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La Place Transform

The name laplace transform is named after it's inventer Simon Laplace. It is an integral transform that converts a function of a real variable 't' to a function of complex variable 's'. It is an important tool for solving differential equations.

Let f(t) be given function for all $t = 0$, Laplace Transform of f(t) is denoted by,

$L\{f(t)\}$ is defined as,

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

- L is known as Laplace Transform operator.
- F(s) is Laplace Transform of f(t).
- s is a complex number.

Linear Property of Laplace Transform.

if f(t) & g(t) be 2 functions whose Laplace exists, Then for any two constant a & b, we have,

$$L\{a\,f(t) + b\,g(t)\} = aL\{f(t)\} + bL\{g(t)\}$$

P.T.O.

Proof LHS $= L\{af(t)+bg(t)\}$

$$= \int_0^\infty e^{-st}\{af(t)+bg(t)\}\,dt$$

$$= a\int_0^\infty e^{-st}f(t)\,dt + b\int_0^\infty e^{-st}g(t)\,dt$$

$$= aL\{f(t)\} + bL\{g(t)\} = \text{RHS}$$

Q. Find $L\{f(t)\}$, where $f(t)=1$.

$$L\{f(t)\} = \int_0^\infty e^{-st}f(t)\,dt = \int_0^\infty e^{-st}(1)\,dt$$

$$= \int_0^\infty e^{-st}\,dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = -\frac{1}{s}\left[\frac{1}{e^\infty}-\frac{1}{e^0}\right] = -\frac{1}{s}[0-1]$$

$$= \frac{1}{s}, \quad s>0.$$

$$\boxed{\text{① } L\{1\} = \frac{1}{s}, \ s>0 \quad \text{or} \quad L\{n\} = \frac{n}{s}, \ s>0}$$

$$\boxed{\text{② } L\{e^{at}\} = \frac{1}{s-a}, \ s>a}$$

Q. $L\{e^{2t}-e^{7t}+e^{-5t}\} = L\{e^{2t}\} - L\{e^{7t}\} + L\{e^{-5t}\}$

$$= \frac{1}{s-2} - \frac{1}{s-7} + \frac{1}{s+5}$$

$\Gamma \rightarrow$ Gamma function.

$$\boxed{\Gamma m = m-1\,\Gamma m-1 \quad \& \quad \Gamma\tfrac{1}{2} = \sqrt{\pi}}$$

Q. Find $L\{t\}$.

soln. $= \int_0^\infty e^{-st}\, t\, dt = \left[\frac{t\, e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^\infty$

$$= \left[(0-0) - \left(0 - \frac{1}{s^2}\right) \right]$$

$$= \frac{1}{s^2}$$

③ $L\{t\} = \frac{1}{s^2}$, $s > 0$ or $L\{t^n\} = \frac{n!}{s^{n+1}}$ when $n \in N$

or $L\{t^n\} = \frac{\Gamma n+1}{s^{n+1}}$ when $n > 0$. $\Gamma n = \int_0^\infty e^{-t} t^{n-1}\, dt$

$L\{t^n\} = \frac{n!}{s^{n+1}}$ when n is a natural number.

Q. $L\{t^3 + t^2 + e^t\} = L\{t^3\} + L\{t^2\} + L\{e^t\}$

$$= \frac{3!}{s^4} + \frac{2!}{s^3} + \frac{1}{s-1} = \frac{6}{s^4} + \frac{2}{s^3} + \frac{1}{s-1}$$

Q. $L\{t^{3/2}\} = \frac{\Gamma \frac{3}{2}+1}{s^{3/2+1}} = \frac{\Gamma\frac{5}{2}}{s^{5/2}} = \frac{\frac{3}{2}\Gamma\frac{3}{2}}{s^{5/2}} = \frac{\frac{3}{4}\sqrt{\pi}}{s^{5/2}}$

④ $L\{\sin at\} = \frac{a}{s^2+a^2}$, $s > 0$

⑤ $L\{\cos at\} = \frac{s}{s^2+a^2}, \quad s > 0$

⑥ $L\{\sinh at\} = \frac{a}{s^2-a^2}, \quad s > a$

⑦ $L\{\cosh at\} = \frac{s}{s^2-a^2}, \quad s > a$

sin at, cos at can be written as circular functions.

