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Sampling Theory
(Population and Sample)

Introduction—

Data are the fundamental base in any statistical investigation and there are two ways to collect statistical data :—(i) Census method, (ii) Sampling method. Before discussing each, we should be familiar with the term—**Population** and **Sample**.

POPULATION (OR UNIVERSE)

The word 'Population' denotes all that area or all those units of the investigation about which, information is to be obtained. For example—

(i) If per capita income of a town is to be computed, relevant population (or universe) will consist of every person of the town.

(ii) If average expenditure made by students of a certain college consisting of 10,000 students is to be computed, the population (or universe), here will consist of all the 10,000 students of the college.

A universe may be **finite** or **infinite**. A finite universe is that in which all of the objects can be counted such as people of a town. An **infinite universe** is one in which number of all the objects cannot be determined. For example, number of stars in the sky.

SAMPLE

If refers to a part of the population. According to ***Simpson*** and ***Kafka***, "A sample is that part of the universe which we select for the purpose of investigation."

For example, if a survey is to be conducted relating to average income of persons in a city and for this purpose, study is done after selecting some persons of that city, it will be called sample.

A sample drawn from a population provides a very valuable information about its parent population. In most cases it gives almost the same results as that of the whole population. It is used to measure and estimate the corresponding characteristics of the parent population.

METHODS OF ENUMERATION
Or
METHODS OF COLLECTING STATISTICAL DATA

As has been pointed out earlier, there are two ways of collecting statistical data :—

(i) Census Method (or Census Enumeration or Census Enquiry)

(ii) Sample Method (or Sample Enumeration or Sample Enquiry)

Census method—

In the census method, information is collected about each and every object comprising the whole. An object may be household, factory, shop or any other similar object in any specified geographical area to be covered under study. The aggregate of all the objects (or units) under consideration is called the population (or universe). *For example*,

(i) Indian census, which is conducted once in every ten years.

(ii) A list of all voters, prepared for election purposes etc.

This method of data collection is also called *complete enumeration* or *100% enumeration* method.

Merits of Census Method—

(1) **More accurate and reliable results**—The data collected through census method are more accurate and reliable because the information is sought from each unit comprised in the population.

(2) **Extensive study**—In census method, information can be obtained from each and every object, hence, more facts relating to the problem under investigation may also be collected. *For example*, population census collects information about not only number of people but also collects information about age, marital status, employment, education, source of income etc.

Demerits of Census Method—

(1) **Costly and time consuming**—Census method requires lots of time, money, manpower and administrative personnel.

(2) **Often not practical**—This method is often impractical to adopt. Only big organizations and governments, having sufficiently large resources at their disposal can afford such method.

(3) **Not useful in urgent conclusion**—This method is useless when results are urgently required. It might be possible that, by the time data collected through census method are available, the conditions changed completely.

(4) **May be unnecessary**—It may be unnecessary if a sample can yield equally reliable results.

Sample Method—

Under this method, some representative objects (or units) are selected from a population. These selected objects are called samples of the population (or universe) from which the conclusions or inferences are drawn upon the whole population. It is to be noted that if the size of the sample is considerably large, and the samples have been properly selected, then various conclusions (or results) obtained from the study of the sample units, also, hold good for the entire population. *For example*,

(i) We may select a sample of 500 students out of 10,000 students studying in a college to study spending habits of all 10,000 students.

(ii) We may test only a small number of accounting entries to verify the accuracy of all the entries in test audit.

Merits of Sample Method (Importance of Sampling)—

Sample method of investigation is very important and popular. Its use has not been limited to scientific and technical investigations only but it is also widely used in our day-to-day life. *For example*,

(i) a grain merchant test the quality of grain just by taking a handful sample from a big lot.

(ii) The physician makes inferences about patient's blood through examination of single drop etc.

Various advantages of sample method as compared to census method are :—

(1) **Less costly**—Survey of the whole population is always more costly than the

survey of a small part of it. Sometimes a census costs so much that the very idea of collecting information may have to be dropped.

Although the amount of expenses involved in collecting information are generally greater per unit of sample, the total cost of the sample survey is expected to be much lower than that of census.

(2) **Saves time and labour**—Since only a part of the entire population is to be inspected and examined, the sample method results in considerable amount of saving in time and labour. There is saving of time and labour not only in conducting the sampling enquiry but also in the processing, editing and analysing of the data.

(3) **Testing of accuracy**—The accuracy of sampling enquiry can be tested by comparing the results of two or more samples from the same population. If samples are drawn on the basis of random sampling, their errors can also be estimated.

(4) **Practical approach**—A true census is impossible if population is infinite or testing procedure destroy the population unit. *For example*,

(i) testing the strength of a bullet,

(ii) testing the quality of match sticks,

(iii) testing the life of a bulb.

In such cases, sampling is the only practical way of assessing the quality of the whole lot. Moreover, if population is too large or infinite or spread over a large geographical area, there is no alternative but to use sample method. *For example*, study of fish in a river, or number of wild animals in a dense forest etc.

(5) **Administrative convenience**—Census method requires a very huge administrative set-up that involves lot of employees, trained investigators and above all the co-ordination between the various operating agencies. Whereas, the organization and administration of sample survey is relatively much convenient as it requires small administrative set-up.

(6) **Detailed and intensive enquiry**—In this method, the number of units under study is kept limited and this makes it possible to study them in detail and with greater precision.

(7) **Reliable results**—With a small data to process, there are fewer chances of non-sampling statistical errors. The sampling errors would, of course, be there but it is possible to estimate and control them, thus conclusions and results obtained by sample method are generally more reliable than those obtained by census method.

(8) **Scientific approach**—The size and method of sampling is scientific and is not based on mere tradition. This method has full justification for the expenditure involved.

(9) **Hypothetical population**—In case of hypothetical population, where the process may continue a large number of times or infinitely, the sample method is the only scientific technique of estimating the parameter of the population. *For example*, in the problem of tossing a coin or of throwing a die.

Demerits of Sample Method—

(1) **Conclusions may be inaccurate and misleading**—If a sample survey is not carefully planned and executed, the results obtained may be inaccurate and misleading.

(2) **Selecting sample is a difficult task**—In order to exhibit the true picture of the population, sampling technique should be scientific and samples should be drawn properly is proper size and on proper grounds which is a difficult task.

(3) **Requires services of qualified and skilled personnel**—An efficient sampling method requires the services of skilled and experienced staff, better supervision, more

sophisticated equipments and statistical techniques. In absence of these, the conclusions drawn may not be reliable.

(4) **Exposed to personal biases**—The sample method is exposed to personal biases and prejudices of the investigator especially with regard to selection of technique and drawing of sampling units.

(5) **If the size of the sample is inadequate**, it may fail to indicate the true characteristics of the population.

(6) **Not appropriate if the information is required about each and every unit of the population**—In such a case complete census method is appropriate.

SAMPLING

Sampling is the process of obtaining samples. The primary objective of sampling is to obtain maximum information about population with minimum effort. Sometimes sampling becomes easier if each unit in the population is numbered for identification.

Sampling Frame—A list of all the units in a population is called sampling frame. If a sample consists of 'n' units of observation, it is called a sample of size 'n'.

Statistics and Parameters—The values, such as mean, median, mode, standard deviation, mean deviation, variance etc., obtained from the study of a sample are known as ***Statistics***. Whereas, such values for the whole population are called ***Parameters***. *For example*, a soap manufacturing company found that the mean sale per week per shop of a particular brand of soap was 150 dozens. After an advertising campaign a sample of 30 shops was taken and the mean sale was found to be 155 dozens. Here 150 dozens is a parameter while 155 dozens is a statistics.

