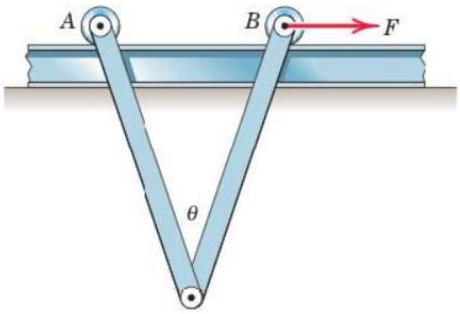
## Tutorial-4

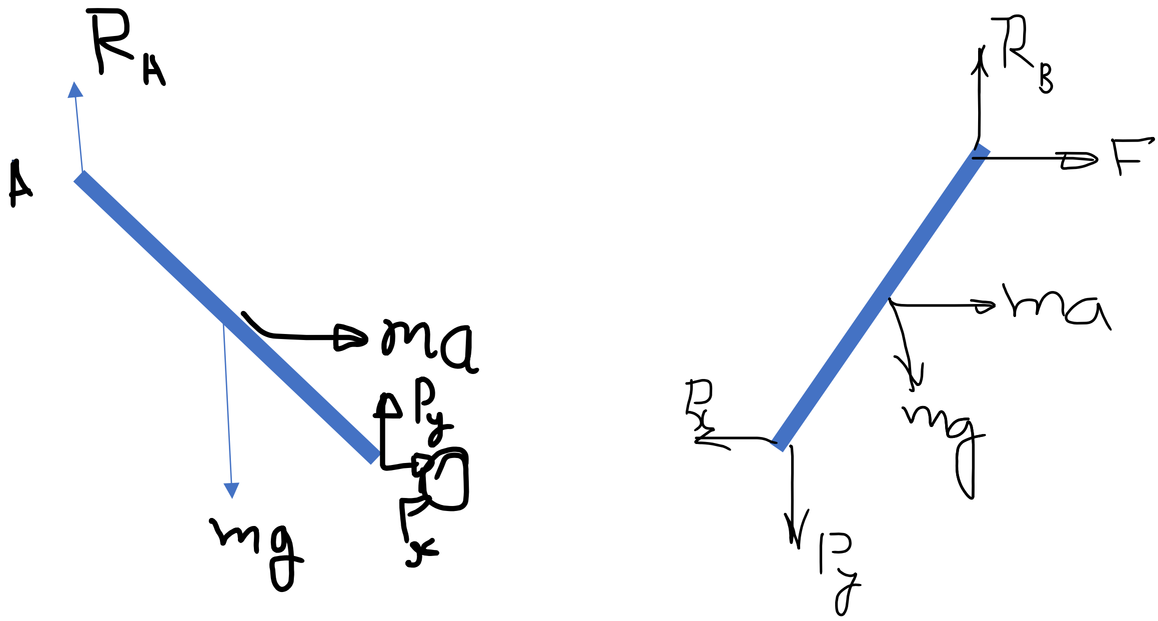
## Solution

**Plane kinetics of rigid bodies** Date: **06/07/2023**

**Q1.** The two uniform identical bars are freely hinged at the lower ends and are supported at the upper ends by small rollers of negligible mass which roll on a horizontal rail. Determine the steady-state angle $\theta$ assumed by the bars when they are accelerating under the action of a constant force F. Also find the vertical forces on the rollers at A and B.




FBD:




$$\sum M_o = mal$$

$R_A l sin\frac{\theta}{2} - mg\frac{l}{2}\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$ ................................ (1)

$-R_B l\sin\frac{\theta}{2} + Fl\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\theta$ ................................ (2)

$$\sum F_y = 0$$

$R_A + R_B = 2mg$ ................................ (3)

Subtract eq (1) from eq (2) and put in eq (3)

$$mg\tan\frac{\theta}{2} = F$$

$$\theta = 2\tan^{-1}\left(\frac{F}{mg}\right)$$

$$F = 2ma$$

$$a = \left(\frac{g}{2}\tan\frac{\theta}{2}\right)$$

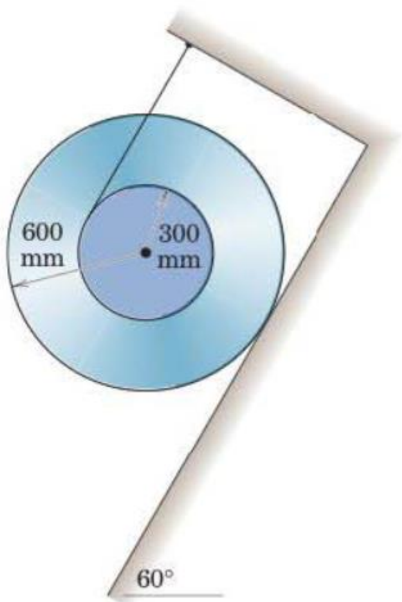
$$mg\tan\frac{\theta}{2}\; l\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B l\sin\frac{\theta}{2} = m\left(\frac{g}{2}\tan\frac{\theta}{2}\right)\frac{l}{2}\cos\theta$$

$$mgl\sin\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - mg\frac{l}{4}\sin\frac{\theta}{2} = R_B l\sin\frac{\theta}{2}$$

