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Finding the Numero UNO! Strategy

Introduction

UNO! is a family friendly card game by Mattel where 2-7 players try to get rid of all of their cards. There are 108 cards in an UNO! deck: 19 Blue, Green, Red, and Yellow cards numbered from 0 to 9 (there is only one 0 per color). The deck also has some special cards. There are two Skip, Reverse, Draw 2 cards for each of the four colors. There are also four Wild and Wild Draw 4 cards in the deck.

The game starts after every player is dealt seven cards. The top card from the deck is moved to the discard deck, and unless it was a special card, the player seated counterclockwise to the dealer plays. A play is completed when a player puts a card that matches the number or value of the discard deck's top card or is a wild card. The game ends when one player runs out of cards.

The non-numeric cards have special properties when flipped before and during gameplay. If a special card was the first card added to the discard pile after dealing:

Skip	The player ccw of the dealer loses a turn
Reverse	The game play starts clockwise instead of counterclockwise
Draw 2	The player ccw of the dealer draws 2 cards and loses a turn
Wild	The player ccw of the dealer picks a color and plays a turn
Wild Draw 4	This card is inserted to the bottom of the deck and a new card is chosen

If a special card is played during the game:

Skip	The next player loses a turn
Reverse	The game play switched from ccw to cw or vice-versa
Draw 2	The next player draws 2 cards and loses a turn

Wild	The player who played Wild picks a color and plays a turn
Wild Draw 4	Only playable if the player cannot play a card of the same color as the top card in the discard pile. The player who played Wild 4 picks a color and plays a turn. The next player draws 4 cards and loses a turn.

Strategies

Ordering Strategies

One class of strategies involves choosing which cards to keep and which to play. Part of the art to making such strategies is that they must fit in the rules. For example, it would be tempting to play Wild 4s given the chance, but they cannot be played if a player has a card that matches the color of the discard deck's top card.

A friend who is very good at UNO suggested this strategy: try to end with a normal card and a Wild or Wild 4 card. The normal card will have more potential matches than a colored special card, and having a wild card will give a player control on a potential second-to-last turn.

This suggests a few potential (anecdotally competitive) strategies. The strategies are named such that the first card code is the first card a player tries to get rid of. The card codes will be: C for matching color, V for matching Value, D for Draw 2, S for Skip, R for Reverse, W for Wild, W4 for Wild 4. The strategies using this scheme are:

DSRVCW4W	Priorities of DSR and W4W maximize the punishment the next player gets
DSRCVW4W	CV, VC seemed like an arbitrary choice
DSRCW4VW	Maximizes punishment for next player but ensures player does not illegally play W4
DSRCW4VW	Maximizes punishment but tries have Wild be the final card

After we find the best order out of these, we can try all 6 permutations of DSR to choose the optimal ordering strategy.

Advanced Strategies

We can also have strategies that depend on the game state. One such strategy that I like to use is to have an attacking strategy when there are few cards left in the game, and a defensive strategy that does not introduce too many Draws, Wild 4s at the start of the game. When I get the chance to choose a color, if I am playing defensively then I choose my most common color. When I get the chance to choose a color while attacking, I choose the most discarded color unless the next player can win and the next player last played a card with the most discarded color.

Another friend suggested this strategy: a player tries to make the final two cards a wild and a normal card but also greedily get rid of cards that are different than the other cards in the player's hand when the game is early. To simulate the game being early, I decided to define early as $e^{-\beta \times \# total\ cards\ played} > 0.5$ where β is a positive constant. As $\beta \to \infty$, only the end-game strategy is played so the colors in the player's hand would tend to be uniform.

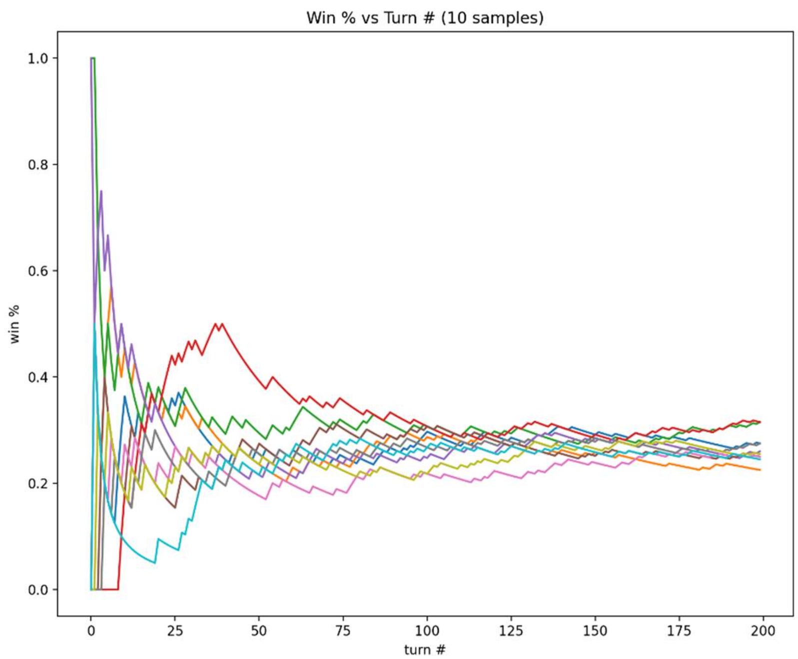
Simulation

All simulation code is hosted on github: [Krishna-Saxena/Uno: Simulate, Test Uno Strategies with Python (github.com)]

