INFERENTIAL STATISTICS BUSINESS REPORT

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GREAT LEARNING PROJECT

Contents

1. Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

Table 1 Injury Status and Playing Position of Players

	Striker	Forward	Attacking Midfielder	Winger	Total
Players Injured	45	56	24	20	145
Players Not Injured	32	38	11	9	90
Total	77	94	35	29	235

The following questions will be answered based on the above data.

1.1 What is the probability that a randomly chosen player would suffer an injury?

Total number of players injured = 145

Total number of players = 235

So, probability that a randomly chosen player is injured is **145 / 235 = 0.617**

1.2 What is the probability that a player is a forward or a winger?

Total number of forwards = 94

Total number of wingers = 29

Total number of players = 235

Probability that a player is a forward or a winger is **(94 + 29) / 235 = 0.5234**

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

Number of injured strikers = 45

Total number of players = 235

Probability that a randomly chosen player is a striker with a foot injury is **45 / 235 = 0.1915**

1.4 What is the probability that a randomly chosen injured player is a striker?

Number of injured strikers = 45

Total number of injured players = 145

Probability that a randomly chosen injured player is a striker is **45 / 145 = 0.3103**

2. Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter.

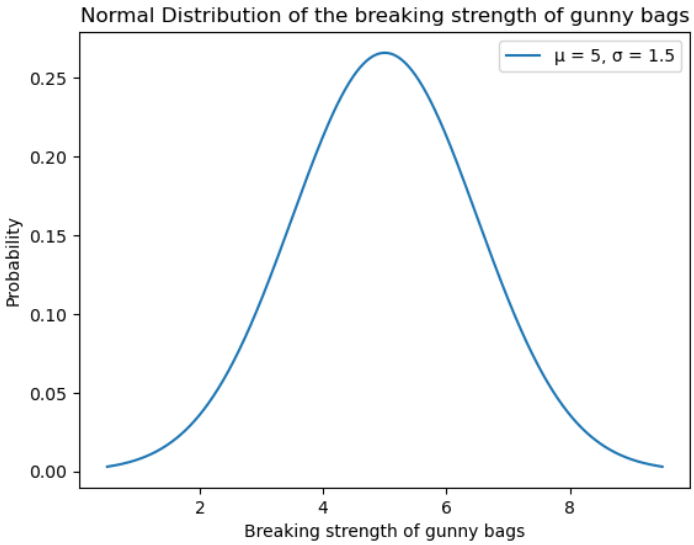

Figure 1 Distribution of breaking strength of gunny bags

The quality team of the cement company has come up with a list of questions which would help them better understand the packaging material wastage or pilferage within the supply chain. These questions will be answered in the following sections.

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

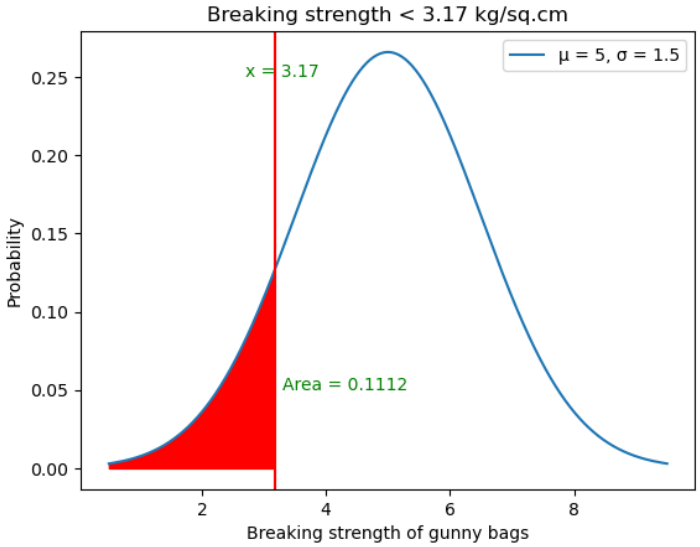

Figure 2 Gunny bags with breaking strength < 3.17 kg/sq.cm

The proportion of the gunny bugs having a breaking strength of less than 3.17 kg per sq cm is the area shaded in red. It evaluates to **0.1112**.

