Asymptotic Complexity Analysis and Empirical Runtime Characterization

Explanation of the Dynamic Programming Algorithm

The algorithm calculates the maximum length of wood one can leave with, given the lengths of the segments. It uses a 2D dynamic programming table where dp[i][j] represents the maximum length of wood one can take from segments i to j. The algorithm fills this table based on the recurrence relation defined in the problem statement.

Theoretical Code and Complexity Analysis

The below is the core code snippet of the dynamic programming algorithm:

```
def T_dp(segment_lengths):
  n = len(segment_lengths)
  prefix_sum = [0]
  dp = [[0] * n for _ in range(n)]

  for i, length in enumerate(segment_lengths):
    prefix_sum.append(prefix_sum[-1] + length)
    dp[i][i] = length

  for length in range(2, n + 1):
    for i in range(n - length + 1):
      j = i + length - 1
      total_length = prefix_sum[j + 1] - prefix_sum[i]
      dp[i][j] = total_length - min(dp[i + 1][j], dp[i][j - 1])

  return dp[0][n - 1]
```

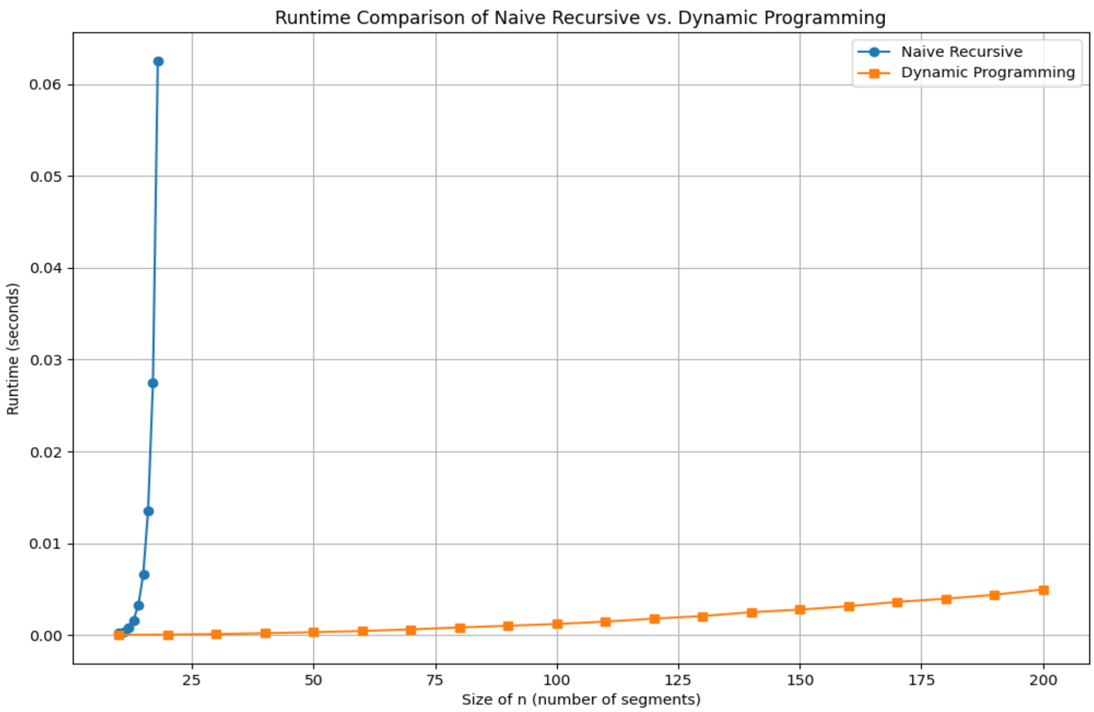
Table for Asymptotic Complexity

Process in Algorithm	Complexity	Reason
DP Table Initialization	O(n^2)	Filling an n x n table with zeroes
Calculating Prefix Sums	O(n)	Single pass through an array
Filling DP Table	O(n^2)	Nested loops each running up to n times

This table shows that while the prefix sum calculation is O(n), the dominant factor is the double nested loop used to fill the DP table, resulting in O(n^2) overall complexity.

Empirical Analysis

I measured the runtime of the algorithms across various sizes of input n. For the dynamic programming approach, I tested sizes ranging from small to large (up to 200), demonstrating its efficiency and scalability. For the naive recursive approach, I limited the measurement to smaller sizes due to its exponential runtime growth. I generated a plot to visually compare the runtimes of both approaches, illustrating the significant performance improvement offered by the dynamic programming solution.



Quadratic Regression Analysis

I performed a quadratic regression on the runtime data for the dynamic programming approach, yielding the equation: -2.12e-07 * n^2 + 4.87e-04 * n - 3.68e-03. This equation empirically supports the O(n^2) complexity, where the quadratic term, despite its small coefficient, is the dominating factor in the runtime's growth, aligning with the theoretical analysis.

Comparison of Theoretical and Empirical Results

The runtime plot shows a clear distinction between the naive recursive and dynamic programming approaches. The recursive approach exhibits exponential growth, making it impractical for larger n, whereas the dynamic programming method grows polynomially, aligning well with the theoretical O(n^2) complexity.

Conclusion

This comparison demonstrates that the dynamic programming solution not only adheres to the expected theoretical runtime but also significantly outperforms the naive approach in practical scenarios.