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Unit :- 1

Set theory

- A set is an evenly defined collection of object, bold the elements for members of set.
- It is denoted by capital letters.
- In set element are ~~return~~ ~~in~~ written in '{}' and seperated by comma.

Ex :- $A = \{a, e, i, o, u\}$

$B = \{2, 4, 6, 8\}$

* Representation of a set :-
  - Roaster method
  - Set - builder method
  - Statement method

* Roaster Method :- In this form all the elements for the set listed, the element being seperated by commas and are enclosed with in brashes.

Ex :- $A = \{0, 1\}$

$B = \{a, e\}$

* Ruled Method for set - builder form :- In this method a set is defined by specifing a property that element of set have formed.

Ex :- $A = \{x ; P(x)\}$

$A = \{1, 2, 3, 4, 5\}$.

$A = \{x ; x \in N ; x < 6\}$

$B = \{\cdots -3, -2, -1, 0, 1, 2, 3 \cdots\}$

$B = \{x ; x \in Z\}$.

* Types of sets :-

1. Finite and Infinite set :-

- A set with finite number of element in it, is called finite set.
- An infinite set is a set which contain infinite number of element.

Ex :- The set of month in a year (finite set)
The set of vowels in english (finite set)
$A = \{1, 1/3, 1/9, 1/2 \because\}$ (infinite set)
$B = \{1, 2, 3 \cdots \infty\}$ (infinite set)

2. Null set (empty (ϕ)) :-

- A set which contain no no. of elements at all is called null set.
- It is denoted by ϕ or $\{\}$.

Ex :- $A \{x;\ x^2+4=0,\ x$ is a real no.$\}$
$B \{x;\ x$ is a multiple of 4, x is odd$\}$

3. Singleton :- A set which has only one elements is called a singleton set.

Ex :- $S = \{a\}$

4. Sub Set :-

- If A and B are sets such that every elements of A is also an element of B, then A is said to be a subset of B. and it is denoted by $(A \subset B)$.
- If A is not a subset of B that is atleast one element of A doesn't belong to B. $(A \not\subset B)$.

* Every set A is a subset of itself. $A \subseteq A$
* The Null set ϕ is considered a subset of any sets. $\phi \subseteq A$
* If A is a subset of B and B is a subset of C then A is subset of C.

  $A \subseteq B$ & $B \subseteq C$ then $A \subseteq C$

Ex:- $A = \{1, 3, 4\}$ is subset of $B = \{1, 2, 3, 4, 5\}$

$A = \{6, 5, 4\}$ & $B = \{4, 5, 6\}$ then A is a subset of B & B is a subset of A.

5. Super set :-

Set A is considered the superset of B in set theory if all of the components of set B are also elements of set A.

Ex :- $A = \{1, 2, 3, 4\}$ & $B = \{1, 3, 4\}$ say that set A is a superset of B.

6. Proper set :-

A is said to be proper subset of another set B if A is a subset of B but their is a least one element of B which doesn't belong to A.

7. Equal set :-

when all the elements in two or more sets are same and the no. of elements is also same is called equal set.

8. Power Set :-

$A = \{1, 2\}$, Power $(A) = ? = 2^n$, where n is no. of elements then, $P(A) = 2^2 = 4 = [\phi, \{1\}, \{2\}, \{1,2\}]$

Q. $S = \{x, y, z\}$ find power of S.

$P(S) = 2^3$

$P(S) = 8$

$P(S) = \{\phi, \{x\}, \{y\}, \{z\}, \{x,y\}, \{y,z\}, \{x,z\}, \{x,y,z\}\}$

Q. $A = [\{a,b\}, \{c\}, \{d,e,f\}]$ find power.

$P(A) = 2^3 = 8$

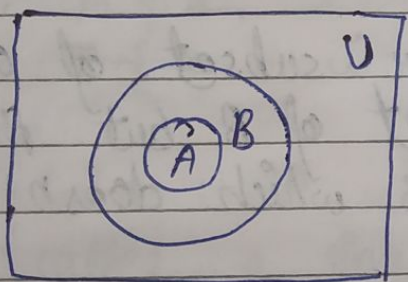
$P(A) = [\phi, \{\{a,b\}\}, \{\{c\}\}, \{\{d,e,f\}\}, \{\{a,b\},\{c\}\}, \{\{c\},\{d,e,f\}\},$

$\{\{a,b\},\{d,e,f\}\}, \{\{a,b\},\{c\},\{d,e,f\}\}]$

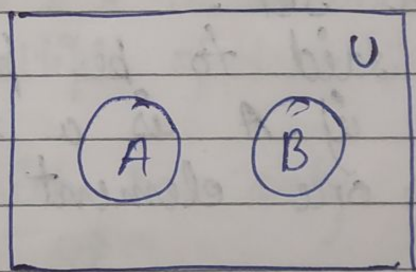
⇒ Representation of sets :-

* Venn diagram :-

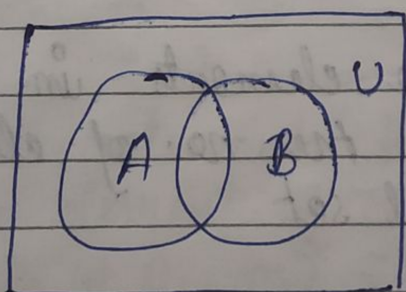
- A Venn diagram is a pictorial representation of sets are represented by enclosed area.
- It is denoted by 'U'.



$A \subseteq B$



A & B are disjoint



A & B have atleast one element common.

⇒ Set Operations :-

* **Union** :- The union of two set A & B denoted by (A ∪ B).

$(A \cup B) = \{x ; x \in A \text{ or } x \in B\}$

* **Intersection** :- The intersection of A and B denoted by $(A \cap B)$, is the set of element which belong to both A and B.

$A \cap B = \{x ; x \in A \text{ and } x \in B\}$.

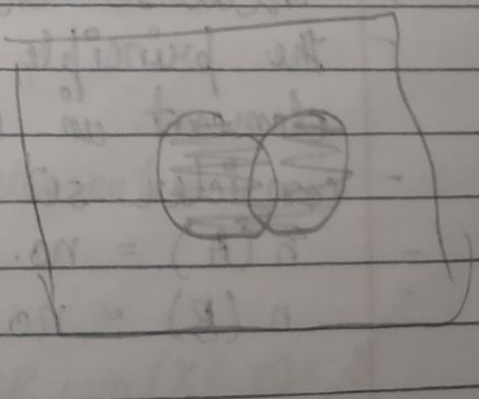
* **Complement** :- Complement of set A denoted by A^c, A', $\bar{A}$ is the sets of element which belongs to U but which don't belong to A.

$A^c \{x ; x \in U, x \notin A\}$

* **Symmetric difference** :- The symmetric difference of set A and B denoted by $A \oplus B$, consist of those elements which belong to A or B but not both.

$$A \oplus B = (A - B) \cup (B - A)$$

⇒ Laws of Algebra of set :-



* **Idemptent law** :-

- $A \cup A = A$
- $A \cap A = A$

* **Associative law** :-

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

* Commutative law :-
- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

* Distributive law :-
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

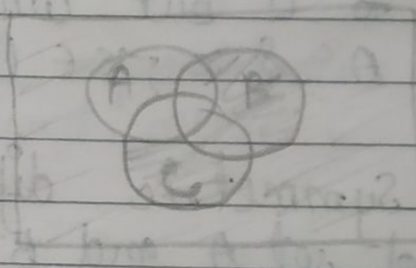
* Identity law :-
- $A \cup \phi = A$
- ~~$A \cup A = A \cup$~~ $A \cup U = U$
- $A \cap \phi = \phi$
- $A \cap U = A$

$A^c \cap (C \cup B)$

$(A^c \cap C) \cup (A^c \cap B)$

* Involution law :-
- $(A^c)^c = A$

* Complement law :-
- $A \cup A^c = U$
- $A \cap A^c = \phi$
- $U^c = \phi$
- $\phi^c = U$



* Demorgan's laws :-
- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$

⇒ Inclusive and Exclusive set :-
- the principle of count the total number of element in the set.
- consider set (finite) A, B
- $n(A)$ = no. of element in A
- $n(B)$ = no. of element in B.

* If A & B are disjoint set :-
$n(A \cup B) = n(A) + n(B)$

* If A & B are not disjoint set :-

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Q. Consider the following data for 120 mathematics student. They are study language French, German Russian, 65 studied French, 45 study German, 42 study Russian, 20 study both French and German, 25 study both French and Russian, 15 study both German and Russian, and 8 study all three languages. Find the number student who study one of three language.

$$n(F \cup G \cup R) = n(F) + n(G) + n(R) + n(F \cap R \cap G) - n(F \cap G) - n(G \cap R) - n(F \cap R)$$

$$= 65 + 45 + 42 + 8 - 20 - 15 - 25$$

$$= 100$$

no. of student who study one of three language $= 120 - 100$

$= \underline{20}$.

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Practice sheet (HW)

Q.1. Total no. of subsets of B having n elements.

Total no. of subsets of B $= 2^n$

Q.2 $n(X \cup Y) = 36$, $n(X) = 20$, $n(Y) = 28$, $n(X \cap Y) = ?$

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$

$n(X \cap Y) = \cancel{36}\ 20 + 28 - 36$

$n(X \cap Y) = 48 - 36$

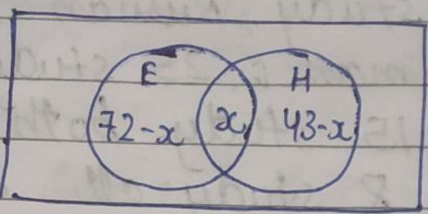
$= 12$

Q.3 100 students, 72 speaks English, 43 speaks Hindi,

i) find no. of students, speaks eng. only.

ii) find no. of students speaks hindi only

iii) find no. of students who can speak both.



$72 - \cancel{x} + \cancel{x} + 43 - x = 100$

$115 - x = 100$

$x = 15$

iii) 15 students can speak both Hindi & english.

i) $72 - 15 = 57$

57 students can speak english only

ii) $43 - 15 = 28$

28 students can speak Hindi only.

Q.5 If $x \in N$ and x is prime, then x is <u>infinite</u> set.

Q.6 Power set of empty or Null set has exactly <u>1</u> subset.

Q.7 How many element in Power set of set $A = \{\{\phi\}, \{\phi, \{\phi\}\}\}$

$n = 2$

$n(P(A)) = 2^n = 2^2 = 4$

There are 4 eliments in Power set of set A.

$\{\phi, \{\{\phi\}\}, \{\{\phi, \{\phi\}\}\}, \{\{\phi\}, \{\phi, \{\phi\}\}\}\}$

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Q.8 cardinality of set of odd positive integers less than 10.

$\{1, 3, 5, 7, 9\}$

cardinality = 5

Q.9 which two sets are equal ?

i) $A = \{1,2\}$ & $B = \{1\}$

ii) $A = \{1,2\}$ & $B = \{1,2,3\}$

iii) $A = \{1,2,3\}$ & $B = \{2,1,3\}$

iv) $A = \{1,2,4\}$ & $B = \{1,2,3\}$

Ans iii) $A = \{1,2,3\}$ & $B = \{2,1,3\}$

Q.10 Prove, $(A \cup B) \cap (A \cup B^c) = A$

we, know that by distributive property

$\{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\}$

So, $(A \cup B) \cap (A \cup B^c) = A \cup (B \cap B^c)$

and $\{\because B \cap B^c = \phi\}$

then, $(A \cup B) \cap (A \cup B^c) = A \cup \phi$

$\{\because A \cup \phi = A\}$

then, $(A \cup B) \cap (A \cup B^c) = A$

Hence, Proved.

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⇒ Conditions :-

* $x \in A \cup B \Rightarrow x \in A$ or $x \in B$
* $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$
* $x \in A \cap B \Rightarrow x \in A$ and $x \in B$
* $x \in A - B \Rightarrow x \in A$ and $x \notin B$
* $x \in A = x \notin A^c$
* $x \in A^c = x \notin A$
* $A = B$, $A \subset B$, $B \subset A$

Q. $(A \cup B)^c = A^c \cap B^c$

Let $x \in (A \cup B)^c$

$x \notin (A \cup B)$

$x \notin A$ and $x \notin B$

$x \in A^c$ and $x \in B^c$

$x \in A^c \cap x \in B^c$

$x \in A^c \cap B^c$

$x \subset A^c \cap B^c$ — (1)

Let $y \in A^c \cap B^c$

$y \in A^c$ and $y \in B^c$

$y \notin A$ and $y \notin B$

$y \notin A \cup B$

$y \in (A \cup B)^c$

$y \subset (A \cup B)^c$ — (2)

⇒ Partitions :-

* Let S be a non-empty set, a partition of S is a sub-division of S into non-overlapping, non-empty subset.
* A partition of S is a collection $\{A_i\}$ of non-empty S.

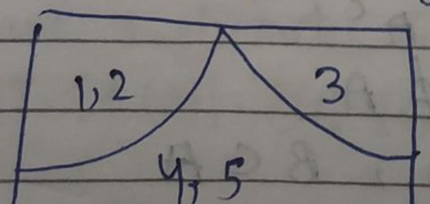
i) a in S belongs to one of the A_i

ii) $\{A_i\}$ are mutually disjoint,

$A_i \neq A_j$ then $A_i \cap A_j = \phi$

Eg:- $S = \{1, 2, 3, 4, 5\}$

$= [\{1, 2\}, \{3\}, \{4, 5\}]$



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Relations

* **Ordered pair :-**

Ordered pairs (a, b) of elements, where a is designed as the first element and b as the second element,

$$(a, b) = (c, d)$$

if and only if $a = c$ and $b = d$.

* **Product sets :-**

Consider two aribitary sets and A 4 B the set of all ordered pairs (A, B), here 'a' belongs to 'A' and 'b' belongs to 'B' then it is called product sets or cartesian product.

$$a \in A$$

$$b \in B$$

$$A \times B = \{ (a, b), x \in A, x \in B \}$$

Eg :- $A = \{1, 2\}$, $B = \{a, b, c\}$

$A \times B = \{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$

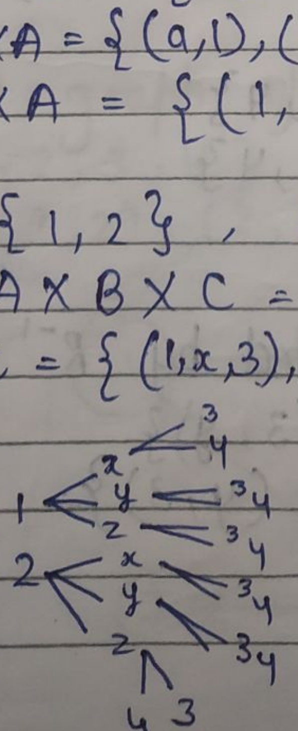
$B \times A = \{ (a,1), (a,2), (b,1), (b,2), (c,1), (c,2) \}$

$A \times A = \{ (1,1), (1,2), (2,1), (2,2) \}$

Eg :- If $A = \{1, 2\}$, $B = \{x, y, z\}$, $C = \{3, 4\}$

find $A \times B \times C$ = 12 elements

$A \times B \times C = \{ (1,x,3), (1,x,4), (1,y,3), (1,y,4), (1,z,3), (1,z,4),$
$(2,x,3), (2,x,4), (2,y,3), (2,y,4),$
$(2,z,3), (2,z,4) \}$



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⇒ Relation :- suppose R is a relation a to b and then R is a set of ordered pair where each first element comes from 'A' and second element comes from 'B'.

i) $(a,b) \in R$ this means aRb

ii) $(a,b) \notin R$ this means $a \not R b$

The domain of relation ~~are~~ R is a set of all first elements of ordered pairs belongs to R and range of R is a set of second element

Ex:- $A = \{1, 2, 3\}$, $B = \{x, y, z\}$

$R = \{(1,y)\ (1, z), (3,y)\}$

$1 \not R x$, $1 R y$, $1 R z$, $3 R y$

Domain $(R) = \{1, 3\}$

Range $(R) = \{x, y, z\}$

Q. Let A be the set $A = \{1, 2, 3, 4\}$ and $R = \{(a, b) \mid a$ is divisible by $b\}$

$R = \{(1,1), (2,1), (3,1), (4,1), (4,2), (2,2), (3,3), (4,4)\}$

Domain $(R) = \{1, 2, 3, 4\}$

Range $(R) = \{1, 2, 3, 4\}$

⇒ Inverse Relation :-

The Inverse of R denoted by R^{-1}

Eg $R = \{(1,y), (1, z), (3, y)\}$

$R^{-1} = \{(y,1), (z,1), (y,3)\}$

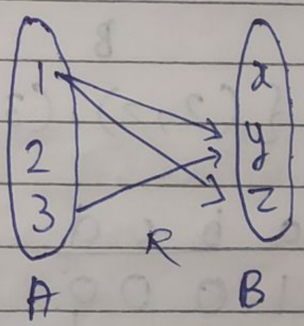
⇒ **Representation of Relation in finite set :-**
Suppose A & B are finite sets, the following are two way for picturing a relation R A to B.

i) From a rectangular array whose row's are lebeled by the element of A & whose columns are lebeled by the element of B, Put 1 or 0 in each position of the array, according as a ∈ A or is not Related to b ∈ B.

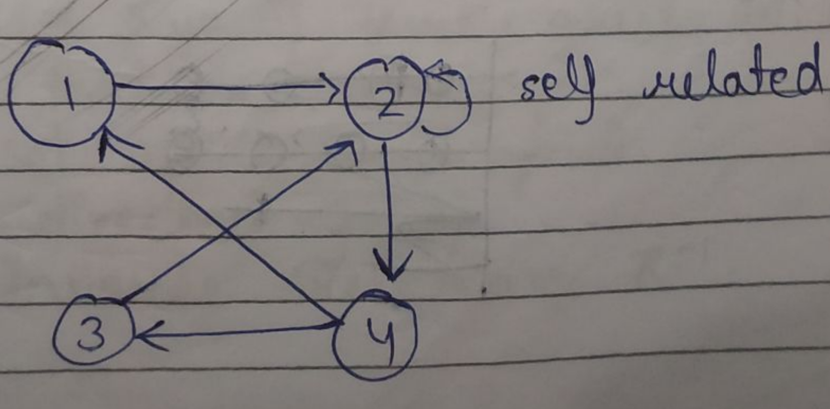
ii) write down the element of A & B in two disjoint disc and then draw an arrow from A to B, is called arrow diagram of the relation.

Eg:- $A = \{1, 2, 3\}$, $B = \{x, y, z\}$
$R = \{(1, y), (1, z), (3, y)\}$

	x	y	z
1	0	1	1
2	0	0	0
3	0	1	0



⇒ **Directed graph of relation on sets :-**
Eg:- $A = \{1, 2, 3, 4\}$
$R = \{(1, 2), (2, 2), (2, 4), (3, 2), (4, 1), (4, 3)\}$



⇒ Composition of Relation :-

Let A, B, C are sets and R is the relation from A to B, S is the relation from B to C

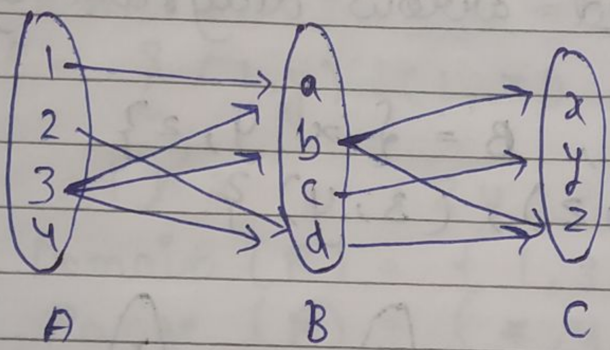
$a(R \circ S)c$ or $R \circ S = \{(a,c)$ exist $b \in B$ for which $(a,b) \in R$ and $(b,c) \in S\}$

Eg:- $A = \{1,2,3,4\}$, $B = \{a,b,c,d\}$

$C = \{x,y,z\}$

$R = \{(1,a),(3,a),(2,d),(3,d),(3,b)\}$

$S = \{(b,x),(b,z),(c,y),(d,z)\}$



$R \circ S = \{(2,z),(3,x),(3,z)\}$

Q.

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array} \qquad \begin{array}{c|ccc} & x & y & z \\ \hline a & 0 & 0 & 0 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 0 \\ d & 0 & 0 & 1 \end{array}$$

MR MS

4×4 4×3

4×3

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

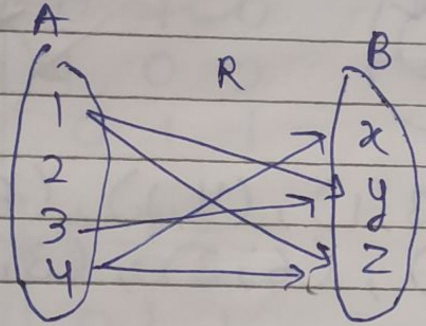
Q. $A = \{1, 2, 3, 4\}$, $B = \{x, y, z\}$,
$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$

a) Determine the matrix of relation
b) Draw the arrow diagram
c) find the inverse relation
d) Determine the domain & Range of R.

a)

$$\begin{array}{c|ccc} & x & y & z \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 \end{array} = M_R$$

b)



c) $R^{-1} = \{(y, 1), (z, 1), (y, 3), (x, 4), (z, 4)\}$

d) Domain (R) $= \{1, \cancel{2}, 3, 4\}$
Range (R) $= \{x, y, z\}$

Q. Let $A = \{1, 2, 3, 4, 6\}$ and let R be the relation on A defined by x divides y
(Note :- x/y if there exist an integer z such that $xz = y$)

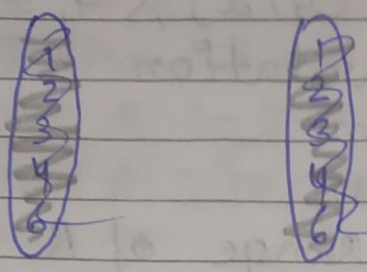
i) R as a set of ordered pair
ii) Draw its directed graph
iii) find the inverse relation R^{-1}.

~~R = {(2, 2, 1)}~~

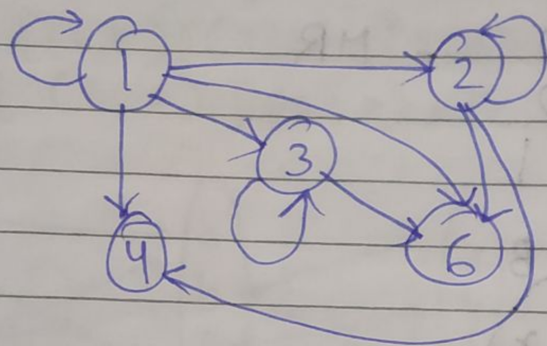
$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$

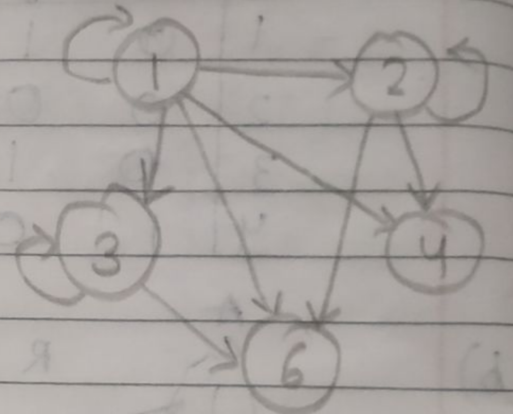
$\frac{2}{4}, \frac{2}{6}, \frac{2}{2}, \frac{3}{3}, \frac{3}{6}$

$x = 2$
$y = 2$
$\frac{x}{y}$
2×1



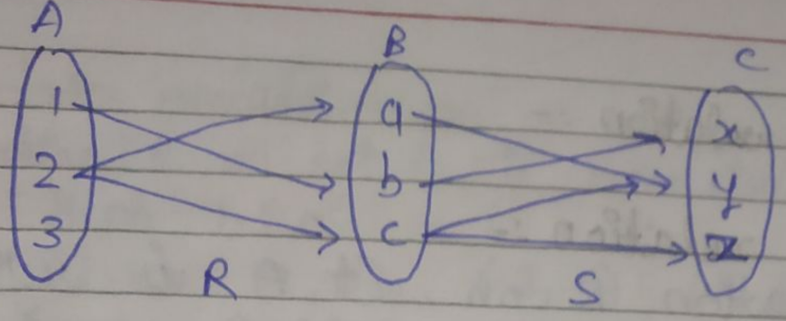
$R = \{(1,1), (1,2), (1,3), (1,4), (3,6), (1,6), (2,2), (2,4), (2,6), (3,3)\}$





$R^{-1} = \{(1,1), (2,1), (3,1), (4,1), (6,3), (6,1), (2,2), (4,2), (6,2), (3,3)\}$

Q. Let $A = \{1, 2, 3\}$, $B = \{a, b, c\}$
$C = \{x, y, z\}$ consider following relation R & S from A to B and from B to C.
$R = \{(1,b), (2,a), (2,c)\}$
$S = \{(a,y), (b,x), (c,y), (c,z)\}$
i) find the composition relation RoS
ii) find the matrices MR, MRoS, MS and MR · MS



i) $R \circ S = \{(1,x),(2,y),(2,z)\}$

ii)
$$\begin{array}{c|ccc} & a & b & c \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{array} = MR$$

$$\begin{array}{c|ccc} & x & y & z \\ \hline a & 0 & 1 & 0 \\ b & 1 & 0 & 0 \\ c & 0 & 1 & 1 \end{array} = MS$$

$$\begin{array}{c|ccc} & x & y & z \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 \end{array} = MR_oS$$

$$MR \cdot MS = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$MR \cdot MS = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

⇒ Types of relation :-

* Reflexive relation :-

A relation R on set A is reflexive if aRa for every $a \in A$ i.e $(a,a) \in R$ for every $a \in A$, ~~and~~ R is not reflexive if there exist where an $a \in A$ such that $(a,a) \notin R$.