The notations used for the parameter in case of population and the statistic in case of sample are:—

	Population	**Sample**
Size	N	n
Mean	μ	$\overline{X}$
Standard Deviation	σ	s
Proportion	P	p

Objectives of Sampling—

(1) **To obtain information about the population**—This is done on the basis of estimation of an unknown parameter of population by using a suitable statistic computed from a sample drawn from such parent population.

(2) **To set-up the limits of accuracy of the estimates of the population parameters computed on the basis of sample statistic**—The estimates of the population parameters obtained on the basis of sample statistic may not give true results. Thus, the limits of accuracy and the degree of confidence should be set-up on such estimates in order to determine how exact are the estimates.

(3) **To test the significance about the population characteristics**—Conclusion about any characteristics of population may be easily drawn on the basis of sample statistics.

Basic Statistical Laws—

A sample should exhibit the characteristics of the population. There are two important laws which are basis of sampling :—

...to be near to that of the population. This law states that, larger the size of the ...number of units from the population. It is because when a ...insignificant. *For example*, if a coin is tossed 20 times, the variation in the aggregate times and tails only 8 times. But if the coin is tossed 2,000 times, we may get heads 12 heads 1,000 times and tails 1,000 times. it is more likely to get

METHODS OF SAMPLING (or TYPES OF SAMPLING)

The various sampling method may be broadly classified into :—

(I) **Random sampling method**—
   (a) Simple (or Unrestricted) random sampling
   (b) Restricted random sampling
      (i) Systematic Random Sampling
      (ii) Stratified Random Sampling
      (iii) Multi-Stage Random Sampling

(II) **Non-Random sampling method**—
   (i) Deliberate (or Judgement or Purposive) Sampling
   (ii) Cluster Sampling
   (iii) Convenience Sampling
   (iv) Quota Sampling

(I) RANDOM SAMPLING

Random sampling is also called as **'Chance Sampling'** or **'Probability Sampling'**. A random sample is one which is selected in such a way that each and every item in the population has an equal chance of being included. Main types of random sampling are :—

a) Simple (or unrestricted) random sampling—

In practice, simple random sampling is called random sampling only. In simple andom sampling all the units of the population have equal chances of being selected. In ractice, it is not easy to ensure true randomness in the selection of items in a sample. Iowever, to ensure randomness in the selection, one may adopt either of the following nethods :—

(i) **Lottery method**—This method invalves preparing different slips of paper of lentical size each bearing number or name of a unit of the population. All these slips are en put in a container and thoroughly mixed. The number of slips required to constitute e sample are then picked up blindfoldedly. Following points should be kept in mind for e reliability of this method :—

— all slips should be homogeneous in shape, size, colour etc.

— the slips should be thoroughly shuffled before each draw of units for sample.

— the work of drawing slips should be done blindfoldedly.

The method is illustrated below by means of an example—Suppose there are 5,000 rsons and we want to select 100 persons out of them. Then we assign the numbers 1 to 000, one each to a person and write these numbers on 5,000 slips (1 to 5,000). These lded slips are then put in a container and thoroughly mixed and then 100 slips are drawn by one. The 100 persons, corresponding to numbers on the slips drawn will constitute

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Test of Hypothesis : Large Sample

Introduction—

The term hypothesis literally means an assumption, or a supposition about the state of affairs of a certain thing. In the terminology of statistics, it means an estimation, or a set of inference that is drawn about certain parameter of a population on the basic of sample study. Such a decision involves an element of risk, the risk of taking a wrong decision. *For example*,

— a phamaceutical company may be interested in finding out if a new drug is really effective for the particular disease.

— a person may want to decide whether given food stuff is really effective in decreasing weight, or

— which of the two brands of a particular product is more effective.

Method of statistics which helps in arriving at the criterion for such decisions is known as **testing of hypothesis**.

TEST OF HYPOTHESIS (OR TEST OF SIGNIFICANCE)

The test of hypothesis is a process of testing of significance regarding parameter of the population (universe) on the basis of sample. In it, a statistic is computed from the sample drawn from a population and on the basis of computed value of statistic, it is seen whether the drawn sample belongs to the parent population or not with specified characteristics (parameter).

The computed value of statistic may differ from the hypothetical value (assumed value) of the parameter due to sampling fluctuations,. If the difference is small, it is considered that the difference has arisen due to sampling fluctuations. Hence, the difference is considered to be insignificant and hypothesis is accepted.

If the difference is considerable (large), it is considered that the difference has not arisen due to sampling fluctutions but due to some other reasons. Hence, the difference is considered to be significant and hypothesis is rejected.

Techniques of hypothesis testing is used if we have an idea about the value of parameter in question. We make hypothesis about the population parameter and then examine the hypothesis by the technique of hypothesis testing. *For example*, suppose a company claims that the average life of a tyre produced by it is 40,000 kms. A consumer's association tests this claim by taking a sample of 40 tyres and examine how many kms. on an average they lasted. If this figure is say 35,000 kms, the consumer's association might feel inclined to reject the company's claim. If, however, the sample mean is 39,000 kms. or 41,000 kms., it would be inclined to accept the company's claim.

Since the decision to reject or accept athe company's claim is based on the result of a sample, the decision is risky indeed. In such cases, hypothesis testing helps us in taking decisions with acceptable risks.

Procedure of Testing a Hypothesis—

The following are the steps involved in test of signifiacnce or hypothesis testing problem :—

(1) Setting up of hypothesis—

A statistical hypothesis, or simply a hypothesis, is a tentative drawn statement or assumption, which may or may not be true, about any parameter of the population. *For example*,

(i) The average children per couple in India is 4.

(ii) The average height of soldiers in the army is 162 cms.

(iii) The number of defects in printing per page is 5.

(iv) A given detergent cleans better than any washing soap.

(v) Male students who particpate in college athletics are taller than other male students.

(vi) The company secretary courses costs less to a candidate thany any other professional course.

Now, all these hypothesis may be verified on the basis of certain sample test. (like Z-test, *t*-test, chi-square test, F-test).

The common way of formulating a hypothesis is that, it is presumed and stated that there is no difference between the sample mean and the population. The term **no difference** means that the difference, if any, is just due to sampling fluctuations not due to any other reasons. Hence, if the statistical test shows that the difference is significant, the hypothesis is rejected.

Null Hypothesis—The term 'null hypothesis' refers to a hypothesis of no difference, or no significance in the difference between any two values under the comparison. It is stated for the purpose of possible acceptance. It is denoted by symbol **Ho**.

According to ***Prof. R.A. Fisher***, "Null Hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true."

For example,

$$H_o \;:\; \mu = 4$$

if it is assumed that the average children per couple in India is 4.

While setting the null hypothesis, the following points should be kept in mind :—

(i) To test the significance of the difference between a statistic and the parameter of population then set up a null hypothesis that, difference is not significant and is just due to sampling fluctuations.

$$H_o \;:\; \mu = \overline{X}$$

For example, if we want to test the null hypothesis that there is significant difference between the average life of 100 sample bulbs with mean 1,570 hours $(\overline{X})$ and all bulbs produced by company with mean 1,600 (μ) hours.

(ii) To test the significance of difference between two sample statistics then we set up a null hypothesis that, difference is not significant and is just due to sampling fluctuations.

$$H_o \;:\; \mu_1 = \mu_2$$

For example, if we want to test the null hypothesis that there is significant difference between the mean yield of crops in the two districts A and B with mean 210 kgs. and 220 kgs. respectively.

(iii) **To test any statement about the population we set-up the null hypothesis that it is true**

$$H_o \ : \ \mu = \mu_o$$

where μ_o is the population mean

For example, if we want to test the null hypothesis that the average children per couple in India is 4 (μ_0).

Alternative Hypothesis—Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis. It is usually denoted by H_1.