$\sin at = \frac{e^{iat} - e^{-iat}}{2i}$ $\qquad \cos at = \frac{e^{iat} + e^{-iat}}{2}$

sin hat, cos hat can be written as hyperbolic functions.

$\sinh at = \frac{e^{at} - e^{-at}}{2}$ $\qquad \cosh at = \frac{e^{at} + e^{-at}}{2}$

Laplace of piecewise continuous function.

Q. Find $L\{f(t)\}$, where $f(t) = \begin{cases} 4, & 0 < t < 1 \\ 3, & t > 1 \end{cases}$

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^1 e^{-st}(4)\,dt + \int_1^\infty e^{-st}(3)\,dt$$

$$= 4\left[\frac{e^{-st}}{-s}\right]_0^1 + 3\left[\frac{e^{-st}}{-s}\right]_1^\infty = +4\left[\frac{e^{-s}}{-s} + \frac{1}{s}\right] + 3\left[0 - \frac{e^{-s}}{-s}\right]$$

$$= -\frac{4e^{-s}}{s} + \frac{3e^{-s}}{s} + \frac{4}{s} = \frac{-e^{-s}}{s} + \frac{4}{s}$$

Q. Find the Lapl

a) $e^{at} - e^{bt}$

i) $3t^4 - 2t^3 + 4e^{3}$

First shi

if $L\{$

$L\{e^{at}$

Q. $L\{e^{at}\sin$

$F(s$

∴ L

Q. $L\{t^{7/2}$

L

∴ L

Q. Find the Laplace transform of the following :-

a) $e^{at} - e^{bt}$ b) $\cos^2 kt$ c) $(5e^{2t} - 8)^2$

d) $3t^4 - 2t^3 + 4e^{3t} - 5\sin 5t + 3\cos 2t$ e) $\frac{\cos\sqrt{t}}{\sqrt{t}}$

Ans First shifting theorem.

if $L\{f(t)\} = F(s)$, then according to FST,

$$\boxed{L\{e^{at} f(t)\} = F(s-a)}$$

Q. $L\{e^{at} \sin bt\}$ Here $f(t) = \sin bt$

$$F(s) = L\{f(t)\} = \frac{b}{s^2 + b^2}$$

$$\therefore L\{e^{at} f(t)\} = \frac{b}{(s-a)^2 + b^2}$$

Q. $L\{t^{7/2} e^{2t}\}$, $f(t) = t^{7/2}$

$$L\{f(t)\} = L\{t^{7/2}\} = \frac{\Gamma 9/2}{s^{9/2}} = \frac{\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}}{s^{9/2}}$$

$$= \frac{105\sqrt{\pi}}{16 s^{9/2}}$$

$$\therefore L\{t^{7/2} e^{2t}\} = \frac{105\sqrt{\pi}}{16(s-2)^{9/2}}$$

Q. $L\{e^{3t}\sin 4t\} = \cancel{\frac{4}{s^2+16}} \quad \frac{4}{(s-3)^2+16}.$

Q. $L\{\cos^2 t\} = L\left\{\frac{1+\cos 2t}{2}\right\} = \frac{1}{2}L\{1\} + \frac{1}{2}L\{\cos 2t\}$

$= \frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{s}{s^2+4} = \frac{1}{2s} + \frac{s}{2s^2+8}$

Change of Scale.

if $L\{f(t)\} = F(s)$, then $\boxed{L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)}$

Q. $L\{3t\} = \frac{1}{3}F\left(\frac{s}{3}\right) = \frac{1}{3} \times \frac{1}{\left(\frac{s}{3}\right)^2} = \frac{3}{s^2}$