Eg:- Consider a following relation on set $A = \{1,2,3,4\}$

i) $R_1 = \{(1,1),(2,3),(4,4)\}$(

ii) $R_2 = \{(1,1),(1,2)(2,1),(2,2),(3,3)$
$(4,4\}$

iii) $R_3 = \{(1,3),(2,1)\}$

iv) $R_4 = \{\phi\}$

v) $R_5 = A \times A$

R_1 is not reflexive
R_2 is reflexive
R_3 is not reflexive
R_4 is not reflexive
R_5 is reflexive

* Irreflexive relation :-

A relation R on a set is irreflexive if $(a,a) \notin R$ for every $a \in A$, R is not irreflexive if there exists $a \in A$ such that $(a,a) \in R$

R_3 is irreflexive
R_4 is irreflexive

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* Symmetric relation :-
A relation R on set A is symmetric if whenever aRb then bRa
$R_1 = \{(1,1), (1,2), (1,3), (4,4)\}$
R_1 is not symmetric.
$R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$
R_2 is symmetric.
$R_4 = \phi$
R_4 is symmetric
$R_5 = A \times A$
R_5 is symmetric.
$\{1,2\} \subseteq \{1\}$ is not symmetric.

* Anti-symmetric relation :-
A relation R on a set A is anti-symmetric if whenever aRb and bRa then a = b
R_4, R_2 & R_5 is anti-symmetric relation.
R_1, R_3, ~~R_4~~ is not anti-symmetric relation.

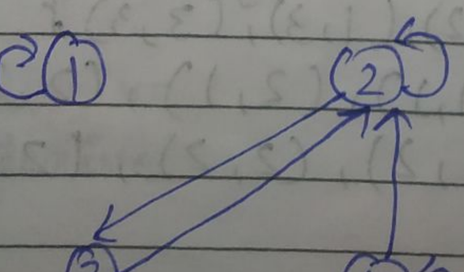
* Transitive relation :-
A relation R on a set A is transitive if aRb and bRc then aRc.
R_1 is transitive.
R_4, R_2, R_5 is transitive

Q. Consider the following relation in set $A = \{1,2,3\}$
$R = \{(1,1), (1,2), (1,3), (3,3)\}$
$S = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$
$T = \{(1,1), (1,2), (2,2), (2,3)\}$
$A \times A = X$

Determine whether or not each of the above relation.
a) reflexive
b) symmetric
c) Transitive
d) Anti - symmetric

a) Reflexive
R is not reflexive
S is reflexive
T is not reflexive
X is reflexive
b) symmetric
S and X is symmetric
R and T are not symmetric
c) Transitive
X is transitive, R is transitive, S i
T is not transitive, X is tr
d) Anti - symmetric
R is anti - symmetric, S is anti - symmetric

Q. $A = \{1, 2, 3, 4\}$
$R = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$
a) Draw its directed graph
b) Is R reflexive, transitive, symmetric and anti - symmetric.
c) RoR

a)


b) R is symmetric only.

c) $R \circ R = \{(2,3), (2,2), (1,1), (4,4), (4,2)\}$

8/9/23 ⇒ Reflexive and symmetric closure :-

i) $R \cup \Delta_A$ is the reflexive closure of A.

ii) $R \cup R^{-1}$ is the symmetric closure of R.

Ex:- $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3)\}$

Reflexive $(R) = R \cup \{(2,2), (4,4)\}$

$= \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3), (2,2), (4,4)\}$

Symmetric $(R) = R \cup \{(4,2), (3,4)\}$

$= \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3), (4,2), (3,4)\}$

Ex:- $A = \{1, 2, 3\}$

$R = \{(1,2), (2,3), (3,3)\}$

$R \circ R = \{(1,3), (2,3), (3,3)\} = R^2$

$R^2 \circ R = \{(1,3), (2,3), (2.$

Q. Consider a set $A = \{a, b, c\}$, and Relation R on A defined by A

$R = \{(a,a), (a,b), (b,c), (c,c)\}$

find Reflexive (R), symmetric (R), Transitive (R

Reflexive $(R) = R \cup \{(b,b)\}$

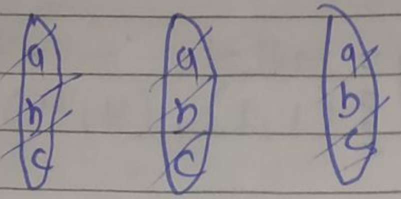
$= \{(a,a), (a,b), (b,c), (c,c), (b,b)\}$

symmetric$(R) = R \cup \{(b,a), (c,b)\}$

$= \{(a,a), (a,b), (b,c), (c,c), (b,a), (c,b)\}$

$R \circ R = \{~~(a,b), (a,c), (b,c), (a,a), (b,b)~~\}$

$R_2 \circ R = \{(a,a), (a,b), (a,c)\}, (b,c), (b,b)\}$



$R \cup \Delta A$

$R \cup R^{-1}$

$R \cup R^2 \cup R^3$

Transitive closure $= R^n$

Transitive $(R) = R \cup R^2 \cup \cdots R^n$

⇒ Equivalence Relation :-

i) for every $a \in S$ aRa

ii) If aRb then bRa

iii) If aRb then bRc then aRc

Ex:- $A = \{1, 2, 3, 4, 5\}$ and let R be Relation on A, such that.

$R = \{(x, y) \mid x+y = 5\}$

$R = \{(1,4), (4,1), (2,3), (3,2)\}$

i) not reflexive

ii) it's symmetric

iii) not transitive

Hence, it is not equivalence relation.

⇒ Equivalence relation and partition

Consider the z of integer and an integer $m > 1$ we say that x is congruent to y modulo m ~~~~ written as

$$x \equiv y \pmod{m}$$

Q. if $x - y$ is divisible by m, show that equivalence relation on z.

* Reflexive : for any x in Z, we have

$x \equiv x \pmod m$ because $x - x = 0$ then $\frac{x-x}{m}$ is divisible by m.

Hence the relation is reflexive.

* Symmetric : suppose,

$x \equiv y \pmod m$, so

$x-y$ is divisible by m

then, $-\frac{(x-y)}{m} = \frac{y-x}{m}$

So ~~then~~, $y \equiv x \pmod m$; thus, the relation is symmetric.

* Transitive : Now suppose,

$x \equiv y \pmod m$ & $y \equiv z \pmod m$,

so, $\frac{x-y}{m}$ & $\frac{y-z}{m}$. then the sum

is $\frac{x-\cancel{y}}{m} + \frac{\cancel{y}-z}{m} \Rightarrow \frac{x-z}{m}$.

then, we have $x \equiv z \pmod m$, thus, the relation is transitive.

According the relation of congruence modulo m is on Z is an equivalence relation

Q. Let A be a set of non-zero integer & let $\approx$ be the relation on $A \times A$ defined by $(a,b) \approx (c,d)$ whenever $ad = bc$, Prove that $\approx$ is an equivalence relation.

* Reflexive :- suppose $(a,b) \approx (a,b)$ $\quad c=a, d=b$

then $ad = bc \Rightarrow ab = ba$

Hence, the relation is reflexive.

* Symmetric :- If $(a,b) \approx (c,d)$,
whenever, $a\cancel{d} = bc$
then, $cb = da$
then $(c,d) \approx (b,a)$
Hence, the relation is symmetric.

* Transitive :-
If $(a,b) \approx (c,d)$ & $(c,d) \approx (e,f)$
then $ad = bc$ & $cf = de$
$a\cancel{d}\cancel{c}f = b\cancel{c}\cancel{d}e$
$af = be$
~~then~~ so, $(a,b) \approx (e,f)$
Hence, the relation is transitive.

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Q. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let $\sim$ be the relation on $A \times A$
$(a,b) \sim (c,d)$
if $a+d = b+c$ then Prove that $\sim$ is an equivalence relation.

* Reflexive :- suppose $(a,b) \sim (a,b)$
Then $a+b = b+a$
Hence, the relation is reflescive.

* Symmetric :- If $(a,b) \approx (c,d)$
$a+d = b+c$
$c+b = d+a$
so, $(c,d) \sim (b,a)$
Hence, the relation is symmetric.

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* Transitive :-

If $(a, b) \sim (c, d)$ & $(c, d) \sim (e, f)$

then $a + d = b + c$ & $c + f = d + e$

$$a + \cancel{d} + \cancel{c} + f = b + \cancel{c} + \cancel{d} + e$$

$$a + f = b + e$$

So, $(a, b) \sim (e, f)$

Hence, the relation is transitive

therefore, the relation is equivalence relation

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→ Partial order relation :-

Reflexive, Asymmetric, transitive.

Unit

Logic & Propositional

⇒ **Proposition :-**

A proposition is a declarative sentence which is either true or false but not both.

Eg:- Paris is in France.

$1+1=2$

$9<6$

$x=2$ is solution of $x^2=4$

Where are you going? } Not proposition.
Do your homework.

⇒ **Compound proposition :-**

Many proposition are composit, i.e composed of sub-proposition and various connection such composit proposition are called compound proposition.

Eg:- Roses are red, and Voilets are blue.

John is intelligent or studies every night.

⇒ **Basic logical operations :-**

i) **Conjunction ($p \wedge q$) :-**

Any two proposition can be combined by the word "and", to form a compound proposition called the conjunctions which is denoted by '$p \wedge q$'.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

ii) Disc-junction :-
Any two proposition can be combined by the word "or" to form a compound proposition called dis-junction, which is denoted by '$p \vee q$'.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

iii) Negation :-
If p is true than $\neg p$ is false.

p	¬p
T	F
F	T

Q. Find the Truth Table for negation $p \wedge \neg$

p	q	¬q	p∧¬q	¬(p∧¬q)
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

* Precidence :- $\neg > \wedge > \vee$

⇒ Tautologies & Contradiction :-

* Some proposition $P(p\ q - \dots)$ contain only T in the last column of their Truth table or in other word they are true for any truth values of their varible such propositions are called Tautologies.
* A proposition $P(p\ q \dots)$ is called a contradiction if it contains only F in the last column of the truth table.

Eg :-

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

$p \vee \neg p$ → Tautologies

$p \wedge \neg p$ → Contradiction

Q. Let p be "it is cold" and q be "it is rainy"

i) $\neg p$ it is not cold.

ii) $p \wedge q$ it is cold and it is rainy.

iii) $p \vee q$ it is cold or it is rainy.

iv) $q \vee \neg p$ it is rainy or it is not cold.

Q. verify that the proposition $p \vee \neg(p \wedge q)$ is tautology.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

All are True in last column, Hence, it is tautogy.

~~Q. Let P (pq ---)~~

→ Logical equivalent :-

Two proposition $P(p, q \ldots)$ and $Q(p, q \ldots)$ are set to be logical equivalent ~~to~~ or equal denoted by $P(p, q \ldots) \equiv Q(p, q \ldots)$ if they have identical truth table.

Eg :- Consider the statemente "it is not the case that Roses are red and voilet are blue.

$\Rightarrow \neg(P \wedge q) = \neg P \vee \neg q$

P	q	$\neg p$	$\neg q$	$P \wedge q$	$\neg(P \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Hence, it is logical equivalent.

Same or identical truth table.

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⇒ Laws of algebra of proposition :-

* Idempotent law :-

a) $P \vee P \equiv P$

b) $P \wedge P \equiv P$

* Associative law :-

a) $(P \vee q) \vee r \equiv P \vee (q \vee R)$

b) $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$

* Commutative law :-

a) $P \vee q \equiv q \vee P$

b) $(P \wedge q) \equiv (q \wedge P)$

* Identity law :-

a) $P \vee T \equiv T$ $\quad$ $P \vee T \equiv T$

b) $P \wedge T \equiv P$ $\quad$ $P \wedge T \equiv P$

c) $P \vee F \equiv P$ $\quad$ $P \vee F \equiv P$

d) $P \wedge F \equiv F$ $\quad$ $P \wedge F = F$

* Complement law :-

a) $P \vee \neg P \equiv T$

b) $P \wedge \neg P = F$

c) $\neg T \equiv F$

d) $\neg F \equiv T$

* Involution law :-

a) $\neg\neg P \equiv P$

* Distributive law :-

a) $P \vee (q \wedge r) = (P \vee r) \wedge (P \vee r)$

b) $P \wedge (q \vee r) \equiv (P \wedge r) \vee (P \wedge r)$

* Demorgan's law :-

a) $\neg(P \vee q) \equiv \neg P \wedge \neg q$

b) $\neg(P \wedge q) \equiv \neg P \vee \neg q$

⇒ Conditional and Biconditional statements :-

* "If p then q" such statements are called conditional statement, denoted by $p \to q$ i.e p implies q or p only if q.
* "p if and only if q" such statements are called biconditional statement.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(1)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(2)

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(3)

* from Table ① & ③ $p \to q \equiv \neg p \vee q$

Eg:- Rewrite the following statement without using the conditional :-

i) If it is cold, he wear a hat.
ii) If productivity increases, then wages rise

i) It is not cold or he wears a hat.
ii) Productivity does not increases, or wages rise.

(Converse) (Contrapositive)
(Conditional) (Inverse)

p	q	¬p	¬q	p→q	q→p	¬p→¬q	¬q→¬p
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Q. Determine the contrapositive of these statement.

i) If John is a poet, then he is poor.
ii) Only if Mark studies, will he pass the test.
i) If John not poor then he is not a poet.
ii) Only if Mark will not pass the test then he does not studies.

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Q. Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is contradiction.

p	q	$p \wedge q$	$p \vee q$	$(p\wedge q) \wedge \neg(p \vee q)$	$\neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	F	T

⇒ Arguments :-

* It is an assertion that a given set of proposition ($P_1, P_2 --- P_n$) ~~for~~ called premises.
* An other set of proposition Q called conclusion, such an argument is denoted by ($P_1, P_2 --- P_n \vdash Q$)
* An argument ($P_1, P_2 --- P_n \vdash Q$) is

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said to be valid if capital Q is true whenever all possible premises $p_1, p_2 \ldots p_n$ are true.

* An argument which is not valid is called fallacy.

Eg:- $p, p \rightarrow q \vdash q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ valid

both premises and conclusion

Q. $p \rightarrow q, q \vdash p$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ valid

→ This is not valid

→ In the first row all are true i.e (Premises and conclusion are) and in third row premises are true but conclusion is not true so it is fallacy condition.

Q. Show that the following argument is fallacy.

$p \rightarrow q, \neg p \vdash \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

→ fallacy

→ Valid

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⇒ Theorem :-

An argument $p_1, p_2 \dots p_n \vdash Q$ is valid if and only if the proposition p_1 and p_2 $(p_1 \wedge p_2 \dots \wedge p_n) \rightarrow q$

Eg :- $p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$ (t)	$(p \rightarrow q) \wedge (q \rightarrow r)$ (s)	$s \rightarrow t$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Q. Consider the following argument.

S_1 : If a man is a bachelor he is unhappy.

S_2 : If a man is unhappy, he dies young.

- -

S_3 : Bachelors die young

p = he is a bachelor.

q = he is unhappy.

r = he dies young

$S_1 : p \rightarrow q$

$S_2 : q \rightarrow r$

$p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$

Q. Test the validity of the following argument,

S_1 : If $7 < 4$ then, 7 is not prime.

S_2 : $7 \nless 4$

- -

S_3 : 7 is a prime no.

$S_1 : p \rightarrow \neg q$

$S_2 : \neg p$

$p \rightarrow \neg q, \neg p \vdash q$

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$(p \rightarrow \neg q) \wedge (\neg p)$	$((p \rightarrow \neg q) \wedge (\neg p)) \rightarrow$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

Fallacy

It is not valid.

Date:

Q. Test the validity of the following argument
S_1 : If two sides of a triangle are equal, the opposites angles are equal.
S_2 : Two sides of a triangle are not equal

S_3 : The opposite angles are not equal.

$$p \to q , \neg p \vdash \neg q$$

p	q	¬p	¬q	p→q	(p→q) ∧ ¬p	[(p→q)∧¬p] → ¬q
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Fallacy

It is not valid

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⇒ Logical implication :-

P(p, q ---) is said to logically imply
i) a proposition Q (p, q ---).
* If q (p. q ----) is true whenever,

$$P \Rightarrow p \vee q$$

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

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⇒ Theorem :-

For any proposition $P(p, q \cdots)$ and $Q(P, q \cdots)$ for ~~any~~ the following three statements are equivalent :-

i) $P(P, q \cdots)$ logically implies $Q(p, q \cdots)$.

ii) The arguments $P(P, q \cdots) \vdash Q(P, q \cdots)$ is valid.

iii) The proposition $P(p, q \cdots) \rightarrow Q(p, q \cdots)$ is a tautology

⇒ Proposition function Quantifiers :-

A propositional function define on A is an expression $P(x)$ which has the property that $P(a)$ is true or false for each $a \in A$.

$$T_P \{ x : x \in A , p(x) \text{ is true} \} \text{ or}$$
$$T_P = \{ x : P(x) \}$$

Ex:- Let $p(x)$ be $x + 2 > 7$ 4 x belongs to N

Find its truth set.

Truth set :-

$$\{ x ; x \in N, x + 2 > 7 \}$$
$$= \{ 6, 7, 8, \cdots \} \text{ is true}$$

Ex:- Let $P(x)$ be $x + 5 < 5$ 4 $x \in N$, find truth set

Truthe set :-

$$\{ x : x \in N, x + 5 < 3 \} = \phi$$

P.T.O

⇒ Universal Quantifiers :-

Let $P(x)$ be a propositional function define on set A, then

$$(\forall x \in A)\, p(x) \quad \text{or} \quad \forall x, p(x)$$

↓

$\left\{\begin{array}{l}\text{for all or} \\ \text{for every value}\end{array}\right\}$ An universal quantifier

$$T_P = \{x, x \in A, p(x)$$

Ex:- The proposition $(\forall n \in N)(n+4>3)$:-
$= \{1, 2, 3 \dots\}$ True for all, it's a universal quantifier.

Ex:- $(\forall n \in N)(n+2>8)$
$= \{7, 8, 9, 10 \dots\}$ True
but for $\{1,2,3,4,5,6\}$ is false so, it's not a universal quantifier.

⇒ Existential Quantifiers :-

Let $P(x)$ be a propositional function define on set A.

$$(\exists x \in A)\, p(x) \quad \text{or} \quad \exists x, p(x)$$

↓

For some or
For at least one

$$T_P = \{x : x \in A, p(x)\} \neq \phi$$

Ex:- The proposition $(\exists n \in N)(n+4<7)$
for $\{1,2\}$, is True, so it's a Extential quantifie[r]

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Ex:- The preposition $(\exists n \in N)(n+6 \leq 4)$
It is equal to ϕ, hence, it is not an Existential quantifier.

Q. Let $A = \{1, 2, 3, 4, 5\}$ determine the truth value of each of the following statements :-

a) $(\exists x \in A)(x + 3 = 10)$
False, Not existential quantifier

b) $(\forall x \in A)(x + 3 < 10)$
True, Universal quantifier.

c) $(\exists x \in A)(x + 3 < 5)$
True for $\{1\}$, so, existential quantifier.

d) $(\forall x \in A)(x + 3 \leq 7)$
Not True for $\{5\}$ so, Not universal quantifier

Q. Determine the truth value of each of the following statement where $U = \{1, 2, 3\}$.

a) $\exists x \forall y,\ x^2 < y + 1$
b) $\forall x \exists y,\ x^2 + y^2 < 12$
c) $\forall x \forall y,\ x^2 + y^2 < 12$

a) True :-
$x = 1$ and $y = \{1, 2, 3\}$
$1^2 < 1+1 \Rightarrow 1 < 2$, $1^2 < 1+2 \Rightarrow 1 < 3$,
$1^2 < 1+3 \Rightarrow 1 < 4$
for if $x = 1$ then, 1, 2, 3 are all solution of $x^2 < y + 1$.

P.T.O

b) True :-

$x = \{1, 2, 3\}$ and $y = \{1\}$

$1^2 + 1^2 < 12 \Rightarrow 2 < 12$, $2^2 + 1^2 < 12 \Rightarrow 4 + 1 < 12 \Rightarrow 5 < 12$

$(3^2 + 1^2) < 12 \Rightarrow 9 + 1 < 12 \Rightarrow 10 < 12$

for if $y = 1$ then 1, 2, 3 are all solution of x in $x^2 + y^2 < 12$.

c) False :-

for $x = 1, 2, 3$ and $y = 1, 2, 3$ then all solution are not valid for $x^2 + y^2 < 12$, like

$x = 2$, $y = 1, 2, 3$

$2^2 + 3^2 < 12$

$4 + 9 < 12$

$\times$ $13 < 12$ (Not True)

Q. If I study hard, I shall succeed.

If I don't succeed, I didn't study hard.

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⇒ Negation of Quantified statement :-
Consider statement on ~~map majors~~
" All math majors are male."

$\forall$ - It is not the case that all the math majors are male. $\neg(\forall x \in M)$ x is male.

$\exists$ - There exist at least one math major who is female. $(\exists x \in (M))$ x is female.

$$\neg(\forall x \in M) \text{ x is male} \equiv (\exists x \in M) \text{ x is female.}$$

* $p(x) \rightarrow$ x is male, then

$$\neg(\forall x \in M)\, p(x) \equiv (\exists x \in M)\, \neg(p(x))$$
$$\neg \forall x\, p(x) \equiv \exists x\, \neg P(x)$$

Q. Negate each of the following statement :-

a) $\exists x \forall y,\ p(x,y)$
b) $\forall x \forall y,\ p(x,y)$
c) $\exists y \exists x \forall z,\ p(x,y,z)$

a) $\neg(\exists x \forall y,\ p(x,y)) \equiv \forall x \exists y,\ \neg p(x,y)$
b) $\neg(\forall x \forall y,\ p(x,y)) \equiv \exists x \exists y,\ \neg p(x,y)$
c) $\neg(\exists y \exists x \forall z,\ p(x,y,z)) \equiv \forall y \forall x \forall z,\ \neg p(x,y,z)$

⇒ Counter example :-
It is an exception to a proposed general rules or laws and often appear as an example, which disproves a universal statement.

Eg:- If p is m odd prime, then p+2 is also a prime

a) 3
b) 5
c) $7 \rightarrow 7+2 = 9$ which is not a prime no., so it is counter example.
d) 9

⇒ Well - formed formula :-

* A grammatically correct expression is called a well formed formula.
* A well formed formula can be generated by the following rules :-

i) All variable and constants are well formed formula.

ii) If p is well formed formula then, $\neg p$ is also a well formed formula.

iii) If P & Q are well formed formula then $(P \wedge q)$, $(P \vee q)$, $(P \rightarrow q)$, $(P \Rightarrow q)$ are all well formed formula.

iv) A statement formula consists of variable, parenthesis, and connectives is recursively well formed formula.

Eg:- $\neg(P \vee q)$ and $(p \rightarrow (P \wedge q))$ are well formed ~~fw~~ formula where $\wedge q$ and $(P \wedge q) \Rightarrow \wedge P$ are not well formed formula.

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Unit :- 2

functions

⇒ Function :-

A function is a special case of relation, let A ~~Bt~~, B be are two non-empty sets and R is a relation from A to B, then R may not relate an element of A to an element of B or it may relate an element of A to more than one element of B. But a function relate ~~eb~~ each element of A to a unique element of B.

