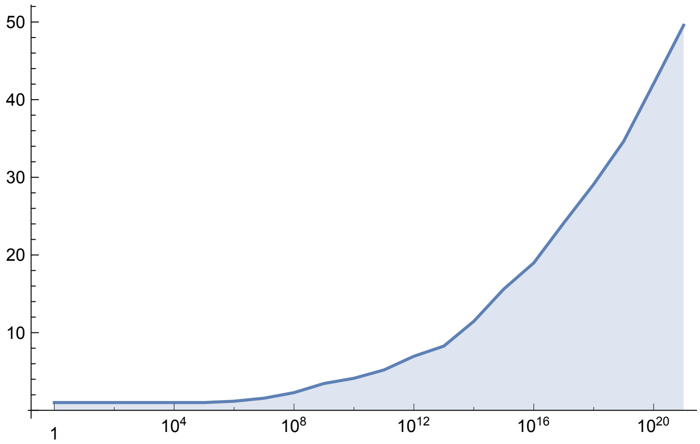
In[18]:=
```
(* List of fast Deleglise-Rivat alpha factors for x ≤ 10^21 found by running
   pi(x) benchmarks using the find_fastest_alpha.sh script *)

alphaDelegliseRivat =
 { (* {x, alpha} *) {1, 1}, {10^1, 1}, {10^2, 1}, {10^3, 1}, {10^4, 1},
  {10^5, 1}, {10^6, 1.172}, {10^7, 1.561}, {10^8, 2.278}, {10^9, 3.455},
  {10^10, 4.125}, {10^11, 5.195}, {10^12, 6.960}, {10^13, 8.272},
  {10^14, 11.462}, {10^15, 15.619}, {10^16, 18.980}, {10^17, 24.119},
  {10^18, 29.115}, {10^19, 34.635}, {10^20, 42.072}, {10^21, 49.575}}
```

Out[18]=
```
{{1, 1}, {10, 1}, {100, 1}, {1000, 1}, {10 000, 1}, {100 000, 1},
 {1 000 000, 1.172}, {10 000 000, 1.561}, {100 000 000, 2.278},
 {1 000 000 000, 3.455}, {10 000 000 000, 4.125}, {100 000 000 000, 5.195},
 {1 000 000 000 000, 6.96}, {10 000 000 000 000, 8.272},
 {100 000 000 000 000, 11.462}, {1 000 000 000 000 000, 15.619},
 {10 000 000 000 000 000, 18.98}, {100 000 000 000 000 000, 24.119},
 {1 000 000 000 000 000 000, 29.115}, {10 000 000 000 000 000 000, 34.635},
 {100 000 000 000 000 000 000, 42.072}, {1 000 000 000 000 000 000 000, 49.575}}
```

In[19]:=
```
ListLogLinearPlot[alphaDelegliseRivat, Filling → Bottom, Joined → True]
```

Out[19]=



In[20]:=
```
(* alpha is a tuning factor that balances
 the compuation of the easy special leaves and the
   hard special leaves. The formula below is
 used in the file src/primecount.cpp to
   calculate a fast alpha factor for the computation of pi(x) *)

NonlinearModelFit[alphaDelegliseRivat,
 a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]
```

Out[20]= FittedModel[$0.802942 + 0.123034\,\mathrm{Log}[x] - 0.0160586\,\mathrm{Log}[x]^2 + 0.000711339\,\mathrm{Log}[x]^3$]

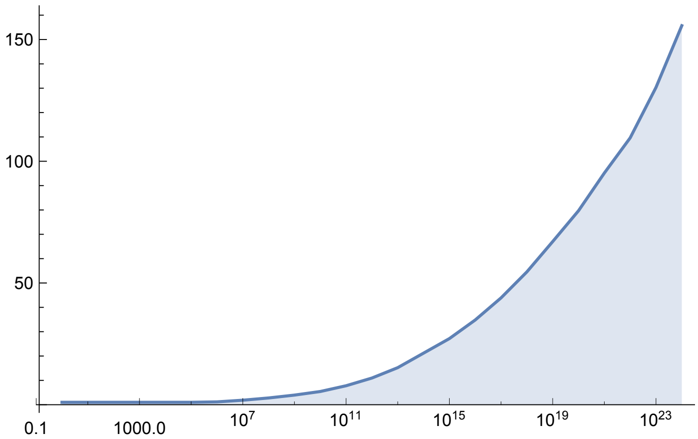
In[55]:=
```
(* List of fast Deleglise-Rivat alpha factors for x > 10^21 found by running
   pi(x) benchmarks. A larger alpha reduces
  CPU cache misses for large pi(x) computations. *)

alphaDelegliseRivatLarge = { (* {x, alpha} *) {1, 1}, {10^1, 1}, {10^2, 1},
  {10^3, 1}, {10^4, 1}, {10^5, 1}, {10^6, 1.172}, {10^7, 1.861},
  {10^8, 2.778}, {10^9, 3.955}, {10^10, 5.426}, {10^11, 7.795},
  {10^12, 10.960}, {10^13, 15.22}, {10^15, 27.16}, {10^16, 34.80},
  {10^17, 43.88}, {10^18, 54.56}, {10^19, 67.01}, {10^20, 79.68},
  {10^21, 95.16}, {10^22, 109.61}, {10^23, 130.33}, {10^24, 155.69}}
```

