3

Test of Hypothesis : Small Samples

Introduction—

In the previous chapter, test of hypothesis relating to large samples have been discussed, which was based on two assumptions :—

(i) The sampling distribution of samples is approximately normal.

(ii) Values obtained by the sample data are sufficiently close to the population values.

These assumptions generally do not hold good in case of small samples. Hence, small samples need a different treatment.

Meaning—

By the term 'sampling of variables of small size', we mean selecting samples of less than 30 units from a population which can assume any value within a wider range of a variable like height, weight, income, expenditure, diameter etc.

Characteristics—

The main characteristics of a small sample may be enumerated as follows :—

(i) The size of sample is conventionally limited to 30 at the maximum.

(ii) It is not possible to substitute any statistic such as Mean, S.D. etc. of such a sample for, the respective parameter of the population for determining the standard error of any difference as it is done in case of large samples.

(iii) With the decrease in size of samples, difference in mean values of samples goes on increasing.

(iv) With every decrease in size of samples, standard deviation also goes on decreasing.

(v) The distribution of any statistic like Mean, S.D. etc. of such samples may or may not take the shape of a normal distribution,.

(vi) Such a sampling is preferred in the fields of biology, physical sciences etc., where it is very expensive to collect a data through costly laboratories or field experiments.

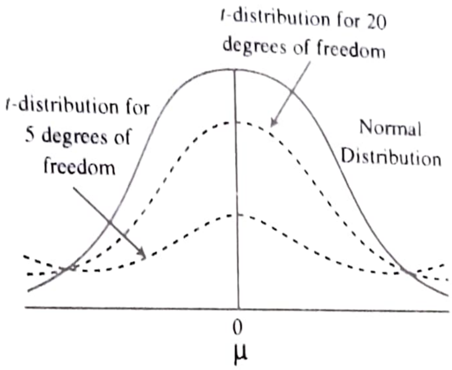
(vii) The test statistic which is extensively used in case of large samples is not applicable in case of small samples. Instead, a special test statistic ***t*** is applied to such samples for testing the significance of differences and various null hypothesis.

Student's *t*-distribution—

If large number of small samples from a population are taken and mean for each sample is calculated and then frequency distribution of these means are plotted, the resulting sampling distribution would be student's *t*-distribution.

A t-distribution is a sampling distribution of small samples only. The theory of such a distribution was developed by ***Sir William S. Gosset*** in 1900 and published in 1905 under his pseudonym, or pen name, 'student', as his employer did not allow him to publish any such research work in his own name. Hence, this distribution is popularly known after his pseudonym as ***'Student's t-distribution'*** or simply t-distribution.

The following diagram illustrates that the t-distribution is similar to a normal curve but a little flatter. This means that 95% limits will lie farther from the mean in the t-distribution than they do in the normal distribution.



Comparison between Normal Curve and Corresponding t-curve

As sample size in sampling distribution of small samples approaches to 30, the t-distribution becomes more and more like the normal curve.

Properties of *t*-distribution—

A t-distribution has the following important properties :—

(i) The value of t-distribution ranges from minus infinity ($-\infty$) to plus infinity ($+\infty$) just as a normal distribution does.

(ii) It gives a normal bell shaped curve which is symmetrical and mono-peaked. However, the difference is that the curve of t-distribution has longer tails with more pointed peak.

(iii) It is higher than the normal distribution at both the tails of the curve but lower in height at the point of the mean (μ).

(iv) Its mean is always zero like that of normal distribution.

(v) The t-distribution has greater variability than normal distribution. As n gets larger (*i.e.* sample size), the t-distribution approaches the normal distribution.

(vi) It has a set of critical values like that of Z, χ^2 and F-distribution etc. These values are referred to, for testing the magnitude of the calculated value of the test statistic t.

(vii) Its techniques can be used for testing the hypothesis not only in small samples, but also in large samples, although, techniques of large samples can not be used in small samples for the purpose.

(viii) The shape of its curve is determined by :—

(a) degree of freedom (d.f.) or

(b) size of the sample (n).

