

Q2 Win $\rightarrow W$
Loss $\rightarrow L$

Transition matrix: $\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$

Let stationary distribution be

$$\pi = [\pi_W, \pi_L]$$

Solving $\pi P = \pi$ gives:

$$\pi_W = 0.6 \quad \text{and} \quad \pi_L = 0.4$$

- (a) Long run proportion of wins = 0.6
(b) Proportion of dinners:

$$0.6 \times 0.7 + 0.4 \times 0.2 = 0.5$$

(c) $E[\text{no. of games for dinner}] = \frac{1}{0.5} = 2$

Q3

(a)

$$\text{Cat's transition matrix} = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

$$\pi_{\text{cat}} = [0.5 \quad 0.5]$$

Mouse's

transition

matrix

$$= \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$

$$\pi_{\text{mouse}} = \left[\frac{2}{3}, \frac{1}{3} \right]$$

(b) Since cat and mouse move independently

$Z_n = (\text{cat}, \text{mouse})$
is a Markov chain in 4th stage