CHAPTER 5

3D-Transformation

5.1 INTRODUCTION

We have already studied about 2D transformations. In a 3D transformations as seen in 2D, we can have two types of views. In the first case, either the object/picture is moved or manipulated directly with the geometric transformation. In the second case, we try to manipulate the object, by changing viewer's coordinates. We can construct the complex objects/pictures by instant transformation. There are four basic transformations in 3D :

1. 3D Translation.
2. 3D Scaling.
3. 3D Rotation.
4. 3D Reflection.

5.2 3D TRANSLATION

In 3D translation we are changing the position of an object in x, y and z directions with respect to origin. Let $P(x, y, z)$ be the point object, we wish to translate this object point to the new position *i.e.*, $P'(x', y', z')$ as in Fig. 5.1.

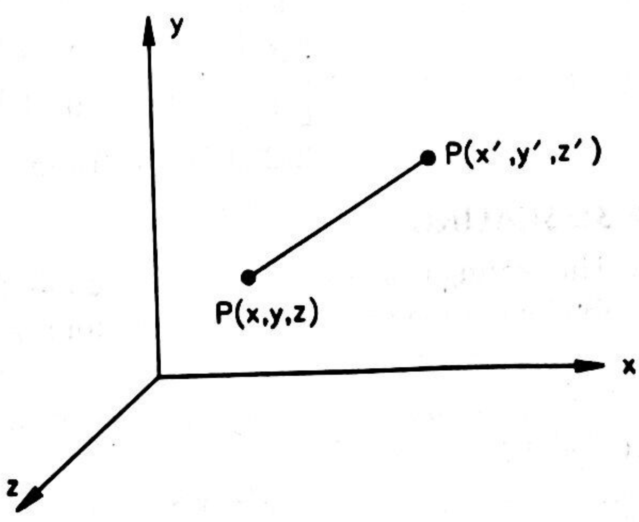

Fig. 5.1.

It is evident from the Fig. that the displacement and direction of the translation is now defined by the vector $v = a\hat{i} + b\hat{j} + c\hat{k}$. Hence we can write new coordinate as—

$$x' = x + tx$$
$$y' = y + ty$$
$$z' = z + tz$$

where tx, ty and tz are translation factors in x, y and z directions.

The matrix representation will be

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Example 5.1 *Translate the pyramid defined by the coordinates A (0, 0, 0), B (1, 0, 0), C (0, 1, 0) and D (0, 0, 1) 2 units each in x, y and z directions.*

Solution. $A(0,0,0)$ $\quad t_x = 2$

$B(1,0,0)$ $\quad t_y = 2$

$C(0,1,0)$ $\quad t_z = 2$

$D(0,0,1)$

$$\text{Object Matrix} = \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$\text{Translation matrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$tx = 2 \qquad ty = 2 \qquad tz = 2$

$$\therefore \quad \text{Translation matrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ $A'(2,2,2)$, $B'(3,2,2)$, $C'(2,3,2)$ and $D'(2,2,3)$

5.3 3D SCALING

The scaling transformation either magnifies or reduces the size of the object depending on the value of scaling factor (If scaling factor is greater than 1 it implies magnifactor and if it is less than 1, it means reduction).

Let $P(x, y, z)$ be the point object. We now scale this object to twice its size 4 the new points be $P(x', y', z')$.

$\therefore$
$$x' = x\,Sx$$
$$y' = y\,Sy$$

$$z' = z\,Sz$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Example 5.2 *Scale the pyramid defined by the coordinates A (0, 0, 0), B (1, 0, 0), C (0, 1, 0) and D (0, 0, 1) 2 units each in x, y and z axis.*

Solution.

$$\text{Object matrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Scale matrix

$S_x = 2 \qquad S_y = 2 \qquad S_z = 2$

$$\therefore \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ $A'(0, 0, 0)$, $B'(2, 0, 0)$, $C'(0, 2, 0)$ and $D'(0, 0, 2)$

5.4 3D ROTATION

As seen in 2D transformation the rotation was prescribed by the angle of rotation and the point of rotation. Since we have now three axis, so the rotation can take place about any one of there axis. Thus we have rotation about x-axis, y-axis and z-axis respectively.

5.4.1 Rotation about *Z*-axis

The rotation about z-axis is defined by the xy plane.

Let $P(x, y, 0)$ be the original point making angle ϕ with respect to origin and $P'(x', y', 0)$ making angle $(\theta + \phi)$ with respect to origin.

$$\left.\begin{aligned} x &= r \cos \phi \\ y &= r \sin \phi \end{aligned}\right\} \quad \ldots (i)$$

and

$$\left.\begin{aligned} x' &= r \cos (\theta + \phi) \\ y' &= r \sin (\theta + \phi) \\ z' &= z \end{aligned}\right\} \quad \ldots (ii)$$

$$\left.\begin{aligned} x' &= r \cos \theta \cos \phi - r \sin \theta \sin \phi \\ y' &= r \sin \theta \cos \phi + r \cos \theta \sin \phi \end{aligned}\right\} \quad \ldots (iii)$$

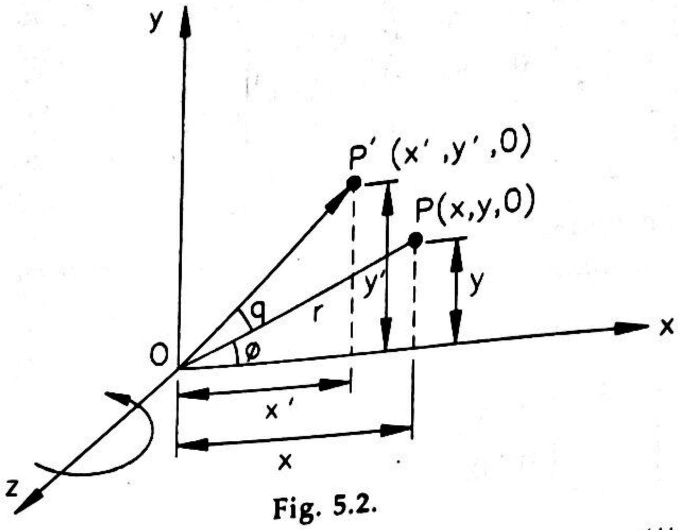

Fig. 5.2.