For example, if we want to test the null hypothesis that the average children per couple in India is 4 *i.e.*

$$H_o \ : \ \mu = 4 = \mu_o \text{ (say)}$$

then the alternative hypothesis could be any of the following :—

(i) $H_1 : \mu \neq \mu_o$ [*i.e.* $\mu > \mu_0$ or $\mu < \mu_0$, a two-tailed alternative]

(ii) $H_1 : \mu > \mu_o$ [a one-tailed alternative, *i.e.* right tailed]

(iii) $H_2 : \mu < \mu_o$ [a one-tailed alternative, *i.e.* left tailed]

Accordingly, the corresponding tests of significance are known as two-tailed, right-tailed and left-tailed tests (discussed later). It may be noted that while a null hypothesis is of simple nature, an alternative hypothesis is of composite nature. (*i.e.* null hypothesis tests a specific value of any measure, whereas the alternative hypothesis examine the value within a range). The acceptance of one hypothesis rejects the other hypothesis.

(2) Computation of Test Statistic—

This step is to compute an appropriate test statistic which is based on an appropriate probability distribution. It is the main yard-stick which is used to test whether the null-hypothesis set-up should be accepted or rejected. For this purpose, generally, we use Z-distribution under normal curve for large samples where the sample size is greater than or equal to 30 ($n \geq 30$) and student's t-distribution for small samples where the sample size is less than 30 ($n < 30$).

In this context, calculation of standard error related to testing statistic is very important for which different formulae are used under different conditions.

Type I Error (α) **and Type II Error** (β)—

In the process of hypothesis testing, we usually come accross some sort of errors, since there is every possibility of rejecting a null hypothesis which is true, and of accepting a null hypothesis which is false, and there is no guarantee that we always accept a null hypothesis (H_o) which is correct and reject the null hypothesis which is wrong. Because the decision is made on the basis of the information supplied by the sample data, there is always a chance of making error. There are two possible types of errors in the test of hypothesis :—

(i) **Type I Error**—It is an error, when a null hypothesis is true but is rejected due to significant difference between observed and expected values.

It is like convicting a person, under the legal system, who is actually innocent.

The probability of making such types of error is denoted by the Greek letter α (alpha).

(ii) **Type II Error**—It is an error, when a null hypothesis is false but is accepted due to insignificant difference between observed and expected values.

It is like letting a person go free, under the legal system, who is actually not innocent.

The probability of making such types of error is denoted by the Greek letter β (beta).

The four possible situations that may arise out of a hypothesis testing may be tabulated dichotomically as under

Reality of the state	Decision	
	H_o Accepted	H_o Rejected
H_o is true	Correct Decision (No error) Probability = $(1 - \alpha)$	Wrong Decision (Type I error) Probability = α
H_o is false	Wrong Decision (Type II error) Probability = β	Correct Decision (No error) Probability = $(1 - \beta)$

While testing a hypothesis, attempts are made to minimize both the types of errors, but it is not at all possible to minimize them both at the same time. This is because, both are contrary to each other, a reduction in one leads to proportionate increase in the other.

In hypothesis testing, it is considered more risky to commit a Type II Error (*i.e.* acceptance of a hypothesis which is false) than to commit a Type I Error (*i.e.* rejection of a hypothesis which is true). Hence, in deciding whether to accept or reject a null hypothesis, we always try to minimize the probability of making Type I Error & the probability of making a correct decision is $(1 - \alpha)$ which is also called ***confidence coefficient***.

(3) Level of significance—

The maximum probability of making Type I error is known as level of significance, denoted by α and should be determined in advance before applying the test.

The commonly used levels of significance are 1% (0·01) and 5% (0·05). If we use 5% level of significance, it means that in 5 out of 100 cases, we are likely to reject a correct null hypothesis (H_0). In other words, this means that we are 95% confident that our decision to reject null hypothesis (H_0) is correct.

We can minimize Type I error, by redcuing the level of significance but it may result in an increase in the probability of type II error.

***Note* :** Usually 5% of significance ($\alpha = 0{\cdot}05$) is used in testing a hypothesis unless otherwise any other level of significance is specifically stated.

(4) Critical region—

A region under a standard normal curve in which if the computed value of the test statistic lies, we reject the null hypothesis, is called the ***critical region*** or ***rejection region***.

The region of standard normal curve that is not covered by the rejection region is called the ***acceptance region***.

When the test statistic computed to test the hypothesis falls in the rejection region, it is reasonable to reject the hypothesis as it is believed to be probably false. Similarly, when the test statistic computed to test the hypothesis falls in the acceptance region, it is reasonable to accept the hypothesis as it is believed to be probably true.

The decision as to which value lie in the rejection region and which fall in the acceptance region is made on the basis of the level of significance (α). In fact, the region of the standard normal curve corresponding to a pre-determined level of significance should be known.

Critical value—The value of the test statistic computed to test the hypothesis is called the ***critical value***. The critical value separates the acceptance region from the rejection region.

The critical value is obtained from specific tables at a particular level of significance. When the sample size is large (*i.e.* $n \geq 30$), it is obtained from the table of Z-distribution and when the sample size is small (*i.e.* $n < 30$), it is obtained from the table of t-distribution. Besides F-table and χ^2 (Chi-square) table are also used, wherever needed.

Tailed tests of Hypothesis

Every standard normal curve has two tails *i.e.* left tail and right tail which approaches to the base in both its left hand side and right hand side. The total area under the curve is taken as 100% or 1, which according to the theory of, test of hypothesis, is divided into two regions *i.e.* acceptance region and rejection region. When these regions are distinctly marked, and kept close to the respective tails of the curve for testing a null hypothesis in terms of the alternative hypothesis, it is called a tailed test of hypothesis.

The critical region may be represented by a portion of the area under the normal curve in two ways :—

(i) Two tailed (ii) One tailed

Thus, tailed test of hypothesis can be of two types :—

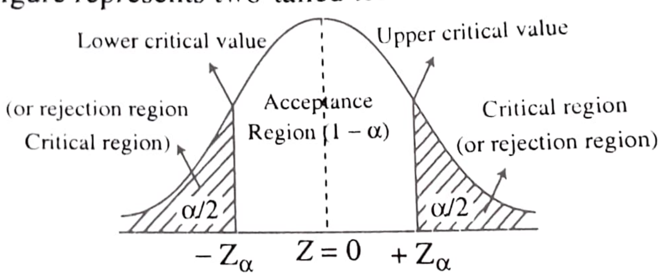
(i) Two-tailed test (ii) One-tailed test

Two-tailed test (or Two-sided test)—When the entire rejection region is divided into two halves, and each half of such a region is shown at each of the two tails of the standard normal curve, and the entire acceptance region is shown in the middle part of the curve, the hypothesis test made thereon is called ***two tailed test*** of hypothesis.

For example, A test of any statistical hypothesis where the alternative hypothesis is two sided, such as—

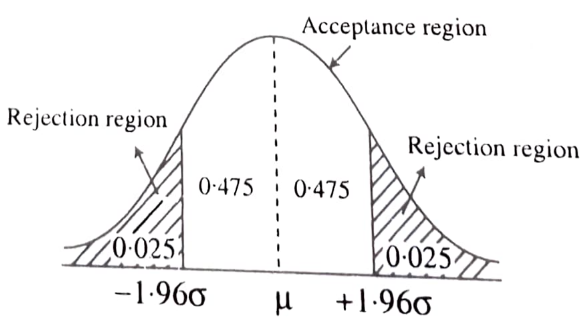
$$H_0 \;:\; \mu = \mu_0$$
$$H_1 \;:\; \mu \neq \mu_0 \; (i.e.\ \mu > \mu_0 \text{ or } \mu < \mu_0)$$

The following figure represents two-tailed test :—

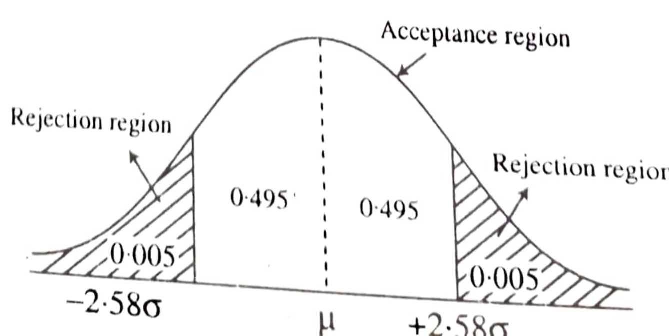


where, α = **Level of Significance**

The following diagram shows the picture of two tailed test of hypothesis at 5% and 1% levels.



