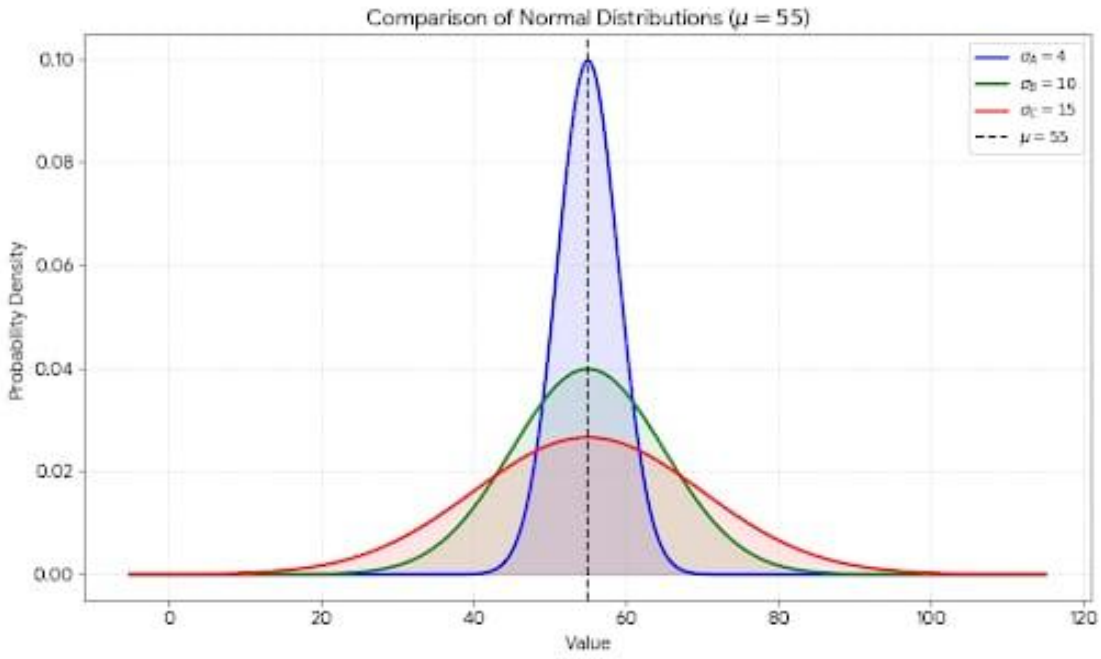
Assignment-3

<u>If μ= 55, σ_{4a}= 4, $\sigma_{4\beta}$=10, σ_4=*c* 15, In this which is better</u>

Solution :

➢ Introduction

In statistics, mean (μ) and standard deviation (σ) are two of the most important measures used to understand and analyze data. The mean tells us the central or average value of the dataset, while the standard deviation indicates how much the data values vary or spread around the mean.



When multiple datasets have the same mean but different standard deviations, the dataset with the smaller standard deviation is considered better, because it shows less variation, more consistency, higher reliability, and better stability.

❖ In this problem, we are given:

- Mean (μ) = 55
- σ_a = 4
- σ_β = 10
- σc = 15

We are asked to determine which case is better and why, using proper statistical reasoning.

1. Understanding Mean (μ)

The mean (μ) is the average value of a dataset and is calculated by dividing the total su m of observations by the number of observations.

$$\mu = \frac{\sum x}{n}$$

Given Mean Value

Here:

$$\mu = 55$$

This means that the average value of all three datasets is 55.

Importance of Mean

❖ The mean represents:

- Central tendency
- Typical value
- Expected value

❖ In real life, mean can represent:

- Average marks of students
- Average temperature
- Average income
- Average product life

Since all three datasets have the same mean, the main factor for comparison becomes the standard deviation.

2. Understanding Standard Deviation (σ)

Standard deviation (σ) is a measure of dispersion that shows how much the values in a dataset differ from the mean.

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{n}}$$

❖ Meaning of σ

- Small σ → Data values are close to mean → More consistency
- Large σ → Data values are far from mean → More variability

Thus, lower standard deviation is always preferred when reliability and consistency are required.

3. Given Data Analysis

We are given:

Case	Mean (μ)	Standard Deviation (σ)
A	55	4
B	55	10
C	55	15

Let us analyze each case.

4. Case A: $\mu = 55$, $\sigma_a = 4$

❖ Interpretation

- Standard deviation is very small (4).
- This means data points are tightly clustered around the mean (55).
- Most values lie between 51 and 59.