Substitute the value of $r \cos \phi$ and $r \sin \phi$ from eqn. (*i*) into eqn. (*iii*)

$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

$$z' = 0$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Example 5.3 *The pyramid defined by the coordinates A(0,0, 0), B(1, 0, 0), C(0, 1, 0) and D(0, 0, 1) is rotated 45° with respect to z-axis. Find the coordinates of rotated object.*

Solution. Object Matrix =

$$\begin{array}{cccc} A & B & C & D \end{array}$$
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Rotation Matrix (for $\theta = 45°$) $= \begin{bmatrix} \cos 45° & -\sin 45° & 0 & 0 \\ \sin 45° & \cos 45° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$

$$A' = (0, 0, 0)$$

$$B' = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$$

$$C' = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$$

$$D' = (0, 0, 1)$$

5.4.2 Rotation about *X*-axis

Let $P(o, y, z)$ be the original point, making angle θ. Now rotate it with angle ϕ to the new point *i.e.* $P'(x', y', z')$.

$$\left.\begin{aligned} y &= r\cos\theta \\ z &= r\sin\theta \end{aligned}\right\} \quad \ldots(i)$$

Now rotate the $P(o, y, z)$ to new point $P'(o, y', z')$.

$$x' = x$$

$$\left.\begin{aligned} y' &= r\cos(\theta + \phi) \\ &= r\cos\theta\cos\phi - r\sin\theta\sin\phi \\ z' &= r\sin(\theta + \phi) \\ &= r\sin\theta\cos\phi + r\cos\theta\sin\phi. \end{aligned}\right\} \quad \ldots(ii)$$

Substitute equation (*i*) in (*ii*), we get

$$y' = y\cos\theta - z\sin\theta$$

$$z' = y\sin\theta + z\cos\theta.$$

and $\qquad x' = x.$

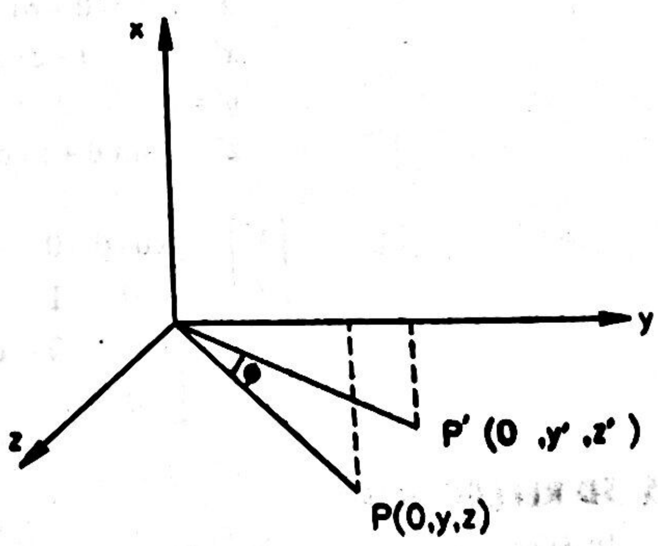

Fig. 5.3

$$\therefore \quad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

5.4.3 Rotation about the Y-axis

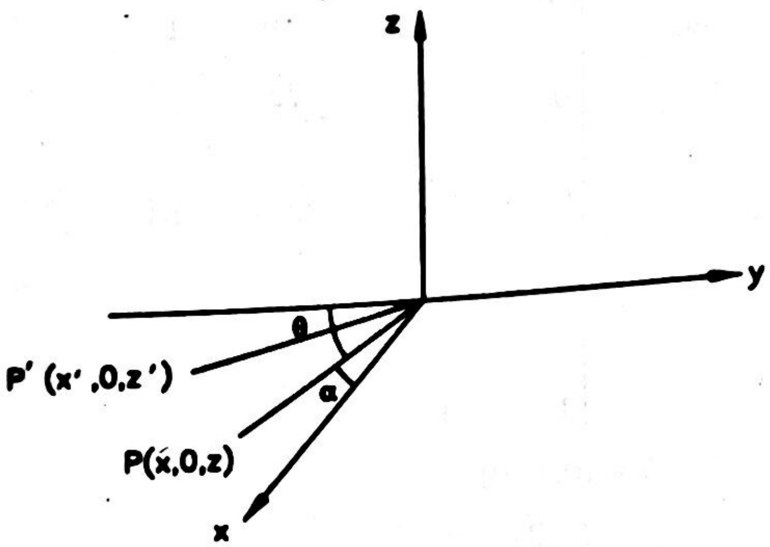

Fig. 5.4

Let $P(x, 0, z)$ be the original point making angle θ. Now rotate it with angle ϕ to get new point $P'(x, 0, z')$.

$$\left.\begin{aligned} x &= r\cos\phi \\ z &= r\sin\phi \end{aligned}\right\} \quad \ldots(i)$$

$$y' = y.$$

$$\left.\begin{aligned} x' &= r\cos(\theta+\phi) \\ z' &= r\sin(\theta+\phi) \end{aligned}\right\} \quad \ldots(ii)$$

$$\left.\begin{aligned} x' &= x\cos\theta - z\sin\theta \\ y' &= y \\ z' &= x\sin\theta + z\cos\theta \end{aligned}\right\} \quad \ldots(iii)$$

$$\therefore \quad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

5.5 3D REFLECTION

In 3*D* reflection we have three cases :

1. Reflection with respect to *xy* plane.
2. Reflection with respect to *yz* plane.
3. Reflection with respect to *xz* plane.

