...he probability distribution of the number of correct ...ers (X) is given in the adjoining Table.

TABLE 14·1 : PROBABILITY DISTRIBUTION OF CORRECT ANSWERS

x	0	1	2	3
$p(x)$	8/27	12/27	6/27	1/27

...xample 14·4. *Suppose that a Central University has to form a committee of 5 members from a list of ...ndidates out of whom 12 are teachers and 8 are students. If the members of the committee are selected ...ndom, what is the probability that the majority of the committee members are students ?*

[*Delhi Univ. B.A. (Econ. Hons.), 2009*]

...olution. In the usual notations we have : $n = 5$;

$$p = \text{Probability of selecting a student member} = \frac{8}{20} = \frac{2}{5}$$

$$q = \text{Probability of selecting a teacher member} = \frac{12}{20} = \frac{3}{5}$$

Let X denote the number of students selected in the committee. Then $X \sim B(n = 5, p = 2/5)$. Hence, by ...mial probability distribution,

$$P(X = r) = p(r) = \binom{5}{r}\left(\frac{2}{5}\right)^r\left(\frac{3}{5}\right)^{5-r} \quad ; \quad r = 0, 1, 2, 3, 4, 5 \qquad \ldots(1)$$

The required probability is given by :

$$P(X \geq 3) = p(3) + p(4) + p(5) = \binom{5}{3}\left(\frac{2}{5}\right)^3 \cdot \left(\frac{3}{5}\right)^2 + \binom{5}{4}\left(\frac{2}{5}\right)^4 \cdot \left(\frac{3}{5}\right) + \binom{5}{5}\left(\frac{2}{5}\right)^5$$

$$= \frac{1}{5^5}\left[10 \times 8 \times 9 + 5 \times 16 \times 3 + 1 \times 32\right] = \frac{720 + 240 + 32}{3125} = \frac{992}{3125} = 0.3174$$

Example 14·5. *The number of tosses of a coin that are needed so that the probability of getting at least ...head being 0·875 is*

(*i*) 2, (*ii*) 3, (*iii*) 4, (*iv*) 5. [*I.C.W.A. (Intermediate), Dec. 2001*]

Solution. Let the required number of tosses of the coin be n. Then

$$P[\text{At least one head in } n \text{ tosses of a coin}] = 1 - P[\text{ No head in } n \text{ tosses of a coin}] = 1 - \left(\frac{1}{2}\right)^n$$

We want n so that this probability is 0·875.

$$\therefore \quad 1 - \left(\frac{1}{2}\right)^n = 0{\cdot}875 \quad \Rightarrow \quad \left(\frac{1}{2}\right)^n = 1 - 0{\cdot}875 = 0{\cdot}125 = (0{\cdot}5)^3 = \left(\frac{1}{2}\right)^3 \quad \Rightarrow \quad n = 3$$

$\therefore$ (*ii*) is the correct answer.

Example 14·6. (*a*) *Find the probability of getting the sum 7 on at least 1 of 3 tosses of a pair of fair ...e.*

(*b*) *How many tosses are needed in order that the probability in (a) is greater than 0.95.*

[*Delhi Univ., B.A. (Econ. Hons.), 2009*]

Solution. (*a*) Let p be the probability of getting the sum of 7 in toss of a pair of fair dice. Then

$$p = \frac{6}{36} = \frac{1}{6} \quad \Rightarrow \quad q = 1 - p = \frac{5}{6}$$

[Exhaustive cases $= 6^2 = 36$; Favourable cases $\{(1, 6), (6, 1) (2, 5) (5, 2), (3, 4), (4, 3)\}$ *i.e.* **six.**]

Let the r.v. X denote the number of times 7 is obtained in 3 tosses of a pair of dice Then

$$X \sim B\left(n = 3, \ p = \frac{1}{6}\right); \text{ so that} \qquad \ldots(i)$$

$$P(X = r) = \binom{3}{r} \cdot \left(\frac{1}{6}\right)^r \cdot \left(\frac{5}{6}\right)^{3-r} \quad ; \quad r = 0, 1, 2, 3$$