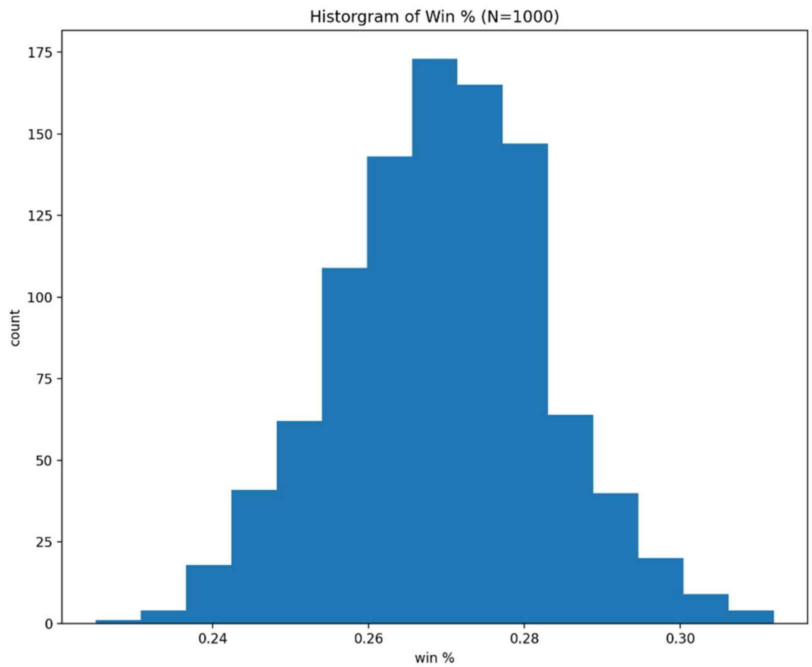
Verifying the Central Limit Theorem

In order to use the Central Limit Theorem to create confidence interval for strategies' winning percentages, we must check that the winning percentage converges to a fixed percent as the number of games approaches infinity and that the distribution of winning percentages is approximately normal after a large number of games have been played.

The winning percentage of a player using the DSRCVW4W ordering strategy vs three players using the DSRCW4WV strategy appears to approach a limit as more games are played:



The distribution of 1,000 winning percentages after 1,000 games between a player using the DSRCVW4W ordering strategy vs three players using the DSRCW4WV strategy appears to approach a normal distribution:



Comparing Ordering Strategies

In order to find the ordering strategy with the best win rate, first we will first have strategies DSRVCW4W, DSRCVW4W, DSRCW4VW, and DSRCW4VW compete each other. Each competition consists of 10 rounds of 20,000 games. We increase the number of games per percentage measurement from 1,000 to 20,000 to get more confidence that the percentage is not due to random changes in how the games played out.

Since we have evidence that the Central Limit Theorem applies, we can use properties of the normal distribution to create confidence intervals using each competition's sample of 10 winning percentages. Because the normal distribution is symmetric, there is only a $\left(\frac{1}{2}\right)^{10}$ chance that all the winning percentages are greater than or all less than the true winning percentage. Thus, there is a $1 - 2\left(\frac{1}{2}\right)^{10} = 1 - \frac{1}{2^9} = 0.998 = 99.8\%$ chance that the true winning percentage is in between the maximum and minimum sample winning percentages.

The confidence intervals of 10 winning percentages found after 20,000 games of each of these strategies

	Other 3 Opponents' Strategy			
Player's Strategy	DSRVCW4W	DSRCVW4W	DSRCW4VW	DSRCW4WV

DSRVCW4W	(0.248,0.253)	(0.248, 0.259)	(0.264, 0.271)	(0.250, 0.263)
DSRCVW4W	(0.241, 0.248)	(0.248, 0.255)	(0.262, 0.272)	(0.255, 0.262)
DSRCW4VW	(0.234, 0.239)	(0.236, 0.455)	(0.243, 0.255)	(0.257, 0.266)
DSRCW4WV	(0.226, 0.232)	(0.227, 0.235)	(0.235, 0.245)	(0.246, 0.255)

Based on these results, DSRVCW4W is the best aggressive ordering strategy. This strategy counters DSRCW4VW and DSRCW4WV but we cannot claim that is does better than random against DSRCVW4W. DSRCVW4W also counters two strategies (also DSRCW4VW and DSRCW4WV) but does worse than random against three players playing our chosen strategy— DSRVCW4W.

To find the best ordering strategy, I compared all the permutations of DSR followed by VCW4W.