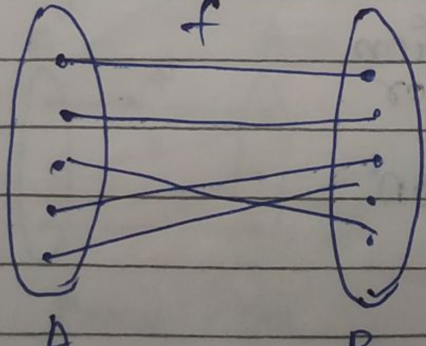
→ Defination :-

* Let A and B are two non-empty sets a function 'f' from A to B is a set of ordered pairs, $f \subseteq A \times B$ with the property that for each element x in A there is a unique element in Y is B ~~by x in B~~ such that X Y belong to f.
* The statement f is a function from A to B

$$f : A \rightarrow B$$

$$A \xrightarrow{f} B$$

* A function can be represented in pictorial form



A B

* If f is function from A to B then A is called a domain of f denoted by "dom f" and set B is called the co-domain.
* If $(x, y) \in f$ then $y = f(x)$, y is called ~~for~~ the image of x and x is the pre-image of y.
* Eg :-

Let $A = \{1, 2, 3, 4, 5\}$

$B = \{0, 1, 2, 3, 5, 7, 9, 12, 13\}$

and

a) $f = \{(1,1), (2,0), (3,7), (4,9), (5,12)\}$

It is a function

Range $= \{1, 0, 7, 9, 12\}$

b) $f = \{(1,3), (2,3), (3,5), (4,9), (5,9)\}$

It's a function

Rang $= \{3, 5, 9\}$

c) $f = \{(1,1), (2,3), (4,7), (5,12)\}$

It is not a function from A to B because the element 3 of A has no image in B.

d) $f = \{(1,1), (2,3), \cancel{(5,7)}, (3,5), (3,7), (4,9)\}$

f is not a function because ~~the element~~ the different pairs $(3,5)$, $(3,7)$ have same first component.

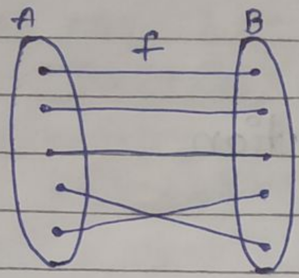
⇒ TYPES OF FUNCTIONS :-

i) One to One function

ii) Injective function or

iii) Many one function

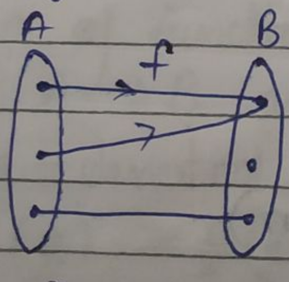
i) One to One function :-
A function in A to B is one to one or injective, if for all elements x_1, x_2 in A, such that $f_{x_1} = f_{x_2}$
$f(x_1) = f(x_2)$ it implies $x_1 = x_2$.



Eg:-
* Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$ and
$f(1) = a$, $f(2) = c$, $f(3) = d$
It's a injective

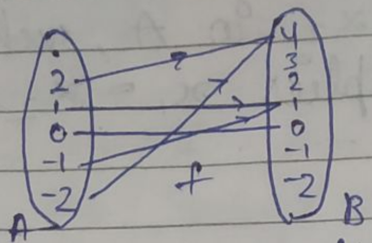
* If $f(x) = 3x - 1$, is one to one function ?
$f(x_1) = 3x_1 - 1$
$f(x_2) = 3x_2 - 1$
then $f(x_1) = f(x_2)$
$\Rightarrow \cancel{3}x_1 \cancel{-1} = \cancel{3}x_2 \cancel{-1}$
$\boxed{x_1 = x_2}$
It's an injective function.

ii) Many one function :-
A function f from $A \to B$ is said to be Many one if and only if two or more element of A have same image in B.



Many one

Eg :- Let $f(x) = x^2$, x is any real number and ~~fx~~ $f : R \rightarrow R$



Many one function

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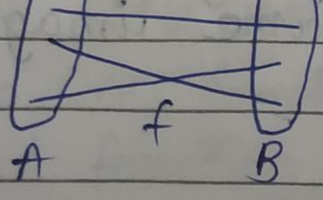
⇒ Onto function :-
A function $f : A \rightarrow B$ is called Into function if and only if there exist atleast one element in B which is not an ~~as~~ image of any element in A, i.e the range of f is a proper subset of co-domain of f.

⇒ Onto function :-
A function $f : A \rightarrow B$ is called Onto or surjective if every element of B is the image of some element in A i.e. if B = range of f

⇒ Bijective function :-
A function $f : A \rightarrow B$ is said to be bijective if f is both injective ~~&~~ surjuctive i.e both one-to-one & onto.



(bijective function)

* One to One, Into
* Many one, Into
* Many one, Onto

Q. A function $f: z \to z$, $f(x) = 5x + 3$ prove that it is one one but not onto.

$f(x_1) = 5x_1 + 3$

$f(x_2) = 5x_2 + 3$

$f(x_1) = f(x_2)$

$\cancel{5}x_1 + \cancel{3} = \cancel{5}x_2 + \cancel{3}$

$x_1 = x_2$

(one to one)

$f(x) = y$

$y = 5x + 3$

$x = \frac{y-3}{5}$

for $y = 6$, $x = \frac{3}{5}$

where x is not integer, hence it is not onto.

Q. $A = \{1, 2, 3\}$ & $B = \{a, b\}$ & $f = \{(1,a), (2,b), (3,a)\}$
f is a many one & onto function.

Q. $f: A \to B$ is defined by $f(x) = x + 1$,
$A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$
one one & into function.

⇒ Composition of function :-
Let $f: A \to B$ and $g: B \to C$, the composition of f & g denoted by gof,

$gof(x) = g(f(x))$

for all x in A.

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Eg:- Let $A = \{1, 2, 3\}$, $B = \{a, b\}$ and $C = \{r, s\}$ $f: A \to B$

$f(1) = a$, $f(2) = (a)$, $f(3) = (b)$

$g: B \to C$, $g(a) = s$, $g(b) = r$

$gof(1) = g(f(x)) = g(f(1))$
$= g(a)$
$= s$

$gof(2) = g(f(x)) = g(f(2))$
$= g(a)$
$= s$

Q. If $f: R \to R$ and $g: R \to R$, $f(x) = x+2$, $g(x) = x^2$, find fog and gof.

$gof(x_1) = g(f(x_1)) = gf(x_1+2)$

$g(x_1+2) = x_1^2 + 2x_1 + 4$

$fog(x) = f(g(x)) = f(x^2)$

$fog = x^2 + 2$

Q. $f(x) = x+9$, $g(y) = y^2+3$ find $fof(a)$ and $gof(b)$, $gof(3)$

$fof(a) = f(a+9) = a+9+9 = a+18$

$gof(b) = g(b+9) = (b+9)^2 + 3$
$= b^2 + 18b + 81 + 3$
$= b^2 + 18b + 84$

$gof(3) = g(12) = 144 + 3$
$= 147$

⇒ Inverse of a function :-
Let $f : A \to B$ is one-one, onto function then a function $f^{-1} : B \to A$ associate each element $b \in B$ to the element $a \in A$ such that $f(a) = b$ is called an inverse function.

Eg:- $f : A \to B$ $f(a) = 1, f(b) = 2, f(c) = 3$ on a set $\{a, b, c\}$ & $\{1, 2, 3\}$ is one-one function then find $f^{-1} : B \to A$

$$f^{-1}(1) = a$$
$$f^{-1}(2) = b$$
$$f^{-1}(3) = c$$

Q. Let $f : R \to R$ be a function defined on $f(x) = 3x + 7$ find $f^{-1}(y)$

$$y = 3x + 7$$
$$x = \frac{y - 7}{3}$$
$$f(y) = \frac{y - 7}{3}$$

Q. Show that the mapping $f : R \to R$ defined by $f(x) = ax + b$ where $\{a, b, x\}$ belong to R $f^{-1}(x)$

$$f(x_1) = ax_1 + b$$
$$f(x_2) = ax_2 + b$$
$$x_1 = x_2$$
$$y = ax + b$$
$$x = \frac{y - b}{a}$$
$$f^{-1}(x) = \frac{y - b}{a}$$

Q. $f(x) \{(1,a)\ (2,a)\ (3,d)\ (4,c)\}$ find $f^{-1}(x)$, show that f is not a function,

$$f^{-1}(x) = \{(a,1), (a,2), (d,3), (c,4)\}$$

Q) $f: R \to R$, $f(x) = 2x+1$ & $g(x) = x^2-2$
fog, gof

$$fog(x^2-2) = f(x^2-2) = 2(x^2-2)+1$$
$$= 2x^2-4+1$$
$$= \underline{2x^2-3}$$

$$gof(x) = gof(2x+1) = g(2x+1)$$
$$= (2x+1)^2-2$$
$$= 4x^2+4x+1-2$$
$$= \underline{4x^2+4x-1}$$

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⇒ Recursively defined function :-
A function is recursively defined, if the func definition refers to itself. The func must have the following two properties ;- * there must be certain arguments called base value, for which the func does not refer to itself.
* Each time the func does refer to itself, the argument that func must be closer to a base value.

Eg:- Let a and b be positive integer and suppose Q is defined recursively.

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$$\phi(a,b) = \begin{cases} 0, & \text{if } a < b \\ \phi(a-b, b)+1, & \text{if } b \le a \end{cases}$$

i) find $\phi(2,5) = 0$

ii) find $(12,5) = \phi(7,5)+1$

$= \phi(2,5)+1+1$

$= 0+1+1$

$= 2$

Q. Let n denote a positive integer suppose L is recursively defined as

$$L(n) = \begin{cases} 0, & \text{if } n=1 \\ L([n/2])+1, & \text{if } n>1 \end{cases}$$

find $L(25)$

$L([25/2]) = 12+1$

$= 13$

$L([13/2]) = 6+1+1 = 8$

$L([7/2]) = 3+1+1 = 4$

$L([4/2]) = 2+1+1 = 3$

$L([3/2]) = 1$

$L(n) = 0$

$L[25/2] = 12+1 = 13$

$L[13/2] = 6+1+1 = 8$

$L[8/2] = 4+1+1+1 = 7$

$L[7$

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Practice sheet :- 2
(Function)

1) Let $A = \{1, 2\}$ and $B = \{3, 6\}$ and f and g be function $f : A \to B$ defined by $f(x) = 3x$ and $g(x) = x^2+2$, show that $f = g$.

$f(1) = 3$, $g(1) = 3$

$f(2) = 6$, $g(2) = 6$

Hence, $f = g$

2) Let $f : R \to R$, $f(x) = x^2+1$ find $f^{-1}(10)$

$f^{-1}(10) = x^2+1 = 10$

$x^2+1 = 10$

$x^2 = 10-1$

$x^2 = 9$

$x = \sqrt{9}$

$x = 3$

$f^{-1}(10) = \pm 3$

3) Let $f : N \to N$ $f(x) = 2x$ for all ~~real no. Let $f : R \to R$~~ x in N. Check whether f is a bijection.

$f(x_1) = 2x_1$

$f(x_2) = 2x_2$

$f(x_1) = f(x_2)$

$\cancel{2}x_1 = \cancel{2}x_2$

Hence, f is one-one

$f(x) = y = 2x$

$x = \frac{y}{2}$ for every $y \in N$ but $\frac{y}{2} \notin N$.

$f(y) = \frac{y}{2}$

hence, the f is not onto, so that f is not a bijection.

4. Let R be the set of all real no. Let $f: R \to R$ $f(x) = \cos x$ and let $g: R \to R$ $g(x) = 3x^2$ show that $(gof) \neq (fog)$

$gof(x) \Rightarrow g(f(x))$

$g(\cos x) = \cancel{3\cos^2}$

$3\cos^2 x$

$f(g(x)) = f(3x^2)$

$f(3x^2) = \cos(3x^2)$

thence, $fog \neq gof$

5. Let $A = \{2, 3, 4, 5\}$ and $B = \{7, 9, 11, 13\}$ and let $f = \{(2,7), (3,9), (4,11), (5,13)\}$ Show that f is invertible & find f^{-1}

A is one-one as every ~~domain have~~ ~~one~~ element of B has single pre image on A.

A is onto as every element of B has pre image on B.

so it is injective hence, it is invertible.

$f^{-1}\{(7,2), (9,3), (11,4), (13,5)\}$

6. Let a and b be integers and suppose $\phi(a,b)$ is defined recursively by

$$\phi(a,b) = \begin{cases} 5 & \text{is } a < b \\ \phi(a-b, b+2) + a & \text{is } a \geq b \end{cases}$$

find $\phi(2,7)$, $\phi(5,3)$ & $\phi(15,2)$

i) $\phi(2,7)$, ~~2<7~~ $2 < 7$

$\phi(2,7) = 5$

ii) $\phi(5,3)$, $5 > 3$

$\phi(5,3) = \phi(2,5) + 5$

$\phi(2,5)$, $2 < 5$

$\phi(2,5) = 5$

$\phi(5,3) = 5 + 5 = 10$

iii) $\phi(15,2)$, $15 > 2$

$\phi(15,2) = \phi(13,4) + 15$

$\phi(13,4)$, $13 > 4$

$\phi(13,4) = \phi(9,6) + 13$

$\phi(15,2) = \phi(9,6) + 28$

$\phi(9,6)$, $9 > 6$

$\phi(9,6) = \phi(3,8) + 9$

$\phi(15,2) = \phi(3,8) + 37$

$\phi(3,8) = 3 < 8$

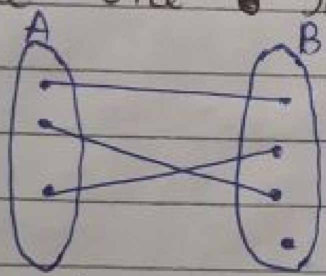
$\phi(3,8) = 5$

$\phi(15,2) = 5 + 37$

$= 42$

7. Define :-

i) One - One Into :-



In this every element of A has image one B and every range in B has one ~~ont~~ pre-image on A and In B atleast one element does not has pre-image in A.

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⇒ Ackermann function :-

It is function with two argument, each of which can be assign any non-negative integer i.e 0,1,2,...

This function is defined as follows :-

* If $m=0$, then $A(m,n) = n+1$
* If $m \neq 0$, but $n=0$ then $A(m,n) = A(m-1,1)$
* If $m \neq 0$ and $n \neq 0$ then $A(m,n) = A(m-1, A(m, n-1))$

Eg:- $A(1,1) = A(0, A(1,0))$

~~$A(0,A(1,0)) = A(0,(1,0))$~~

$A(1,1) = A(0, A(1,0))$

$A(1,0) = A(0,1)$

$A(0,1) = 2$

$A(1,0) = 2$

$A(1,1) = A(0,2)$

$A(0,2) = 3$

$A(1,1) = 3$

Q. $A(1,3)$

$= A(0, A(1,2))$

$= A(1,2) = A(0, A(1,1))$

$= A(1,1) = A(0, A(1,0))$

$= A(1,0) = A(0,1)$

$= A(0,1) = 2$

$= A(1,1) = A(0,2)$

$= A(1,2) = A(0, A(0,2))$

$= A(0,2) = 3$

$A(1,2) = \cancel{A(0,3)}\ A(0,3)$

$= A(0,3) = 4$

$= A(0,4) = 5$

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13/10/23 ~~Suppose~~

→ Ordered Set & lattice :-

* Ordered set :-

Suppose R is a relation set S satisfy the following three points :-

* Reflexive :- for any $a \in S$ where aRa

* Anti-symmetric :- If aRb & bRa then $a=b$

* Transitive :- If aRb and bRc then aRc

R is called a partial order or simply an order relation the set S with the partial order is called a partially ordered set, or ~~Po-set~~ (POSET).

$a \leq b \rightarrow$ a precedes b

$a < b \rightarrow$ a strictly precedes b

$b \geq a \rightarrow$ b succeeds a

$b > a \rightarrow$ b strictly ~~s~~ succeeds a

* Dual ordered :-

Let $\leq$ is any partial ordering of a set S, ~~and suppose $a, b \in A$~~. the relation $\geq$ i.e a succeeds b is also a partially ordering of a set, it is called the dual ordering of set.

the dual order is the inverse of the relation $\leq$ ~~is~~ i.e ($\leq = \geq^{-1}$)

* Ordered subset :-

Let A is a subset of an ordered set S and suppose $a, b \in A$ define $a \leq b$ as element of A whenever ~~if~~ $a \leq b$ as element of S, this defined a partial ordering of A, called the induced ~~ored~~ order on A.

the subset A with the induced order is called an ordered subset of S.

⇒ Quasi order :-

Suppose $<$ is a relation on a set S satisfy the ~~property~~ following two properties :-

i) Irreflexive :- for any $a \in A$ we have $a \nless a$

ii) Transitive :- if $a < b$ and $b < c$ then $a < c$, then $<$ is ~~then $<$ is~~ called a Quasi order on S

→ Comparability linear, linearly ordered set :-

* suppose a & b are element in a partially ordered set S, we say a & b are comparable if $a \preceq b$ or $b \preceq a$ i.e if one of them preceeds the other.

-* a & b are non-comparable or $a \parallel b$ if neither $a \preceq b$ nor $b \preceq a$.

* Linearly ordered set is also known as totally ordered set.

Eg :- Suppose set $N = \{1, 2, 3 \cdots\}$ of positive integers is ordered by divisibility, Insert the symbol $<, >, \parallel$ between each pair of number.

i) $2 \preceq 8$

ii) $18 \parallel 24$

iii) $9 \succeq 3$

iv) $5 \preceq 15$

Eg :- Let $N = \{1, 2, 3 \cdots\}$ ordered by divisibility state whether each of following cubset of N are linearly ordered.

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i) $\{(24, 2, 6\}$ $2 < 6 < 24$, linearly ordered

ii) $\{3, 5, 15\}$ $3 \nmid 5$ & $5 < 15$, $3 < 15$, non-linearly ordered

iii) $N = \{1, 2, 3 - - \}$ Non - linearly ordered set

iv) $\{2, 8, 32, 4\}$ $2 < 4 < 8 < 32$, linearly order

v) $\{7\}$ linearly order set

vi) $\{15, 5, 30\}$ $5 < 15 < 30$, linearly ordered set

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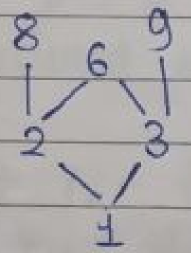
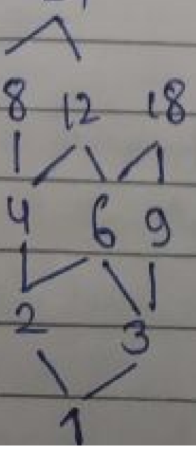
Hasse diagram and partial

* Let S is a partially ordered set suppose, a & b is belong to S, a is an imidiate predecessor of b or b is an imidiate successor, a << b
* The hasse diagram of finite partially ordered S is the directed graph whose vertices ~~of~~ are the element of S, and there is a directed Edge from a to b, instead of drawing an arrow a to b, we ~~at~~ sometime place b higher than a and draw a line between them.

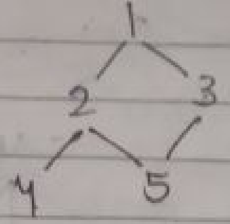
Eg:- Let A = {1, 2, 3, 6, 8, 9} is ordered by the relation x divides y



Q. Let A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} odered by divisibility



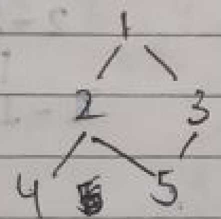
Q. Let $A = \{1, 2, 3, 4, 5\}$ be ordered by hasse diagram,



Insert the correct symbol, $<$, $>$, $\|$ between each pair of element.

a) $1 \underline{>} 5$
b) $2 \underline{\|} 3$
c) $4 \underline{<} 1$
d $3 \underline{\|} 4$

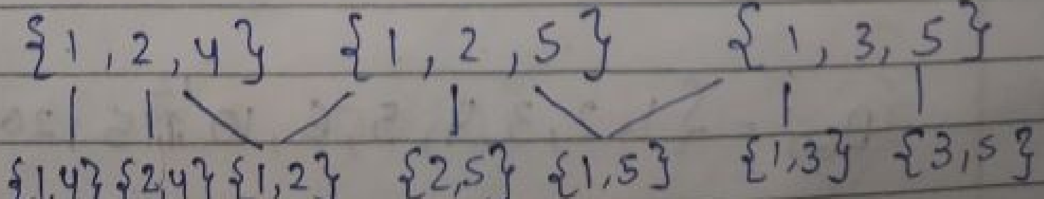
Q. Consider the ordered set $A = \{1, 2, 3, 4, 5\}$ of figure



Let $L(A)$ denote the collection of all linearly set A with two or more element., Let $L(A)$ be ordered by set inclusion. Draw the hasse diagram of $L(A)$

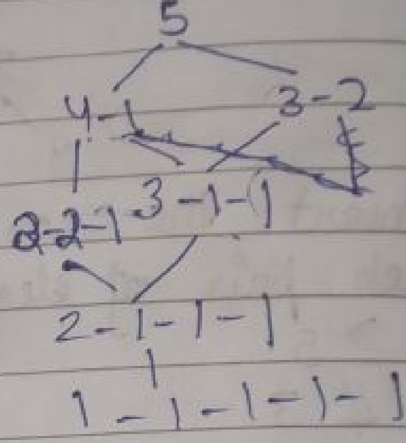
$L(A) = \{1,2\}\{1,3\}\{1,4\}\{1,5\}\{2,4\}\{2,5\}\{3,5\}$
$\{1,2,4\}\{1,2,5\}\{1,3,5\}$

Hasse diagram :-

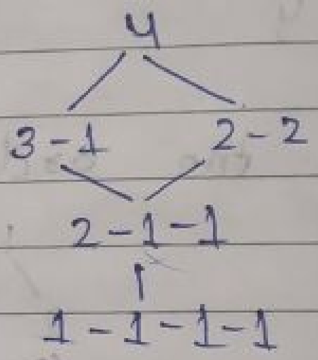


Q. A ~~partition~~ of a positive integers m is a set of positive integers whose sum is m for $m = 5$ when

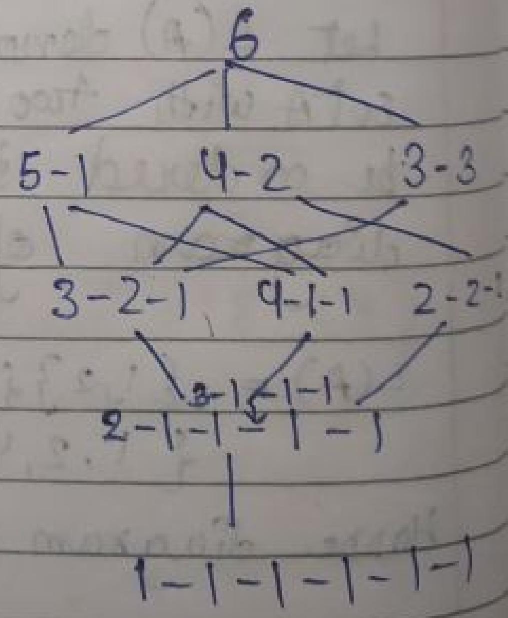
$m = 5$
4-1
3-2
3-1-1
2-2-1
2-1-1-1
1-1-1-1-1



$m = 4$
3-1
2-2
2-1-1
1-1-1-1



$m = 6$
5-1
4-2
3-3
3-2-1
4-1-1
2-2-2
3-1-1-1
2-2-1-1
2-1-1-1-1
1-1-1-1-1-1



$D_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$

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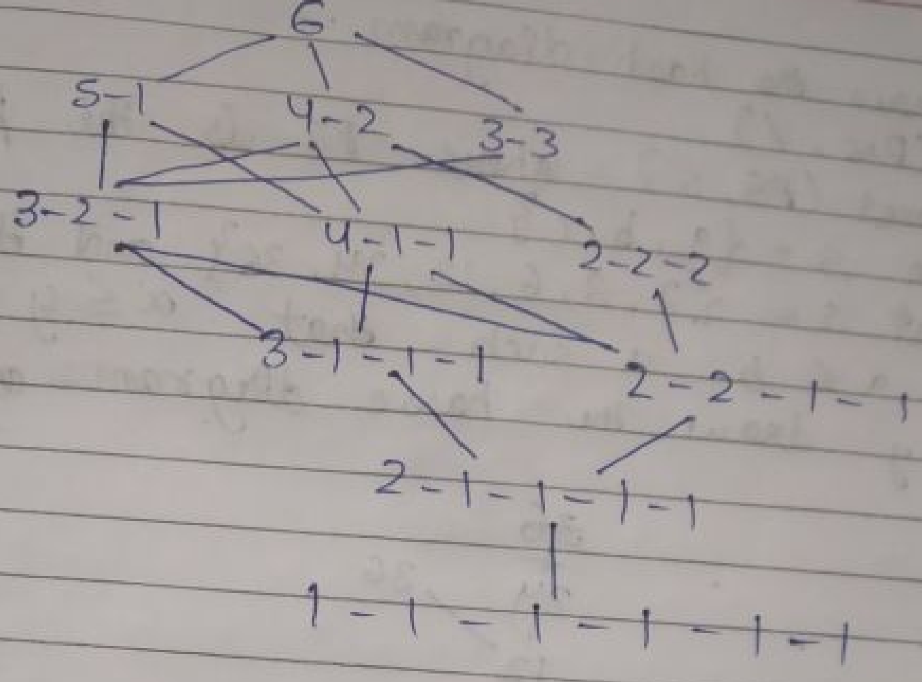
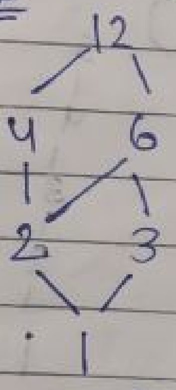
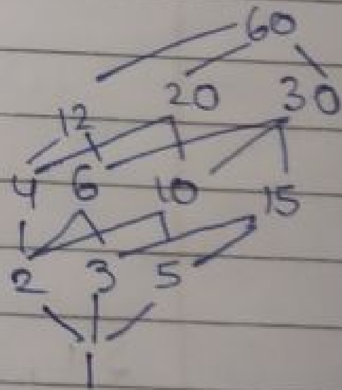


