

Assignment 1 - Problem 3

Given, the normalized Widrow Hoff Learning Rule,

$$\Delta w^{(k)} = \eta \left[t^{(k)} - w^{(k)} x^{(k)} \right] \frac{x^{(k)}}{\|x^{(k)}\|^2}$$

$$\Delta w^{(k)} = w^{(k+1)} - w^{(k)} \rightarrow \textcircled{1}$$

$x^{(k)}$ \rightarrow Input vector,

$t^{(k)}$ \rightarrow target and

$w^{(k)}$ \rightarrow weight vector at iteration (k)

Rewriting equation $\textcircled{1}$

$$w^{(k+1)} = \Delta w^{(k)} + w^{(k)} \rightarrow \textcircled{2}$$

Given that same input $x^{(k)}$ is given at iteration $(k+1)$, this implies the target is also the same $t^{(k)}$

$$\left. \begin{aligned} x^{(k+1)} &= x^{(k)} \\ t^{(k+1)} &= t^{(k)} \end{aligned} \right\} \rightarrow \textcircled{3}$$

Substituting these into the given rule.
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$\textcircled{2}$ and $\textcircled{3}$

$$\Delta \omega^{(k+1)} = \eta \left[t^{(k+1)} - \omega^{(k+1)} x^{(k+1)} \right] \frac{x^{(k+1)}}{\|x^{(k+1)}\|^2}$$

$$= \eta \left[t^{(k)} - (\Delta \omega^{(k)} + \omega^{(k)}) x^{(k)} \right] \frac{x^{(k)}}{\|x^{(k)}\|^2}$$

$$= \eta \left[t^{(k)} - \omega^{(k)} x^{(k)} \right] \frac{x^{(k)}}{\|x^{(k)}\|^2} - \eta \Delta \omega^{(k)} \frac{x^{(k)} / x^{(k)}}{\cancel{\|x^{(k)}\|^2}}$$

Hence

$$\Delta \omega^{(k+1)} = \Delta \omega^{(k)} - \eta \Delta \omega^{(k)} = (1 - \eta) \Delta \omega^{(k)}$$