5.5.1 Reflection with Respect to *xy* Plane

Let $P(x, y, z)$ be the original point now reflect the point $P(x, y, z)$ with respect to xy plane.

$$x' = x$$
$$y' = y$$
$$z' = -z$$

$$\therefore \text{ Reflection Matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = \text{Ref. } x, y \,.\, P.$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

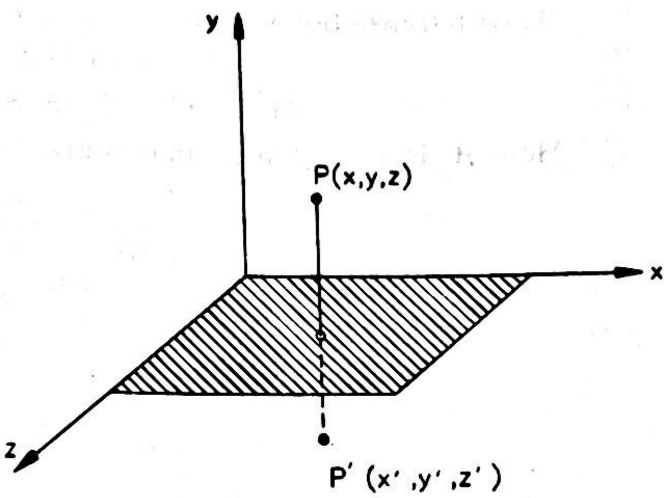

Fig. 5.5

Example 5.4 *The pyramid defined by the coordinates A(0, 0, 0), B(1, 0, 0), C(0, 1, 0) and D(0, 0, 1) is to be reflected about the xy plane. Find the new coordinate values of pyramid.*

Solution. Object Matrix $= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$$\text{Ref. } x, y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\therefore \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad A' = (0, 0, 0)$$
$$B' = (1, 0, 0)$$
$$C' = (0, 1, 0)$$
$$D' = (0, 0, -1)$$

5.6 MICRON REFLECTION WITH RESPECT TO AN ARBITRARY PLANE

Let the plane of reflection be specified by a normal vector N and a reference point $P_0(x_0, y_0, z_0)$. To reduce the reflection to a mirror reflection with respect to the xy plane.

1. Translate P_0 to the origin

2. Align the normal vector N with the vector K normal to the xy plane.
3. Perform the mirror reflection in the xy plane.
4. Reverse steps 1 and 2.

So with translation vector

$$V = -x_o I - y_o J - z_o k$$

$$M_{N, P_o} = T_v^{-1} A_N^{-1} M A_N \cdot T_V$$

Here, A_N is the alignment matrix given as

$$A_N = \begin{bmatrix} \frac{\lambda}{|N|} & \frac{-n_1 n_2}{\lambda |N|} & \frac{-n_1 n_3}{\lambda |N|} & 0 \\ 0 & \frac{n_3}{\lambda} & \frac{-n_2}{\lambda} & 0 \\ \frac{n_1}{|N|} & \frac{n_2}{|N|} & \frac{n_3}{|N|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, if the vector $N = n_1 I + n_2 J + n_3 k$, then

$$(N) = \sqrt{n_1^2 + n_2^2 + n_3^2} \quad \text{and} \quad \lambda = \sqrt{n_2^2 + n_3^2}$$

$$\tau_v = \begin{bmatrix} 1 & 0 & 0 & -x_o \\ 0 & 1 & 0 & -y_o \\ 0 & 0 & 1 & -z_o \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\tau_v^{-1} = \begin{bmatrix} 1 & 0 & 0 & x_o \\ 0 & 1 & 0 & y_o \\ 0 & 0 & 1 & z_o \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

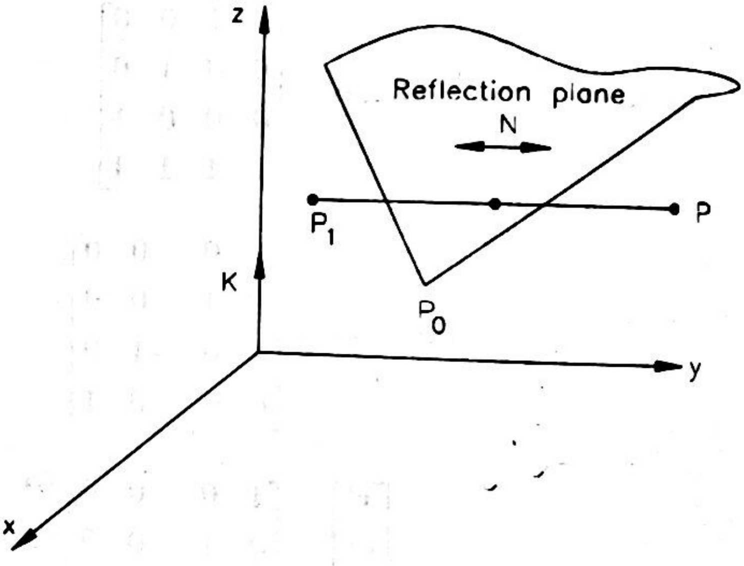

Fig. 5.6

5.7 SHEARING TRANSFORMATION

Shear along any pain of axis is proportional to the third axis. For instances to shear along z-axis in 3D. x and y values are altered by an amount proportional to the value of z, leaving z unchanged. Let $Sh\ zx$, $Sh\ zy$ is the shear due to z along x and y directions respectively and one real values. Then the matrix representation is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ Sh\ zx & Sh\ zy & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Shear for x, y axis is similar to that of z. The general form of shear is given by

