Assignment 4 – problem 2

Given fuzzy set about the nomial x-gyro bias (x_{gb})

$$A = \left\{ \frac{0.2}{x_{gb}-3\delta_x} + \frac{0.4}{x_{gb}-2\delta_x} + \frac{0.6}{x_{gb}-\delta_x} + \frac{0.8}{x_{gb}} + \frac{0.6}{x_{gb}+\delta_x} + \frac{0.4}{x_{gb}+2\delta_x} + \frac{0.2}{x_{gb}+3\delta_x} \right\}$$

When $x_{gb} = 2°/hour$ and $\delta_x = 0.1$. We get the gyro bias in the x direction as

$$A = \left\{ \frac{0.2}{1.7} + \frac{0.4}{1.8} + \frac{0.6}{1.9} + \frac{0.8}{2.0} + \frac{0.6}{2.1} + \frac{0.4}{2.2} + \frac{0.2}{2.3} \right\}$$

Given fuzzy set B, describing accelerator bias in x direction as $B = \left\{ \frac{0.1}{0.25} + \frac{0.4}{0.27} + \frac{0.9}{0.3} + \frac{0.4}{0.33} + \frac{0.1}{0.35} \right\}$

a) Given classical implication operator

$$\mu_R = \max[\min(\mu_A, \mu_B), (1-\mu_A)] \quad \text{—} ①$$

Given Relation R: If A THEN B

According to propositional calculus,

$R: A \rightarrow B : \bar{A} \vee (A \wedge B)$ where $\vee, \wedge$ are S-norm and T-norm operators respectively — ②

we have

$$\mu_A = \left\{ \frac{0.2}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{0.6}{x_5} + \frac{0.4}{x_6} + \frac{0.2}{x_7} \right\}$$

$$\mu_B = \left\{ \frac{0.1}{y_1} + \frac{0.4}{y_2} + \frac{0.9}{y_3} + \frac{0.4}{y_4} + \frac{0.1}{y_5} \right\}$$

From ① and ②

$\min(\mu_A, \mu_B)$ is $(A \wedge B)$ and we are using cartesian product as T-norm operator here

$\min(\mu_A, \mu_B)$:

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\
\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{array} &
\begin{bmatrix}
0.1 & 0.2 & 0.2 & 0.2 & 0.1 \\
0.1 & 0.4 & 0.4 & 0.4 & 0.1 \\
0.1 & 0.4 & 0.6 & 0.4 & 0.1 \\
0.1 & 0.4 & 0.8 & 0.4 & 0.1 \\
0.1 & 0.4 & 0.6 & 0.4 & 0.1 \\
0.1 & 0.4 & 0.4 & 0.4 & 0.1 \\
0.1 & 0.2 & 0.2 & 0.2 & 0.1
\end{bmatrix}
\end{array}
$$

$(1-\mu_A)$: $[0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8]$

- we observe that $\min(\mu_A, \mu_B)$ is 2-D and $(1-\mu_A)$ is 1-D.

Hence, we are considering cylindrical extension of $(1-\mu_A)$

$$
C.E(1-\mu_A) = \begin{bmatrix}
0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\
0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\
0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\
0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\
0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\
0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\
0.8 & 0.8 & 0.8 & 0.8 & 0.8
\end{bmatrix}
$$

R: If A THEN B:-

$$\mu_R = \max\left[\min(\mu_A, \mu_B), (1-\mu_A)\right]$$

$$= \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.8 & 0.4 & 0.2 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix}$$

[Considering maximum of the two matrices $\min(\mu_A, \mu_B)$ and C.E. $(1-\mu_A)$]

b) Given new gyro with fuzzy bias

$$A' = \left\{ \frac{0}{1.7} + \frac{0.5}{1.8} + \frac{0.7}{1.9} + \frac{0.95}{2.0} + \frac{0.7}{2.1} + \frac{0.5}{2.2} + \frac{0}{2.3} \right]$$

(i) Max-min Composition $T = A' \circ R$:

$$A' = \begin{bmatrix} 0 \\ 0.5 \\ 0.7 \\ 0.95 \\ 0.7 \\ 0.5 \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.8 & 0.4 & 0.2 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix}$$

$$T = \underset{\text{rows}}{\max}\left[\underset{\text{columns}}{\min}\left[\begin{bmatrix} 0 \\ 0.5 \\ 0.7 \\ 0.95 \\ 0.7 \\ 0.5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.8 & 0.4 & 0.2 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix}\right.\right.$$

$$= \max_{rows} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.8 & 0.4 & 0.2 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = A' o R = [0.5 \ \ 0.5 \ \ 0.8 \ \ 0.5 \ \ 0.5]$$

The associated acclerometer bias using Max-min composition is $\left\{ \frac{0.5}{0.25} + \frac{0.5}{0.27} + \frac{0.8}{0.3} + \frac{0.5}{0.33} + \frac{0.5}{0.35} \right\}$

ii) Max-product composition $T = A' o R$

$$T = \max_{rows} \left[[0 \ \ 0.5 \ \ 0.7 \ \ 0.95 \ \ 0.7 \ \ 0.5] \cdot \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.2 & 0.4 & 0.8 & 0.4 & 0.2 \\ 0.4 & 0.4 & 0.6 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{bmatrix} \right]$$

$$= \max_{rows} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.28 & 0.28 & 0.42 & 0.28 & 0.28 \\ 0.19 & 0.38 & 0.76 & 0.38 & 0.19 \\ 0.28 & 0.28 & 0.42 & 0.28 & 0.28 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= [0.3 \ \ 0.38 \ \ 0.76 \ \ 0.38 \ \ 0.3]$$

The associated acclerometer bias using Max-product Composition is $\left\{ \frac{0.3}{0.25} + \frac{0.38}{0.27} + \frac{0.76}{0.3} + \frac{0.38}{0.33} + \frac{0.3}{0.35} \right\}$