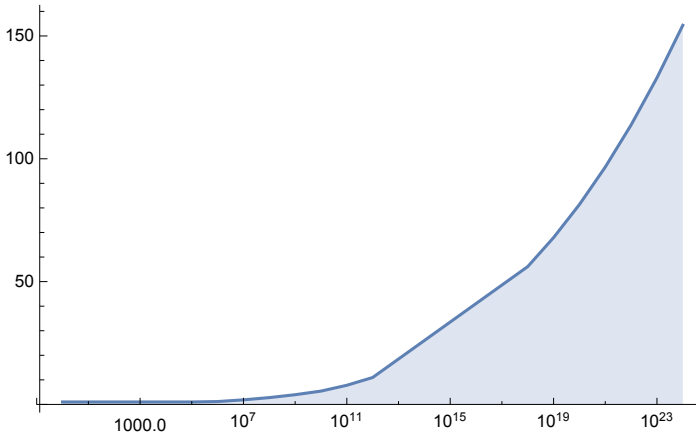
In[311]:=
```
(* List of fast Deleglise-
 Rivat alpha factors determined by running pi(x) benchmarks *)

alphaDelegliseRivat = { (* {x, alpha} *) {1, 1}, {10^1, 1}, {10^2, 1},
  {10^3, 1}, {10^4, 1}, {10^5, 1}, {10^6, 1.172}, {10^7, 1.861},
  {10^8, 2.778}, {10^9, 3.955}, {10^10, 5.426}, {10^11, 7.795},
  {10^12, 10.960}, {10^18, 56.11}, {10^19, 67.92}, {10^20, 81.38},
  {10^21, 96.63}, {10^22, 113.78}, {10^23, 132.97}, {10^24, 154.34}}
```

Out[311]=
```
{{1, 1}, {10, 1}, {100, 1}, {1000, 1}, {10 000, 1}, {100 000, 1},
 {1 000 000, 1.172}, {10 000 000, 1.861}, {100 000 000, 2.778},
 {1 000 000 000, 3.955}, {10 000 000 000, 5.426}, {100 000 000 000, 7.795},
 {1 000 000 000 000, 10.96}, {1 000 000 000 000 000 000, 56.11},
 {10 000 000 000 000 000 000, 67.92}, {100 000 000 000 000 000 000, 81.38},
 {1 000 000 000 000 000 000 000, 96.63}, {10 000 000 000 000 000 000 000, 113.78},
 {100 000 000 000 000 000 000 000, 132.97}, {1 000 000 000 000 000 000 000 000, 154.34}}
```

In[318]:=
```
ListLogLinearPlot[alphaDelegliseRivat, Filling → Bottom, Joined → True]
```

Out[318]=



In[313]:=
```
NonlinearModelFit[alphaDelegliseRivat,
 a (Log[x])^3 + b (Log[x])^2 + c Log[x] + d, {a, b, c, d}, x]
```

Out[313]=
$$\text{FittedModel}\left[0.964413 + 0.150855\,\text{Log}[x] - 0.0299373\,\text{Log}[x]^2 + 0.00140126\,\text{Log}[x]^3\right]$$

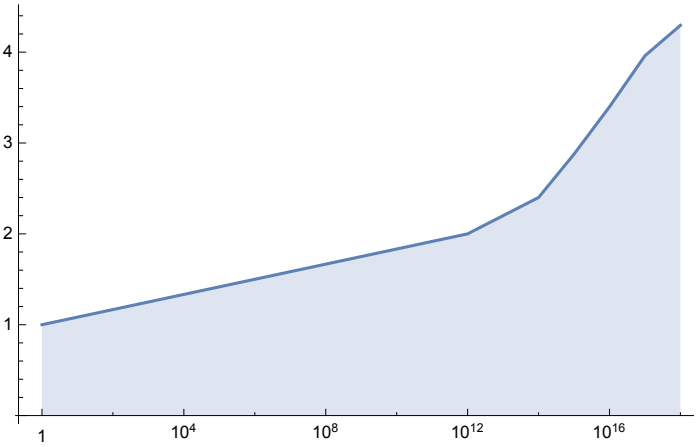
In[314]:=
```
(* List of fast Lagarias-Miller-
 Odlyzko alpha factors determined by running pi(x) benchmarks  *)

alphaLMO = {(* {x, alpha} *) {1, 1}, {10^12, 2}, {10^13, 2.2}, {10^14, 2.4},
  {10^15, 2.877}, {10^16, 3.398}, {10^17, 3.960}, {10^18, 4.295}}
```

Out[314]=
```
{{1, 1}, {1 000 000 000 000, 2}, {10 000 000 000 000, 2.2}, {100 000 000 000 000, 2.4},
 {1 000 000 000 000 000, 2.877}, {10 000 000 000 000 000, 3.398},
 {100 000 000 000 000 000, 3.96}, {1 000 000 000 000 000 000, 4.295}}
```

In[310]:=
```
ListLogLinearPlot[alphaLMO, Filling → Bottom, Joined → True]
```

Out[310]=



In[315]:=
```
NonlinearModelFit[alphaLMO, a (Log[x])^2 + b Log[x] + c, {a, b, c}, x]
```

Out[315]=
```
FittedModel[ 1.00454 - 0.0656652 Log[x] + 0.00352628 Log[x]^2 ]
```