

# 5G NR LDPC Encoder and Decoder

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## CONTENTS

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>5G NR LDPC Encoder</b>	<b>1</b>
2.1	5G-NR LDPC Channel coding . . . .	1
2.2	LDPC Base Graphs . . . . .	2
2.3	LDPC Base Graph selection procedure	2

## 1. INTRODUCTION

5G NR (New Radio) is the latest wireless communication standard that supports higher data rates, lower latency, and increased reliability. LDPC (Low-Density Parity-Check) is a forward error correction (FEC) technique used in 5G NR to improve the reliability of data transmission.

5G NR (New Radio) uses Low-Density Parity-Check (LDPC) codes for forward error correction (FEC) to improve the reliability of data transmission over wireless channels. LDPC codes are linear block codes that have a sparse parity-check matrix. The parity-check matrix is designed to have a low density of 1's, which means that only a small fraction of the bits in the matrix are 1's.

The 5G NR LDPC codes are designed to have a high coding gain and low error rates while maintaining low latency and complexity. The coding gain is the ratio of the signal-to-noise ratio (SNR) required for a non-coded system to achieve a certain bit error rate (BER) compared to the SNR required for a coded system to achieve the same BER. The higher the coding gain, the more robust the code is against channel noise and interference.

The 5G NR LDPC codes are specified by the 3GPP (Third Generation Partnership Project) and include several code rates ranging from 1/5 to 5/6. The code rates determine the amount of redundancy added to the original data to create the codeword. Higher code rates add more redundancy, which results in a more robust code but also increases the latency of the transmission.

The 5G NR LDPC codes have a block size of 8448 bits and are designed to be flexible and adaptable to different channel conditions. The code rate and the number of iterations used in the decoding process can be adjusted based on the channel conditions to optimize the trade-off between coding gain, latency, and complexity.

Overall, the 5G NR LDPC codes are a crucial component of the 5G NR communication system, and their performance

has a significant impact on the reliability and efficiency of data transmission over wireless channels.

## 2. 5G NR LDPC ENCODER

The 5G NR (New Radio) encoder is responsible for encoding information bits into a larger codeword to improve the reliability of data transmission. The 5G NR standard uses a specific type of forward error correction (FEC) technique called LDPC (Low-Density Parity-Check) for error correction. The 5G NR LDPC encoder adds redundant bits to the original information bits to create the codeword.

The 5G NR LDPC encoder uses a parity-check matrix to determine the redundant bits to add to the information bits. The matrix is designed to have a low-density of 1's, which means that only a small fraction of the bits in the matrix are 1's. This reduces the complexity of the encoder and decoder while still providing high coding gain and low error rates.

The 5G NR LDPC encoder can operate at different code rates, which determines the amount of redundancy added to the information bits. The higher the code rate, the more redundancy is added, which results in a more robust codeword but also increases the latency of the transmission.

The 5G NR encoder also includes other features such as rate matching and code block segmentation. Rate matching is used to adjust the size of the codeword to match the modulation and channel coding scheme used for transmission. Code block segmentation is used to divide the information bits into smaller code blocks, which can be processed and encoded separately.

Overall, the 5G NR encoder is a critical component of the communication system, and its performance impacts the reliability and efficiency of data transmission over 5G networks.

### A. 5G-NR LDPC Channel coding

Based on my interpretation from [3GPP Specification 38.212](#) Rel 15 (Multiplexing and channel coding), I had put together the procedure on how LDPC Base Graph selection and coding happens.

- 1) For transmission of a DL transport block, a transport block CRC is first appended to provide error detection, followed by a LDPC base graph selection.
- 2) NR supports two LDPC base graphs, one for small transport blocks and one for larger transport blocks.
- 3) Then transport block is segmented into code blocks and code block CRC attachment is performed.

- 4) Each code block is individually LDPC encoded. The LDPC coded blocks are then individually rate matched.
- 5) Finally, code block concatenation is performed to create a codeword for transmission. Up to 2 code words can be transmitted simultaneously.

