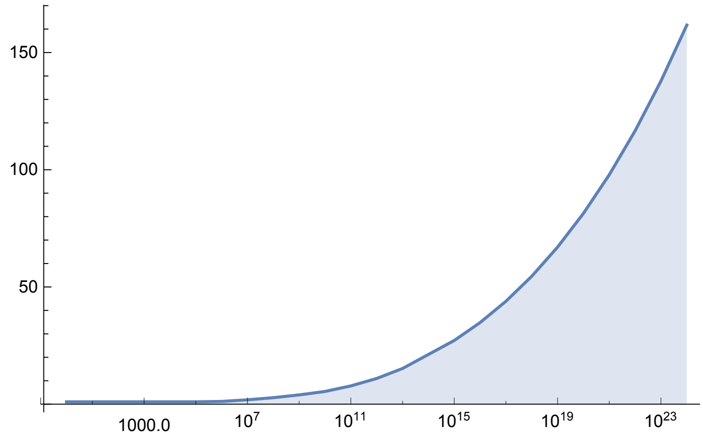
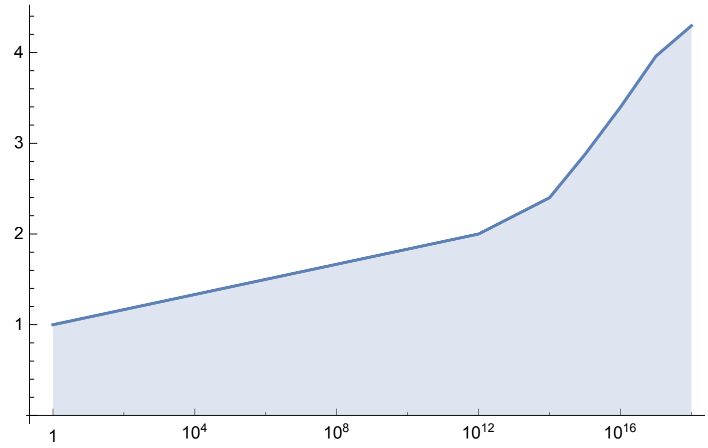
```
In[11]:= (* List of fast Deleglise-
          Rivat alpha factors determined by running pi(x) benchmarks *)

         alphaDelegliseRivat = { (* {x, alpha} *) {1, 1}, {10^1, 1}, {10^2, 1},
           {10^3, 1}, {10^4, 1}, {10^5, 1}, {10^6, 1.172}, {10^7, 1.861},
           {10^8, 2.778}, {10^9, 3.955}, {10^10, 5.426}, {10^11, 7.795},
           {10^12, 10.960}, {10^13, 15.22}, {10^15, 27.16}, {10^16, 34.80},
           {10^17, 43.88}, {10^18, 54.56}, {10^19, 67.01}, {10^20, 81.38},
           {10^21, 97.86}, {10^22, 116.61}, {10^23, 137.83}, {10^24, 161.69}}

Out[11]= {{1, 1}, {10, 1}, {100, 1}, {1000, 1}, {10 000, 1}, {100 000, 1},
          {1 000 000, 1.172}, {10 000 000, 1.861}, {100 000 000, 2.778},
          {1 000 000 000, 3.955}, {10 000 000 000, 5.426}, {100 000 000 000, 7.795},
          {1 000 000 000 000, 10.96}, {10 000 000 000 000, 15.22},
          {1 000 000 000 000 000, 27.16}, {10 000 000 000 000 000, 34.8},
          {100 000 000 000 000 000, 43.88}, {1 000 000 000 000 000 000, 54.56},
          {10 000 000 000 000 000 000, 67.01}, {100 000 000 000 000 000 000, 81.38},
          {1 000 000 000 000 000 000 000, 97.86}, {10 000 000 000 000 000 000 000, 116.61},
          {100 000 000 000 000 000 000 000, 137.83}, {1 000 000 000 000 000 000 000 000, 161.69}}

In[12]:= ListLogLinearPlot[alphaDelegliseRivat, Filling → Bottom, Joined → True]
```

Out[12]=




```
In[13]:= NonlinearModelFit[alphaDelegliseRivat,
          a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]

Out[13]= FittedModel[ 0.0672202 + 0.483613 Log[x] - 0.0508992 Log[x]^2 + 0.0017154 Log[x]^3 ]

         (* Below is another formula which is quite accurate for calculating
           the Deleglise-Rivat alpha factor in primecount. The constant
           1200 has been obtained by running many pi(10^20) benchmarks. *)

         alpha[x_] := (Log[x])^3 / (1200 (Log[Log[10^20]] / Log[Log[x]])^3)


         (* List of fast Lagarias-Miller-
          Odlyzko alpha factors determined by running pi(x) benchmarks  *)

         alphaLMO = {(* {x, alpha} *) {1, 1}, {10^12, 2}, {10^13, 2.2}, {10^14, 2.4},
           {10^15, 2.877}, {10^16, 3.398}, {10^17, 3.960}, {10^18, 4.295}}

         {{1, 1}, {1 000 000 000 000, 2}, {10 000 000 000 000, 2.2}, {100 000 000 000 000, 2.4},
          {1 000 000 000 000 000, 2.877}, {10 000 000 000 000 000, 3.398},
          {100 000 000 000 000 000, 3.96}, {1 000 000 000 000 000 000, 4.295}}
```

```
ListLogLinearPlot[alphaLMO, Filling → Bottom, Joined → True]
```




```
NonlinearModelFit[alphaLMO, a (Log[x]) ^2 + b Log[x] + c, {a, b, c}, x]
```

FittedModel[$1.00454 - 0.0656652\,\text{Log}[x] + 0.00352628\,\text{Log}[x]^2$]