Out[55]=
```
{{1, 1}, {10, 1}, {100, 1}, {1000, 1}, {10 000, 1}, {100 000, 1},
 {1 000 000, 1.172}, {10 000 000, 1.861}, {100 000 000, 2.778},
 {1 000 000 000, 3.955}, {10 000 000 000, 5.426}, {100 000 000 000, 7.795},
 {1 000 000 000 000, 10.96}, {10 000 000 000 000, 15.22},
 {1 000 000 000 000 000, 27.16}, {10 000 000 000 000 000, 34.8},
 {100 000 000 000 000 000, 43.88}, {1 000 000 000 000 000 000, 54.56},
 {10 000 000 000 000 000 000, 67.01}, {100 000 000 000 000 000 000, 79.68},
 {1 000 000 000 000 000 000 000, 95.16}, {10 000 000 000 000 000 000 000, 109.61},
 {100 000 000 000 000 000 000 000, 130.33}, {1 000 000 000 000 000 000 000 000, 155.69}}
```

In[56]:=
```
ListLogLinearPlot[alphaDelegliseRivatLarge, Filling → Bottom, Joined → True]
```

Out[56]=



In[57]:=
```
(* alpha is a tuning factor that balances
 the compuation of the easy special leaves and the
   hard special leaves. The formula below is
 used in the file src/primecount.cpp to
   calculate a fast alpha factor for the computation of pi(x) *)

NonlinearModelFit[alphaDelegliseRivatLarge,
 a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]
```

Out[57]= FittedModel[$0.536167 + 0.294183\,\mathrm{Log}[x] - 0.0378846\,\mathrm{Log}[x]^2 + 0.00149385\,\mathrm{Log}[x]^3$]

```
(* Below is another formula which is quite accurate for calculating the
   Deleglise-Rivat alpha factor in primecount. The constant 2200 has
   been obtained by running many pi(10^20) benchmarks. *)

alpha[x_] := (Log[x])^3 / (2200 (Log[Log[10^20]] / Log[Log[x]])^3)
```

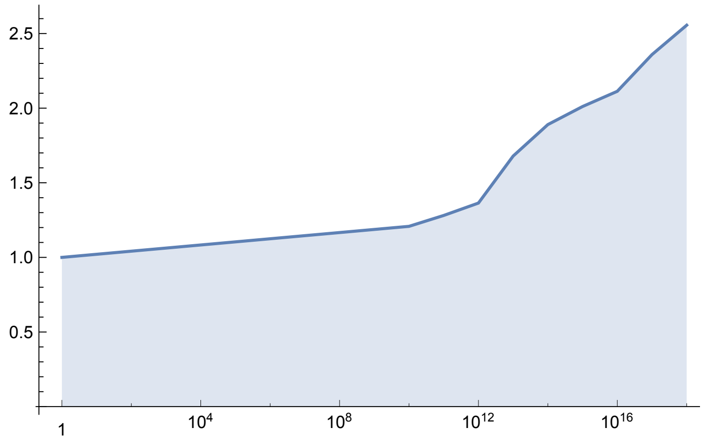
In[24]:=
```
(* List of fast Lagarias-Miller-
 Odlyzko alpha factors found by running pi(x) benchmarks  *)

alphaLMO = {(* {x, alpha} *) {1, 1}, {10^10, 1.208},
   {10^11, 1.281},  {10^12, 1.364}, {10^13, 1.679}, {10^14, 1.890},
   {10^15, 2.011}, {10^16, 2.113}, {10^17, 2.359}, {10^18, 2.556}}
```

Out[24]=
```
{{1, 1}, {10 000 000 000, 1.208}, {100 000 000 000, 1.281},
 {1 000 000 000 000, 1.364}, {10 000 000 000 000, 1.679}, {100 000 000 000 000, 1.89},
 {1 000 000 000 000 000, 2.011}, {10 000 000 000 000 000, 2.113},
 {100 000 000 000 000 000, 2.359}, {1 000 000 000 000 000 000, 2.556}}
```

In[25]:=
```
ListLogLinearPlot[alphaLMO, Filling → Bottom, Joined → True]
```

Out[25]=




```
(* alpha is a tuning factor that balances
 the compuation of the easy special leaves and the
   hard special leaves. The formula below is
 used in the file src/primecount.cpp to
   calculate a fast alpha factor for the computation of pi(x) *)
```

In[26]:=
```
NonlinearModelFit[alphaLMO, a (Log[x])^2 + b Log[x] + c, {a, b, c}, x]
```

Out[26]= FittedModel[$0.990948 - 0.0261411\,\mathrm{Log}[x] + 0.00156512\,\mathrm{Log}[x]^2$]