### B. LDPC Base Graphs

There are two types of Base Graphs standardized in the specification, [3GPP Specification 38.212](#) (Multiplexing and channel coding). Base Graph is a Matrix where each of the entries can be further expanded based on the expansion factor  $Z_c$ .

- 1) Base Graph 1 (BG1) : With Matrix size 46X68 entries For Large Transport Block.

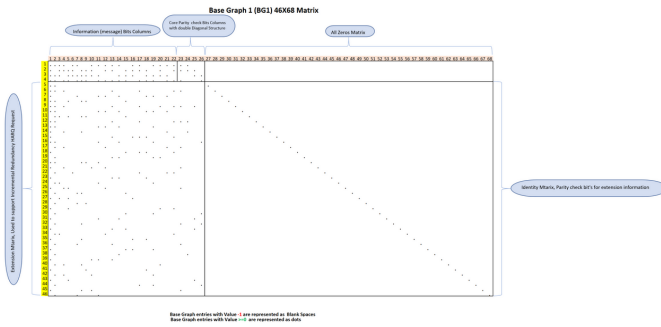


Fig. 1: High Level Visualization of fully Populated BG1 Matrix

- 2) Base Graph 2 (BG2) : With matrix size 42X52 entries For Smaller Transport Block.

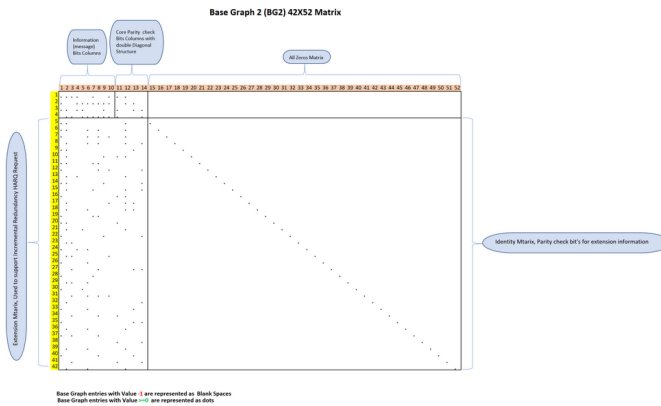


Fig. 2: High Level Visualization of fully Populated BG2 Matrix

### C. LDPC Base Graph selection procedure

I have put together the following example of constructing the LDPC parity check matrix for a given information block size  $K$  and code rate  $R = K/N$ .

For simplicity I have considered a small TBS of size 20bits to illustrate below example,  $K=20$  and  $R=0.25$

- 1) Obtain the base graph BG1 or BG2 for the given  $K$  (Transport Block )and  $R$  (Code Rate), Refer [3GPP Specification 38.212](#) for LDPC base graph selection. As per the specification
  - a) if  $K \leq 3824$  and  $R \leq 0.67$  then BG2 is selected.
  - b) If  $K \leq 292$  then BG2 is selected
  - c) if  $R \leq 0.25$  then BG2 is selected.
  - d) Else BG1 is selected

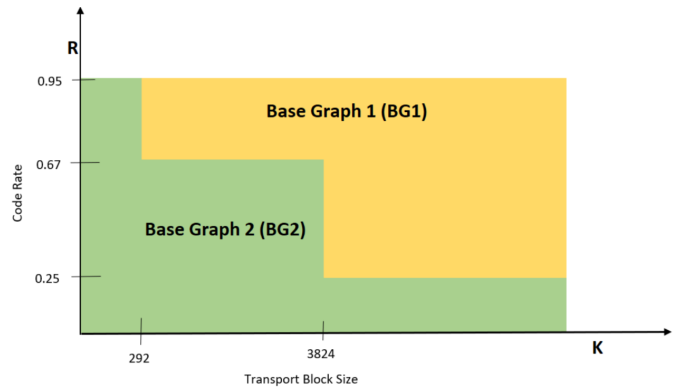


Fig. 3: Graphical Representation of Base Graph Selection

- 2) Determine the value of  $K_b$  for the given  $K$  (Transport Block) and  $R$  (Code Rate) Ref [3GPP Specification 38.212, 5.2.2](#)

$K_b$  denotes the number of information bit columns for the lifting size  $Z_c$ .