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

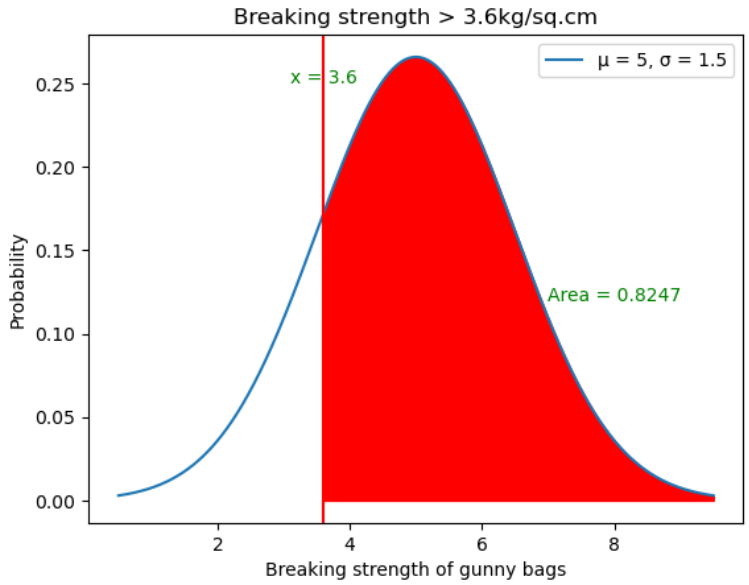

Figure 3 Gunny bags with breaking strength greater than 3.6 kg/sq cm

The proportion of the gunny bugs having a breaking strength of at least 3.6 kg per sq cm is the area shaded in red. It evaluates to **0.8247**.

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

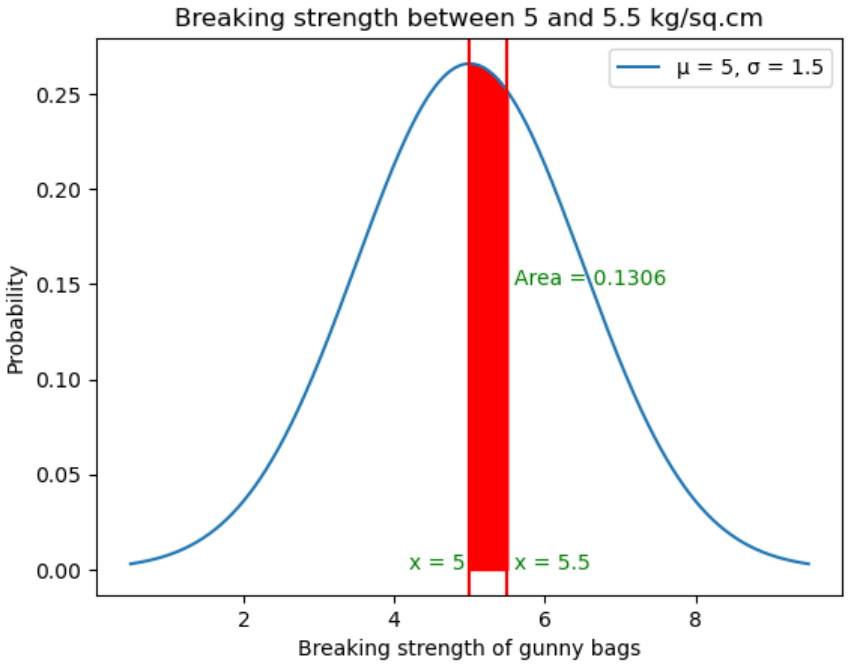

Figure 4 Gunny bags with breaking strength between 5 and 5.5 kg/sq cm

The proportion of the gunny bugs having a breaking strength between 5 and 5.5 kg per sq cm is the area shaded in red. It evaluates to **0.1306**.

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

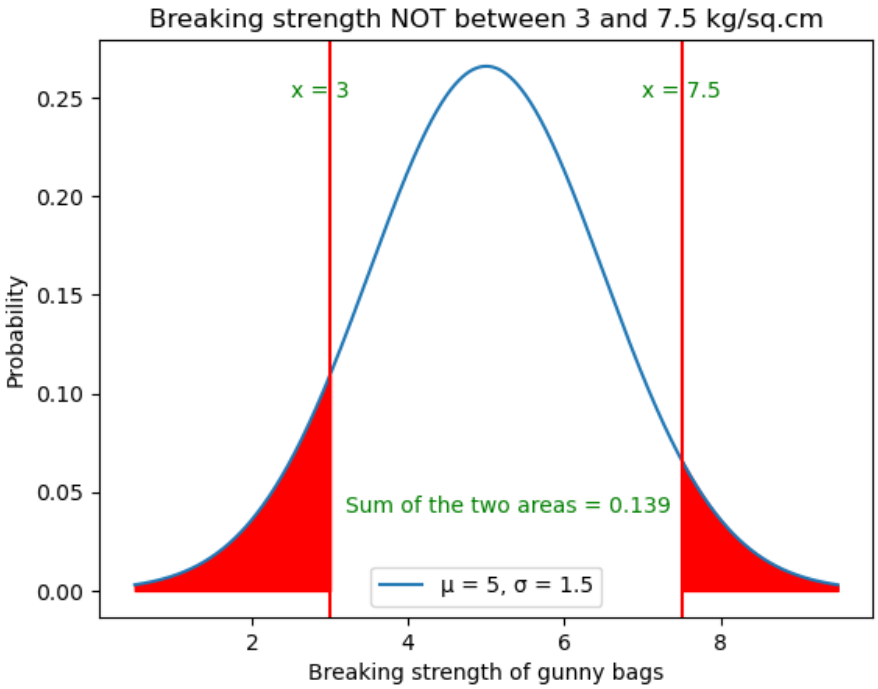

Figure 5 Gunny bags with breaking strength not between 3 and 7.5 kg/sq cm

The proportion of the gunny bugs having a breaking strength not between 3 and 7.5 kg per sq cm is the area shaded in red. It evaluates to **0.139**.

3. Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface needs to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. The test results are shown in the figure below. The test data from the sample will be analysed assuming a 5% significance level.

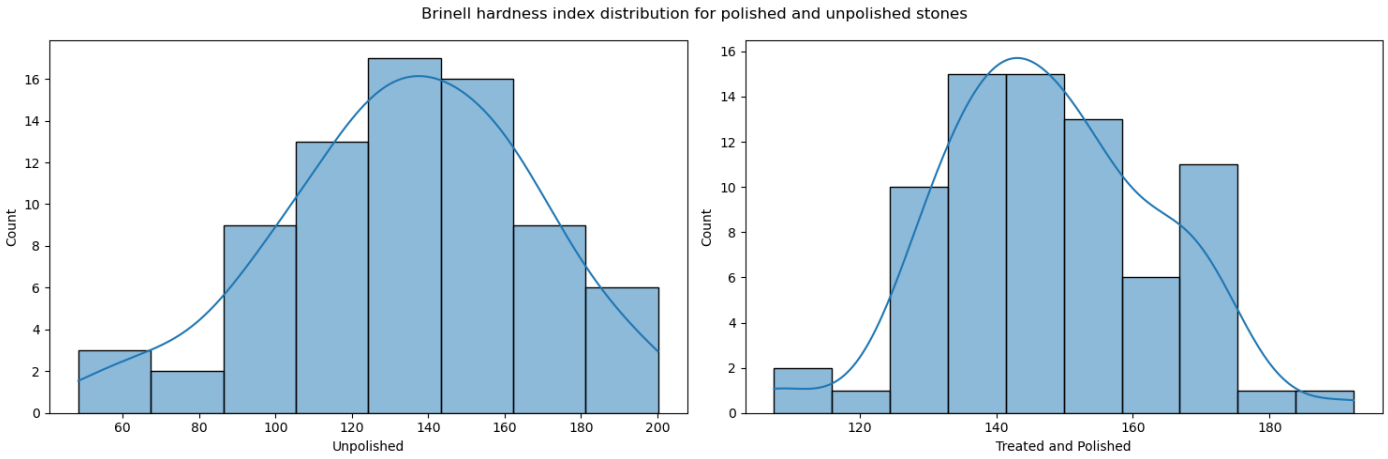

Figure 6 Brinell hardness index of polished and unpolished stones

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

This question can be answered through hypothesis testing. The suitability for testing is assessed from the Brinell hardness index. If that number is at least 150, we can conclude that the stones are suitable for printing. So, the null and alternate hypothesis are stated as follows.

Null hypothesis: The mean hardness index of stones is greater than or equal to 150.
Alternate hypothesis: The mean hardness index of stones is less than 150.

The appropriate test for this scenario is the **1-sample t-test**. The p-value obtained from this test is **4.17×10^{-5}**, which is less than the level of significance of 0.05. Hence, we reject the null hypothesis in favour of the alternate hypothesis.

Conclusion: The Brinell hardness index of unpolished stones is less than 150. As a result, Zingaro is justified to believe unpolished stones are not suitable for printing.

3.2 Is the mean hardness of the polished and unpolished stones the same?

This question can be answered through hypothesis testing. The null and alternate hypothesis are stated as follows.

Null hypothesis: The mean hardness of the polished and unpolished stones is the same.
Alternate hypothesis: The mean hardness of the polished and unpolished stones is not the same.

The appropriate test for this scenario is the **2-independent sample t-test**. The p-value obtained from this test is **0.0016**, which is less than the level of significance of 0.05. Hence, we reject the null hypothesis in favour of the alternate hypothesis.

Conclusion: The Brinell hardness index of polished and unpolished stones is not the same.

4. Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest. The data will be analysed based on a level of significance of 0.05. Alloys 1 and 2 will be studied separately.

4.1 How does the hardness of implants vary depending on dentists?

To study this scenario, the response for the different dentists were plotted.

Alloy 1

The box plot shows the means, medians and ranges of response values for the 5 different dentists when they used Alloy 1.