A change in such a parameter brings a change in the shape of curve (as shown above).

Degrees of freedom (d.f.)—

If we refer to t-distribution tables, it will be noticed that t-values are given for various significance levels over a range of **'degrees of freedom'** instead of sample size (n).

For student's t-distribution, the number of degrees of freedom is the sample size minus one (i.e. $(n - 1)$).

Since t distribution is symmetric around $t = 0$, the significance values at 'α' level of significance for a single-tailed test (left or right) can be obtained from the **table of two tailed test** by looking at the value at level of significance 2α. *For example*, tv (0·05) for single-tailed test = tv (0·10) for two-tailed test, and tv (0·01) for single-tailed test = tv (0·02) for two-tailed test.

For example, following are the critical t-values at 5% level of significance from table :—

Sample size (n)	One-tailed	Two-tailed
5	2·02	2·57
10	1·81	2·23
15	1·75	2·13
20	1·73	2·09
25	1·71	2·06
30	1·70	2·04
∞	1·65	1·96

Assumptions for Student's *t*-test—

While dealing with a t-distribution for hypothesis testing, the following assumption are made about the distribution :—

(1) It is assumed that the parent population from which the sample is drawn is normal.

(2) It is assumed that the samples have been drawn at random *i.e.* the given sample is drawn by random sampling method.

(3) It is assumed that the population standard deviation (σ) is not known.

APPLICATION OF t-DISTRIBUTION

The following are some of the examples to explain the areas in which t-distribution is normally used to test the significance of various results obtained from small samples :—

(i) Test of significance of Mean—Small Sample.

(ii) Test of significance of difference between two means—Small Samples.

(iii) Paired t-test for difference of means.

(I) Test of Significance of a Mean—Small Sample—

In case of testing the significance of the mean of a small sample ($n < 30$), steps are same as in case of testing the significance of a mean of large sample. The only difference is the use of t-values, instead of Z-values as test statistic and critical value from t-table according to degree of freedom (d.f.). The following steps are taken in testing the hypothesis of a sample mean in case of small sample :—

Step 1. Null Hypothesis, H_0—

State the null hypothesis in any one of the following forms :—

(i) The population has the specified mean value *i.e.*

$$\mu = \mu_0$$

(ii) These is no significant difference between the population mean (μ) and the sample mean ($\overline{X}$).

(iii) The given sample has been drawn from the normal population with mean (μ_0).

Step 2. Computation of Test Statistic

Compute test statistic for small sample as :—

$$\boxed{t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n-1}}}}$$

where n = Sample size $\overline{X}$ = Sample mean

μ = Population mean s = Sample S.D.

Step 3. Degree of freedom (d.f.)— $n - 1$.

Step 4. Level of significance (α) Usually 5% level of significance ($\alpha = 0{\cdot}05$) is used unless otherwise any other level of significance is specifically stated such as 1%, 2% etc.

Step 5. Critical value—Tabulated or critical value of t for $(n - 1)$ d.f. at certain level of significance is t_α (from t-table).

Step 6. Decision—If computed value of $|t|$ is less than the critical value of $|t|$, it falls in the acceptance region and the null hypothesis is accepted at the pre-determined significance level.

If the computed value of $|t|$ is greater than the critical value of $|t|$, it falls in the rejection region and the null hypothesis is rejected at the per-dertermined significance level.

Illustration 1.

Ten cartons are taken at random from an automatic filling machine. The mean net weight of the 10 cartons is 11·8 kg. and standard deviation is 0·15 kg. Does the sample mean differs significantly from the intended weight of 12 kg. You are given that for $\nu = 9$, $t_{0{\cdot}95} = 2{\cdot}26$.

Solution :

Here, $n = 10$, $\overline{X} = 11{\cdot}8$ kg., $s = 0{\cdot}15$ kg., $\mu = 12$ kg.

Step 1. Null hypothesis—$H_0 : \mu = 12$ *i.e.* the sample mean does not differs significantly from the intended weight of 12 kg.