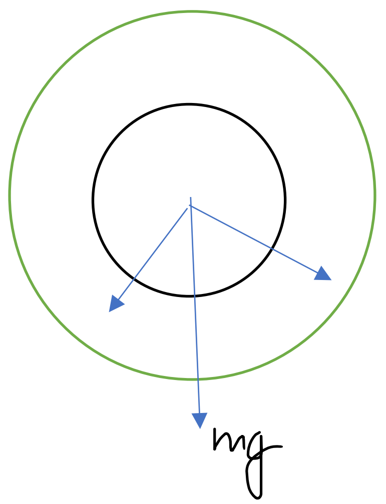
$$R_B = \frac{5}{4}mg$$

$$R_A = \frac{3}{4}mg$$

**Q2.** The wheel and its hub have a mass of 30 kg with a radius of gyration about the center of 450 mm. A cord wrapped securely around its hub is attached to the fixed support, and the wheel is released from rest on the incline. If the coefficients of static and kinetic friction between the wheel and the incline are 0.40 and 0.30, respectively, calculate the acceleration a of the center of the wheel. First prove that the wheel slips.



Solution:




$\sum M_A = 0$

$$0.9\,F - mg\sin\theta \times 0.3 = 0$$

$$F = \frac{mg}{3}\sin\theta$$

$$N = mg\cos\theta$$

$$\mu = \frac{F}{N} = \frac{\frac{mg}{3}\sin\theta}{mg\cos\theta}$$

$$\mu = \frac{\tan\theta}{3} = 0.577$$

$$\sum M_G = I\alpha = mr_G^2\alpha$$

$$0.3T - 0.6F = \frac{mr_G^2 a}{0.3}$$

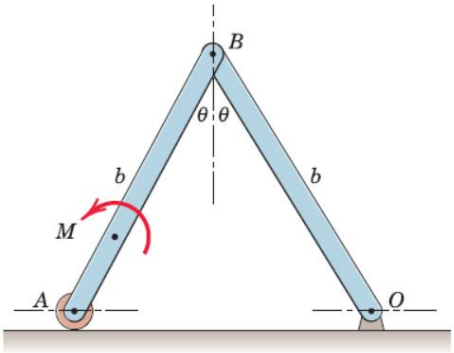
$$F = \mu_k N = 0.3mgsin\theta$$

$$0.3T - 0.18mgsin\theta = \frac{mr_G^2}{0.3}$$

$$mgsin\,\theta - T - 0.3mg\sin\theta = ma$$

$$a = 0.7gsin\theta - \frac{T}{m}$$

**Q3.** The two slender bars each of mass $m$ and length $b$ are pinned together and move in the vertical plane. If the bars are released from rest in the position shown and move together under the action of a couple $M$ of constant magnitude applied to $AB$, determine the velocity of $A$ as it strikes $O$.




Solution:

$$\Delta V_g = 2mg\frac{b}{2}(1 - cos\theta)$$

$$\Delta T_{BO} = \frac{1}{2}I_o\omega^2 = \frac{mv_B^2}{3}$$

$$OB = AB = OB'$$

$$V_A = \omega.2b$$

$$V_B = \omega.b$$

$$V_A = 2V_B$$

$$\Delta T_{AB} = \frac{1}{2}I_c\omega^2$$

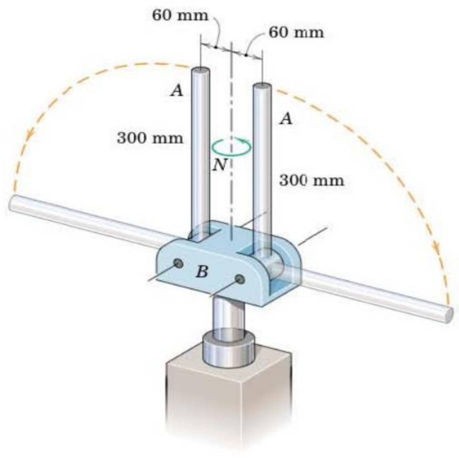
$$\Delta T_{AB} = \frac{7}{24}mV_A^2$$

$$\Delta V_g + \Delta T = M$$

$$mgb(1 - \cos\theta) + \frac{7}{24}mV_A^2 + \frac{1}{24}mV_A^2 = M_A$$

$$v_A = \sqrt{3}\sqrt{\frac{M_A}{m} - 2b(1 - cos\theta)}$$

**Q4.** Each of the two 300-mm uniform rods *A* has a mass of 1.5 kg and is hinged at its end to the rotating base *B*. The 4-kg base has a radius of gyration of 40 mm and is initially rotating freely about its vertical axis with a speed of 300 rev/min and with the rods latched in the vertical positions. If the latches are released and the rods assume the horizontal positions, calculate the new rotational speed *N* of the assembly.