Q. Let D_m denote the positive divisor of m ordered by divisibility, draw the hasse diagram.

$\{1, 2, 3, 6, 4, 12\}$

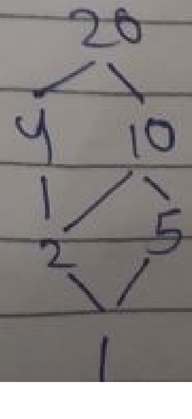
~~12~~





$D_{16} = \{1, 2, 4, 8, 16\}$

$D_{20} = \{1, 2, 4, 5, 10, 20\}$



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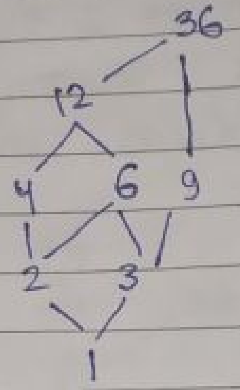
Practice set :- 3

1. Draw the hasse diagram of $(D_{36}, |)$?

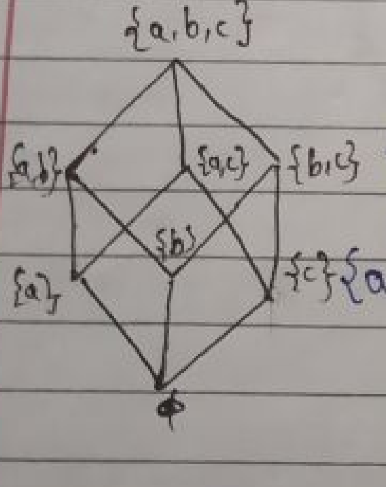
$D_{36} = \{1, 2, 3, 4, 6, 9, 12, 36\}$

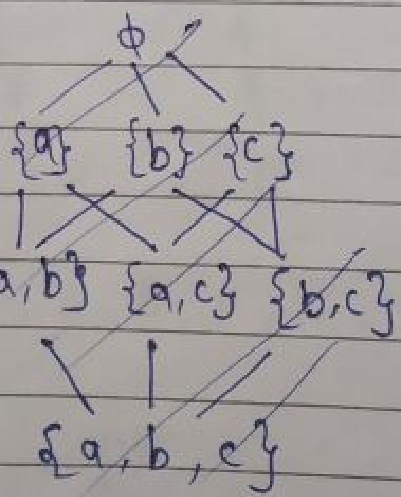


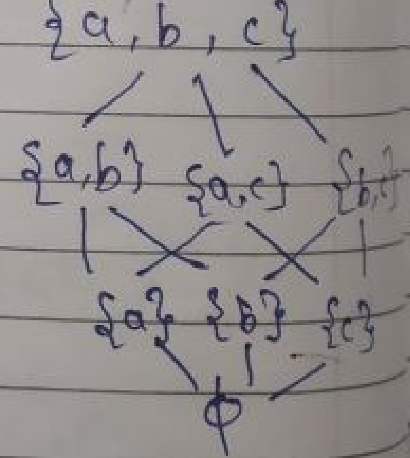
36 ~~pre~~ succeeds
1 preceeds

3) Draw the hasse diagrame for the poset $(P(S), \subseteq)$ where $P(S)$ is the power set on $S = \{a, b, c\}$

$P(S) = \{\phi, \{a,b\}, \{b,c\} \{a,c\}, \{a\}, \{b\}, \{c\}, \{a,b,c\}\}$



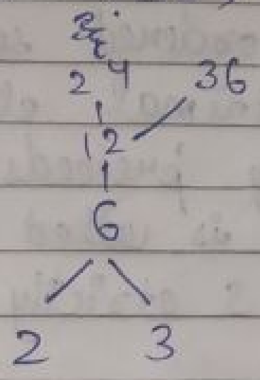




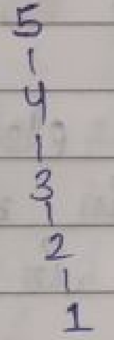
4) Let $x = \{2, 3, 6, 12, 24, 36\}$ and the relation "$\leq$" be such that $x \leq y$ if x divides y draw the hasse diagram of $(x, \leq)$.

{(2, 6) (2, 12), (2, 24), (2, 36), (3, 6, (3, 12), (3, 24) (3, 36) }

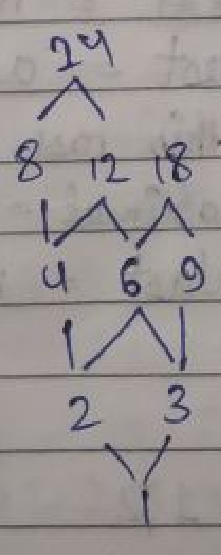
Q.



Q. {1, 2, 3, 4, 5}, ≤



Q. {1, 2, 3, 4, 6, 8, 9, 12, 18, 24}, /



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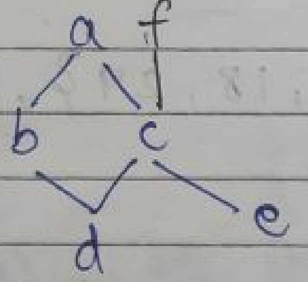
Minimal & Maximal

* Let S be a partially ordered set. An element a in S is called a minimal element, if no other element of S strictly preceeds a.
* An element b in S is called a maximal element if no element of S strictly succeeds b.

→ first and last element :-

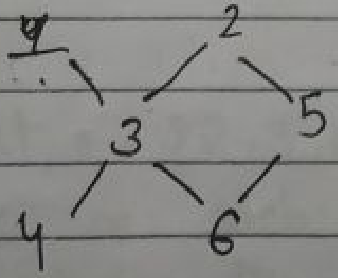
* An element a in S is called a first element if $a \precsim x$ for every element x in S, if a preceeds every other element in S.
* An 'b' in S is called a last element if $y \precsim b$ for every element y in x, if b succeeds every other element in S.

Eg :-



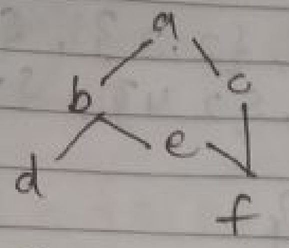
Maximal = a
Minimal = d, e
first = Not exist
last = a
In this case :-
Maximal = a, f
last = not exist

(A)



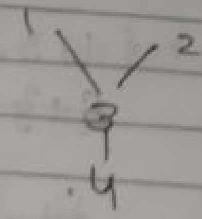
Max = 1,2
Min = 4,6
first = X
last = X

(B)



Max = a
Min = d,f
first = No
last = a

(C)



Max = 1, 2
Min = 4
first = 4
last = No

⇒ Consistent Enumeration :-

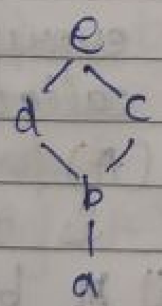
Suppose S is the finite partially ordered set, we want to assign positive integer to the element of S in such a way that the order is preserved.

$$f : S \to N$$

if $a < b$ then $f(a) < f(b)$

Such a function is called a consistent Enumeration of S.

Eg:-

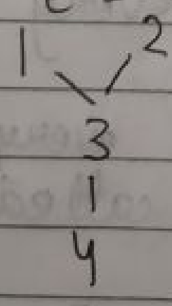


$f(a)=1, f(b)=2, f(c)=3, f(d)=4, f(e)=5$

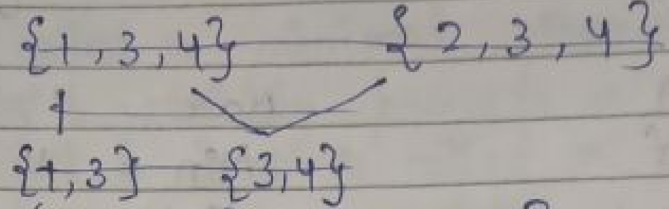
$g(a)=1, g(b)=2, g(c)=4, g(c)=3, g(e)=5$

2 Consistent Enumeration

Q. Let $C = \{1, 2, 3, 4\}$ be ordered in the figure $L(C)$ denote the collection all non-empty linearly ordered subsets of C ordered by set inclusion draw a diagram of hasse diagram of $L(C)$.

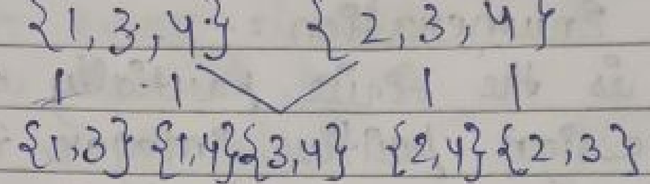


$L(C) = \{1,3,4\}\ \{2,3,4\}\ \{1,3\}, \{2,3\}, \{3,4\}$
$\cancel{\{1\}, \{2\}, \{3\}, \{4\}}, \{2,4\}, \{1,4\}$



Max = $\{1,3,4\}, \{2,3,4\}$
Min = $\{1,3\}, \{1,4\}, \{3,4\}, \{2,4\}, \{2,3\}$

2 Consistent Enumeration.



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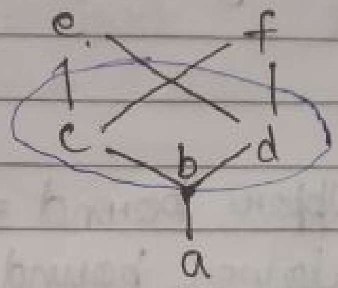
⇒ Supremum and Infimum :-

* Let A be a subset of partially ordered set S, an element M in S is called an upper bound of A, if M succeeds every element of A. i.e for every x in A, we have $x \preceq M$.
* If an upper bound of A preceeds every other upper bound of A then it is called Supremum of A denoted sup(A). or lub (A)
* An element m in a POSET S is called a lower bound of a subset A, if m preceeds every element of A i.e for every y in A we have $m \preceq y$.
* If a lower bound of A succeeds every other lower bound of A then it is called infimum of A i.e inf(A). or glb (A).
* Supremum is called least upper bound and

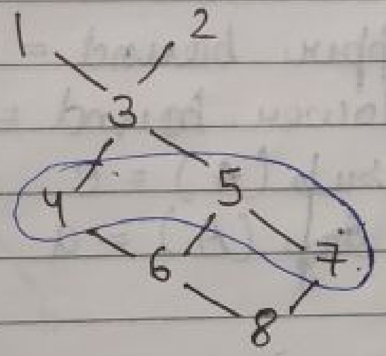
Infimum is also known as greatest lower bound

Eg:- $S = \{a, b, c, d, e, f\}$, $A = \{b, c, d\}$



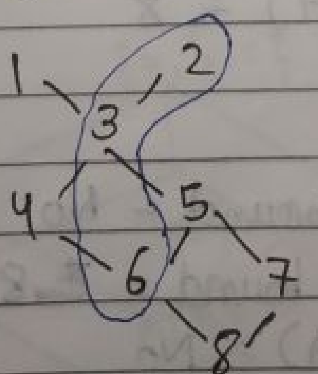
upper bound = e,f
lower bound = a, b
sup (A) = No
Inf (A) = b

Q.



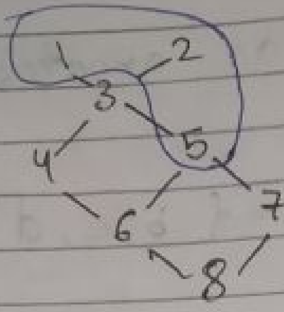
upper bound = 1, 2, 3
lower bound = 8
sup (A) = 3
Inf (A) = 8

Q



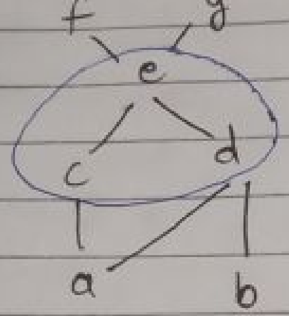
upper bound = 2
lower bound = 8, 6
sup (A) = 2
Inf (A) = 8

Q.



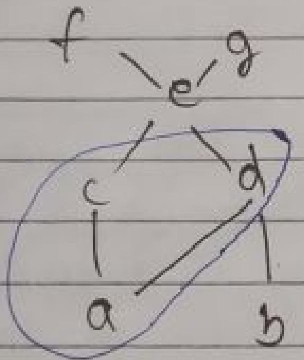
Upper bound = No
lower bound = 5, 6, 7, 8
Sup (A) = No
Inf (A) = 5

Q.



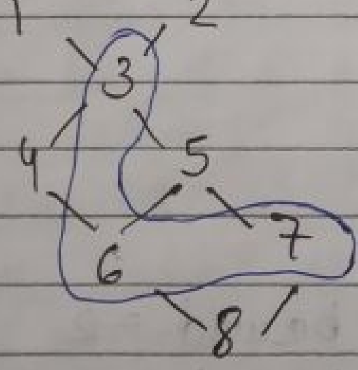
Upper bound = e, f, g
lower bound = a
sup (A) = e
Inf (A) = a

Q.

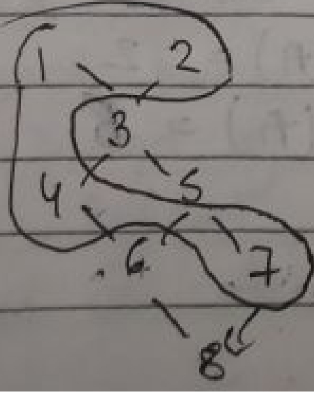


Upper bound = e, f, g
lower bound = a
sup (A) = e
Inf (A) = a

Q.



Upper bound = 1, 2, 3
lower bound = 8
sup (A) = 3
Inf (A) = 8

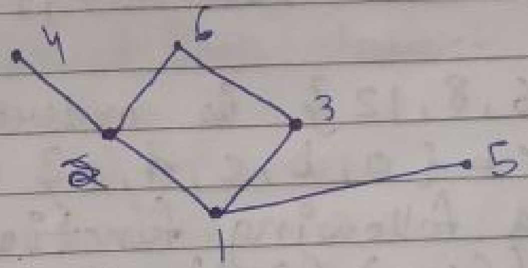


Upper bound = No
lower bound = 8
Sup (A) = No
Inf (A) = 8

a)

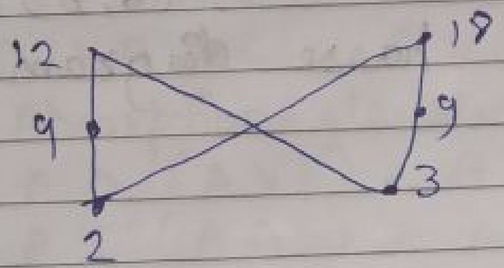
Minimal = 1
Maximal = 27
~~fist~~ first = 1
last = 27

b)

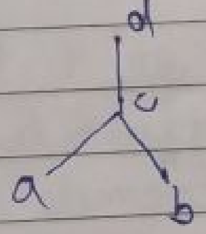


minimal = 1
maxim

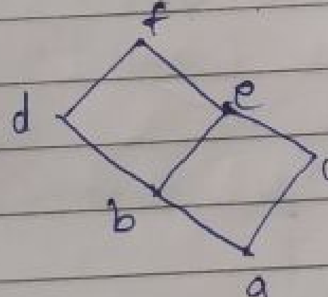
c)



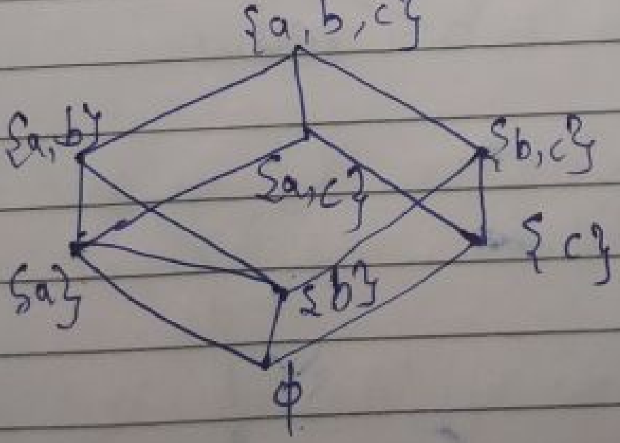
d)



e)



f)



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Date:

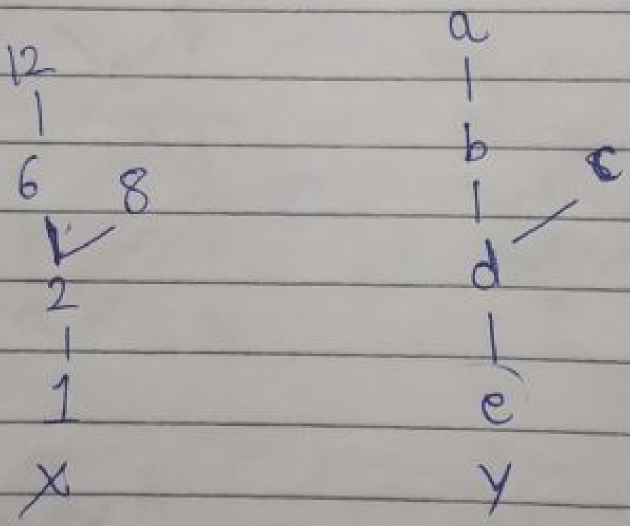
⇒ Isomorphic (similar) ordered set :-

suppose X and Y are Partially ordered set, a one to one (Injective function) $f: X \to Y$ is called a similar mapping from $(X \times Y)$

* ~~f: x → y~~ If $a \le a'$ then $f(a) \le f(a')$.

* If $a \parallel a$ (non-comparable) then $f(a) \parallel f(b)$.

Eg: Suppose $X = \{1, 2, 6, 8, 12\}$ is ordered by divisibility and $Y = \{a, b, c, d, e\}$ is isomorphic to X, a following function f is similar to $f = \{(1, e), (2, d), (6, b), (8,$ $(12, a)\}$ draw the hasse diagram.



LATTICES

→ **Lattice** :- Let L be a non-empty set closed under two binary operations, meet and join by ($\wedge$ and $\vee$) then L is called a lattice, if the following axiums are

i) Commutative law :-

$$A \wedge B = B \wedge A$$
$$A \vee B = B \vee A$$

ii) Associative law :-

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$
$$(A \vee B) \vee C = A \vee (B \vee C)$$

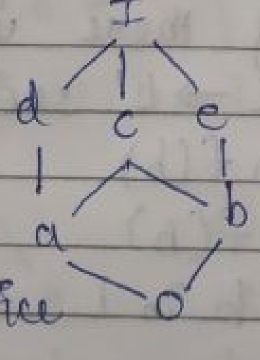
iii) Absorption law :-

$$A \wedge (A \vee B) = A$$
$$A \vee (A \wedge B) = A$$

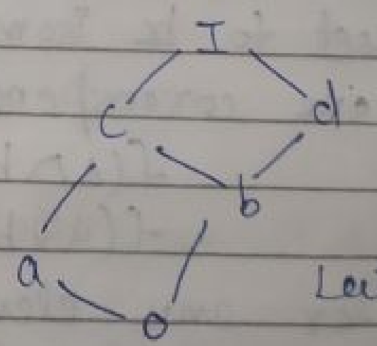
* Let P be a POSET such that infimum ~~of~~ (A, B) and supremum ~~of~~ (A, B) exist for any pair of element in A & B

$$A \wedge B = \inf(A, B) \text{ and } A \vee B = \sup(A \cdot B)$$

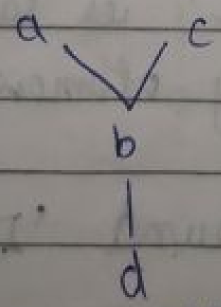
Eg:-

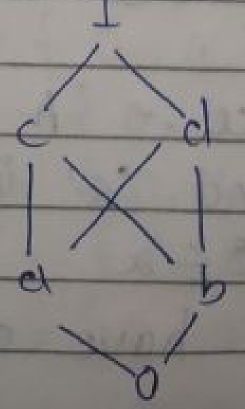


Lattice



Lattice

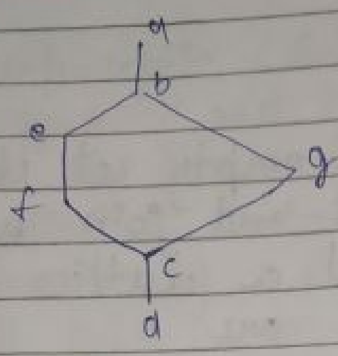




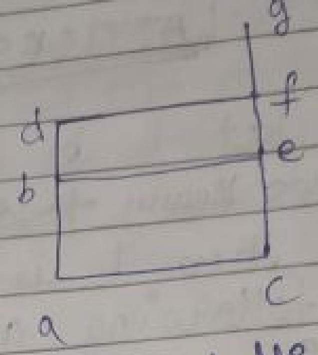
(a,b) has no supremum

So, it's not a lattice.

* not a lattice



lattice



not lattice.

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⇒ Duality :- The dual of any statement in a lattice $(\leq, \wedge, \vee)$ is defined by interchanging of and $\wedge$ and $\vee$.

Eg :- $a \wedge (b \vee a) = a \vee a$

dual is - $a \vee (b \wedge a) = a \wedge a$

⇒ Sub lattice :- Supreme M is a non-empty subset of lattice L. M is a sub lattice of L if M itself is a lattice. M is a sublattice of L if and only if M is closed under the operation of $\wedge$ and $\vee$ of L

⇒ Isomorphic lattice :- Two lattice L and L' are set to be isomorphic if there is a one to one corraspondence $f : L \to L'$ such that f(a

$$f(a \wedge b) = f(a) \wedge f(b)$$
$$f(a \vee b) = f(a) \vee f(b)$$

for any element a, b in L.

⇒ Bounded lattice :- A lattice 'L' is said to have a lower bound 'a' if for any element x in L

$$0 \leq x$$

'L' is said to have an upperbound 'I' if

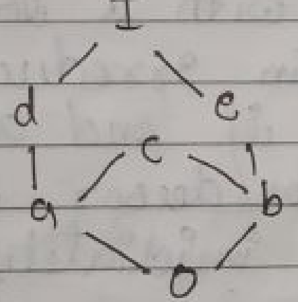
for any x in L we have, $x \leq I$
It means 'L' is bounded if 'I' has both a lowerbound '0' and upperbound 'I'.

$$a \vee I = I$$
$$a \wedge I = a$$
$$a \vee 0 = a$$
$$a \wedge 0 = 0$$

⇒ Distributive lattice :-
Properties for distributive law :-

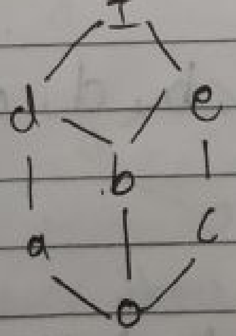
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

Q.



$L_3 = \{a, c, d, I\}$ Sub lattice
$L_4 = \{0, c, d, I\}$ Not sub lattice
$L_2 = \{0, a, e, I\}$
$L_1 = \{0, a, b, I\}$

Q.



find sub lattice consist of 5 elements :

$L_1 = \{0, a, b, d, I\}$ ✓
$L_2 = \{0, b, c, e, I\}$ ✓
$L_3 = \{0, b, d, e, I\}$ ✓
$L_4 = \{a, \cancel{I}, c, e, d\}$ ✓
$L_5 = \{0, a, \cancel{b}, \cancel{d}c, e, I\}$ ✓
$L_6 = \{0, a, d, e, I\}$ ✓
$L_7 = \{0, a, c, d, I\}$ ✓

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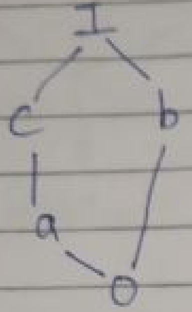
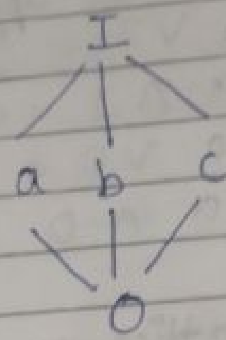
⇒ Distributive lattice :-

L is said to be non-distributive





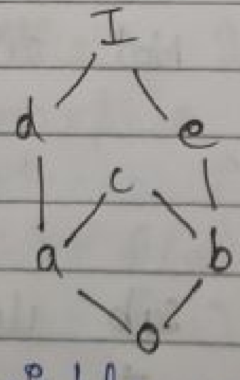
$$a \vee (b \wedge c) = a \vee 0 = a$$
$$(a \vee b) \wedge (a \vee c) = I \wedge C = C$$

⇒ Join Irreducible elements, Atoms :-

* Let L be a lattice with a lower bound 0 clearly, 0 is a Join Irreducible. if a≠0 is Join Irreducible if and only if a has a unique imidiate predecesor. #

* Those element which imidiately succeed 0 is atom

⇒ Eg :-



Join Irreducible = 0, a, b, d, e

Atoms = a, b

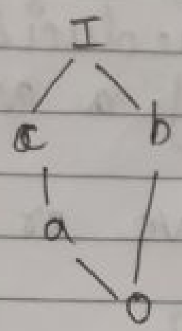
⇒ Complements :-

Let ~~Capito~~ 'L' be a bounded lattice with lower bound zero and upper bound I if let 'a' be an element of L. An element 'x' in L is called a complement of 'a':

$$a \vee x = I$$
$$a \wedge x = 0$$

Complements need not exist and need not be unique.