At 5% level of significance where $\alpha = 0{\cdot}05$, lower critical value = – 1·96 and upper critical value = + 1·96.



At 1% level of significance where $\alpha = 0{\cdot}01$, lower critical value = $-2{\cdot}58$ and upper critical value = $+2{\cdot}58$.

One-tailed test (or One-sided test)—When the entire rejection region is shown at the one end, and the entire acceptance region at the other end of standard normal curve, the hypothesis test made thereon is called one-tailed test of hypothesis. On the basis of side of tail, it may be :—

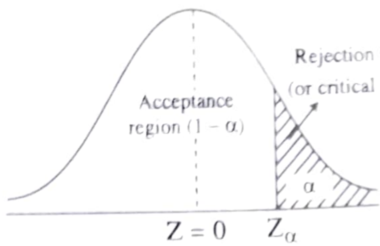
(a) Left-tailed test (or Lower-tailed test)

(b) Right-tailed test (or Upper-tailed test)

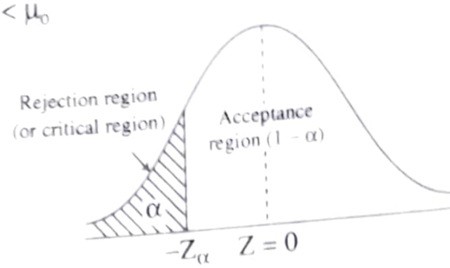
For example, A test of any statistical hypothesis where the alternative hypothesis is one sided, such as :—

(I) $H_0 : \mu = \mu_0$ (II) $H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$ or $H_1 : \mu < \mu_0$



Right Tailed Test



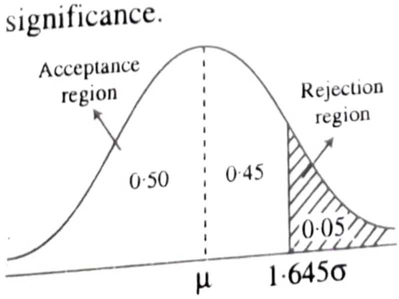
Left Tailed Test

(a) Right-tailed Test—When the entire rejection region is shown at the extreme right hand side of the standard normal curve, and the entire acceptance region is shown just to the left of rejection region, the test is called right-tailed test of hypothesis.

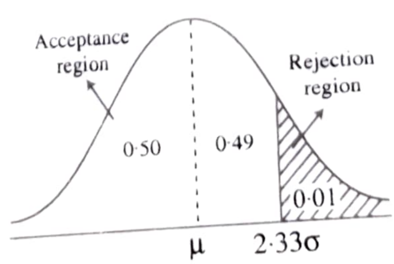
For example, A test of any statistical hypothesis where the alternative hypothesis speaks that the value of the outcome is more than the proposed value. *i.e.*

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

The following diagram shows the picture of right tailed test at 5% and 1% levels of significance.



At 5% level of significance where $\alpha = 0{\cdot}05$, and upper critical value = $+ 1{\cdot}645$



At 1% level of significance where $\alpha = 0{\cdot}01$, and lower critical value = $+ 2{\cdot}33$

(b) Left-tailed test—When the entire rejection is shown at the extreme left hand side of the standard normal curve, and the entire acceptance region is shown just to the right of rejection region, the test is called left-tailed test of hypothesis.

For example, A test of any statistical hypothesis where the alternative hypothesis speaks that the value of the outcome is less than the proposed value *i.e.*

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu < \mu_0$$

- Samples
  - Small Samples ($n < 30$)
  - Large samples ($n \geq 30$)
    - Samples of variables
    - Samples of Attributes

In this chapter, test of hypothesis related to large samples have been discussed.

(I) TEST OF HYPOTHESIS IN SAMPLING OF VARIABLES

By the term **'sampling of variables'**, we mean those facts which can be measured directly in quantitative terms such as height, weight, age, income, production etc.

Objectives—

(1) To form an idea about parent population by estimating some of its characteristics like mean, standard deviation etc. through sampling.

(2) To make a comparison between observed and expected values and to test the significance of the difference between them using appropriate test statistic at a specific level of confidence.

(3) To test the reliability of estimates.

Assumptions—

(1) It is assumed that sampling units have been drawn from one parent population and all units were drawn independently and randomly *i.e.* **simple sampling**.

(2) It is assumed that the sample distribution of various statistic ($\overline{X}$, S.D. etc.) confirms to the properties of a normal distribution, whether the population distribution is normal or not.

(3) It is assumed that statistic are considered best possible and unbiased estimates of parameter.

The various tests of hypothesis (large sample) in sampling of variables are discussed under the following three broad heads :—

(1) Test of significance of a mean.

(2) Test of significance of difference between two means.

(3) Test of significance of difference between two standard deviations.

(1) Test of significance of a mean (large sample) [or Standard Error of the Mean]—

The following steps are taken in testing the hypothesis of a sample mean *in case of large sample*.

Step 1. Null Hypothesis, H_0 :

State the null hypothesis in any one of the following forms :—

(i) The population has the specified mean value, *i.e.*

$$\mu = \mu_0$$

(ii) There is no significant difference between the population mean (μ) and the sample mean ($\overline{X}$).

(iii) The sample has been drawn from the given population with mean (μ_0) and standard deviation (σ).

Step 2. Computation of Test Statistic :

Compute test statistic for large sample as :—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$$

where n = sample size

σ = S.D. of population

If the S.D. of population is not known, then S.D. of sample (*i.e.* s) is used.

Step 3. Level of significance (α) ;

Usually 5% level of significance ($\alpha = 0{\cdot}05$) is used unless otherwise any other level of significance is specifically stated such as 1% (0·01), 2% (0·02) etc.

Step 4. Critical value :

The critical value of the test statistic Z at the pre-determined level of significance can be obtained from the following table :—

TABLE FOR CRITICAL VALUES OF Z

Critical values (Z_α)	Level of Significance (α)				
	1%	2%	4%	5%	10%
Left-tailed test	– 2·33	– 2·05	– 1·75	– 1·645	– 1·28
Right-tailed test	+ 2·33	+ 2·05	+ 1·75	+ 1·645	+ 1·28
Two-tailed test	± 2·58	± 2·33	± 2·05	± 1·96	± 1·645

Note : Critical value of Z for one-tailed test (left or right) at any level of significance (α) is same as the critical value of Z for two-tailed test at level of significance (2α).

Step 5. Decision—Finally, the decision to accept or reject null hypothesis is to be made by comparing the computed value and critical value of $|Z|$.

(i) If computed value of $|Z|$ is less than the critical value of $|Z|$, it means that the computed value of Z falls in the acceptance region. Hence, the null hypothesis (H_0) is accepted at the pre-determined significance level and it may reasonably be concluded that there is no significant difference between the sample mean and the population mean or the given sample has been drawn from the population; or

(ii) if the computed value of $|Z|$, is more than the critical value of $|Z|$, it means that the computed value of Z falls in the rejection region. Hence, the null hypothesis (H_0) is rejected (*i.e.* Alternative hypothesis (H_1) is accepted) at the pre-determined significance level and it may reasonably be concluded that there is a significant difference between the sample mean and the population mean or the given sample has not been drawn from the population.