Altenative hypothesis—$H_1 : \mu \neq 12$ *i.e.* the sample mean differs significantly from the intended weight of 12 kg. (two-tailed test).

Step 2. Test Statistic—Under H_0, the test statistic is :—

$$t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n-1}}} = \frac{11{\cdot}8 - 12}{\sqrt{\frac{(0{\cdot}15)^2}{9}}} = \frac{-0{\cdot}2}{0{\cdot}05} = -4$$

Step 3. Degree of freedom—

$$n - 1 = 10 - 1 = 9$$

Step 4. Level of significance—$\alpha = 0{\cdot}05$ (5%)

Step 5. Critical value—At 5% level of significance and 9 degree of freedom, the critical value of $t = 2{\cdot}26$ (given).

Step 6. Decision—Since the computed value of $|t| = 4$ is greater than the critical value of $|t| = 2{\cdot}26$, it falls in the rejection region. Hence, null hypothesis is rejected and ***it may be concluded that the sample mean differs significantly from the intended weight of* 12 kg**.

Illustration 2.

The mean weekly sale of the Chocolate bar in candy stores was 146·3 bars per store. After an advertising compaign, the mean weekly sales in 22 stores for a typical week increased to 153·7 and showed a standard deviation of 17·2. Was the advertising successful ?

Solution :

Here, $n = 22$, $\overline{X} = 153{\cdot}7$, $s = 17{\cdot}2$, $\mu = 146{\cdot}3$

Step 1. Null hypothesis—$H_0 : \mu = 146{\cdot}3$ *i.e.* there is no significant difference between sample mean and population mean. In other words, advertising was not successful.

Altenative hypothesis—$H_1 : \mu > 146{\cdot}3$ (Right-tailed test) *i.e.* there is a significant difference between sample mean and population mean. In other words, advertising was successful.

Step 2. Test statistic—Under H_0, the test statistic is :—

$$t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n-1}}}$$

$$= \frac{153{\cdot}7 - 146{\cdot}3}{\sqrt{\frac{(17{\cdot}2)^2}{21}}} = \frac{7{\cdot}4}{3{\cdot}75} = 1{\cdot}97$$

Step 3. Degree of freedom—

$$\text{d.f.} = n - 1 = 22 - 1 = 21$$

Step 4. Level of significance—$\alpha = 0{\cdot}05$ (5%)

Step 5. Critical value—At 5% level of significance and 21 d.f., the critical value of t = 1·72 (for right-tailed test).

Step 6. Decision—Since the computed value of $|t| = 1{\cdot}97$ is greater than the critical value of $|t| = 1{\cdot}72$, it falls in the rejection region. Hence, null hypothesis is rejected and ***it may be concluded that advertising was successful***.

Illustration 3.

The quality control department of a food processing firm specified that the mean net weight for package of a certain food must be 20 gm. Experience has shown that the weights are approximately distributed with a standard deviation of 1·5 gm. If a random sample of 15 packages yields a mean weight of 19·5 gm., is this sufficient evidence to indicate that the true mean weight of the package has decreased ? Use 5% significance level.

Solution :

Here, $n = 15$, $\overline{X} = 19{\cdot}5$ gms., $\mu = 20$ gm., $s = 1{\cdot}5$ gm.

Step 1. Null hypothesis—$H_0 : \mu = 20$ gm *i.e.* there is no significant difference between the sample mean and population mean. In other words, the true mean weight of the package has not decreased.

Altenative hypothesis—$H_1 : \mu < 20$ gm. *i.e.* there is a significant difference between the sample mean and population mean. In other words, the true mean weight of the package has decreased. (one-tailed test)

Step 2. Test statistic—Under H_o, the test statistic is :—

$$t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n-1}}}$$

$$= \frac{19 \cdot 5 - 20}{\sqrt{\frac{(1 \cdot 5)^2}{14}}} = \frac{-0 \cdot 5}{0 \cdot 40} = -1 \cdot 25$$

Step 3. Degree of freedom—

$$n - 1 = 15 - 1 = 14$$

Step 4. Level of significance—$\alpha = 0 \cdot 05$ (5%)

Step 5. Critical value—At 5% level of significance and 14 degrees of freedom, the critical value of t is—

$$t = \pm 1 \cdot 76 \text{ (for one-tailed test)}$$

Step 6. Decision—Since the computed value of $|t| = 1 \cdot 25$ is less than the critical value of $|t| = 1 \cdot 76$, it falls in the acceptance region. Hence, null hypothesis is accepted and ***it may be concluded that the true mean weight of the package has not decreased***.