Eg :-

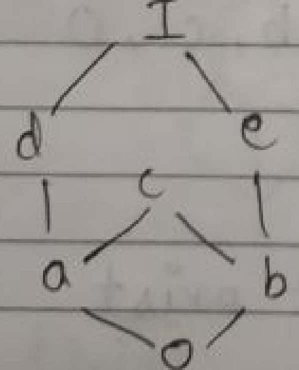


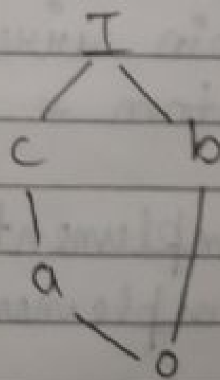
Complement of b = a, c

⇒ Complemented lattice :-

A lattice L is said to be complemented if L is bounded and every element in L has a complement.

→ Eg:-





here, there is no complement of c so, it's not a complemented lattice.

here, there is complement of every element, so it's a complemented lattice.

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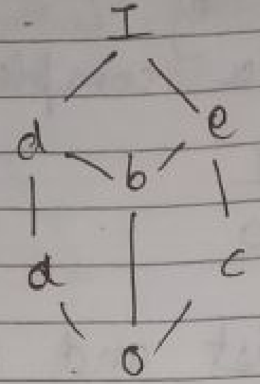
Q.



a) find all sub lattice with 5 elements.
b) find all Join irreducible, atoms
c) find complements of a and b if they exist.
d) Is L is distributive or complementive.

a) $L_1 = \{I, d, b, e, 0\}$
$L_2 = \{0, a, b, d, I\}$ ✓
$L_3 = \{0, c, b, e, I\}$ ✓
$L_4 = \{0, a, c, d, I\}$
$L_5 = \{0, a, d, e, I\}$ ✓
$L_6 = \{0, a, c, e, I\}$
$L_7 = \{$

b) Join irreducible = a, b, c, 0
Atom = a, b, c

c) complement of a = e, c
complement of b = not exist

d) Not distributive { symmetric to non-distributive
Not complementive { b does not have any complement }

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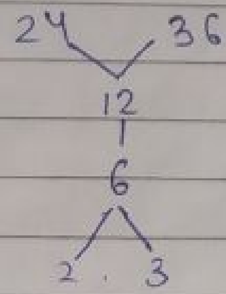
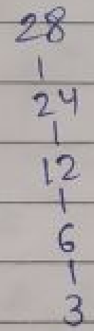
Practice set :- 4

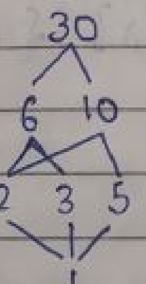
1. i) $A = \{2, 3, 6, 12, 24, 36\}$



ii) $A = \{3, 6, 12, 24, 48\}$



iii) $D_{30} = \{1, 2, 3, 5, 6, 10, 30\}$



iv) $D_{17} = \{1, 17\}$

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Unit :- 5

Graphs

* A Graph 'G' consist of two things :-

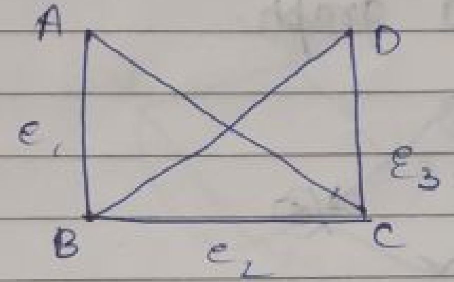
i) A set V = V(G) whose elements are called vertices or points of G or nodes of G.

ii) A set E = E(G) of unordered pairs of distinct vertices called edges of G.

iii) G = (V, E), set of vertices and edges.

V(G) = (V_0, V_1 - - - V_n) set of vertices

E(G) = (E_0, E_1 - - - E_n) set of edges



V(G) = {A, B, C, D}

E(G) = e_1 = (A, B) = e_2 (B, C) - - -

* A graph can be of two types :-

i) directed graph

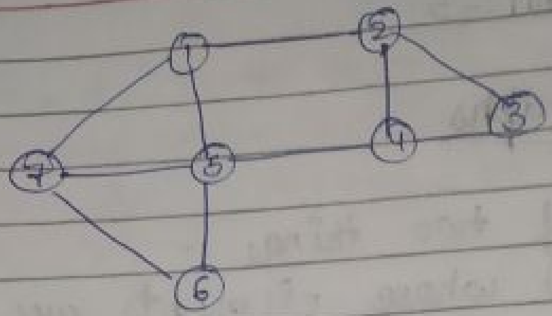
ii) Undirected graph.

→ Undirected graph :-

If the pair of vertices are unordered, then graph 'G' is called an undirected graph.

Eg :-

P.T.O

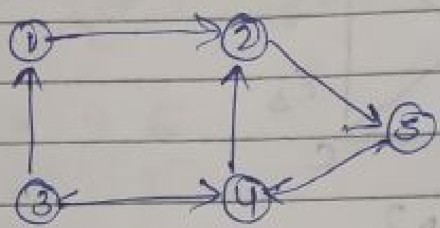


$V(G) = \{1, 2, 3, 4, 5, 6, 7\}$

$E(G) = \{(1,2), (2,3), (2,4), (4,5), (1,5), (1,7), (7,5), (7,6), (5,6)\}$

→ Directed graph :- Di-graph

If the pair of vertices are ordered then graph G is called directed graph.



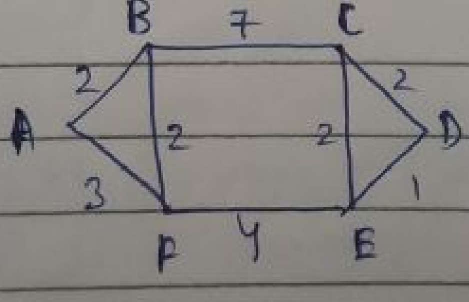
$V(G) = \{1, 2, 3, 4, 5\}$

$E(G) = \{(1,2), (2,5), (5,4), (4,2), (3,4), (3,1)\}$

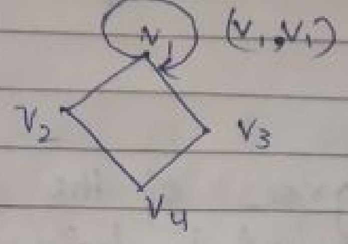
→ Terminology :-

* Weighted graph :-

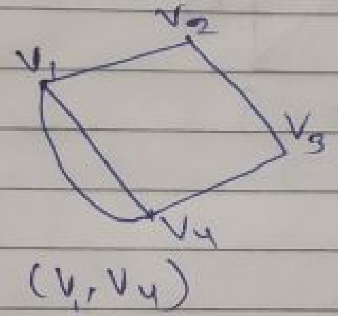
A graph is said to be ~~waited~~ weighted graph if each edge of graph is assign a non-negative ~~neg~~ no. called the weighted graph.



* Self loop :-
A loop is an edge that connect a vertex to itself whose starting & ending points are same.



* parallel edge :-
If there are more than ~~one~~ one edge between same pair of vertices then they are known as parallel edge.

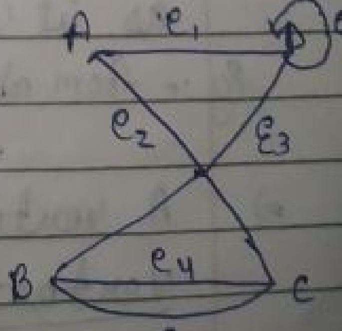


* Adjacent vertices :-
A vertex v_1 is adjacent to vertex v_2, if there is an edge from v_1 to v_2.

$$V_1 \longrightarrow V_2$$

- V_1 is adjacent to V_2
- V_2 is adjacent from V_1

* Multi-graph :-
The edges e_4 & e_5 are multiple edges since they connect the end point and the edge e_6 is called a loop since it's end point are the same vertex.



Note:- A graph permit neither multiple edges nor loops.

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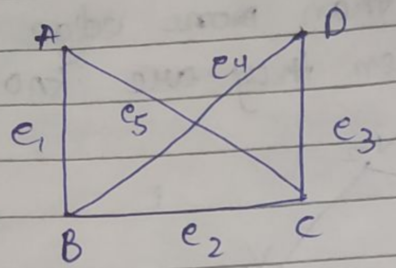
⇒ Degree of a vertex :-
The degree of a vertex 'v' in a graph 'G' written by 'deg (v)', is equal to the no. of edges in G

⇒ Theorem :-
The sum of the degrees of the vertices of a graph of 'G' is equal to twice the no. edges in 'G'.



deg (A) = 2
deg (B) = 3
deg (C) = 3
deg (D) = 2
10

Total edges = 5
~~deg of~~ degree of edges = 2×5 = 10

The sum of the degrees = 10 which is ~~the~~ twice the no. of edges.

⇒ A vertex is said to be even or odd according as it's degree is an even no. or odd no.
Eg :- from above fig., B & C is odd & A & D is even.

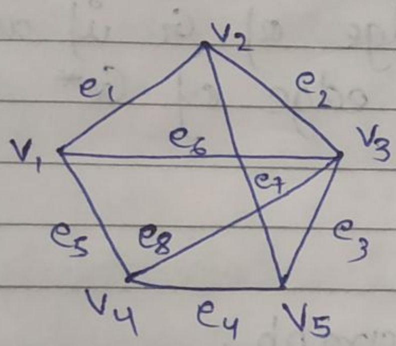
⇒ A vertex of degree 0 is called an isolated vertex.

→ Trivial graph :- (isolated vertex)
Finite graph with one vertex and no edges i.e a single point is called a trivial graph.

→ Sub - graph :-
Given two graphes G and G_1, we say that G_1 is a sub graph of G if the following condition hold :-

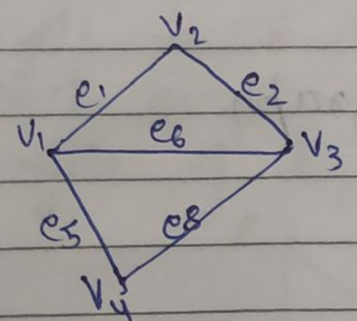
* All the vertices and all the edges of G_1 are in G.
* Each edge of G_1 has the same end vertices in G as in G_1

Eg :-



$G(V, E)$
$V = \{V_1, V_2, V_3, V_4, V_5\}$
$E = \{e_1, e_2, e_3 \dots e_8\}$

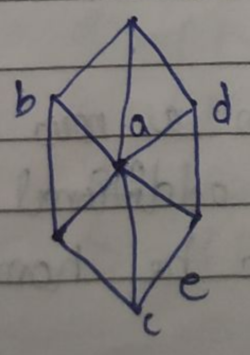
from vertex V_1 to V_4



$G_1(V_1, E_1)$
$V_1 = \{V_1, V_2, V_3, V_4\}$
$E_1 = \{e_1, e_2, e_5, e_6, e_8\}$

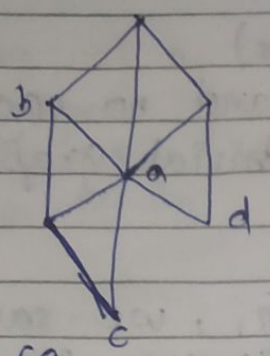
Hence G_1 is a sub graph of G.

Eg :-



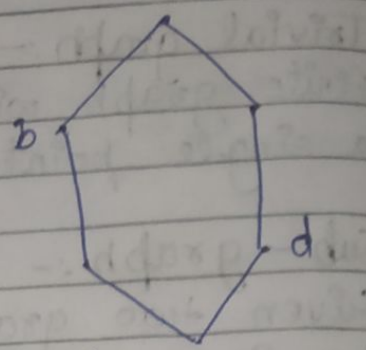
i) $(G - a)$
ii) $(G - a)$

i)



(G-e)

ii)



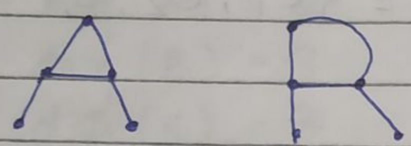
(G-a)

⇒ Isomorphic graph :-

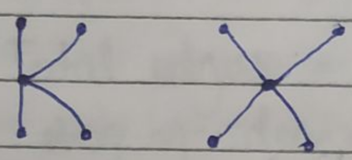
Graph $G(V, E)$ and $G^*(V^*, E^*)$ is said to be isomorphic if there exist a one-to-one corrospond. i.e $f: V \to V^*$.

such that $\{u, v\}$ is an edge of G if and only if $\{f(u), f(v)\}$ is an edge of G^*

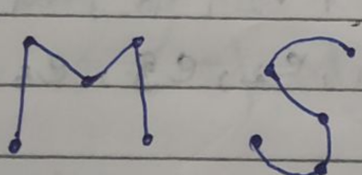
Eg:-



A & R are isomorphic graph.



K & X are isomorphic graph.



M & S are also isomorphic.

⇒ Homeomorphic graph :-

In any graph 'G', we can obtain a new graph by dividing an edge of G with additional vertices. Two graph G & G^* are said to be homeomorphi

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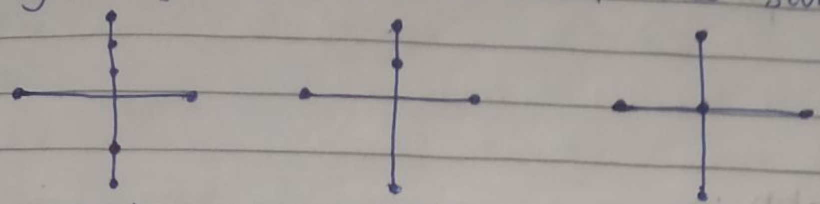
if they can obtain from the same graph.



(a) (b) (c)

a & b is homoeomorphic to c.

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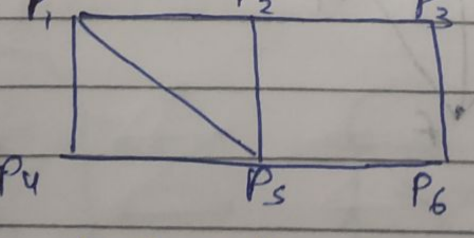
→ Paths, connectivity :-

* A path in a multi-graph G consist of an alternating sequence of vertices and edges in the form of $v_0, e_1, v_1, e_2, v_2, e_3, \ldots e_n\ v_i$
where each edge e_i contain the vertices v_{i-1}, v_i
* The no. 'n' of edges is called the length of the path when there is no ambiguity.
* The path is said to be closed if $v_0 = v_n$.
* A <u>simple path</u> is a path in which all vertices are distinct.
* A path in which all edges are distinct will be ~~to~~ called a trail.
* A cycle is a closed path in which all vertices are distinct except $v_0 = v_n$.
* A cycle of length k is called k-cycle.

Eg :-

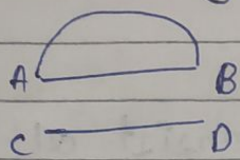


$\alpha = (P_4, \underline{P_1 P_2}, P_5, \underline{P_1, P_2}, P_3, P_6)$ edges repeated
$\beta = (P_4, P_1, P_5, P_2, P_6)$
$\gamma = (P_4, P_1, \underline{P_5}, P_2, \underline{P_5}, P_6)$ vertices repeated
$\delta = (P_4, P_1, P_2, P_3, P_6)$

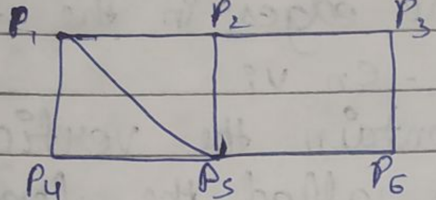
α is path but not trail.
β is not a path

⇒ Connected graph :-
A graph 'G' is connected if there is a path between any two of its vertices.

Eg:-



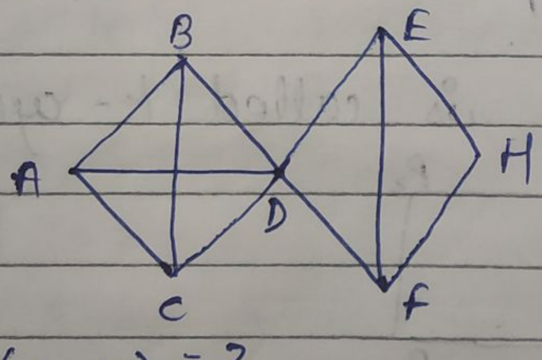
Disconned graph.



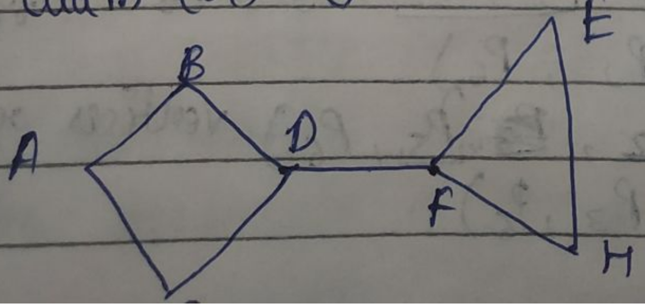
Connected graph.

→ Distance and diameter :-
* The distance between vertices 'u' & 'v' in 'G' written $d(u, v)$ is the length of the shortest path between u & v.
* The diameter of 'G' written diam (G), is the maximum distance between any two points in G.

Eg:-



$d(A, F) = 2$
$diam(G) = 3$

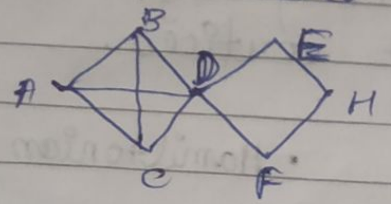


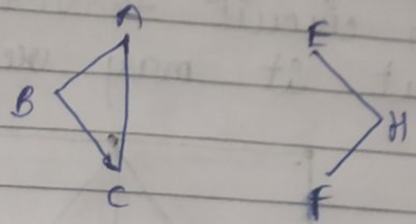
$d(A,F) = 3$
$diam(G) = 4$

⇒ Cut points and Bridges :-

* Let 'G' be a connected graph, a vertex v in 'G' is called a cut point, if G − v is disconnected.
* An edge 'e' of 'G' is called a bridge, if G − e is disconnected.

Eg :-

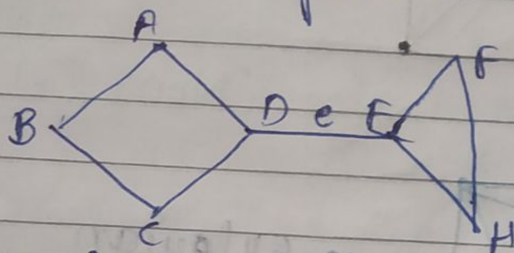


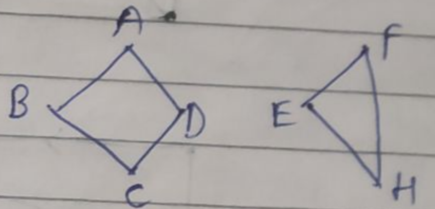


D is a cut point.

(G − D)

Eg.



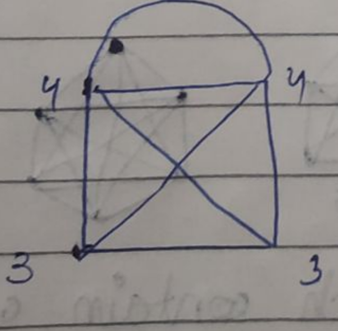


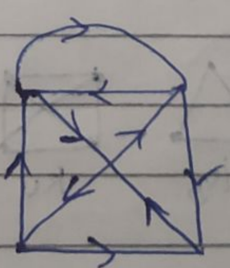
DE is a bridge

(G − e)

⇒ ~~Traversalable~~ Traversable ~~trail~~ Multi-graph :-

* A multi-graph is said to be Traversable if it can be drawn without any break in the curve and without repeating any edges i.e. if there is a path which include all vertices and uses each edge exactly once. Such a path must be a trail and will be called a traversable trail.



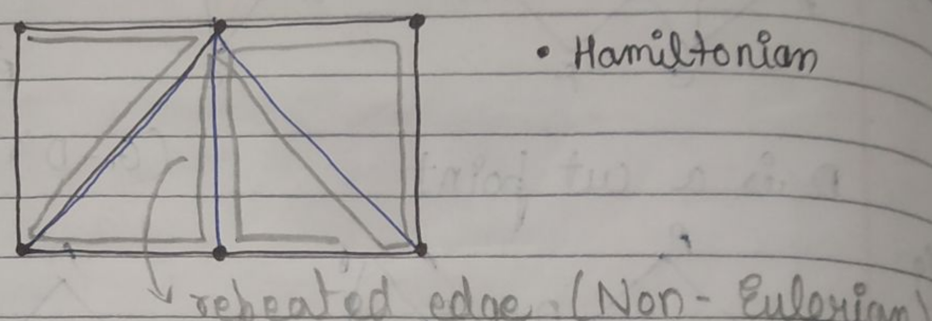


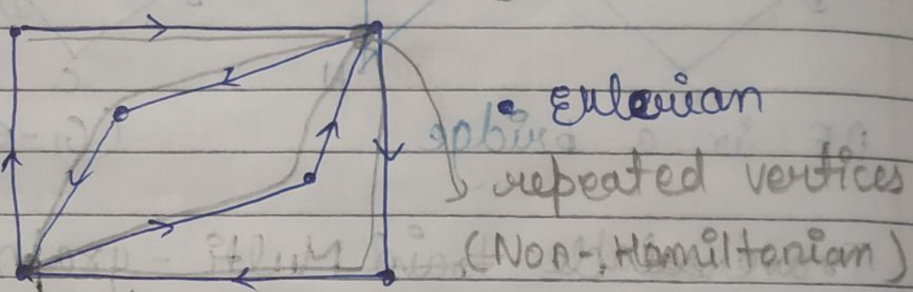
If a vertex q is odd, the traversable trail must begin or end at q. A multigraph with more than two odd vertices can not be traversable.

⇒ Hamiltonian and Eulerian graph :-

* A Hamiltonian circuit in a graph G, is a closed path that visit every vertex in G exactly once but it may repeat edges.
* Eulerian circuit travers every edge at exactly once but it may repeat vertices.

Eg:-

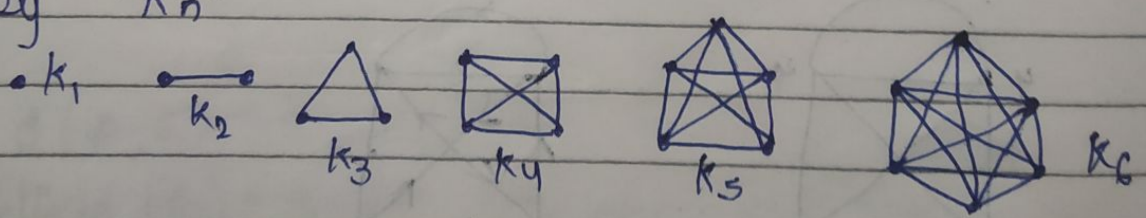




⇒ Complete graph :-

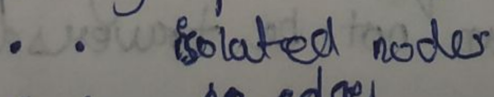
* A graph 'G' is said to be complete, if every vertex in 'G' is connected to every other vertex in 'G', thus a complete graph 'G' must be connected.
* The complete graph with 'n' vertices is denoted by 'K_n'

Eg:-



⇒ Null graph :- A graph which contain only isolated node is called a NULL graph ie the set of edges in a null graph is empty.

Eg :-

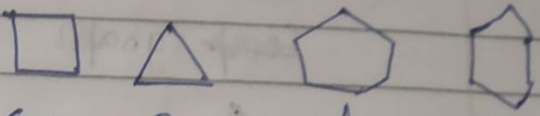


⇒ **Regular graph :-**

A graph 'G' is regular of degree k or k - regular if every vertex has degree k. In other words, a graph is regular if every vertex have a same degree.