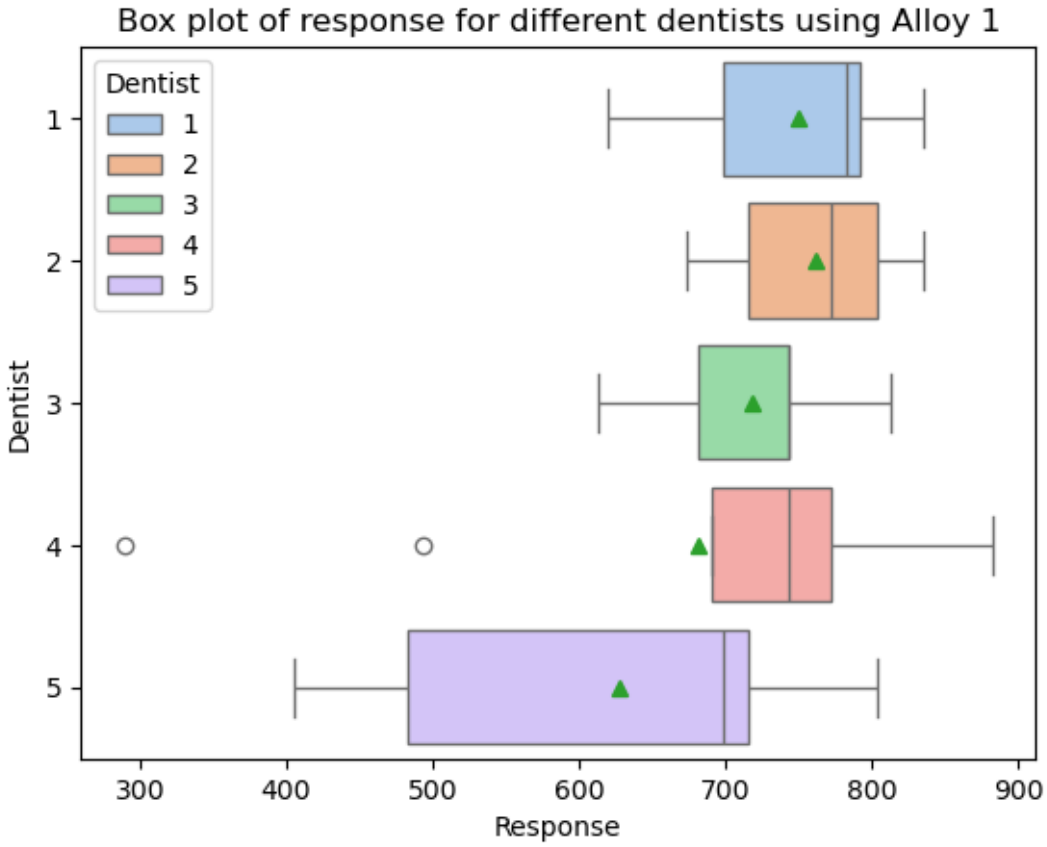

Figure 7 Response of Alloy 1 for different dentists

We will study this data by using the ANOVA test. Here, the null hypothesis and alternate hypothesis are
Null hypothesis: The mean response value is the same for all dentists.
Alternate hypothesis: At least one response is different from the rest.

Assumptions:

1. Populations are normally distributed: No! Shapiro-Wilk's test does not indicate a normal distribution. However, we will still proceed with the test.
2. Samples are independent simple random samples – Yes
3. Population variances are equal - Yes, the homogeneity of variance is verified using the Levene's test.

The ANOVA test produced a p-value of **0.1166**, which is greater than the level of significance of 0.05. So, we fail to reject the null hypothesis.

Conclusion: The mean response value is the same for all dentists, when Alloy 1 is used.

Alloy 2

The box plot shows the means, medians and ranges of response values for the 5 different dentists when they used Alloy 2.