Example 14·9. *In a binomial distribution with 6 independent trials, the probability of 3 and 4 successes is found to be 0·2457 and 0·0819 respectively. Find the parameters p and q of the binomial distribution.* [Delhi Univ. B.Com. (Hons.), 2002; 1998]

Solution. Let $X \sim B(n = 6, p)$ where X denotes the number of successes. Then, by binomial probability the probability of r successes is given by :

$$p(r) = P(X = r) = {}^6C_r\, p^r q^{6-r}\,; \quad r = 0, 1, 2, \ldots, 6; \quad (q = 1 - p). \qquad \ldots(*)$$

Putting $r = 3$ and 4 in (*), we get respectively :

$$p(3) = {}^6C_3\, p^3 q^3 = 20\, p^3 q^3 = 0{\cdot}2457 \text{ (Given)} \qquad \ldots(**)$$

$$p(4) = {}^6C_4\, p^4 q^2 = 15\, p^4 q^2 = 0{\cdot}0819 \text{ (Given)} \qquad \ldots(***)$$

$$\left[\because\ {}^6C_3 = \frac{6 \times 5 \times 4}{3\,!} = 20\ ;\ {}^6C_4 = {}^6C_2 = \frac{6 \times 5}{2} = 15\right]$$

Dividing (***) by (**), we get :

$$\frac{p(4)}{p(3)} = \frac{15p^4 q^2}{20\, p^3 q^3} = \frac{0{\cdot}0819}{0{\cdot}2457} = \frac{1}{3} \quad \Rightarrow \quad \frac{3}{4} \cdot \frac{p}{q} = \frac{1}{3}$$

$$9p = 4q = 4(1 - p) \quad \Rightarrow \quad 13p = 4 \quad \Rightarrow \quad p = \frac{4}{13}$$

$$\therefore \quad q = 1 - p = 1 - \frac{4}{13} = \frac{9}{13}$$

Example 14·10. (*a*) *Comment on the following :*

For a binomial distribution, mean = 7 and variance = 11. [Delhi Univ. B.Com. (Hons.), 2009]

(*b*) *A binomial variable on 100 trials has 6 as its standard deviation. This statement is :*
(*i*) *valid,* (*ii*) *invalid* (*iii*) *cannot say.* *Choose the correct alternative.*
[I.C.W.A. (Intermediate), June 1999]

Solution. (*a*) For a binomial distribution with parameters n and p.

$$\text{Mean} = np = 7 \quad \ldots(i) \quad ; \quad \text{Variance} = npq = 11 \quad \ldots(ii)$$

Dividing (*ii*) by (*i*), we get : $q = \frac{11}{7} = 1{\cdot}6$, which is impossible, since q being the probability, must lie between 0 and 1. Hence, the given statement is wrong.

(*b*) We are given : $X \sim B(n, p)$, where $n = 100$
and $s.d.\ (\sigma) = 6 \Rightarrow \text{Variance } (\sigma^2) = 36.$

We know that if $X \sim B(n, p)$, then the maximum value of variance (X) is $n/4$. *i.e.*,

$$\text{Var}(X) \leq \frac{n}{4} = \frac{100}{4} = 25. \qquad \text{But, we are given Var}(X) = 36.$$

Hence, the given statement is invalid *i.e.*, (*ii*) is the correct answer.

Example 14·11. *If the probability of a defective bolt is 1/10, find* (*i*) *the mean* ; (*ii*) *variance* ; (*iii*) *moment coefficient of skewness;* (*iv*) *kurtosis, for the distribution of defective bolts in a total of 400.* [Delhi Univ. B.Com. (Hon.), 2005]

Solution. In the usual notations, we have : $n = 400, \quad p = \frac{1}{10} = 0{\cdot}1, \quad q = 1 - p = 0{\cdot}9$

According to Binomial probability law :

(*i*) Mean $= np = 400 \times 0{\cdot}1 = 40$; (*ii*) Variance $= npq = 400 \times 0{\cdot}1 \times 0{\cdot}9 = 36.$

(*iii*) The moment coefficient of skewness

$$\beta_1 = \frac{(q-p)^2}{npq} = \frac{(0{\cdot}8)^2}{36} = \frac{0{\cdot}64}{36} = 0{\cdot}01777 \simeq 0{\cdot}018 \quad \Rightarrow \quad \gamma_1 = +\sqrt{\beta_1} = \frac{q-p}{\sqrt{npq}} = \sqrt{0{\cdot}018} = 0{\cdot}134$$