0 regular

1 regular

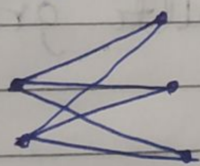


⟵ 2 regular ⟶

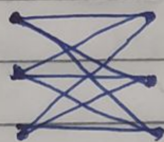
3-regular

Imp ⇒ **Bipartite Graph :-**

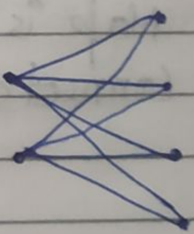
A graph 'G' is said to be Bipartite if its vertices 'V' can be partitioned into two sub set N & M such that each edges of 'G' connects a vertex of M to a vertex of N. This graph is denoted by $K_{m,n}$ where small m is the no. of verticles in 'M' & 'n' is the no. of vertices in 'N'. For this $m \leq n$.



$K_{2,3}$



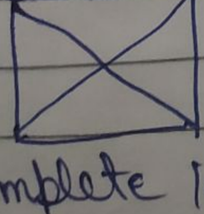
$K_{3,3}$.



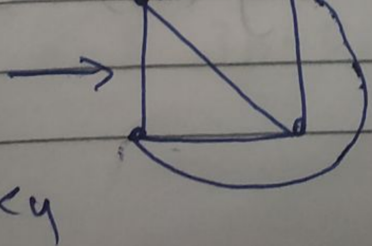
$K_{2,4}$

⇒ **Planar Graph :-**

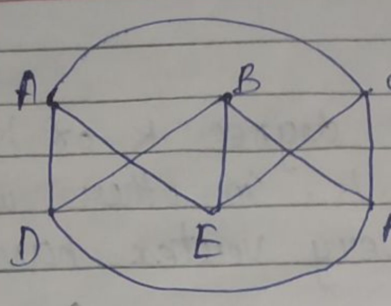
A graph ~~of~~ or multi-graph which can be drawn in the plane so that its edges do not cross is said to be planar graph.



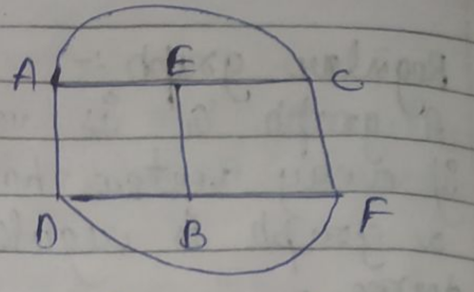
Complete K_4



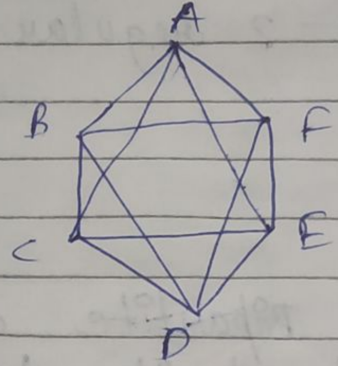
Planar K_4.

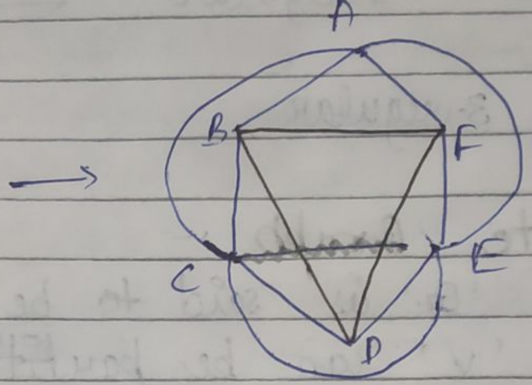


~~Comp~~ graph



Planar Graph





Planar Graph

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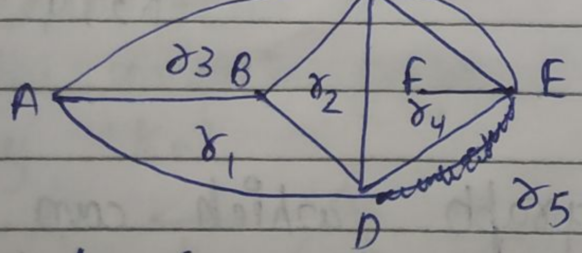
⇒ Maps, region :-
A particular Planar representation of a finite planar multi graph is called a map.
Map is connected if the multi graph is connected.

Eg :-



$deg(r_1) = 3$

vertices $= 6$

edges $= 9$

$deg(r_2) = 3$

$deg(r_3) = 4$

$deg(r_4) = 5$

$deg(r_5) = 3$

→ Euler's formula :-

Euler gave the formula which connect the no. of v of vertices, the no. e of edges and the no. R of regions of any connected map.

$$V - E + R = 2$$

Let 'G' be a connected planar graph with p vertices and q edges where p ~~is q~~ ≥ 3 then $q \leq 3p - 6$.

The theorem is not true for K_1 where, $p = 1$, $q = 0$ and is not true for K_2 where $p = 2$, $q = 1$

proof :-

Let r be the no. of region in planar representation of G, by Euler's formula,

$$p - q + r = 2$$

Now, the sum of the degrees of the region equals to $2q$ ($r = 2q$), but each region has degree 3 or more. Hence,

$$2q \leq 3r$$

$$r \geq \frac{2q}{3}$$

substituting this in the Euler's formula

$$2 = p - q + r$$

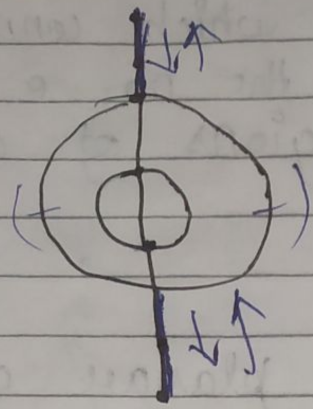
$$\leq p - q + \frac{2q}{3}$$

$$2 \leq p - \frac{q}{3}$$

$$6 \leq 3p - q$$

⇒ ~~Not~~ planar :-

Q :-



vertices = 6
edges = 9
region = 5
degree = 6 (1+ ( ))

⇒ Non - planar graph :-

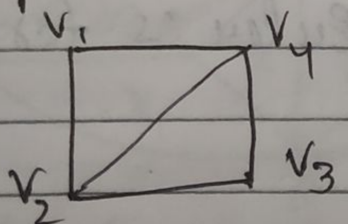
$\cancel{q \le 3p - 6}$
$\cancel{q = 5}$
$\cancel{p = 10}$
$\cancel{5 \le 30 - 6}$
$\cancel{5 \le}$

$q \le 3p - 6$
$p = 5$
$q = 10$
$10 \le 15 - 6$
$10 \nleq 9$

Thus, this is not a planar graph.

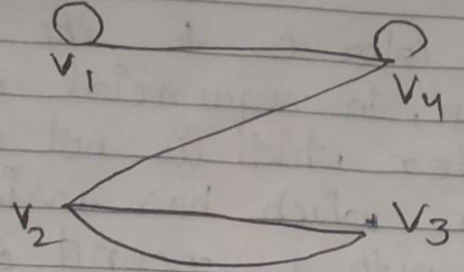
→ Graph :-

Eg :-



$$\begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 0 & 1 & 0 & 1 \\ V_2 & 1 & 0 & 1 & 1 \\ V_3 & 0 & 1 & 0 & 1 \\ V_4 & 1 & 1 & 1 & 0 \end{array}$$

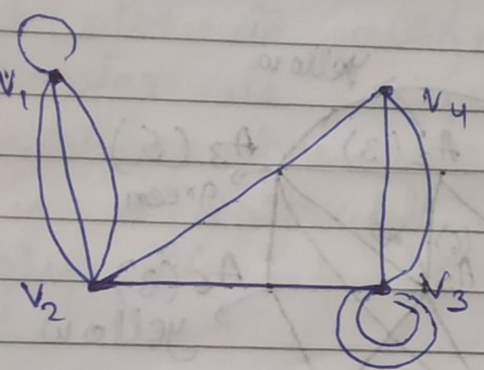
Eg:-



$$\begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 1 & 0 & 0 & 1 \\ V_2 & 0 & 0 & 2 & 1 \\ V_3 & 0 & 2 & 0 & 0 \\ V_4 & 1 & 1 & 0 & 1 \end{array}$$

Eg:-

$$A = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 1 & 3 & 0 & 0 \\ V_2 & 3 & 0 & 1 & 1 \\ V_3 & 0 & 1 & 2 & 2 \\ V_4 & 0 & 1 & 2 & 0 \end{array}$$



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⇒ Graph ~~colouring~~ coloring :-

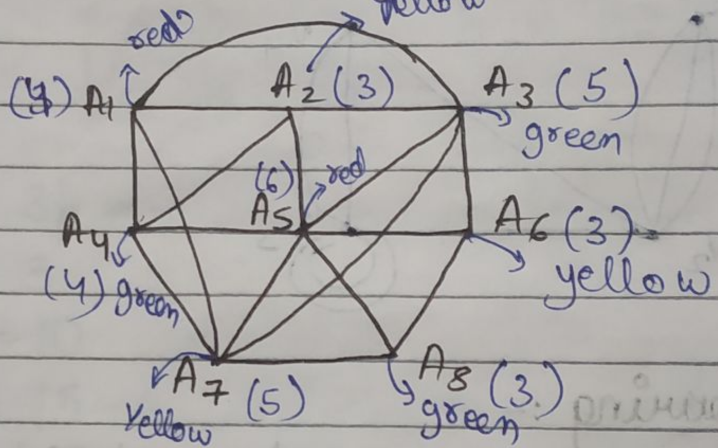
Consider a graph 'G'. A vertex coloring, or a coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different color. G is n-colorable, if there exist a coloring of G which uses n-colors, the minimum no. of colors needed to paint G is called ~~a~~ the chromatic number of G and is denoted by '$\chi(G)$'

→ Algo (welch _ Powell)

The input is Graph 'G'.

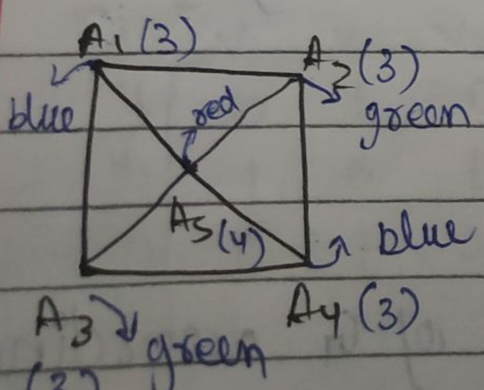
* Step 1 :- Order the vertices of G according to decreasing degrees.

* **Step 2 :-** Assign the first color C_1 to the first vertex and then, in sequencial order, assign C_1 to each vertex which is not adjacent to a previous vertex which has assigned C_1.
* **Step 3 :-** Repeat step 2 with a second color C_2 and the subsequence of non-color vertices.
* **Step 4 :-** Repeat step 3 with a third color C_3, then a fourth color C_4 and so on until all vertices are colored.
* **Step 5 :-** Exit.

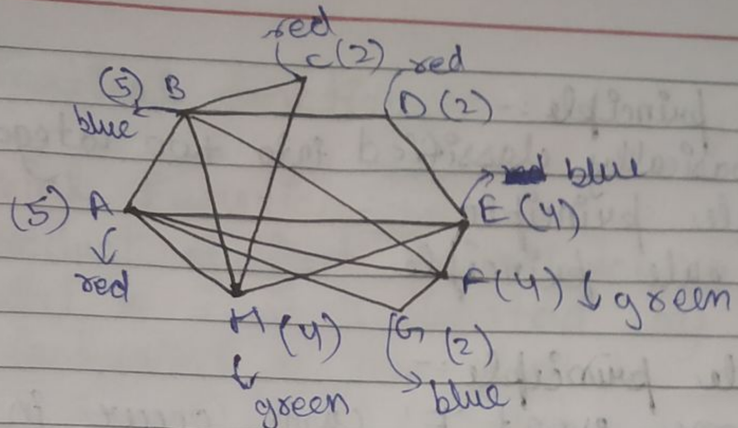


Y. A_5, A_3, A_7, A_4, A_1, A_2, A_6, A_8
red, green, Yellow, green, red, Yellow, Yellow, green

- No. of color used = chromatic number
- chromatic number of graph is 3
- It's a 3-colorable graph.



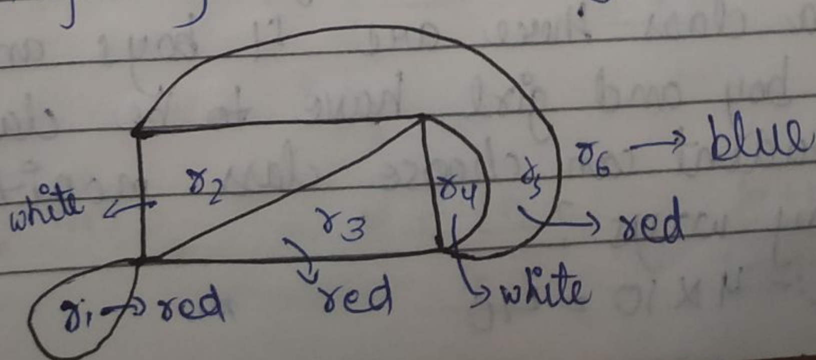
Chromatic no. = 3



⇒
1. $A_5, A_3, A_7, A_4, A_1, A_2, A_6, A_8$
2. The first color is assign to vertices A_5, A_1.
3. The second color is assign to vertices A_3, A_4, A_8.
4. The third color is assign to vertices A_2, A_7, A_6
5. All the vertices has been assignod a color and so G is 3-colorable.
6. Accordingly, $\chi(G) = 3$ (Red, Green, Yellow).

⇒ Theorem :-
The following are equivalent for a graph G :-
1. G is 2-colorable
2. G is bipertite
3. Every cycle of G has even length

⇒ ~~Dt~~ Dual maps :-
'M' is planar representation of a planar multigraph. Two regions of M are said to be adjacent, if they have an edge in common.



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⇒ Counting principle :-
It is basically classified into two categories,
i) Sum rule principle
ii) Product rule principle

i) Sum rule principle :-
* Assume some event 'E' came occur in 'm' ways and a second event 'F' came occur in 'n' ways and suppose both events can not occur simultaneously
* then E or F can occur in '$m \times n$' ways,
* If there are n event and not two events occur at same time then the event can occur in $n_1 + n_2 - - - n$ ways.

Eg :- If eight male professor and 5 female professor teaching maths. then students can choose, ~~professor can choose~~ professor in how many ways

$8 + 5 = 13$

Q.

ii) Product rule principle :-
Suppose there is event E which occur in m ways and independent of this event, there is a second event F which can occur in n ways then combination of E and F can occur in $m \times n$ ways.
If there are n event occuring independently then all events can occur in the order indicated as $n_1 \times n_2 \times n_3 - - - n$ ways.

Eg :- In a class there are 4 boys and 10 girls, if a boy and girl have to be class monitor the student can choose class monitor in how many ways ?

Sol :- $4 \times 10 = 40$

⇒ Mathematical function :-

i) Factorial Function :-
The product of first n natural no. is called factorial n, it is denoted by $\lfloor n$.
The factorial n can also be written as
$\lfloor n = n(n-1)(n-2) \cdots 1$

Eg. Find the value of $\lfloor 10 / \lfloor 8$.
$$\frac{10 \times 9 \times \cancel{\lfloor 8}}{\cancel{\lfloor 8}} = 90.$$

ii) Binomial coefficient :-
It is represented by ${}^nC_r$ where n and r are positive integer with $r \leq n$,
$${}^nC_r = \frac{\lfloor n}{\lfloor n-r \; \lfloor r}$$

Eg :- Find ${}^8C_2$
$${}^8C_2 = \frac{\lfloor 8}{\lfloor 6 \; \lfloor 2} = \frac{\cancel{\lfloor 6} \times 7 \times \cancel{8}\,4}{\cancel{\lfloor 6} \times \cancel{2} \times 1} = 28$$

iii) Permutation & combination :-
* An arrangement of a set of n object in a given order is called permutation of objects.
* An arrangement of any $r \leq n$ of these object in a given order is called an r-permutation or a permutation of n object taken r at a time
* It is denoted by $\underbrace{P(n, r)}_{\downarrow \; {}^nP_r} = \frac{\lfloor n}{\lfloor n-r}$

Eg $4 \times {}^nP_3 = {}^{n+1}P_3$ $\quad 4 \times \frac{\lfloor n}{\lfloor n-3} = \frac{\lfloor n+1}{\lfloor n+1-3}$

$$\frac{4 \times \lfloor r}{\lfloor n-3} = \frac{\lfloor r \times (n+1)}{\lfloor n-2}$$

$$\frac{4}{(n-3) \times \lfloor n-2} = \frac{n+1}{\lfloor n-2}$$

$$\frac{4}{\lfloor n-3} = \frac{n+1}{\lfloor n-3}$$

$$\cancel{4 = n^2 + n - 3n - 3}$$

$$\cancel{n^2 - 2n - 7 = 0}$$

$$4 = n + 1$$

$$n = 3$$

⇒ Permutation with restriction :-

The no. of permutation of n different objects taken r at a time in which k particular object does not occur is ${}^{n-k}P_r$.

The no. of permutation of n different objects taken r at a time in which k particular object are present is ${}^{n-k}P_{r-k} \times {}^{r}P_k$

Eg:- how many six digit no. can be formed by using digits 0, 1, 2, 3, 4, 5, 6, 7, 8, if every no is to start with '30' with no digit repeated.

Permutation with restriction

${}^{n-k}P_r$

$${}^{9-2}P_4 = {}^{7}P_4 = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 120 \times 7$$

$$= 840$$

Q. How many no. of two digits can be form with digits 1, 2, 3, if repeatation of digits are not allowed.

$$^nP_r = {}^3P_2 = \frac{\lfloor 3}{\lfloor (3-2)} = 3 \times 2 = 6$$

Q. If $^nP_2 = 72$, find the value of n.

$$^nP_2 = \frac{\lfloor n}{\lfloor n-2} = 72$$

$$\frac{n(n-1)\cancel{\lfloor n-2}}{\cancel{\lfloor n-2}} = 72$$

$$n^2 - n = 72$$

$\cancel{n = 72}$ $\quad n^2 - n - 72$

$\cancel{n = 72}$ $\quad n^2 - 9n + 8n - 72$

$$n(n-9) + 8(n-9)$$

$$(n-9)(n+8)$$

$$\boxed{n = 9}, \ n = -8$$

Q. How many different no. line between 100 and 1000 can be formed with the digit 1, 2, 3, 4, 5, no digit is repeated.

$$^nP_r = {}^5P_3 = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} = 60$$

Q. Three prizes are to be awarded among 10 candidat
In how many ways can the prizes be given so tha
no candidate may get more than one prize.

$$^{10}P_3 = \frac{10!}{7!} = \cancel{\frac{10 \times 8 \times 7!}{7!}} \cancel{= 80} \times 9 = 720$$

$$= \cancel{\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!}}$$

$$\frac{10!}{7!} = \cancel{10 \times 9 \times 8} = 720$$

Q. In how many of permutation of ten things taken 4 at a time.
a) Two things always occur
b) Never occur

$n = 10, \; r = 4$

a) $K = 2$

$^{10-2}P_2 \times {}^4P_2$

$= \frac{8 \times 7 \times \cancel{6!}}{\cancel{6!}} \times \frac{4 \times 3 \times \cancel{2!}}{\cancel{2!}}$

$= 56 \times 12$

$= 672$

b) $^{10-2}P_4 = {}^8P_4 = \frac{8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}}$

$= 56 \times 30$

$= 1680$

Q. In how many ways can 7 boys and 5 girls be seated in row so that no two girls can seat

8 ways of seat girl, 7 ways of seat boys.

$n = 8, \; r = 5, \; nPr = 8P5 = \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor 8}{\lfloor 8-5}$

$= \frac{\lfloor 8}{\lfloor 3} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 6720 \times \lfloor 7$

$= 33,868,800$

Q. In how many ways can 4 boys and 4 girls can be seated in row so that boys and girls are alter note.

Case 1:- BG BG BG BG $\qquad$ Case 2:- GB GB GB GB

$\lfloor 4 \times \lfloor 4 \quad + \quad \lfloor 4 \times \lfloor 4$

$= 24 \times 24 + 24 \times 24$

$= 576 + 576 = 1152$

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Q. ~~Girls~~ Given 10 people $p_1, p_2, p_3 \cdots p$ time in how many ways can people will line up in the row.

→ how many lines up are there if P_2, P_6, P_9 want to stand together..

Ans:- $\underline{|10}$ ways without any restriction 10 people can be line up in $\underline{|10}$ ways.

treating P_2, P_6, P_9 as one, 8 people can be Arranged in the row in $\underline{|8}$ ways.

P_2, P_6, P_9 can be arranged among them selves in $\underline{|3}$ ways.

$$\text{so, } \underline{|8} \times \underline{|3} = 40320 \times 6 = 241920$$

Q. How many n no. of 4 digit can be formed with 1, 2, 3, 4, 5 and repeatition is not allowed.

$$n = 5,\ r = 4,\ {}^nP_r = \frac{\underline{|n}}{\underline{|n-r}} = \frac{\underline{|5}}{\underline{|5-4}} = \frac{\underline{|5}}{\underline{|1}} = 120$$

Q. find the no. of permutation that had letter DAUGHTER.

i) taking all the letter together,

$$\underline{|8} \text{ ways} = 40320$$

ii) biggining with D, $\underline{|7}$ ways $= 5040$

iii) beginning with D & ending with R.

$$\underline{|6} \text{ ways} = 720$$

iv) vowels being always together

$$A, U, E = 1,\ \underline{|5+1} = \underline{|6} \times \underline{|3} = 4320$$

v) Not all vowels together

$$\underline{|8} - \underline{|6} \times \underline{|3} \Rightarrow 40320 - 4320 = 36000$$

vi) Not 2 vowels together.

$${}^6P_3 \times \underline{|5}$$

$$\frac{\underline{|n}}{\underline{|n-r}} = \frac{\underline{|6}}{\underline{|6-3}} = \frac{\underline{|6}}{\underline{|3}} = \frac{6 \times 5 \times 4 \times \cancel{\underline{|3}}}{\cancel{\underline{|3}}} = 120 \times \underline{|5}$$

$$\Rightarrow 120 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 14400$$

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$\Rightarrow$ Permutation of thing not all different :-

To find the no. of ways in which n things may be arranged among themselves, taking all of them at a time, when 'p' of things ~~are~~ alike of one kind, 'q' of them alike of second kind, 'r' of them of a third kind and the rest all different

$$x = \frac{\lfloor n}{\lfloor p \lfloor q \lfloor r}$$

Q. How many permutation of the letter of the word 'BANANA'.

$$x = \frac{\lfloor 6}{\lfloor 3 \lfloor 2} = \frac{\cancel{6}^{3} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times \cancel{2}} = 60$$

there are 6 letters in the word three are alike of one kind (3A's), two are alike of second kind (2N's) and the rest one letter is different.

Q. Find the no. of possible ways in which the letter 'COTTON' can be arranged so, that the two T's don't come together.

$$x = \frac{\lfloor 6}{\lfloor 2 \lfloor 2} = \frac{\cancel{6}^{3} \times 5 \times 4 \times 3 \times \cancel{2}}{\cancel{2} \times \cancel{2}} = 180$$

Consider two T as one,

$$x = \frac{\lfloor 5}{\lfloor 2} = \frac{5 \times 4 \times 3 \times \cancel{2}}{\cancel{2}} = \cancel{120}\ 60$$

$$180 - 120 = 60$$

Q. A coin is tossed ~~six~~ six time, in how many ways can be obtain 4 Heads and 2 Tails

$$x = \frac{\lfloor 6}{\lfloor 4 \lfloor 2} = \frac{\cancel{6}^{3} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}} = 15$$

Q. There are three copies each of four different book find the no. of ways of arranging them on a ~~sell~~ shelf.

$$x = \frac{\lfloor 12}{\lfloor 3 \lfloor 3 \lfloor 3 \lfloor 3} \quad \cancel{= 12 \times 11 \times 10 \times 9}$$

$$x = \frac{\cancel{12} \times 11 \times 10 \times \cancel{9} \times 8 \times 7 \times \cancel{6} \times 5 \times 4 \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{3} \times \cancel{2} \times \cancel{3} \times \cancel{2}}$$

$x = 66 \times 30 \times 56 \times 20$

$= 66 \times 600 \times 56$

$= 3696 \times 100 = 369600$

$= \cancel{2217600}$

$66 \times 56 = 396 + 3300 = 3696$; $3696 \times 6 = 22176$

⇒ point:–

* The no. of permutation of n different things taken r at a time when thing may be repeated. n^r

Q In How many ways three prizes can be distributed among four boys when ~~bo~~ a boy can get any no. of prizes.

$$\cancel{3^4 = 27} \quad 4^3 = 64$$

$56 \times 36 = 336 + 1680 = 2016$

⇒ Circular permutation :– = $\lfloor n-1$ ways

Q. In how many ways five boys and four girls can be seated at a round table

i) If there is no restriction

ii) All the four girls sit together

iii) All the four girls do not sit together.

$$n = 9 = \lfloor n-1 = \lfloor 8$$

$$\underbrace{8 \times 7}_{56} \times \cancel{6} \times \underbrace{5 \times \cancel{4}}_{20} \times \underbrace{3 \times 2}_{36} = 40320$$

ii) $\frac{\lfloor 6-1}{} = \lfloor 5 \times \lfloor 4 = 120 \times 24 = 2880$

iii) ~~40320~~ $40320 - 2880 = 37440$

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⇒ Combination :-

* The different groups or selection that can be made out of the given set of things by taking some or all of them at a time irrespective of the order
* are called combination.

