The Capacity of the Hopfield Associative Memory

Subject of research

The paper's main subject, "The Capacity of the Hopfield Associative Memory," is studying rigorously the capacity of the Hopfield associative memory using techniques from coding theory, predominantly random coding and sphere hardening. Techniques to improve the capacity and the maximum memories that can be stored and retrieved from the ‘m’ fundamental memories were presented. Extensions to capacity were made by considering alternate ways of forming a connection matrix, and the results were discussed. Capacities, when certain wrong moves are permitted, were also mentioned.

Introduction

The paper first introduces Neuroanatomical models of brain functioning before explaining the actual mathematical formulations. A simple associative structure where each neuron can have a state + 1 or -1 was assumed, the connections between the neurons are fixed, and the connection matrix to be symmetric with a zero diagonal. Logical computations happen at each neuron by a simple threshold rule, +1 if the sign of the weighted sum of the binary states of other neurons equals or exceeds zero and -1 otherwise.

The state of a neuron can be changed in two ways:

1) Through synchronous operation where all the neurons perform the computation in parallel and change their states and
2) Through asynchronous operation where a neuron is chosen at random, and the state is updated, followed by other neurons (one at a time).

The main features of a computational system are to achieve a high degree of parallelism, robustness, simple nodes performing computational tasks, and a distributed system.

The chosen memories are considered to be fixed points in the neural network. These memories are referred to as attractors as they have a region of influence around them to map similar memories. The hamming distance was used for similarity. With this distance concept, the associative memory acts as a decoder which allows us to correct the errors in the initial state. There were two issues discussed when memory and sequence of the association are introduced in the network, the nature of the memory encoding rule and capacity of the system to recall the memories. The weights are updated with this memory encoding rule.

Contributions and Key ideas

"What is the capacity of the Hopfield neural network for information storage" is the main question that was answered through this paper. If there are 'm' fundamental memories to be stored, then how large this m can be is the main subject of the discussion.

This information retrieval process starts with finding a connection matrix T which is called the sum of outer products. A stored memory is retrieved using this connection matrix T along with an initial state vector and some network iterations. The fundamental memories (m) are considered stable, and when m is less than the number of total states in a memory (n), the outer product algorithm performed well.

There are other ways of constructing connection matrices where the m fundamental memories are considered to be fixed. And these constructions require 'm' memories to be eigenvectors of the connection matrix. To reach a capacity 'n,' asymptotically negative diagonal elements can be introduced, leading to instability in the network. Also, if we want to add a new memory, additional complicated calculations would be involved to compute the connection matrix. Capacity can also be increased if a fraction of errors is allowed in the final stable state.

An example was discussed, and the following problems with Hopfield networks were discussed. The resulting fixed point may not be one of the memories, or it might not be the nearest memory. Also, if the memory is not fixed, then it can never be recalled.

Various kinds of memory stability like the forced choice model where we guess the component values, clamping where certain components are fixed were discussed. These could add stability to the network but do not help in increasing the capacity. Three possibilities of stability convergence were discussed.

1) If the memory is within the radius of attraction of a fundamental memory, then any change made to the component is in the right direction.
2) With high probability, any random step will be in the right direction.
3) After finite back and forth changes, the system must settle down at a fixed point in asynchronous mode.

And this concept of the system settling down at a fixed point in asynchronous mode has been derived. If few fundamental memories are chosen, they reach the center of the sphere, i.e., they become stable in both synchronous and asynchronous modes.

Two concepts of capacity were discussed,

1) With high probability, all the 'm' fundamental memories are fixed
2) With high probability, most of the 'm' fundamental memories are fixed

Conclusions:

- A heuristic derivation for the maximum capacity of a network has been discussed, and the asymptotic value of m has been derived. The value of m is closer to n/2logn. If more than this number of m's are stored, then memories will not be fixed points.
- Lemma 1 is about finding the probability (p1) of a component moving in the wrong direction when it was correct before the change. And the expression for the probability p1 was derived.
- Lemma 2 helps in proving the Poisson results by finding the row sum failures.
- The big theorem, which makes use of lemma 1 and 2, derives solutions for the expected number of memories that have a radius with memory at the center and the probability of vectors in the radius around memories.
- After these assumptions, derivations, and proofs, it was finally stated that the capacity of a Hopfield network with m fundamental memories and n neurons is m= n/2logn if a fraction of memories is needed and m=n/4logn if all memories are to be remembered.