Illustration 1.

A sample of size 400 was drawn and the sample mean was found to be 99. Test whether this sample could have come from a normal population with mean 100 and variance 64 at 5% level of significance.

Solution :

Here, $n = 400$, $\overline{X} = 99$, $\mu = 100$, $\sigma^2 = 64$ *i.e.* $\sigma = 8$

Step 1. Null Hypothesis—$H_0 : \mu = 100$, *i.e.* the sample has come from a normal population with mean $\mu = 100$ and variance $\sigma^2 = 64$.

Alternative Hypothesis—$H_1 : \mu \neq 100$ (Two-tailed test) *i.e.* the sample has not come from a normal population with mean $\mu = 100$ and variance $\sigma^2 = 64$.

Step 2. Test Statistic—Under H_0, since sample is large, the test statistic is—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$$

$$= \frac{99 - 100}{\sqrt{\frac{64}{400}}} = \frac{(-1) \times 20}{8} = -2.5$$

Step 3. Level of Significance—

$$\alpha = 0.05$$

Step 4. Critical Value—The critical value of Z at 0·05 level of significance is—

$$Z = \pm 1.96 \text{ (from table)}$$

Step 5. Since the computed value of $|Z| = 2.5$ is greater than the critical value of $|Z| = 1.96$, it falls in the rejection region. Hence, the null hypothesis at 0·05 level of significance is rejected and it may be concluded that ***the sample has not come from a normal population with mean 100 and variance 64.***

Illustration 2.

The average time taken for mixing various ingredients of a detergent powder is 70 minutes with a standard deviation of 8 minutes. It is known that the quality of the product can be improved by altering one of the ingredients, but it is not known whether this would affect the mixing time. 40 batches are made of the 'new' detergent powder, and it is discovered that their average time is 78 minutes.

Does this indicate, at the 0·05 significance level, the new ingredient affects the mixing time ?

Solution :

Here, $n = 40$, $\mu = 70$ min., $\sigma = 8$ min., $\overline{X} = 78$ min.

Step 1. Null Hypothesis—$H_0 : \mu = 70$ min., *i.e.* the new ingredient does not affects the mixing time.

Alternative Hypothesis—$H_1 : \mu \neq 70$ min. (Two-tailed test) *i.e.* the new ingredient affects the mixing time.

Step 2. Test Statistic—Under H_0, since sample size is large ($n \geq 30$), the test statistic is—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{78 - 70}{\sqrt{\frac{(8)^2}{40}}} = \frac{8 \times 6.32}{8} = 6.32$$

Step 3. Level of Significance—

$$\alpha = 0.05$$

Step 4. Critical Value—The critical value of Z at 0·05 level of significance is—

$$Z = \pm 1.96 \text{ (from table)}$$

Step 5. Decision—Since the computed value of $|Z| = 6.32$ is greater than the critical value of $|Z| = 1.96$, it falls in the rejection region. Hence, the null hypothesis at 0·05 level of significance is rejected and it may be concluded that ***the new ingredient affects the mixing time.***

Illustration 3.

A machine part was designed to withstand an average pressure of 120 units. A random sample if size 100 from a large batch was tested and it was found that the average pressure which these parts can withstand is 105 units with a standard deviation of 20 units. Test whether the batch meets the specification.

Solution :

Here, $n = 100$, $\mu = 120$, $\overline{X} = 105$, Sample S.D. $(s) = 20$

Step 1. Null Hypothesis—$H_o : \mu = 120$ *i.e.* the batch meets the specification.

Alternative Hypothesis—$H_1 : \mu \neq 120$ (Two-tailed test) *i.e.* the batch does not meets the specification.

Step 2. Test Statistic—Under H_o, since sample size is large, the test statistic is—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \simeq \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n}}} \quad \text{[Since population S.D. is not given]}$$

$$= \frac{105 - 120}{\sqrt{\frac{(20)^2}{100}}} = \frac{-15 \times 10}{20} = -7\cdot5$$

Step 3. Level of Significance—

$$\alpha = 0\cdot05$$

Step 4. Critical Value—The critical value of Z at 0·05 level of significance is—

$$Z = \pm 1\cdot96 \text{ (from table)}$$

Step 5. Decision—Since the computed value of $|Z| = 7\cdot5$ is greater than the critical value of $|Z| = 1\cdot96$, it falls in the rejection region. Hence, the null hypothesis at 0·05 level of significance is rejected and it may be concluded that *the batch does not meets the specification.*

Illustration 4.

Daily sales figures of 40 shop-keepers showed that their average sales and standard deviation were ₹ 528 and ₹ 60 respectively. Is the assertion that daily sale on the average is ₹ 400 contradicted at 5% level of significance by the sample ?

Solution :

Here, $n = 40$, $\overline{X} = 528$, Sample S.D. $(s) = 60$, $\mu = 400$

Step 1. Null Hypothesis—$H_o : \mu = 400$ *i.e.* the assertion that daily sale on the average is ₹ 400 is not contradicted at 5% level of significance by the sample.

Alternative Hypothesis—$H_1 : \mu \neq 400$ *i.e.* $\mu > 400$ (right-tailed test), the assertion that daily sale on the average is ₹ 400 contradicted at 5% level of significance by the sample.

Step 3. Level of Significance—

$$\alpha = 0.05$$

Step 4. Critical Value—The critical value of Z at 0·05 level of significance is—

$$Z = 1.645 \text{ (from table)} \qquad \text{[Right-tailed test]}$$

Step 5. Decision—Since the computed value of $|Z| = 13.5$ is greater than the critical value of $|Z| = 1.645$, it falls in the rejection region. Hence, the null hypothesis at 0·05 level of significance is rejected and it may be concluded that ***the assertion that daily sale on the average is ₹ 400 contradicted at 5% level of significance by the sample***.

Illustration 5.

A sample of 400 male students is found to have a mean height of 171·38 cm. Can it be reasonably regarded as a sample from a large population with mean height 171·17 cm. and standard deviation 3·30 cms.

Solution :

Here, $n = 400$, $\overline{X} = 171.38$, $\mu = 171.17$, $\sigma = 3.30$

Step 1. Null Hypothesis—$H_0 : \mu = 171.17$ *i.e.* the sample has been drawn from a large population with mean height 171·17 cm. and standard deviation 3·30 cm.

Alternative Hypothesis—$H_1 : \mu \neq 171.17$ (two-tailed test), *i.e.* the sample has not been drawn from a large population with mean height 171·17 cm. and standard deviation 3·30 cms.

Step 2. Test Statistic—Under H_0, since sample size is large, the test statistic is—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$$

$$= \frac{171.38 - 171.17}{\sqrt{\frac{(3.30)^2}{400}}} = \frac{0.21 \times 20}{3.30} = 1.27$$

Step 3. Level of Significance—

$$\alpha = 0.05$$

Step 4. Critical Value—The critical value of Z at 0·05 level of significance is—

$$Z = \pm 1.96 \text{ (from table)}$$

Step 5. Decision—Since the computed value of $|Z| = 1.27$ is less than the critical value of $|Z| = 1.96$, it falls in the acceptance region. Hence, the null hypothesis at 0·05 level of significance is accepted and it may be concluded that the sample has ***been drawn from a large population with mean height 171·17 cm. and standard deviation 3·30 cm.***

Illustration 6.

An educator claims that average IQ of American College students is at most 110, and that in a study made to test this claim, 150 American college students, selected at random, had an average IQ of 111·2 with standard deviation of 7·2. Use a level of significance of 0·01 to test the claim of the educator.