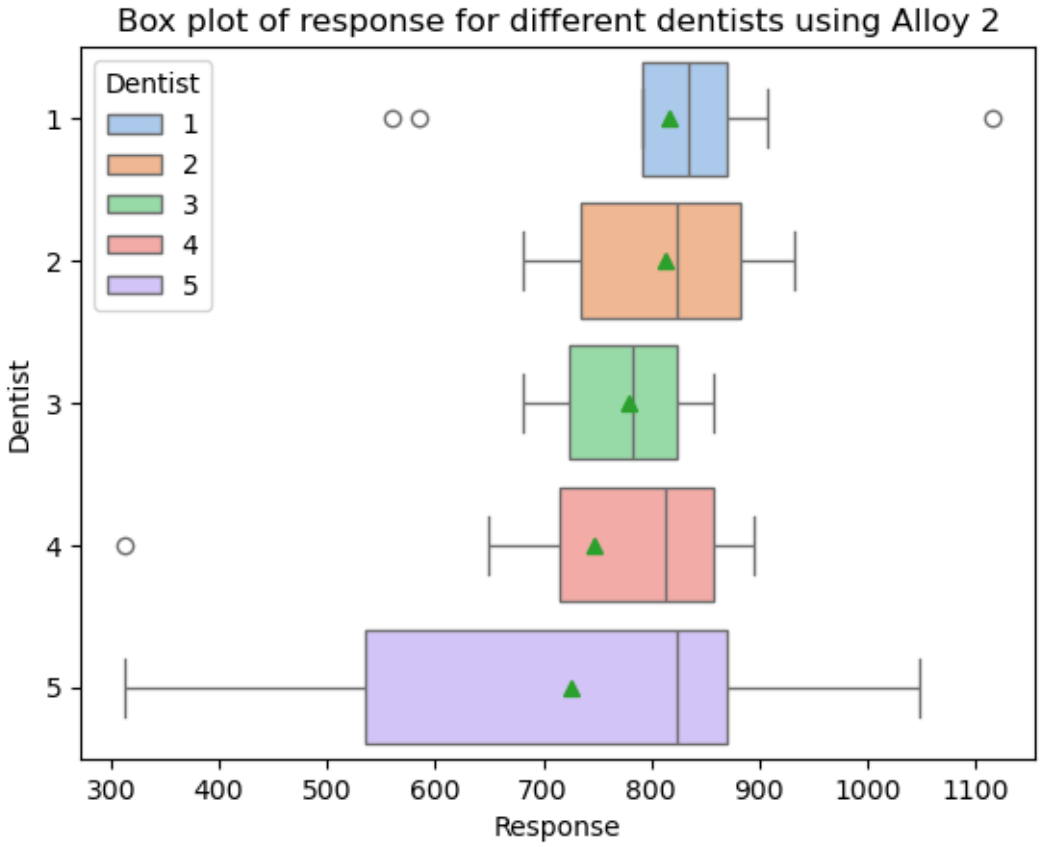

Figure 8 Response of Alloy 2 for different dentists

We will study this data by using the ANOVA test. Here, the null hypothesis and alternate hypothesis are
Null hypothesis: The mean response value is the same for all dentists.
Alternate hypothesis: At least one response is different from the rest.

Assumptions:

1. Populations are normally distributed: No! Shapiro-Wilk's test does not indicate a normal distribution. However, we will still proceed with the test.
2. Samples are independent simple random samples – Yes
3. Population variances are equal - Yes, the homogeneity of variance is verified using the Levene's test.

The ANOVA test produced a p-value of **0.718**, which is greater than the level of significance of 0.05. So, we fail to reject the null hypothesis.

Conclusion: The mean response value is the same for all dentists, when Alloy 2 is used.

4.2 How does the hardness of implants vary depending on methods?

To study this scenario, the response for the different methods were plotted.

Alloy 1

The box plot shows the means, medians and ranges of response values for the 3 different methods when Alloy 1 was used.

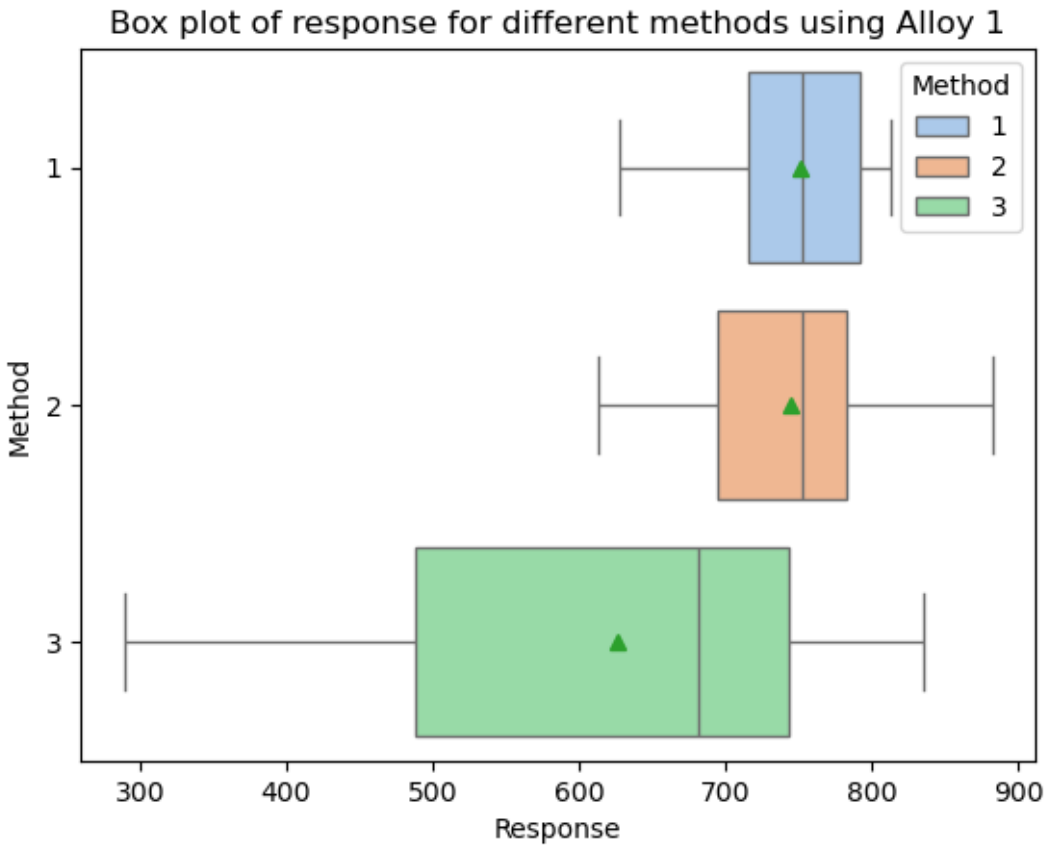

Figure 9 Response of Alloy 1 for different methods

We will study this data by using the ANOVA test. Here, the null hypothesis and alternate hypothesis are
Null hypothesis: The mean response value is the same for all methods.
Alternate hypothesis: At least one response is different from the rest.

