Hierarchical-Clustering

January 14, 2025

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[]: ['''
         Hierarchical Clustering -->
         Hierarchical clustering is a method of cluster analysis that seeks to build
         a hierarchy of clusters. It is widely used in data mining and statistics
         to group objects based on their similarities.
         This approach can be divided into two main types -->
             Agglomerative Hierarchical Clustering (Bottom-Up Approach) -->
             Starts with each data point as its own cluster.
             Gradually merges the closest clusters step by step until all points_{\sqcup}
      ⇒belong to a single cluster.
             Commonly visualized using a dendrogram
           Divisive Hierarchical Clustering (Top-Down Approach) -->
             Starts with all data points in a single cluster.
             Splits the cluster into smaller clusters recursively.
             Continues until each data point is in its own cluster or a stopping \Box
      ⇔criterion is met.
[]: '''
         Steps (Agglomerative HC) ->
         Compute the distance (or similarity) matrix for all pairs of points.
         Identify the two closest clusters and merge them.
         Recompute the distance matrix considering the newly formed cluster.
         Repeat steps 2 and 3 until only one cluster remains.
         Linkage Criteria ->
         Single Linkage: Minimum distance between points in two clusters.
         Complete Linkage: Maximum distance between points in two clusters.
         Average Linkage: Average distance between points in two clusters.
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,,, []: ''' Advantages --> Does not require specifying the number of clusters in advance. Can produce a dendrogram that visually represents the data's structure. Suitable for small to medium-sized datasets. Disadvantages --> Computationally expensive for large datasets (time complexity: $O(n^3)$). Sensitive to noise and outliers. Requires careful selection of linkage criteria and distance metric. *Applications* --> Biology: Grouping species or genes with similar characteristics. Market Segmentation: Grouping customers with similar buying patterns. Image Analysis: Identifying similar regions in an image. ,,, []: [''' Dendogram --> A dendrogram is a tree-like diagram used to represent the arrangement of \Box \hookrightarrow clusters formed through hierarchical clustering. It visually shows the steps in_{\sqcup} \hookrightarrow which clusters are merged or split. Each branch of the tree represents a cluster, and the \sqcup at which two branches merge indicates the distance or dissimilarity between \sqcup \hookrightarrow the clusters. Parts of a Dendrogram --> Leaves (Bottom): Represent individual data points or initial clusters. Branches (Middle): Show how clusters are merged at each step. Height (Vertical Axis): Indicates the distance (or dissimilarity) between ⊔ ⇔clusters when they are merged. Horizontal Axis: Represents the data points or their order. How to Interpret a Dendrogram --> Horizontal lines: Represent cluster merging.

Centroid Linkage: Distance between centroids of two clusters.

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The height of these lines shows the distance or dissimilarity at which

⇔clusters are joined.

Vertical lines: Connect data points or clusters to the merging step.

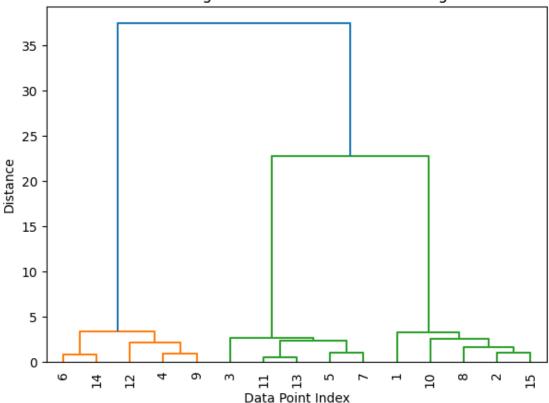
Cut the dendrogram: To determine the clusters, draw a horizontal line

⇒across the dendrogram.