	Other 3 Opponents' Strategy					
Player's Strategy	DSRVCW4W	DRSVCW4W	RDSVCW4W	RSDVCW4W	SDRVCW4W	SRDVCW4W
DSRVCW4W	(0.248, 0.255)	(0.248, 0.255)	(0.245, 0.254)	(0.244, 0.258)	(0.245, 0.257)	(0.248, 0.254)
DRSVCW4W	(0.247, 0.253)	(0.245, 0.256)	(0.247, 0.252)	(0.247, 0.253)	(0.242, 0.257)	(0.244, 0.252)
RDSVCW4W	(0.242, 0.258)	(0.245, 0.253)	(0.245, 0.255)	(0.247, 0.252)	(0.245, 0.257)	(0.245, 0.254)
RSDVCW4W	(0.248, 0.255)	(0.244, 0.253)	(0.246, 0.254)	(0.249, 0.257)	(0.245, 0.253)	(0.245, 0.252)
SDRVCW4W	(0.246, 0.254)	(0.246, 0.253)	(0.243, 0.254)	(0.245, 0.253)	(0.244, 0.256)	(0.243, 0.254)
SRDVCW4W	(0.247, 0.253)	(0.242, 0.251)	(0.244, 0.252)	(0.244, 0.253)	(0.248, 0.257)	(0.244, 0.254)

After running these strategies against each other 10 times for 10,000 games, none were significantly better than another. I predict that this is the case because one specific player does not get all of Draw, Skip, or Reverse multiple times during a typical game. Thus, the differences of strategies for the player we are measuring are too dependent on the shuffling and other players' moves relative to dependency on the strategy.

For the rest of the analysis, we will keep DSRVCW4W as the ordering strategy that the advanced strategies play against because it is most similar to how other players I play against play. We will also incorporate DSRVCW4W as the base strategy for our friend's algorithm which will switch to DSRVCW4W as the game progresses.

Now, we need to choose the best value of β from $\{0.005, 0.01, 0.02, 0.03\}$ for that strategy.

	Other 3 Player's Strategy			
Player's Strategy	$\text{Friend}_{0.005}$	$\text{Friend}_{0.01}$	$\text{Friend}_{0.02}$	$\text{Friend}_{0.03}$
$\text{Friend}_{0.005}$	(0.242, 0.251)	(0.244, 0.256)	(0.248, 0.255)	(0.247, 0.256)
$\text{Friend}_{0.01}$	(0.247, 0.254)	(0.244, 0.252)	(0.242, 0.254)	(0.248, 0.256)
$\text{Friend}_{0.02}$	(0.246, 0.253)	(0.247, 0.258)	(0.246, 0.255)	(0.244, 0.253)
$\text{Friend}_{0.03}$	(0.247, 0.255)	(0.247, 0.257)	(0.246, 0.257)	(0.246, 0.255)

These strategies all neutralize each other, what if we vary β by different orders of magnitude? This should show the effect of β.

	Other 3 Player's Strategy			
Player's Strategy	$\text{Friend}_{0.0002}$	$\text{Friend}_{0.002}$	$\text{Friend}_{0.02}$	$\text{Friend}_{0.2}$
$\text{Friend}_{0.0002}$	(0.244, 0.255)	(0.245, 0.252)	(0.246, 0.254)	(0.242, 0.252)
$\text{Friend}_{0.002}$	(0.246, 0.254)	(0.244, 0.252)	(0.242, 0.254)	(0.248, 0.256)
$\text{Friend}_{0.02}$	(0.247, 0.256)	(0.247, 0.258)	(0.246, 0.255)	(0.244, 0.253)
$\text{Friend}_{0.2}$	(0.248, 0.256)	(0.247, 0.257)	(0.246, 0.257)	(0.246, 0.255)

The parameter's order of magnitude does not change the winning percentage after many runs. I will continue the analysis with $\beta = 0.02$ because this causes a switch after 34 turns which in my experience is approximately the halfway point of a UNO! game. Finally, we will have the best ordering strategy, my friend's strategy with $\beta = 0.02$, and my strategy face off.

	Other 3 Player's Strategy		
Player's Strategy	DSRVCW4W	$\text{Friend}_{0.02}$	Mine
DSRVCW4W	(0.246, 0.257)	(0.248, 0.256)	(0.286, 0.297)
$\text{Friend}_{0.02}$	(0.239, 0.253)	(0.246, 0.255)	(0.284, 0.296)
Mine	(0.214, 0.222)	(0.216, 0.224)	(0.242, 0.254)

Conclusions

Based on the final games, my friend's strategy is significantly better than mine. However, my friend's idea of switching strategies as the game progressed did not make a significant difference compared to always sticking to a DSRVCW4W strategy.

As a result of what felt like over 24 hours of compute time— I had to run different game configurations simultaneously using Google Colab's cloud-based machines— I have a new strategy for winning UNO! games: sort my hand in an DSRVCW4W order. As the game progresses, I will try to keep an approximately even color distribution for my hand because first matching by value agrees with both winning strategies.