Assumptions:

1. Populations are normally distributed: No! Shapiro-Wilk's test does not indicate a normal distribution. However, we will still proceed with the test.
2. Samples are independent simple random samples – Yes
3. Population variances are equal - No! The Levene test indicates that there is no homogeneity of variance. However, we will still proceed with the ANOVA.

The ANOVA test produced a p-value of **0.0042**, which is less than the level of significance of 0.05. So, we reject the null hypothesis. The mean response value is different for different methods.

A further study can be performed using the multiple comparison test to test the differences between all pairs of means. This test shows that the mean response is similar for method 1 and method 2. However, the mean response for method 3 is different from both method 1 and method 2.

Conclusion:

- The mean response value is different for different methods when Alloy 1 is used.
- The mean response for method 3 is different from both method 1 and method 2.

Alloy 2

The box plot shows the means, medians and ranges of response values for the 3 different methods when Alloy 2 was used.

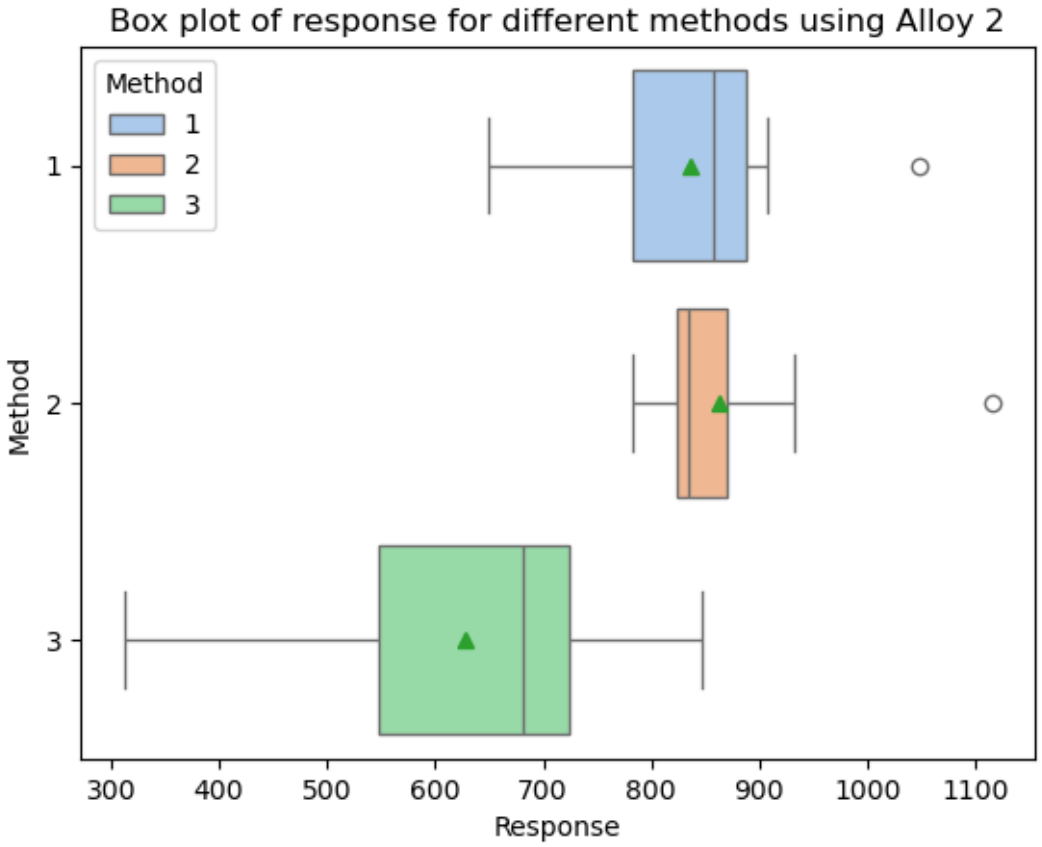

Figure 10 Response of Alloy 2 for different methods

We will study this data by using the ANOVA test. Here, the null hypothesis and alternate hypothesis are
Null hypothesis: The mean response value is the same for all methods.
Alternate hypothesis: At least one response is different from the rest.

Assumptions:

1. Populations are normally distributed: No! Shapiro-Wilk's test does not indicate a normal distribution. However, we will still proceed with the test.
2. Samples are independent simple random samples – Yes
3. Population variances are equal - No! The Levene test indicates that there is no homogeneity of variance. However, we will still proceed with the ANOVA.

The ANOVA test produced a p-value of **5.4e-6**, which is less than the level of significance of 0.05. So, we reject the null hypothesis. The mean response value is different for different methods.