The no. of combination of n different things taken $r \le n$ at a time is denoted by $C(n, r)$ or ${}^nC_r$

$${}^nC_r = \frac{\lfloor n}{\lfloor n-r \ \lfloor r}$$

Eg:- abc, two at a time

$${}^3P_2 = \frac{\lfloor 3}{\lfloor 1} = 6 \qquad ab, ac, ba,$$

$${}^3C_2 = \frac{\lfloor 3}{\lfloor 1 \ \lfloor 2} = 3.$$

$${}^3C_2 = \frac{3 \times \cancel{2}}{\cancel{2} \times 1} = 3, \quad {}^9C_3 = \frac{{}^3\cancel{9} \times \overset{4}{\cancel{8}} \times 7}{\cancel{3} \times \cancel{2}} =$$

Q. Find the value of n ${}^nC_{n-2} = 10$

$$\frac{\lfloor n}{\lfloor \cancel{n}-\cancel{n}-2 \ \lfloor n-2} = \frac{\lfloor n \times n-1 \ \cancel{\lfloor n-2}}{\lfloor 2 \ \cancel{\lfloor n-2}}$$

$n^2 - n = 20$

$n^2 - n - 20$

$n^2 - 5n + 4n - 20 = 0$

$n(n-5) + 4(n-5) = 0 \qquad n = \cancel{-4}, 5$

1365
126

Date: / /

Q. In how many ways can 4 questions be selected from 7 question.

$$^7C_4 = \frac{7 \times \cancel{6} \times 5 \times \cancel{4}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 35$$

Q. If there are 12 person in a party and if ~~two~~ each two of them shake hands with each other, then how many shake hands happen in the party.

$$^{12}C_2 = \frac{{}^{6}\cancel{12} \times 11}{\cancel{2}} = 66$$

Q. In how many ways can a committee of 5 ~~students~~ teachers and 4 student be choosen from 9 teachers and 15 students.

$$^9C_5 \times {}^{15}C_4 = \frac{9 \times \overset{2}{\cancel{8}} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} \times \frac{{}^{15}\cancel{15} \times \overset{7}{\cancel{14}} \times 13 \times \cancel{12}}{\cancel{4} \times \cancel{3} \times \cancel{2}}$$

$$= 126 \times 1365$$

$$= 171990$$

$15 \times 13 = 195$, $195 \times 7 = 1365$

Q. A committee of 5 people is to be form from a group of 4 man and 7 women, how many possible committee can be form if atleast 3 women are on the ~~to~~ committee.

$$^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1 + {}^7C_5 \times {}^4C_0$$

$$= \left(\frac{7 \times \cancel{6} \times 5}{\cancel{3} \times \cancel{2}} \times \frac{{}^{2}\cancel{4} \times 3}{\cancel{2}}\right) + \left(\frac{7 \times \cancel{6} \times 5 \times \cancel{4}}{\cancel{4} \times \cancel{3} \times \cancel{2}} \times 4\right) + \left(\frac{7 \times \cancel{6} \times \cancel{5} \times \cancel{4} \times 3}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}\right)$$

$$= (35 \times 6) + \cancel{(28)} + 21$$

$$= 210 + \cancel{28} + 21 + 140$$

$$= \cancel{259} \quad 371$$

$35 \times 4 = 140$

Q. How many automobile liceanse plate can be made if each plate contain two different letters followed by 3 different digits? Solve the problem if the first digit can not be zero

$\overset{26}{\longleftrightarrow}$ $\overset{10}{\longleftrightarrow}$

$\square\square\ \square\square\square$

$^{26}C_1 \times {}^{25}C_1 \times {}^{9}C_1 \times {}^{9}C_1 \times {}^{8}C_1$

$= 26 \times 25 \times 9 \times 9 \times 8$

$= 421200$

Q. From 10 program how many can ~~be se~~ 5 be selected when

i) A particular programmer include every time.

ii) A particular programmer not include all time

i) $^{9}C_4 = \dfrac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6}}{\cancel{4} \times \cancel{3} \times \cancel{2}} = 126$

ii) $^{9}C_5 = \dfrac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} = 126$

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→ Pigeonhole principle :-

If 'n' pigeons are assigned to 'm' pigeonhole then at least one pigeonhole contain two or more pigeons (m < n).

Proof :- Let 'm' pigeonhole be numbered with the numbers 1 to m.

Beginning with the pigeon 1, each pigeon is assigned in order to the pigeon hole with the same no.

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since $(m < n)$ i.e the no. pigeon hole is less than the no. of pigeons, $n-m$ are left without having assigned a pigeon hole.

Thus, at least one pigeon hole will be assigned to a more than one pigeon

Eg:- If 6 colours are used to paint 37 home, show that at least 7 home of them will ~~of~~ be of same colour.

$$\frac{\cancel{37}\ 36}{\cancel{6}} = 1$$

→ Six home each of 6 colours but reminder 1 will be a colour from the 6.

→ 7 home may have a same cycle

~~DSA~~ - 2, 5

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Unit :- 4

Algebric Structures

⇒ Binary operation : Let G be a non-empty set then
$G * G = \{(a,b) : a \in G, b \in G\}$
If $f : G \times G \to G$ then f is ~~sait~~ said to be ~~empty~~ binary operation on G.
Thus, a binary operation on G is a function that assign each ordered pair of elements of G ~~on~~

Eg :- $\langle N,+\rangle$, $\langle Z,+\rangle$ these are the binary operations.
$\langle N,-\rangle$, $\langle Z,/\rangle$ these are not binary operations.
$\langle N,+\rangle$ $\forall$ $a,b \in G$
$a+b \in G$

⇒ Algebric structure :- A non-empty set together with one or more than one binary operation is called binary algebric structure.

Eg :- $\langle N,+\rangle$, $\langle Z,+\rangle$, $\langle R,+\rangle$ all are the algebric structure.

⇒ Group :- Let $(G,*)$ be an algebric structure, where '$*$' is a binary operation, then $(G,*)$ is called a group under this operation if the following condition are satisfied.

⇒ Closure law :- The binary '$*$' is closed operation i.e $a*b \in G$ $\forall$ $a,b \in G$

⇒ Associative law :- The binary operation '$*$' is an associative operation i.e $a*(b*c) = (a*b)*c$ for all $a,b,c \in G$.

⇒ Identity element :- There exist an identity element i.e for some $e \in G$, $e*a = a*e = a$, ~~$\forall$~~ $a \in G$

→ Inverse element :- For each a in G, there exist an element a' i.e $a * a' = e$.

* A group G is said to be abelian if the commutative law holds. $(a*b = b*a)$.
* A group with addition binary operation is known as additive group and that with multiplication binary operation is known as multiplicative group.

Q. Let G = 0, 1, 2, 3, 4, 5 is a set prove that G is an abelian group under operation additive %6

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

* Closure property :- all elements belong to G so, $(G, +_6)$ is algebric structure. $(0 +_6 1) = 1 \in G$
* Associative law :-

$0 +_6 (1 +_6 2) = (0 +_6 1) +_6 2$

$0 +_6 3 = 1 +_6 2$

$3 = 3$

* Identity element :-

$1 +_6 0 = 1$, $4 +_6 0 = 4$

* Inverse element :-

Let a = 0

$0 +_6 0 = 0$

$1 +_6 5 = 0$

$2 +_6 4 = 0$

$3 +_6 3 = 0$

$4 +_6 2 = 0$

$5 +_6 1 = 0$

* Commutative law :-

$0 +_6 1 = 1 +_6 0$

$1 = 1$

Hence, $(G, +_6)$ is an abelian group.

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Q. Prove that $G = 1, 2, 3, 4, 5, 6$ is an abelian group under operation $(*_7)$.

$*_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

* closure property :-
all element belongs to G so, $(G, *_7)$ is algebric

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Q. The set Q^+ of all positive rational no form an abelian group under the composition * defined by.

$$a * b = ab/2 \quad \forall a, b \in Q^+$$

* Closure :- If $a, b \in Q^+$ then $ab/2$ is also a positive rational no., therefore $a * b = ab/2 \in Q^+ \ \forall a, b \in Q^+$

Eg:- $\frac{5}{7} \times 3 = \frac{\frac{5}{7} \times 3}{2} = \frac{15}{14}$ (ration no.)

* Associative :-

for, $a, b, c \in Q^+$ we have

$$a * (b * c) = (a * b) * c$$

$$a * \frac{bc}{2} = ab/2 * c$$

$$\frac{abc}{4} = \frac{abc}{4}$$

Therefore, $a * (b * c) = (a * b) * c$

* Identity law :-

$$a * e = a$$

$$\frac{\cancel{a}e}{2} = \cancel{a}$$

$$e = 2$$

for eg :- $\frac{5}{7} * 2 = \frac{\frac{5}{7} * \cancel{2}}{\cancel{2}} = \frac{5}{7}$

hence $\frac{5}{7} * 2 = \frac{5}{7}$

* Inverse law :-

$$a * a' = e$$

$$\frac{aa'}{2} = e$$

$$aa' = 2e$$

$$aa' = 2 \times 2$$

$$a' = \frac{4}{a}$$

→ Check :-

$\frac{5}{7} * \frac{4}{5/7} = 2$

$\cancel{\frac{5}{7} * \frac{28}{5}}$ $\frac{\frac{\cancel{5}}{\cancel{7}} * \frac{\cancel{28}^{4}}{\cancel{5}}}{\cancel{2}} \cancel{4}\ 2 = 2$

hence $(a * a' = e)$.

* Commutative law :-

$$a * b = b * a$$
$$\frac{ab}{2} = \frac{ba}{2}$$

Check :-

$$\frac{\frac{5}{7} \times 3}{2} = \frac{3 \times \frac{5}{7}}{2}$$

hence, $(a * b = b * a)$.

hence, the set Q^+ form an abelian group under the operation.

Q. Let $G = R - \{-1\}$, Prove that $(G, *)$ is a group where $*$ is defined by $a * b = a + b + ab$ $\forall\, a, b \in G$.

* closure property :-

Let $a, b \in G$ where a and b are real no.

$a + b + ab$ is also a real no.

* Associative law :-

for $a, b, c \in G$ we have

$$a * (b * c) = a * (b + c + bc)$$
$$= a + (b + c + bc) + (ab + ac + abc)$$
$$(a * b) * c = (a + b + ab) * c$$
$$= (a + b + ab) + c + (ac + bc + abc)$$

hence, $a * (b * c) = (a * b) * c$.

* Identity law :-

$$a * e = a$$
$$a + e + ae = a$$
$$e + ae = 0$$
$$e(1 + a) = 0$$
$$e = 0$$
$$a + 0 + 0 = a$$
$$a = a$$

hence, $a * e = a$.

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* Inverse law:-

$a * a' = e$

$a + a' + aa' = e$

$a'(1+a) = e - a \quad , e = 0$

$$\boxed{a' = \frac{-a}{(1+a)}}$$

$$a - \frac{a}{1+a} - \frac{a^2}{(1+a)} = e$$

$$\frac{\cancel{a} + \cancel{a^2} - \cancel{a} - \cancel{a^2}}{(1+a)} = 0$$

hence, $a * a' = e$.

* Commutative law:-

$a * b = b * a$

$a + b + ab = b + a + ab$

hence, $a * b = b * a$

Therefore, the ~~set~~ group G form an abelian group under the operation.

2) ~~Order of group:-~~

Q. Prove that the four roots of $\{1, -1, i, -i\}$ form an abelian multiplicative group.

*	1	-1	i	-i
1	$\boxed{1}$	-1	i	-i
-1	-1	$\boxed{1}$	-i	+i
i	i	-i	-1	$\boxed{1}$
-i	-i	i	$\boxed{+1}$	-1

* Closure law:-

Since, all the entries in the table are the element of G and hence, G is closed with respect to multiplicative.

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* Associative law :-

$a * (b * c) = (a * b) * c$

Eg :- $1 * (-1 * i) = (1 * -1) * i$

$-i = -i$

* Identity element :-

1 belong to G is identity element i.e ($e = 1$)

$a * e = a$

$a * 1 = a$

* Inverse law :-

Inverse of $1, -1, i, -i$ are $1, -1, i, -i$ respecti[vely]

all belongs to G.

* Commutative property :-

$ab = ba \quad \forall\ a, b \in G$

hence, this group is a abelian multiplicati[ve] group.

). Prepare the composition table for multiplicati[on] on the element in the set $A = 1, \omega, \omega^2$, where $\omega = \sqrt[3]{1}$ show that this form an abelian group.

$\boxed{\omega^3 = 1}$

*	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

Closure property :-

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⇒ Order of an element of group :-
Let G be a multiplicative group & $a \in G$ is any element of G then an non-negative smallest integer n is said to be order of element a, if $a^n = e$ where e is identity element of Group.

Q. Find the order $\{1, \omega, \omega^2\}$

$(\omega = \sqrt[3]{1}$
$\omega^3 = 1$

$(1)^1 = 1$
$O(1) = 1$
$\omega^3 = 1$
$O(\omega) = 3$
$(\omega^2)^3 = 1$

$O(\omega^2) = 3$

Q. $\{1, -1, i, -i\}$
$(1)^1 = 1$
$O(1) = 1$
$(-1)^2 = 1$
$O(-1) = 2$
(i)

$G = \{0, 1, 2, 3\}$
under $+_4$
$2^1 = 2$
$2^2 = 2 +_4 2$
$\cancel{1^2 = 1 +_4}$
$1^3 = 1 +_4 3$
$= \frac{4}{4}$
$\boxed{O(1) = 3}$
$2^2 = 2 +_4 2$
$\boxed{O(2) = 2}$
$3^1 = 3 +_4 1$
$\boxed{O(3) = 1}$

$0^1 = 0$
$\boxed{O(0)}$

$\cancel{3 +_4 3 = 6}$ $2 +_4 3$

$3 +_4 3 = 2$
$2 +_4 3 = 1$

⇒ Sub group :-

* Let H be a sub set of group G then H is a called a sub group of G if H itself is a group under the operation of G.

* ~~Let H be~~ A subset H of group G if the following condition hold :-

i) The identity element $e \in H$

ii) It is closed under the operation of G if $(a,b) \in H$ then $(a*b) \in H$

iii) H is closed under inverse if $a \in H$ then $a^{-1} \in H$

iv) Associative law.

Eg:- Consider the group $\langle Z_4, +_4 \rangle$. $Z_4 = \{0,1,2,3,\}$

a) $H_1 = \{0,1\}$

b) $H_2 = \{0,3\}$

c) $H_3 = \{0,2\}$

which of the following is a sub group.

a) $H_1 = \{0,1\}$

$+_4$	0	1
0	0	1
1	1	2

$0 \in H_1$

$1 \in H_1$

$2 \notin H_1$, so, it's not a sub group.

b)

$\cancel{+_4}$	0	3
0	0	3
3	3	2

$0 \in H_2$

$3 \in H_2$

$2 \notin H_2$, so, it's not a sub group.

c)

$+_4$	0	2
0	0	2
2	2	0

~~$(0*2)*3 = 0*(2*3)$~~

~~$(0+_4 2)+_4 2 = 0+_4(2+_4 2)$~~

$2 +_4 2 = 0 +_4 0$

$= 0$

3) Homomorphism

Let $\langle G, * \rangle$, $\langle G', O \rangle$ be two group then a mapping $f: G \to G'$ is called homomorphism if for each $(a, b) \in G$ we have ~~f(a*b) = f~~

$$f(a*b) = f(a) \, O \, f(b)$$

Q Let $P = \{0, 1, 2, 3\}$ and $Q = \{1, 3, 7, 9\}$ are two sets, $\langle P, +_4 \rangle$ and $\langle Q, *_{10} \rangle$ are algebric structure. A mapping $f: P \to Q$ is such f is defined as $f(0) = 1$, $f(1) = 3$, $f(2) = 9$, $f(3) = 7$, decide the homomorphism between two given algebric structure.

P

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Q

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

$$f(a*b) = f(a) \, O \, f(b)$$
$$f(2 +_4 3) = f(2) \times_{10} f(3)$$
$$f(1) = 9 \times_{10} 7$$
$$3 = 3$$

Hence, Homomorphism

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Eq:- Let $(G, \times)$ be a group defined by $G = \{1, -1, -i, i\}$ and $(I, +)$ be a group, prove that $f: I \to G$ is a homomorphism where $f(n) = i^n \; \forall \; n \in I$

Let $n_1, n_2 \in I$

$$f(n_1 + n_2) = i^{n_1 + n_2}$$
$$f(n_1 + n_2) = i^{n_1} \times i^{n_2}$$
$$f(n_1 + n_2) = f(n_1) \times f(n_2)$$

Hence, $n_1, n_2 \in I$, so, $f: I \to G$ is a homomorphism

Q. Let $(Z, +)$ be group and $(G, \times)$ be another group. G can be defined as $G = \{2^n, n \in Z\}$ a function $f: Z \to G$ be defined by $f(n) = 2^n \ \forall \ n \in Z$. Show that $f: Z \to G$ is a homomorphism.

Let $n_1, n_2 \in Z$

$$f(n_1 + n_2) = 2^{n_1 + n_2}$$

$$f(n_1 + n_2) = 2^{n_1} \cdot 2^{n_2}$$

$$f(n_1 + f_2) = f(n_1) \cdot f(n_2)$$

⇒ Cyclic Group:-

* Let G be any group, if for $a \in G$, every element $x \in G$ can be generated by a

$$x = a^n, \text{ where } n \in I$$

then G is called a cyclic group, and a is called its generator.

* A cyclic group G with generater a is denoted by $G = \langle a \rangle$ or $G\{a\}$.

Q. Prove that $G = \{1, -1, i, -i\}$ is cyclic and find its generater.

$(i)^1 = i$

$(i)^2 = -1$ $\qquad G = \{i^4, i^2, i, i^3\}$

$(i)^3 = -i$ $\qquad G = \langle i \rangle$

$(i)^4 = 1$ $\qquad G$ is a cyclic

<u>Note</u> :- The inverse of generator is also a generater.

so inverse of i is $-i$.

$(-i)^1 = -i$ $\qquad G = \{-i^4, -i^2, -i^3, -i^1\}$

$(-i)^2 = -1$

$(-i)^3 = i$ $\qquad G = \langle -i \rangle$

$(-i)^4 = 1$

Q. Prove that multiplication group of $\{\omega, 1, \omega^2\}$ is a cyclic group and find its group.

Q. $G = \{\{0,1,2,3,4,5\} +_6\}$ is cyclic group find generator.

$(1)^1 = 1$

$(1)^2 = 1 +_6 1 = 2$

$(1)^3 = 1 +_6 1 +_6 1 = 3$

$(1)^4 = 1 +_6 1 +_6 + 6 1 +_6 1 = 4$

$(1)^5 = 1 +_6 1 +_6 1 +_6 1 +_6 1 = 5$

$(1)^6 = 1 +_6 1 +_6 1 +_6 1 +_6 1 +_6 1 = 0$

G is cyclic group

$G = \langle 1 \rangle$

Inverse of $1 = 1 +_6 5 = 0$ (Identity element)

$G = \langle 5 \rangle$

Q. Consider the group $G = \{1,2,3,4,5,6\}$ under $\times_7$

a) Find Multiplicant of G.

b) Find 2 inverse

find 3, 6 inverse

c) Find orders and subgroup generated by 2 and 3.

d) Is G cycle.

(a)

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	2
4	4	1	5	2	6	3
5	5	3	1	6	3	2
6	6	5	4	3	2	1

b) $2 \times_7 4 = 1 \ (2^{-1} = 4)$

$3 \times_7 5 = 1 \ (3^{-1} = 5)$

$6 \times_7 6 = 1 \ (6^{-1} = 6)$

c) $a^n = e$

c) $2^1 = 2$

$2^2 = 2 \times_7 2 = 4$

$2^3 = 2 \times_7 2 \times_7 2 = 1$

$|2| = 3$

$gp(2) = \{1, 2, 4\}$

$3^1 = 3$

$3^2 = 3 \times_7 3 = 2$

$3^3 = 3 \times_7 3 \times_7 3 = 6$

$3^4 = 3 \times_7 3 \times_7 3 \times_7 3 = 4$

$3^5 = 4 \times_7 3 = 5$

$3^6 = 5 \times_7 3 = 1$

$|3| = 6$

$gp(3) = \{1, 2, 3, 4, 5, 6\}$

d) $G = \langle 3 \rangle$

Q. $G = \{1, 2, 4, 7, 8, 11, 13, 14\}$ under $\times_{15}$.

a) find multiplacation table of G

b) find 2, 7, 11 inverse.

c) find the order and sub group generated 2, 7, 11

d) Is G is cycle ?

$\times_{15}$	1	2	4	7	8	11	13	14
1	1	2	4	7	8	11	13	14
2	2	4	8	14	1	7	11	13
4	4	8	1	13	2	14	7	11
7	7	14	13	4	11	2	1	8
8	8	1	2	11	4	13	14	7
11	11	7	14	2	13	1	8	4
13	13	11	7	1	14	8	4	2
14	14	13	11	8	7	4	2	1

b) $2 \times_{15} 8 = 1 \quad (2^{-1} = 8)$

$7 \times_{15} 13 = 1 \quad (7^{-1} = 13)$

$11 \times_{15} 11 = 1 \quad (11^{-1} = 11)$

c) $2^1 = 2$

$2^2 = 2 \times_{15} 2 = 4$

$2^3 = 2 \times_{15} 2 \times_{15} 2 = 8$

$2^4 = 8 \times_{15} 2 = 1$

$|2| = 4$

$gp(2) = \{1, 2, 4, 8\}$

$7^1 = 7$

$7^2 = 7 \times_{15} 7 = 4$

$7^3 = 4 \times_{15} 7 = 13$

$7^4 = 13 \times_{15} 7 = 1$

$|7| = 4$

$gp(7) = \{1, 4, 7, 13\}$

$11^1 = 11$

$11^2 = 11 \times_{15} 11 = 1$

$|11| = 2$

$gp(11) = \{11, 2\}$

d) G is not cycling.

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⇒ Permutation group :-
Let A be a finite set then a function $f : A \to A$ is said to be a permutation of A :-
i) If f is one-one
ii) If f is onto
i.e A bijection from A to ~~ind~~ itself is called a permutation of A.
The no. of distinct elements in the finite set A is called the degree of the permutation.

$$f = \begin{pmatrix} a_1, a_2 \cdots a_n \\ f(a_1), f(a_2) \cdots f(n) \end{pmatrix}$$

⇒ Equality of two permutation :-
Let f & g be two permutation on a set X then f = g if and only if $f(x) = g(x) \ \forall \ x$ in X

Eg :-
$$f = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix} \quad g = \begin{bmatrix} 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$f(1) = 2 = g(1)$
$f(2) = 3 = g(2)$
$f(3) = 4 = g(3)$
$f(4) = 1 = g(4)$

So, $f = g$

⇒ Identity permutation :-
If each element of a permutation be replaced by itself then it is called the identity permutation and it is denoted by I

Eg :-
$$I = \begin{bmatrix} a & b & c \\ a & b & c \end{bmatrix}$$

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⇒ Product of permutation or Composition of permutation :-

The product of two permutation f and g of same degree. is denoted by fog or fg.

Eg :- $f = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix}$ $g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{bmatrix}$ $I = \begin{bmatrix} a & b & c \\ a & b & c \end{bmatrix}$

Q. find the product of two permutation and show that it is not commutative.

$$fg = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

$$gf = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

$fg \neq gf$ so, it is not a commutative

⇒ Inverse permutation :-

A permutation is one-one ~~on~~ onto map and hence it is inversible.

Eg :- $f = \begin{bmatrix} a_1, a_2 \cdots a_n \\ b_1, b_2 \cdots b_n \end{bmatrix}$, $f^{-1} = \begin{bmatrix} b_1, b_2 \cdots b_n \\ a_1, a_2 \cdots a_n \end{bmatrix}$

⇒ Total no. of permutation :-

Let X be a set consisting of n distinct elements then the element of x can be permuted in $\lfloor n$ distinct ways.

If S_n is the set consisting of all permutation of degree n, this set S_n is called the symmetric set of permutation of degree n.

