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Assignment-5

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Problem Statement:

Find the equations of circle passing through (-4,3) and touching the lines x+y=2 and x-y=2 .

SOLUTION:

Given:

Equation of lines

$$\mathbf{n_1}^\top \mathbf{x} = \mathbf{c_1} \tag{1}$$

$$\mathbf{n_2}^\top \mathbf{x} = \mathbf{c_2} \tag{2}$$

Point on the circle

$$\mathbf{P} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{3}$$

To Find

Equations of circle passing through **P** and touching the lines

STEP-1

$$x + y = 2 \tag{4}$$
$$x - y = 2 \tag{5}$$
$$\implies \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{6}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -1 & 2 \end{array}\right) \tag{7}$$
$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -2 & 0 \end{array}\right) \tag{8}$$
$$\xleftrightarrow{R_2 \leftarrow R_2/(-2)} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 0 \end{array}\right) \tag{9}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right) \implies \mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{10}$$

STEP-2

Let **O** be the centre of the circle.

Angle of Bi-sectors for the given line equations are:

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{11}$$

$$\mathbf{m_2} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{12}$$

$$\mathbf{m_1} - \mathbf{m_2} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{13}$$

The equation of line passing through **x** with normal vector **n** is given by

$$\mathbf{n}^\top (\mathbf{X} - \mathbf{x}) = 0 \tag{14}$$

where

$$\mathbf{n} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{15}$$

$$(\mathbf{X} - \mathbf{x}) = \begin{pmatrix} x - 2 \\ y - 0 \end{pmatrix} \tag{16}$$

on substituting (16), (15) in (14) we get

y=0

which means the direction vector on X-axis and hence the centre of the circle will also lie on X-axis

Let d be the distance from centre **O** to line (1)

$$d = \frac{\left|\mathbf{n}^\top \mathbf{O} - c\right|}{\|\mathbf{n}\|} \tag{17}$$

where d=r
So,

$$r = \frac{\left|\mathbf{n}^\top \mathbf{O} - c\right|}{\|\mathbf{n}\|} \tag{18}$$

Where **n** is normal vector of line (1) given by

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \; and \;\; c = 2 \tag{19}$$

Substituting **n** , c and **O** in (18)

Distance from **P** to the centre **O** is given by

$$d = \|\mathbf{P} - \mathbf{O}\| \tag{20}$$

where d=r
Squaring on both sides

$$r^2 = \|\mathbf{P} - \mathbf{O}\|^2 \tag{21}$$

substituting **O** and **P** in (21)
compare (18) and(21) we get

$$\mathbf{O} = \begin{pmatrix} -2.65 \\ 0 \end{pmatrix} \textit{ and } \mathbf{O_1} = \begin{pmatrix} -17.35 \\ 0 \end{pmatrix} \tag{22}$$

From (18) we get the radius as:

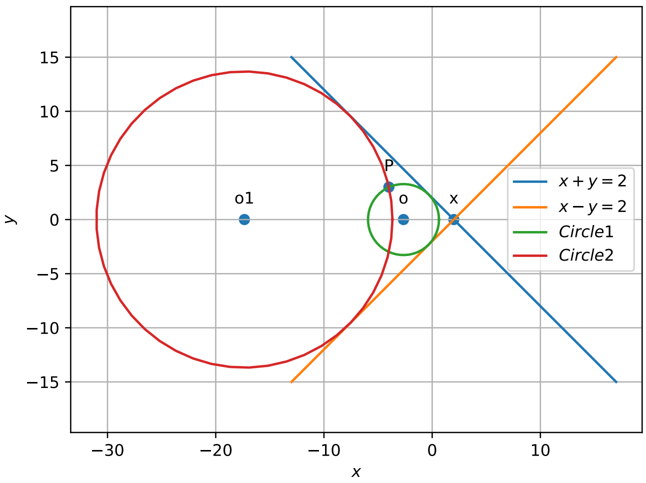
$$\text{r} = 3.78 \ ,10.66$$

The required equations of the circles are:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0 \tag{23}$$
$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f_2 = 0 \tag{24}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{25}$$
$$\mathbf{u_1} = \begin{pmatrix} -2.65 \\ 0 \end{pmatrix} \tag{26}$$
$$\mathbf{u_2} = \begin{pmatrix} -17.35 \\ 0 \end{pmatrix} \tag{27}$$
$$f_1 = -7.26 \ \ , f_2 = 187.38 \tag{28}$$



Construction

vertex	coordinates
P	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Download the code
Github link: Assignment-5.