Illustration 4.

The nine items of a sample had the following values :—

45, 47, 50, 52, 48, 47, 49, 53, 51

Does the mean of the nine items differ significantly from the assumed population mean 47·5 ? (Given t_8 (0·05) = 2·31) *[CCS Univ., M.Sc., 2012]*

Solution :

Here, $n = 9$, $\mu = 47 \cdot 5$

COMPUTATION OF MEAN and S.D.

X	Deviation from 49 (dx)	d^2x
45	– 4	16
47	– 2	4
50	1	1
52	3	9
48	– 1	1
47	– 2	4
49	0	0
53	4	16
51	2	4
	$\Sigma dx = 1$	$\Sigma d^2x = 55$

Arithmetic mean ($\overline{X}$)

$$= A + \frac{\Sigma dx}{n}$$

$$= 49 + \frac{1}{9} = 49 \cdot 1$$

Standard deviation (s)

$$= \sqrt{\frac{\Sigma d^2x}{n} - \left(\frac{\Sigma dx}{n}\right)^2}$$

$$= \sqrt{\frac{55}{9} - \left(\frac{1}{9}\right)^2}$$

$$= \sqrt{6 \cdot 11 - 0 \cdot 01} = \sqrt{6 \cdot 10}$$

$$= 2 \cdot 47$$

Step 1. Null hypothesis—$H_o : \mu = 47 \cdot 5$ *i.e.* the mean of nine items does not differ significantly from the assumed population mean of 47·5.

Altenative hypothesis—$H_1 : \mu \neq 47 \cdot 5$ *i.e.* the mean of nine items differ significantly from the assumed population mean of 47·5 (two-tailed test).

0·040 inch. Compute the statistic you would use to test whether the work is meeting the specification. Also state how you would proceed further.

Solution :

Here, $n = 10$, $\overline{X}$ 0·742, $s = 0{\cdot}040$, $\mu = 0{\cdot}700$

Step 1. Null hypothesis—$H_0 : \mu = 0{\cdot}700$ *i.e.* there is no significant difference between the sample mean and population mean. In other words, the work is meeting the specification.

Altenative hypothesis—$H_1 : \mu \neq 0{\cdot}700$ *i.e.* there is a significant difference between the sample mean and population mean. In other words, the work is not meeting the specification.

Step 2. Test statistic—Under H_0, the test statistic is :—

$$t = \frac{\overline{X} - \mu}{\sqrt{\dfrac{s^2}{n-1}}}$$

$$= \frac{0{\cdot}742 - 0{\cdot}700}{\sqrt{\dfrac{(0{\cdot}040)^2}{9}}} = \frac{0{\cdot}042}{0{\cdot}013} = 3{\cdot}23$$

Step 3. Degree of freedom—

$$\text{d.f.} = n - 1 = 10 - 1 = 9$$

Step 4. Level of significance—$\alpha = 0{\cdot}05$ (5%)

Step 5. Critical value—At 5% level of significance and 9 d.f., the critical value of t is—

$$t = \pm\, 2{\cdot}26 \text{ (two-tailed test)}$$

Step 6. Decision—Since the computed value of $|t| = 3{\cdot}23$ is greater than the critical value of $|t| = 2{\cdot}26$, it falls in the rejection region. Hence, null hypothesis is rejected and ***it may be concluded that the work is not meeting the specification.***

Illustration 6.