$a=3 \quad f(t)=t$

$F(s) = \frac{1}{s^2}$

Q. if $L\{f(t)\} = \frac{e^{-1/s}}{s}$, Find $L\{e^{-t}f(3t)\}$.

$L\{f(t)\} = \frac{e^{-1/s}}{s}$

By change of scale,

$L\{f(3t)\} = \frac{1}{3}F\left(\frac{s}{3}\right) = \frac{1}{\cancel{3}}\frac{e^{-1/s/3}}{s/\cancel{3}} = \frac{e^{-3/s}}{s}$

$\therefore L\{e^{-t}f(3t)\} = \frac{e^{-\frac{3}{s+1}}}{s+1}$

Laplace Transform of Derivative.

$$L\{f'(t)\} = sL\{f(t)\} - f(0)$$

$$L\{f''(t)\} = s^2 L\{f(t)\} - sf(0) - f'(0)$$

In general,

$$L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) \ldots f^{n-1}(0) \ldots$$

Q. Find $L\{f'''(t)\}$, if $f(t) = t^3 + 1$.

$$L\{f'''(t)\} = s^3 L\{f(t)\} - s^2 f(0) - sf'(0) - f''(0)$$

$f(t) = t^3 + 1$, $f(0) = 1$

$f'(t) = 3t^2$, $f'(0) = 0$

$f''(t) = 6t$, $f''(0) = 0$

$$L\{f(t)\} = L\{t^3\} + L\{1\}$$

$$= \frac{3!}{s^4} + \frac{1}{s}$$

$$L\{f'''(t)\} = s^3\left[\frac{6}{s^4} + \frac{1}{s}\right] - s^2(1) - s(0) - 0$$

$$= \frac{6}{s} + s^2 - s^2 = \frac{6}{s}$$

Laplace of Integral of function.

$$L\left\{\int_0^t f(u)\,du\right\} = \frac{F(s)}{s}, \quad s>0$$

$$L\left\{\int_0^t\int_0^t f(u)\,du\,du\right\} = \frac{F(s)}{s^2}, \quad s>0$$

In general.

$$L\left\{\int_0^t\int_0^t\int_0^t\int_0^t \overset{n \text{ times}}{\cdots} f(u)\,du \ldots du\right\} = \boxed{\frac{F(s)}{s^n}, \quad s>0}$$

Q. $L\left\{\int_0^t e^{3u}\,du\right\}$

$f(u) = e^{3u}$

$L\{f(u)\} = L\{e^{3u}\}$

$= \frac{1}{s-3} = F(s)$

$= \frac{1}{s(s-3)}$

Q. $L\left\{\int_0^t\int_0^t\int_0^t \cos 3u\,du\,du\,du\right\} = \frac{s}{s^3(s^2+9)}$

$f(u) = \cos 3u$

$F(u) = \frac{s}{s^2+9}$

Multiplication of t.

if $L\{f(t)\} = F(s)$, $\boxed{L\{t\,f(t)\} = -\frac{d}{ds}F(s)}$

In general. $\boxed{L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)}$

Q. $L\{t \sin 2t\}$ Here $f(t) = \sin 2t$

$F(s) = \frac{2}{s^2+4}$

$L\{t \sin 2t\} = -\frac{d}{ds} F(s) = -2 \frac{d}{ds}(s^2+4)^{-1}$

$= \boxed{\frac{4s}{(s^2+4)^2}}$

Q. $L\{t^2 \cos 3t\}$ $f(t) = \cos 3t$ $F(s) = \frac{s}{s^2+9}$

$(-1)^2 \frac{d^2}{ds^2}\left(\frac{s}{s^2+9}\right) = \frac{d^2}{ds^2}\left(\frac{s}{s^2+9}\right)$

$$L^{-1}\left\{\frac{1}{(s+a)^{n+1}}\right\} = e^{-at}\frac{t^n}{n!}$$

Division by t.

if $L\{f(t)\} = F(s)$, then $\boxed{L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(u)\,du}$