$$\begin{bmatrix} 1 & Sh\,xy & Sh\,xz & 0 \\ Sh\,yx & 1 & Sh\,yz & 0 \\ Sh\,zx & Sh\,zy & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SOLVED PROBLEMS

Problem 5.1. *The pyramid defined by the coordinates A(0, 0, 0), B(1, 0, 0), C(0, 1, 0) and D(0, 0, 1) is rotated 45° about the line L that has the direction V = J + k and passing through point C(0, 1, 0). Find the coordinate of the rotated figure.*

Solution. The rotation Matrix $R_{\theta,L}$ can be found by concatenating the matrices

$$R_{\theta,L} = T_{-p}^{-1} = A_v^{-1}\, R\theta, k\; A_v \,.\, T_{-p}$$

with $p = (0, 1, 0)$, then

$$T_{-p} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now $V = J + k$, So $\left.\begin{aligned} a &= 0 \\ b &= 1 \\ c &= 1 \end{aligned}\right\}$ Magnitude of i, J and k.

We find $\lambda = \sqrt{2}$, $|V| = \sqrt{2}$

and

$$A_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} & 0 \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_v^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ 0 & \dfrac{-1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$f\,45^\circ, k = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{-P'}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then $$R_{\theta, L} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{2+\sqrt{2}}{4} & \frac{2-\sqrt{2}}{4} & \frac{2-\sqrt{2}}{4} \\ \frac{-1}{2} & \frac{2-\sqrt{2}}{4} & \frac{2+\sqrt{2}}{4} & \frac{\sqrt{2}-2}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, to find the coordinates of the rotated figure, we apply the rotation matrix $R_{\theta, L}$ to the matrix of homogeneous coordinates of the vertices A, B, C and D.

$$C = ABCD = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R_{\theta, L} \cdot C = \begin{bmatrix} \frac{1}{2} & \frac{1+\sqrt{2}}{2} & 0 & 1 \\ \frac{2-\sqrt{2}}{4} & \frac{4-\sqrt{2}}{4} & 1 & \frac{2-\sqrt{2}}{2} \\ \frac{\sqrt{2}-2}{4} & \frac{\sqrt{2}-4}{4} & 0 & \frac{\sqrt{2}}{2} \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The rotated coordinates are :

$$A' = \left(\frac{1}{2}, \frac{2-\sqrt{2}}{4}, \frac{\sqrt{2}-2}{4}\right)$$

$$B' = \left(\frac{1+\sqrt{2}}{2}, \frac{4-\sqrt{2}}{4}, \frac{\sqrt{2}-4}{4}\right)$$

$$C' = (0, 1, 0)$$

$$D' = \left(1, \frac{2-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right).$$

Problem 5.2. *The pyramid defined by the coordinates A(0, 0, 0), B(1, 0, 0), C(0, 1, 0) and D(0, 0, 1) is rotated 90° about the line L that has direction vector $v = i + j + k$ and passing through the origin. Find the coordinates of rotated figures :*

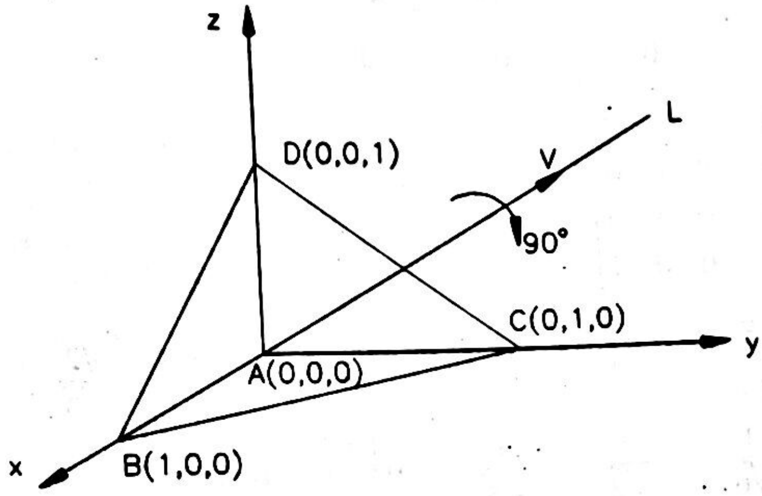

Fig. 5.7

Solution. Because direction vector V does not along normal vector, so align it with normal vector k(*i.e.* along z-axis) then rotate by an angle $\theta = 90°$ then rotate by an angle $\theta = 90°$ then align it with original direction. Therefore,

$$R_{\theta, L} = A_v^{-1} \cdot R_{\theta, k} \cdot A_v$$

here $\quad a = b = c = 1 \quad \Rightarrow \quad |V| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$$\lambda = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$A_v = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{-(1)(+1)}{\sqrt{2}\sqrt{2}} & \frac{-1}{2} \cdot \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_v = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_v^{-1} = A_v^{-1} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{90^\circ, k} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