A sample of size 10 drawn from a normal population has a mean 31 and a variance 2·25. Is it reasonable to assume that the mean of the population 30 ? (Use 1% level of significance, given that $P(|t| > 3{\cdot}25) = 0{\cdot}01$ for 9 d.f.). [*C.A. Foundation, May*, 2002]

Solution :

Here, $n = 10$, $\overline{X} = 31$, $s^2 = 2{\cdot}25$, $\mu = 30$

Step 1. Null hypothesis—$H_0 : \mu = 30$ *i.e.* there is no significant difference between the sample mean and population mean. (Mean of population is 30)

Altenative hypothesis—$H_1 : \mu \neq 30$ *i.e.* there is a significant difference between the sample mean and population mean (two-tailed test). (Mean of population is not 30).

Step 2. Test statistic—Under H_0, the test statistic is :—

$$t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n-1}}}$$

$$= \frac{31 - 30}{\sqrt{\frac{2{\cdot}25}{9}}} = \frac{1}{0{\cdot}5} = 2$$

Step 3. Degree of freedom—

$$\text{d.f.} = n - 1 = 10 - 1 = 9$$

Step 4. Level of significance—$\alpha = 0{\cdot}01$ (1%) given

Step 5. Critical value—Critical value of t at 1% level of significance and 9 d.f. is = 3·25 (given).

Step 6. Decision—Since the computed value of $|t| = 2$ is less than the critical value of $|t| = 3{\cdot}25$, it falls in the acceptance region. Hence, null hypothesis is accepted and ***it may be concluded that there is no significant difference between the sample mean and population mean and mean of population is 30***.

Illustration 7.

A fertilizer mixing machine is set to give 12 kg. of nitrate for every quintal bag of fertilizer. Ten 100 kg. bags are examined. The percentages of nitrate are as follows :—

11, 14, 13, 12, 13, 12, 13, 14, 11, 12

Is there reason to believe that the machine is defective ? Value of t for 9 degrees of freedom is 2·262.

Solution :

Here, $n = 10$, $\mu = 12$

COMPUTATION OF MEAN and S.D.

X	Deviation from 12 (dx)	d^2x
11	– 1	1
14	2	4
13	1	1
12	0	0
13	1	1
12	0	0
13	1	1
14	2	4
11	– 1	1
12	0	0
	$\Sigma dx = 5$	$\Sigma d^2x = 13$

Arithmetic mean ($\overline{X}$)

$$= A + \frac{\Sigma dx}{n}$$

$$= 12 + \frac{5}{10} = 12{\cdot}5$$

Standard deviation (s)

$$= \sqrt{\frac{\Sigma d^2x}{n} - \left(\frac{\Sigma dx}{n}\right)^2}$$

$$= \sqrt{\frac{13}{10} - \left(\frac{5}{10}\right)^2}$$

$$= \sqrt{1{\cdot}3 - 0{\cdot}25}$$

$$= 1{\cdot}02$$

Step 1. Null hypothesis—$H_o : \mu = 3 \cdot 50$ *i.e.* there is no significant difference between the sample mean and population mean. In other words, the true average yield for this kind of alfalfa is 3·50 tonnes per hectare.

Altenative hypothesis—$H_1 : \mu \neq 3 \cdot 50$ *i.e.* there is a significant difference between the sample mean and population mean. In other words, the true average yield for this kind of alfalfa is not 3·50 tonnes per hectare.

Step 2. Test statistic—Under H_o, the test statistic is :—

$$t = \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n-1}}}$$

$$= \frac{3 \cdot 75 - 3 \cdot 50}{\sqrt{\frac{(0 \cdot 99)^2}{5}}} = \frac{0 \cdot 25}{0 \cdot 44} = 0 \cdot 57$$

Step 3. Degree of freedom—

$$\text{d.f.} = n - 1 = 6 - 1 = 5$$

Step 4. Level of significance—$\alpha = 0 \cdot 05$ (5%)

Step 5. Critical value—At 5% level of significance and 5 degrees of freedom, the critical value of $t = 2 \cdot 57$.

