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Laplace formulas.

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

$$L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$$

$$L\{n\} = \frac{n}{s}, \quad s > 0$$

$$L\{e^{at}\} = \frac{1}{s-a}, \quad s > a$$

$$L\{t^n\} = \frac{n!}{s^{n+1}} \quad \text{when } n \text{ is a natural number.}$$

$$L\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}} \quad \text{when } n > 0 \qquad \Gamma n = \int_0^\infty e^{-t} t^{n-1}\,dt$$

$$L\{\sin at\} = \frac{a}{s^2+a^2}, \quad s > 0$$

$$L\{\cos at\} = \frac{s}{s^2+a^2}$$

$$L\{\sinh at\} = \frac{a}{s^2-a^2}$$

$$L\{\cosh at\} = \frac{s}{s^2-a^2}$$

$$L\{e^{at} f(t)\} = F(s-a) \qquad \{\text{First shifting}\}$$

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right) \qquad \{\text{Change of scale}\}$$

$$L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) \dots f^{n-1}(0) \dots$$ {LT of Derivatives}

$$L\left\{\int_0^t\int_0^t\int_0^t \text{n times } f(u)\,du\,du \dots du\right\} = \frac{F(s)}{s^n}$$ {LT of Integrals}

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$ {Multiplication of t}

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)\,du$$ {Division by t}

Inverse Laplace

$$L^{-1}\{F(s)\} = f(t)$$

$$L^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$L^{-1}\left\{\frac{1}{s^2}\right\} = t$$

$$L^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{n! \text{ or } \Gamma n}$$

$$L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$$

$$L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at$$

$$L^{-1}\{F(s-a)\} = e^{at} f(t)$$

$$L^{-1}\{F(ks)\} = \frac{1}{k} f\left(\frac{t}{k}\right)$$

$$L^{-1}\left\{\frac{d^n}{ds^n}(s)\right\} = (-1)^n t^n f(t)$$

$$L^{-1}\{sF(s)\} = \frac{df}{dt}$$

$$L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(u)\,du$$

Lagrange's PDE.

$$P(x,y,z)p + Q(x,y,z)q = R(x,y,z)$$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

2x Integrate any 2

$\downarrow \quad \downarrow$

$$F(u,v) = 0$$

Charpit's

$f(x,y,z,p,q) = 0$ $\qquad \underbrace{f_p = \frac{\partial f}{\partial p}} \qquad \underbrace{f_q = \frac{\partial f}{\partial q}} \ldots$

Sub. eqn,

$$\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dz}{pf_p + qf_q} = \frac{dp}{-(f_x + pf_z)} = \frac{dq}{-(f_y + qf_z)}$$

Homogeneous Linear Eqn with Constant Coefficients.

Case I. m_1 & m_2 are real & distinct

$$C.F = \phi_1(y + m_1 x) + \phi_2(y + m_2 x)$$

Case 2. m_1 & m_2 are real & equal

$$C.F = \phi_1(y + m_1 x) + x\phi_2(y + m_1 x)$$

Case II. if m_1 & m_2 are complex $m_1 = a + ib$, $m_2 = a - ib$

$$CF = \phi_1(y + ax + ibx) + \phi_1(y + ax - ibx) + i\{\phi_2(y + ax + ibx) - \phi_2(y + ax - ibx)\}$$

Rules for Particular Integral.

1. $P.I = \dfrac{1}{f(D_x, D_y)} e^{ax+by} = \dfrac{1}{f(a, b)} e^{ax+by}$

2. $P.I = \dfrac{1}{f(D_x^2, D_xD_y, D_y^2)} \sin(ax + by)$ or $\cos(ax+by)$ $= \dfrac{1}{f(-a^2, -ab, -b^2)} \sin(ax + by)$

3. $P.I = \dfrac{1}{f(D_x, D_y)} x^m y^n = [f(D_x, D_y)]^{-1}(x^m y^n)$

4. $P.I = \dfrac{1}{f(D_x, D_y)} f(x, y)$ for $ax + by$, $\dfrac{1}{f(a, b)} \underbrace{\iiint}_{n \text{ times}} \phi(u)\, du \ldots du$

5. $P.I = \dfrac{1}{f(D_x, D_y)} e^{ax+by} v(x, y) = e^{ax+by} \dfrac{1}{f(D_x + a, D_y + b)} v(x, y)$

2nd order PDE.

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0$$

when $B^2 - 4AC < 0 \longrightarrow$ Elliptic

When $B^2 - 4AC = 0 \longrightarrow$ Parabolic

When $B^2 - 4AC > 0 \longrightarrow$ Hyperbolic

Heat Eqn → $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ (P)

Laplace Eqn → $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (E)

Poisson Eqn → $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x,y)$ (E)

Wave Eqn → $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (H)

Non linear PDE of first order.

Form I. $f(p,q) = 0$ put $\boxed{p = a, \quad q = b}$,

$$\boxed{z = ax + \phi(a)y + c}$$

Form II. $f(z,p,q) = 0$ put $\boxed{q = ap}$

$p = f(z)$, $dz = p\,dx + q\,dy$.

Form III. $f(x,p) = g(y,q) = a$

$f(x,p) = a \qquad g(y,q) = a$

$p = \phi(x) \qquad y = \phi(q)$

$dz = p\,dx + q\,dy$.

Form IV $z = px + qy + f(p,q)$

Then, $\boxed{z = ax + by + f(a,b)}$ $p = a, q = b$.

General integration & diff. formulas.

$\int dx = x + c$ $\qquad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int \frac{dx}{x} = \ln|x| + c$ $\qquad$ $\int e^x dx = e^x + c$

$\int \sin x \, dx = -\cos x + c$ $\qquad$ $\int \tan x \, dx = -\ln|\cos x| + c$

$\int \cos x \, dx = \sin x + c$ $\qquad$ $\int \cot x \, dx = -\ln|\sin x| + c$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$ $\qquad$ $\int \csc x \, dx = -\ln|\csc x + \cot x| + c$

$\int \sec^2 x \, dx = \tan x + c$ $\qquad$ $\int \csc^2 x \, dx = -\cot x + c$

$\int \sec x \tan x \, dx = \sec x + c$ $\qquad$ $\int \csc x \cot x \, dx = -\csc x + c$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ $\qquad$ $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\frac{|x|}{a} + c$

Just remember these lol.