Q. $L\left\{\frac{\sin t}{t}\right\} = \int_s^{\infty} F(u)\,du$

$f(t) = \sin t \qquad F(s) = \frac{1}{s^2+1} \qquad F(u) = \frac{1}{u^2+1}$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^{\infty}\frac{1}{u^2+1}\,du = \left[\tan^{-1}u\right]_s^{\infty}$$

$$= \tan^{-1}\infty - \tan^{-1}s$$

$$= \frac{\pi}{2} - \tan^{-1}s$$

Inverse Laplace Transform.

if $L\{f(t)\} = F(s)$, then $\boxed{L^{-1}\{F(s)\} = f(t)}$

(1) $L^{-1}\left\{\frac{1}{s}\right\} = 1$ (2) $L^{-1}\left\{\frac{1}{s^2}\right\} = t$

(3) $L^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{\Gamma n}$ (4) $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

(5) $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$ (6) $L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at$

$L^{-1}\left\{\frac{1}{s^2}\right\} = t \qquad L^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}, \quad n \in \mathbb{N}$

First shifting property. if $L^{-1}\{F(s)\} = f(t)$ then.

$$\boxed{L^{-1}\{F(s-a)\} = e^{at} f(t)}$$

Q. Find inverse LT of $\frac{1}{(s+1)^3}$ & $\frac{s+2}{(s+2)^2+4^2}$

$$L^{-1}\left\{\frac{1}{(s+1)^3}\right\} = L^{-1}\left\{\frac{1}{(s-(-1))^3}\right\}_{a=-1} = e^{-t} L^{-1}\left\{\frac{1}{s^3}\right\}$$

$$= \boxed{e^{-t}\frac{t^2}{2!}}$$

$$L^{-1}\left\{\frac{s+2}{(s+2)^2+4^2}\right\} = L^{-1}\left\{\frac{s-(-2)}{(s-(-2))^2+4^2}\right\} = e^{-2t} L^{-1}\left\{\frac{s}{s+4^2}\right\}$$

$$= \boxed{e^{-2t}\cos 4t}$$

Change of scale property.

if $L^{-1}\{F(s)\} = f(t)$, then, $\boxed{L^{-1}\{F(ks)\} = \frac{1}{k} f\left(\frac{t}{k}\right)}$

eg. $L^{-1}\left\{\frac{1}{(3s)^2}\right\} = \frac{1}{3} f\left(\frac{t}{3}\right) = \frac{1}{3} \times \frac{t}{3} = \frac{t}{9}$

P.T.O.

Imagine, $L^{-1}\left\{\frac{1}{(s+1)^2}\right\}_{a=-1}$ then, $\underbrace{L^{-1}\left\{\frac{1}{s^2}\right\}}_{\text{Solve this differently}}$, $= e^{-t}t$ take e^{-t} outside

$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$ $\quad L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at.$

Q. $L^{-1}\left\{\frac{s+1}{s^2-6s+25}\right\} = L^{-1}\left\{\frac{s+1}{s^2-6s+9+16}\right\} = L^{-1}\left\{\frac{(s-3)+4}{(s-3)^2+4^2}\right\}$

$\Rightarrow L^{-1}\left\{\frac{s-3}{(s-3)^2+4^2}\right\} + L^{-1}\left\{\frac{4}{(s-3)^2+4^2}\right\}$

$= e^{3t}\cos 4t + e^{3t}\sin 4t$

Inverse Laplace of Derivatives.

if $L^{-1}\{F(s)\} = f(t)$, Then $L^{-1}\left\{\frac{d^n}{ds^n}(s)\right\} = (-1)^n t$

$$\boxed{L^{-1}\left\{\frac{d^n}{ds^n}(s)\right\} = (-1)^n t^n f(t)}$$

Q. ILT of $F(s) = \frac{s}{(s^2+1)^2}$

$\frac{s}{(s^2+1)^2} = \frac{-1}{2}\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$

$L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = L^{-1}\left\{\frac{-1}{2}\frac{d}{ds}\left(\frac{1}{s^2+1}\right)\right\}$

$= \frac{-1}{2}(-1)^1 t\sin t$

$= \frac{t\sin t}{2}$

I.L.T by Method of Partial Fractions.