Solution :

Here, $n = 150$, $\mu = 110$, $\overline{X} = 111.2$, Sample S.D. $(s) = 7.2$,

Step 1. Null Hypothesis—$H_0 : \mu \leq 110$ *i.e.* the average IQ of American college students is at most 110.

Alternative Hypothesis—$H_1 : \mu > 110$ (right tailed test), *i.e.* the average IQ of American college students is more than 110.

Step 2. Test Statistic—Under H_0, since sample size is large, the test statistic is—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \simeq \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n}}}$$

$$= \frac{111 \cdot 2 - 110}{\sqrt{\frac{(7 \cdot 2)^2}{150}}} = \frac{1 \cdot 2 \times 12 \cdot 25}{7 \cdot 2} = 2 \cdot 04$$

Step 3. Level of Significance—

$$\alpha = 0 \cdot 01$$

Step 4. Critical Value—The critical value of Z at 0·01 level of significance is—

$$Z = 2 \cdot 33$$ [Right-tailed test]

Step 5. Decision—Since the computed value of $|Z| = 2 \cdot 04$ is less than the critical value of $|Z| = 2 \cdot 33$, it falls in the acceptance region. Hence, the null hypothesis is accepted and it may be concluded that ***the average IQ of American college students is at most 110***.

Illustration 7.

A random sample of boots worn by 40 combat soldiers in a desert region showed an average life of 1·08 years with standard deviation of 0·05 years. Under standard conditions the boots are known to have an average life of 1·28 years. Is there reasons to assert at a level of significance of 0·05 that use in the desert causes the mean life of such boots to decrease ?

Solution :

Here, $n = 40$, $\overline{X} = 1 \cdot 08$ years, Sample S.D. $(s) = 0 \cdot 05$ years, $\mu = 1 \cdot 28$ years.

Step 1. Null Hypothesis—$H_0 : \mu = 1 \cdot 28$ years, *i.e.* the use in the desert does not cause the mean life of such boots to decrease at a level of significance of 0·05.

Alternative Hypothesis—$H_1 : \mu < 1 \cdot 28$ years (left-tailed test) *i.e.* the use in the desert causes the mean life of such boots to decrease at a level of significance of 0·05.

Step 2. Test Statistic—Under H_0, since sample size is large, the test statistic is—

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \simeq \frac{\overline{X} - \mu}{\sqrt{\frac{s^2}{n}}}$$

$$= \frac{1 \cdot 08 - 1 \cdot 28}{\sqrt{\frac{(0 \cdot 05)^2}{40}}} = \frac{(-0 \cdot 2) \times 6 \cdot 32}{0 \cdot 05} = -25 \cdot 28$$

Step 3. Level of Significance—

$$\alpha = 0 \cdot 05$$

Step 4. Critical Value—The critical value of Z at 0·05 level of significance is—

$$Z = -1 \cdot 645$$ [Left-tailed test]

Step 5. Decision—Since the computed value of $|Z| = 25 \cdot 28$ is greater than the critical value of $|Z| = 1 \cdot 645$, it falls in the rejection region. Hence, the null hypothesis is

(i) Two samples have been drawn from the same parent population.

(ii) There is no difference between the means of two population *i.e.* the difference is not significant although samples are drawn from two different population.

i.e. $H_0 : \mu_1 = \mu_2$

Alternative Hypothesis :

$H_1 : \mu_1 \neq \mu_2$ (Two-tailed test)

Step 2. Computation of Test Statistic—Compute test statistic for large sample as—

$$\boxed{Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}}}$$

where, σ_1 and σ_2 = Population S.D. of population I and II respectively

n_1 and n_2 = Sample size of population I and II respectively

$\overline{X}_1$ and $\overline{X}_2$ = Sample mean of population I and II respectively.

***Note* :** If two samples come from same population then—

$$\sigma_1 = \sigma_2 = \sigma$$

where σ = S.D. of population.

If S.D. of the population is not known, then S.D. of samples (*i.e.* s_1 and s_2) will be used, since samples are large.

Hence, in this case, the test statistic becomes—

$$\boxed{Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}}$$

Step 3. (Level of significance) and

Step 4. Critical value will be same as those followed in testing the significance of mean for a large sample.

Step 5. Decision—

(i) If the computed value of $|Z|$ is less than the critical value of $|Z|$, it means that the computed value of Z falls in the acceptance region. Hence, the null hypothesis is accepted and it may be concluded that two samples have been drawn from the same population or there is no difference between the mean of two populations.

(ii) If the computed value of $|Z|$ is more than the critical value of $|Z|$, it means that the computed value of Z falls in the rejection region. Hence, the null hypothesis is rejected and it may be concluded that two samples have been drawn from the different population or there is a significant difference between the mean of two populations.

Illustration 11.

A random sample of 50 male employees is taken at the end of a year and the mean number of hours of absenteeism for the year is found to be 63 hours. A similar sample of

50 female employees has a mean of 66 hours. Could these samples be drawn from a population with the same mean and S.D. 10 hours ? State clearly the assumptions you make.

Solution :

Here, $n_1 = 50, n_2 = 50, \overline{X}_1 = 63, \overline{X}_2 = 66, \sigma_1 = \sigma_2 = \sigma = 10, \mu_1$ and μ_2 are population means.

Step 1. Null Hypothesis—$H_0 : \mu_1 = \mu_2$ *i.e.* there is no significant difference between the mean of two samples and these samples are drawn from a population with same mean and S.D. of 10 hours.

Alternative Hypothesis—$H_1 : \mu_1 \neq \mu_2$ (Two-tailed test) *i.e.* these samples are drawn from a population with different mean and there is a significant difference between the mean of two samples.

Step 2. Test Statistic—Under H_0, the test statistic for large sample is :—

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}}$$

$$= \frac{63 - 66}{\sqrt{\frac{10^2}{50} + \frac{10^2}{50}}} = \frac{-3}{10\sqrt{\frac{1}{25}}} = \frac{-3}{2} = -1{\cdot}5$$

Step 3. Level of Significance—$\alpha = 0{\cdot}05$ (5%)

Step 4. Critical Value—The critical value of Z at 5% level of significance is—

$$Z = \pm 1{\cdot}96$$

Step 5. Decision—Since the computed value of $|Z| = 1{\cdot}5$ is less than the critical value of $|Z| = 1{\cdot}96$, it falls in the acceptance region. Hence, the null hypothesis is accepted and it may be concluded that ***these samples are drawn from a population with the same mean and S.D. of 10 hours.***

Illustration 12.

A potential buyer of light bulbs bought 50 bulbs of each of two brands A and B. Upon testing the bulbs he finds that brand A had a mean life of 1,282 hours with a standard deviation of 80 hours whereas brand B had a mean life of 1,208 hours with a standard deviation of 70 hours. Can the buyer be quite certain that the mean life of brand A is higher than that of brand B. Use 5% level of significance.

Solution :

Here,	**Brand A**	**Brand B**
	$n_1 = 50$	$n_2 = 50$
	$\overline{X}_1 = 1{,}282$	$\overline{X}_2 = 1{,}208$
(Sample S.D.)	$s_1 = 80$	$s_2 = 70$
	μ_1 = Population mean	μ_2 = Population mean

Step 1. Null Hypothesis—$H_0 : \mu_1 = \mu_2$ *i.e.* there is no significant difference between the mean life of two brands and their mean life is equal.

Alternative Hypothesis—$H_1 : \mu_1 \neq \mu_2$ (two-tailed test) *i.e.* there is a significant difference between the mean life of two brands and the mean life of brand A is higher than that of brand B.