A further study can be performed using the multiple comparison test to test the differences between all pairs of means. This test shows that the mean response is similar for method 1 and method 2. However, the mean response for method 3 is different from both method 1 and method 2.

Conclusion:

- The mean response value is different for different methods when Alloy 2 is used.
- The mean response for method 3 is different from both method 1 and method 2

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

To study the interaction effect between the dentists and the methods on the response, the interaction plots are shown for Alloy 1 and Alloy 2.

Alloy 1

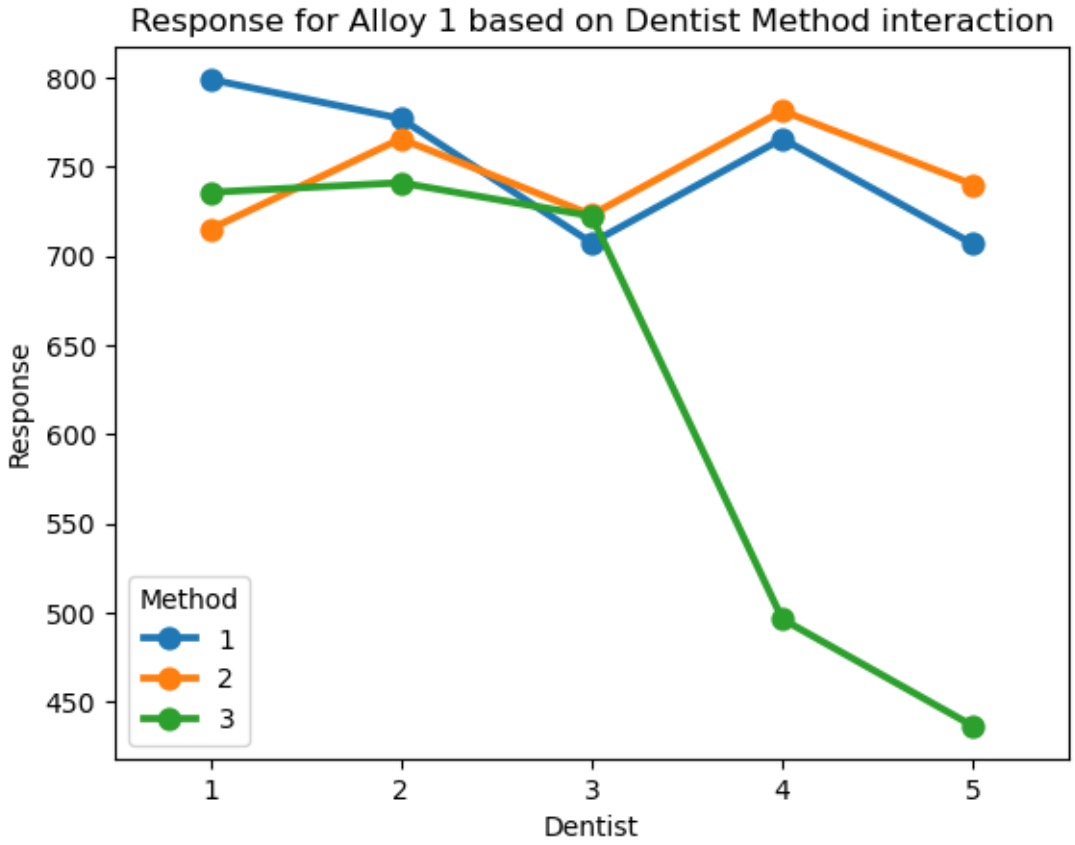

Figure 11 Response for Alloy 1 based on Dentist Method interaction

Observations and Inferences from interaction plot

- Method 1: Dentist 1 has the highest response of ~800. Dentists 3 and 5 have low response of ~710. The range is roughly 90.
- Method 2: Dentist 1 has the lowest response of ~710. Dentist 4 has the highest response of ~780. The range is roughly 70.
- Method 3: Dentists 1,2,3 have a high response of ~740, but the response for Dentist 4 and Dentist 5 is very low at 500 and 450 respectively.
- Dentist 3 has the smallest variability in response value, irrespective of the method used. Dentist 2 is not far behind in this regard.
- Except for “Dentist 4 – Method 3” and “Dentist 5 – Method 3”, all the response values are approximately between 700 and 800.
- From the business point of view, the above combinations need to be studied further.