❖ Range Interpretation

Using ±1σ:

$$55 \pm 4 = 51 \text{ to } 59$$

❖ Meaning

- High consistency
- Low variability
- High reliability
- Stable dataset

❖ Real-Life Meaning (Student Marks Example)

If the average marks of students is 55 and σ = 4:

- Most students scored between 51 and 59.
- Performance is uniform and predictable.

❖ Conclusion for Case A

✓ Very stable
✓ Highly reliable
✓ Best consistency
✓ Best choice

5. Case B: $\mu = 55, \sigma_\beta = 10$

❖ Interpretation

- Standard deviation is moderate (10).
- Data values are moderately spread.
- Values range between 45 and 65.

❖ Range Interpretation

$$55 \pm 10 = 45 \text{ to } 65$$

❖ Meaning

- Medium variability
- Moderate consistency
- Moderate reliability

❖ Real-Life Meaning

If students' average marks is 55 and σ = 10:

- Scores range widely from 45 to 65.
- Students' performance varies noticeably.

❖ Conclusion for Case B

✓ Acceptable
✓ Moderate performance stability
❌ Less consistent than Case A

6. Case C: μ = 55, σ*c* = 15

❖ Interpretation

- Standard deviation is very large (15).
- Data values are highly spread out.
- Values range between 40 and 70.

❖ Range Interpretation

$$55 \pm 15 = 40 \text{ to } 70$$

❖ Meaning

- High variability
- Low consistency
- Low reliability

❖ Real-Life Meaning

If students' average marks is 55 and σ = 15:

- Marks range from 40 to 70.
- Performance varies greatly.
- High unpredictability

❖ Conclusion for Case C

❌ Highly unstable
❌ Poor consistency
❌ Least reliable

7. Graphical Interpretation (Conceptual)

If we draw normal distribution curves for these three cases:

- σ = 4 → Tall and narrow curve
- σ = 10 → Medium height and width
- σ = 15 → Short and wide curve

❖ Meaning

- Narrow curve → Less spread → More accuracy

- Wide curve → More spread → Less accuracy

Thus:

$$\sigma_4 < \sigma_{10} < \sigma_{15}$$

8. Comparison Table

Feature	Case A (σ = 4)	Case B (σ = 10)	Case C (σ = 15)
Mean	55	55	55
Spread	Very Low	Medium	Very High
Stability	Very High	Medium	Very Low
Reliability	Very High	Moderate	Poor
Risk	Very Low	Medium	High
Performance	Best	Average	Worst

9. Practical Example: Student Performance Analysis

Suppose 3 schools have the same average marks (55):

School	Mean	Standard Deviation
School A	55	4
School B	55	10
School C	55	15

❖ Which School is Best?

- School A → Students perform consistently
- School B → Moderate variation
- School C → Highly inconsistent performance

Thus, School A is best.

10. Why Lower Standard Deviation is Better?

- Indicates uniformity

- Shows predictability
- Improves control and planning
- Reduces risk
- Improves decision making

11. Applications of Low Standard Deviation

- Education – Consistent student results
- Manufacturing – Uniform product quality
- Medical – Stable patient health indicators
- Finance – Lower investment risk
- AI & ML – Reliable predictions

12.Final Conclusion

❖ Given:

- $\mu = 55$
- $\sigma_a = 4$
- $\sigma_\beta = 10$
- $\sigma c = 15$

Best Choice: $\sigma_a = 4$

❖ Because:

- It has lowest variability
- Highest consistency
- Best reliability
- Most stable performance

❖ Final Answer:

- Case A ($\sigma = 4$) is the best.

13.Final Summary

- Same mean → Compare standard deviation
- Lower σ → Better dataset
- σ = 4 is best
- σ = 10 is moderate
- σ = 15 is worst

➢ Conclusion

In this problem, all three datasets have the same mean value μ = 55, but different standard deviations σ_a = 4, σ_β = 10, and σc = 15. Since the mean is identical, the best dataset is determined by comparing the standard deviation, which measures the spread and consistency of the data.

Among the three cases, Case A (σ = 4) is the best, because it has the lowest standard deviation, indicating minimum variability, high consistency, and greater reliability. Case B (σ = 10) shows moderate variability, while Case C (σ = 15) has very high variability, making it the least reliable.

Therefore, we conclude that a dataset with lower standard deviation is always preferable, and hence σ = 4 represents the best and most stable distribution.