Step 6. Decision—Since the computed value of $|t| = 0 \cdot 57$ is less than the critical value of $|t| = 2 \cdot 57$, it falls in the acceptance region. Hence, null hypothesis is accepted and ***it may be concluded that the true average yield for thus kind of alfalfa is 3·50 tonnes per hectare.***

(II) Test of Significance of difference between two means—Small Samples—

In case of testing the significance of difference between two means-small samples ($n < 30$), steps are same as in case of testing the significance of difference between two means–large samples. The only difference is the use of t-values, instead of Z-values as test statistic and critical value from t-table according to degree of freedom (d.f.). The following steps are taken in testing the hypothesis of difference between two means–small samples.

Step 1. Null hypothesis—H_o State the null hypothesis in any one of the following forms :—

(i) Two samples have been drawn from the same parent population.

(ii) Two samples have been drawn from two different parent populations having the same mean or the two population means do not differ significantly *i.e.*

$$H_o : \mu_1 = \mu_2$$

Altenative hypothesis—$H_1 : \mu \neq \mu_2$ (Two-tailed test)

Step 2. Computation of test statistic—Under the assumption that population variances are unknown but equal ($\sigma_1^2 = \sigma_2^2 = \sigma^2$), the test statistic for H_o is—

$$\boxed{t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\hat{s}^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

and $$\hat{s}^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

where $\overline{X}_1$ and $\overline{X}_2$ = Means of two samples

n_1 and n_2 = Number of observations in two samples

s_1^2 and s_2^2 = Variances of two samples

Step 3. Degree of freedom It is determined on the basis of $n_1 + n_2 - 2$.

Step 4. (Level of significance), and **Step 5** (Critical value) will be same as those followed in testing the significance of a mean for small sample.

Step 6. Decision

(i) if the computed value of $|t|$ is less than the critical value of $|t|$, it means that the computed value of t falls in the acceptance region. Hence, null hypothesis is accepted and it may be concluded that two samples have been drawn from the same parent population or there is no significant difference between the means of two populations.

(ii) If the computed value of $|t|$ is greater than the critical value of $|t|$, it means that the computed value of t falls in the rejection region. Hence, null hypothesis is rejected and it may be concluded that ***two samples have been drawn from the different parent population or there is a significant difference between the means of two populations.***

Assumptions—

The above model of the t-statistic has been formulated on the following assumptions :—

(i) Parent populations from which samples are drawn are normally distributed.

(ii) Population variances are not known but equal *i.e.* $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

(iii) The two samples have been drawn independently, and at random.

Illustration 9.

The mean life of a sample of 10 electric bulbs was found to be 1456 hours with standard deviation of 423 hours. A second sample of 17 bulbs chosen from a different batch showed a mean life of 1280 hours with standard deviation of 398 hours. Is there a significant difference between the means of the two batches ?

Solution :

Here, $n_1 = 10$ $\quad n_2 = 17$

$\overline{X}_1 = 1456$ hrs. $\quad \overline{X}_2 = 1280$ hrs.

$s_1 = 423$ hrs. $\quad s_2 = 398$ hrs.

Step 1. Null hypothesis—$H_0 : \mu_1 = \mu_2$ *i.e.* there is no significant difference between the means of two batches.

Alternative hypothesis—$H_1 : \mu_1 \neq \mu_2$ *i.e.* there is a significant difference between the means of two batches.

Step 2. Test statistic—Under H_0, the test statistic is—

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\hat{s}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where,

$$\hat{s}^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{10 \times (423)^2 + 17 \times (398)^2}{10 + 17 - 2}$$

$$= \frac{17,89,290 + 26,92,868}{25}$$

$$= \frac{44,82,158}{25} = 1,79,286\cdot32$$

$$\therefore \quad t = \frac{1456 - 1280}{\sqrt{1,79,286\cdot32\left(\frac{1}{10}+\frac{1}{17}\right)}}$$

$$= \frac{176}{\sqrt{1,79,286\cdot32 \times 0\cdot159}} = \frac{176}{168\cdot84} = 1\cdot04$$