=) Cyclic permutation :- circular permutation
A permutation which replace an object cyclically is called a cyclic permutation.

$$P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

$$P = [1\ 2\ 3\ 4]$$
$$= [2\ 3\ 4\ 1]$$
$$= [3\ 4\ 1\ 2]$$
$$= [4\ 1\ 2\ 3]$$

=) The no. of element permutated by a cycle is set to be its length and the disjoint cycle are those which has no common element,

$$t = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix}$$

$(1,2)$, $(3,4,6)$, (5)
2 length, 3 length, 1 length

Q. If $A = (1,2,3,4,5)$ and $B(2,3)(4,5)$ find ~~AB~~

$(1,3,5)$

$$\cancel{\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 5 & 4 \end{bmatrix}} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{bmatrix}$$

Q. Express the permutation $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{bmatrix}$
find product of transposition.
$(1,6)$, $(2,5,3)$, (4)

⇒ Even and Odd permutation :-

Q. Show that the permutation $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 4 & 1 & 3 \end{bmatrix}$ is odd while the permutation $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 4 & 5 & 2 & 1 \end{bmatrix}$ is even.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 4 & 1 & 3 \end{bmatrix} = (1,5)(263)(4)$$
$$\underbrace{(1,5)\ (26)\ (23)}_{\text{odd}}\ (4)$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 4 & 5 & 2 & 1 \end{bmatrix} = (16)(2,3,4,5)$$
$$\underbrace{(16)\ (23)\ (24)\ (25)}_{\text{even}}$$

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⇒ Cosets :-

If H is a ~~subset~~ group of G and $a \in G$ then the set $H_a = \{ha ; h \in H\}$ is called a right coset of H, aH is called left coset of H

⇒ Normal sub group :-

A sub group H of G is normal subgroup, if $a^{-1}Ha \subseteq H$ for every $a \in G$.

H is normal if; $aH = Ha$.

Eg:- Consider the permutation group S_3 of degree 3.

The set $H = \{\varepsilon, \sigma_1\}$ is a subgroup of S_3

$S_3 = \lfloor 3 = 6$

$H\phi_1 = \{\phi_1, \sigma_2\}$ $\qquad$ $\phi H = \{\phi_1, \sigma_3\}$

$\varepsilon\phi_1 = \{\phi_1\}$ $\qquad$ $\phi_1\varepsilon = \{\phi_1\}$

$\sigma_1\phi_1 = \{\sigma_2\}$ $\qquad$ $\phi_1\sigma_1 = \{\sigma_3\}$

Right coset $\neq$ left coset

It's not a normal sub group.

⇒ Lagrange's theorem :-
The order of each sub-group of a group in a divisor of the order of group.

Proof :- Let 'G' be a finite group of order 'n' consider a sub-group 'H' of group 'n' such that order of $H = (n)$
$O(H) = m$.

Consider $h_1, h_2, \cdots h_m \in H$, the finite elements of sub group H.

Let an element $H \in G$ then Ha is a right coset of H in G, and $Ha = \{h_1 a, h_2 a, \cdots h_m a\}$ has m distinct element a,

$$h_i = h_j$$

Thus, each right coset of H in G has (m) distinct element.

Any two right ~~group~~ coset of H in G are disjoint. G is a finite group, to the no. of right cosets of H in G will also be finite.

Let it be k, then the union of k - right cosets of H in G is equal to h .. i.e if $Ha_1, Ha_2, \cdots Ha_k$ are the right cosets of H in G.

$$G = Ha_1 \cup Ha_2 \cup \cdots Ha_k$$

no. of elements in G = no. of element in Ha_1 + no. of element in Ha_2 .... no. of element in Ha

$$O(G) = O(Ha_1) + O(H \cdot a_2) \cdots O(Hak)$$

$$n = m + m + \cdots\cdots m \text{ (k times)}$$

$$n = m.k$$

$$k = \frac{n}{m} \text{ or } \frac{O(G)}{O(H)}$$

⇒ Rings :- (It performs under addition and multiplication operation $(R, +, \cdot)$

R is called ring if the following axioms are satisfied :-

[R_1] For any $a, b, c \in R$, we have $(a+b)+c = a+(b+c)$.

[R_2] There exists an element $0 \in R$, called the zero element, such that $a+0=a$ $a+0 = 0+a = a$ for every $a \in R$.

[R_3] For each $a \in R$ there exists an element $-a \in R$, called the negative of a, such that $a+(-a) = (-a)+a = 0$.

[R_4] For any $a, b \in R$, we have $a+b = b+a$.

[R_5] For any $a, b, c \in R$ we have $(ab)c = a(bc)$.

[R_6] For any $a, b, c \in R$, we have (i) $a(b+c) = ab+ac$ and
(ii) $(b+c)a = ba+ca$

The axiom [R_1] through [R_4] may be summarized by saying that R is an abelian group addition.

Q. Prove that a set $R = \{0, 1, 2, 3, 4\}$ be a ring with respect to $+_5$, $\cdot_5$, modulo 5

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$\cdot_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

* Closure property :- All the element belongs to R.
* Associative property :-

$a +_5 (b +_5 c) = (a +_5 b) +_5 c$

$0 +_5 (1 +_5 2) = (0 +_5 1) +_5 2$

$3 = 3$

Hence, it follows associative property.

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* identity :-

0 belongs to R and is identy element

$a +_5 e = a$

$1 +_5 0 = 1$

* Inverse :-

~~$a + a^{-1} =$~~

The inverse of 0, 1, 2, 3, 4 are 0, 4, 3, 2, 1 respectively and all belongs to R.

* Commutative :-

$a +_5 b = b +_5 a \quad , \quad a, b \in R$

Hence, this Group is abelian under Addition operation.

R_4 for any $a, b, c \in R$, we have $(ab)c = a(bc)$

$(a \cdot_5 b) \cdot_5 c = a \cdot_5 (b \cdot_5 c)$

$(0 \cdot_5 1) \cdot_5 2 = 0 \cdot_5 (1 \cdot_5 2)$

$0 \cdot_5 2 = 0 \cdot_5 2$

$0 = 0$

R_5 i) $a(b+c) = ab + ac$, ii) $(b+c)a = ba + ca$

i) $a \cdot_5 (b +_5 c) = a \cdot_5 b + a \cdot_5 c$

$1 \cdot_5 (2 +_5 3) = 1 \cdot_5 2 +_5 1 \cdot_5 3$

$1 \cdot_5 0 = 2 +_5 3$

$0 = 0$

ii) $(2 +_5 3) \cdot_5 1 = 2 \cdot_5 1 +_5 3 \cdot_5 1$

$0 \cdot_5 1 = 2 +_5 3$

$0 = 0$

Boolean Algebra

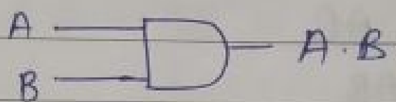
* The set of rules use to specify the given logical expression with changing the functionality.
* It is used when no. of variables are less.

→ Rules :-

1) Complement Rules :- $\bar{A}$ or A' (Not A)
2) Double Inversion :- $(A')' = A$
3) AND :- $A \cdot A = A$, $A \cdot \bar{A} = 0$

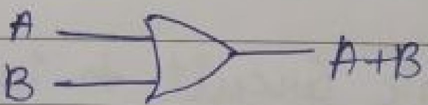
4)

A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1



4) OR :- $A + A = A$, $A + \bar{A} = 1$

A	A	B	A+B
0	0	0	0
0	0	1	1
1	1	0	1
1	1	1	1



5) Distributive :-

{ Dual, OR → AND
AND → OR }

$A \cdot (B + C) = AB + AC$

$A + (B \cdot C) = (A + B) \cdot (A + C)$

$A + \bar{A} \cdot B$	$\bar{A} + AB$
$(A + \bar{A}) \cdot (A + B)$	$(\bar{A} + A) \cdot (\bar{A} + B)$
$= (A + B)$	$= (\bar{A} + B)$

6) Commutative :-

$A+B = B+A$

$A \cdot B = B \cdot A$

7) Associative :-

$A+(B+C) = (A+B)+C$

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$

8) absorption law :-

$A + A \cdot B = A$

$A(1+B)$

$= A$

Dual $= A \cdot (A+B) = A$

$= A \cdot A + AB$

$= A + AB$

$= A$

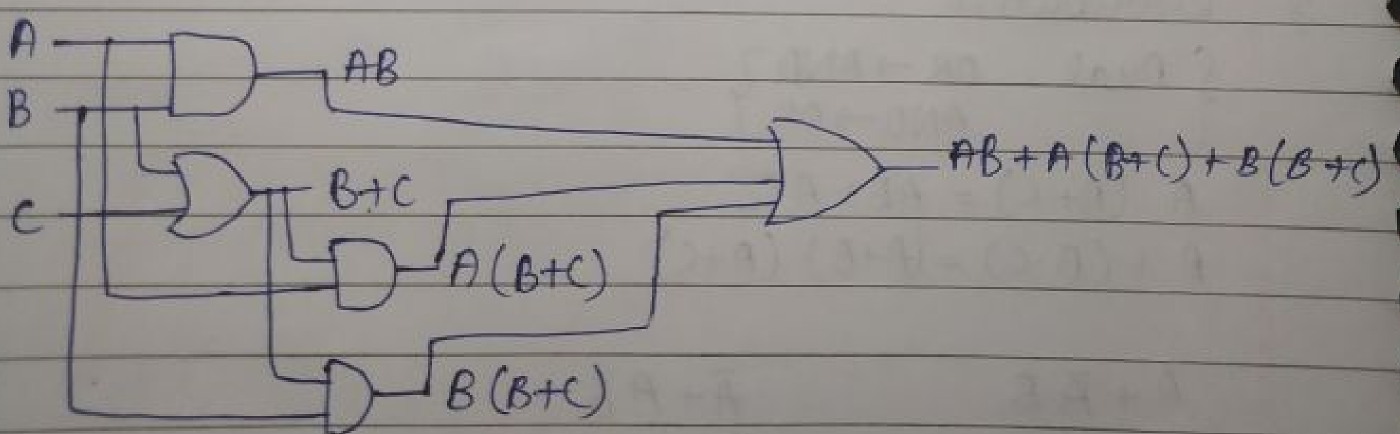
9) Demorgan's law :-

$\overline{A \cdot B} = \overline{A} + \overline{B}$

$\overline{A+B} = \overline{A} \cdot \overline{B}$

Eg:- Draw the logic circuit to implement the logic operation and it minimise form.

$Y = AB + A(B+C) + B(B+C)$



$= AB + A(B+C) + B(B+C)$

$= AB + AB + AC + BB + BC$

$= A(B+B) + AC + B + BC$

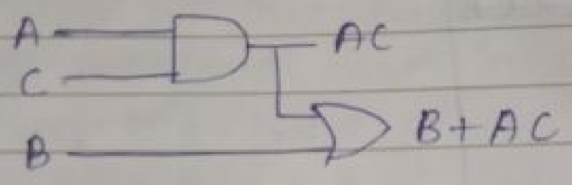
$= AB + AC + B + BC$

$= AB + AC + B(1+C)$

$= AB + AC + B$

$= B(A+1) + AC$

$= B + AC$



Eg:- Prove that $A + \bar{A}B + AB = A+B$

$= A + \bar{A}B + AB$

$= A + B(\bar{A}+A)$

$= A+B$

Eg:- $ABCD + A\bar{B}CD = ACD$

$= ACD(B+\bar{B})$

$= ACD$

Eg:- Evaluate the value of $\overline{(\overline{AB} + \bar{A} + AB)}$

$= \overline{\bar{A}+\bar{B}+\bar{A}+AB}$

$= \overline{\bar{A}+\bar{A}+\bar{B}+AB}$

$= \overline{\bar{A}+\bar{B}+AB}$

$= \bar{\bar{A}}\cdot\bar{\bar{B}}\cdot\overline{AB}$

$= AB\cdot\overline{AB}$

$= 0$

Q. $(A+B)\cdot(\bar{A}+C)\cdot(B+C) = A+B\cdot(\bar{A}+C)$

$= (A+B)\cdot(\bar{A}B + \bar{A}C + CB + CC)$

$= (A+B)\cdot(\bar{A}B + \bar{A}C + CB + C$

$(A+B)\cdot(\bar{A}B+\bar{A}C+CB+C)$

$(A+B)\cdot(\bar{A}B+\bar{A}C+C(B+1))$

$(A+B)\cdot(\bar{A}B+\bar{A}C+C)$

$(A+B)\cdot(\bar{A}B+C(\bar{A}+1))$

$(A+B)\cdot(\bar{A}B+C)$

$A\bar{A}B+AC+B\bar{A}B+BC$

$AC+B\bar{A}+BC+A\cdot\bar{A}$

$\cancel{AC+B(1+C)}$ $\cancel{AC+BA}$

$\cancel{= \boxed{AC+B}}$

RHS :- $A+B\cdot(\bar{A}+C)$

$= A+B\bar{A}+BC$

Q. $Y=\bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}\bar{C}+AB\bar{C}$

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⇒ SOP and POS :-

$AB + BC + AC \rightarrow$ SOP (sum of Product) $\bar{A} \rightarrow 0, A = 1$

$(A+B) \cdot (B+C) \cdot (A+C) \rightarrow$ POS (Product of sum) $\bar{A} \rightarrow 1, A = 0$

Eg :- $\bar{A}\bar{B} + \bar{A}B$

$00 + 01$

$(0, 1)$

⇒ Min term & Max term :-

* Each individual term in standard SOP form is called Min term.

Eg :- $AB + A\bar{B}\bar{C} + \bar{A}BC$ (mi)

* Each individual term in the standard POS form is called Max term.

Eg :- $Y = (A+B) \cdot (A+\bar{B})$ (Mi)

A	B	min term	max term
0	0	$\bar{A} \cdot \bar{B} \rightarrow m_0$	$A+B \rightarrow M_0$
0	1	$\bar{A}B \rightarrow m_1$	$A+\bar{B} \rightarrow M_1$
1	0	$A\bar{B} \rightarrow m_2$	$\bar{A}+B \rightarrow M_2$
1	1	$AB \rightarrow m_3$	$\bar{A}+\bar{B} \rightarrow M_3$

Q. Convert SOP to POS, $\bar{A}\bar{B} + AB$

$\bar{A}\bar{B} + AB$

SOP → $\underbrace{0\,0}_{m_0} + \underbrace{1\,1}_{m_3}$

$\Sigma m(0, 3)$

$\pi M(1, 2) \rightarrow$ POS

$\quad\;\; 01 \;\; 10$

$= (A+\bar{B}) \cdot (\bar{A}+B)$

$POS = (A+\bar{B}) \cdot (\bar{A}+B)$

Q. Convert SOP to POS :-

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$

$\quad 000 + 010 + 011 + 101 + 111$

$Y = m_0 + m_2 + m_3 + m_5 + m_7$

$F = \sum m(0, 2, 3, 5, 7)$

$F = \pi M(\underset{010}{1}, \underset{100}{4}, \underset{110}{6})$

$POS = (A+\bar{B}+C)\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{B}+C)$

⇒ Steps to make standard SOP:-

1. Multiply by $A + \bar{A}$ term of missing variable.
2. To make standard POS, $(A \cdot \bar{A})$ term of missing variable.

Q. Represent or expand $f(A,B) = \bar{A} + \bar{B}$ in SOP & POS and find min term and max term.

$f(A,B) = \bar{A} + \bar{B}$

$= \bar{A}(B+\bar{B}) + \bar{B}(A+\bar{A})$

$= \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B}$

$SOP = \bar{A}B + \bar{A}\bar{B} + A\bar{B}$

$\quad = 01 + 00 + 10$

$= m_1 + m_0 + m_2$

$F = \sum m(0, 1, 2)$

$F = \pi M(\underset{11}{3})$

$POS = \bar{A}+\bar{B}$

Q. Expand $A + B\bar{C} + AB + ABC$

$f(A,B,C) = A(B+\bar{B})(C+\bar{C}) + (A+\bar{A})B\bar{C} + AB(C+\bar{C}) + ABC$

$=(AB+A\bar{B})(C+\bar{C}) + AB\bar{C} + \bar{A}B\bar{C} + ABC + AB\bar{C} + ABC$

$= ABC + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C} + ABC + AB\bar{C} + ABC$

$SOP = ABC + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C}$

$\quad\quad 111 \quad 100 \quad 110 \quad 010$

$= m_7 + m_4 + m_6 + m_2$

$F = \Sigma m(2,4,6,7)$

$F = \pi M(0,1,3,5)$

$\quad 000 \quad 001 \quad 011 \quad 101$

$POS = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+\bar{C})$

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Q. $ABC\cdot D + A\bar{B}CD$

$(ACD)(B+\bar{B})$

$= ACD$

Q. $\overline{(\overline{AB}+\bar{A}+AB)}$

$= \overline{(\bar{A}+\bar{B}+\bar{A}+AB)}$

$= \overline{\bar{A}+\bar{B}+AB}$

$= \bar{\bar{A}}\cdot\bar{\bar{B}} + \overline{AB}$

$= AB + \overline{AB}$

$= AB\cdot(\bar{A}+\bar{B})$

$= A\bar{A}B + AB\bar{B}$

$= 0$

Q. $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= \bar{A}\bar{C}(\bar{B}+B) + A\bar{C}(\bar{B}+B)$

$= \bar{A}\bar{C} + A\bar{C}$

$= \bar{C}(\bar{A}+A)$

$= \bar{C}$

Q. $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$

$\underbrace{000}_{0} \quad \underbrace{010}_{2} \quad \underbrace{011}_{3} \quad \underbrace{101}_{5} \quad \underbrace{111}_{7}$

$f = \Sigma m(0,2,3,5,7)$

$f = \Sigma M(1,4,6)$

$\quad 001 \quad 100 \quad 110$

$POS = (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

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Q. $\bar{A} + \bar{B}$

$= \bar{A}(B+\bar{B}) + \bar{B}(A+\bar{A})$

$= \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B}$

$= \underbrace{\bar{A}B}_{\substack{01 \\ 1}} + \underbrace{\bar{A}\bar{B}}_{\substack{00 \\ 0}} + \underbrace{A\bar{B}}_{\substack{10 \\ 2}}$

$f = \Sigma m(0,1,2)$

$f = \pi M(\underbrace{3}_{11})$

~~SOP~~ POS $= (\bar{A}+\bar{B})$

Q. $A(\bar{B}+A)B$

~~$A+(B\cdot\bar{B})(A+\bar{B})\ B+(A\cdot\bar{A})$~~

~~(AB~~

$(A+B\cdot\bar{B})(A+\bar{B})(A\cdot\bar{A}+B)$

$(A+B)(A+\bar{B})(A+\bar{B})(A+B)(\bar{A}+B)$

$\underbrace{(A+B)}_{\substack{00 \\ 0}}\ \underbrace{(A+\bar{B})}_{\substack{01 \\ 1}}\ \underbrace{(\bar{A}+B)}_{\substack{10 \\ 2}}$

$f = \pi M(0,1,2)$

$f = \Sigma m(\underbrace{3}_{11})$

SOP $=$ ~~$\bar{A}\bar{B}$~~ AB

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⇒ SOP and POS with K-map :-

Minimize SOP with K-map

Two possibilities

Min term, Max term will ~~begin~~ be given in ques
$\Sigma m(1,2,3,5,7,10)$
$\pi M(1,2,4,5)$

Boolean expression will be given in ques
$F(A,B,C,D) = A\bar{B} + AB\bar{C} + C\bar{D}$

Q. Find k-map and simplify the eq^n.

i) $A\bar{B} + \bar{A}\bar{B}$

ii) $A\bar{B} + \bar{A}B$

iii) $A\bar{B} + \bar{A}B + \bar{A}\bar{B}$

i)

B \ A	$\bar{A}$ 0	A 1
$\bar{B}$ 0	1	1
B 1		

$= \bar{B}$

ii)

B \ A	$\bar{A}$ 0	A 1
$\bar{B}$ 0		1
B 1	1	

$= A\bar{B} + \bar{A}B$ (Already simplified) does not contain any adjacent containing 1 so expression can not simplified

iii)

B \ A	$\bar{A}$ 0	A 1
$\bar{B}$ 0	1	1
B 1	1	

$= \bar{A} + \bar{B}$

Q. Simplify boolean function using K-map in SOP and POS forms and implement with gates.

i) $f(A,B,C) = \Sigma m(0,2,3,4,5,6)$

Note :- $\Sigma m() \rightarrow$ min term $\rightarrow$ SOP when $f=1$, so, put 1 in k-map.

$\pi M() \rightarrow$ max term $\rightarrow$ POS when $f=0$, so, put 0 in k-map.

A \ BC	00	01	11	10
0	1 (0)	(1)	1 (3)	1 (2)
1	1 (4)	1 (5)	(7)	1 (6)

$= \bar{C} + A\bar{B} + \bar{A}B$

A \ BC	00	01	11	10
0	(0)	(1) 0	(3)	(2)
1	(4)	(5)	(7) 0	(6)

$= \bar{A}\bar{B}C + ABC \Rightarrow (A+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

1) $X = AB\bar{C} + ABC$

$\underbrace{110}_{6} \quad \underbrace{111}_{7}$

$\Sigma m (6, 7)$

$\cancel{A+}\ \pi M (0, 1, 2, 3, 4, 5)$

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7 1	6 1

$= AB$

A \ BC	00	01	11	10
0	0 0	1 0	3 0	2 0
1	4 0	5 0	7	6

$= \bar{B} + \bar{A}$

$= AB$

2) $X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

$\underbrace{000}_{0} \quad \underbrace{100}_{4}$

$\Sigma m (0, 4)$

$\cancel{A}M (1, 2, 3, 5, 6, 7)$

A \ BC	00	01	11	10
0	0 1	1	3	2
1	4 1	5	7	6

$= \overline{B}\overline{C}$

A \ BC	00	01	11	10
0	0	1 0	3 0	2 0
1	4	5 0	7 0	6 0

$= \bar{B}C + B$

$= (B + \bar{C})(\bar{B})$

3) $K(A, B, C) = \Sigma m (1, 3, 6\ 7)$

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→ K - Map with 4 variable :-

Q. Simplify the following questions :-

$f(A,B,C,D) = A\bar{B}C + \bar{A}BC\bar{D} + AB\bar{C}D + ABC$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	7	6 1
AB 11	12	13 1	15 1	14 1
$A\bar{B}$ 10	8	9	11 1	10 1

$SOP = \underbrace{AC}_{10} + \underbrace{ABD}_{13} + \underbrace{BC\bar{D}}_{6}$

$\Sigma m(10, 13, 6)$

$\pi M(0,1,2,3,4,5,7,8,9,11,12,14,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	
AB	0			
$A\bar{B}$	0	0		

$SOP = \bar{A}\bar{B} + \bar{C}\bar{D} + \bar{A}D + A\bar{B}\bar{C}$

$POS = (A+B)(C+D)(A+\bar{D})(\bar{A}+B+C)$

Q. $K(PQRS) = \Sigma m(0,2,5,7,8,9,10,13,15)$

PQ \ RS	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
$\bar{P}\bar{Q}$	1			1
$\bar{A}Q$		1	1	
PQ		1	1	
$P\bar{Q}$	1			1

$SOP = \cancel{\bar{P}\bar{Q}\bar{S} + QS + P\bar{Q}\bar{S}}$

$= \bar{Q}\bar{S} + QS$

PQ \ RS	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
PQ		0	0	
$\bar{P}Q$	0			0
PQ	0			0
$P\bar{Q}$		0	0	

$SOP = Q\bar{S} + PS$

$POS = (\bar{Q}+S)(\bar{P}+S)$

Q. $f(A,B,C,D) = \sum m(0,2,5,7,13,15)$

$\pi M(1,$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		0	0	
$\bar{A}B$	0			0
AB	0			0
$A\bar{B}$	0	0	0	0

$SOP = B\bar{D} + \bar{B}D + \bar{A}\bar{B}$

$POS = (\bar{B}+D)(B+\bar{D})(\bar{A}+B)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$		1	1	
AB		1	1	
$A\bar{B}$				

$SOP = \bar{A}\bar{B}\bar{D} + BD$

Q. $f(A,B,C,D) = ACD + \bar{A}B + \bar{D}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$	1	1	1	1
AB	1		1	1
$A\bar{B}$	1		1	1

$SOP = \bar{D} + \bar{A}B + AC$

Q. $f(A,B,C,D) = \sum m(4,5,6,7,8,9,10,11,12,13,14,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

$\Rightarrow \bar{A}B + A$

1) $f(A,B,C) = \pi M(0,3,6,7)$

C\AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$ 0	0		0	
C 1			0	0

SOP = $\bar{A}\bar{B}\bar{C} + AB + CA$

POS = $(A+B+C)(\bar{A}+\bar{B})(\bar{A}+\bar{C})$

2) $f(A,B,CD) = \pi M(3,5,7,8,10,11,12,13)$

AB\CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			0	
$\bar{A}B$		0	0	
AB	0		0	
$A\bar{B}$	0		0	0

SOP = $CD + \bar{A}BD + A\bar{C}\bar{D}\cancel{A} + A\bar{B}C$

POS = $(\bar{C}+\bar{D})(A+\bar{B}+\bar{D})(\bar{A}+C+D)(\bar{A}+B+\bar{C})$