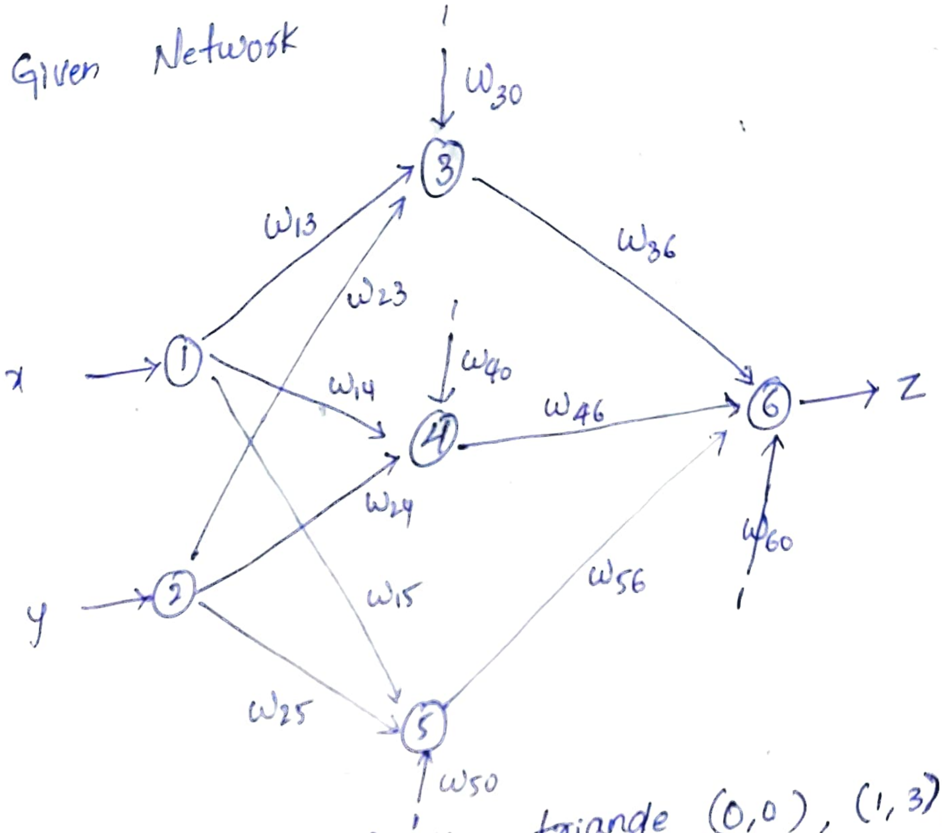
Assignment 1 - Problem 2

Given Network



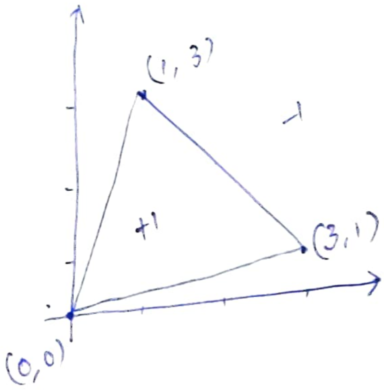
Given Vertices of the triangle $(0,0), (1,3), (3,1)$

Equations of lines passing through these points

$(0,0)$ & $(3,1)$ $\Rightarrow$ $x - 3y = 0 \longrightarrow$ ①

$(0,0)$ & $(1,3)$ $\Rightarrow$ $3x - y = 0 \longrightarrow$ ②

$(3,1)$ & $(1,3)$ $\Rightarrow$ $x + y - 4 = 0 \longrightarrow$ ③



Points inside triangle are considered of class 1 and Points on and outside triangle are class -1.

Input for node 3 is

$$w_{13}x + w_{23}y + w_{30}(1) \rightarrow ④$$

for node 4 $\Rightarrow$ $w_{14}x + w_{24}y + w_{40}(1) \rightarrow ⑤$

for node 5 $\Rightarrow$ $w_{15}x + w_{25}y + w_{50}(1) \rightarrow ⑥$

Comparing equations ① ② & ③ with ④, ⑤ and ⑥ respectively

$w_{13} = 3$, $w_{23} = -1$, $w_{30} = 0$

$w_{14} = 1$, $w_{24} = -3$, $w_{40} = 0$

$w_{15} = 1$, $w_{25} = 1$, $w_{50} = -4$

If a point lies inside the triangle then the point should lie above the line $x - 3y = 0$ and below the lines $x + y - 4 = 0$, $3x - y = 0$.

We have, a point $A(x_1, y_1)$ is above a line $ax + by + c = 0$

if, (i) $ax_1 + by_1 + c > 0$ and $b > 0$

(ii) $ax_1 + by_1 + c < 0$ and $b < 0$

a point $A(x_1, y_1)$ is below a line $ax + by + c = 0$ if,

(i) $ax_1 + by_1 + c < 0$ and $b > 0$

(ii) $ax_1 + by_1 + c > 0$ and $b < 0$

Hence, for a

Point inside a triangle, it should be ① below

$3x - y = 0$ ⇒ output of node 3 should be > 0

since $b < 0$

② above $x - 3y = 0$ ⇒ output of node 4 should be < 0

since $b < 0$

③ below $x + y - 4 = 0$ ⇒ output of node 5 should be $\not< 0$

since $b > 0$

we considered signum functions as activation functions at nodes 3, 4, 5 & 6.

Hence we get

$$W_{36} = 1, \quad W_{46} = -1, \quad W_{56} = -1$$

to make outputs of nodes 3, 4, 5 +ve, -ve and -ve respectively.

Input to node 6 is $f_3 W_{36} + f_4 W_{46} + f_5 W_{56} + W_{60}$

This equation is greater than 0 for point inside triangle.

$$f_3 W_{36} + f_4 W_{46} + f_5 W_{56} + W_{60} > 0$$

$$1 \times 1 + (-1)(-1) + (-1)(-1) + W_{60} > 0$$

$$W_{60} > -3$$

Considering W_{60} to be -2.

Here are the final weights.

$W_{13} = 3$	$W_{14} = 1$	$W_{15} = 1$	$W_{36} = 1$
$W_{23} = -1$	$W_{24} = -3$	$W_{25} = 1$	$W_{46} = -1$
$W_{30} = 0$	$W_{40} = 0$	$W_{50} = -4$	$W_{56} = -1$
			$W_{60} = -2$