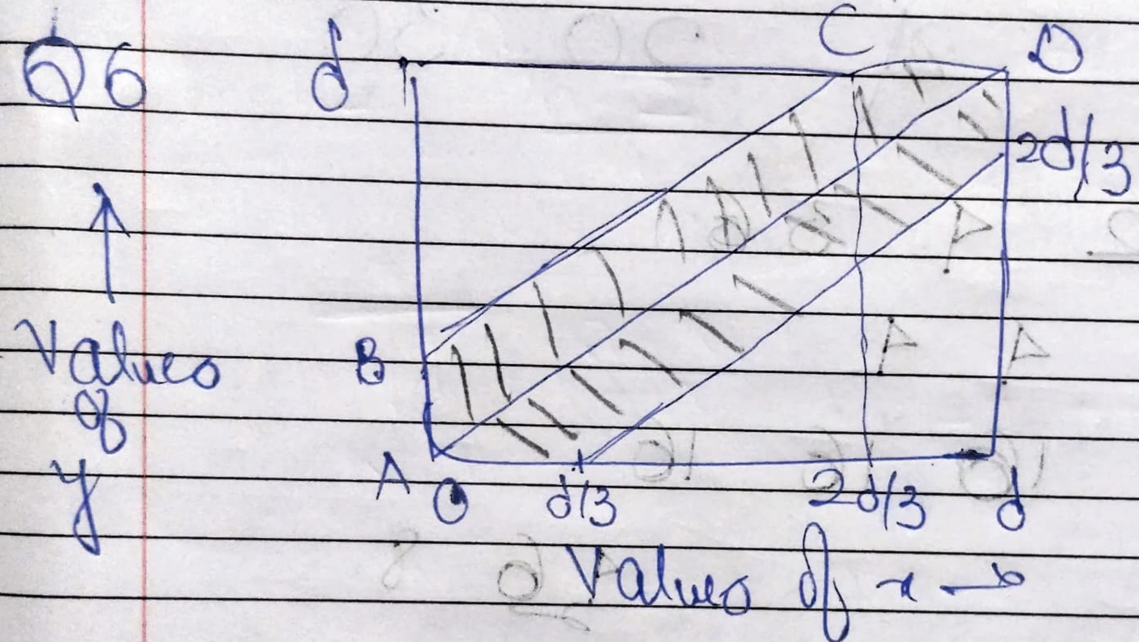
Case I → $x \geq y$

$\Rightarrow$ (i) $y \geq x - \frac{d}{3}$

Case II

$x \leq y$

Q6



$|x-y| < \frac{d}{3}$

$\boxed{\text{Case I}} \rightarrow x \geq y$

$\Rightarrow$ (i) $y \geq x - \frac{d}{3}$

$\boxed{\text{Case II}} \rightarrow y \geq x$

$x \geq y - \frac{d}{3}$

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Shaded area = $2 \times ar(ABCD)$

$$= 2 \times \frac{1}{2} \times \left(\frac{d}{3} \times \frac{1}{\sqrt{2}}\right)\left(\sqrt{2}d - 2 \times \frac{d}{3\sqrt{2}}\right)$$

$$= \frac{5}{9} d^2$$

$\therefore$ ar(Rectangle) = d^2

$\boxed{\text{Probability} = \frac{5}{9}}$

Q7. The current holder will choose from n remaining ~~habitant~~ inhabitants.

[Case-I]

(a) ~~For the first~~

At a particular time the ~~for~~ person who wants to tell the rumour will not choose himself and the originator.

$\therefore$ P[above written line happens] = $\frac{n-1}{n}$

$\therefore$ For that to happen "r" time = $\left(\frac{n-1}{n}\right)^r$ [$\because$ they are independent]

(b) $\boxed{\text{Start}}$

Originator $\longrightarrow$ Person 1

n people remaining

n-1 people remaining

~~Pf~~

E_x :- Event that person "x" will tell the rumor to someone who doesn't know it.

$$P(E_1) = \frac{n-1}{n}$$

$$P(E_2) = \frac{n-2}{n}$$

$$\boxed{P(\text{no repeat in first } r \text{ steps}) = \prod_{k=1}^{r} \left(\frac{n-k}{n}\right)}$$

$\because$ Because events are independent

(Case-II)

(a) Probability that originator is not there in group of "N" people $= \dfrac{{}^{n-1}C_N}{{}^{n}C_N}$

Probability $= \dfrac{\dfrac{(n-1)!}{N!\,(n-1-N)!}}{\dfrac{n!}{N!\,(n-N)!}}$

$$= \frac{n-N}{n}$$

$$\boxed{P\left(\begin{array}{c}\text{no return} \\ \text{in } r \text{ steps}\end{array}\right) = \left(\frac{n-N}{n}\right)^{r}}$$

(b) Carrying the same notations from [case-I] part (b)

$$P(E_1) = {}^{n-1}C_N$$

$$P(E_2) = {}^{n-1-N}C_N$$

$$P(E_3) = {}^{n-1-2N}C_N$$

$$\therefore \boxed{\text{Total probability} = \prod_{k=1}^{r} {}^{n-1-(k-1)N}C_N}$$

Q8. As A_1, A_2 etc. are independent.

$$P\left(\bigcap_{i=1}^{n} A_i^c\right) = \prod_{i=1}^{n} P(A_i^c)$$

Their complements will also be independent.

$$\cancel{A' \cap B' = 1 - (A \cup B)}$$

$$\cancel{= 1 - A - B + A \cap B}$$

$$\boxed{\begin{aligned} P(A' \cap B') &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)\,P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A').\,P(B') \end{aligned}}$$

$$\therefore \quad P\left(\bigcap_{i=1}^{n} A_i^c\right) = \prod_{i=1}^{n} P(A_i^c) = \prod_{i=1}^{n} [1 - P(A_i)]$$

We know for real x: $1 - x \leq e^{-x}$

$$\therefore \quad \boxed{P\left(\bigcap_{i=1}^{n} A_i^c\right) \leq \prod_{i=1}^{n} e^{-\sum_{i=1}^{n} P(A_i)}}$$

Q5 $M = \max\{draw_1, draw_2, \dots, draw_n\}$

For $k = 1, 2, \dots, N$

$$P(M \le k) = \left(\frac{k}{N}\right)^n$$

$$P(M = k) = P(M \le k) - P(M \le k-1)$$

$$= \left(\frac{k}{N}\right)^n - \left(\frac{k-1}{N}\right)^n$$

$$E(M) = \sum_{k=1}^{N} P(M \ge k) = \sum_{k=1}^{N} \left[1 - P(M \le k-1)\right]$$

$$= \sum_{k=1}^{N} \left[1 - \left(\frac{k-1}{N}\right)^n\right]$$

$$\therefore \quad \boxed{E(M) = N - \frac{1}{N^n} \sum_{k=1}^{N} (k-1)^n}$$