Step 2. Test Statistic Under H_o, the test statistic for large sample is

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \simeq \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

[Since S.D. of population is not given]

$$= \frac{1,282 - 1,208}{\sqrt{\frac{(80)^2}{50} + \frac{(70)^2}{50}}} = \frac{74}{\sqrt{\frac{6,400}{50} + \frac{4,900}{50}}} = \frac{74}{15.03} = \mathbf{4{\cdot}92}$$

Step 3. Level of Significance—$\alpha = 0{\cdot}05$ (5%)

Step 4. Critical Value—The critical value of Z at 5% level of significance is—

$$Z = \pm 1{\cdot}96$$

Step 5. Decision—Since the computed value of $|Z| = 4{\cdot}92$ is greater than the critical value of $|Z| = 1{\cdot}96$, it falls in the rejection region. Hence, the null hypothesis is rejected and it may be concluded that ***mean life of brand A is higher than that of brand B***.

Illustration 13.

The sales manager of a large company conducted a sample inquiry to examine the significance of the difference between the sales performance of his salesmen posted in the two states : Gujarat and Andhra Pradesh. The results of the enquiry are shown in the following table :—

State	Arithmetic mean	Standard deviation	Sample size
Gujarat	₹ 10,400	₹ 140	500
Andhra Pradesh	₹ 11,200	₹ 80	500

Test at 5% level of significance that the average performance of the salesmen in the two states is the same and does not differ significantly. [*UPTU, MBA*, 2003-04]

Solution :

Here,

	Gujarat	Andhra Pradesh
	$n_1 = 500$	$n_2 = 500$
	$\overline{X}_1 = 10{,}400$	$\overline{X}_2 = 11{,}200$
(Sample S.D.)	$s_1 = 140$	$s_2 = 80$
	μ_1 = Population mean	μ_2 = Population mean

Step 1. Null Hypothesis—$H_o : \mu_1 = \mu_2$ *i.e.*, the average performance of the salesmen in the two states is the same and does not differ significantly.

Alternative Hypothesis—$H_1 : \mu_1 \neq \mu_2$ (two-tailed test) *i.e.* the average performance of the salesmen in the two states is not same and differ significantly.

Step 2. Test Statistic—Under H_o, the test statistic for large sample is :—

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \simeq \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{10{,}400 - 11{,}200}{\sqrt{\frac{(140)^2}{500} + \frac{(80)^2}{500}}} = \frac{-800}{\sqrt{52}} = -\frac{800}{7{\cdot}21} = \mathbf{-110{\cdot}95}$$

Step 3. Level of Significance—$\alpha = 0{\cdot}05$ (5%)

Step 4. Critical Value—The critical value of Z at 5% level of significance is—

$$Z = \pm 1{\cdot}96$$

Step 5. Decision—Since the computed value of $|Z| = 110{\cdot}95$ is greater than the critical value of $|Z| = 1{\cdot}96$, it falls in the rejection region. Hence, null hypothesis is rejected and it may be concluded that ***the average performance of the salesmen in the two states is not same and differ significantly***.

Illustration 14.

The average hourly wage of a sample of 150 workers in a plant 'A' was ₹ 2·56 with a standard deviation of ₹ 1·08. The average wage of a sample of 200 workers in plant 'B' was ₹ 2·87 with a standard deviation of ₹ 1·28. Can an applicant safely assume that the hourly wages paid by plant 'B' are higher than those paid by plant 'A'.

Solution :

Here,

	Plant 'A'	Plant 'B'
	$n_1 = 150$	$n_2 = 200$
	$\overline{X}_1 = 2{\cdot}56$	$\overline{X}_2 = 2{\cdot}87$
(Sample S.D.)	$s_1 = 1{\cdot}08$	$s_2 = 1{\cdot}28$
	μ_1 = Population mean	μ_2 = Population mean

Step 1. Null Hypothesis—$H_o : \mu_1 = \mu_2$ *i.e.*, the hourly wages paid by plant A and B does not differ significantly, means the hourly wages paid by plant A and plant B are same.

Alternative Hypothesis—$H_1 : \mu_1 \neq \mu_2$ (two-tailed test) *i.e.* the hourly wages paid by plant 'B' are higher than those paid by plant 'A'.

Step 2. Test Statistic—Under H_o, the test statistic for large sample is :—

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \simeq \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

[Since in large sample, sample S.D. (s) is closed to population S.D. (σ)]

$$= \frac{2{\cdot}56 - 2{\cdot}87}{\sqrt{\frac{(1{\cdot}08)^2}{150} + \frac{(1{\cdot}28)^2}{200}}} = \frac{-0{\cdot}31}{0{\cdot}13} = \mathbf{-2{\cdot}38}$$

Step 3. Level of Significance—$\alpha = 0{\cdot}05$ (5%)

Step 4. Critical Value—The critical value of Z at 5% level of significance is—

$$Z = \pm 1{\cdot}96$$

Step 5. Decision—Since the computed value of $|Z| = 2{\cdot}38$ is greater than the critical value of $|Z| = 1{\cdot}96$, it falls in the rejection region. Hence, null hypothesis is rejected and it may be concluded that ***the hourly wages paid by plant 'B' are higher than those paid by plant 'A'***.

Illustration 15.

An examination was given to 50 students at college A and to 60 students at college B. At A, the mean grade was 75 with a standard deviation of 9. At B, the mean grade was 79 with a standard deviation of 7. Is there a significant difference between the performance of the students at A and those at B, given that $\alpha = 0{\cdot}05$? $\alpha = 0{\cdot}01$?

(ii) Test of significance for difference between two sample proportions.

1. Test of Significance of a Sample Proportion (or Test of significance for difference in proportion of successes)—

It is another test of significance, which is of great practical utility. The following steps are used to test the significance of a sample proportion (or percentage)—

Step 1. Null Hypothesis—The sample has been drawn from a population (or universe) with proportion P *i.e.*

$$H_0 : P = P_0$$

Alternative Hypothesis—The sample has not been drawn from a population with proportion P *i.e.*

$$H_1 : P \neq P_0 \text{ (Two-tailed test)}$$

Step 2. Test Statistic—Under H_0, the test statistic is—

$$Z = \frac{p - P}{\text{S.E. }(p)}$$

$$\boxed{Z = \frac{p - P}{\sqrt{\frac{P(1-P)}{n}}}} \qquad \left[\because \text{S.E. }(p) = \sqrt{\frac{P(1-P)}{n}}\right]$$

where,

P = Population proportion of success

$1 - P$ = Population proportion of failure

n = Sample size or total number of observations

p = Observed proportion of success

Step 3, 4 and 5 in testing the significance of sample proportion will be the same as followed in previous sections.

Illustration 23.

A coin is tossed 900 times, and heads appear 490 times. Does this result support the hypothesis that the coin is unbiased ?

Solution :

Here,

$$n = 900$$

$$p = \text{Observed proportion of success} = \frac{490}{900} = 0{\cdot}544$$

Step 1. Null Hypothesis—$H_0 : P = 1/2 = 0{\cdot}5$ *i.e.* coin is unbiased. (Since proportion of getting head is 0·5).

Alternative Hypothesis—$H_1 : P \neq 1/2 \neq 0{\cdot}5$ *i.e.* coin is biased.

Step 2. Test Statistic—Under H_0, the test statistic for large sample is—

$$Z = \frac{p - P}{\sqrt{\frac{P(1-P)}{n}}}$$

$$= \frac{0{\cdot}544 - 0{\cdot}5}{\sqrt{\frac{(0{\cdot}5)(1-0{\cdot}5)}{900}}} = \frac{0{\cdot}044}{0{\cdot}017} = 2{\cdot}59$$

Step 3. Level of Significance—

$$\alpha = 0{\cdot}05 \ (5\%)$$

Step 4. Critical Value—The critical value of Z at 5% level of significance is—

$$Z = \pm 1{\cdot}96 \text{ (from table)}$$

Step 5. Decision—Since computed value of $|Z| = 2.59$ is greater than critical value of $|Z| = 1.96$, it falls in the rejection region. Hence, null hypothesis is rejected at 5% level of significance and it may be concluded that ***the coin is biased*** (*i.e.* coin is not unbiased).