The number of vertical lines intersected gives the number of clusters.
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[1]: # Dendogram -->
     import numpy as np
     import matplotlib.pyplot as plt
     from scipy.cluster.hierarchy import dendrogram, linkage
     from sklearn.datasets import make_blobs
     # Generate synthetic data
     X, _ = make_blobs(n_samples=15, centers=3, random_state=42)
     # Perform hierarchical clustering
     linked = linkage(X, method='ward')
     # Plot dendrogram with a width of 500px
     plt.figure(figsize=(7, 5)) # Width: 5 inches, Height: 3 inches
     dendrogram(linked,
                labels=np.arange(1, len(X)+1),
                leaf_rotation=90,
                leaf_font_size=10)
     plt.title('Dendrogram for Hierarchical Clustering')
     plt.xlabel('Data Point Index')
     plt.ylabel('Distance')
     plt.show()
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Visual inspection of dendrogram to find optimal clusters -->

Step 1: Create a dendrogram using hierarchical clustering.
Step 2: Observe the vertical lines (linkages) that connect clusters.
Step 3: Identify the longest vertical line (largest distance)
that you can "cut" without crossing more than one horizontal line.
Step 4: Draw a horizontal line across the dendrogram where this longest vertical line exists. The number of vertical lines (clusters) that the horizontal line intersects indicates the number of clusters.

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[8]: # Importing Libraries -->
import pandas as pd
from sklearn.cluster import AgglomerativeClustering
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[5]: # Importing Data -->
data = pd.read_csv('Data/Mall_Customers.csv')
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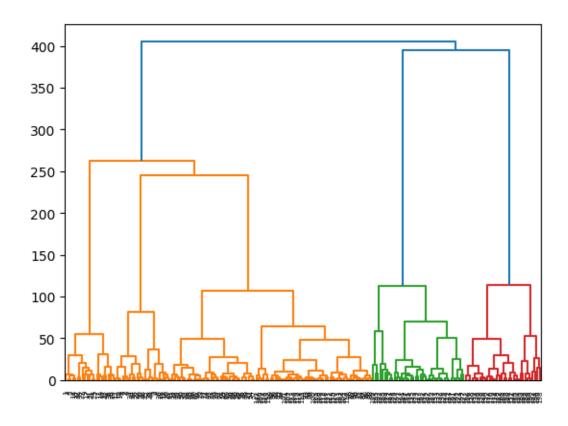
data.head(10) [5]: Annual Income (k\$) Spending Score (1-100) CustomerID Genre Age 0 1 Male 19 15 39 2 Male 1 21 15 81 6 2 3 Female 20 16 3 Female 23 16 77 Female 4 5 31 17 40 5 6 Female 22 17 76 Female 7 6 35 18 6 7 94 Female 23 18 8 8 9 Male 64 19 3 9 72 Female 30 19 10 [6]: x_data = data.iloc[:,[3,4]].values x_data [6]: array([[15, 39], [15, 81], [16, 6], [16, 77], [17, 40], [17, 76], [18, 6], [18, 94], [19, 3], [19, 72], [19, 14], [19, 99], [20, 15], [20, 77], [20, 13], [20, 79], [21, 35], 66], [21, [23, 29], 98], [23, [24, 35], [24, 73], [25, 5], [25, 73], [28, 14], [28, 82], [28, 32], [28, 61], [29, 31], [29, 87],

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            [137, 83]], dtype=int64)
[7]: # Optimal Clusters by Dendrogram -->
     dendro = dendrogram(linkage(x_data, method='ward'))
     plt.show()
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[11]: #
                            Training the model -->
                model = AgglomerativeClustering(n_clusters=5, linkage='ward')
                y_pred = model.fit_predict(x_data)
                y_pred
[11]: array([4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3,
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                                   0, 2], dtype=int64)
[14]: #
                            Visualizing The Clusters -->
                plt.scatter(x_data[y_pred==0,0], x_data[y_pred==0,1], s=50, c='magenta')
                plt.scatter(x_data[y_pred==1,0], x_data[y_pred==1,1], s=50, c='blue')
                plt.scatter(x_data[y_pred==2,0], x_data[y_pred==2,1], s=50, c='green')
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plt.scatter(x_data[y_pred==3,0], x_data[y_pred==3,1], s=50, c='red')
plt.scatter(x_data[y_pred==4,0], x_data[y_pred==4,1], s=50, c='cyan')
plt.xlabel('Annual Income ($)')
plt.ylabel('Spending Score (1-100)')
plt.show()
```