(*iv*) Coefficient of kurtosis is given by :

$$\beta_2 = 3 + \frac{1 - 6pq}{npq} = 3 + \frac{1 - 6 \times 0{\cdot}1 \times 0{\cdot}9}{36} = 3 + \frac{0{\cdot}46}{36} = 3 + 0{\cdot}013 = 3{\cdot}013 \quad \Rightarrow \quad \gamma_2 = \beta_2 - 3 = 0{\cdot}013$$

4. (a) Define a binomial variate with parameters n and p and obtain ... the binomial distribution in terms of the number of trials and the probability of success.
(b) Obtain an expression for the mean of the binomial probability distribution, and hence find β_1 and
(c) Obtain the first four moments about mean for the binomial probability distribution, and hence find β_1 and β_2. Prove that as $n \to \infty$, $\beta_1 \to 0$ and $\beta_2 \to 3$.

5. (a) Obtain the expression for the mean and variance of a binomial distribution with parameters n and p. Hence show that for the binomial distribution, variance is less than mean.
(b) Obtain the variance of a binomial distribution $B(n, p)$. What is its upper bound ?

6. (a) What is binomial distribution ? State its important properties.
(b) Enumerate some real life situations where binomial distribution is applicable.

7. "A binomial distribution need not necessarily be a symmetrical distribution." Do you agree with the statement ? Give reasons.

8. 12% of the items produced by a machine are defective. What is the probability that out of a random sample of 20 items produced by the machine, 5 are defective ? (Simplification is not necessary).

Ans. $^{20}C_5 \,.\, (0{\cdot}12)^5 \,.\, (0{\cdot}88)^{15}$.

9. The average number of defective pieces, in the manufacturing of an article, is 1 in 10. Find the probability of getting exactly 3 defective articles in a packet of 10 articles selected at random. [*Delhi Univ. B.A. (Econ. Hons.), 2000*]

Ans. $^{10}C_3\ (0{\cdot}1)^3\ (0{\cdot}9)^7$.

10. The probability that a student will graduate is 0·4. Determine the probability that out of 5 students : (i) none; (ii) 1; (iii) at least 1 ; and (iv) all, will graduate. [*Delhi Univ. B.Com (Hons.), 1997*]

Ans. (i) 0·07776 (ii) 0·2592 (iii) 0·92224 (iv) 0·01024

11. Suppose that the probability is $\frac{1}{2}$ that a car stolen in Delhi will be recovered. Find the probability that at least one out of 20 cars stolen in the city on a particular day will be recovered. [*Delhi Univ. B.A. (Econ. Hons.), 2002*]

Ans. $1 - \left(\frac{1}{2}\right)^{20}$.

12. It is observed that 80% of television viewers watch "Aap Ki Adalat" programme. What is probability that at least 80% of the viewers in a random sample of five, watch this programme ? [*I.C.W.A. (Intermediate), Dec. 1996*]

Ans and Hint. Required Probability = $P(X \geq 80\ \%$ of $5) = P(X \geq 4) = 0{\cdot}7373$; $X \sim B\,(n = 5, p = 0{\cdot}8)$

13. If the probability of male birth is 0·5, then the probability that in a family of 4 children there will be at least 1 boy, is

(i) $\frac{4}{16}$, (ii) $\frac{4}{16}$, (iii) $\frac{11}{16}$, (iv) $\frac{15}{16}$.

[*I.C.W.A. (Intermediate), June 19*

Ans. (iv).

14. The merchant's file of 20 accounts contains 6 delinquent and 14 non-delinquent accounts. An auditor randomly selects 5 of these accounts for examination.

(i) What is the probability that the auditor finds exactly 2 delinquent cases ?
(ii) Find the expected number of delinquent accounts in the sample selected.

Ans. (i) $^5C_2\ (0{\cdot}3)^2\,(0{\cdot}7)^3 = 0{\cdot}3087$ (iii) $np = 5 \times 0{\cdot}3 = 1{\cdot}5$

15. An oil exploration firm finds that 5% of the test wells it drills, yield a deposit of natural gas. If the firm dri... wells, what is the probability that

(i) exactly 2 wells, (ii) at least one well ; yield gas ? [*I.C.W.A. (Intermediate), June*

Ans. (i) 0·0305 (ii) $1 - (0{\cdot}95)^6 = 0{\cdot}2649$.