Illustration 24.

A person threw 10 dice 500 times and obtained 2,560 times 4, 5 and 6. Can this be attributed to fluctuations in sampling ?

Solution :

If occurrence of 4, 5 or 6 is termed as a success then, in the usual notations we are given—

$$n = 10 \times 500 = 5{,}000$$

$$p = \text{Observed proportion of success} = \frac{2{,}560}{5{,}000} = 0.512$$

Step 1. Null Hypothesis—$H_0 : P = 1/2 = 0.5$ *i.e.* Dice is unbiased (since proportion of getting 4, 5 or 6 in throw of a dice is 0·5).

Alternative Hypothesis—$H_1 : P \neq 1/2 \neq 0.5$ *i.e.* Dice is biased.

Step 2. Test Statistic—Under H_0, the test statistic for large sample is—

$$Z = \frac{p - P}{\sqrt{\frac{P(1-P)}{n}}}$$

$$= \frac{0.512 - 0.5}{\sqrt{\frac{(0.5)(1-0.5)}{5{,}000}}} = \frac{0.012}{0.0071} = 1.69$$

Step 3. Level of Significance—

$$\alpha = 0.05 \ (5\%)$$

Step 4. Critical Value—The critical value of Z at 5% level of significance is—

$$Z = \pm 1.96 \text{ (from table)}$$

Step 5. Decision—Since computed value of $|Z| = 1.69$ is less than the critical value of $|Z| = 1.96$, it falls in the acceptance region. Hence, null hypothesis is accepted, at 5% level of significance and it may be concluded that ***the dice is unbiased*** (*i.e.* there are fluctuations in sampling).

Illustration 25.

While throwing 5 dice 30 times, a person obtained success 23 times, securing a 6 which was considered a success. Can we consider the difference between the observed and the expected results as being significantly different ?

Solution :

If securing a 6 was considered a success, then, in the usual notations we are given :—

$$n = 5 \times 30 = 150$$

$$p = \text{Observed proportion of success} = \frac{23}{150} = 0.153$$

Step 1. Null Hypothesis—$H_0 : P = 1/6 = 0.17$ *i.e.* there is no significant difference between the observed and expected results.

5. What do you ...
6. Explain the meaning of Null Hypothesis and Acceptance region.
7. Differentiate between Critical Region and Acceptance region.
8. What do you mean by Test Statistic in test of hypothesis.
9. Explain Critical Value.
10. Differentiate between one-tailed and two-tailed test. [C.A. Found., May, 2000]

Long Answer Questions—

1. What is the "Test of Hypothesis" ? What is its purpose ? [MTU, MBA, 2011-12]
2. What are the steps in a test of significance problem ? [MTU, MBA, 2012-13]
3. What is the major purpose of hypothesis testing ? [UPTU, MBA, 2011-12]
4. Define Type I error and Type II error in tests of significance. [C.A. Foundation, May, 2004]
5. Discuss the concept of one-tailed and two-tailed tests with the help of diagrams.
6. Discuss the limitations of tests of significance.
7. How do you test the significance of the difference between the means of the two samples.
8. What is test of significance ? Discuss different tests of significance for the cases when the size of sample is large.
9. What are the various tests of significance generally used in sampling of attributes ? Discuss briefly.
10. Explain two types of errors in testing of hypothesis. [C.A. Foundation, May, 2005]

NUMERICAL QUESTIONS

Test of Significance of a mean (Large Sample)—

1. A random sample of 400 flower stems has an average length of 10 cm. Can this be regarded as a sample from a large population with mean of 10·2 cm. and a Standard deviation of 2·25 cm. ? **[Ans. : Z = – 1·78]**

2. The mean life time of sample of 100 fluorescent light tubes produced by a company is computed to be 1,570 hours with a standard deviation of 120 hours. The company claims that the average life of the tubes produced by the company is 1,600 hours. Using the level of significance of 0·05 is the claim acceptable ? **[Ans. Z = – 2·5]**

3. A sample of 400 items is taken from a normal population, whose mean is 4 and whose variance is also 4. If the sample mean is 4·45, can the sample be regarded as a truly random sample ? Give necessary justification for your conclusion. **[Ans. : Z = 4·5]**

4. A manufacturer of steel bars claims that his products have a mean breaking strength of 48 units. For a sample of 144 bars produced by his firm, the mean breaking strength is 46·2 units and the standard deviation is 8·7 units. Make a test at the 5% level of significance and state whether this evidence supports his claim. **[Ans. : Z = – 2·483]**

5. From the following data obtained from a sample of 1,000 persons, calculate the standard error of the mean :—

Daily earnings :	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of persons :	50	100	150	200	200	100	100	100

If the average of the population was ₹ 42, what conclusion can you arrive at about the reliability of the sample ? **[Ans. : $\overline{X}$ = 41, S.D. (s) = 19·34, Z = – 1·635]**

6. The average number of defective articles in a certain factory is claimed to be less than the average for all the factories. The average for all the factories is 30·5. A random sample of 100 defective articles showed the following distribution :—

Class-limits :	16—20	21—25	26—30	31—35	36—40
Number :	12	22	20	30	16

Calculate the mean and standard deviation of the sample and use it to claim that the average is less than the figure for all the factories at 5% level of significance. Give Z (– 1·645) = 0·95. **[Ans. : $\overline{X}$ = 28·8, S.D. (s) = 6·35, Z = – 2·68]**

Hint :

Null Hypothesis—There is no significant difference between the average number of defective articles in a certain factory and the average number of defective articles of all the factories *i.e.*

$$H_0 = \mu \geq 30{\cdot}5$$

Alternative Hypothesis—

$$H_1 : \mu < 30{\cdot}5 \text{ (left-tailed test)}$$

Critical value—At 5% level of significance (*i.e.* at 95% confidence level, Z = – 1·645 (given in question).

7. The mean breaking strength of the cables supplied by a manufacturer is 1,800 with a standard deviation 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cables have increased. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1,850. Can we support the claim at 0·01 level of significance. **[Ans. : Z = 3·535]**

8. A certain machine part manufactured in millions is to withstand a pressure of 100 units. The variability measured by standard deviation on the basis of thousands of parts was found to be 20 units. A random sample of size 400 was taken and the result of testing the machine parts showed that the average strength was 96·9 units. Test the hypothesis that the machine parts have an average strength of 100 units. State clearly the statistical hypothesis with which you have started and any basic theorem on sampling distribution that you have used. **[Ans. : Z = – 3·1]**

9. A sample of 100 iron bars is said to be drawn from a large number of bars, whose lengths are normally distributed with mean 4 ft. and standard deviation 0·6 ft. If the sample mean is 4·2 ft., can the sample be regarded as a truly random sample ? (Null hypothesis and assumptions should be stated clearly). **[Ans. : Z = 3·33]**

10. A random sample of 400 tins of vegetable oil, labelled "5 kg. net weight" gave a mean net weight of 4·98 kgm. with a standard deviation of 0·22 kgm. Do we reject the hypothesis of net weight of 5 kgm. per tin on the basis of this sample, at 1 percent level of significance ? **[Ans. : Z = – 1·82]**

11. The mean life time of 100 electric bulbs produced by a manufacturing company is estimated to be 1,570 hours with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis μ = 1,600 hours, against the alternative hypothesis μ ≠ 1,600 hours, using a level of significance of 0·05. **[Ans. : Z = – 2·5]**