Step 3. Degree of freedom—

$$d.f. = n_1 + n_2 - 2$$

$$= 10 + 17 - 2 = 25$$

Step 4. Level of significance—

$$\alpha = 0\cdot05\ (5\%)$$

Step 5. Critical value—At 5% level of significance and 25 degrees of freedom—

$$t_{25}\,(0\cdot05) = 2\cdot06 \text{ (from table)}$$

Step 6. Decision—Since the computed value of $|t| = 1\cdot04$ is less the critical of $|t| = 2\cdot06$, it falls in the acceptance region. Hence, null hypothesis is accepted and it may be concluded that ***there is no significant difference between the mean life of two batches.***

Illustration 10.

You are given the following data about the life of the two brands of bulbs:—

	Mean life	Standard deviation	Size of sample
Brand A	2000 hrs.	250 hrs.	12
Brand B	2230 hrs.	300 hrs.	15

Do you think there is a significant difference in the two bulbs ?

Solution :

Here, $n_1 = 12 \qquad n_2 = 15$

$\overline{X}_1 = 2000 \qquad \overline{X}_2 = 2230$

$s_1 = 250 \qquad s_2 = 300$

Step 1. Null hypothesis—$H_o : \mu_1 = \mu_2$ *i.e.* there is no significant difference between the mean life of two bulbs.

Alternative hypothesis—$H_1 : \mu_1 \neq \mu_2$ *i.e.* there is a significant difference between the means life of two bulbs.

Step 2. Test statistic—Under H_o, the test statistic is—

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\hat{s}^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where,

$$\hat{s}^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{12 \times (250)^2 + 15 \times (300)^2}{12 + 15 - 2}$$

$$= \frac{7,50,000 + 13,50,000}{25}$$

$$= \frac{21,00,000}{25} = 84,000$$

$$\therefore \quad t = \frac{2,000 - 2,230}{\sqrt{84,000\left(\frac{1}{12} + \frac{1}{15}\right)}} = \frac{-230}{\sqrt{84,000 \times 0.15}}$$

$$= \frac{-230}{112.25} = -\mathbf{2.05}$$

Step 3. Degree of freedom—

$$d.f. = n_1 + n_2 - 2$$
$$= 12 + 15 - 2 = 25$$

Step 4. Level of significance—

$$\alpha = 0.05\ (5\%)$$

Step 5. Critical value—At 5% level of significance and 25 degrees of freedom, the critical value of t is—

$$t_{25}\ (0.05) = 2.06 \text{ (from table)}$$

Step 6. Decision—Since the computed value of $|t| = 2.05$ is less the critical of $|t| = 2.06$, it falls in the acceptance region. Hence, null hypothesis is accepted and it may be concluded that ***there is no significant difference between the mean life of two bulbs.***

Illustration 11.

Two types of batteries X and Y are tested for their length of life and the following results are obtained:—

Battery	Sample size	Mean (hrs.)	Variance (hrs.)
A	10	1000	100
B	12	1020	121

Is there a significant difference in the two means ? (*i.e.* would you say that the two types of batteries are having the same mean life ?

Solution :

Here, $n_1 = 10$ $\quad$ $n_2 = 12$

$\overline{X}_1 = 1000$ $\quad$ $\overline{X}_2 = 1020$

$s_1^2 = 100$ $\quad$ $s_2^2 = 121$

Step 1. Null hypothesis—$H_o : \mu_1 = \mu_2$ *i.e.* there is no significant difference in the two means *i.e.* we can say that the two types of batteries are having the same mean life.

Alternative hypothesis—$H_1 : \mu_1 \neq \mu_2$ *i.e.* there is a significant difference in the two means *i.e.* we can say that the two types of batteries are not having the same mean life.

Step 2. Test statistic—Under H_o, the test statistic is—

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\hat{s}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where,

$$\hat{s}^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{10 \times 100 + 12 \times 121}{10 + 12 - 2}$$

$$= \frac{1,000 + 1,452}{20} = \frac{2,452}{20} = 122.6$$