16. 20% of the bolts produced by a machine are defective. Obtain the probability distribution of the numb... efectives in a sample of 5 bolts chosen at random.

Ans. $p(x) = {}^5C_x \cdot (1/5)^x\ (4/5)^{5-x}$; $x = 0, 1, 2, 3, 4, 5$.

17. Four coins are tossed simultaneously. What is the probability of getting (i) 2 heads and 2 tails, (ii) at least two heads, and (iii) at least one head.

Ans. (i) $\frac{3}{8}$, (ii) $\frac{11}{16}$, (iii) $\frac{15}{16}$.

Example 14·26. *Fit a Poisson distribution to the following data and calculate the theoretical frequencies.*

x	:	0	1	2	3	4
f	:	123	59	14	3	1

Solution.

x	0	1	2	3	4	
f	123	59	14	3	1	$\Sigma f = 200$
fx	0	59	28	9	4	$\Sigma fx = 100$

$\therefore \quad \bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{100}{200} = 0{\cdot}5$

Thus, the mean (m) of the theoretical (Poisson) distribution is $m = \bar{x} = 0{\cdot}5$. By Poisson probability law, the theoretical frequencies are given by :

$$f(r) = Np(r) = 200.\frac{e^{-m}\, m^r}{r\,!} \quad ; \quad r = 0, 1, 2, 3, \ldots$$

$$\therefore \quad f(0) = N\,p(0) = 200 \times e^{-m} = 200 \times e^{-0.5} = 200 \times 0{\cdot}6065 = 121{\cdot}3.$$

TABLE 14·7 : **COMPUTATION OF EXPECTED FREQUENCIES**

x	*Expected Poisson Frequencies N.p(x)*			
0	$N\,p(0) = 121{\cdot}3$			$\simeq 121$
1	$N\,p(1) = N\,p(0) \times m$	$= 121{\cdot}3 \times 0{\cdot}5$	$= 60{\cdot}65$	$\simeq 61$
2	$N\,p(2) = N\,p(1) \times \dfrac{m}{2}$	$= \dfrac{60{\cdot}65 \times 0{\cdot}5}{2}$	$= 15{\cdot}3125$	$\simeq 15$
3	$N\,p(3) = N\,p(2) \times \dfrac{m}{3}$	$= \dfrac{15{\cdot}3125 \times 0{\cdot}5}{3}$	$= 2{\cdot}552$	$\simeq 3$
4	$N\,p(4) = N\,p(3) \times \dfrac{m}{4}$	$= \dfrac{2{\cdot}552 \times 0{\cdot}5}{4}$	$= 0{\cdot}32$	$\simeq 0$
Total				200

Example 14·27. *A systematic sample of 100 pages was taken from the Concise Oxford Dictionary and the observed frequency distribution of foreign words per page was found to be as follows :*

No. of foreign words per page (X)	:	0	1	2	3	4	5	6
Frequency (f)	:	48	27	12	7	4	1	1

Calculate the expected frequencies using Poisson distribution. Also compute the mean and variance of fitted distribution.

Solution.

TABLE 14·8 : **FITTING OF POISSON DISTRIBUTION**

x	f	fx
0	48	0
1	27	27
2	12	24
3	7	21
4	4	16
5	1	5
6	1	6
	$\Sigma f = 100$	$\Sigma fx = 99$

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{99}{100} = 0{\cdot}99$$

If the above distribution is approximated by a Poisson distribution, then the parameter (m) of Poisson distribution is given by $m = \bar{x} = 0{\cdot}99$ and by Poisson probability law, the frequency (number) of pages containing r foreign words is given by :

$$f(r) = Np(r) = N.P(X = r) = 100 \times \frac{e^{-0.99}\,(0{\cdot}99)^r}{}$$

Example 14·31. *The hourly wages of 1,000 workmen are normally distributed around a mean of Rs. 70 with a standard deviation of Rs. 5. Estimate the number of workers whose houly wages will be :*

(i) between Rs. 69 and Rs. 72 (ii) more than Rs. 75 ; (iii) less than Rs. 63.