Alloy 2

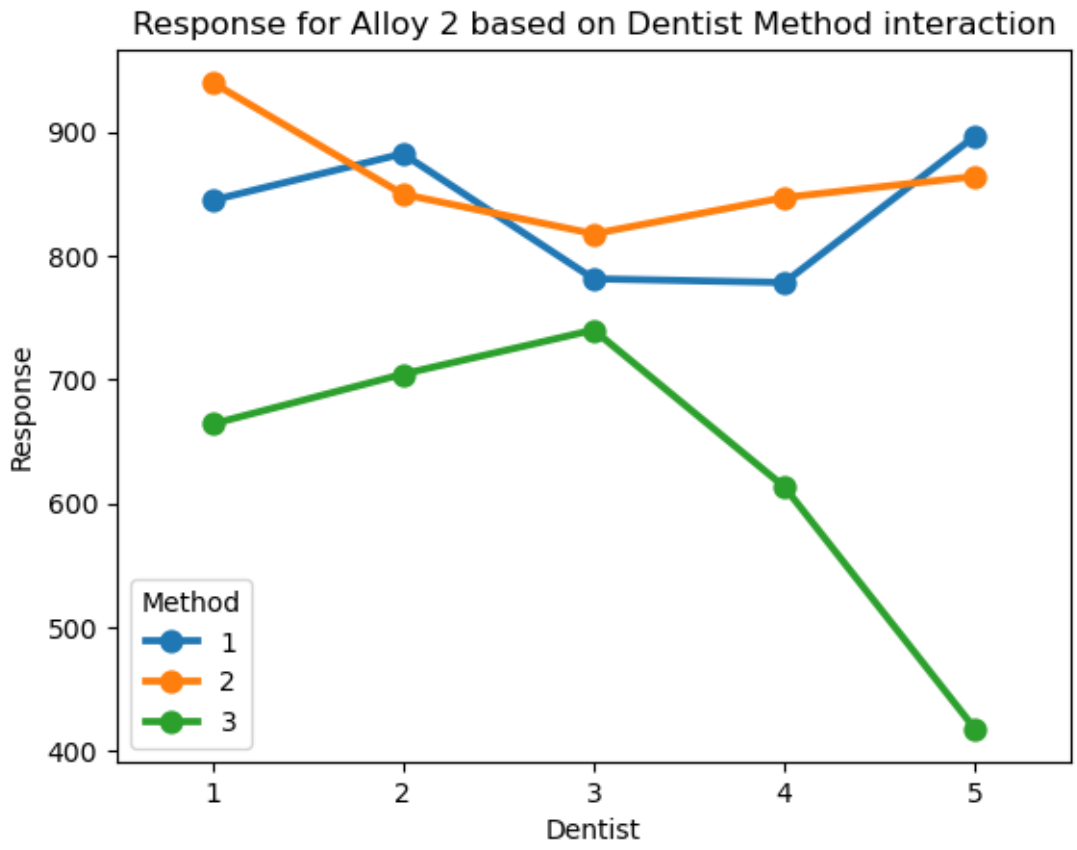

Figure 12 Response for Alloy 2 based on Dentist Method interaction

Observations and Inferences from interaction plot

- Method 1: Dentist 5 has the highest response of ~900. Dentists 3 and 4 have low response of ~790. The range is roughly 110.
- Method 2: Dentist 1 has the highest response of ~920. Dentist 3 has the highest response of ~810. The range is roughly 110.
- Method 3: Dentists 3 has a high response of ~730, but the response for Dentist 5 is much lower at around 400.
- The response value for method 3 is lower than methods 1 and 2 for all dentists.
- For alloy 2 also, Dentist 3 has the smallest variability in response value.
- In this scenario also, “Dentist 4 – Method 3” and “Dentist 5 – Method 3” combinations produce much lower response values compared to other combinations.

4.4 How does the hardness of implants vary depending on dentists and methods together?

As usual, we will handle Alloy 1 and Alloy 2 separately.

Alloy 1

Let us start with the null hypothesis and alternate hypothesis.
Null hypothesis: There is no interaction effect between dentists and methods.
Alternate hypothesis: An interaction effect exists.

Assumptions:

1. Populations are normally distributed: No! Shapiro-Wilk's test does not indicate a normal distribution. However, we will still proceed with the test.
2. Samples are independent simple random samples – Yes
3. Population variances are equal - Yes, the homogeneity of variance is verified using the Levene's test.

An n-way ANOVA test will be used to test the hypotheses. This test produced a p-value of **0.0068**, which is less than the level of significance of 0.05. So, we reject the null hypothesis.
Conclusion:

- There is an interaction between methods and dentists for Alloy 1.
- Figure 11 proves that the response is lower for method 3 when used by dentist 4 and dentist 5.

Alloy 2

Let us start with the null hypothesis and alternate hypothesis.
Null hypothesis: There is no interaction effect between dentists and methods.
Alternate hypothesis: An interaction effect exists.

Assumptions:

1. Populations are normally distributed: No! Shapiro-Wilk's test does not indicate a normal distribution. However, we will still proceed with the test.
2. Samples are independent simple random samples – Yes
3. Population variances are equal - Yes, the homogeneity of variance is verified using the Levene's test.

An n-way ANOVA test will be used to test the hypotheses. This test produced a p-value of **0.0932**, which is greater than the level of significance of 0.05. So, we fail to reject the null hypothesis.

Conclusion:

- There is no interaction between methods and dentists for Alloy 2.
- Figure 12 suggests the response is lower for method 3 when used by dentist 5. The p-value of 0.0932 tells us that we are 90.68% sure of this interaction. However, we are not 95% sure. The evidence is not strong enough to conclude that there is an interaction.