then $$R_{\theta, L} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{90^\circ, L} = \begin{bmatrix} \frac{1}{3} & \frac{1-\sqrt{3}}{3} & \frac{1+\sqrt{3}}{3} & 0 \\ \frac{1+\sqrt{3}}{3} & \frac{1}{3} & \frac{1-\sqrt{3}}{3} & 0 \\ \frac{1-\sqrt{3}}{3} & \frac{1-\sqrt{3}}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, the points after rotate can be obtained by the following equation

$$P'(A', B', C', D') = R_{90^\circ, L} \cdot P(A, B, C, D)$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1-\sqrt{3}}{3} & \frac{1+\sqrt{3}}{3} & 0 \\ \frac{1+\sqrt{3}}{3} & \frac{1}{3} & \frac{1-\sqrt{3}}{3} & 0 \\ \frac{1-\sqrt{3}}{3} & \frac{1-\sqrt{3}}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1-\sqrt{3}}{3} & \frac{1+\sqrt{3}}{3} \\ 0 & \frac{1+\sqrt{3}}{3} & \frac{1}{3} & \frac{1-\sqrt{3}}{3} \\ 0 & \frac{1-\sqrt{3}}{3} & \frac{1-\sqrt{3}}{3} & \frac{1}{3} \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore \quad A' = (0, 0, 0)$

$$B' = \left(\frac{1}{3}, \frac{1+\sqrt{3}}{3}, \frac{1-\sqrt{3}}{3}\right)$$

$$C' = \left(\frac{1-\sqrt{3}}{3}, \frac{1}{3}, \frac{1-\sqrt{3}}{3}\right),$$

$$D' = \left(\frac{1+\sqrt{3}}{3}, \frac{1-\sqrt{3}}{3}, \frac{1}{3}\right).$$

Problem 5.3. *Translate the unit cube 2 units in x, y and z direction.*

Solution. Consider the cube as shown in Fig. 5.8.

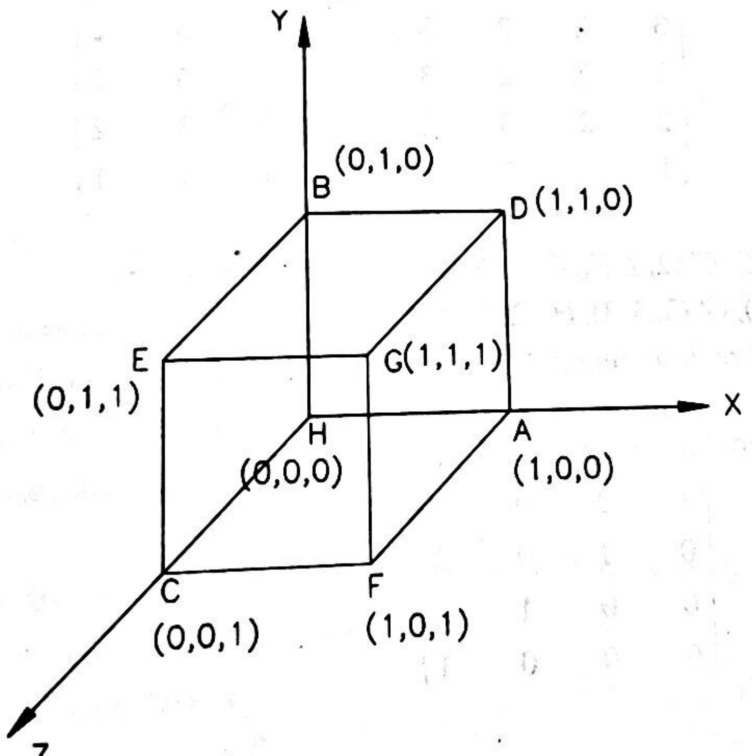

Fig. 5.8

Its vertices are $A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$, $D(1, 1, 0)$, $E(0, 1, 1)$, $F(1, 0, 1)$, $G(1, 1, 1)$ and $H(0, 0, 0)$.

We have, translating factors are

$$tx = 2, ty = 2 \text{ and } tz = 2$$

we know that, translation matric T_V is

$$T_V = \begin{pmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

i.e.,

$$T_V = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Also, object matrix Δ is

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

After translation, the required matrix Δ' is given by

$$\Delta' = T_V \cdot \Delta$$

$$= \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 & 2 & 3 & 2 & 3 & 3 & 2 \\ 2 & 3 & 2 & 3 & 3 & 2 & 3 & 2 \\ 2 & 2 & 3 & 2 & 3 & 3 & 3 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

So, new points are

$A'(3, 2, 2)$, $B'(2, 3, 2)$, $C'(2, 2, 3)$, $D'(3, 3, 2)$

$E'(2, 3, 3)$, $F'(3, 2, 3)$, $G'(3, 3, 3)$, $H'(2, 2, 2)$.