We use this when, $\boxed{deg\ (Num) \leq deg\ (Den)}$

Some formulas;

Denominator | Partial Fraction.

a) Non-repeated Linear Function

$$\frac{1}{(ax+b)(cx+d)} \qquad \frac{A}{ax+b} + \frac{B}{cx+d}$$

b) Repeated

$$\frac{1}{(ax+b)^n} \qquad \frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} \dots \frac{A_n}{(ax+b)^n} +$$

c) Non-repeated quadratic

$$\frac{1}{ax^2+bx+c} \qquad \frac{Ax+B}{ax^2+bx+c}$$

d)

$$\frac{1}{(ax^2+bx+c)^n} \qquad \frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots$$

Q. $L^{-1}\left\{\frac{s+2}{(s-3)(s-2)}\right\}$

$$\frac{s+2}{(s-3)(s-2)} = \frac{A}{s-3} + \frac{B}{s-2} = \frac{A(s-2)+B(s-3)}{(s-3)(s-2)}$$

$\Rightarrow s+2 = As - 2A + Bs - 3B$

$\Rightarrow s+2 = (A+B)s - 2A - 3B$

$A+B = 1$ $\quad$ $-2A-3B = 2$

$\boxed{A = 1-B}$ $\quad$ $\boxed{B = -4}$

$\boxed{A = 5}$

$$L^{-1}\left\{\frac{s+2}{(s-3)(s-2)}\right\} = L^{-1}\left\{\frac{5}{s-3}\right\} - L^{-1}\left\{\frac{4}{s-2}\right\}$$

$$= 5\,L^{-1}\left\{\frac{1}{s-3}\right\} - 4\,L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= 5e^{3t} - 4e^{2t}.$$

Shortcut method, $L^{-1}\left\{\frac{s+2}{(s-3)(s-2)}\right\}$

$$\frac{s+2}{(s-3)(s-2)} = \frac{3+2}{(s-3)(3-2)} + \frac{2+2}{(s-2)(2-3)}$$

$$= \frac{5}{s-3} - \frac{4}{(s-2)}$$

Very Easy.

Q. Find $L^{-1}\left\{\dfrac{3s+7}{s^2-2s-3}\right\} = L^{-1}\left\{\dfrac{3s+7}{s^2-3s+s-3}\right\}$

(3)

$$L^{-1}\left\{\frac{3s+7}{s(s-3)+1(s-3)}\right\} = L^{-1}\left\{\frac{3s+7}{(s+1)(s-3)}\right\}$$

$$\frac{3s+7}{(s+1)(s-3)} = \frac{-3+7}{(s+1)(-1-3)} + \frac{9+7}{(s-3)(4)}$$

$$L^{-1}\left\{\frac{3s+7}{(s+1)(s-3)}\right\} = -L^{-1}\left\{\frac{1}{s+1}\right\} + L^{-1}\left\{\frac{4}{s-3}\right\}$$

$$= -e^{-t} + 4e^{3t}.$$

Q. $L^{-1}\left\{\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

$$\frac{2s^2-4}{(s+1)(s-2)(s-3)} = \frac{2-4}{(s+1)(-3)(-4)} + \frac{8-4}{(s-2)(3)(-1)} + \frac{14}{(s-3)(1)(4)}$$

$$= -\frac{\cancel{2}}{(s+1)\cancel{(12)}\,6} + \frac{4}{(s-2)(-3)} + \frac{\cancel{14}\,7}{(s-3)\cancel{4}\,2}$$

$$= \frac{-1}{6}e^{-t} - \frac{4}{3}e^{2t} + \frac{7}{2}e^{3t}$$

$$L^{-1}\left\{\frac{1}{(s+1)^2}\right\} = te^{-t}$$

-(-1)

