Assignment

1. A drawer contains two coins. One is an unbiased coin, which when tossed, is equally likely to turn up heads or tails. The other is a biased coin, which will turn up heads with probability p and tails with probability $1-p$. One coin is selected (uniformly) at random from the drawer. Two experiments are performed:

   (i) The selected coin is tossed n times. Given that the coin turns up heads k times and tails $n-k$ times, what is the probability that the coin is biased?

   (ii) The selected coin is toss repeatedly until it turns up heads k times. Given that the coin is tossed n times in total, what is the probability that the coin is biased?

2. Which of the following functions are probability density functions:

   (i) $f(x) = x(2-x)$, $0 < x < 2$, and 0 elsewhere.

   (ii) $f(x) = \frac{1}{\lambda} e^{-(x-\theta)/\lambda}$, $x > \theta$, and 0 elsewhere, $\lambda > 0$.

   (iii) $f(x) = \sin x$, $0 < x < \pi/2$, and 0 elsewhere.

3. A workstation consists of three machines, M_1, M_2 and M_3, each of which will fail after an amount of time T_i which is an independent exponentially distributed random variable, with parameter 1. Assume that the times to failure of the different machines are independent. The workstation fails as soon as both of the following have happened:

   (i) Machine M_1 has failed.

   (ii) Atleast one of the machines M_2 or M_3 has failed.

   Find the expected value of the time to failure of the workstation.

4. A and B throw alternatively with a pair of balanced dice. A wins if he throws a sum of 6 points before B throws a sum of 7 points, while B wins if he throws a sum of 7 points before A throws a sum of 6 points. Suppose that A begins the game. What is the probability that A will win the game?

5. A post office has 2 clerks. Nikita enters the post office while 2 other customers, Rohan and Ankita, are being served by the 2 clerks. She is next in line. Assume that the time a clerk spends serving a customer has the Exponential(λ) distribution.

   (a) What is the probability that Nikita is the last of the 3 customers to be done being served?

   (b) What is the expected total time that Nikita needs to spend at the post office?