Problem 5.4. *Translate the pyramid A (1, 0, 0), B (0, 1, 0), C (0, 0, 1) and D (0, 0, 0) 2 units in x, y and z direction.*

Solution. Again, translation matrix is

$$T_V = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Object matrix Δ is

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

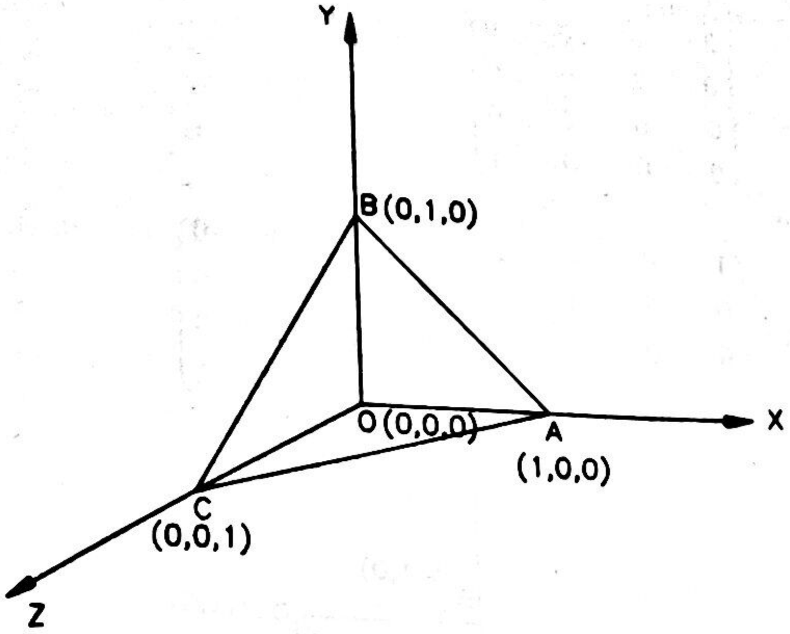

Fig. 5.9

After translation, $\Delta' = T_V \cdot \Delta$

i.e.
$$\Delta' = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, new coordinates are

$A'(3, 2, 2)$, $B'(2, 3, 2)$, $C'(2, 2, 3)$, $D'(2, 2, 2)$.

Problem 5.5. *Scale the unit cube 2 units in x, y and z directions.*

Solution. Consider the cube as shown in Fig. 5.10.

Its vertices are

$A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$, $D(1, 1, 0)$, $E(0, 1, 1)$, $F(1, 0, 1)$, $G(1, 1, 1)$ and $H(0, 0, 0)$.

Also, scaling factors are $S_x = S_y = S_z = 2$

We know that, scaling matrix is

$$S_{x,y,z} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$S_{2,2,2} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Also, object matrix is

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

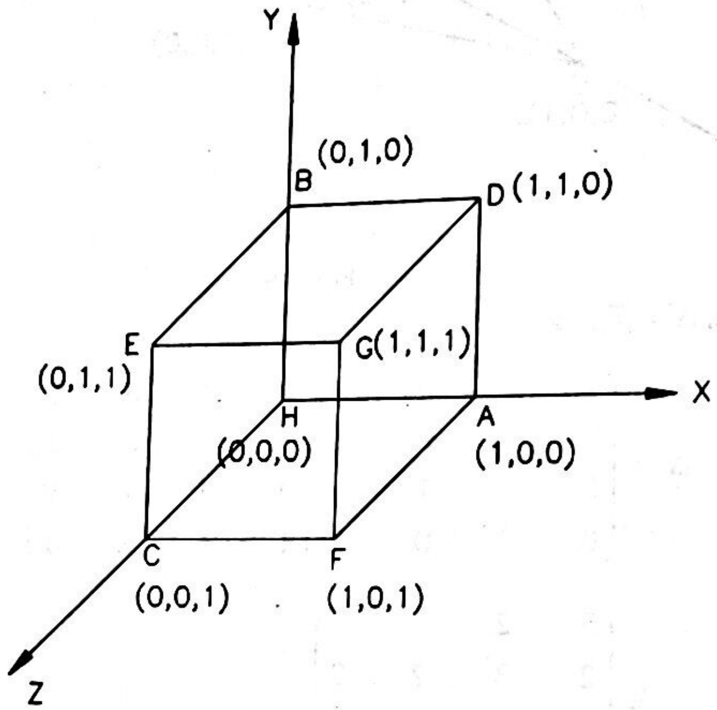

Fig. 5.10

Now $\Delta' = S_{2,2,2} \cdot \Delta$

$$= \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 & 2 & 0 & 2 & 2 & 0 \\ 0 & 2 & 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 2 & 2 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

So, new points of Δ' are :

$A'(2,0,0)$, $B'(0,2,0)$, $C'(0,0,2)$, $D'(2,2,0)$, $E'(0,2,2)$, $F'(2,0,2)$, $G'(2,2,2)$ and $H'(0,0,0)$.

Problem 5.6. *Scale the pyramid $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$ and $D(0,0,0)$ 2 units in x, y and z directions.*

Solution. Here, scaling factors are

$$S_x = S_y = S_z = 2.$$

So, scaling matrix is

$$S_{2,2,2} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Also, object matrix Δ is given by