(iv) Also estimate the lowest hourly wages of the 100 highest paid workers.

Solution. Let the random variable X denote the hourly wages in Rupees. Then X is a normal variable with mean $\mu = 70$ and $\sigma = 5$. The standard normal variable corresponding to X is

$$Z = \frac{X-\mu}{\sigma} = \frac{X-70}{5}$$

X	63	69	72	75
$Z = \frac{X-70}{5}$	$\frac{63-70}{5} = -1\cdot4$	$-0\cdot2$	$0\cdot4$	1

... (*)

(i) $P(69 < X < 72) = P(-0\cdot2 < Z < 0\cdot4)$ [From (*)]

$= P(-0\cdot2 < Z < 0) + P(0 < Z < 0\cdot4)$

$= P(0 < Z < 0\cdot2) + P(0 < Z < 0\cdot4)$ (By symmetry)

$= 0\cdot0793 + 0\cdot1554 = 0\cdot2347$

Hence, the required number of workers is : $1000 \times 0\cdot2347 = 234\cdot7 \simeq 235$.

(ii) We want $P(X > 75)$.

$\therefore$ $P(X > 75) = P(Z > 1)$ [From (*)]

$= 0\cdot5 - P(0 < Z < 1)$ [From Fig. 14·11]

$= 0\cdot5 - 0\cdot3413 = 0\cdot1587$

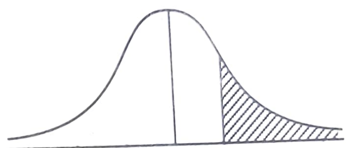

Fig. 14·11

Thus, the number of workers with hourly wages more than Rs. 75 is :

$1000 \times 0\cdot1587 = 158\cdot7 \simeq 159$

(iii) $P(X < 63) = P(Z < -1\cdot4)$ [From (*)]

$= P(Z > 1\cdot4)$ [By symmetry, Fig. 14·12]

$= 0\cdot5 - P(0 < Z < 1\cdot4)$

$= 0\cdot5 - 0\cdot4192 = 0\cdot0808$.

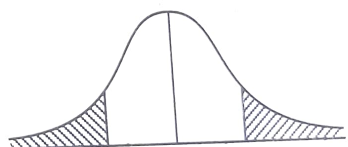

Fig. 14·12

Hence, the number of workers with hourly wages less than Rs. 63 is : $1000 \times 0\cdot0808 = 80\cdot8 \simeq 81$.

(iv) Proportion of the 100 highest paid workers is : $\frac{100}{1000} = \frac{1}{10} = 0\cdot10$

We want to determine $X = x_1$, say, such that $P(X > x_1) = 0\cdot10$ ... (**)

When $X = x_1$, $Z = \frac{x_1 - 70}{5} = z_1$, (say).

Then $P(Z > z_1) = 0\cdot10 \Rightarrow P(0 < Z < z_1) = 0\cdot5 - 0\cdot1 = 0\cdot40$

From the Normal Probability Table VI and (**), we get

$$z_1 = \frac{x_1 - 70}{5} = 1\cdot28 \text{ (approx.)} \Rightarrow x_1 = 70 + 5 \times 1\cdot28 = 70 + 6\cdot40 = 76\cdot40$$

Hence, the lowest hourly wages of the 100 highest paid workers are Rs. 76·40.

Example 14·32. *Time taken by the crew, of a company, to construct a small bridge is a normal variate with mean 400 labour hours and standard deviation of 100 labour hours.*

(i) What is the probability that the bridge gets constructed between 350 to 450 labour hours ?

(ii) If the company promises to construct the bridge in 450 labour hours or less and agrees to pay a penalty of Rs. 100 for each labour hour spent in excess of 450, what is the probability that the company pays a penalty of at least Rs. 2000 ? [Delhi Univ. B.A. (Econ. Hons.), 1998]

Solution. Let X denote the time (in labour hours) to construct the bridge. Then, in the usual notations, we are given : $X \sim N(\mu, \sigma^2)$ where $\mu = 400$ hrs, $\sigma = 100$ hrs.