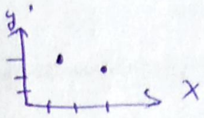
unit 2

Point representation:-

The Point representation consist of definition of a specific variety of Parameters such as the number of bits for each component, the base of exponent, the range and representation of the significant and of the exponent.

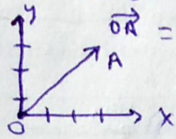
Example



Vector representation:-

vector are geometric representation of magnitude and direction which are often represented by straight arrow, starting at one point on a Coordinate axis and ending at a different point.

$\vec{OA} = (2,3)$.



Matrices and its operation

Matrix is a set of numbers arranged in rows and columns so as to form a rectangular array.

matrices order $m \times n$.

$$A = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix}.$$

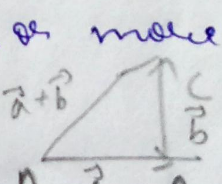
matrices operation → Addition, Subtraction, multiplication.

Vector Addition

vector addition means putting two or more vector together.

$\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$

$\vec{a} + \vec{b} = \vec{AB} + \vec{BC} = \vec{AC}$



subtraction $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

Multiplication

product of vectors

① Scalar or dot product

② vector or cross product $\times$.

Let $\vec{a}$ and $\vec{b}$ are two vector then

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\, \hat{n}$

Scalar Multiple

If $\vec{a}$ is a vector and λ is a scalar, then $\lambda\vec{a}$ is a vector.

Scalar product -

when $\vec{a}$ and $\vec{b}$ are in component form.

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$
$$\vec{a}\cdot\vec{b} = a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2$$

Vector product

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{j} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

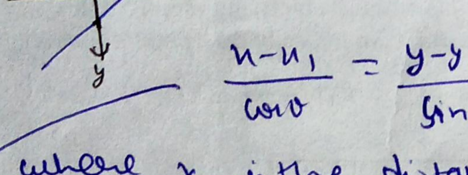
Example $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b} = -j + 2\hat{j} + \hat{k}$

$$\vec{a}\cdot\vec{b} = (2\times -1) + (1\times 2) + 3\times 1$$
$$= -2 + 2 + 3 = 3$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ -1 & 2 & 1 \end{vmatrix}$$
$$= \hat{j}(1-6) - \hat{j}(2+3) + \hat{k}(4+1)$$
$$= -5\hat{j} - 5\hat{j} + 5\hat{k}$$

Equation of line Parametric / distance form



$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

where r is the distance of point (x_1, y_1) and (x, y)

$x - x_1 = r\cos\theta$

$\boxed{x = x_1 + r\cos\theta}$

$\frac{y - y_1}{\sin\theta} = r$

$y - y_1 = r\sin\theta$

$\boxed{y = y_1 + r\sin\theta}$

Q A line passes through the point (3, 4) and make angle 60° along x-axis. find the coordinates of line which lies on a distance 5 unit from given point.

$(3, 4)$, $\theta = 60°$, $r = 5$ find :- $x = ?$
x_1, y_1 $y = ?$

Equation of line in distance / parametric form

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

$$\frac{x - 3}{\cos 60°} = \frac{y - 4}{\sin 60°} = 5$$

$x - 3 = 5\cos 60°$

$x = \frac{5}{2} + 3$

$\boxed{x = \frac{11}{2}}$

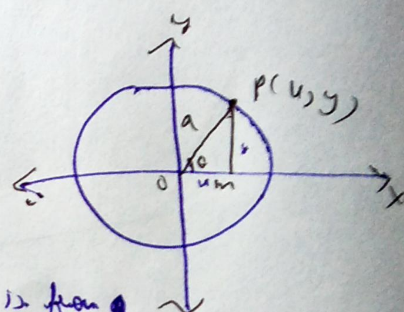
$y - 4 = 5\sin 60$

$\boxed{y = \frac{5\sqrt{3}}{2} + 4}$

∴ point is $\left(\frac{11}{2}, \frac{5\sqrt{3}}{2} + 4\right)$.

Parametric form of conics

(i) Parametric form of circle,
$x^2 + y^2 = a^2$.



Join OP and let make an angle θ with x-axis
draw PM perpendicular to x-axis from P
$x = OM = a\cos\theta$

$y = MP = a\sin\theta$

Thus coordinates of any point on the given circle $(a\cos\theta, a\sin\theta)$.

then $\frac{x}{a} = \cos\theta$, —(1) $\frac{y}{a} = \sin\theta$ —(2)

squaring and adding (1) and (2)

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = \cos^2\theta + \sin\theta = 1$$

$\boxed{x^2 + y^2 = a^2}$ the equation to circle with centre (0, 0) and radius a unit.

• circle equation with centre.

$x^2+y^2+2gx+2fy+c=0$ centre $(-g, -f)$

Radius $(a) = \sqrt{g^2+f^2-c}$.

Intercept on $x = 2\sqrt{g^2-c}$

Intercept on $y = 2\sqrt{f^2-c}$

(ii) Parametric form of Parabola $y^2 = 4ax$

focus $x = a$

Directrix $x = -A$

axis x-axis i.e $y=0$

vertex $\overset{x\ y}{(0, 0)}$

Length of lactum Rectum $= 4a$

Parametric coordinate

$(x = at^2, \ y = 2at)$

Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b} = 1.$ $(a > B)$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

focus $(\pm ae, 0)$

Directix $x = \pm \frac{a}{e}$

distance b/w focus $= 2ae$

Distance b/w directrix $= \frac{2a}{e}$

length of major axis $= 2a$

length of minor axis $= 2b$

Parametric for

$x(a\cos\theta, b\sin\theta)$

Length of latus Rectum $= \frac{2b^2}{a}$

• parametric of hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

centre $(0,0)$

foci $(\pm ae, 0)$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Length of latus rectum $= \frac{2b^2}{a}$

Parametric coordinates

$(a\sec\theta, b\tan\theta)$.