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

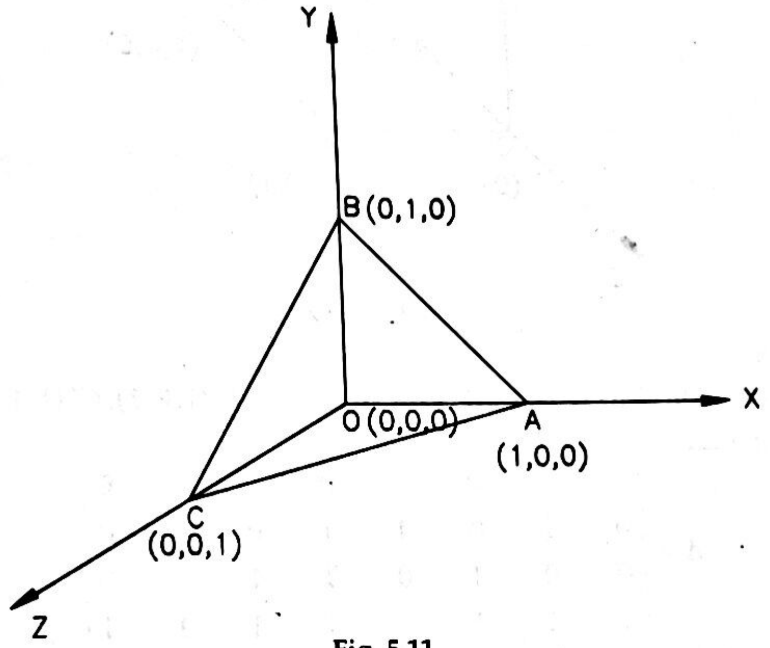

Fig. 5.11

$\therefore$ New matrix Δ' is given by

$$\Delta' = S_{2,2,2} \cdot \Delta$$

$$= \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, new points are :

$A'(2,0,0)$, $B'(0,2,0)$, $C'(0,0,2)$, $D'(0,0,0)$.

Problem 5.7. *Rotate a unit cube in x-direction with an angle of 45° (anticlockwise).*

Solution. Consider the cube as shown in Fig. 5.12.

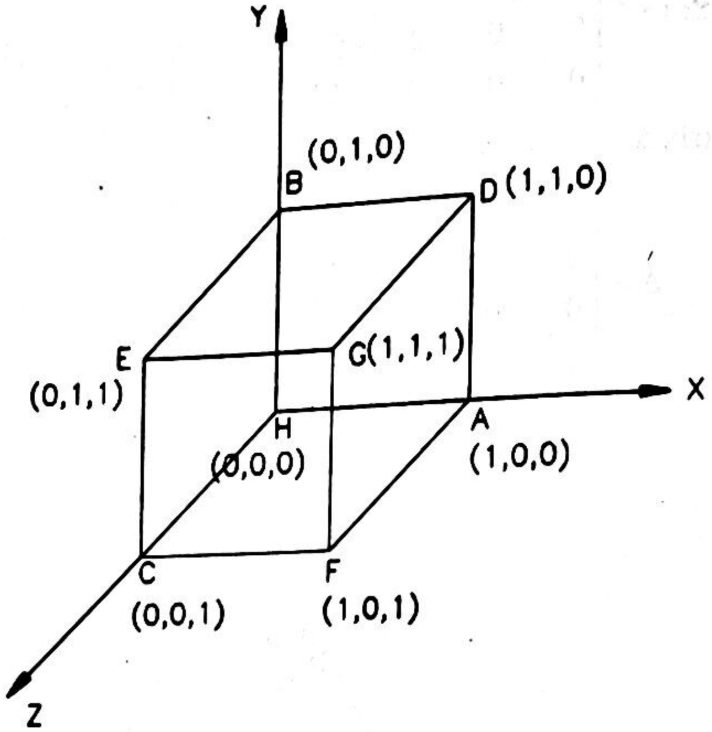

Fig. 5.12

Its vertices are

$A'(1,0,0), B'(0,1,0), C'(0,0,1), D'(1,1,0), E'(0,1,1), F'(1,0,1), G'(1,1,1), H(0,0,0)$.

So, object matrix Δ is given by

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Also, rotation matrix is given by (about x-axis)

$$R_\theta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

we have, $\theta = 45°$

$$\therefore \quad R_{45°} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now $\quad \Delta' = R_{45°} \,.\, \Delta$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2} & \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

So, new points are :

$A'(1, 0, 0)$, $B'\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $C'\left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $D'\left(1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $E'(0, 0, \sqrt{2})$, $F'\left(1, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $G'(1, 0, \sqrt{2})$ and $H'(0, 0, 0)$.

Problem 5.8. *Rotate the pyramid $A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$ and $D(0, 0, 0)$ about z-axis by an angle of 45°.*

Solution. The coordinates of the pyramid are

$A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$, $D(0, 0, 0)$.

$\therefore$ Object matrix is

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

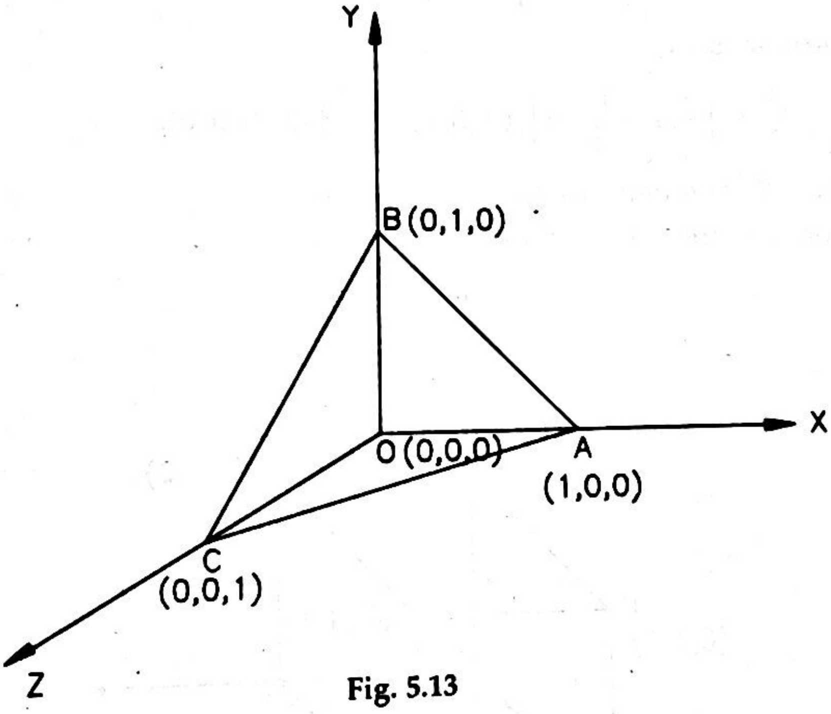

Fig. 5.13

So, rotation matrix about z-axis is

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Here, $\theta = 45°$.

