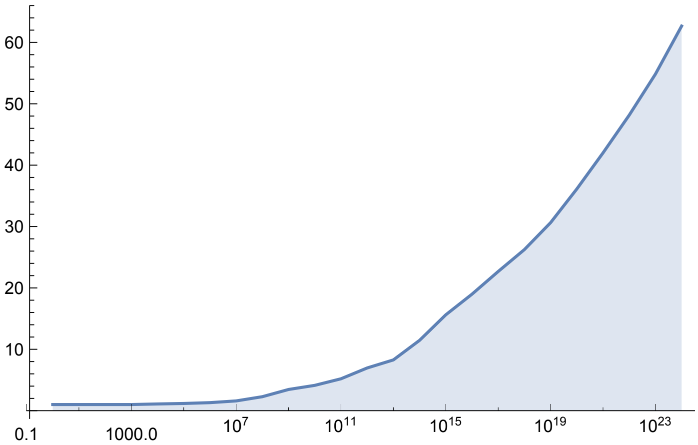
```
In[6]:= (* List of fast Deleglise-Rivat alpha factors for x ≤ 10^23 found by
          running pi(x) benchmarks using the find_fastest_alpha.sh script *)

        alphaDelegliseRivat = { (* {x, alpha} *) {1, 1}, {10^1, 1}, {10^2, 1},
          {10^3, 1}, {10^4, 1.095}, {10^5, 1.174}, {10^6, 1.310}, {10^7, 1.591},
          {10^8, 2.278}, {10^9, 3.455}, {10^10, 4.125}, {10^11, 5.195}, {10^12, 6.960},
          {10^13, 8.272}, {10^14, 11.462}, {10^15, 15.619}, {10^16, 18.980},
          {10^17, 22.677}, {10^18, 26.246}, {10^19, 30.635}, {10^20, 36.120},
          {10^21, 42}, {10^22, 48.148}, {10^23, 54.832}, {10^24, 62.66} }

Out[6]= {{1, 1}, {10, 1}, {100, 1}, {1000, 1}, {10 000, 1.095}, {100 000, 1.174},
         {1 000 000, 1.31}, {10 000 000, 1.591}, {100 000 000, 2.278},
         {1 000 000 000, 3.455}, {10 000 000 000, 4.125}, {100 000 000 000, 5.195},
         {1 000 000 000 000, 6.96}, {10 000 000 000 000, 8.272}, {100 000 000 000 000, 11.462},
         {1 000 000 000 000 000, 15.619}, {10 000 000 000 000 000, 18.98},
         {100 000 000 000 000 000, 22.677}, {1 000 000 000 000 000 000, 26.246},
         {10 000 000 000 000 000 000, 30.635}, {100 000 000 000 000 000 000, 36.12},
         {1 000 000 000 000 000 000 000, 42}, {10 000 000 000 000 000 000 000, 48.148},
         {100 000 000 000 000 000 000 000, 54.832}, {1 000 000 000 000 000 000 000 000, 62.66}}

In[9]:= ListLogLinearPlot[alphaDelegliseRivat, Filling → Bottom, Joined → True]
```

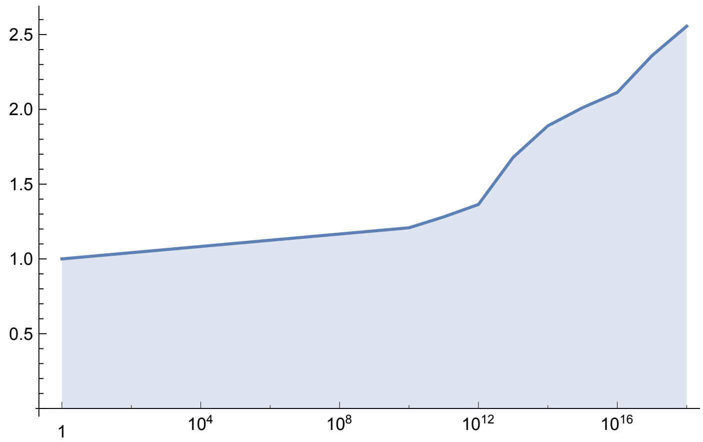
Out[9]=




```
In[11]:=
        (* alpha is a tuning factor that balances the compuation
         of the easy special leaves and the hard special leaves. The
         formula below is used in the file src/primecount.cpp to
         calculate a fast alpha factor for the computation of pi(x). *)

        NonlinearModelFit[alphaDelegliseRivat,
         a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]

Out[11]= FittedModel[ 1.39952 - 0.125227 Log[x] + 0.00263762 Log[x]^2 + 0.000356618 Log[x]^3 ]
```

```
(* Below is another formula which is quite accurate for calculating the
   Deleglise-Rivat alpha factor in primecount. The constant 2200 has
   been obtained by running many pi(10^20) benchmarks. *)

alpha[x_] := (Log[x])^3 / (2200 (Log[Log[10^20]] / Log[Log[x]])^3)
```

```
(* List of fast Lagarias-Miller-
 Odlyzko alpha factors found by running pi(x) benchmarks. *)

alphaLMO = {(* {x, alpha} *) {1, 1}, {10^10, 1.208},
  {10^11, 1.281}, {10^12, 1.364}, {10^13, 1.679}, {10^14, 1.890},
  {10^15, 2.011}, {10^16, 2.113}, {10^17, 2.359}, {10^18, 2.556}}

{{1, 1}, {10 000 000 000, 1.208}, {100 000 000 000, 1.281},
 {1 000 000 000 000, 1.364}, {10 000 000 000 000, 1.679}, {100 000 000 000 000, 1.89},
 {1 000 000 000 000 000, 2.011}, {10 000 000 000 000 000, 2.113},
 {100 000 000 000 000 000, 2.359}, {1 000 000 000 000 000 000, 2.556}}

ListLogLinearPlot[alphaLMO, Filling → Bottom, Joined → True]
```




```
(* alpha is a tuning factor that balances the compuation
 of the easy special leaves and the hard special leaves. The
 formula below is used in the file src/primecount.cpp to
 calculate a fast alpha factor for the computation of pi(x). *)

NonlinearModelFit[alphaLMO, a (Log[x])^2 + b Log[x] + c, {a, b, c}, x]

FittedModel[ 0.990948 - 0.0261411 Log[x] + 0.00156512 Log[x]^2 ]
```