Solution:

$$\Delta H = 0$$

Initial angular momentum

$$H_{rod} = 2I\omega = 2(1.5)(0.06)^2 \times \frac{300}{60} \times 2\pi$$

$$H_{lat} = mk^2\omega = 4 \times \frac{(0.04)^2 300}{60} \times 2\pi$$

Final angular momentum

$$H_{rod} = 2(I + ml^2)\omega = \frac{2m\left(\frac{l^2}{12} + d^2\right)2\pi N}{60}$$

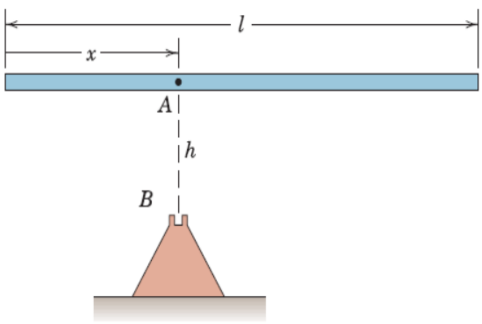
$$H_{rod} = 0.154\left(\frac{2\pi N}{60}\right)$$

$$(H_{rod} + H_{lat})_{inial} = (H_{rod} + H_{lat})_{final}$$

$$\{(0.06)^2 + 4 \times (0.04)^2\} \times 300 = (0.154 + 0.064)N$$

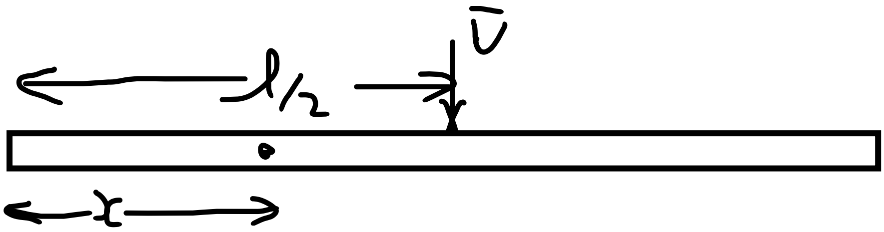
$$N = 32\frac{rad}{min}$$

**Q5.** The slender bar of mass *m* and length *l* is released from rest in the horizontal position shown. If point *A* of the bar becomes attached to the pivot at *B* upon impact, determine the angular velocity *ω* of the bar immediately after impact in terms of the distance *x*. Evaluate your expression for *x* = 0, *l* /2, and *l*.




Solution: Velocity of CG

$$\bar{v} = \sqrt{2gh}$$



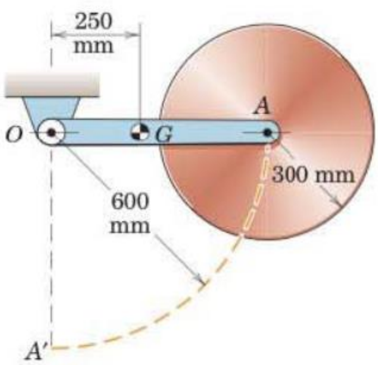

$$\Delta H = 0$$

$$I_B\omega = m\bar{v}\left(\frac{l}{2} - x\right)$$

$$\omega = m\left(\frac{l}{2} - x\sqrt{2gh}\right)/(m\left(\frac{l^2}{3} - lx + x^2\right)$$

$$\omega_{x=0} = \frac{3}{2l}\sqrt{2gh}$$

**Q6.** The link OA and pivoted circular disk are released from rest in the position shown and swing in the vertical plane about the fixed bearing O. The 6-kg link OA has a radius of gyration about O of 375 mm. The disk has a mass of 8 kg. The two bearings are assumed to be frictionless. Find the force $F_0$ exerted at O on the link (a) just after release and (b) as OA swings through the vertical position OA.




Solution: $a_1 = \alpha$

$$m_1 a_1 = m_1 \alpha r_G$$

$$I\alpha = (m r_G^2)\alpha$$

$$\sum M_o = I + \sum mad$$