$$\therefore \quad R_{45°} = \begin{pmatrix} \cos 45° & -\sin 45° & 0 & 0 \\ \sin 45° & \cos 45° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now, $\Delta' = R_{45^\circ} . \Delta$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, new coordinates are :

$A'\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), B'\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), C'(0, 0, 1)$ and $D'(0, 0, 0)$.

Problem 5.9. *Reflect the unit cube about the xy-plane.*

Solution. Consider the cube as shown in Fig. 5.14.

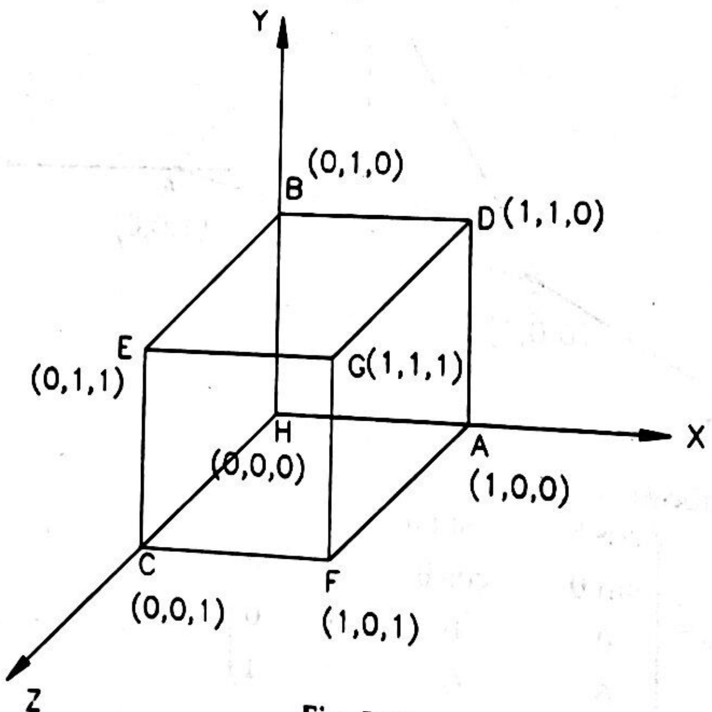

Fig. 5.14

Its vartices are :

$A(1, 0, 0), B(0, 1, 0), C(0, 0, 1), D(1, 1, 0), E(0, 1, 1), F(1, 0, 1), G(1, 1, 1)$ and $H(0, 0, 0)$.

$\therefore$ Object matrix is

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Also, reflection matrix about x-y plane is

$$\text{Ref}_{xy} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now, $\Delta' = \text{Ref}_{xy} \cdot \Delta$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

So, after, reflection, new points are
$A'(1, 0, 0)$, $B'(0, 1, 0)$, $C'(0, 0, -1)$, $D'(1, 1, 0)$, $E'(0, 1, -1)$, $F'(1, 0, -1)$, $G'(1, 1, -1)$ and $H'(0, 0, 0)$,

Problem 5.10. *Reflect the pyramid $A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$ and $D(0, 0, 0)$ about yz-plane.*

Solution. The co-ordinates of the pyramid are
$A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$ and $D(0, 0, 0)$.
So, object matrix is

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

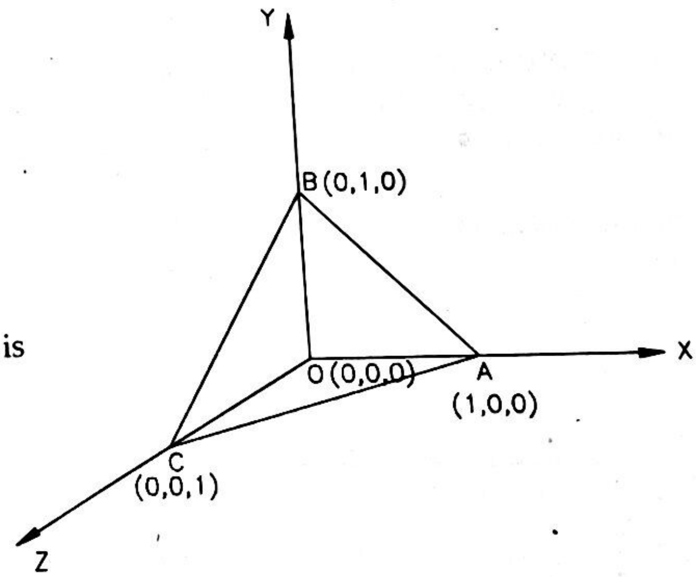

Fig. 5.15

Also, the reflection matrix about the yz-plane is

$$\text{Ref}_{yz} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now, $\Delta' = \text{Ref}_{yz} \cdot \Delta$

$$= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$\therefore$ New coordinates after reflection are
$A'(-1, 0, 0)$, $B'(0, 1, 0)$, $C'(0, 0, 1)$ and $D'(0, 0, 0)$