Multiplication by s.

if $L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\{sF(s)\} = \frac{df}{dt}$

Eg. Find $L^{-1}\left\{\frac{s}{(s+1)^2}\right\}$

$L^{-1}\{sF(s)\}$, where $F(s) = \frac{1}{(s+1)^2}$

$$f(t) = L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$L^{-1}\{sF(s)\} = L^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \frac{d f(t)}{dt} = \frac{d}{dt}(te^{-t})$$

$$= -te^{-t} + e^{-t}$$

Q. $L^{-1}\left\{\frac{s^2}{s^2+1}\right\} = L^{-1}\{sF(s)\}$.

$$f(t) = L^{-1}\{F(s)\} = L^{-1}\left\{\frac{s}{s^2+1}\right\} = \cos t$$

$$\frac{d}{dt}f(t) = \frac{d}{dt}\cos t = -\sin t.$$

Division by s.

if $L^{-1}\{F(s)\} = f(t)$, then, $L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(u)\,du$

similarly, $L^{-1}\left\{\frac{F(s)}{s^2}\right\} = \int_0^t\int_0^t f(u)\,du\,du$

In general, $L^{-1}\left\{\frac{F(s)}{s^n}\right\} = \int_0^t\int_0^t \underset{n \text{ times}}{\cdots} \int_0^t f(u)\,du^n$

Q. $L^{-1}\left\{\frac{1}{s^3(s+1)}\right\} = L^{-1}\left\{\frac{F(s)}{s^3}\right\}$ where $F(s) = \frac{1}{s+1}$

$f(t) = L^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$ $\qquad f(u) = e^{-u}$

$L^{-1}\left\{\frac{F(s)}{s}\right\} = L^{-1}\left\{\frac{1}{s(s+1)}\right\} = \int_0^t e^{-u}\,du = \left[\frac{e^{-u}}{-1}\right]_0^t$

$= \underline{1 - e^{-t}}$

$L^{-1}\left\{\frac{F(s)}{s^2}\right\} = \int_0^t (1-e^{-u})\,du = \left[u + e^{-u}\right]_0^t = t + e^{-t} - 1$

$L^{-1}\left\{\frac{F(s)}{s^3}\right\} = \int_0^t (u + e^{-u} - 1)\,du = \left(\frac{u^2}{2} - e^{-u} - u\right)_0^t$

$= \frac{t^2}{2} - e^{-t} - t + 1$

$$\boxed{L\{y\} = \bar{y}}^{*} \qquad \boxed{L^{-1}L = 1}$$

Application of LT to Differential Equation.

Q: Solve, $y'' - 2y' - 8y = 0$, $y(0) = 3$, $y'(0) = 6$

$$\frac{d^2y}{dt^2} - \frac{2dy}{dt} - 8y = 0$$

Apply LT on both sides,

$$L\{y''\} - 2L\{y'\} - 8L\{y\} = 0$$

$$[s^2\bar{y} - sy(0) - y'(0)] - 2[s\bar{y} - y(0)] - 8\bar{y} = 0$$

$$(s^2\bar{y} - 3s - 6) - 2s\bar{y} + 6 - 8\bar{y} = 0$$

$$s^2\bar{y} - 2s\bar{y} - 8\bar{y} - 3s = 0$$

$$\bar{y}(s^2 - 2s - 8) - 3s = 0$$

$$\boxed{\bar{y} = \frac{3s}{s^2 - 2s - 8}}$$

$$\frac{1}{s+2} + \frac{2}{s-4}$$

$$L\{y\} = \frac{3s}{s^2 - 2s - 8} \Rightarrow y = L^{-1}\left\{\frac{3s}{(s-4)(s+2)}\right\}$$

$$y = L^{-1}\left\{\frac{1}{s+2}\right\} + 2L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$y = e^{-2t} + 2e^{4t}.$$