As per the Specification

For LDPC BG1:

- a)  $K_b = 22$

For LDPC BG2:

- a) if  $K$  is between  $640 < K \leq 3824$  then  $K_b = 10$
- b) if  $K$  is between  $560 < K \leq 640$  then  $K_b = 9$
- c) if  $K$  is between  $192 < K \leq 560$  then  $K_b = 8$
- d) If  $K$  is  $\leq 192$  then  $K_b = 6$

- 3) Determine the base matrix expansion factor  $Z_c$  by selecting the minimum  $Z_c$  value in below Table, such that  $K_b * Z_c \geq K$ . Sets of LDPC lifting size  $Z_c$  in the specification I have populated below  $Z_c$  table.

a

Zc	2	3	5	7	9	11	13	15
0	2	3	5	7	9	11	13	15
1	4	6	10	14	18	22	26	30
2	8	12	20	28	36	44	52	60
3	16	24	40	56	72	88	104	120
4	32	48	80	112	144	196	208	240
5	64	96	160	224	288			
6	128	192	320					
7	256	384						

$$Zc = a * 2^j$$

For K=20 , Zc = 4, this satisfies the condition  $Kb * Zc \geq K$  ,  $6 * 4 = 24$  ,  $24 > 20$  and this is the minimum Zc value from the above table that satisfies this condition.

- 4) After Zc is determined, the corresponding shift coefficient matrix set need to be selected from below Table., Ref for this table is [3GPP Specification 38.212,5.3.2](#)

Set Index(iLS)	Set of Lifting Sizes(Zc)
0	2,4,8,16,32,64,128,256
1	3,6,12,24,48,96,192,384
2	5,10,20,40,80,160,320
3	7,14,28,56,112,224
4	9,18,36,72,144,288
5	11,22,44,88,196
6	13,26,52,104,208
7	15,30,60,120,240

Since Zc = 4, Set Index (iLS) "0" is considered.

- 5) Determine the entries values in the base matrix based on the Zc, Calculate the shifting coefficient value P(i,j) by the modular Zc operation.

$$P(i,j) = f(V_{i,j}, Zc) = \text{mod}(V_{i,j}, Zc)$$

Referral tables to calculate P(i,j) are available in the specification [3GPP Specification 38.212,5.3.2](#)

H <sub>BG</sub>	V <sub>i,j</sub>	H <sub>BG</sub>	V <sub>i,j</sub>
Row index i	Column index j	Row index i	Column index j
Set index i <sub>LS</sub>	Set index i <sub>LS</sub>	Set index i <sub>LS</sub>	Set index i <sub>LS</sub>
0	0 1 2 3 4 5 6 7	0	0 1 2 3 4 5 6 7
1	0 260 307 73 223 211 284 0 135	1	1 96 2 280 120 0 348 6 138
2	1 69 19 15 16 198 118 0 227	2	10 65 210 60 131 163 15 81 220
3	2 220 60 103 84 188 167 0 156	3	13 63 318 130 209 108 81 182 173
4	3 159 369 49 91 186 330 0 134	4	18 76 65 184 209 68 176 53 142
5	4 100 181 240 74 219 207 0 84	5	25 179 269 51 81 64 113 46 40
6	5 10 216 39 10 4 166 0 83	6	37 0 0 0 0 0 0 0 0
7	6 59 317 15 0 29 243 0 53	7	1 64 13 69 154 270 190 88 78
8	7 228 288 162 205 144 250 0 225	8	3 49 336 140 164 13 283 198 152
9	8 110 109 215 216 119 1 0 205	9	11 49 57 45 43 69 332 160 84
10	9 191 17 164 21 216 339 0 128	10	20 51 289 115 189 54 331 122 5
11	10 228 288 162 205 144 250 0 225	11	22 154 57 300 101 0 114 182 205
12	11 110 109 215 216 119 1 0 205	12	38 0 0 0 0 0 0 0 0
13	12 191 17 164 21 216 339 0 128	13	0 7 280 257 66 163 110 91 183
14	13 3 357 133 215 115 201 0 75	14	14 164 303 147 110 137 228 184 112
15	14 186 215 288 14 233 53 0 135	15	16 59 61 128 200 0 247 30 106
16	15 23 108 110 70 144 347 0 217	16	17 1 358 51 63 0 116 3 219
17	16 190 242 113 141 95 304 0 220	17	21 144 375 228 4 162 190 155 129
18	17 35 180 16 198 216 167 0 90	18	39 0 0 0 0 0 0 0 0
19	18 239 330 189 104 73 47 0 105	19	1 42 130 260 199 181 47 1 183
20	19 31 346 32 81 261 188 0 137		
21	20 239 330 189 104 73 47 0 105		
22	21 31 346 32 81 261 188 0 137		
23	22 1 1 1 1 1 1 0 1		
24	23 0 0 0 0 0 0 0 0		

Fig. 4: LDPC base graph 1 (BG 1) and its parity check matrices

H <sub>BG</sub>	V <sub>i,j</sub>	H <sub>BG</sub>	V <sub>i,j</sub>
Row index i	Column index j	Row index i	Column index j
Set index i <sub>LS</sub>	Set index i <sub>LS</sub>	Set index i <sub>LS</sub>	Set index i <sub>LS</sub>
0	0 1 2 3 4 5 6 7	16	0 1 2 3 4 5 6 7
1	0 9 174 0 72 3 156 143 145	17	0 26 0 0 0 0 0 0 0
2	1 117 97 0 110 26 143 19 131	18	0 27 58 0 35 79 13 110 39
3	2 204 166 0 23 53 14 176 71	19	1 20 42 158 138 28 135 124 84
4	3 26 66 0 181 35 3 165 21	20	0 185 156 154 86 41 145 52 88
5	4 189 71 0 95 115 40 196 23		29 0 0 0 0 0 0 0 0
6	5 205 172 0 8 127 123 13 112		1 26 76 0 6 2 128 196 117
7	6 0 0 0 0 0 0 0 0		4 105 61 148 20 103 62 35 227
8	7 0 0 0 0 0 0 0 0		11 29 153 104 141 78 173 114 6
9	8 0 0 0 0 0 0 0 0		30 0 0 0 0 0 0 0 0
10	9 0 0 0 0 0 0 0 0		
11	10 0 0 0 0 0 0 0 0		
12	11 0 0 0 0 0 0 0 0		
13	12 0 0 0 0 0 0 0 0		
14	13 0 0 0 0 0 0 0 0		
15	14 0 0 0 0 0 0 0 0		

Fig. 5: LDPC base graph 2 (BG 2) and its parity check matrices

For K=20 , Base Graph = 2, Zc= 4 and SetIndex iLS = 0, from above Table LDPC base graph 2 Using the equation  $P(i,j) = f(V_{i,j}, z) = \text{mod}(V_{i,j}, z)$  all the possible base graph matrix entries with the shifting coefficient are determined.

Below I have illustrated how the Base Matrix entries P(i,j) are populated, I have considered only the first row for below illustration, like wise the full matrix is built.

H <sub>BG</sub>	V <sub>i,j</sub>
Row index i	Column index j
Set index i <sub>LS</sub>	Set index i <sub>LS</sub>
0	0 1 2 3 4 5 6 7
1	0 9 174 0 72 3 156 143 145
2	1 117 97 0 110 26 143 19 131
3	2 204 166 0 23 53 14 176 71
4	3 26 66 0 181 35 3 165 21
5	4 189 71 0 95 115 40 196 23
6	5 205 172 0 8 127 123 13 112
7	6 0 0 0 0 0 0 0 0
8	7 0 0 0 0 0 0 0 0
9	8 0 0 0 0 0 0 0 0
10	9 0 0 0 0 0 0 0 0
11	10 0 0 0 0 0 0 0 0
12	11 0 0 0 0 0 0 0 0
13	12 0 0 0 0 0 0 0 0
14	13 0 0 0 0 0 0 0 0
15	14 0 0 0 0 0 0 0 0