Q. $y'' + 2y' + 5y = e^{-t}\sin t \quad , \quad y(0) = 0 \, , \; y'(0) = 1$

Apply LT on both sides,

$$L\{y''\} + 2L\{y'\} + 5L\{y\} = L\{e^{-t}\sin t\}$$

$$[s^2\bar{y} - sy(0) - y'(0)] + 2[s\bar{y} - y(0)] + 5\bar{y} = \frac{1}{(s+1)^2+1}$$

$$[s^2\bar{y} - 0 - 1] + 2s\bar{y} + 5\bar{y} = \frac{1}{(s+1)^2+1}$$

$$(s^2+2s+5)\bar{y} - 1 = \frac{1}{(s+1)^2+1}$$

$$y = \frac{1}{s^2+2s+5} + \frac{1}{(s^2+2s+5)((s+1)^2+1)}$$

Apply Inverse LT on both sides.

$$y = L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\} + L^{-1}\left\{\frac{1}{(s^2+2s+5)(s^2+2s+2)}\right\}$$

$$y = \frac{e^{-t}\sin 2t}{2} + L^{-1}\left\{\frac{1}{(s^2+2s+5)(s^2+2s+2)}\right\}$$

X

P.T.O.

$$\frac{1}{(s^2+2s+5)(s^2+2s+2)} = \frac{As+B}{(s^2+2s+5)} + \frac{Cs+D}{(s^2+2s+2)}$$

$$1 = As+B(s^2+2s+2) + (Cs+D)(s^2+2s+5)$$

$$1 = As^3 + As^2 + 2As + Bs^2 + 2Bs + 2B + Cs^3 + 2Cs^2 + 5Cs + Ds^2 + 2Ds + 5D$$

$A+C = 0$, $A = -C \longrightarrow s^3$

$2A + B + 2(+1) = 0 \longrightarrow s^2$

$2A + 2B + 5C + 2D = 0 \longrightarrow s$

$1 = 2B + 5D$

$B = \frac{1-5D}{2}$

$$-2C + \frac{1-5D}{2} + 2C + D = 0$$

$1 - 5D + 2D = 0$

$\boxed{D = \frac{1}{3}}$ $\boxed{B = \frac{-1}{3}}$

$\boxed{C = 0}$ $\boxed{A = 0}$

$$L^{-1}\left\{\frac{1}{(s^2+2s+5)(s^2+2s+2)}\right\} = L^{-1}\left\{\frac{-1}{3(s^2+2s+5)}\right\} +$$

$$L^{-1}\left\{\frac{1}{3(s^2+2s+2)}\right\}$$

$$= -\frac{1}{3} L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\} + \frac{1}{3} L^{-1}\left\{\frac{1}{(s+1)^2+1^2}\right\}$$

$$= -\frac{1}{3} e^{-t}\sin 2t + \frac{1}{3} e^{-t}\sin t$$

$$y = e^{-t}\sin t - \frac{e^{-t}\sin 2t}{6} + \frac{1}{3}e^{-t}\sin t$$

Q. Solve $y''' - 3y'' + 3y' - y = t^2e^t$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = -2$.

$$L\{y'''\} - 3L\{y''\} + 3L\{y'\} - L\{y\} = L\{t^2e^t\}$$

$$\bar{y} = \frac{s^2 - 3s + 1}{(s-1)^3} + \frac{2}{(s-1)^6}$$

$L^{-1}\left\{\frac{1}{(s-a)^n}\right\} = \frac{e^{at}t^{n-1}}{\Gamma n}$, $n > 0$

$$\bar{y} = \frac{(s-1)^2 - (s-1) - 1}{(s-1)^3} + \frac{2}{(s-1)^6}$$

when $n \in \mathbb{N}$ $\boxed{n!}$

$$\bar{y} = \frac{1}{(s-1)} - \frac{1}{(s-1)^2} - \frac{1}{(s-1)^3} + \frac{2}{(s-1)^6}$$

Applying ILT

$$y = e^t - te^t - \frac{t^2e^t}{2} + \frac{t^5e^t}{60} \quad \text{Ans.}$$

Solve this properly !!