Fig. 6: LDPC parity check matrix selection

From Step 3 and 4 Set Index iLS = 0 and Zc =4

1<sup>st</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "1" (0) under SetIndex "0" = 9, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 9 \text{ mod } 4 = 1$   
2<sup>nd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "2" (1) under SetIndex "0" = 117, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 117 \text{ mod } 4 = 1$   
3<sup>rd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "3" (2) under SetIndex "0" = 204, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 204 \text{ mod } 4 = 0$   
4<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "4" (3) under SetIndex "0" = 26, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 26 \text{ mod } 4 = 2$   
5<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "5" (4) under SetIndex "0" = 143, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 143 \text{ mod } 4 = 3$   
6<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "6" (5) under SetIndex "0" = 19, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 19 \text{ mod } 4 = 3$   
7<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "7" (6) under SetIndex "0" = 131, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 131 \text{ mod } 4 = 3$   
8<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "8" (7) under SetIndex "0" = 145, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 145 \text{ mod } 4 = 1$   
9<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "9" (8) under SetIndex "0" = 156, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 156 \text{ mod } 4 = 0$   
10<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "10" (9) under SetIndex "0" = 165, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 165 \text{ mod } 4 = 1$   
11<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "11" (10) under SetIndex "0" = 176, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 176 \text{ mod } 4 = 0$   
12<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "12" (11) under SetIndex "0" = 71, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 71 \text{ mod } 4 = 3$   
13<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "13" (12) under SetIndex "0" = 21, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 21 \text{ mod } 4 = 1$   
14<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "14" (13) under SetIndex "0" = 165, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 165 \text{ mod } 4 = 1$   
15<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "15" (14) under SetIndex "0" = 23, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 23 \text{ mod } 4 = 3$   
16<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "16" (15) under SetIndex "0" = 112, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 112 \text{ mod } 4 = 0$   
17<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "17" (16) under SetIndex "0" = 11, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 11 \text{ mod } 4 = 3$   
18<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "18" (17) under SetIndex "0" = 123, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 123 \text{ mod } 4 = 3$   
19<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "19" (18) under SetIndex "0" = 13, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 13 \text{ mod } 4 = 1$   
20<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "20" (19) under SetIndex "0" = 1, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 1 \text{ mod } 4 = 1$   
21<sup>st</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "21" (20) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
22<sup>nd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "22" (21) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
23<sup>rd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "23" (22) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
24<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "24" (23) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
25<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "25" (24) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
26<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "26" (25) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
27<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "27" (26) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
28<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "28" (27) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
29<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "29" (28) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
30<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "30" (29) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
31<sup>st</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "31" (30) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
32<sup>nd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "32" (31) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
33<sup>rd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "33" (32) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
34<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "34" (33) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
35<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "35" (34) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
36<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "36" (35) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
37<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "37" (36) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
38<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "38" (37) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
39<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "39" (38) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
40<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "40" (39) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
41<sup>st</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "41" (40) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
42<sup>nd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "42" (41) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
43<sup>rd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "43" (42) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
44<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "44" (43) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
45<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "45" (44) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
46<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "46" (45) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
47<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "47" (46) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
48<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "48" (47) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
49<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "49" (48) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
50<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "50" (49) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
51<sup>st</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "51" (50) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
52<sup>nd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "52" (51) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
53<sup>rd</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "53" (52) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
54<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "54" (53) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
55<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "55" (54) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
56<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "56" (55) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
57<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "57" (56) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
58<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "58" (57) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
59<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "1" (0) & Column Index "59" (58) under SetIndex "0" = 0, now using the formula  $P(i,j) = \text{mod}(V_{i,j}, Zc) = 0 \text{ mod } 4 = 0$   
60<sup>th</sup> entry of Row1 = Select a V<sub>i,j</sub> value that corresponds to Row Index "

Fig. 8: Base Graph2 for  $iLS=0$  and  $Z_c=4$